Free Quality School Education

Ministry of Basic and Senior Secondary Education

Pupils' handbook for

JSS Mathematics

JSS 3

Term 2

STRICTLY NOT FOR SALE

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Please see final page for further information.

FOREWORD

The production of Teachers' Guides and Pupils' handbooks in respect of English and Mathematics for Junior Secondary Schools (JSSs) in Sierra Leone is an innovation. This would undoubtedly lead to improvement in the performance of pupils in the Basic Education Certificate Examination in these subjects. As Minister of Basic and Senior Secondary Education, I am pleased with this development in the educational sector.

The Teachers' Guides give teachers the support they need to utilize appropriate pedagogical skills to teach; and the Pupils' Handbooks are designed to support self-study by the pupils, and to give them additional opportunities to learn independently.

These Teachers' Guides and Pupils' Handbooks had been written by experienced Sierra Leonean and international educators. They have been reviewed by officials of my Ministry to ensure that they meet specific needs of the Sierra Leonean population.

I call on the teachers and pupils across the country to make the best use of these educational resources.

This is just the start of educational transformation in Sierra Leone as pronounced by His Excellency, the President of the Republic of Sierra Leone, Brigadier Rtd. Julius Maada Bio. I am committed to continue to strive for the changes that will make our country stronger and better.

I do thank the Department for International Development (DFID) for their continued support. Finally, I also thank the teachers of our country - for their hard work in securing our future.

Timbo

Mr. Alpha Osman Timbo
Minister of Basic and Senior Secondary Education

The Ministry of Basic and Senior Secondary Education, Sierra Leone, policy stipulates that every printed book should have a lifespan of 3 years.

To achieve this DO NOT WRITE IN THE BOOKS.

Table of contents

Introduction

to the Pupils' Handbook

These practice activities are aligned to the lesson plans in the Teachers' Guide, and are based on the National Curriculum and the West Africa Examination Council syllabus guidelines. They meet the requirements established by the Ministry of Education, Science and Technology.

1. The practice activities will not take the whole term, so use any extra time to revise material or re-do activities where you made mistakes.
2. Use other textbooks or resources to help you learn better and practise what you have learned in the lessons.
3. Read the questions carefully before answering them. After completing the practice activities, check your answers using the answer key at the end of the book.
4. Make sure you understand the learning outcomes for the practice activities and check to see that you have achieved them. Each lesson plan shows these using the symbol to the right.

Learning Outcomes

5. Organise yourself so that you have enough time to complete all of the practice activities. If there is time, quickly revise what you learned in the lesson before starting the practice activities. If it is taking you too long to complete the activities, you may need more practice on that particular topic.
6. Seek help from your teacher or your peers if you are having trouble completing the practice activities independently.
7. Make sure you write the answers in your exercise book in a clear and systematic way so that your teacher can check your work and you can refer back to it when you prepare for examinations.
8. Congratulate yourself when you get questions right! Do not worry if you do not get the right answer – ask for help and continue practising!

Lesson Title: Review of Transformations	**Theme:** Geometry
Practice Activity: PHM-09-046	**Class:** JSS 3

Learning Outcome
By the end of the lesson, you will be able to identify and perform translation, reflection, and rotation.

Overview

This lesson is a review of 3 transformations: translation, reflection, and rotation.

Transformation changes the position or size of objects. The objects are usually plane shapes, drawn on a Cartesian plane. In this lesson, you will move these objects to a new position.

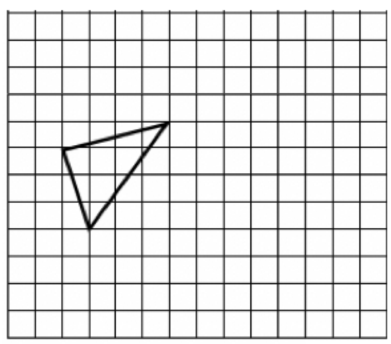
Translation moves an object up, down, left or right without changing its size or shape. See Solved Example 1.

Reflection creates an image of an object of the same size and shape, across a mirror line or line of symmetry. See Solved Example 2.

Rotation turns an object around a fixed point, called the centre of rotation, without changing its size or shape. See Solved Example 3.

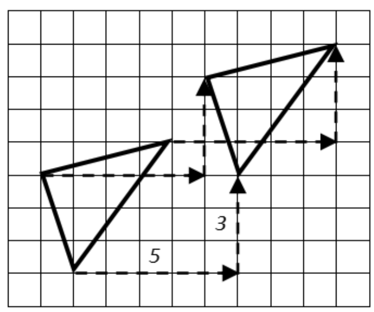
Solved Examples

1. Translate the shape 5 units right and 3 units up:



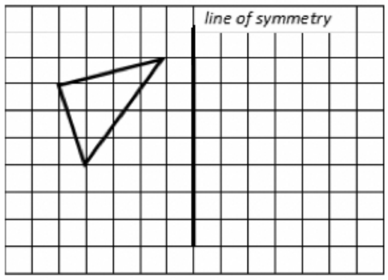
Solution

When we translate an object, every part of the object moves the same amount in the same direction.



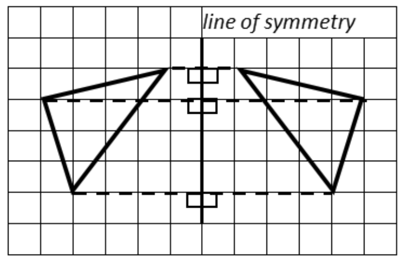
Select points on the object. For a triangle, select the 3 angles. From each angle, count 5 units right and 3 units up. Mark each point of the new triangle. Join the 3 points to draw a triangle.

2. Reflect the shape in the line of symmetry shown:



Solution

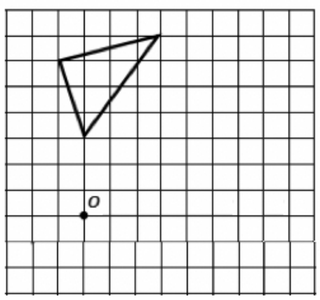
Remember that a line of symmetry is a mirror line. The triangle will have the same size, but it will face the opposite direction.



Use the fact that every point in the image is the same distance from the line of symmetry as the original object.

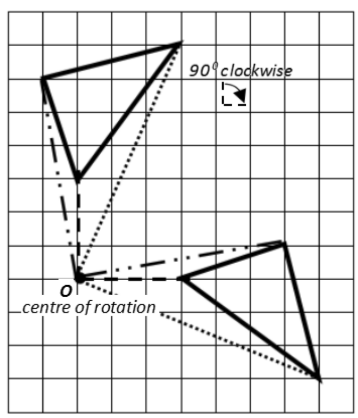
Choose points on the object. In this case, choose the 3 angles of the triangle. Draw a line at 90^o to the line of symmetry from each point on the original object to the other side of the line of symmetry. Mark on the 90^o line the **same distance** from the line of symmetry as the original object. That is the new point of the reflected object. Join the points to draw the shape at its new position after reflection.

3. Rotate the shape 90° clockwise about the point shown:



Solution

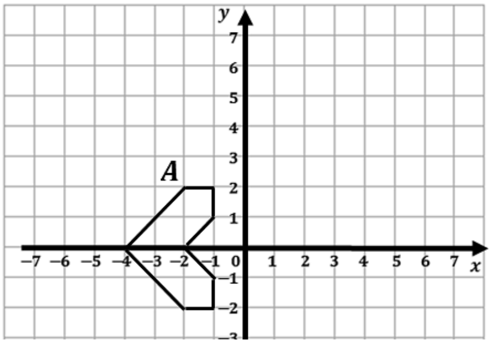
You are given 3 pieces of information in order to rotate our shape – the centre of rotation (O), the angle of rotation (90°) and the direction of rotation (clockwise).



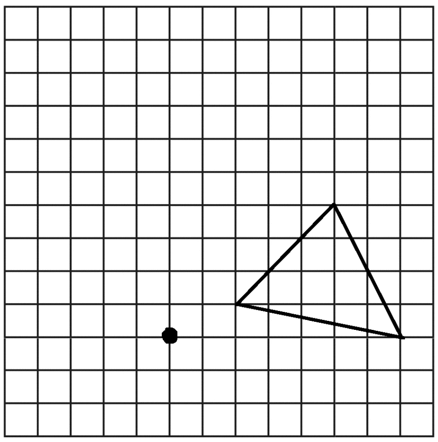
Draw a straight line from one of the angles of the triangle to the centre of rotation. Measure the line. Draw a line at an angle of 90^o to the first line. Mark the measurement on the second line. This is the new point of the object after rotation. Follow the same steps for the other 2 angles of the triangle.

Practice

1. Translate the shape A 5 units right and 1 unit up:

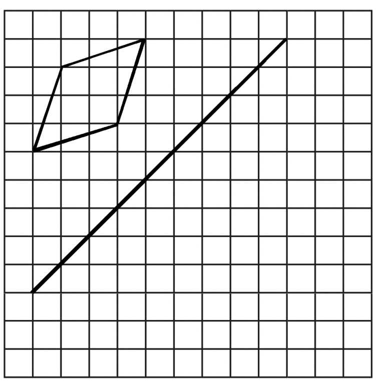


2. Rotate the triangle 90° counter-clockwise around the point shown:

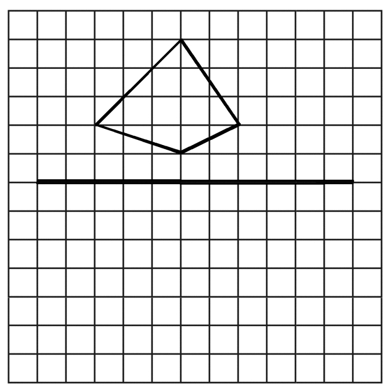


3. Reflect each shape about the line of symmetry shown:

a.



b.



Lesson Title: Combining Transformations	Theme: Geometry
Practice Activity: PHM-09-047	**Class:** JSS 3

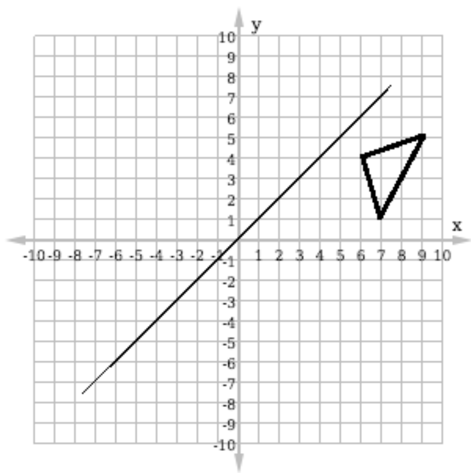
Learning Outcomes
By the end of the lesson, you will be able to:
1. Carry out combinations of translation, reflection, and rotation.
2. Describe and compare the 3 transformations.

Overview

The 3 transformations we looked at in the last lesson can be combined by doing one transformation and then another. For instance, we can translate an object, then reflect the translated object. Or we can rotate, then reflect or translate the resultant shape. When combining transformations, it is often easier to see what has taken place if we use a Cartesian plane to show the transformations.

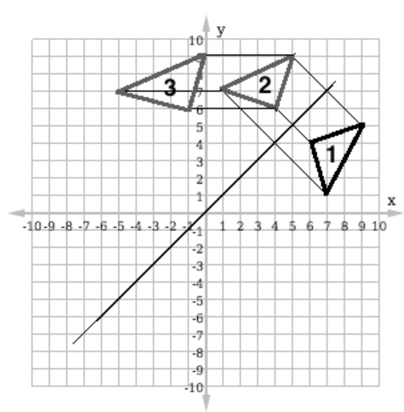
Solved Examples

1. Reflect the shape in the line of symmetry shown. Then transform it to the left 6 units.

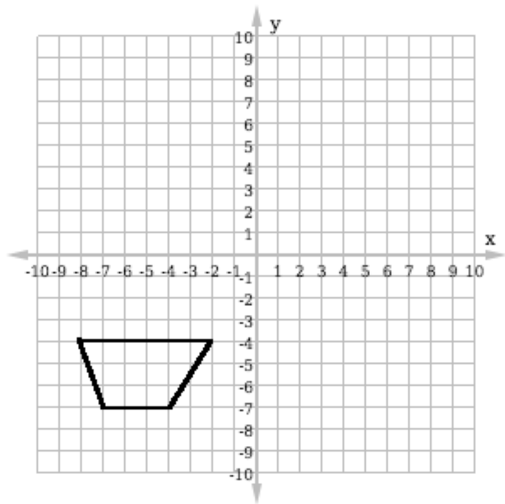


Solution
Do the transformations in the order in which they are given in the problem. Remember to perform the transformations on each of the 3 angles of the triangle.

- Reflect the shape about the diagonal line, which is the line of symmetry. This gives triangle 2.
- Transform the shape 6 units to the left. This gives triangle 3.

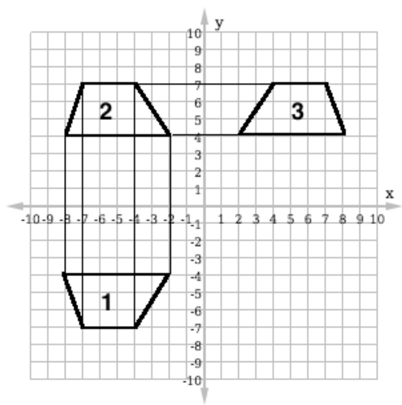


2. Reflect the shape shown in the x-axis. Then, reflect it in the y-axis.

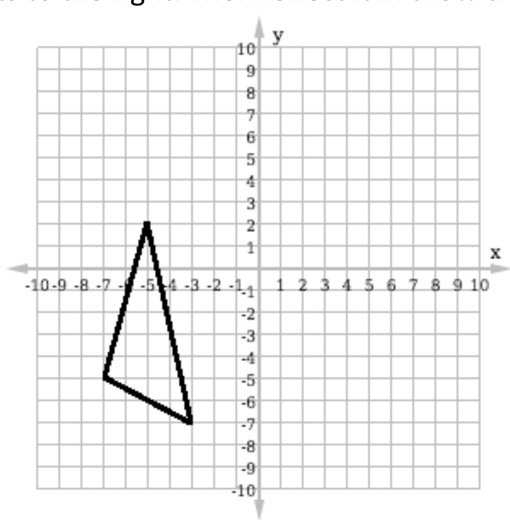


Solution

This shape has 4 corners. Reflect each of them about each axis.

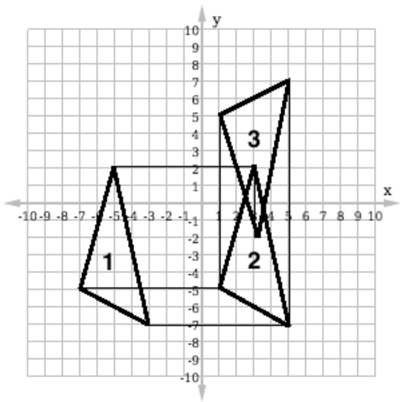


3. Transform the shape 8 units to the right. Then reflect it in the x-axis.



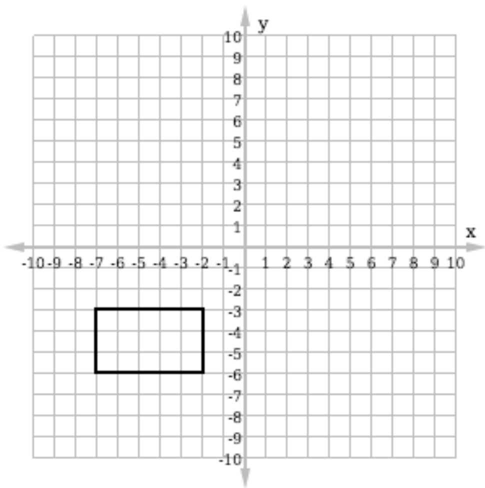
Solution

In the second step, note that one angle of the triangle is reflected from above to below the x-axis. The other 2 angles are reflected from below to above the x-axis.

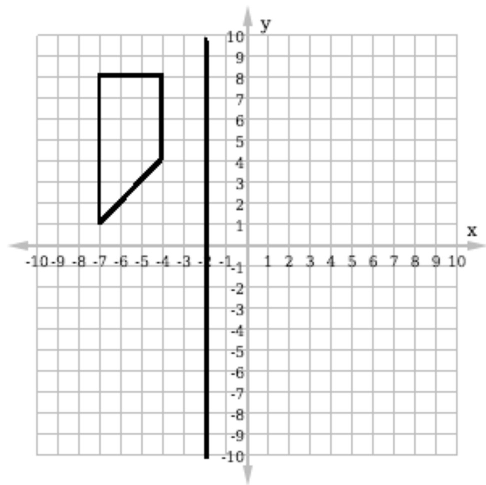


Practice

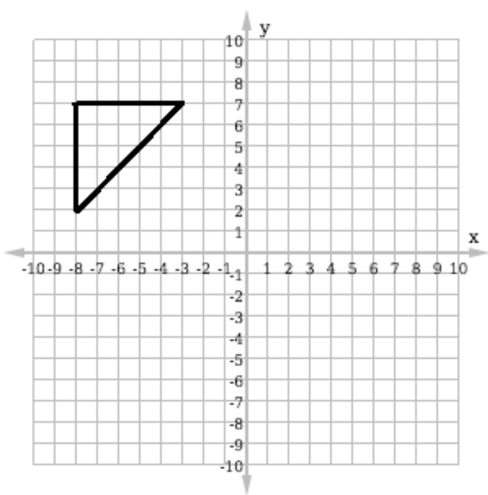
1. Rotate the shape 90° in the clockwise direction about the point $(-2,-2)$. Then transform it 10 units to the right and 1 unit down.



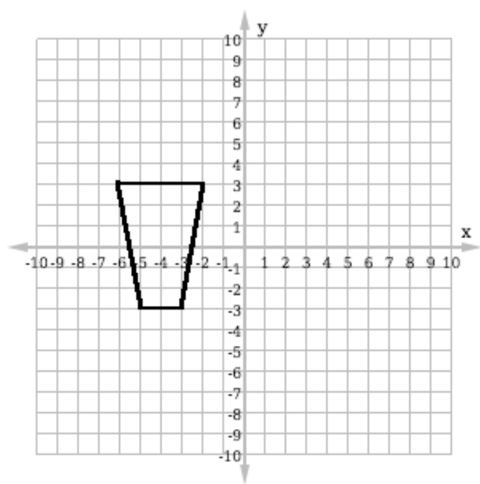
2. Reflect the shape in the line of symmetry shown. Then rotate it 90° in the clockwise direction about the origin.



3. Reflect the shape shown in the y-axis. Then transform it down 5 units and to the left 4 units.



4. Transform the shape 5 units to the right and 2 units down. Then reflect it in the x-axis.



Lesson Title: Congruency	Theme: Geometry
Practice Activity: PHM-09-048	**Class:** JSS 3

Learning Outcome
By the end of the lesson, you will be able to compare 2 shapes that have undergone reflection, rotation and translation and identify them as congruent.

Overview

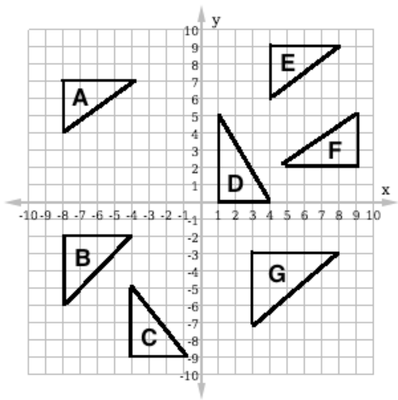
In this lesson, you will compare 2 shapes that have undergone reflection, rotation and translation and identify them as congruent.

Congruent shapes are exactly the same size and shape. They are identical in every way. If you put one on top of the other, they will make an exact fit. You can flip, turn, or move a shape to fit it on top of the other.

It can be helpful to imagine that the shapes are pieces of paper that you can pick up, turn and flip. If one paper shape could fit exactly over the other, the shapes are congruent. When we translate, rotate, or reflect an object, it maintains the same shape and size. It only changes its location or the direction it is facing. Shapes are **always** congruent after making any of these 3 transformations.

Solved Examples

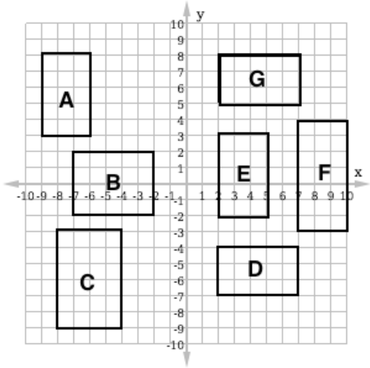
1. Among the triangles shown below, identify those that are congruent to A:



Solution
First, note the characteristics of A. It is a right-angled triangle with base 3 units and height 4 units. Any triangles with a longer base and height are not congruent.

These triangles are congruent to A: C, E, F.

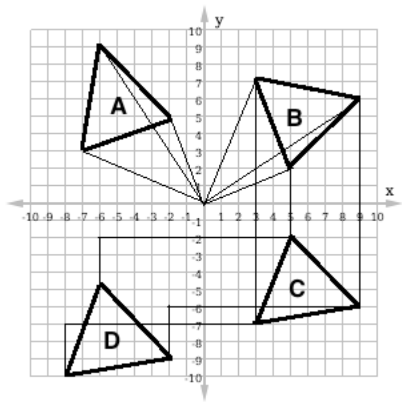
2. Identify all of the congruent shapes in the diagram below:



Solution

Look for rectangles of the same shape and size. Note that A, D, E and G are all the same shape and size. They all have length 5 units and width 3 units. Among the other rectangles there are none with the same shape and size. Rectangles A, D, E and G are congruent.

3. In the diagram below, A is the original triangle. The other triangles are transformations from A.
    a. Describe the transformations that created each of the triangles B, C and D.
    b. Which shapes are congruent to A?

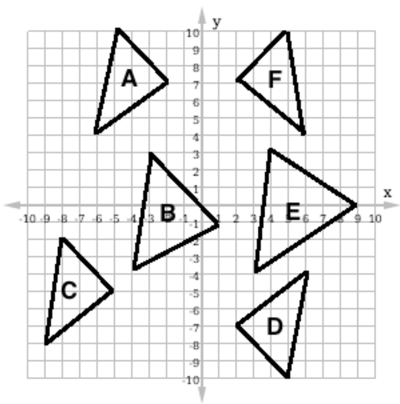


Solutions

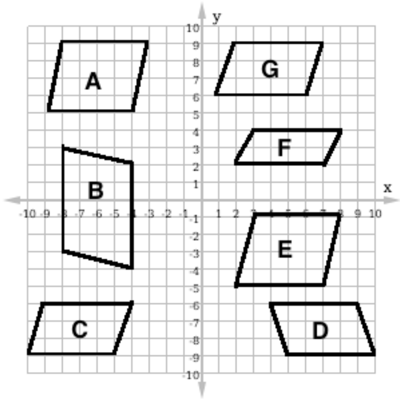
a. A is rotated to create B. B is reflected in the x-axis.
   to create C. C is transformed to the left 11 units and down 3 units to create D.
b. **All** 3 triangles created by rotating, reflecting, and translating A are congruent to A.

Practice

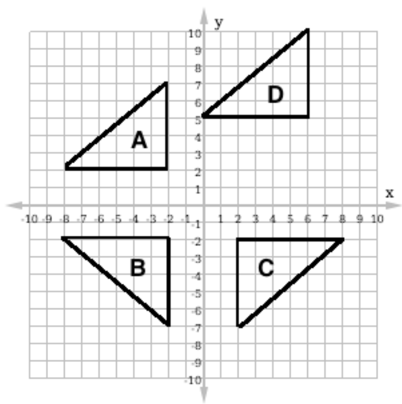
1. Among the triangles shown below, identify those that are congruent to A:



2. Identify all of the congruent shapes in the diagram below.



3. In the diagram below, A is the original triangle. The other triangles are transformations from A.
    a. Describe the transformations that created each of the triangles B, C and D.
    b. Which shapes are congruent to A?



Lesson Title: Practice with Congruency	Theme: Geometry
Practice Activity: PHM-09-049	**Class:** JSS 3

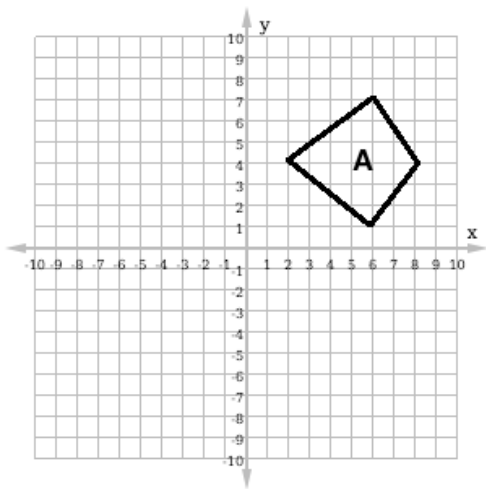
Learning Outcome
By the end of the lesson, you will be able to create congruent shapes by performing transformations.

Overview

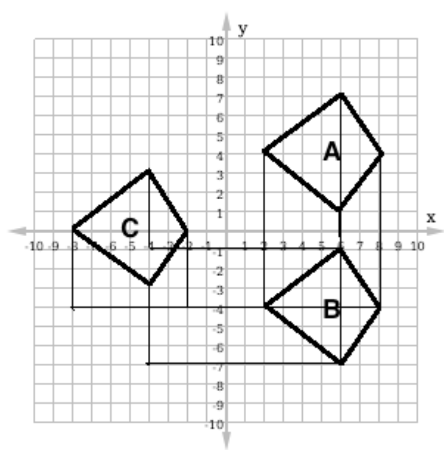
In the previous lesson, you learned to recognise congruent shapes. In this lesson, you will create congruent shapes by performing transformations. Remember that the result of a rotation, reflection or translation is congruent to the original shape.

Solved Examples

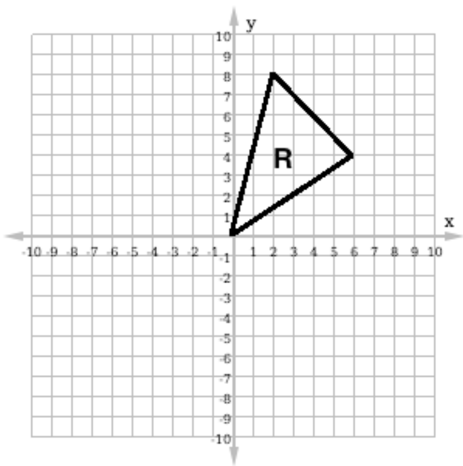
1. The diagram below gives shape A.
   a. Create shape B by reflecting A in the x-axis.
   b. Create shape C by transforming B 10 units to the left and 4 units up.
   c. Which shapes in your diagram are congruent?



Solutions
   a. See shape B in the diagram at right.
   b. See shape C in the diagram at right.
   c. All 3 shapes are congruent.

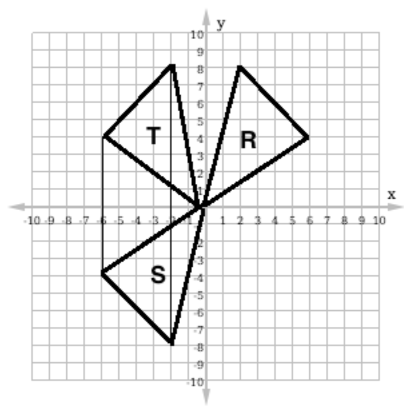


2. The diagram below gives shape R.
    a. Create shape S by rotating R 180° in the clockwise direction about the origin.
    b. Create shape T by reflecting S in the x-axis.
    c. Which shapes in your diagram are congruent?

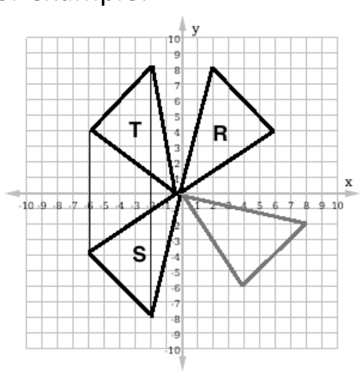


Solutions

a. See shape S in the diagram at right.
b. See shape T in the diagram at right.
c. All 3 shapes are congruent.

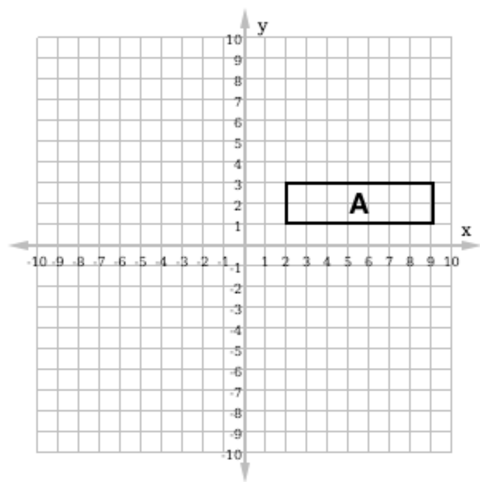


Note that when you are asked to rotate a shape 180°, it can be less confusing to rotate it 90° twice. Rotate R 90°, then rotate the result 90° more to get S. For example:

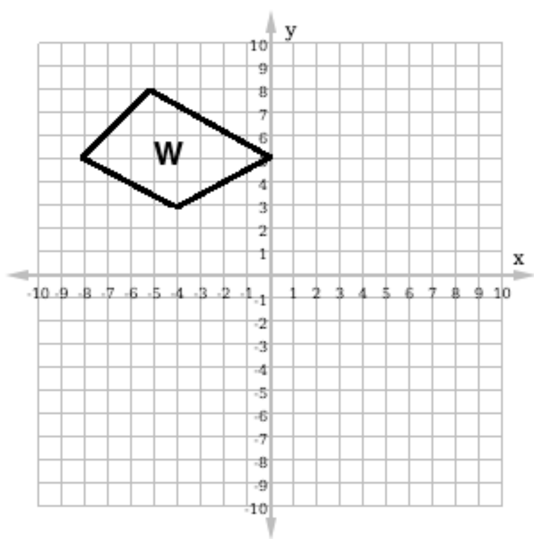


Practice

1. The shape below is A.
    a. Create shape B by rotating A about the origin 90° in the counter-clockwise direction.
    b. Create shape C by transforming B 11 units down and 5 units to the right.
    c. Which shapes in your diagram are congruent?



2. The shape below is W.
    a. Create shape X by reflecting W in the y-axis.
    b. Create shape Y by translating shape X 8 units down and 2 units to the right.
    c. Create shape Z by reflecting shape W in the x-axis.
    d. Which shapes in your diagram are congruent?



Lesson Title: Length Measurement of 2 Congruent Shapes	**Theme:** Geometry
Practice Activity: PHM-09-050	**Class:** JSS 3

Learning Outcome

By the end of the lesson, you will be able to recognise that length measurements (length, area, perimeter, and so on) of congruent shapes are maintained.

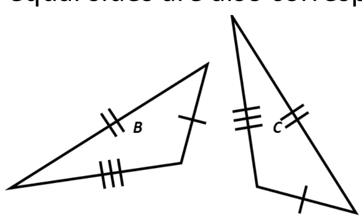
Overview

Remember the definition of **congruent shapes:** they are shapes that are exactly the same size and shape.

In this lesson, you will identify that the measurements of the length of congruent shapes are maintained. Up to now, we have been relying on the fact that we are transforming the same shape to make other congruent shapes. Suppose we have 2 shapes and we want to find out if they are congruent. We can check for congruency by measuring the lengths of their sides.

When checking for congruency, it is important that you measure corresponding sides to check if they have the same length. **Corresponding sides** are sides in the same position in shapes.

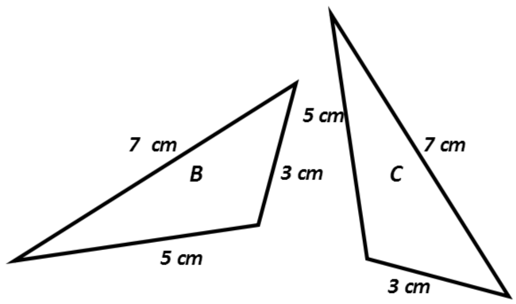
Remember that we show sides of the same length using small lines. For example, in the diagram below the sides with 1 mark are equal, the sides with 2 marks are equal, and the sides with 3 marks are equal. The equal sides are also corresponding sides.



There is another test for congruency which applies only to right-angled triangles. If we have two right-angled triangles, we do not need to measure all the sides. If the lengths of the hypotenuse and a corresponding side of two right-angled triangles are equal, then the triangles are congruent.

Solved Examples

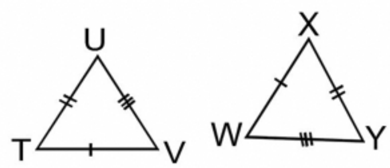
1. Triangles B and C are shown in the diagram below. Are they congruent? Give your reasons.



Solution

Yes, the triangles are equal because the corresponding sides are equal. The sides marked 3 cm correspond to each other, the sides marked 7 cm correspond to each other, and the sides marked 5 cm correspond to each other.

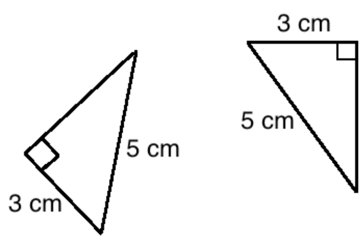
2. Triangles TUV and WXY are given in the diagram below. Are they congruent? Give your reasons.



Solution

Yes, the shapes are congruent. The corresponding sides are equal. In this example, TV=WX, UV=WY, and TU=XY.

3. Two right-angled triangles are given in the diagram below. Are they congruent? Give your reasons.



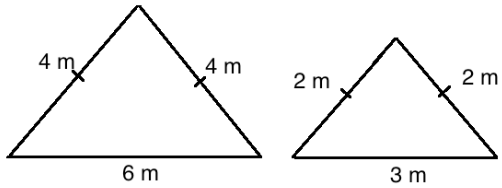
Solution

Yes, the shapes are congruent.

If the lengths of the hypotenuse and a corresponding side of 2 right-angled triangles are equal, then the triangles are congruent.

In these 2 triangles, the hypotenuse is 5 cm. The length of 1 set of corresponding sides is 3 cm. This tells us that the shapes are congruent.

4. Two triangles are given below. Are they congruent? Give your reasons.

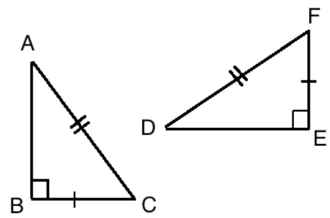


Solution

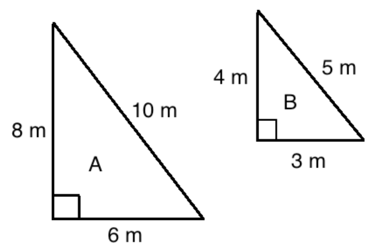
No, the triangles are not congruent. They have the same shape, but they have a different size.

Practice

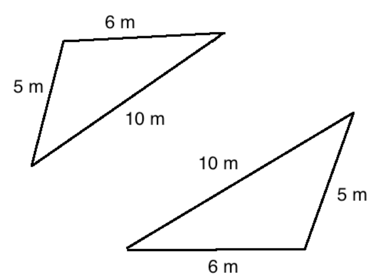
1. Triangles ABC and DEF are given in the diagram below. Are they congruent? Give your reasons.



2. Triangles A and B are given below. Are they congruent? Give your reasons.



3. Are the triangles below congruent? Give your reasons.



Lesson Title: Angles of Congruent Shapes	**Theme:** Geometry
Practice Activity: PHM-09-051	**Class:** JSS 3

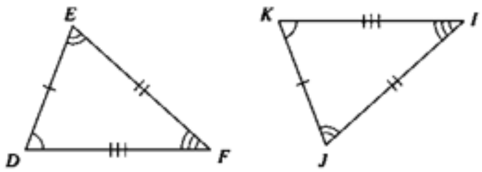
Learning Outcome

By the end of the lesson, you will be able to recognise that angle measurements of congruent shapes are maintained.

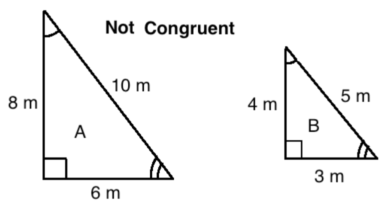
Overview

In this lesson, you will learn that angle measurements of congruent shapes are maintained. If two shapes are congruent, then their corresponding angles have the same measure.

For example, consider triangles DEF and JKL below. We can see that they **are congruent**, because they have 3 corresponding sides that are equal. Also notice that their corresponding angles are equal. That is, $\angle D = \angle K$, $\angle E = \angle J$ and $\angle F = \angle I$.



Although all congruent shapes have the same angles, it is **not true** that shapes with the same angles are congruent. For example, the triangles A and B below have equal angles. However, they are **not congruent** because their sides are different lengths.



There are 4 ways to identify whether triangles are congruent. The first 2 involve only sides, and were covered in the previous lesson. The second 2 involve sides and angles.

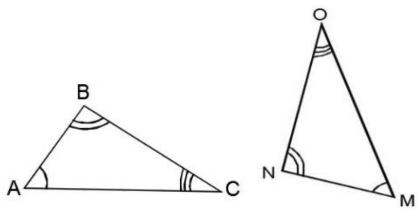
Two triangles are congruent if:

- The three sides of one are respectively equal to the three sides of the other (SSS).
- They are right-angled, and have the hypotenuse and another side of one respectively equal to the hypotenuse and another side of the other (RHS).
- Two sides and the included angle of one triangle are respectively equal to two sides and the included angle of the other (SAS).
- Two angles and a side of one triangle are respectively equal to two angles and the corresponding side of the other (ASA or AAS).

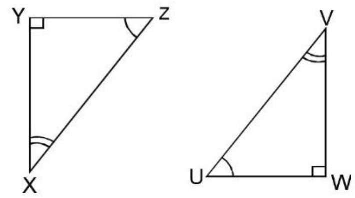
Solved Examples

1. Given the triangles below, identify the equal angles and the corresponding sides. Identify whether the triangles are congruent.

   a.



   b.

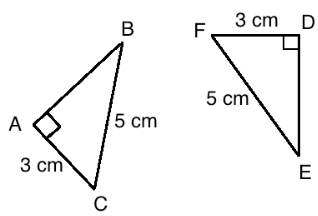


   Solutions

   a. $\angle A = \angle M$, $\angle B = \angle N$, $\angle C = \angle O$
   $|BC|$ corresponds to $|NO|$; $|AC|$ corresponds to $|OM|$; $|AB|$ corresponds to $|MN|$.
   There is not enough information. We cannot determine if the triangles are congruent because we do not know any side lengths.

   b. $\angle Y = \angle W$, $\angle Z = \angle U$, $\angle X = \angle V$
   $|ZX|$ corresponds to $|UV|$;$|XY|$ corresponds to $|VW|$; $|YZ|$ corresponds to $|WU|$
   There is not enough information. We cannot determine if the triangles are congruent because we do not know any side lengths.

2. Show that the 2 triangles below are congruent by naming the corresponding sides and angles.

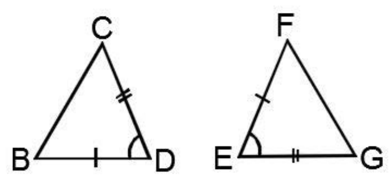


   Solution

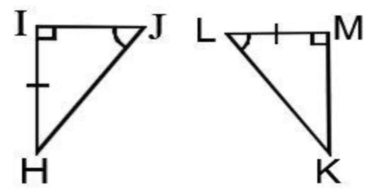
   First, verify that the triangles are congruent. They are congruent because they are right-angled triangles, and they have the hypotenuse and 1 corresponding side equal.
   Because they are congruent, all of the corresponding sides and angles must be equal. This is true although they are not marked.
   $|AB|$ corresponds to $|DE|$; $|BC|$ corresponds to $|EF|$; $|CA|$ corresponds to $|FD|$.
   The following angles are corresponding: $\angle A = \angle D$, $\angle B = \angle E$, $\angle C = \angle F$

3. Establish whether each pair of triangles are congruent or not.

a.



b.



Solutions

a. In △BCD and △EFG:

- $|DB| = |EF|$ (equal sides)
- $|CD| = |GE|$ (equal sides)
- And ∠D = ∠E (equal angles)

Hence △BCD and △EFG **are congruent** (side angle side, SAS)

b. In △HIJ and △LMK:

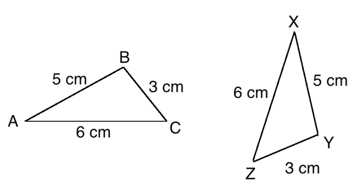
- ∠I = ∠M (equal angles)
- ∠J = ∠L (equal angles)
- $|HI| = |LM|$ (equal sides)

Hence △HIJ and △LMK **are congruent** (AAS or ASA)

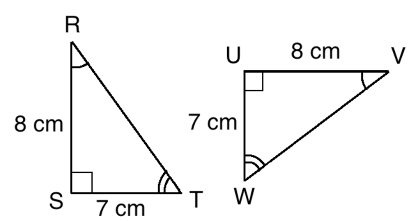
Practice

1. Each set of triangles below is congruent. Name each of the corresponding sides and angles.

a.

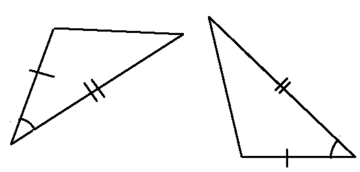


b.

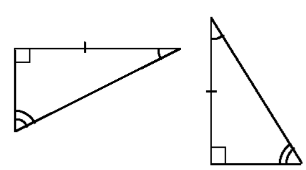


2. Identify whether each set of triangles is congruent, and give your reasons.

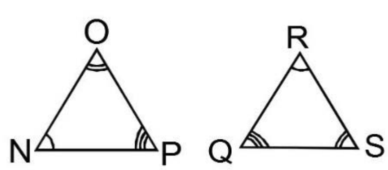
a.



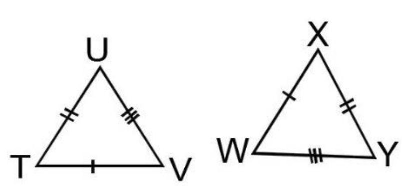
b.



c.



d.



Lesson Title: Enlargement	**Theme:** Geometry
Practice Activity: PHM-09-052	**Class:** JSS 3

Learning Outcomes

By the end of the lesson, you will be able to:

1. Identify that enlargement creates an object of the same shape, but a different size.
2. Recognise and perform enlargement.

Overview

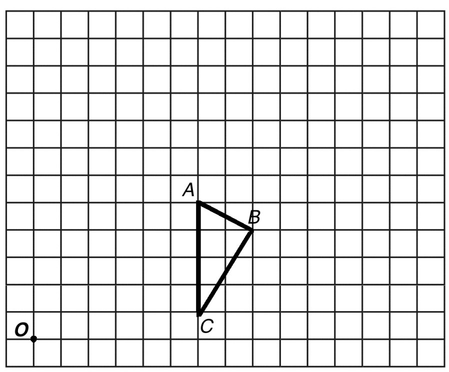
In previous lessons, you have performed 3 transformations: translation, reflection, and rotation. In this lesson, you will perform another transformation, **enlargement**. To enlarge a shape means to make it larger. It keeps the same shape, but becomes a different size.

The size of the final object will depend on the **scale factor** that we use. A scale factor tells us how many times larger to make the object. For example, a scale factor of 2 will make the shape twice as large. A scale factor of 3 will make the shape 3 times as large.

If a scale factor is a fraction, the shape becomes smaller. For example, a scale factor of $\frac{1}{2}$ will make the shape half the size of the original.

Solved Examples

1. Draw an enlargement to the triangle ABC with a scale factor of 2. Use the point O as the centre of enlargement.

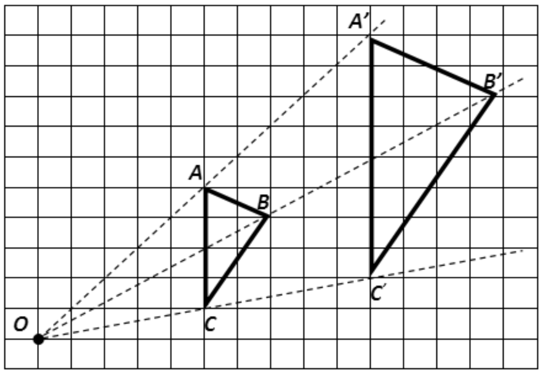


Solution

To enlarge the triangle by a scale factor of 2, we will make it twice as large.

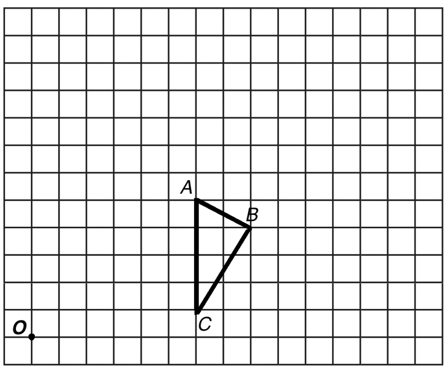
Follow these steps to draw the enlargement:

- Draw a line from O through point A on the triangle. Extend it at least twice as far as the distance from O to A.
- Measure the distance from O to A.
- Measure the same distance on the line past A, and draw a point. This is A'.
- Follow the same process for points B and C to make B' and C'.



Note that the distance from O to each point on the enlarged triangle is twice as far as the distance from O to any point on the first triangle. For example, $\mathrm{OA}' = 2 \times \mathrm{OA}$.
Note that the length of each side of the new triangle is twice as long as each side of the first triangle. For example, $\mathrm{A}'\mathrm{B}' = 2 \times \mathrm{AB}$.

2. Draw an enlargement to the triangle ABC with a scale factor of $\frac{1}{2}$. Use the point O as the centre of enlargement.

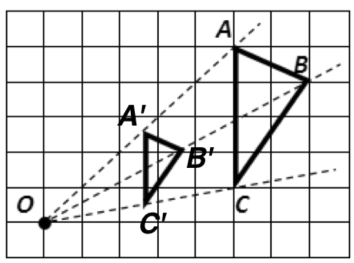


Solution

To enlarge the triangle by a scale factor of $\frac{1}{2}$, we will make it half as large.

Follow these steps to draw the enlargement:

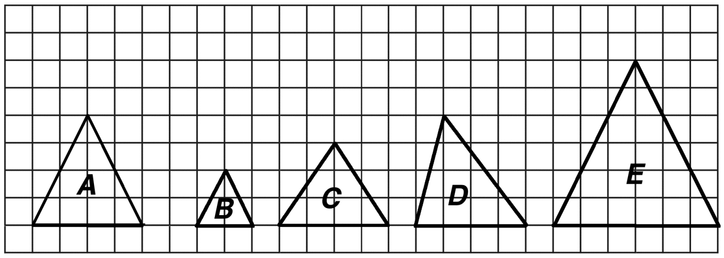
- Draw a line from O through point A on the triangle.
- Measure the distance from O to A.
- Find half of the distance on the line from O to A. This is A'.
- Follow the same process for points B and C to make B' and C'.



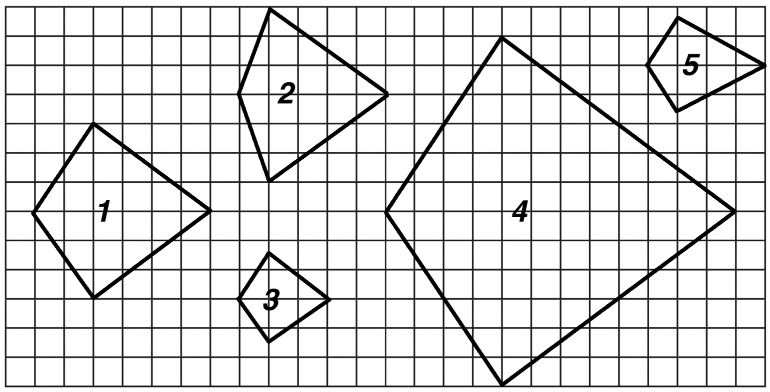
Note that the length of each side of the new triangle is half as long as each side of the first triangle. For example, $\mathrm{A}'\mathrm{B}' = \frac{1}{2} \times \mathrm{AB}$.

Practice

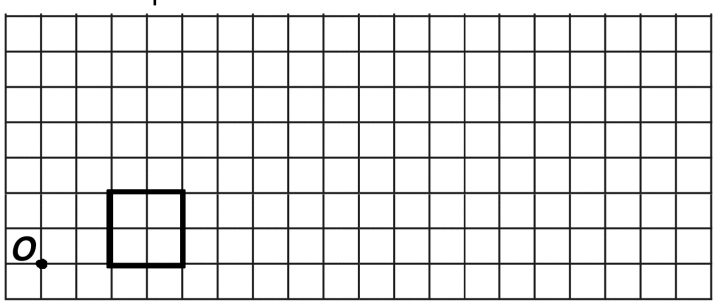
1. Which of the triangles shown below are enlargements of shape A? Give the scale factor for each enlargement.



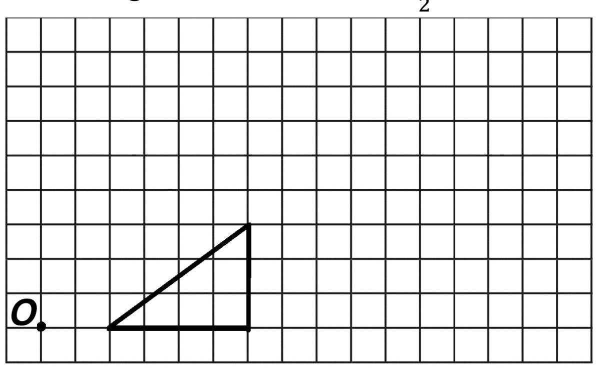
2. Which of the shapes shown below are **not** enlargements of shape 1?



3. Draw an enlargement of the square with a scale factor of 3.



4. Draw an enlargement of the triangle with scale factor $1\frac{1}{2}$.

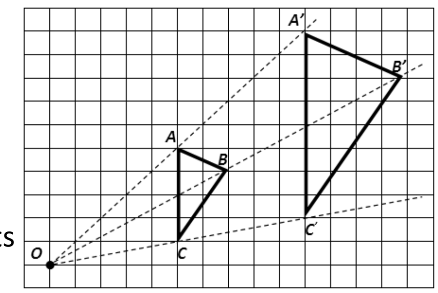


Lesson Title: Similarity	Theme: Geometry
Practice Activity: PHM-09-053	Class: JSS 3

Learning Outcome
By the end of the lesson, you will be able to identify that enlarged shapes are similar because angles are preserved but lengths are not.

Overview

When we say objects are **similar** in maths, we mean that they have the same shape but have different sizes.



Similar objects are enlargements of each other. We can increase or decrease any one of them and we will get another one like it. Look back in your exercise books at the enlargements we did in the last lesson. All of the enlargements you did created **similar** shapes.

The angles of similar shapes are always exactly the same. Only the lengths of the sides change, according to the scale factor.

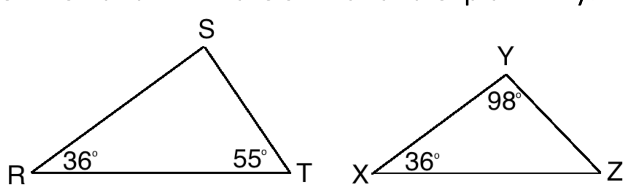
If we know the lengths of the sides of one object and we also know the scale factor by which it was enlarged, we can find the lengths of the enlarged object. The reverse also works. We can use the same method to find the lengths of the original shape.

In maths, we often handle similar triangles. Similar triangles have the same shape, but are of different size. All of their angles are equal, but the sides are of different lengths. In the diagrams below, $\angle B = \angle E$, $\angle C = \angle F$, and $\angle A = \angle D$.

Unlike congruent shapes, similar shapes do not fit on top of each other if they are turned, moved or flipped.

Solved Examples

1. Establish whether the △RST and △XYZ are similar and explain why.



Solution

Remember that similar triangles may have sides of different lengths, but the corresponding angles should be the same measure. One set of corresponding angles is known: $R = X = 36°$.

Solve for the unknown angles in the triangles (S and Z) to check if the other corresponding angles are equal.

Subtract R and T from 180° to find the measure of S:

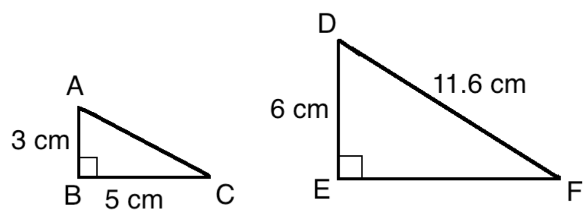
$$\begin{aligned} S &= 180° - R - T \\ &= 180° - 36° - 55° \\ &= 89° \end{aligned}$$

Subtract X and Y from 180° to find the measure of Z:

$$\begin{aligned} Z &= 180° - X - Y \\ &= 180° - 36° - 98° \\ &= 45° \end{aligned}$$

The corresponding angles in the triangles are not equal ($S \neq Y$ and $T \neq Z$). The triangles are **not** similar.

2. In the diagram below, △ABC has been enlarged to △DEF.
   a. Find the scale factor of the enlargement.
   b. Calculate the lengths of sides AC and EF.



Solutions

a. We know the triangles are similar, so we can find the scale factor using any 2 sides. Note AB and DE are corresponding sides. DE is twice as long as AB, so the scale factor is 2: $\frac{\mathrm{DE}}{\mathrm{AB}} = \frac{6}{3} = 2$.

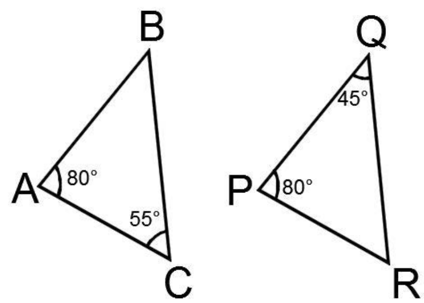
b. Use the scale factor. Multiplying the scale factor by any side of △ABC will give the length of the corresponding side of △DEF.

$$\begin{aligned} EF &= 2 \times BC \\ &= 2 \times 5 \\ &= 10 \text{ cm} \end{aligned} \qquad \begin{aligned} DF &= 2 \times AC \\ 11.6 &= 2 \times AC \\ \tfrac{11.6}{2} &= AC \\ 5.8 \text{ cm} &= AC \end{aligned}$$

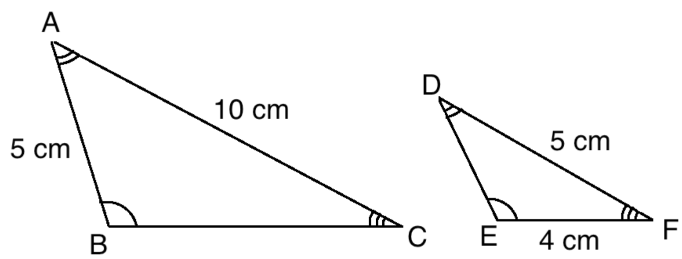
Answer: $AC = 5.8$ cm and $EF = 10$ cm

Practice

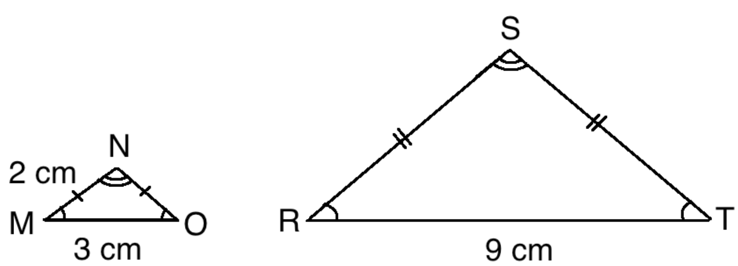
1. Establish whether the triangles below are similar and explain why.



2. In the diagram below, △ABC has been enlarged to △DEF.
   a. Find the scale factor of the enlargement.
   b. Calculate the lengths of sides BC and DE.



3. In the diagram below, △MNO has been enlarged to △RST.
   a. Find the scale factor of the enlargement.
   b. Calculate the lengths of sides NO, RS and ST.



Lesson Title: Comparing Congruent and Similar Shapes	**Theme:** Geometry
Practice Activity: PHM-09-054	**Class:** JSS 3

Learning Outcome
By the end of the lesson, you will be able to differentiate between congruency and similarity of shapes.

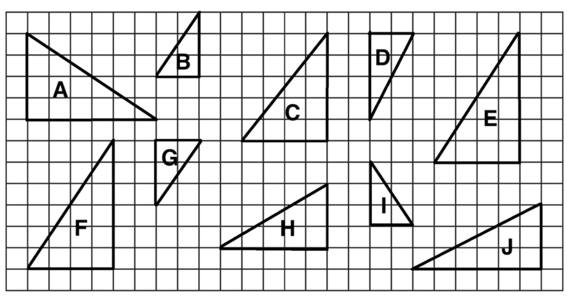
Overview

In the previous lessons, you have learned about two relationships between shapes: congruency and similarity. In this lesson, you will differentiate between the congruency and similarity of shapes. Remember that **congruent** shapes have the same shape and size. **Similar** shapes have the same shape but a different size.

Remember that in both congruent shapes and similar shapes the angles should be equal. The difference is in the lengths of the sides. In congruent triangles, the lengths of all 3 sides are the same. In similar triangles, the lengths of the sides are in proportion according to the scale factor. This is also called the similarity ratio. We can find missing lengths using this ratio.

Solved Examples

1. In the diagram below, identify which shapes are similar and which are congruent. Give your reasons.



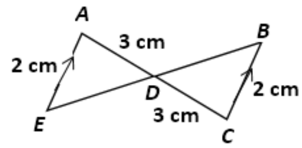
Solution

A, E and F are congruent because they have exactly the same shape and size. They are only turned in different directions.

B, G and I are congruent because they have exactly the same shape and size.
Triangles A, E and F are enlargements of triangles B, G and I. Therefore, these 2 sets of triangles are similar. The scale factor is 2.

Triangles C, D, H and J are not similar or congruent to any other triangle in the diagram.

2. In the diagram below, AE and BC are parallel. Two transversal lines AC and EB intersect to form triangles ADE and BCD. Are the 2 triangles similar or congruent? Give your reasons.



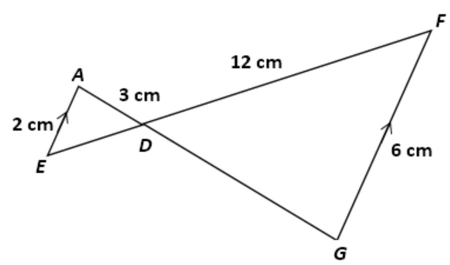
Solution

$\triangle$ADE and $\triangle$BCD are **congruent**. Since the 2 lines AE and BC are parallel, it means that $\angle EAD = \angle BCD$. Remember that SAS is one of the rules for congruency. If 2 sets of corresponding sides are equal and the included angle is also equal, then the triangles are congruent. In triangles ADE and BCD, we have:

- $AD = DC = 3 \text{ cm}$
- $\angle EAD = \angle BCD$
- $AE = BC = 2 \text{ cm}$

This gives us 2 sides and the included angle (SAS) and tells us that the triangles are congruent.

3. In the diagram below, AE and FG are parallel. Two transversal lines AG and EF intersect to form triangles ADE and FGD.
   a. Are the 2 triangles similar or congruent? Give your reasons.
   b. Find the lengths of DE and DG.



Solutions

a. $\triangle$ADE and $\triangle$FGD are **similar**.
   We can show that all 3 corresponding angles of the triangles are equal. Since the 2 lines AE and FG are parallel, it means that $\angle EAD = \angle FGD$. Similarly, $\angle DFG = \angle DEA$. We also know that $\angle ADE = \angle FDG$ because they are opposite angles in intersecting lines. The sides are of different lengths. Note that AE corresponds to FG. FG is 3 times as long as AE. This means that the triangles are similar, with a scale factor of 3.
b. Use the scale factor, 3, to find the lengths of the unknown sides. Note that AD corresponds to DG and DE corresponds to DF.

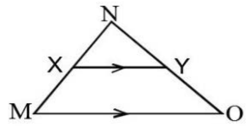
$$DG = 3 \times AD \qquad\qquad DF = 3 \times DE$$

$$= 3 \times 3 \qquad\qquad 12 = 3 \times DE$$

$$= 9 \text{ cm} \qquad\qquad \frac{12}{3} = DE$$

$$4 \text{ cm} = DE$$

4. Identify the similar triangles in the figure below, and give your reasons.

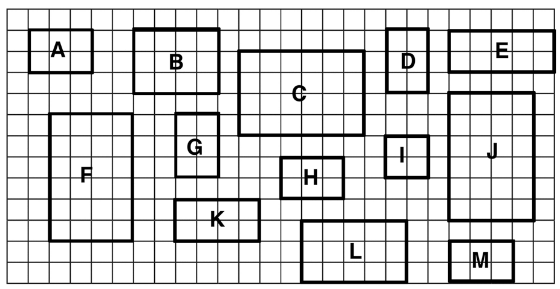


Solution

ΔMNO is similar to ΔXNY. We know this because all 3 angles of these triangles are equal. Note that $\angle NMO = \angle NXY$ because these are corresponding angles. The line MN is a transversal of the parallel lines XY and MO. For the same reason, $\angle NOM = \angle NYX$.

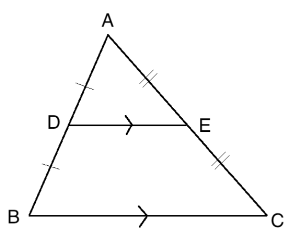
Practice

1. In the diagram below, identify which shapes are similar and which are congruent. Give your reasons.

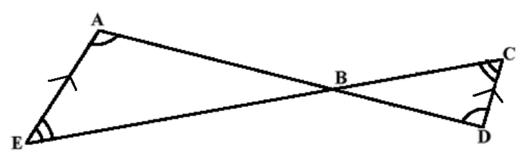


2. Identify whether any of the triangles in the diagrams below are similar or congruent. Give your reasons.

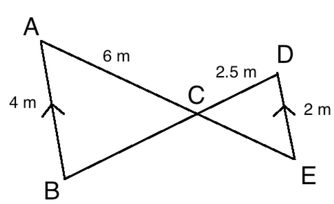
a.



b.



3. In the diagram below, AB and DE are parallel. Two transversal lines AE and BD intersect to form triangles ABC and CDE.
   a. Are the 2 triangles similar or congruent? Give your reasons.
   b. Find the lengths of CB and CE.



Lesson Title: Transformation Practice	Theme: Geometry
Practice Activity: PHM-09-055	**Class:** JSS 3

Learning Outcomes

By the end of the lesson, you will be able to:

1. Carry out combinations of the 4 common transformations.
2. Identify shapes as either congruent or similar after carrying out a combination of transformations.

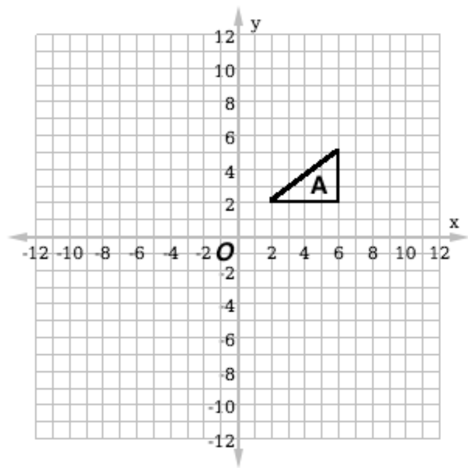
Overview

Recall the 4 transformations that you have learned to perform: translation, reflection, rotation and enlargement. In this lesson, you will carry out combinations of the 4 common transformations. You will then identify shapes as either congruent or similar after carrying out a combination of transformations.

Remember that performing translation, reflection or rotation results in a congruent shape because the shape does not change size. Enlargement results in a similar shape because the size changes but the shape stays the same.

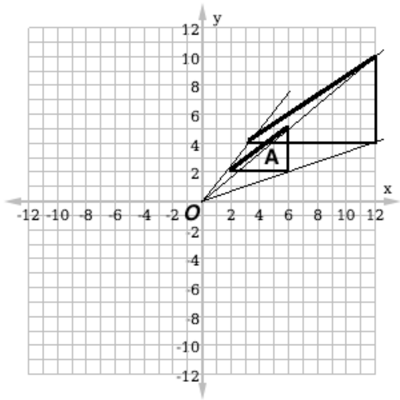
Solved Examples

1. Use the triangle in the diagram to complete the following:
    a. Enlarge the triangle with a scale factor of 2 using O as the centre of enlargement.
    b. Translate the triangle 15 units down and 10 units to the left.
    c. Is the final shape congruent or similar to triangle A?

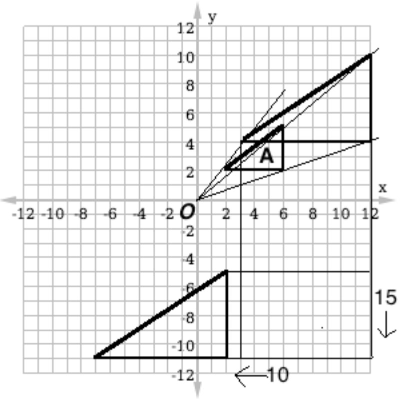


Solution

a. Use O as the centre of enlargement and draw a triangle 2 times as large as A:

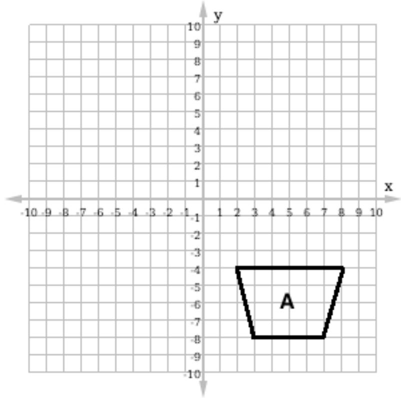


b. Translate each angle of the triangle 15 units down and 10 units to the left:



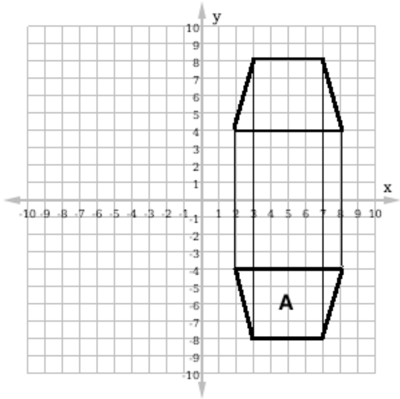
c. The final shape is similar to triangle A. It has the same shape but a different size.

2. Reflect the shape shown in the x-axis. Then, enlarge it with a scale factor of $\frac{1}{2}$, with the origin as the centre of enlargement. Is the resulting shape congruent or similar to shape A?

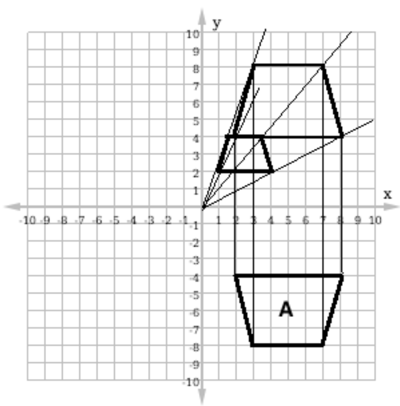


Solution

Step 1. Reflect the shape:



Step 2. Enlarge the shape by $\frac{1}{2}$. Use the origin (0, 0) as the centre of enlargement:

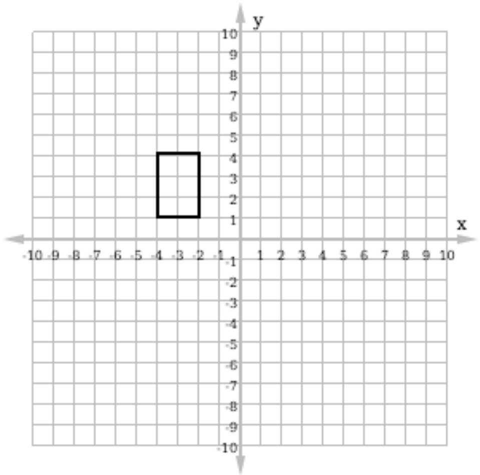


The final shape is similar to shape A. They have the same shape but a different size.

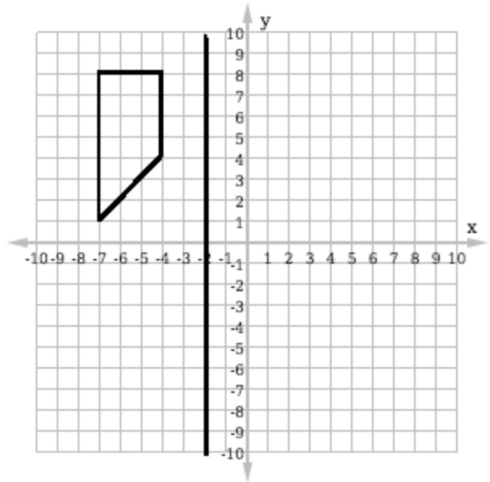
Practice

Perform each transformation given below. Then, determine whether the result is congruent or similar to the original shape.

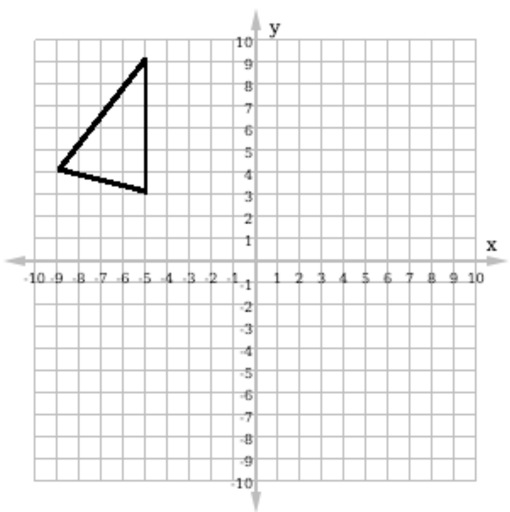
1. Rotate the shape 90° in the clockwise direction about the origin. Then enlarge it by a scale factor of 2, with the origin as the centre of enlargement.



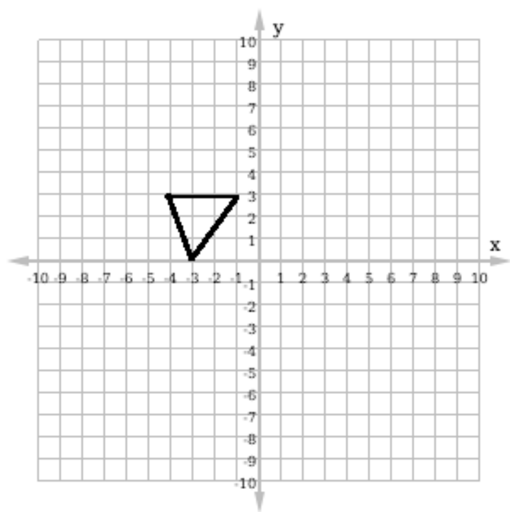
2. Reflect the shape in the line of symmetry shown. Then enlarge it by a scale factor of $\frac{1}{2}$, with the origin as the centre of enlargement.



3. Reflect the shape shown in the y-axis. Then transform it down 8 units and to the left 11 units.



4. Transform the shape 7 units to the right and 5 units down. Then enlarge it with a scale factor of 2, using the origin as the centre of enlargement.



Lesson Title: Introduction to Trigonometry	**Theme:** Geometry
Practice Activity: PHM-09-056	**Class:** JSS 3

Learning Outcomes

By the end of the lesson, you will be able to:

1. Identify the right and acute angles of a right-angled triangle.
2. Identify the relative sides of a right-angled triangle (adjacent, opposite, hypotenuse).
3. Identify SOHCAHTOA as a rule for remembering trigonometric ratios.

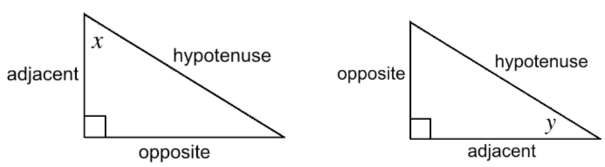
Overview

In this lesson, you will use right-angled triangles to identify **trigonometric ratios**. Remember that a right-angled triangle is one with 1 right angle. The other 2 angles are acute (less than 90°).

Trigonometric ratios are used to find the measures of unknown sides in a triangle. You will see how this works in the following lessons. Remember that ratios are fractions.

Trigonometric ratios can be applied to the angle and sides of right-angled triangles. We use 3 types of sides (adjacent, opposite and hypotenuse) in trigonometric ratios. The sides "adjacent" and "opposite" are determined by their relationship to the angle in question. The hypotenuse is fixed. It does not depend on the position of the angles.

See the examples below. In the first diagram, the sides are labeled according to their relationship with x. In the second diagram, they are labeled according to their relationship with y. In both cases, the hypotenuse is fixed.



The 3 trigonometric ratios in this lesson are:

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{\text{O}}{\text{H}}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{\text{A}}{\text{H}}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{\text{O}}{\text{A}}$$

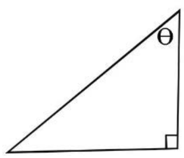
Sin, cos, and tan are the abbreviations that we use for sine, cosine, and tangent. We apply these 3 ratios to angles. The theta symbol (θ) is shown here, and it is often used to represent angles.

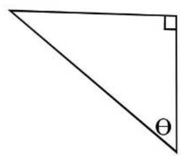
We use the term SOHCAHTOA as a way of remembering the ratios:

- SOH stands for "sine equals opposite over hypotenuse".
- CAH stands for "cosine equals adjacent over hypotenuse".
- TOA stands for "tangent equals opposite over adjacent".

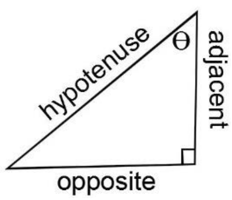
Solved Examples

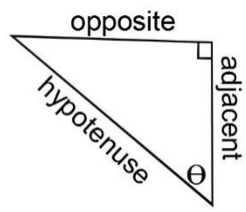
1. Identify and label the sides as "opposite", "adjacent" or "hypotenuse" based on their relative position to θ.

a. 

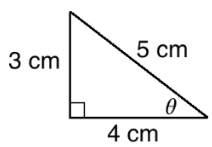
b. 

Solutions

a. 

b. 

2. Apply the trigonometric ratios to θ:



Solution

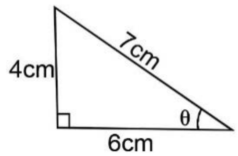
Use the term SOHCAHTOA and try to write each ratio from memory. Write the ratio for each using the corresponding sides in the triangle.

$$\sin\theta = \frac{O}{H} = \frac{3}{5}$$

$$\cos\theta = \frac{A}{H} = \frac{4}{5}$$

$$\tan\theta = \frac{O}{A} = \frac{3}{4}$$

3. Apply the trigonometric ratios to angle θ on the triangle:



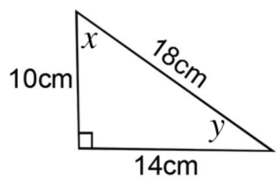
Solution

$$\sin\theta = \frac{O}{H} = \frac{4}{7}$$

$$\cos\theta = \frac{A}{H} = \frac{6}{7}$$

$$\tan\theta = \frac{O}{A} = \frac{4}{6} = \frac{2}{3}$$

4. For the triangle below, apply the trigonometric ratios to both angles x and y.



Solution

$$\sin x = \frac{O}{H} = \frac{14}{18} = \frac{7}{9} \qquad \sin y = \frac{O}{H} = \frac{10}{18} = \frac{5}{9}$$

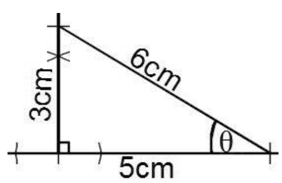
$$\cos x = \frac{A}{H} = \frac{10}{18} = \frac{5}{9} \qquad \cos y = \frac{A}{H} = \frac{14}{18} = \frac{7}{9}$$

$$\tan x = \frac{O}{A} = \frac{14}{10} = \frac{7}{5} = 1\frac{2}{5} \qquad \tan y = \frac{O}{A} = \frac{10}{14} = \frac{5}{7}$$

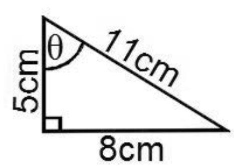
Practice

1. For each of the triangles below, apply each of the trigonometric ratios to angles θ.

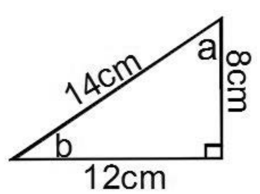
a.



b.



2. For the triangle below, apply the trigonometric ratios to both angles a and b. Simplify your answers.



Lesson Title: Sine	**Theme:** Geometry
Practice Activity: PHM-09-057	**Class:** JSS 3

Learning Outcome

By the end of the lesson, you will be able to apply the sine ratio to solve for an unknown side.

Overview

In the previous lesson, you learned to identify the 3 trigonometric ratios using a right-angled triangle. In this lesson, you will solve for unknown sides in a triangle using the sine ratio. The sine ratio is $\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$.

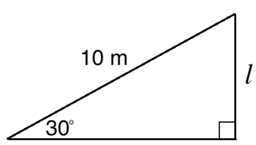
Each angle has a value for its sine ratio. The sine ratios of some common angles are given in the table below. You will use these in this lesson. These can be found using calculators or sine tables. This is the topic of a later lesson.

Angle (θ)	sin θ
30	0.5000
35	0.5736
40	0.6428
45	0.7071
50	0.7660
55	0.8192
60	0.8660

You will be given triangles with 1 of the acute angles given, and the length of either the opposite side or the hypotenuse given. That is, one of the sides from the sine ratio $\left(\frac{\text{opposite}}{\text{hypotenuse}}\right)$ will be given.

Use the value of the sine ratio for the given angle, and the length of the given side. Solve for the unknown side.

For example, look at this triangle:



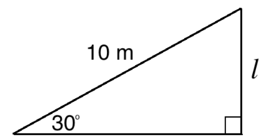
One angle is given (30°) and the hypotenuse is given (10 m). You can use this information to find the length of side l. Substitute this information into the sine formula. Note from the table that $\sin 30° = 0.5$.

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} \quad \rightarrow \quad \sin 30° = \frac{l}{10 \text{ m}} \quad \rightarrow \quad 0.5 = \frac{l}{10 \text{ m}}$$

If you solve this equation, you will find the length of the side labeled l. The solution of this problem is in Solved Example 1.

Solved Examples

1. Find the length of side l in the triangle:



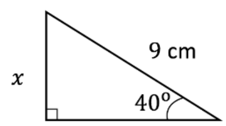
Solution

Substitute $\theta = 30°$ and hypotenuse = 10 m into the sine ratio, and solve for l.

$\sin\theta$	$=$	$\frac{\text{opposite}}{\text{hypotenuse}}$	Sine ratio
$\sin 30°$	$=$	$\frac{l}{10}$	Substitute $30°$ and 10 m
$10 \times \sin 30°$	$=$	l	Multiply both sides by 10
10×0.5	$=$	l	Substitute $\sin 30° = 0.5$
5 m	$=$	l	Multiply

The length of side l is 5 m.

2. Find the measure of missing side x. Give your answer to 1 decimal place.



Solution

Substitute the known angle and side into the formula $\sin\theta = \frac{\text{O}}{\text{H}}$. Use $\sin 40^{\text{o}} = 0.6428$ from the table in Overview.

$$\sin 40^{\text{o}} = \frac{x}{9}$$

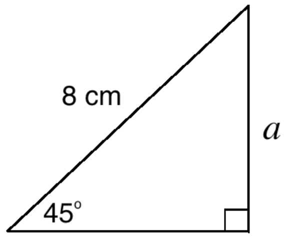
$$9 \times \sin 40^{\text{o}} = x$$

$$9 \times 0.6428 = x$$

$$x = 5.7852$$

$$x = 5.8\text{ cm to 1 d.p.}$$

3. Find the measure of a. Give your answer to 2 decimal places.



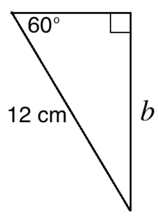
Solution

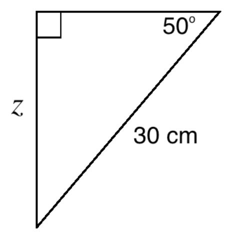
Substitute the known angle and side into the formula $\sin\theta = \frac{\text{O}}{\text{H}}$. Use $\sin 45^{\text{o}} = 0.7071$ from the table in Overview.

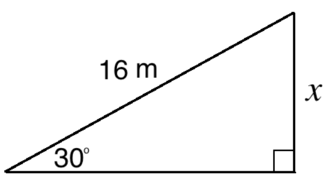
$$\begin{aligned} \sin 45^{\text{o}} &= \frac{a}{8} \\ 8 \times \sin 45^{\text{o}} &= a \\ 8 \times 0.7071 &= a \\ a &= 5.6568 \\ a &= 5.66 \text{ cm to 2 d.p.} \end{aligned}$$

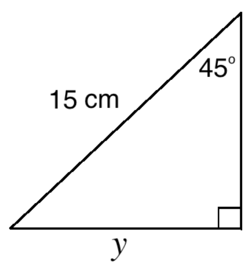
Practice

Use the sine ratio to find the measure of each side marked with a letter. Give your answers to 1 decimal place.

1. 

2. 

3. 

4. 

Lesson Title: Cosine	Theme: Geometry
Practice Activity: PHM-09-058	**Class:** JSS 3

Learning Outcome
By the end of the lesson, you will be able to apply the cosine ratio to solve for an unknown side.

Overview

In this lesson, you will solve for unknown sides in a triangle using the cosine ratio.
The cosine ratio is $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$.

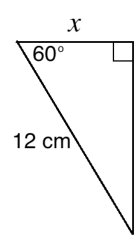
Each angle has a value for its cosine ratio. The cosine ratios of some common angles are given in the table below. You will use these in this lesson. These can be found using calculators or cosine tables, which is the topic of a later lesson.

Angle (θ)	cos θ
30	0.8660
35	0.8192
40	0.7660
45	0.7071
50	0.6428
55	0.5736
60	0.5000

You will be given triangles with 1 of the acute angles given, and the length of either the adjacent side or the hypotenuse given. That is, one of the sides from the cosine ratio $\left(\frac{\text{adjacent}}{\text{hypotenuse}}\right)$ will be given.

Use the value of the cosine ratio for the given angle, and the length of the given side. Solve for the unknown side.

For example, look at this triangle:



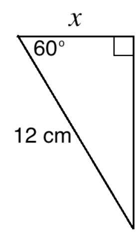
One angle is given (60°) and the hypotenuse is given (12 m). You can use this information to find the length of side x. Substitute this information into the cosine formula. Note from the table that $\cos 60° = 0.5$.

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \quad \rightarrow \quad \cos 60° = \frac{x}{12 \text{ m}} \quad \rightarrow \quad 0.5 = \frac{x}{12 \text{ m}}$$

If you solve this equation, you will find the length of the side labeled x. The solution of this problem is in Solved Example 1.

Solved Examples

1. Find the length of side x in the triangle:



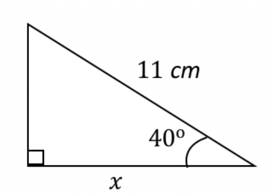
Solution

Substitute $\theta = 60°$ and hypotenuse = 12 cm into the cosine ratio, and solve for x.

$\cos\theta$	$=$	$\frac{\text{adjacent}}{\text{hypotenuse}}$	Cosine ratio
$\cos 60°$	$=$	$\frac{x}{12}$	Substitute $60°$ and 12 m
$12 \times \cos 60°$	$=$	x	Multiply both sides by 12
12×0.5	$=$	x	Substitute $\cos 60° = 0.5$
$6\ cm$	$=$	x	Multiply

The length of side x is 6 cm.

2. Find the length of the side marked x in the triangle below. Give your answer correct to 1 decimal place.

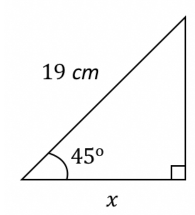


Solution

Substitute $\theta = 40°$ and hypotenuse = 11 cm into the cosine ratio, and solve for x.

$\cos\theta$	$=$	$\frac{\text{adjacent}}{\text{hypotenuse}}$	Cosine ratio
$\cos 40°$	$=$	$\frac{x}{11}$	Substitute $40°$ and 11 cm
$11 \times \cos\ 40°$	$=$	x	Multiply both sides by 11
11×0.7660	$=$	x	Substitute $\cos 40° = 0.7660$
x	$=$	8.426 m	Multiply
x	$=$	8.4 m to 1 d.p.	

3. Find the length of the side marked x in the triangle below. Give your answer correct to 1 decimal place.



Solution

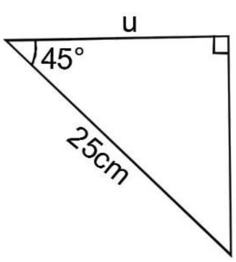
Substitute $\theta = 45°$ and hypotenuse = 19 cm into the cosine ratio, and solve for x.

$\cos\theta$	$=$	$\frac{\text{adjacent}}{\text{hypotenuse}}$	Cosine ratio
$\cos 45°$	$=$	$\frac{x}{19}$	Substitute $45°$ and 19 cm
$19 \times \cos 45°$	$=$	x	Multiply both sides by 19
19×0.7071	$=$	x	Substitute $\cos 45° = 0.7071$
x	$=$	13.4349 m	Multiply
x	$=$	13.4 m to 1 d.p.	

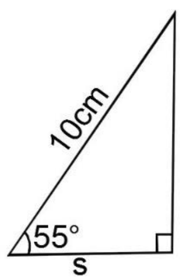
Practice

Use the cosine ratio to find the measure of each side marked with a letter. Give your answers to 1 decimal place.

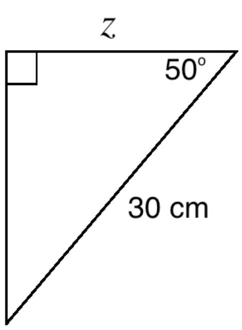
1.



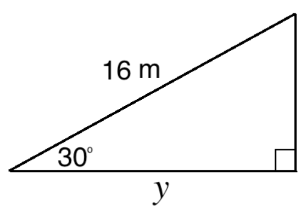
2.



3.



4.



Lesson Title: Tangent	**Theme:** Geometry
Practice Activity: PHM-09-059	**Class:** JSS 3

Learning Outcomes

By the end of the lesson, you will be able to:

1. Apply the tangent ratio to solve for an unknown side.
2. Identify that tangent is a ratio of sine and cosine: $\tan\theta = \frac{\sin\theta}{\cos\theta}$

Overview

In this lesson, you will solve for unknown sides in a triangle using the tangent ratio. The tangent ratio is $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$.

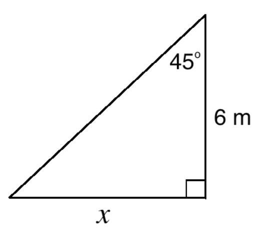
Each angle has a value for its tangent ratio. The cosine ratios of some common angles are given in the table below. You will use these in this lesson. These can be found using calculators or tangent tables, which is the topic of a later lesson.

Angle (θ)	tan θ
30	0.5774
35	0.7002
40	0.8391
45	1.000
50	1.192
55	1.428
60	1.732

You will be given triangles with 1 of the acute angles given, and the length of either the opposite or adjacent side given. That is, one of the sides from the tangent ratio $\left(\frac{\text{opposite}}{\text{adjacent}}\right)$ will be given.

Use the value of the tangent ratio for the given angle, and the length of the given side. Solve for the unknown side.

For example, look at this triangle:



One angle is given (45°) and the hypotenuse is given (6 m). You can use this information to find the length of side x. Substitute this information into the tangent formula. Note from the table that $\tan 45° = 1$.

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} \quad \rightarrow \quad \tan 45° = \frac{x}{6\text{ m}} \rightarrow \quad 1 = \frac{x}{6\text{ m}}$$

If you solve this equation, you will find the length of the side labeled x. The solution of this problem is in Solved Example 1.

There is a special relationship between sine, cosine, and tangent. The tangent ratio is the sine ratio divided by the cosine ratio: $\tan\theta = \frac{\sin\theta}{\cos\theta}$

You can see how this is true by dividing the ratios:

$\frac{\sin\theta}{\cos\theta} = \frac{\frac{\text{opposite}}{\text{hypotenuse}}}{\frac{\text{adjacent}}{\text{hypotenuse}}}$		Divide the 2 fractions
$= \frac{\text{opposite}}{\text{hypotenuse}} \times \frac{\text{hypotenuse}}{\text{adjacent}}$		Multiply by the reciprocal of the 2nd fraction
$= \frac{\text{opposite}}{\text{adjacent}}$		Simplify
$= \tan\theta$		

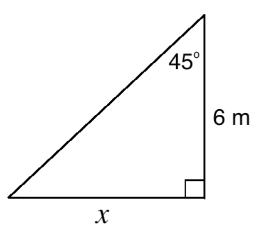
This is true for any value of θ. Consider $\theta = 45°$. From the tables in the previous lessons, we have: $\sin\theta = 0.7071$ and $\cos\theta = 0.7071$

Substitute these values in the equation: $\tan 45° = \frac{\sin 45°}{\cos 45°} = \frac{0.7071}{0.7071} = 1$

You can see from the table above that $\tan 45° = 1$ is true.

Solved Examples

1. Find the length of side x in the triangle:



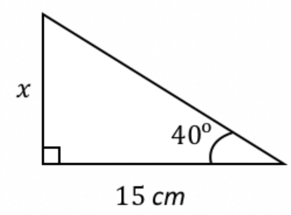
Solution

Substitute $\theta = 45°$ and adjacent = 6 m into the tangent ratio, and solve for x.

$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$		Tangent ratio
$\tan 45° = \frac{x}{6}$		Substitute 45° and 6 m
$6 \times \tan 45° = x$		Multiply both sides by 6
$6 \times 1 = x$		Substitute $\tan 45° = 1$
$6\text{ m} = x$		Multiply

The length of side x is 6 m.

2. Find the length of the side marked x in the triangle. Give your answer correct to 1 decimal place.



Solution

Substitute $\theta = 40°$ and adjacent = 15 cm into the tangent ratio, and solve for x.

$$\begin{aligned}
\tan\theta &= \frac{\text{opposite}}{\text{adjacent}} && \text{Tangent ratio}\\
\tan 40° &= \frac{x}{15} && \text{Substitute } 40° \text{ and 15 cm}\\
15 \times \tan 40° &= x && \text{Multiply both sides by 15}\\
15 \times 0.8391 &= x && \text{Substitute } \tan 40° = 0.8391\\
12.5865\ cm &= x && \text{Multiply}\\
x &= \text{12.6 cm to 1 d.p.}
\end{aligned}$$

3. Verify that $\tan 35° = 0.7002$ by using the sine and cosine ratios of 35°.

Solution

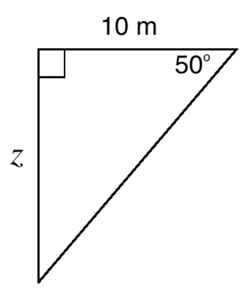
Find sin 35° and cos 35° in the tables from the previous lessons. Substitute the values into the formula $\tan 35° = \frac{\sin 35°}{\cos 35°}$.

$$\begin{aligned}
\tan 35° &= \frac{\sin 35°}{\cos 35°}\\
&= \frac{0.5736}{0.8192} && \text{Substitute values of sine and cosine}\\
&= 0.7002 && \text{Divide}
\end{aligned}$$

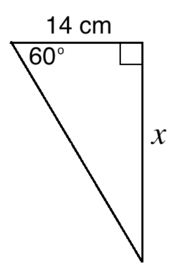
Practice

1. Use the tangent ratio to find the measures of the sides marked with a letter. Give your answers to 1 decimal place.

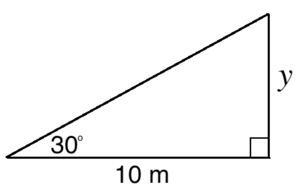
a.



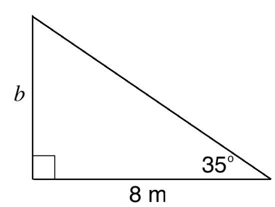
b.



c.



d.



2. Verify that $\tan 30° = 0.5774$ by using the sine and cosine ratios of 30°.

Lesson Title: Applying the Trigonometric Ratios	Theme: Geometry
Practice Activity: PHM-09-060	**Class:** JSS 3

Learning Outcome

By the end of the lesson, you will be able to find the lengths of the sides of a triangle using sine, cosine, and tangent of given angles.

Overview

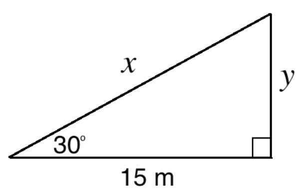
In this lesson, you will find the lengths of the sides of a triangle using sine, cosine, and tangent ratios of given angles. The problems will not tell you which ratio to use, but you will decide for yourself. Remember the rule, SOHCAHTOA.

Use the values of trigonometric ratios that were introduced in previous lessons:

Angle (θ)	$\sin\theta$	$\cos\theta$	$\tan\theta$	Angle (θ)	$\sin\theta$	$\cos\theta$	$\tan\theta$
30	0.5000	0.8660	0.5774	50	0.7660	0.6428	1.192
35	0.5736	0.8192	0.7002	55	0.8192	0.5736	1.428
40	0.6428	0.7660	0.8391	60	0.8660	0.5000	1.732
45	0.7071	0.7071	1.000	65	0.9063	0.4226	2.145

Solved Examples

1. Find the measures of x and y in the triangle below. Give your answers to 1 decimal place.



Solution

We are given 1 acute angle and the length of its adjacent side. We are asked to find the measures of the other 2 sides. We will use 2 trigonometric ratios to find them. Use the 2 ratios that contain “adjacent”: $\cos\theta = \frac{A}{H}$ and $\tan\theta = \frac{O}{A}$.

Apply the cosine ratio to find x:

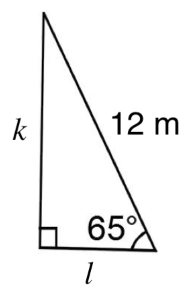
$\cos \theta$	$=$	$\frac{\text{adjacent}}{\text{hypotenuse}}$	Cosine ratio
$\cos 30°$	$=$	$\frac{15}{\text{x}}$	Substitute $30°$ and 15 m
$x \times \cos 30°$	$=$	15	Multiply both sides by x
x	$=$	$\frac{15}{\cos 30°}$	Divide both sides by cos 30°
x	$=$	$\frac{15}{0.8660}$	Substitute $\cos 30° = 0.8660$
x	$=$	17.3 m	Divide

Apply the tangent ratio to find y:

$\tan \theta$	$=$	$\frac{\text{opposite}}{\text{adjacent}}$	Tangent ratio
$\tan 30°$	$=$	$\frac{y}{15}$	Substitute $30°$ and 15 m
$15 \times \tan 30°$	$=$	y	Multiply both sides by 15
15×0.5774	$=$	y	Substitute $\tan 40° = 0.8391$
8.7 m	$=$	y	Multiply

The missing sides are $x = 17.3$ m and $y = 8.7$ m.

2. Find the lengths of sides k and l in the diagram below. Give your answers to the nearest whole number.



Solution

We are given 1 acute angle and the length of the hypotenuse. We are asked to find the measures of the other 2 sides. We will use 2 trigonometric ratios to find them. Use the 2 ratios that contain "hypotenuse": $\sin \theta = \frac{O}{H}$ and $\cos \theta = \frac{A}{H}$.

Apply the sine ratio to find k:

$\sin \theta$	$=$	$\frac{\text{opposite}}{\text{hypotenuse}}$	Sine ratio
$\sin 65°$	$=$	$\frac{k}{12}$	Substitute $65°$ and 12 m
$12 \times \sin 65°$	$=$	k	Multiply both sides by 12
12×0.9063	$=$	k	Substitute $\sin 65° = 0.9063$
k	$=$	$10.88 = 11 \text{ m}$	

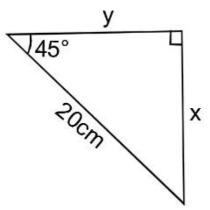
Apply the cosine ratio to find l:

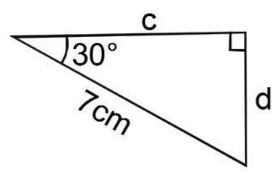
$\cos 65^{o} = \frac{\text{adjacent}}{\text{hypotenuse}}$	Sine ratio
$\cos 65^{o} = \frac{l}{12}$	Substitute 65° and 12 m
$12 \times \cos 65° = l$	Multiply both sides by 12
$12 \times 0.4226 = l$	Substitute $\cos 65° = 0.4226$
$l = 5.0712 = 5$ m	

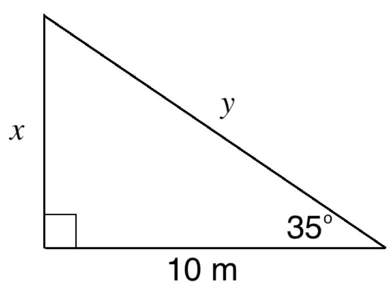
The unknown sides are $k = 11$ m and $l = 5$ m.

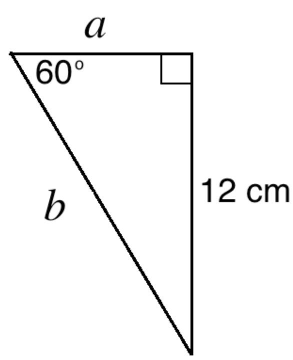
Practice

Find the measure of each marked side in the problems below. Give your answers to the nearest whole number.

1. 

2. 

3. 

4. 

Lesson Title: Trigonometric Tables for Sine	**Theme:** Geometry
Practice Activity: PHM-09-061	**Class:** JSS 3

Learning Outcome

By the end of the lesson, you will be able to use trigonometric tables to find the sine of an angle.

Overview

So far you have found lengths of unknown sides using the sine, cosine and tangent of angles given in a table. These are the trigonometric ratios for some of the more common angles, such as 45° and 60°. In everyday life, we will come across many different angles. For example, you may measure an angle that is 10.52°.

In this lesson, you will use trigonometric tables to find the **sine** of an angle. The trigonometric tables are at the end of this book. Part of the sine table is below. This shows the sines of the angles from 0 to 10, including decimal numbers. The decimal digit in the **tenths** place is given in the first 10 columns of numbers, marked .0 through .9. The digits in the "ADD Differences" columns are digits in the **hundredths** place.

Sine Table

											ADD Differences								
x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9
0	0·0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	2	3	5	7	9	10	12	14	16
1	0·0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	2	3	5	7	9	10	12	14	16
2	0·0349	0366	0384	0401	0419	0436	0454	0471	0488	0506	2	3	5	7	9	10	12	14	16
3	0·0523	0541	0558	0576	0593	0610	0628	0645	0663	0680	2	3	5	7	9	10	12	14	16
4	0·0698	0715	0732	0750	0767	0785	0802	0819	0837	0854	2	3	5	7	9	10	12	14	16
5	0·0872	0889	0906	0924	0941	0958	0976	0993	1011	1028	2	3	5	7	9	10	12	14	16
6	0·1045	1063	1080	1097	1115	1132	1149	1167	1184	1201	2	3	5	7	9	10	12	14	16
7	0·1219	1236	1253	1271	1288	1305	1323	1340	1357	1374	2	3	5	7	9	10	12	14	16
8	0·1392	1409	1426	1444	1461	1478	1495	1513	1530	1547	2	3	5	7	9	10	12	14	16
9	0·1564	1582	1599	1616	1633	1650	1668	1685	1702	1719	2	3	5	7	9	10	12	14	15
10	0·1736	1754	1771	1788	1805	1822	1840	1857	1874	1891	2	3	5	7	9	10	12	14	15

To find the sine of an angle, follow the steps below. $\sin 10.52°$ is used as an example.

Step	Example: $\sin 10.52°$
1. Find the **whole number** part of the angle in the left-hand column.	Our example has whole number **10**. The second column gives $\sin 10° = 0.1736$.
2. Find the column for the digit in the **tenths** place.	Our example has 5 in the tenths place. The column marked .5 gives $\sin 10.5° = 0.1822$.
3. Find the column for the digit in the **hundredths** place. Add this number to the decimal digits found in Step 2.	Our example has 2 in the hundredths place. The column marked 2 under “ADD Differences” gives the number 3. Add this 3 to the digits from Step 2: $1{,}822 + 3 = 1{,}825$ We have $\sin 10.52° = 0.1825$.

We have found the sine of our example number: $\sin 10.52° = 0.1825$.

If there are no digits in the hundredths place, skip Step 3. If there are no digits in the tenths or hundredths places, skip Steps 2 and 3.

Trigonometric tables give the value of a trigonometric ratio to 4 decimal places. These are rounded numbers. Most trigonometric ratios have many decimal places. You can use a calculator to find trigonometric ratios to more decimal places.

Solved Examples

1. Use the sine table to find $\sin 42°$

Solution

Identify the row for 42° in the sine table:

											ADD Differences								
x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9
⋮																			
42	0·6691	6704	6717	6730	6743	6756	6769	6782	6794	6807	1	3	4	5	6	8	9	10	12

The column marked .0 gives the sine of the whole number 42.

Answer: $\sin 42° = 0.6691$.

2. Use the sine table to find $\sin 14.57°$.

Solution

Identify the row for 14° in the sine table:

x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	ADD Differences								
											1	2	3	4	5	6	7	8	9
⋮																			
14	0·2419	2436	2453	2470	2487	2504	2521	2538	2554	2571	2	3	5	7	8	10	12	14	15

Find the digits in the column for 5 tenths, and 7 hundredths. They are circled below:

x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	ADD Differences								
											1	2	3	4	5	6	7	8	9
⋮																			
14	0·2419	2436	2453	2470	2487	2504	2521	2538	2554	2571	2	3	5	7	8	10	12	14	15

Add the digits for 5 tenths and 7 hundredths: $2{,}504 + 12 = 2{,}516$

This is actually a decimal number, 0.2516

Answer: $\sin 14.57° = 0.2516$

3. Use a sine table to find the values of the following:

a. $\sin 68°$ b. $\sin 68.2°$ c. $\sin 68.35°$

Solutions

Find the row for 68° in the sine table. This is what it looks like:

x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	ADD Differences								
											1	2	3	4	5	6	7	8	9
⋮																			
68	0·9272	9278	9285	9291	9298	9304	9311	9317	9323	9330	1	1	2	3	3	4	4	5	6

a. $\sin 68°$ is given by the first number in the table:

$\sin 68° = 0.9272$

b. To find $\sin 68.2°$, find the column for 2 tenths. The digits in that column are 9,285. This is actually a decimal number:

$\sin 68.2° = 0.9285$

c. To find $\sin 68.25°$, identify the columns for 2 tenths and 5 hundredths. These give $9{,}285 + 3 = 9{,}288$.

Write this as a decimal number: $\sin 68.25° = 0.9288$

Practice

Find the values of the following using the sine table:

1. a. $\sin 5°$ b. $\sin 72°$
2. $\sin 6.9°$
3. $\sin 21.05°$
4. $\sin 70.5°$
5. a. $\sin 81.28°$; b. $\sin 25.25°$; c. $\sin 77.7°$
6. a. $\sin 14\frac{1}{2}°$; b. $\sin 28\frac{3}{4}°$

Lesson Title: Trigonometric Tables for Cosine	**Theme:** Geometry
Practice Activity: PHM-09-062	**Class:** JSS 3

Learning Outcome
By the end of the lesson, you will be able to use trigonometric tables to find the cosine of an angle.

Overview

In this lesson, you will use trigonometric tables to find the **cosine** of an angle. The trigonometric tables are at the end of this book. Part of the cosine table is below. This shows the cosine of the angles from 0 to 10, including decimal numbers. The decimal digit in the **tenths** place is given in the first 10 columns of numbers, marked .0 through .9. The digits in the "SUBTRACT Differences" columns are digits in the **hundredths** place.

Cosine Table

											SUBTRACT Differences								
x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9
0	1·0000	1·000	1·000	1·000	1·000	1·000	·9999	·9999	·9999	·9999									
1	0·9998	·9998	·9998	·9997	·9997	·9997	·9996	·9996	·9995	·9995									
2	0·9994	·9993	·9993	·9992	·9991	·9990	·9990	·9989	·9988	·9987	0	0	0	0	0	0	1	1	1
3	0·9986	·9985	·9984	·9983	·9982	·9981	·9980	·9979	·9978	·9977	0	0	0	0	1	1	1	1	1
4	0·9976	·9974	·9973	·9972	·9971	·9969	·9968	·9966	·9965	·9963	0	0	0	1	1	1	1	1	1
5	0·9962	·9960	·9959	·9957	·9956	·9954	·9952	·9951	·9949	·9947	0	0	1	1	1	1	1	1	2
6	0·9945	·9943	·9942	·9940	·9938	·9936	·9934	·9932	·9930	·9928	0	0	1	1	1	1	1	2	2
7	0·9925	·9923	·9921	·9919	·9917	·9914	·9912	·9910	·9907	·9905	0	0	1	1	1	1	2	2	2
8	0·9903	·9900	·9898	·9895	·9893	·9890	·9888	·9885	·9882	·9880	0	1	1	1	1	2	2	2	2
9	0·9877	·9874	·9871	·9869	·9866	·9863	·9860	·9857	·9854	·9851	0	1	1	1	1	2	2	2	3
10	0·9848	·9845	·9842	·9839	·9836	·9833	·9829	·9826	·9823	·9820	0	1	1	1	2	2	2	3	3

Using a cosine table is very similar to using a sine table. The difference is that you will **subtract** the difference. For the sine ratio, you added the differences.

To find the cosine of an angle, follow the steps below. cos $10.52°$ is used as an example.

Step	Example: $\cos 10.52°$
1. Find the **whole number** part of the angle in the left-hand column.	Our example has whole number **10**. The second column gives $\cos 10° = 0.9848$.
2. Find the column for the digit in the **tenths** place.	Our example has 5 in the tenths place. The column marked .5 gives $\cos 10.5° = 0.9833$.
3. Find the column for the digit in the **hundredths** place. Subtract this number from the decimal digits found in step 2.	Our example has 2 in the hundredths place. The column marked 2 under “SUBTRACT Differences” gives the number 1. Subtract this 1 to the digits from step 2: $9{,}833 - 1 = 9{,}832$ We have $\cos 10.52° = 0.9832$.

We have found the cosine of our example number: $\cos 10.52° = 0.9832$.

If there are no digits in the hundredths place, skip Step 3. If there are no digits in the tenths or hundredths places, skip Steps 2 and 3.

Solved Examples

1. Use the cosine table to find $\cos 58\,°$

Solution

Identify the row for 58° in the cosine table:

											SUBTRACT Differences								
x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9
⋮																			
58	0·5299	·5284	·5270	·5255	·5240	·5225	·5210	·5195	·5180	·5165	1	3	4	6	7	9	10	12	13

The column marked .0 gives the cosine of the whole number 58.

Answer: $\cos 58\,° = 0.5299$.

2. Use the cosine table to find $\cos 12.37\,°$.

Solution

Identify the row for 12° in the cosine table:

											SUBTRACT Differences								
x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9
⋮																			
12	0·9781	·9778	·9774	·9770	·9767	·9763	·9759	·9755	·9751	·9748	0	1	1	2	2	2	3	3	3

Find the digits in the column for 3 tenths, and 7 hundredths. They are circled below:

x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	SUBTRACT Differences 1	2	3	4	5	6	7	8	9
12	0·9781	·9778	·9774	·9770	·9767	·9763	·9759	·9755	·9751	·9748	0	1	1	2	2	2	3	3	3

Subtract the digits for hundredths from the digits for tenths: $9770 - 3 = 9767$

This is actually a decimal number, 0.9767

Answer: $\cos 12.37° = 0.9767$

3. Use a cosine table to find the values of the following:

   a. $\cos 34°$ b. $\cos 34.8°$ c. $\cos 34.86°$

   Solutions

   Find the row for 34° in the cosine table. This is what it looks like:

x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	SUBTRACT Differences 1	2	3	4	5	6	7	8	9
34	0·8290	·8281	·8271	·8261	·8251	·8241	·8231	·8221	·8211	·8202	1	2	3	4	5	6	7	8	9

   a. $\cos 34°$ is given by the first number in the table:

   $\cos 34° = 0.8290$

   b. To find $\cos 34.8°$, find the column for 8 tenths. The digits in that column are 8,211. This is actually a decimal number:

   $\cos 34.8° = 0.8211$

   c. To find $\cos 34.86°$, identify the columns for 8 tenths and 6 hundredths. These give $8{,}211 - 6 = 8{,}205$.

   Write this as a decimal number: $\cos 34.86° = 0.8205$

Practice

Find the values of the following using the sine table:

1. a. $\cos 5°$ b. $\cos 72°$
2. $\cos 6.9°$
3. $\cos 21.05°$
4. $\cos 70.5°$
5. $\cos 81.28°$
6. $\cos 25.25°$
7. $\cos 77.7°$
8. a. $\cos 42\frac{1}{2}°$ b. $\cos 52\frac{3}{4}°$

Lesson Title: Trigonometric Tables for Tangent	**Theme:** Geometry
Practice Activity: PHM-09-063	**Class:** JSS 3

Learning Outcome
By the end of the lesson, you will be able to use trigonometric tables to find the tangent of an angle.

Overview

In this lesson, you will use trigonometric tables to find the **tangent** of an angle. The trigonometric tables are at the end of this book. Part of the tangent table is below. This shows the tangents of the angles from 0 to 10, including decimal numbers. The decimal digit in the **tenths** place is given in the first 10 columns of numbers, marked .0 through .9. The digits in the "ADD Differences" columns are digits in the **hundredths** place.

Tangent Table

											ADD Differences								
x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9
0	0·0000	·0017	·0035	·0052	·0070	·0087	·0105	·0122	·0140	·0157	2	3	5	7	9	10	12	14	16
1	0·0175	·0192	·0209	·0227	·0244	·0262	·0279	·0297	·0314	·0332	2	3	5	7	9	10	12	14	16
2	0·0349	·0367	·0384	·0402	·0419	·0437	·0454	·0472	·0489	·0507	2	3	5	7	9	10	12	14	16
3	0·0524	·0542	·0559	·0577	·0594	·0612	·0629	·0647	·0664	·0682	2	4	5	7	9	11	12	14	16
4	0·0699	·0717	·0734	·0752	·0769	·0787	·0805	·0822	·0840	·0857	2	4	5	7	9	11	12	14	16
5	0·0875	·0892	·0910	·0928	·0945	·0963	·0981	·0998	·1016	·1033	2	4	5	7	9	11	12	14	16
6	0·1051	·1069	·1086	·1104	·1122	·1139	·1157	·1175	·1192	·1210	2	4	5	7	9	11	12	14	16
7	0·1228	·1246	·1263	·1281	·1299	·1317	·1334	·1352	·1370	·1388	2	4	5	7	9	11	12	14	16
8	0·1405	·1423	·1441	·1459	·1477	·1495	·1512	·1530	·1548	·1566	2	4	5	7	9	11	12	14	16
9	0·1584	·1602	·1620	·1638	·1655	·1673	·1691	·1709	·1727	·1745	2	4	5	7	9	11	13	14	16
10	0·1763	·1781	·1799	·1817	·1835	·1853	·1871	·1890	·1908	·1926	2	4	5	7	9	11	13	14	16

To find the tangent of an angle, follow the steps below. $\tan 10.52°$ is used as an example.

Step	Example: $\tan 10.52°$
1. Find the **whole number** part of the angle in the left-hand column.	Our example has whole number **10**. The second column gives $\tan 10° = 0.1763$.
2. Find the column for the digit in the **tenths** place.	Our example has 5 in the tenths place. The column marked .5 gives $\tan 10.5° = 0.1853$.
3. Find the column for the digit in the **hundredths** place. Add this number to the decimal digits found in step 2.	Our example has 2 in the hundredths place. The column marked 2 under "ADD Differences" gives the number 4. Add this 4 to the digits from step 2: $1{,}853 + 4 = 1{,}857$ We have $\tan 10.52° = 0.1857$.

We have found the tangent of our example number: $\tan 10.52° = 0.1857$

If there are no digits in the hundredths place, skip Step 3. If there are no digits in the tenths or hundredths places, skip Steps 2 and 3.

Solved Examples

1. Use the tangent table to find $\tan 41°$.

   Solution
   Identify the row for 41° in the tangent table:

											ADD Differences								
x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9
41	0.8693	·8724	·8754	·8785	·8816	·8847	·8878	·8910	·8941	·8972	3	6	9	12	16	19	22	25	28

   The column marked .0 gives the tangent of the whole number 41.
   Answer: $\tan 41° = 0.8693$.

2. Use the tangent table to find $\tan 13.18°$.

   Solution
   Identify the row for 13° in the tangent table:

											ADD Differences								
x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9
13	0·2309	·2327	·2345	·2364	·2382	·2401	·2419	·2438	·2456	·2475	2	4	6	7	9	11	13	15	17

   Find the digits in the column for 1 tenth, and 8 hundredths. They are circled below:

											ADD Differences								
x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9
13	0·2309	·2327	·2345	·2364	·2382	·2401	·2419	·2438	·2456	·2475	2	4	6	7	9	11	13	15	17

   Add the digits for 1 tenth and 8 hundredths: $2{,}327 + 15 = 2{,}342$
   This is actually a decimal number, 0.2342
   Answer: $\tan 13.18° = 0.2342$

3. Use a tangent table to find the values of the following:
   a. $\tan 61°$ b. $\tan 61.5°$ c. $\tan 61.51°$

Solution

Find the row for 61° in the tangent table. This is what it looks like:

											ADD Differences								
x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9
61	1·804	1·811	1·819	1·827	1·834	1·842	1·849	1·857	1·865	1·873	1	2	2	3	4	5	5	6	7

a. $\tan 61\,°$ is given by the first number in the table:
   $$\tan 61\,° = 1.804$$
b. To find $\tan 61.5\,°$, find the column for 5 tenths. The digits in that column are 1.842. This gives:
   $$\tan 61.5\,° = 1.842$$
c. To find $\tan 61.51\,°$, identify the columns for 5 tenths and 1 hundredth. These give $1842 + 1 = 1843$.
   Write this as a decimal number: $\tan 61.51\,° = 1.843$

Practice

Find the values of the following using the tangent table:

1. $\tan 7\,°$
2. $\tan 12.9\,°$
3. $\tan 25.05\,°$
4. $\tan 81\,°$
5. $\tan 72.9\,°$
6. $\tan 30.28\,°$
7. $\tan 41.25\,°$
8. $\tan 88.8\,°$
9. $\tan 36\frac{1}{2}\,°$
10. $\tan 42\frac{3}{4}\,°$

Lesson Title: Trigonometric Practice	Theme: Geometry
Practice Activity: PHM-09-064	**Class:** JSS 3

Learning Outcomes
By the end of the lesson, you will be able to:
1. Determine which trigonometric function should be applied to a given problem.
2. Apply the trigonometric functions.

Overview

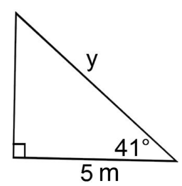
In this lesson, you will determine and apply the correct trigonometric function to a given problem. Now that you know how to read the trig tables, you can answer a wide variety of problems involving sine, cosine and tangent.

In the previous exercises that you did, you were told which ratio to use. Now you have to decide for yourself. Use the rule SOHCAHTOA to decide which ratio to use based on the sides you are given.

After deciding which ratio to use, use the correct table to find the trigonometric ratio for the angle.

Solved Examples

1. Find the length of the side marked y in the diagram below, correct to 1 decimal place.



Solution
You will apply the cosine ratio (CAH) because the adjacent side is given and you want to find the hypotenuse. Find $\cos 41^o$ in the cosine table. It is equal to 0.7547.
Apply the ratio:

$$\cos 41^o = \frac{\text{adjacent}}{\text{hypotenuse}}$$

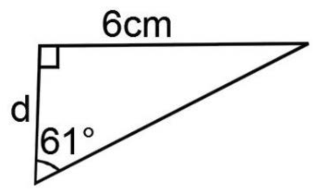
$$\cos 41^o = \frac{5}{y}$$

$$0.7547 \times y = 5$$

$$y = \frac{5}{0.7547}$$

$$y = 6.6 \text{ m}$$

2. Find the length of d in the diagram below, correct to the nearest whole number.



Solution

You will apply the tangent ratio (TOA) because the opposite side is given and you want to find the adjacent side. Find $\tan 61^o$ in the tangent table. It is equal to 1.804.

$$\tan 61^o = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan 61^o = \frac{6}{d}$$

$$d \tan 61° = 6$$

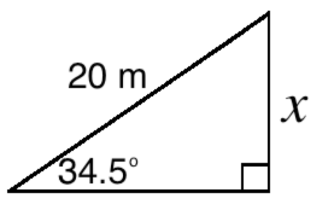
$$d \times 1.804 = 6$$

$$d = \frac{6}{1.804}$$

$$d = 3.3259$$

$$d = 3 \text{ cm}$$

3. Find the length of x in the diagram below, correct to 1 decimal place.



Solution

You will apply the sine ratio (SOH) because the hypotenuse is given and you want to find the opposite side. Find $\sin 34.5^o$ in the sine table. It is equal to 0.5664.

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin 34.5° = \frac{x}{20}$$

$$20 \times \sin 34.5° = x$$

$$20 \times 0.5664 = x$$

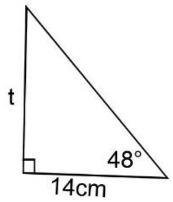
$$x = 11.328$$

$$= 11.3 \text{ m}$$

Practice

Use the correct trigonometric ratios and tables to find the measure of each missing side in the triangles below. Give your answers to 1 decimal place.

1.



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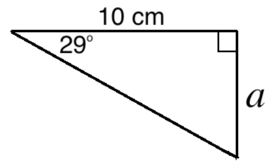

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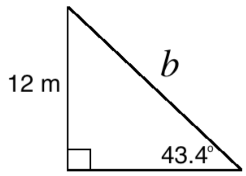

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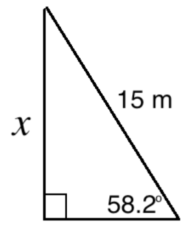

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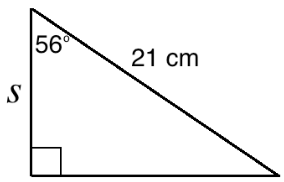

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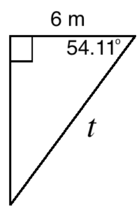


Lesson Title: Trigonometric Word Problems	Theme: Geometry
Practice Activity: PHM-09-065	**Class:** JSS 3

Learning Outcome
By the end of the lesson, you will be able to solve trigonometry word problems with and without diagrams.

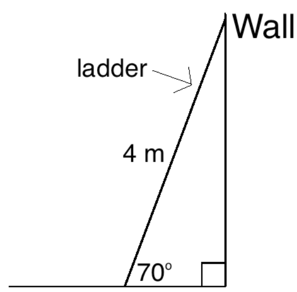
Overview

In this lesson, you will solve trigonometry word problems with and without diagrams. If there is not a diagram for a problem, it is often helpful to draw one of your own.
You will continue using the trigonometry tables to find values.

Solved Examples

1. A ladder of length 4 m leans against a wall. It forms an angle of 70° with the ground.
   a. Draw a diagram for the problem.
   b. How far is the bottom of the ladder from the wall? Give your answer to 1 decimal place.
   c. How far is the top of the ladder from ground level? Give your answer to 1 decimal place.

   Solutions
   a. Draw the diagram as shown:



   b. To find the distance from the bottom of the ladder to the wall, use cosine (CAH). We want to find the length adjacent to 70°, and we know the hypotenuse.

$$\cos 70^{\circ} = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos 70^{\circ} = \frac{a}{4}$$

$$4 \times \cos 70^{\circ} = a$$

$$4 \times 0.3420 = a \qquad \text{Substitute } \cos 70^{\circ} = 0.3420$$

$$1.368 = a$$

The distance from the bottom of the ladder to the wall is 1.4 metres.

c. To find the distance from the ground, use sine (SOH). We want to find the length opposite of 70°, and we know the hypotenuse.

$$
\begin{aligned}
\sin \theta &= \frac{\text{opposite}}{\text{hypotenuse}} \\
\sin 70° &= \frac{h}{4} \\
4 \times \sin 70° &= h \\
4 \times 0.9397 &= h \\
3.7588 &= h
\end{aligned}
$$

The height of the top of the ladder from the ground is 3.8 metres.

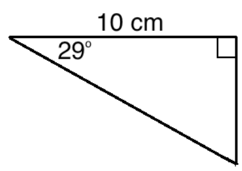
2. If $\cos A = 0.5878$, find the value of A.

Solution

A is an unknown angle. The cosine of A is equal to 0.5878. Look for this number in the cosine table. Numbers in the cosine table are in descending order, so simply follow them until you reach 0.5878.

The angle that corresponds to 0.5878 is 54°. Thus, $A = 54°$.

3. Calculate the area of the triangle shown. Give your answer correct to 1 decimal place.



Solution

Recall the formula for area of a triangle: $A = \frac{1}{2}bh$. Base and height are the 2 perpendicular sides of the triangle. Take base to be 10 cm. We must find the length of the height. Use the tangent ratio (TOA), because we are given a side adjacent to 29, and we want to find the opposite side.

$$
\begin{aligned}
\tan 29^{\text{o}} &= \frac{\text{opposite}}{\text{adjacent}} \\
\tan 29^{\text{o}} &= \frac{h}{10 \text{ cm}} \\
10 \times \tan\ 29° &= h \\
10 \times 0.5543 &= h \\
5.543 &= h
\end{aligned}
$$

Substitute the base and height into the area formula:

$$\begin{aligned} A &= \tfrac{1}{2}bh \\ &= \tfrac{1}{2}(10)(5.543) \\ &= 27.715 \\ &= 27.7 \text{ cm}^2 \end{aligned}$$

The area of the triangle is 27.7 cm^2.

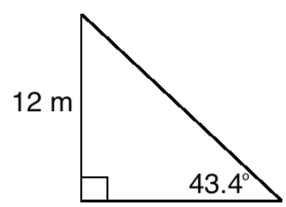
4. Find the value of x if $x = \sin 45° + \tan 45°$.

   Solution
   Use the sine table and tangent table to find the values of the 2 ratios in the problem.
   $\sin 45° = 0.7071$ and $\tan 45° = 1.000$
   Add the values of the 2 ratios to find x:

$$\begin{aligned} x &= \sin 45° + \tan 45° \\ &= 0.7071 + 1.000 \\ &= 1.7071 \end{aligned}$$

Practice

1. Calculate the area of the triangle shown. Give your answer correct to 1 decimal place.



2. Find the value of x if $x = \sin 60° + \cos 30°$.
3. A ladder leans against a wall. It forms an angle of 75° with the ground, and the bottom of the ladder is 3 metres from the wall.
   a. Draw a diagram for the problem.
   b. What is the length of the ladder? Give your answer to 1 decimal place.
   c. How far is the top of the ladder from ground level? Give your answer to 1 decimal place.
4. If $\tan x = 1.072$, find the value of x.
5. If $\sin y = 0.3437$, find the value of y.

Lesson Title: Changing the Subject of a Formula	Theme: Algebra
Practice Activity: PHM-09-066	**Class:** JSS 3

Learning Outcome
By the end of the lesson, you will be able to balance an equation using addition, subtraction, multiplication, and division.

Overview

A formula is an equation where we use letters to represent quantities. It always has at least 2 variables and gives the relationship between the variables. The plural of formula is formulae or formulas.

$C = 2\pi r$ is an example of a formula. It relates the circumference of a circle to its radius. This means that if we know the radius of a circle, we can find its circumference.

C is called the **subject** of the formula. It stands by itself on one side of the equal sign. It can be calculated directly by substituting known values of the radius r. In maths, the subject of a formula or equation is usually written on the left-hand side of the equal sign.

If we know the circumference of a circle and want to find its radius, we change the subject of the formula from C to r. To do this, we rearrange the letters so that r stands by itself on one side of the equal sign. See Solved Example 1 for how to make r the subject of the equation. When changing the subject, it is very important that you **do the same thing to both sides of the equation**.

Do the inverse of the sign or operation to cancel it. These are inverses:

- Adding and subtracting
- Multiplying and dividing
- Square and square root
- Powers and related roots (e.g. $x^3$and $\sqrt[3]{x}$)

Solved Examples

1. Make r the subject of $C = 2\pi r$.

   Solution
   Note that on the right-hand side, r is multiplied by 2π. Divide both sides by 2π to cancel it, and get r by itself.

$$\begin{aligned} C &= 2\pi r && \text{Formula} \\ \frac{C}{2\pi} &= \frac{2\pi r}{2\pi} && \text{Divide throughout by } 2\pi \\ \frac{C}{2\pi} &= r \\ r &= \frac{C}{2\pi} \end{aligned}$$

2. Make x the subject of each formula:

   a. $y = 4x$; b. $y = x + 2$; c. $y = \frac{1}{2}x$; d. $y = \frac{x-1}{2}$

   Solutions

   a. To get x by itself, divide throughout by 4:

$$\begin{aligned} y &= 4x \\ \frac{y}{4} &= \frac{4x}{4} \\ \frac{y}{4} &= x \end{aligned}$$

   b. To solve for x, subtract 2 from both sides:

$$\begin{aligned} y &= x + 2 \\ y - 2 &= x + 2 - 2 \\ y - 2 &= x \end{aligned}$$

   c. To solve for x, multiply both sides by 2:

$$\begin{aligned} y &= \frac{1}{2}x \\ 2y &= 2\left(\frac{1}{2}x\right) \\ 2y &= x \end{aligned}$$

   d. To write the formula with x as the subject, we must take 2 steps:

$$\begin{aligned} y &= \frac{x-1}{2} \\ 2y &= 2\left(\frac{x-1}{2}\right) && \text{Multiply both sides by 2} \\ 2y &= x - 1 \\ 2y + 1 &= x - 1 + 1 && \text{Add 1 to both sides} \\ 2y + 1 &= x \end{aligned}$$

3. Make x the subject of the formula $z = 4x + 4y$.

   Solution

   We must take multiple steps to get x by itself:

$$\begin{aligned} z &= 4x + 4y \\ z - 4y &= 4x + 4y - 4y && \text{Subtract } 4y \text{ from both sides} \\ z - 4y &= 4x \\ \tfrac{z-4y}{4} &= \tfrac{4x}{4} && \text{Divide both sides by 4} \\ \tfrac{z-4y}{4} &= x \end{aligned}$$

4. Make b the subject of the formula $p = \sqrt{a+b}$.

   Solution

$$\begin{aligned} p &= \sqrt{a+b} \\ p^2 &= \left(\sqrt{a+b}\right)^2 && \text{Square both sides} \\ p^2 &= a + b \\ p^2 - a &= b && \text{Subtract } a \text{ from both sides} \end{aligned}$$

5. Solve for x in the formula $y + 3 = x^2 + 1$

   Solution

$$\begin{aligned} y + 3 &= x^2 + 1 \\ y + 3 - 1 &= x^2 && \text{Subtract 1 from both sides} \\ y + 2 &= x^2 \\ \sqrt{y+2} &= \sqrt{x^2} && \text{Take the square root of both sides} \\ \sqrt{y+2} &= x \end{aligned}$$

Practice

1. Make b the subject of each formula:
   a. $a = 13b$; b. $a = b - 3$ c. $a = \frac{1}{4}b$ d. $a = 2b - 8$
2. Solve for r: $p = r^2 + 1$
3. Make t the subject of the formula $x = \sqrt{t+u}$.
4. Make a the subject in the formula $p = 2(a+b)$.
5. Make F the subject of the formula $C = \frac{5}{9}(F - 32)$.

Lesson Title: Combining Like Terms	**Theme:** Algebra
Practice Activity: PHM-09-067	**Class:** JSS 3

Learning Outcome
By the end of the lesson, you will be able to identify and combine like terms.

Overview

In algebra, **like terms** are terms that have the same variable, and the variables have the same power. For example, $2a, 3a, 5a$ are all like terms, with the variable a to the power 1. As another example, $5p^2$ and $8p^2$ are like terms. The numbers can be different, but the rest of the term must be exactly the same.

Like terms can be combined by adding or subtracting to give a single term. The result will have the same variable with the same power. The coefficients of the terms are added or subtracted. For example, consider these combinations:

$$3a + 2a = (3 + 2)a = 5a$$
$$4a - a = (4 - 1)a = 3a$$
$$4a + a - 2a = (4 + 1 - 2)a = 3a$$

Unlike terms are two or more terms that are not like terms. That is, they do not have the same variables or powers. Examples of unlike terms are:

$$2a + 3b + 4c$$
$$3m^2 + 2m^3$$

Unlike terms cannot be combined.

Solved Examples

1. Simplify: $8x - 2y - 4x + 5y$

   Solution

$8x - 2y - 4x + 5y$	$=$	$8x - 4x + 5y - 2y$	Collect like terms
	$=$	$(8 - 4)x + (5 - 2)y$	Combine like terms
	$=$	$4x + 3y$	

2. Simplify: $7p - 3q + 3p + 2q$

Solution

$$\begin{aligned} 7p - 3q + 3p + 2q &= 7p + 3p + 2q - 3q && \text{Collect like terms} \\ &= (7+3)p + (2-3)q && \text{Combine like terms} \\ &= 10p - q \end{aligned}$$

3. Simplify the following algebraic expressions:
   a. $12e + 5f - 4e - 2f$
   b. $2m + 5n - 3m - 4n$
   c. $11x - 10y - 10x + 12y$
   d. $3u - 3 + 4v - 2u + 7 - 2v$

   Solutions

   a.

$$\begin{aligned} 12e + 5f - 4e - 2f &= 12e - 4e + 5f - 2f && \text{Collect like terms} \\ &= (12-4)e + (5-2)f && \text{Combine like terms} \\ &= 8e + 3f \end{aligned}$$

   b.

$$\begin{aligned} 2m + 5n - 3m - 4n &= 2m - 3m + 5n - 4n && \text{Collect like terms} \\ &= (2-3)m + (5-4)n && \text{Combine like terms} \\ &= -m + n \end{aligned}$$

   c.

$$\begin{aligned} 11x - 10y - 10x + 12y &= 11x - 10x + 12y - 10y \\ &= (11-10)x + (12-10)y \\ &= x + 2y \end{aligned}$$

   d.

$$\begin{aligned} 3u - 3 + 4v - 2u + 7 - 2v &= 3u - 2u + 4v - 2v + 7 - 3 \\ &= (3-2)u + (4-2)v + (7-3) \\ &= u + 2v + 4 \end{aligned}$$

4. Simplify the following algebraic expressions:
    a. $pq + 12xp + 5pq - 3xp$
    b. $a + 3ab + 6 + 7a - 2ab - 1$

Solutions

a.

$$\begin{aligned} pq + 12xp + 5pq - 3xp &= pq + 5pq + 12xp - 3xp \\ &= (1+5)pq + (12-3)xp \\ &= 6pq + 9xp \end{aligned}$$

b.

$$\begin{aligned} a + 3ab + 6 + 7a - 2ab - 1 &= a + 7a + 3ab - 2ab + 6 - 1 \\ &= (1+7)a + (3-2)ab + (6-1) \\ &= 8a + ab + 5 \end{aligned}$$

Practice

Simplify the following algebraic expressions:

1. $4y - 3x + 5x - 3y$
2. $9a + 4b - 11a + 3b$
3. $6m + 11n - 4m + 2n - m + n$
4. $2xy - 4xp + 3xy + 3xp$
5. $8mn + 9m + 4mn - 10m$
6. $4pq - 5pq + 8qr + 4qr + 3pq - 4qr$
7. $12p^2q - 4pq^2 + pq^2 - 4p^2q$
8. $3xyz - 4xz + 6xyz + 3xz$
9. $4ab + 7 - 3a - 3 - 2ab + 7a$

Lesson Title: Solving Linear Equations	**Theme:** Algebra
Practice Activity: PHM-09-068	**Class:** JSS 3

Learning Outcome

By the end of the lesson, you will be able to solve linear equations in one variable by balancing the equation and combining like terms.

Overview

Remember that a **variable** is a letter in an algebraic expression, like x or y. It is a letter that represents a number we do not know.

A **linear equation** in one variable is an equation with 1 variable, such as x. The variable may appear more than once. The highest power of the variable in a linear equation is 1. For example, if the variable is squared (x^2), it is not a linear equation.

For example, these are all linear equations:

$$x + 5 = 7 \qquad 8 = 3x - 7 \qquad 7y = 2y + 20 \qquad 4 - 7y = 3y + 4$$

We use the balancing method to solve for the variable in a linear equation. To use the balancing method, apply the same operation to both sides of an equal to sign. Our goal is to get the variable by itself. These are the same steps you took when changing the subject of an equation.

These are possible steps in using the balancing method:

- Add the same quantity to each side.
- Subtract the same quantity from each side.
- Multiply each side by the same quantity.
- Divide each side by the same quantity.

You often need to perform multiple operations to get the answer. When balancing, perform addition or subtraction before multiplication and division. Combine any like terms.

It is good practice to check the answer by substituting the solution into the equation. The right-hand side (RHS) should be equal to the left-hand side (LHS). You will know when you have made a mistake if you do not get RHS = LHS.

Solved Examples

1. Solve for x in the equation $x + 5 = 7$.

Solution

The variable is x. We want to get it by itself by balancing the equation. Subtract 5 from both sides.

$$\begin{aligned} x+5 &= 7 \\ x+5-5 &= 7-5 && \text{Subtract 5 from both sides} \\ x+0 &= 2 \\ x &= 2 \end{aligned}$$

Check your answer by substituting $x = 2$ into the equation:

$$\begin{aligned} x+5 &= 7 \\ (2)+5 &= 7 \\ 7 &= 7 \\ LHS &= RHS \end{aligned}$$

The answer is correct, because the LHS and RHS are equal.

2. Solve $8 = 3x - 7$.

Solution

This problem requires multiple steps. Remember to add/subtract before multiplying/dividing.

$$\begin{aligned} 8 &= 3x-7 \\ 8+7 &= 3x-7+7 && \text{Add 7 to both sides} \\ 15 &= 3x \\ \tfrac{15}{3} &= \tfrac{3x}{3} && \text{Divide both sides by 3} \\ 5 &= x \end{aligned}$$

3. Solve $5y = 40$.

Solution

$$\begin{aligned} 5y &= 40 \\ \tfrac{5y}{5} &= \tfrac{40}{5} && \text{Divide both sides by 5} \\ y &= 8 \end{aligned}$$

4. Solve for x if $5x - 3 = 3x + 7$.

Solution

In this problem, there are 2 terms that contain x. We want to combine them. Get them together on one side of the equation, and solve.

$$\begin{aligned}
5x - 3 &= 3x + 7 \\
5x - 3 - 3x &= 3x + 7 - 3x && \text{Subtract } 3x \text{ from both sides} \\
2x - 3 &= 7 \\
2x - 3 + 3 &= 7 + 3 && \text{Add 3 to both sides} \\
2x &= 10 \\
\frac{2x}{2} &= \frac{10}{2} && \text{Divide both sides by 2} \\
x &= 5
\end{aligned}$$

5. Solve: $7y = 2y + 20$

   Solution

$$\begin{aligned}
7y &= 2y + 20 \\
7y - 2y &= 2y - 2y + 20 && \text{Subtract } 2y \text{ from both sides} \\
5y &= 20 \\
\frac{5y}{5} &= \frac{20}{5} && \text{Divide both sides by 5} \\
y &= 4
\end{aligned}$$

6. Solve: $4 - 7y = 3y + 4$

   Solution

$$\begin{aligned}
4 - 7y &= 3y + 4 \\
4 - 7y - 3y &= 3y - 3y + 4 && \text{Subtract } 3y \text{ from both sides} \\
4 - 10y &= 4 \\
4 - 4 - 10y &= 4 - 4 && \text{Subtract 4 from both sides} \\
-10y &= 0 \\
\frac{-10y}{-10} &= \frac{0}{-10} && \text{Divide both sides by} -10 \\
y &= 0
\end{aligned}$$

Practice

Solve the following equations:

1. $5 + m = 12$
2. $4y = 16$
3. $3n - 15 = 45$
4. $5y - 4 = 11$
5. $x + 4 = 7 - 2x$
6. $0 = 10 - 8y$
7. $3 - 4x = 5x + 12$
8. $1 + 7m = 5m + 1$
9. $5y + 6 = 3y + 20$

Lesson Title: Substitution	Theme: Algebra
Practice Activity: PHM-09-069	**Class:** JSS 3

Learning Outcome
By the end of the lesson, you will be able to find the value of an algebraic expression by substituting values.

Overview

In this lesson, you will substitute given values in an algebraic expression and evaluate the answers.

In substitution, variables are replaced by numbers, and we evaluate the expression to find a final result. Substitution is a useful tool in algebra to find a value or to rewrite equations in terms of a single variable.

In substitution, you will need to observe the following rules:

- When substituting negative numbers put them in brackets () so that you can get the calculation right.
- Two like signs (–)(–) or (+)(+) become a positive sign and two unlike signs (+)(–) or (–)(+) become a negative sign. This follows the rule for multiplying negative numbers.
- Remember to use the correct order of operations (BODMAS).

Solved Examples

1. Find the value of $x - 2$ if $x = 6$.

   Solution
   We can substitute 6 for x. Instead of $x - 2$, we will have $6 - 2$.
   Answer: $x - 2 = 6 - 2 = 4$

2. If $x = 3$ and $y = 4$, what is $x + xy$?

   Solution
   Remember that two variables written together in a term (as in xy) means they are multiplied together.

$x + xy$	$= 3 + (3)(4)$	Substitute $x = 3$ and $y = 4$
	$= 3 + 12$	Multiply
	$= 15$	Add

3. Evaluate the expression $x^2 + xy$ given $x = 3$ and $y = 4$.

Solution

$x^2 + xy$	$= 3^2 + (3)(4)$	Substitute $x = 3$ and $y = 4$
	$= 9 + 12$	Multiply
	$= 21$	Add

4. Evaluate $x^2 + 2x + 1$ given $x = 5$.

Solution

$x^2 + 2x + 1$	$= 5^2 + 2(5) + 1$	Substitute $x = 5$
	$= 25 + 10 + 1$	Multiply
	$= 36$	Add

5. Evaluate $2 - a + a^2$ given $a = -2$

Solution

$2 - a + a^2$	$= 2 - (-2) + (-2)^2$	Substitute $a = -2$
	$= 2 + 2 + 4$	Simplify
	$= 8$	

6. Find the value of $\frac{abc}{ab-bc}$ if $a = 2,\ b = 4$ and $c = -1$.

Solution

$\frac{abc}{ab-bc}$	$= \frac{(2)(4)(-1)}{(2)(4)-(4)(-1)}$	Substitute $a = 2,\ b = 4, c = -1$
	$= \frac{-8}{8+4}$	Multiply
	$= \frac{-8}{12}$	Add denominator
	$= -\frac{2}{3}$	Simplify fraction

Practice

1. If $a = 7$ and $b = 3$, calculate the value of:
   a. $a + b$
   b. $3a - b$
   c. $a^2 + b^2$
2. If $x = 3$ and $y = -1$, calculate the value of:
   a. $x - y$
   b. $x - xy$
   c. $x + y$
   d. $x^2 + y$
   e. $\frac{x}{y}$
3. What is the value of $\frac{str}{st+tr}$ if $s = -6,\ t = 2$ and $r = 3$
4. Evaluate the value of C from the equation $C = \frac{5}{9}(F - 32)$ given
   a. $F = 59$
   b. $F = 95$
5. Given that $p = 1, q = \frac{1}{2}$, find the value of the expression $\frac{p^2-q^2}{(p-q)^2}$
6. Evaluate $\frac{3a^2b}{2b-c}$ given that $a = 4, b = 2$ and $c = -2$.
7. If $m = -3, n = 2$ and $p = -5$, find:
   a. $m^2 + n^2 + p^2$
   b. $\frac{4n+p}{mnp}$

Lesson Title: Practice Solving Algebraic Expressions	**Theme:** Algebra
Practice Activity: PHM-09-070	**Class:** JSS 3

Learning Outcome
By the end of the lesson, you will be able to solve algebraic expressions using various techniques.

Overview

You have learned techniques for solving different types of algebraic expressions in the last few lessons. In this lesson, you will practice solving algebraic expressions using these various techniques. You will collect like terms, substitute values into algebraic expressions, and solve linear equations.

Solved Examples

1. Collect the like terms and simplify:
    a. $2a + 5b + 7c + 2b + c$
    b. $2xy + x - 4y + xy - 2x$
    c. $3x^2 + x + 19 - x^2 + 5x$

    Solutions
    Collect the like terms in each expression. Remember that like terms have exactly the same variables and powers. Add or subtract the coefficients on the like terms.

    a.
    $$\begin{aligned} 2a + 5b + 7c + 2b + c &= 2a + 5b + 2b + 7c + c && \text{Collect like terms} \\ &= 2a + (5+2)b + (7+1)c && \text{Combine} \\ &= 2a + 7b + 8c \end{aligned}$$

    b.
    $$\begin{aligned} 2xy + x - 4y + xy - 2x &= 2xy + xy + x - 2x - 4y && \text{Collect like terms} \\ &= (2+1)xy + (1-2)x - 4y && \text{Combine} \\ &= 3xy - x - 4y \end{aligned}$$

    c.
    $$\begin{aligned} 3x^2 + x + 19 - x^2 + 5x &= 3x^2 - x^2 + x + 5x + 19 && \text{Collect like terms} \\ &= (3-1)x^2 + (1+5)x + 19 && \text{Combine} \\ &= 2x^2 + 6x + 19 \end{aligned}$$

2. Substitute the given values and evaluate the expressions below: $x = 2, y = 3, z = -1$.
    a. $x + y + z$
    b. $2x + y - z^2$
    c. $\frac{x+z}{x-y}$

Solutions

Substitute the given values into each expression and evaluate:

a.

$x + y + z$	$=$	$2 + 3 + (-1)$	Substitute
	$=$	$2 + 3 - 1$	Simplify
	$=$	4	Add/Subtract

b.

$2x + y - z^2$	$=$	$2(2) + 3 - (-1)^2$	Substitute
	$=$	$4 + 3 - 1$	Multiply
	$=$	6	Add/Subtract

c.

$\frac{x+z}{x-y}$	$=$	$\frac{2+(-1)}{2-3}$	Substitute
	$=$	$\frac{2-1}{2-3}$	Simplify
	$=$	$\frac{1}{-1}$	Subtract
	$=$	-1	

3. For the equation $x + 9 = y - 2$, substitute $x = 5$ and solve for y.

Solution

It takes multiple steps to solve this problem. First, substitute the given value for x. Simplify, and solve for y by balancing the equation.

$x + 9$	$=$	$y - 2$	Equation
$5 + 9$	$=$	$y - 2$	Substitute $x = 5$
14	$=$	$y - 2$	Simplify LHS
$14 + 2$	$=$	$y - 2 + 2$	Add 2 to both sides
16	$=$	y	

Answer: $y = 16$

4. Evaluate $xy - (x - y)$ if $x = \frac{1}{2}$ and $y = -2$.

Solution

Substitute the values into the expression and evaluate:

$xy - (x - y)$	$=$	$\left(\frac{1}{2}\right)(-2) - \left(\frac{1}{2} - (-2)\right)$	Substitute
	$=$	$-\left(\frac{2}{2}\right) - \left(\frac{1}{2} + 2\right)$	Simplify
	$=$	$-1 - 2\frac{1}{2}$	Remove Brackets
	$=$	$-3\frac{1}{2}$	Subtract

Practice

1. Collect the like terms and simplify:
    a. $x - z + 5x + 8y - x + z$
    b. $2ab + bc - 4bc + 5ab$
    c. $2x^2 + 4x - x^2 - 5x + 12$
2. If $y = \frac{a^2}{a+bc}$, find the value of y when $a = -6$, $b = 4$ and $c = 3$.
3. For the equation $2x - 4 = y + 5$, substitute $x = 6$ and solve for y.
4. Substitute the given values and evaluate the expressions below: $x = 1, y = -1, z = 4$.
    a. $x + y + z$
    b. $x^2 + y + z^2$
    c. $\frac{x+y}{z}$
5. If $S = ut + \frac{1}{2}at^2$, find S given $a = 30, t = 15$ and $u = 20$.

Lesson Title: Multiplying an Algebraic Expression by an Integer	**Theme:** Algebra
Practice Activity: PHM-09-071	**Class:** JSS 3

Learning Outcome

By the end of the lesson, you will be able to expand an algebraic expression by multiplying an expression by an integer.

Overview

In today's lesson, you will handle algebraic expressions that contain brackets. For example, $2(5x-4)$. In order to simplify such expressions, we must first remove the brackets. In removing brackets, **multiply** the term outside the bracket by each of the terms inside the bracket.

We must be very careful with signs when removing brackets. When there is a positive (+) number before the bracket, the sign inside the brackets does not change when the brackets are removed. When there is a negative number (–) in front of the brackets, the signs inside the bracket change when the brackets are removed. This is because of the rules of multiplication.

Remember the rules for multiplying negative numbers:

- Negative x Positive = Negative
- Positive x Negative = Negative
- Negative x Negative = Positive

Remember that when you multiply a number by a term with a coefficient, the 2 numbers are multiplied. For example: $2\times 5x=10x$.

Solved Examples

1. Simplify: $2(5x-4)$

   Solution

   Multiply 2 by each term inside brackets. The terms inside brackets are $5x$ and -4.

   $$\begin{aligned} 2(5x-4) &= (2\times 5x)+(2\times -4) && \text{Multiply each term by 2} \\ &= 10x-8 \end{aligned}$$

2. Simplify: $-4(2y-3)$

Solution

This is an example of a problem with a negative number in front of the brackets.

$$\begin{aligned} -4(2y-3) &= (-4\times 2y)+(-4\times -3) && \text{Multiply each term by –4} \\ &= -8y+12 \end{aligned}$$

3. Simplify: $-3(2q+2p)$

Solution

$$\begin{aligned} -3(2q+2p) &= (-3\times 2q)+(-3\times 2p) && \text{Multiply each term by } -3 \\ &= -6q-6p \end{aligned}$$

4. Simplify: $-3(2m+3n)$

Solution

$$\begin{aligned} -3(2m+3n) &= (-3\times 2m)+(-3\times 3n) && \text{Multiply each term by } -3 \\ &= -6m-9n \end{aligned}$$

5. Simplify: $-(a+4b)$

Solution

If there is a negative sign in front of the brackets, it is the same as having -1 in front of the bracket. Multiply each term inside brackets by -1. This changes the sign on each term. In other words, the negative sign is distributed to each term inside brackets.

$$\begin{aligned} -(a+4b) &= (-1\times a)+(-1\times 4b) && \text{Multiply each term by } -1 \\ &= -a-4b \end{aligned}$$

Practice

Remove brackets and simplify the following algebraic expressions:

1. $5(x-4)$
2. $-7(3y-4)$
3. $-2(m+n)$
4. $3(2v+3)$
5. $-(2x-4y)$
6. $8(-3m+2n)$
7. $-2(-2a-3)$
8. $10(a-3b)$

Lesson Title: Multiplying Variables	**Theme:** Algebra
Practice Activity: PHM-09-072	**Class:** JSS 3

Learning Outcome
By the end of the lesson, you will be able to multiply 2 monomials with variables, applying the rules of indices.

Overview

A monomial is an expression that consists of only one term. 'Mono' means one. These are examples of monomials:

$$x \qquad y \qquad 3 \qquad x^2 \qquad 3xy \qquad 2x^2y \qquad 7xyz$$

Monomials can be numbers, variables, or a mixture of both.

A number in a monomial that also has a variable is called a **coefficient**. In $3xy$, 3 is the coefficient of the monomial. Monomials cannot have negative or fractional indices or powers. They have no operations like addition or subtraction in them.

In this lesson, you will multiply 2 monomials with variables using the laws of indices. Recall the first law of indices. When 2 indices with the same base are multiplied, their powers are added: $a^m \times a^n = a^{m+n}$.

When 2 monomials are multiplied together, the coefficients are multiplied and the variable parts are multiplied. Only combine variables if they have the same base.

Remember that raising a number or expression to a power is the same as multiplying it by itself. You may also calculate the power of a monomial. Apply the fourth and fifth laws of indices: $(a^m)^n = a^{mn}$ and $(a \times b)^n = a^n \times b^n$. If a monomial is raised to a power, each part of the monomial is raised to that power. Find the power of the coefficient part and the power of the variable part. Evaluate the power on any coefficient. For example, you would change 2^2 to 4.

Solved Examples

1. Multiply: $x^4 \times x^7$

   Solution
   Apply the law of indices. Add the powers: $x^4 \times x^7 = x^{4+7} = x^{11}$

2. Multiply: $2y \times x^7$

   Solution
   The variables are different, so they cannot be combined. Remember when variables are written next to each other, it means they are multiplied. The answer is $2y \times x^7 = 2yx^7$.

3. Multiply: $2y \times 3y^2$

   Solution
   Multiply the coefficient part and the variable part separately.

$$\begin{aligned} 2y \times 3y^2 &= (2 \times 3)(y \times y^2) \\ &= 6y^{1+2} \\ &= 6y^3 \end{aligned}$$

4. Multiply: $4x^2 \times 3xy$

   Solution
   Multiply the coefficient part and the variable part separately. Remember to combine variables only if they are the same.

$$\begin{aligned} 4x^2 \times 3xy &= (4 \times 3)(x^2 \times x)y \\ &= 12x^{2+1}y \\ &= 12x^3y \end{aligned}$$

5. Multiply: $(2xy)^3$

   Solution
   The power 3 is applied to each part of the monomial: 2, x, and y.

$$\begin{aligned} (2xy)^3 &= 2^3x^3y^3 \\ &= 8x^3y^3 \end{aligned}$$

6. Multiply: $4(xy)^2$

   Solution
   In this case, the 4 is outside of bracket for the power. Only the variables are raised to the power. The answer is $4(xy)^2 = 4x^2y^2$.

7. Multiply: $(-3x^4)^2$

   Solution

   Remember to multiply the powers when an index is raised to a power. Also, note that a negative number squared becomes positive: $(-3)^2 = -3 \times -3 = 9$

$$\begin{aligned}(-3x^4)^2 &= (-3)^2(x^4)^2 \\ &= 9x^{4\times 2} \\ &= 9x^8\end{aligned}$$

Practice

Simplify each expression by multiplying:

1. $y^5 \times y^2$
2. $y^3 \times x^2$
3. $2y^5 \times 3xy$
4. $2x^2 \times 5x^3y$
5. $xy \times xy$
6. $xy^2 \times 5x^2y$
7. $(4xy)^3$
8. $(-6xy^3)^2$
9. $5(xy)^2$
10. $9(x^2y^4)^3$

Lesson Title: Multiplying an Algebraic Expression by a Variable	**Theme:** Algebra
Practice Activity: PHM-09-073	**Class:** JSS 3

Learning Outcome
By the end of the lesson, you will be able to expand an algebraic expression by multiplying an expression by a variable.

Overview

In this lesson, you will multiply algebraic expressions by variables. For example, consider $x(x+3)$. The variable x is multiplied by the expression $x+3$. Multiplying a variable by an expression is also called **expansion**.

Use the same method that you used in PHM-09-071, when you multiplied an expression by a number. Multiply each term in brackets by the variable.

Use the techniques you learned in the previous lesson to multiply variables.

Solved Examples

1. Multiply: $x(x+3)$

   Solution
   Multiply x by each term inside brackets. The terms inside brackets are x and 3.

   $$\begin{aligned} x(x+3) &= (x \times x) + (x \times 3) && \text{Multiply each term by } x \\ &= x^2 + 3x \end{aligned}$$

2. Expand: $x(x^2-2)$

   Solution
   In this case, 'expand' means to multiply. Remember to apply the laws of indices to multiply the variables.

   $$\begin{aligned} x(x^2-2) &= (x \times x^2) + (x \times -2) && \text{Multiply each term by } x \\ &= x^{1+2} - 2x && \text{Apply the law of indices} \\ &= x^3 - 2x \end{aligned}$$

3. Multiply: $-x(4-5x)$

Solution

Each term in brackets is multiplied by −x. Remember the rules for multiplying negative values.

$$\begin{aligned} -x(4-5x) &= (-x \times 4) + (-x \times -5x) \quad \text{Multiply each term by } -x \\ &= -4x + 5x^{1+1} \\ &= -4x + 5x^2 \end{aligned}$$

4. Multiply: $x(x+y-4)$

Solution

There are 3 terms in brackets. The variable x is multiplied by each of them.

$$\begin{aligned} x(x+y-4) &= (x \times x) + (x \times y) + (x \times -4) \quad \text{Multiply each term by } x \\ &= x^2 + xy - 4x \end{aligned}$$

5. Expand: $3x(x-4)$

Solution

In this case, the expression is multiplied by a monomial with both a number (3) and a variable (x). $3x$ is multiplied by both terms in brackets. Use the techniques you learned in the previous lesson on multiplying monomials.

$$\begin{aligned} 3x(x-4) &= (3x \times x) + (3x \times -4) \quad \text{Multiply each term by } 3x \\ &= 3x^2 - 12x \end{aligned}$$

6. Expand: $x(x+1) - 2x(3-x)$

Solution

There are 2 sets of brackets in this problem. Multiply each set of brackets separately, then combine any like terms.

$$\begin{aligned} x(x+1) - 2x(3-x) &= (x \times x) + (x \times 1) + (-2x \times 3) + (-2x \times -x) \\ &= x^2 + x - 6x + 2x^2 \\ &= x^2 + 2x^2 + x - 6x \\ &= (1+2)x^2 + (1-6)x \\ &= 3x^2 - 5x \end{aligned}$$

Practice

Expand the following expressions:

1. $x(x-2)$
2. $x(4-x)$
3. $x(2x-1)$
4. $x(x^2-3x)$
5. $-x(x+12)$
6. $-x(9-x+y)$
7. $3x(x-4)$
8. $-2x(4x+1)$
9. $4x(x^2+x)$
10. $x(3-x)+x(2x-4)$

Lesson Title: Algebraic Expression Story Problems	Theme: Algebra
Practice Activity: PHM-09-074	**Class:** JSS 3

Learning Outcome
By the end of the lesson, you will be able to write and simplify algebraic expressions for situations in story problems.

Overview

This lesson is on writing algebraic expressions from story problems. There are many types of word problems which involve relations among known and unknown numbers. These can be written in the form of expressions. The unknown values in story problems are assigned variables, such as x.

You will use maths topics from other lessons, such as finding perimeter and area of plane shapes. Identify the formula, apply operations as you normally would. Instead of applying operations to numbers, you will apply the operations to algebraic expressions.

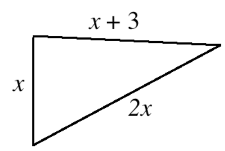
Always simplify your expressions if possible by expanding or combining like terms.

Solved Examples

1. Write an expression for the perimeter of a triangle with sides x, $2x$, and $x + 3$.

   Solution
   Remember that a triangle is a 3-sided shape. We find its perimeter by adding the lengths of its 3 sides.

   It may help to draw a picture. For example:



   Add the lengths of the 3 sides, and simplify:

$$
\begin{aligned}
P &= x + 2x + (x + 3) & \\
&= x + 2x + x + 3 & \text{Add the 3 sides} \\
&= (1 + 2 + 1)x + 3 & \text{Group like terms} \\
&= 4x + 3 & \text{Combine like terms}
\end{aligned}
$$

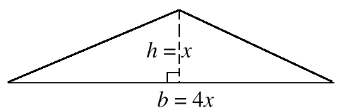
   The expression for the perimeter of this triangle is $P = 4x + 3$.

2. Write an expression for the area of a triangle with base $4x$ and height x.

   Solution

   Remember that the formula for area of a triangle is $A = \frac{1}{2}bh$. We are given the base and height in this problem.

   It may help to draw a picture. A triangle with base $4x$ and height x has a base 4 times longer than its height. For example:



   Substitute the base and height into the formula, and multiply:

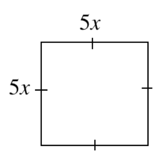
$$\begin{aligned} A &= \tfrac{1}{2}bh && \text{Formula} \\ &= \tfrac{1}{2}(4x)(x) && \text{Substitute } b = 3x \text{ and } h = x \\ &= \tfrac{4}{2}x^2 && \text{Multiply} \\ &= 2x^2 && \text{Simplify} \end{aligned}$$

   The expression is $A = 2x^2$.

3. Write an expression for the area of a square with sides of length $5x$.

   Solution

   Remember that the formula for area of a square is $A = l^2$. We are given the length.
   It may help to draw a picture. For example:



   Substitute the length into the formula, and multiply:

$$\begin{aligned} A &= l^2 && \text{Formula} \\ &= (5x)^2 && \text{Substitute } l = 5x \\ &= 25x^2 && \end{aligned}$$

   The expression is $A = 25x^2$.

4. Write an expression for the combined ages of 4 friends aged x, $x+3$, $x-1$ and $x+2$.

   Solution

   The words "combined age" tell us to add their ages. Add all 4 of the given expressions together:

$$
\begin{aligned}
\text{Combined age} &= x + (x+3) + (x-1) + (x+2) && \text{Add ages} \\
&= x + x + 3 + x - 1 + x + 2 \\
&= x + x + x + x + 3 - 1 + 2 && \text{Group like terms} \\
&= 4x + 4
\end{aligned}
$$

   The expression for their combined age is $4x + 4$.

Practice

1. Write an expression for the area of a rectangle with length $x+3$ and width x.
2. Write an expression for the perimeter of a quadrilateral with sides of length x, $2x$, $x+8$ and $x+2$.
3. Bockarie is 11 years older than Kallon. If Kallon is x years old, write an expression for Bockarie's age.
4. Hawa is twice as old as Musa. If Musa is $x+3$ years old, write an expression for Hawa's age.
5. Three friends measured their weights on a scale. If their weights are x, $x+5$ and $x-7$, write an expression for their combined weight.

Lesson Title: Introduction to Quadratic Equations	**Theme:** Algebra
Practice Activity: PHM-09-075	**Class:** JSS 3

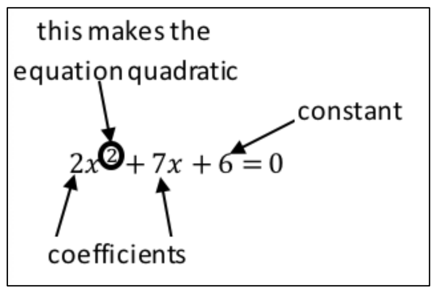
Learning Outcome
By the end of the lesson, you will be able to identify a quadratic equation as one of the form $ax^2 + bx + c = 0$.

Overview

This is the first lesson on quadratic equations. Remember that you have learned linear equations, which are equations with a variable to the power of 1. Quadratic equations are equations with a variable to the power of 2.

A quadratic equation is identified by these main features:

- A variable, usually x.
- The 'highest' power of x in the equation is 2. This x^2 term is what makes it a quadratic equation. Without this term it will not be a quadratic equation.
- It usually has 3 terms, though that is not always the case, it can also have 2 or just 1.
- The numbers 2 and 7 are examples of coefficients of each term. They are used to multiply the variables.



- The 6 is the constant term. It does not change when x changes.

The general form of a quadratic equation is $ax^2 + bx + c = 0$, where x is a variable and a, b and c represent numbers. a cannot be zero as there will be no x^2 term if it is zero. In some cases b and c are zero.

Note that quadratic equations must have an equals sign. If they do not have an equals sign, they may be called "quadratic expressions". For example, $x^2 + 2x + 3$ is not an equation, but is a quadratic expression.

Solved Examples

1. Identify the values of a, b and c in the quadratic equation $2x^2 + 6 = 0$.

   Solution
   $a = 2$, because that is the coefficient on x^2.
   $b = 0$ because there is no x term.
   $c = 6$ because this is the constant.

2. Identify whether each of the following equations is a quadratic equation. If it is a quadratic equation, identify the values of a, b and c.
   a. $x^2 - x + 12 = 0$
   b. $x + 6 = 0$
   c. $2y^2 - 4 = 0$
   d. $y - 6x + 3 = 0$

   Solutions
   a. The equation is quadratic. The values are $a = 1$, $b = -1$, and $c = 12$.
   b. The equation is **not** quadratic. There is no x^2 term.
   c. The equation is quadratic. Quadratic equations usually use the variable x, but that is not always the case. Any variable can be used. $a = 2, b = 0$, and $c = -4$.
   d. The equation is **not** quadratic. There is no x^2 term.

3. Identify whether each of the following equations is a quadratic equation. If it is a quadratic equation, identify the values of a, b and c.
   a. $x(x + 4) = 0$
   b. $x(6 - y) = 0$
   c. $x^2(x + 4) = 0$
   d. $x^2 = 8$

   Solutions
   Quadratic equations may not always be written in the form $ax^2 + bx + c = 0$. The terms may be rearranged. If the equation can be changed so that it takes the form of a quadratic equation, then it is already considered a quadratic equation. For example, if you can expand an expression to put it in the form $ax^2 + bx + c = 0$, it is quadratic.
   a. Expand the left-hand side: $x(x + 4) = x^2 + 4x = 0$
      This is a quadratic equation where $a = 1$, $b = 4$, and $c = 0$.
   b. Expand the left-hand side: $x(6 - y) = 6x - xy = 0$
      This is **not** a quadratic equation because it does not have a variable with a power of 2.
   c. Expand the left-hand side: $x^2(x + 4) = x^3 + 4x^2 = 0$
      This is **not** a quadratic equation because it has a variable with a power higher than 2.
   d. Subtract 8 from both sides:

$$\begin{aligned} x^2 &= 8 \\ x^2 - 8 &= 8 - 9 \\ x^2 - 8 &= 0 \end{aligned}$$

   This is a quadratic equation where $a = 1, b = 0$, and $c = -8$.

Practice

1. Identify whether each of the following equations is a quadratic equation. If it is a quadratic equation, identify the values of a, b and c.
    a. $x^3 - 2x + 15 = 0$
    b. $x - y - 8 = 0$
    c. $12y^2 - 7y + 8 = 0$
    d. $20x + x^2 + 35 = 0$

2. Identify whether each of the following equations is a quadratic equation. If it is a quadratic equation, identify the values of a, b and c.
    a. $5(x^2 + x) = 0$
    b. $x^2 = 8 - x$
    c. $x(x - 1) = 0$
    d. $x^2(y + x) = 0$

Lesson Title: Multiplying 2 Binomials	**Theme:** Algebra
Practice Activity: PHM-09-076	**Class:** JSS 3

Learning Outcomes
By the end of the lesson, you will be able to:
1. Identify the FOIL (First Outside Inside Last) method as a rule for multiplying (expanding) 2 binomials.
2. Multiply 2 binomials.

Overview

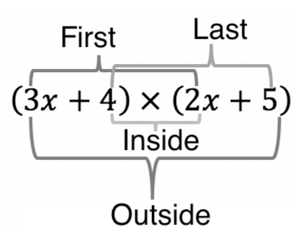
Remember that a monomial is an expression with 1 term. A binomial is an expression with 2 terms. For example, these are binomials:

$$x + 1 \qquad 2x + 5 \qquad 7y - 12 \qquad x^2 - 1 \qquad 3xy + 2x$$

In this lesson, you will multiply 2 binomials. For example: $(3x + 4)(2x + 5)$

To multiply 2 binomials, we need to multiply each term in the first binomial by each term in the second binomial. There is a rule that helps us to do this, called FOIL.

FOIL stands for First, Outside, Inside, Last. We can multiply the terms of the 2 binomials in this order. FOIL gives their positions in the expression:



If we complete the steps of FOIL, we will have multiplied each term in the first binomial by each term in the second.

When we multiply 2 expressions, we also call that “expanding”. When we say to expand an expression or equation, we mean to multiply out the brackets.

After multiplying, always collect like terms together and simplify.

Solved Example

1. Expand the expression $(3x + 4)(2x + 5)$.

   Solution

   Apply the FOIL method. Multiply the First, Outside, Inside and Last terms:

F	Multiply $3x \times 2x$	$=$	$6x^2$
O	Multiply $3x \times 5$	$=$	$15x$
I	Multiply $4 \times 2x$	$=$	$8x$
L	Multiply 4×5	$=$	20

   Add the terms together: $(3x + 4)(2x + 5) = 6x^2 + 15x + 8x + 20$

   Combine like terms: $(3x + 4)(2x + 5) = 6x^2 + 23x + 20$

2. Multiply: $(x + 5)(x + 10)$

   Solution

   Note that it is not necessary to write out the steps of FOIL for each solution. You can multiply the terms of FOIL in your head, and write the 4 terms out together.

   $(x + 5)(x + 10) \quad = x^2 + 10x + 5x + 50$ Apply FOIL

   $= x^2 + 15x + 50$ Combine like terms

3. Expand: $(y - 4)(y - 3)$

   Solution

   $(y - 4)(y - 3) \quad = y^2 - 3y - 4y + 12$ Apply FOIL

   $= y^2 - 7y + 12$ Combine like terms

4. Expand: $(3a + b)(2a - b)$

   Solution

   $(3a + b)(2a - b) \quad = 6a^2 - 3ab + 2ab - b^2$ Apply FOIL

   $= 6a^2 - ab - b^2$ Combine like terms

5. Multiply: $(3mn - 4)(2mn + 3)$

   Solution

   $(3mn - 4)(2mn + 3) \quad = 6m^2n^2 + 9mn - 8mn - 12$ Apply FOIL

   $= 6m^2n^2 + mn - 12$ Combine like terms

Practice

Expand and simplify the following:

1. $(7x + 2)(x + 4)$
2. $(p - 5)(p - 6)$
3. $(y - 5)(y + 5)$
4. $(m + 3)(m - 2)$
5. $(2x - 4)(4x - 2)$
6. $(y + 10)(2y - 12)$
7. $(xy - 2)(2xy + 5)$

Lesson Title: Practice with Multiplying 2 Binomials	**Theme:** Algebra
Practice Activity: PHM-09-077	**Class:** JSS 3

Learning Outcome
By the end of the lesson, you will be able to multiply (expand) 2 binomials to form a quadratic equation.

Overview

In the last lesson, we looked at multiplying 2 binomial expressions. Most of the solutions we got were in the form of a quadratic expression. Remember that quadratic equations have the form $ax^2 + bx + c = 0$. Quadratic equations were the topic of PHM-09-075.

There is a very important relationship between binomials and quadratic equations. You will use this relationship in later lessons to solve quadratic equations.

In this lesson, you will look at more examples of expanding 2 binomials to form a quadratic equation.

Solved Examples

1. Expand: $(x-5)^2 = 0$

   Solution
   Note that the square of a binomial is that binomial multiplied by itself.

   $$\begin{aligned} (x-5)^2 &= (x-5)(x-5) && \text{Rewrite} \\ &= x^2 - 5x - 5x + 25 && \text{Apply FOIL} \\ &= x^2 - 10x + 25 && \text{Combine like terms} \end{aligned}$$

   Answer: $x^2 - 10x + 25 = 0$

2. Expand: $(x+3)(x+1) + (x+4)(x+1)$

   Solution
   Apply the order of operations, BODMAS. The first step is to remove brackets. You will remove each set of brackets separately. Apply FOIL twice. Then, group and combine like terms.

   $$\begin{aligned} (x+3)(x+1) + (x+4)(x+1) &= x^2 + x + 3x + 3 + x^2 + x + 4x + 4 && \text{Apply FOIL} \\ &= x^2 + x^2 + x + 3x + x + 4x + 3 + 4 && \text{Group terms} \\ &= 2x^2 + 9x + 7 && \text{Combine} \end{aligned}$$

   The expression $(x+3)(x+1) + (x+4)(x+1)$ has been expanded and simplified to give the quadratic expression $2x^2 + 9x + 7$.

3. Write $(3x+6)(2x+1)-(x+3)(3x+1)=0$ as a quadratic equation.

Solution

Follow the same steps as the previous example. Make sure to apply the subtraction sign to each term of the second set of binomials

$$\begin{aligned}(3x+6)(2x+1)-(x+3)(3x+1) &= 6x^2+3x+12x+6-(3x^2+x+9x+3)\\ &= 6x^2+3x+12x+6-3x^2-x-9x-3\\ &= 6x^2-3x^2+3x+12x-x-9x+6-3\\ &= 3x^2+5x+3\end{aligned}$$

Answer: $3x^2+5x+3=0$

Practice

Expand and simplify the equations:

1. $(x-4)^2=0$
2. $(n+3)^2=0$
3. $(x+4)(x+5)+(x+2)(x+3)=0$
4. $(y+3)(y-2)+(y+8)(y-4)=0$
5. $(2x+12)(x+4)-(x+1)(x+9)=0$

Lesson Title: Review of Factorisation: Integers	**Theme:** Algebra
Practice Activity: PHM-09-078	**Class:** JSS 3

Learning Outcomes

By the end of the lesson, you will be able to:

1. Identify that factorisation involves using division to break an expression into parts.
2. Identify and factor integers that are common factors in an algebraic expression.

Overview

In this lesson, you will identify that **factorisation** involves using division to break an expression into parts. You will identify and factor integers that are common factors in an algebraic expression.

Factorisation is the **opposite** of expanding a bracket. Remember that we multiply to expand a bracket: $2(x+3) = 2 \times x + 2 \times 3 = 2x + 6$.

In this lesson, you will take an expression like $2x+6$ and use division to find all its factors. This process is called factorisation. You will use factorisation to get from $2x+6$ to $2(x+3)$.

Use these steps to factor an expression:

1. Find the highest common factor (HCF) of the terms in the expression.
2. Write the HCF outside of empty brackets.
3. Divide each term in the expression by the HCF, and write the result in the brackets.

Always check your result to make sure the expression in brackets cannot be factorised further. If you choose a factor that is not the HCF, you may need to factorise more than once to complete the factorisation.

Solved Examples

1. Factorise the expression $2x+6$.

Solution

First, look for the HCF of the expression. It is 2. This is the largest number which can divide $2x$ and 6.

$$\begin{aligned} 2x+6 &= 2(\quad) && \text{Factor the HCF, 2} \\ &= 2(x+3) && \text{Divide each term in } 2x+6 \text{ by 2} \end{aligned}$$

Answer: $2(x+3)$

Check your answer by expanding the brackets:

$$\begin{aligned} 2(x+3) &= 2\times x + 2\times 3 && \text{Multiply each term in brackets by 2} \\ &= 2x+6 && \text{Check that this is the original expression} \end{aligned}$$

2. Factorise the expression $12x - 24$.

Solution

Note that the HCF of the expression is 12.

$$\begin{aligned} 12x-24 &= 12(\quad) && \text{Factor the HCF, 12} \\ &= 12(x-2) && \text{Divide each term in } 12x-24 \text{ by 12} \end{aligned}$$

Answer: $12(x-2)$

Check your answer by expanding the brackets:

$$\begin{aligned} 12(x-2) &= 12\times x + (12\times -2) && \text{Multiply each term in brackets by 12} \\ &= 12x-24 && \text{Original expression} \end{aligned}$$

3. Factorise $15 - 25y$.

Solution

Note that the HCF of the expression is 5.

$$\begin{aligned} 15-25y &= 5(\quad) && \text{Factor the HCF, 5} \\ &= 5(3-5y) && \text{Divide each term in } 15-25y \text{ by 5} \end{aligned}$$

Answer: $5(3-5y)$

Check your answer by expanding the brackets:

$$\begin{aligned} 5(3-5y) &= 5\times 3 + (5\times -5y) && \text{Multiply each term in brackets by 5} \\ &= 15-25y && \text{Original expression} \end{aligned}$$

4. Factorise $10 + 3y - 2 + y$.

Solution

Note that there is no HCF for the 4 terms of this expression. However, there are like terms in the expression. Combine the like terms first, then try to factorise the expression.

$$\begin{aligned} 10+3y-2+y &= 10-2+3y+y && \text{Collect like terms} \\ &= 8+4y && \text{Combine like terms} \\ &= 4(\quad) && \text{Factor the HCF, 4} \\ &= 4(2+y) && \text{Divide each term in } 8+4y \text{ by 4} \end{aligned}$$

Check your answer by expanding the brackets:

$$\begin{aligned} 4(2+y) &= 4 \times 2 + (4 \times y) && \text{Multiply each term in brackets by 4} \\ &= 8 + 4y && \text{Original expression} \end{aligned}$$

5. Factorise $5x^3 + 15x^2 + 35x + 20$.

Solution

Note that the HCF of the expression is 5. This expression has higher powers of x. However, the factorisation process is the same. Divide each term by the HCF (5). The result will have the same powers of x.

$$\begin{aligned} 5x^3 + 15x^2 + 35x + 20 &= 5(\qquad) && \text{Factor the HCF, 5} \\ &= 5(x^3 + 3x^2 + 7x + 4) && \text{Divide each term by 5} \end{aligned}$$

Check your answer by expanding the brackets:

$$5(x^3 + 3x^2 + 7x + 4) = 5x^3 + 15x^2 + 35x + 20$$

Practice

Factorise the expressions below. Please check all answers.

1. $4x + 12$
2. $7x - 21y$
3. $14 - 2x$
4. $20x + 30$
5. $4y - 6$
6. $10s + 12t - 4t$
7. $9 - 18p + 3$
8. $3x^2 + 12x + 30$
9. $9x^2 - 12$
10. $2x^3 + 40x^2 + 12x + 24$

Lesson Title: Review of Factorisation: Variables	Theme: Algebra
Practice Activity: PHM-09-079	**Class:** JSS 3

Learning Outcome
By the end of the lesson, you will be able to identify and factor variables that are common factors in an algebraic expression.

Overview

In the previous lesson, you factorised numbers from algebraic expressions. In this lesson, you will factorise variables.

When you factorise a variable, divide each term in the expression by that variable. You will apply the second law of indices. This says that when dividing indices, subtract their powers: $a^m \div a^n = a^{m-n}$.

If 2 terms contain the same variable, then that variable is a common factor. Any common factor can be factorised out of the expression.

For example, consider $x^2 + 2x$. The variable x is a common factor because it is in both terms. Write x outside of brackets, and divide each term by x: $x^2 + 2x = x(x + 2)$

It is not necessary to show the division each time you factorise an expression, but the division of each term of this example is shown below:

$\frac{x^2}{x} = x^{2-1} = x^1 = x$
$\frac{2x}{x} = 2$, x cancels

In some cases, you can factor both an integer and a variable. Divide the expression by both of them at the same time. For example, consider $10x^2 + 20x$. The common factors are 10 and x. The HCF of this expression is found by multiplying the common factors. The HCF is $10x$. When you factorise the expression you will divide by the HCF:
$10x^2 + 20x = 10x(x + 2)$.

Solved Examples

1. Factorise $x^2 + 5x$.

   Solution

   Note that the HCF of the expression is x.

$x^2 + 5x$	$=$	$x(\quad)$	Factor the HCF, x
	$=$	$x(x + 5)$	Divide each term in $x^2 + 5x$ by x

   Answer: $x(x + 5)$

   Check your answer by expanding the brackets:

$x(x + 5)$	$=$	$x \times x + (x \times 5)$	Multiply each term by x
	$=$	$x^2 + 5x$	Original expression

2. Factorise $3x^3 + 2x^2 + x$.

   Solution

   Note that the HCF of the expression is x.

$3x^3 + 2x^2 + x$	$=$	$x(\quad)$	Factor the HCF, x
	$=$	$x(3x^2 + 2x + 1)$	Divide each term by x

   Answer: $x(3x^2 + 2x + 1)$

   Check your answer by expanding the brackets:

   $$\begin{aligned} x(3x^2 + 2x + 1) &= (x \times 3x^2) + (x \times 2x) + (x \times 1) \\ &= 3x^3 + 2x^2 + x \end{aligned}$$

3. Factorise $12x^2 - 6x$.

   Solution

   Note that there are 2 common factors in the expression, 6 and x. This means the HCF is $6x$. Bring $6x$ outside of brackets.

$12x^2 - 6x$	$=$	$6x(\quad)$	Factor the HCF, $6x$
	$=$	$6x(2x - 1)$	Divide each term by $6x$

   Check your answer by expanding the brackets:

   $$\begin{aligned} 6x(2x - 1) &= (6x \times 2x) + (6x \times -1) \\ &= 12x^2 - 6x \end{aligned}$$

4. Factorise $a^2 + a + 7a + 3a^2$

Solution

Note that there are like terms in this expression. Remember to combine like terms before factorising the expression.

$a^2 + a + 7a + 3a^2$	$= a^2 + 3a^2 + a + 7a$	Collect like terms
	$= 4a^2 + 8a$	Combine like terms
	$= 4a(\quad)$	Factor the HCF, $4a$
	$= 4a(a+2)$	Divide each term by $4a$

Check your answer by expanding the brackets:

$$\begin{aligned} 4a(a+2) &= (4a \times a) + (4a \times 2) \\ &= 4a^2 + 8a \end{aligned}$$

Practice

Factorise the expressions below. Please check all answers.

1. $xy + y$
2. $xy + yz$
3. $2a^2 - a$
4. $3x^2 + 8x$
5. $y^3 + y^2$
6. $x^3 + 7x^2 - 3x$
7. $3x^3 + 9x^2 - 18x$
8. $5x^3 - 15x^2$
9. $9a^2 + 13a - 3a - 4a^2$
10. $5x^3 + 12x^2 + 9x^3 - 5x^2$

Lesson Title: Factorisation of Quadratic Equations	Theme: Algebra
Practice Activity: PHM-09-080	**Class:** JSS 3

Learning Outcome
By the end of the lesson, you will be able to identify the factorisation method of factoring a quadratic equation into 2 binomials.

Overview

Remember that you have multiplied 2 binomials to make a quadratic equation. For example, $(x+3)(x+5) = x^2+8x+15$. In this lesson, you will do the opposite. You will factorise a quadratic equation into 2 binomials. In other words, you will be given $x^2+8x+15$ and you will find the answer $(x+3)(x+5)$.

Remember that a quadratic expression has the general form ax^2+bx+c where a, b and c are integers and x is a variable.

The general rule for factorising a quadratic expression is $ax^2+bx+c = (x+p)(x+q)$ where $p+q=b$ and $p\times q = c$ when $a=1$.

To factorise a quadratic expression, we can split the middle term so that it is 2 terms. This gives an expression with 4 terms, which can be factored using the same process we have used before. The middle term should be split to px and qx, where p and q sum to give b of the middle term and multiply to give c of the last term (as in the general rule above).

Solved Examples

1. Factorise $y^2-7y+12$

   Solution
   p and q can be added to get $b=-7$. They can be multiplied to get $c=12$.

   To find p and q, first note the factors of 12. They may both be positive, or both be negative: 1×12, 3×4, 2×6, -1×-12, -3×-4, -2×-6

   Note that the 2 factors p and q must sum to -7. The numbers are -3 and -4.

$y^2-7y+12$	$=$	$y^2-3y-4y+12$	Split the middle term
	$=$	$y(y-3)-4y+12$	Factorise the first two terms
	$=$	$y(y-3)-4(y-3)$	Factorise the last two terms
	$=$	$(y-3)(y-4)$	Factorise the common factor of $x-3$

   Therefore, $y^2-7y+12 = (y-3)(y-4)$.

2. Factorise $x^2 + 6x + 8$

Solution

Factors of 8	$= 1, 2, 4, 8$
Product of factors	$c = 2 \times 4 = 8$
Sum of factors	$b = 2 + 4 = 6$

$x^2 + 6x + 8$	$=$	$x^2 + 2x + 4x + 8$	Split the middle term.
	$=$	$x(x+2) + 4(x+2)$	Factorise the first 2 and last 2 terms
	$=$	$(x+2)(x+4)$	Factorise the common factor of $x + 2$

3. Factorise $m^2 + 8m - 33$

Solution

When you encounter large numbers or a mix of positive and negative numbers, the values of p and q will not be obvious. Start by writing the factors of c: -33: -1×33, 1×-33, -3×11, 3×-11

Note that $c = -3 \times 11 = -33$ and $b = -3 + 11 = +8$.
Use $p = -3$ and $q = 11$ to factorise:

$$\begin{aligned} m^2 + 8m - 33 &= m^2 - 3m + 11m - 33 \\ &= m(m-3) + 11(m-3) \\ &= (m+11)(m-3) \end{aligned}$$

4. Factorise $x^2 - 6x - 27$

Solution

Factors of 27	$= 1, 2, 3, 9, 27$
Product of factors	$c = 3 \times (-9) = -27$
Sum of factors	$b = 3 - 9 = -6$
$x^2 - 6x - 27$	$= x^2 + 3x - 9x - 27$
	$= x(x+3) - 9(x+3)$
	$= (x+3)(x-9)$

5. One of the factors of the quadratic expression $y^2 - 14y + 45$ is $(y - 5)$. Find the other factor.

Solution

You are given the quadratic expression, so you may factor it as usual. You should find that one factor is $(y - 5)$, which is given in the problem. The other factor will be your answer.

Product of factors:	$c = (-9) \times (-5) = +45$
Sum of factors	$b = -9 - 5 = -14$

Factorise:

$$y^2 - 14y + 45 \quad = y^2 - 9y - 5y + 45$$
$$= y(y-9) - 5(y-9)$$
$$= (y-5)(y-9)$$

Hence the other factor is $(y-9)$.

Practice

1. Factorise the following:
    a. $x^2 + 9x + 20$
    b. $P^2 - 11p + 24$
    c. $y^2 - 10y + 25$
    d. $n^2 + n - 6$

2. One of the factors of $x^2 - 10x + 21$ is $(x-3)$. Find the other factor.

3. One of the factors of $u^2 - 5u - 6$ is $(u-6)$ find the other factor.

Lesson Title: Practice with Factorisation of Quadratic Equations	**Theme:** Algebra
Practice Activity: PHM-09-081	**Class:** JSS 3

Learning Outcome
By the end of the lesson, you will be able to identify the factorisation method to factor a quadratic equation into 2 binomials.

Overview

In this lesson, you will practice the skills you learned in the previous lesson to solve problems.

You will solve problems where $a > 1$ in the quadratic equation. In other words, the coefficient on x^2 is greater than 1. For example, you will factorise $2x^2 + 11x + 12$.

Remember that when $a = 1$ in the quadratic expression $ax^2 + bx + c$, we find 2 numbers p and q such that $p + q = b$ and $p \times q = c$.

When $a > 1$, we follow a similar process. We find 2 numbers p and q such that $p + q = b$ and $p \times q = \boldsymbol{ac}$**.** Note that a is multiplied by c to find $p \times q$.

Solved Examples

1. Factorise $2x^2 + 11x + 12$.

   Solution
   Using the values of a, b and c in the expression, we have $p + q = b = 11$ and $p \times q = a \times c = 2 \times 12 = 24$.

   To find p and q, we want to find the factors of 24 that sum to 11. We can now follow the process that we used in the previous lesson.

   Factors of 24 $= 1, 2, 3, 4, 6, 8, 12, 24$

   Product of factors $= 3 \times 8 = 24$

   Sum of factors $= 3 + 8 = 11$

   The 2 numbers p and q we are looking for are 3 and 8.

$$\begin{aligned} 2x^2 + 11x + 12 &= 2x^2 + 8x + 3x + 12 && \text{Split the middle term.} \\ &= 2x(x + 4) + 3(x + 4) && \text{Factorise the first 2 and last 2 terms} \\ &= (x + 4)(2x + 3) && \text{Factorise the common factor of } x + 4 \end{aligned}$$

You can always check your answer by multiplying the 2 binomials together:

$$
\begin{aligned}
(x+4)(2x+3) &= 2x^2+3x+8x+12 && \text{Multiply} \\
&= 2x^2+11x+12 && \text{Combine like terms}
\end{aligned}
$$

The answer is $(x+4)(2x+3)$.

2. Factorise $6x^2+5x-6$.

   Solution
   Using the values of a, b and c in the expression, we have $p+q=b=5$ and $p \times q = a \times c = 6 \times -6 = -36$.

   To find p and q, we want to find the factors of -36 that sum to 5. We can now follow the process that we used in the previous lesson. Note that for p and q, one of them must be negative and the other positive to give -36 and 5.

Factors of -36	$= (\pm)1, 2, 3, 4, 6, 9, 12, 18, 36$
Product of factors	$= -4 \times 9 = -36$
Sum of factors	$= -4 + 9 = 5$

   The 2 numbers p and q we are looking for are -4 and 9.

$$
\begin{aligned}
6x^2+5x-6 &= 6x^2+9x-4x-6 && \text{Split the middle term} \\
&= 3x(2x+3)-2(2x+3) && \text{Factorise the first 2 and last 2 terms} \\
&= (2x+3)(3x-2) && \text{Factorise the common factor of } 2x+3
\end{aligned}
$$

   The answer is $(2x+3)(3x-2)$.

3. Factorise x^2-4.

   Solution
   Using the values of a, b and c in the expression, we have $p+q=b=0$ and $p \times q = a \times c = 1 \times -4 = -4$.

   To find p and q, we want to find the factors of -4 that sum to 0. We can now follow the process that we used in the previous lesson. Note that for p and q, one of them must be negative and the other positive to give -4.

Factors of -36	$= (\pm)1, 2, 4$
Product of factors	$= -2 \times 2 = -4$
Sum of factors	$= -2 + 2 = 0$

   The 2 numbers p and q we are looking for are -2 and 2.

$x^2 - 4$	$=$	$x^2 + 2x - 2x - 4$	Split the middle term.
	$=$	$x(x+2) - 2(x+2)$	Factorise the first 2 and last 2 terms
	$=$	$(x+2)(x-2)$	Factorise the common factor of $2x+3$

The answer is $(x+2)(x-2)$.

You will notice a pattern. You will see problems where $b = 0$, and c is a negative perfect square. In such cases, p and q are the positive and negative square root of positive c. See Practice problems 7 and 8 for more examples.

Practice

Factorise the following quadratic expressions:

1. $x^2 + 11x + 18$
2. $2x^2 - x - 3$
3. $x^2 + 6x + 5$
4. $x^2 - 12x + 11$
5. $2x^2 + 7x + 3$
6. $x^2 + 9x - 22$
7. $x^2 - 1$
8. $y^2 - 36$
9. $1 - x - 2x^2$
10. $2 - x - 6x^2$

Lesson Title: Factorisation by Completing the Squares Method	**Theme:** Algebra
Practice Activity: PHM-09-082	**Class:** JSS 3

Learning Outcome
By the end of the lesson, you will be able to identify the 'completing the squares' method of factoring a quadratic equation into 2 binomials.

Overview

Consider the expression $x^2 + 4x + 2$. We cannot use the method we learned in the previous two lessons to factorise this quadratic expression. We have to use a process called **completing the square** to solve this problem.

We start the lesson by expanding expressions such as $(x + 2)^2$. Such expressions are called perfect squares because they are squares of binominals. They are easy to expand and easy to factorise back into 2 binomial factors.

Completing the square changes a quadratic expression to the sum of a perfect square and a number.

Compare $x^2 + 4x + 2$ with the perfect square $(x + 2)^2 = x^2 + 4x + 4$. There is a difference of 2. Thus, we can rewrite $x^2 + 4x + 2$ using the perfect square:

$$\begin{aligned} x^2 + 4x + 2 &= x^2 + 4x + 4 - 2 \\ &= (x + 2)^2 - 2 \end{aligned}$$

Generally, every imperfect square in the form $ax^2 + bx + c$ can be written as $ax^2 + bx + c = (x + p)^2 + q$ where p and q are constants. Complete the square by finding p and q from the quadratic equation, and substituting them into the formula $(x + p)^2 + q$.

Find the value of p using the formula $p = \frac{b}{2}$. Then, find the value of q using the formula $q = c - p^2$.

Solved Examples

1. Factorise $x^2 + 4x + 3$ by completing the square.

   Solution
   Note that in this equation, $a = 1, b = 4$ and $c = 3$. $x^2 + 4x + 3$ is an imperfect square, so we will write in the form: $x^2 + 4x + 3 = (x + p)^2 + q$

Use the formulae to find p and q. Calculate p:

$$\begin{aligned} p &= \tfrac{b}{2} \\ &= \tfrac{4}{2} \\ &= 2 \end{aligned}$$

Calculate q:

$$\begin{aligned} q &= c - p^2 \\ &= 3 - 2^2 \\ &= 3 - 4 \\ &= -1 \end{aligned}$$

Substitute $p = 2$ and $q = -1$ into the formula, $(x + p)^2 + q$.

$$\begin{aligned} x^2 + 4x + 3 &= (x + p)^2 + q \\ &= (x + 2)^2 - 1 \end{aligned}$$

Answer: The factorisation of $x^2 + 4x + 3$ is $(x + 2)^2 - 1$.

2. Complete the square by writing the quadratic expression $x^2 + 6x + 2$ in the form $(x + p)^2 + q$. Use the equations $p = \frac{b}{2}$ and $q = c - p^2$.

   Solution

   Note that in this equation, $a = 1, b = 6$ and $c = 2$. $x^2 + 6x + 2$ is an imperfect square, so we will write in the form: $x^2 + 6x + 2 = (x + p)^2 + q$

   Use the formulae to find p and q. Calculate p:

$$\begin{aligned} p &= \tfrac{b}{2} \\ &= \tfrac{6}{2} \\ &= 3 \end{aligned}$$

Calculate q:

$$\begin{aligned} q &= c - p^2 \\ &= 2 - 3^2 \\ &= 2 - 9 \\ &= -7 \end{aligned}$$

Substitute $p = 3$ and $q = -7$ into the formula, $(x + p)^2 + q$.

$$\begin{aligned} x^2 + 6x + 2 &= (x + p)^2 + q \\ &= (x + 3)^2 - 7 \end{aligned}$$

Answer: The factorisation of $x^2 + 6x + 2$ is $(x + 3)^2 - 7$.

Practice

1. Factorise $x^2 - 8x + 2$ using completing the square method.
2. Factorise the quadratic equation by completing the square: $x^2 - 10x + 7$.
3. Write $x^2 + 4x - 5$ in the form $(x + p)^2 + q$ by completing the square.
4. Factorise $x^2 + 6x + 6$ by completing the square.
5. Factorise $x^2 - 4x + 5$ by completing the square.

Lesson Title: Practice with Completing the Squares Method	**Theme:** Algebra
Practice Activity: PHM-09-083	**Class:** JSS 3

Learning Outcome
By the end of the lesson, you will be able apply the factorisation method to factor a quadratic equation into 2 binomials.

Overview

This is the second lesson on factoring a quadratic expression using the completing the square method. You will continue to practice what you learned in the previous lesson.

When you are asked to factorise a quadratic equation, you will first try the factoring method you have learned in PHM-09-080 and PHM-09-081. When an expression cannot be factorised easily, you can factorise it by completing the square.

Solved Examples

1. Factorise $x - 2x + 9$ by completing the square. Check your answer.

   Solution
   Note that in this equation, $a = 1, b = -2$ and $c = 9$. $x - 2x + 9$ is an imperfect square, so we will write in the form $x - 2x + 9 = (x + p)^2 + q$

   Use the formulae to find p and q. Calculate p:

$$\begin{aligned} p &= \tfrac{b}{2} \\ &= \tfrac{-2}{2} \\ &= -1 \end{aligned}$$

   Calculate q:

$$\begin{aligned} q &= c - p^2 \\ &= 9 - (-1)^2 \\ &= 9 - 1 \\ &= 8 \end{aligned}$$

   Substitute $p = -1$ and $q = 8$ into the formula, $(x + p)^2 + q$.

$$\begin{aligned} x - 2x + 9 &= (x + p)^2 + q \\ &= (x - 1)^2 + 8 \end{aligned}$$

   Answer: The factorisation of $x - 2x + 9$ is $(x - 1)^2 + 8$.

Check: remember that you can always check your factorisation by expanding, or multiplying, your answer.

$$\begin{aligned}(x-1)^2+8 &= (x-1)(x-1)+8\\ &= x^2-x-x+1+8\\ &= x^2-2x+9\end{aligned}$$

The answer is correct because when expanded it gives the original expression.

2. Complete the square by writing the quadratic expression $x^2+8x+10$ in the form $(x+p)^2+q$. Check your answer.

Solution
Note that in this equation, $a=1, b=8$ and $c=10$. $x^2+8x+10$ is an imperfect square, so we will write in the form $x^2+8x+10=(x+p)^2+q$

Use the formulae to find p and q. Calculate p:

$$\begin{aligned}p &= \tfrac{b}{2}\\ &= \tfrac{8}{2}\\ &= 4\end{aligned}$$

Calculate q:

$$\begin{aligned}q &= c-p^2\\ &= 10-4^2\\ &= 10-16\\ &= -6\end{aligned}$$

Substitute $p=4$ and $q=-6$ into the formula, $(x+p)^2+q$.

$$\begin{aligned}x^2+8x+10 &= (x+p)^2+q\\ &= (x+4)^2-6\end{aligned}$$

Answer: The factorisation of $x^2+8x+10$ is $(x+4)^2-6$.

Check: remember that you can always check your factorisation by expanding, or multiplying, your answer.

$$\begin{aligned}(x+4)^2-6 &= (x+4)(x+4)-6\\ &= x^2+4x+4x+16-6\\ &= x^2+8x+10\end{aligned}$$

The answer is correct because when expanded it gives the original expression.

Practice

Factorise each expression by completing the square. Check your answers.

1. $x^2 + 6x + 1$
2. $x^2 - 2x - 5$
3. $x^2 + 2x - 1$
4. $x^2 + 10x + 12$
5. $x^2 - 6x + 4$
6. $x^2 + 10x + 25$
7. $x^2 - 12x + 36$

Lesson Title: Practice with Factorisation	**Theme:** Algebra
Practice Activity: PHM-09-084	**Class:** JSS 3

Learning Outcome
By the end of the lesson, you will be able to identify and apply the best method to factor a given algebraic expression, including quadratic expressions.

Overview

During the previous 6 lessons, you have factorised expressions using different methods. In this lesson, you will identify and apply the best method to factor a given algebraic expression, including quadratic expressions.

There are solved examples for each type of factorization that you have learned. Look at the previous lessons if you need to revise these.

Solved Examples

1. Factorise $4x - 20$.

   Solution
   The terms of the expression have a common factor of 4. Factorise 4 from the expression:
   $4x - 20 = 4(x - 5)$

2. Factorise $x^2 - 2x$.

   Solution
   The terms of the expression have a common factor of x. Factorise x from the expression:
   $x^2 - 2x = x(x - 2)$

3. Factorise $x^2 - 4x + 3$.

   Solution
   Check for p and q that can be added to get $b = -4$ and multiplied to get $c = 3$.

   To find p and q, first note the factors of 3. They may both be positive, or both be negative: 1×3, -1×-3

   Note that the 2 factors p and q must sum to -4. The numbers are -1 and -3.

$x^2 - 4x + 3$	$=$	$x^2 - x - 3x + 3$	Split the middle term
	$=$	$x(x - 1) - 3(x - 1)$	Factorise 2 sets of terms
	$=$	$(x - 1)(x - 3)$	Factorise the common factor of $x - 1$

   Therefore $x^2 - 4x + 3 = (x - 1)(x - 3)$.

4. Factorise $x^2 - 2x + 8$.

Solution

There are no terms p and q that can be added to get $b = -2$ and multiplied to get $c = 8$. We must use the completing the squares method.

$x^2 - 2x + 8$ is an imperfect square, so we will write in the form $x^2 + 8x + 10 = (x + p)^2 + q$

Use the formulae to find p and q. Calculate p:

$$\begin{aligned} p &= \frac{b}{2} \\ &= \frac{-2}{2} \\ &= -1 \end{aligned}$$

Calculate q:

$$\begin{aligned} q &= c - p^2 \\ &= 8 - (-1)^2 \\ &= 8 - 1 \\ &= 7 \end{aligned}$$

Substitute $p = -1$ and $q = 7$ into the formula, $(x + p)^2 + q$.

$$\begin{aligned} x^2 - 2x + 8 &= (x + p)^2 + q \\ &= (x - 1)^2 + 7 \end{aligned}$$

Answer: The factorisation of $x^2 - 2x + 8$ is $(x - 1)^2 + 7$.

Practice

Factorise each expression using the best method:

1. $3x + 6y - 12$
2. $2y^2 + 10y$
3. $x^2 + 7x + 12$
4. $2x^2 - x - 1$
5. $12x - 18$
6. $x^2 - 2x + 12$
7. $x^2 + 3x - 18$
8. $3x^2 - 9x$
9. $2 - 9x - 5x^2$
10. $4 - y^2$

Lesson Title: Story Problems with Quadratic Expressions	**Theme:** Algebra
Practice Activity: PHM-09-085	**Class:** JSS 3

Learning Outcome
By the end of the lesson, you will be able to write quadratic expressions for situations in story problems.

Overview

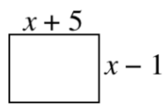
In this lesson, you will write quadratic expressions for situations in story problems.

Solved Examples

1. A rectangle has length $x + 5$ and width $x - 1$. Write a quadratic expression for its area.

   Solution
   Recall that the formula for area of a rectangle is $A = l \times w$. We can find the quadratic expression for its area by multiplying the expressions that we are given for length and width.

   It can be helpful to draw a picture:

   $$\begin{aligned} A &= l \times w \\ &= (x+5)(x-1) \\ &= x^2 - x + 5x - 5 \\ &= x^2 + 4x - 5 \end{aligned}$$

   The area of the rectangle is $A = x^2 + 4x - 5$

2. Two consecutive integers in a pattern are denoted x and $x + 3$. Find an expression for their product.

   Solution
   The product is found by simply multiplying the two expressions: $x(x + 3) = x^2 + 3x$.

3. Find an expression for the combined age of 5 children aged $x + 3$, x^2, $x - 1$, $x^2 + 2$, and $x + 2$.

   Solution
   To 'combine' means to add. Add the 5 expressions together, then simplify:

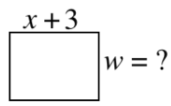
   $$\begin{aligned} x + 3 + x^2 + x - 1 + x^2 + 2 + x + 2 &= x^2 + x^2 + x + x + x + 3 - 1 + 2 + 2 \\ &= 2x^2 + 3x + 6 \end{aligned}$$

4. The area of a rectangle is given by the quadratic expression $x^2 + x - 6$. If the length of the rectangle is $x + 3$, what is its width?

   Solution

   Recall that the formula for the area of a rectangle is $A = l \times w$. If we have a quadratic expression for its area, we can factorise the expression to find the length and width. If we factorise $x^2 + x - 6$, we should find that one binomial factor is the given length, $x + 3$. The other factor will be the width.

   It can be helpful to draw a picture:



   Check for p and q that can be added to get $b = 1$ and multiplied to get $c = -6$.

   To find p and q, first note the factors of -6. One must be positive, and the other negative: -1×6, 1×-6, -2×3, 2×-3

   Note that the 2 factors p and q must sum to 1. The numbers are -2 and 3.

$x^2 + x - 6$	$=$	$x^2 - \mathbf{2x} + \mathbf{3x} - 6$	Split the middle term
	$=$	$x(x - 2) + 3(x - 2)$	Factorise 2 sets of terms
	$=$	$(x - 2)(x + 3)$	Factorise the common factor of $x - 2$

   Therefore $x^2 + x - 6 = (x - 2)(x + 3)$.

   The width of the rectangle is $x - 2$.

Practice

1. Find a quadratic expression for the product of 2 numbers denoted $x + 7$ and $x - 3$.
2. The area of a rectangle is given by the quadratic expression $x^2 + 5x + 6$. Find the length and width of the rectangle in terms of x.
3. Find an expression for the combined age of 4 children aged $2x - 1$, $x + 4$, $x^2 - 3$ and $2x$.
4. The area of a square is given by the quadratic expression $x^2 + 8x + 16$. Find the length of its sides.
5. The product of two numbers in terms of x is $x^2 + 7x + 12$. If one of them is $(x + 3)$, find the other.
6. The area of a rectangular field is $2x^2 + 11x - 6$. If one side has length $x + 6$, what is the length of the other side?

Lesson Title: Introduction to Linear Equations in 2 Variables	**Theme:** Algebra
Practice Activity: PHM-09-086	**Class:** JSS 3

Learning Outcome
By the end of the lesson, you will be able to identify a simple linear equation in 2 variables and the form its solutions take: (x, y).

Overview

Linear equations are equations with 1 or 2 variables to the power 1. The variables are usually x and y. Equations with variables to higher powers are not linear equations. These are examples of linear equations:

$x + y = 5$ $\qquad$ $x + 3 = 7$ $\qquad$ $y = 2x - 6$ $\qquad$ $7y - 4 = 0$

The **standard form** of a linear equation is $ax + by + c = 0$, where a and b cannot both be zero at the same time, and c is a constant term which can be zero.

You will often see linear equations written in the form $y = mx + c$. This is called **slope-intercept form**. In this equation, x and y are the variables and m and c are constants. You will use this form in later lessons to graph lines.

Any linear equation can be written in standard form or slope-intercept form by balancing the equation.

The solution to a linear equation in 2 variables takes the form (x, y). A linear equation has infinitely many solutions. This means there are many different solutions that satisfy the equation.

For example, consider a real-life situation: The number of girls and boys in a JSS 3 class is 82. We can write the linear equation $x + y = 82$, where x is the number of girls and y is the number of boys.

There are many different numbers that could make this situation true. For example, there could be 40 girls and 42 boys. We would have $40 + 42 = 82$. We could also have 80 girls and 2 boys. We would have $80 + 2 = 82$. There are many possibilities! Each set of possibilities is a solution to the equation.

The first example solution can be written $(40, 42)$ and the second example can be written $(80, 2)$. Each of these is called an **ordered pair**. An ordered pair is always written in the form (x, y). We always write the x value first, and the y value second.

If you know the value of one of the variables in the ordered pair, you can use the equation to find the other. You will often be given the value of x, and you will find the value of y.

Solved Examples

1. Determine whether each of the following equations is a **linear equation in 2 variables**. Give your reasons.
   a. $x + 5y = 7$
   b. $y - 8 = 12$
   c. $3x - 8 = 0$
   d. $y = 4x + 6$
   e. $x^2 - y = 0$

   Solution
   Remember that a linear equation in 2 variables has 2 variables with a power of 1. The order of the terms does not matter.
   a. $x + 5y = 7$ is a linear equation in 2 variables. It has 2 variables with a power of 1, x and y.
   b. $y - 8 = 12$ is **not** a linear equation in 2 variables. It is a linear equation in 1 variable, y.
   c. $3x - 8 = 0$ is **not** a linear equation in 2 variables. It is a linear equation in 1 variable, x.
   d. $y = 4x + 6$ is a linear equation in 2 variables. It has 2 variables with a power of 1, x and y.
   e. $x^2 - y = 0$ is **not** a linear equation in 2 variables. It has 2 variables, but one of them has a power of 2. This is not a linear equation at all. It is a quadratic equation.

2. Hawa solved the linear equation $y = 3x + 1$. She found that $y = 7$ and $x = 2$. Help her write her solution as an ordered pair.

   Solution
   Remember that the ordered pair is in the format (x, y). Hawa's solution is $(2, 7)$.

3. Use the equation $y = x + 3$ to find the value of y when $x = 0$. Write the solution as an ordered pair.

   Solution
   Substitute the x -value into the equation and solve for y.

y	$=$	$x + 3$	Equation
	$=$	$0 + 3$	Substitute $x = 0$
	$=$	3	Solve

   When $x = 0$, we have $y = 3$. The ordered pair is $(0, 3)$.

Practice

1. The following are solutions to a linear equation. Write each solution as an ordered pair:
    a. $x = -1, y = 5$
    b. $x = 0, y = -0.8$
    c. $y = 0, x = 0$
    d. $y = -\frac{1}{2}, x = \frac{1}{2}$
2. Determine whether each of the following equations is a **linear equation in 2 variables**. Give your reasons.
    a. $x - y = 0$
    b. $y^2 - 8 = 12$
    c. $3x + 8y = 21$
    d. $y = 4x$
    e. $x + 1 = 13$
3. Martin solved the linear equation $2x + y = 0$. He found that $x = -3$ and $y = 6$. Help him write his solution as an ordered pair.
4. Sia found that $x = 0$ and $y = -1$ is a solution to a linear equation. Write her solution as an ordered pair.
5. Use the equation $y = 3x$ to find the value of y when $x = 1$. Write the solution as an ordered pair.
6. Use the equation $y = x - 5$ to find the value of y when $x = 10$. Write the solution as an ordered pair.

Lesson Title: Verifying Solutions to Linear Equations	**Theme:** Algebra
Practice Activity: PHM-09-087	**Class:** JSS 3

Learning Outcome
By the end of the lesson, you will be able to verify solutions to equations in 2 variables by substitution.

Overview

In this lesson, you will be given an ordered pair. You will verify that the ordered pair is a solution to a given linear equation. We verify a solution by substituting the values of the ordered pair into the equation. This is a check that our answer is correct. If the ordered pair is a solution, you will find that the left-hand side equals the right-hand side of the equation.

When the ordered pair is a solution to an equation, we say it 'satisfies' the equation.

Solved Examples

1. Verify that $(2, 0)$ is a solution to the linear equation $y = 3x - 6$.

   Solution
   Substitute $x = 2$ and $y = 0$ into the given equation. Evaluate and verify that LHS = RHS.

y	$=$	$3x - 6$	Equation
0	$=$	$3(2) - 6$	Substitute $x = 2$ and $y = 0$
0	$=$	$6 - 6$	Evaluate
0	$=$	0	
LHS	$=$	RHS	

   $(2, 0)$ is a solution because it satisfies the equation.

2. Is $(-1, 3)$ a solution to the linear equation $y = x + 4$?

   Solution
   Substitute $x = -1$ and $y = 3$ into the given equation. Check whether LHS = RHS.

y	$=$	$x + 4$	Equation
3	$=$	$-1 + 4$	Substitute $x = -1$ and $y = 3$
3	$=$	3	Evaluate
LHS	$=$	RHS	

   Yes, $(-1, 3)$ is a solution to the equation.

3. Is $(5, 6)$ a solution to the linear equation $y = 2x - 3$?

Solution
Substitute $x = 5$ and $y = 6$ into the given equation. Check whether LHS = RHS.

y	$=$	$2x - 3$	Equation
6	$=$	$2(5) - 3$	Substitute $x = 5$ and $y = 6$
6	$=$	$10 - 3$	Evaluate
6	$\neq$	7	
LHS	$\neq$	RHS	

No, $(5, 6)$ is **not** a solution to the equation.

4. Identify whether $(-2, -10)$ satisfies the equation $y = 3x - 4$.

Solution
Remember that an ordered pair 'satisfies' an equation if it is a solution. Follow the same process as above to check whether $(-2, -10)$ is a solution to the equation:

Substitute $x = -2$ and $y = -10$ into the given equation. Check whether LHS = RHS.

y	$=$	$3x - 4$	Equation
-10	$=$	$3(-2) - 4$	Substitute $x = -2$ and $y = -10$
-10	$=$	$-6 - 4$	Evaluate
-10	$=$	-10	
LHS	$=$	RHS	

Yes, $(-2, -10)$ satisfies the equation.

Practice

1. Verify that $(1, 3)$ is a solution to the linear equation $y = 4x - 1$.
2. Is $(1, -3)$ a solution to the linear equation $y = -2x + 5$?
3. Is $(2, -4)$ a solution to the linear equation $y = -x - 2$?
4. Identify whether each of the ordered pairs satisfies the equation $y = 6 - 2x$.
    a. $(-1, 8)$
    b. $(0, 6)$
    c. $(1, 4)$
    d. $(2, 4)$
5. Verify whether each ordered pair satisfies the given linear equation:
    a. $(0, 0)$ $\quad x + y = 1$
    b. $(1, 10)$ $\quad y = 2x + 8$
    c. $(-2, 11)$ $\quad y = 9 - x$

Lesson Title: Finding Solutions to Linear Equations I	Theme: Algebra
Practice Activity: PHM-09-088	**Class:** JSS 3

Learning Outcome
By the end of the lesson, you will be able to find solutions to equations in 2 variables by substituting a value for one variable and solving for the other.

Overview

In this lesson, you will find solutions to equations in 2 variables by substituting a value for one variable and solving for the other.

You may be given a value for x and be asked to solve for y. Alternatively, you may be given a value for y and be asked to solve for x. Substitute the value that you are given into the equation, and solve for the value of the other variable. You might need to balance the equation. Write your solution as an ordered pair (x, y).

Solved Examples

1. Solve $y = 2x + 5$ when $x = 3$.

   Solution
   Substitute $x = 3$ into the equation and solve for y:

$$\begin{aligned} y &= 2x+5 && \text{Equation} \\ y &= 2(3)+5 && \text{Substitute } x = 3 \\ y &= 6+5 && \text{Evaluate} \\ y &= 11 && \end{aligned}$$

   The solution to the equation is $(3, 11)$.

2. Solve $y = 2x + 5$ when $y = 9$.

   Solution
   Substitute $y = 9$ into the equation and solve for x. You will need to balance the equation:

$$\begin{aligned} y &= 2x+5 && \text{Equation} \\ 9 &= 2x+5 && \text{Substitute } y = 9 \\ 9-5 &= 2x && \text{Subtract 5 from both sides} \\ 4 &= 2x && \\ \tfrac{4}{2} &= \tfrac{2x}{2} && \text{Divide both sides by 2} \\ 2 &= x && \end{aligned}$$

   The solution to the equation is $(2, 9)$.

3. Solve $y = 2(x - 3)$ when:
    a. $x = 4$
    b. $y = 8$

Solutions
Substitute each value into the equation and solve for the other variable:

a. Substitute $x = 4$ into the equation and solve for y:

y	$=$	$2(x-3)$	Equation
y	$=$	$2(4-3)$	Substitute $x = 4$
y	$=$	$2(1)$	Evaluate
y	$=$	2	

The solution to the equation is $(4, 2)$.

b. Substitute $y = 8$ into the equation and solve for x:

y	$=$	$2(x-3)$	Equation
8	$=$	$2(x-3)$	Substitute $y = 8$
$\frac{8}{2}$	$=$	$\frac{2(x-3)}{2}$	Divide both sides by 2
4	$=$	$x-3$	
$4+3$	$=$	x	Add 3 to both sides
7	$=$	x	

The solution to the equation is $(7, 8)$.

Practice

1. Solve $y = -x + 7$ when $x = 2$.
2. Solve $y = -x + 7$ when $y = 6$.
3. Solve $y = 3(x - 1)$ when:
    a. $x = 2$
    b. $x = 0$
    c. $y = 9$
    d. $y = 0$
4. Solve $y = 2x - 5$ when:
    a. $x = -1$
    b. $x = 3$
    c. $y = 5$
    d. $y = -3$

Lesson Title: Finding Solutions to Linear Equations II	**Theme:** Algebra
Practice Activity: PHM-09-089	**Class:** JSS 3

Learning Outcome
By the end of the lesson, you will be able to solve linear equations where the variable appears on both sides of the equation by balancing the equation and combining like terms.

Overview

This is the second lesson on finding solutions to linear equations. In this lesson you will solve linear equations where the variable appears on both sides of the equation. These equations may have 1 or 2 variables. For example:

$5x + 2 = 4x + 5$ linear equation in 1 variable
$3x - y = x + 2$ linear equation in 2 variables

You will balance the equation and combine like terms to find the value of the given variable. For linear equations in 2 variables, substitute the given value for the variable as you did in the previous lesson. Solutions to linear equations in 2 variables are always ordered pairs.

Solved Examples

1. Solve for y in the equation $2(3y - 5) = 5y - 9$.

Solution
Solve for y by balancing the equation:

$$\begin{aligned} 2(3y-5) &= 5y-9 && \text{Equation} \\ 6y-10 &= 5y-9 && \text{Expand LHS} \\ 6y-5y-10 &= -9 && \text{Subtract } 5y \text{ from both sides} \\ y-10 &= -9 && \\ y &= -9+10 && \text{Add 10 to both sides} \\ y &= 1 && \end{aligned}$$

Answer: $y = 1$

2. Solve $y - x = 2x + 3$ when $x = 1$.

Solution
Substitute $x = 1$ into the equation and solve for y by balancing the equation:

$$\begin{aligned} y-x &= 2x+3 && \text{Equation} \\ y-1 &= 2(1)+3 && \text{Substitute } x=1 \\ y-1 &= 2+3 && \text{Evaluate} \\ y-1 &= 5 && \\ y &= 5+1 && \text{Add 1 to both sides} \\ y &= 6 && \end{aligned}$$

Answer: $(1, 6)$

3. Solve $x + 2y = 3 - y$ when $y = 4$.

   Solution

   Substitute $y = 4$ into the equation and solve for x by balancing the equation:

$x + 2y$	$=$	$3 - y$	Equation
$x + 2(4)$	$=$	$3 - 4$	Substitute $y = 4$
$x + 8$	$=$	-1	Evaluate
x	$=$	$-1 - 8$	Subtract 8 from both sides
x	$=$	-9	

   Answer: $(-9, 4)$

4. Solve the equation $\frac{1}{x-1} = \frac{1}{2x}$.

   Solution

   When the variable is in the denominator of a fraction, you may multiply both sides by the denominator to cancel it.

$\frac{1}{x-1}$	$=$	$\frac{1}{2x}$	Equation
$\frac{(x-1)}{x-1}$	$=$	$\frac{(x-1)}{2x}$	Multiply both sides by $x - 1$
1	$=$	$\frac{x-1}{2x}$	$x - 1$ cancels on the LHS
$2x$	$=$	$\frac{(x-1)(2x)}{2x}$	Multiply both sides by $2x$
$2x$	$=$	$x - 1$	$2x$ cancels on the RHS
$2x - x$	$=$	-1	Subtract x from both sides
x	$=$	-1	

   Answer: $x = -1$

Practice

1. Solve for y in the equation $2(4 - y) = 2y - 4$.
2. Solve for x in the equation $3x = \frac{1}{2}(x - 10)$.
3. Solve for x in the equation $x - 1 = 2x + 7$.
4. Solve $2x + y = 2 + 5y$ when $x = 3$.
5. Solve $2y = 12 - y + x$ when $y = 4$.
6. Solve $x - y = y + 6$ when $x = 4$.
7. Solve the equation $\frac{1}{x-5} = \frac{1}{2x+3}$
8. Solve the equation $\frac{1}{x+1} = \frac{1}{2x-3}$
9. Solve the equation $\frac{1}{3}(1 + 3x) - \frac{1-3x}{3} = 2$.

Lesson Title: Practice Solving Linear Equations	**Theme:** Algebra
Practice Activity: PHM-09-090	**Class:** JSS 3

Learning Outcome
By the end of the lesson, you will be able to solve any linear equation in 2 variables.

Overview

In this lesson, you will continue finding solutions to linear equations. For each linear equation in 2 variables, substitute the given value for the variable and solve for the other variable. Solutions to linear equations in 2 variables are always ordered pairs. Remember that you can check solutions by substituting both values in the ordered pair into the linear equation.

Solved Examples

1. Solve $y = 4(x + 1)$ when:
   a. $x = -2$ b. $y = 16$ c. $y = x - 5$

   Solutions
   Substitute each value or expression into the linear equation and find the value of the other variable.

   a. Substitute $x = -2$ into the equation and solve for y:

$$\begin{aligned} y &= 4(x+1) && \text{Equation} \\ y &= 4(-2+1) && \text{Substitute } x = -2 \\ y &= 4(-1) && \text{Evaluate} \\ y &= -4 && \end{aligned}$$

   The solution to the equation is $(-2, -4)$.

   b. Substitute $y = 16$ into the equation and solve for x:

$$\begin{aligned} y &= 4(x+1) && \text{Equation} \\ 16 &= 4(x+1) && \text{Substitute } y = 16 \\ \tfrac{16}{4} &= \tfrac{4(x+1)}{4} && \text{Divide both sides by 4} \\ 4 &= x+1 && \\ 4-1 &= x && \text{Add 3 to both sides} \\ 3 &= x && \end{aligned}$$

   The solution to the equation is $(3, 16)$.

c. Substitute $y = x - 5$ by exchanging y on the left-hand side of the equation $y = 4(x+1)$ with $x - 5$, the expression given for y.

$$\begin{array}{rcll} y &=& 4(x+1) & \text{Equation} \\ x-5 &=& 4(x+1) & \text{Substitute } y = x-5 \\ x-5 &=& 4x+4 & \text{Expand RHS} \\ -5 &=& 4x-x+4 & \text{Subtract } x \text{ from both sides} \\ -5 &=& 3x+4 & \\ -5-4 &=& 3x & \text{Subtract 4 from both sides} \\ -9 &=& 3x & \\ \frac{-9}{3} &=& \frac{3x}{3} & \text{Divide by 3} \\ -3 &=& x & \end{array}$$

Now that we have the x-value, we need to find the y-value. Substitute it into the formula for y:

$$\begin{array}{rcll} y &=& 4(x+1) & \text{Equation} \\ y &=& 4(-3+1) & \text{Substitute } x = -3 \\ y &=& 4(-2) & \\ y &=& -8 & \end{array}$$

The solution is $(-3, -8)$.

2. Verify that $(-3, -3)$ is a solution to the equation $2y - 3 = y + 2x$.

Solution

Substitute the given values into the equation, and evaluate. You should find that LHS = RHS.

$$\begin{array}{rcll} 2y-3 &=& y+2x & \text{Equation} \\ 2(-3)-3 &=& -3+2(-3) & \text{Substitute } x=-3,\ y=-3 \\ -6-3 &=& -3-6 & \text{Evaluate} \\ -9 &=& -9 & \\ \text{LHS} &=& \text{RHS} & \end{array}$$

$(-3, -3)$ is a solution.

Practice

1. Verify that $(-1, -1)$ is a solution to the equation $3x - y = 2x + 3y + 3$.
2. Verify that $(-1, 2)$ is a solution to the equation $4(y-2) = 3(x+1)$.
3. Solve $y = -(x+5)$ when:
   a. $x = -3$ b. $y = 12$ c. $y = 2x + 1$
4. Solve $y = -2(3x+2)$ when:
   a. $x = -1$ b. $y = 5$ c. $y = -2x$

Lesson Title: Solving Linear Equation Story Problems I	**Theme:** Algebra
Practice Activity: PHM-09-091	**Class:** JSS 3

Learning Outcome
By the end of the lesson, you will be able to solve simple story problems by creating and solving linear equations in 2 variables.

Overview

This lesson is on solving equations from story problems. There are many types of word problems which involve relations among known and unknown numbers. These can be written in the form of equations.

To solve a story problem, read the problem carefully and note what is given and what is required. Assign a variable to represent each unknown. Write the equation representing the situation, and solve the equation for the unknown variable or variables. Always check your answers.

Solved Examples

1. Mustapha is 21 years older than his son, Issa. Let Issa's age be x and Mustapha's age be y.
    a. Write an equation for the relationship between their ages.
    b. If Mustapha is 44 years old, how old is Issa?

    Solutions
    a. If Mustapha (y) is 21 years older than Issa (x), the equation is $y = x + 21$.
    b. Substitute $y = 44$, and solve for x.

y	$=$	$x + 21$	Equation
44	$=$	$x + 21$	Substitute $y = 44$
$44 - 21$	$=$	x	Subtract 21 from both sides
23	$=$	x	

Issa is 23 years old.

2. Hawa is 5 years older than twice Sia's age. Let Hawa's age be y and Sia's age be x.
    a. Write an equation for the relationship between their ages.
    b. If Sia is 7 years old, how old is Hawa?

Solutions

a. If Hawa (y) is 5 years older than twice Sia's age (x), the equation is $y = 2x + 5$.

b. Substitute $x = 7$, and solve for y.

$$\begin{aligned} y &= 2x + 5 && \text{Equation} \\ y &= 2(7) + 5 && \text{Substitute } x = 7 \\ y &= 14 + 5 && \text{Evaluate} \\ y &= 19 \end{aligned}$$

Hawa is 19 years old.

3. The width of a rectangle is x and the length is $2x$.
   a. Write an equation for the perimeter, P, in terms of x.
   b. Find the perimeter when $x = 3$ cm.
   c. Find the value of x when the perimeter is 24 cm.

Solutions

a. Remember that perimeter is the distance around the shape. The formula for perimeter of a rectangle is $P = 2l + 2w$. Substitute the length ($2x$) and width (x) into the formula:

$$\begin{aligned} P &= 2l + 2w && \text{Equation} \\ &= 2(2x) + 2x && \text{Substitute } l = 2x, w = x \\ &= 4x + 2x && \text{Evaluate} \\ &= 6x \end{aligned}$$

The equation is $P = 6x$

b. Substitute $x = 3$ cm to find P:

$$\begin{aligned} P &= 6x && \text{Formula} \\ &= 6(3 \text{ cm}) \\ &= 18 \text{ cm} \end{aligned}$$

The perimeter is 18 cm.

c. Substitute $P = 24$ cm to find the value of x:

$$\begin{aligned} P &= 6x && \text{Formula} \\ 24 &= 6x \\ \tfrac{24}{6} &= \tfrac{6x}{6} \\ 4 &= x \end{aligned}$$

The value of x is 4 cm.

Practice

1. Martin is 8 years older than Fatu. Let Fatu's age be x and Martin's age be y.
    a. Write an equation for the relationship between their ages.
    b. If Fatu is 9 years old, how old is Martin?
2. David is 2 years younger than three times Aminata's age. Let David's age be x and Aminata's age be y.
    a. Write an equation for the relationship between their ages.
    b. If Aminata is 12 years old, how old is David?
3. Three angles of a shape are given by x, $2x$, $x + 60$.
    a. Write an equation for y, the sum of the 3 angles.
    b. If $y = 180$, find the value of x.
4. The width of a rectangle is $2x$ cm and the length is $(x + 4)$ cm.
    a. Write an equation for the perimeter, P, in terms of x.
    b. Find the perimeter when $x = 2$ cm.
    c. Find the value of x when the perimeter is 68 cm.

Lesson Title: Solving Linear Equation Story Problems II	**Theme:** Algebra
Practice Activity: PHM-09-092	**Class:** JSS 3

Learning Outcome
By the end of the lesson, you will be able to solve more difficult story problems by creating and solving linear equations in 2 variables.

Overview

This is the second lesson on solving story problems involving linear equations in 2 variables.

Solved Examples

1. A rectangle is $(4q + 5)$ cm long and $(2q - 3)$ cm wide. Find the perimeter of the rectangle in terms of q.

   Solution
   Remember that the formula for perimeter of a rectangle is $= 2l + 2w$. Substitute the length $(4q + 5)$ and width $(2q - 3)$ into the formula:

P	$=$	$2l + 2w$	Equation
	$=$	$2(4q + 5) + 2(2q - 3)$	Substitute l and w
	$=$	$8q + 10 + 4q - 6$	Multiply
	$=$	$8q + 4q + 10 - 6$	Collect like terms
	$=$	$(8 + 4)q + 10 - 6$	Combine like terms
	$=$	$12q + 4$	

   The equation is $P = 12q + 4$.

2. The angles of a triangle are $(5x - 30)°$, $(x - 10)°$ and $(2x + 20)°$.
   a. Write a linear equation for the sum of the angles in the triangle, where the sum is y.
   b. Find the value of x if $y = 180°$.

   Solutions
   a. The equation for the sum of the angles in the triangle is:

   $$\begin{aligned} y &= (5x - 30)° + (x - 10)° + (2x + 20)° \\ &= 5x + x + 2x - 30° - 10° + 20° \\ &= 8x - 20° \end{aligned}$$

b. Remember that the sum of the angles of a triangle is 180°. Substitute $y = 180°$ in the equation and solve for x.

$$\begin{aligned} y &= 8x - 20° \\ 180° &= 8x - 20° && \text{Substitute } y = 180° \\ 180° + 20° &= 8x && \text{Add } 20° \text{ to both sides} \\ 200° &= 8x \\ \tfrac{200°}{8} &= \tfrac{8x}{8} && \text{Divide both sides by 8} \\ 25° &= x \end{aligned}$$

3. Two consecutive odd numbers are n and $n + 2$. Their sum is denoted by S.
   a. Write the equation for S in terms of n.
   b. If $S = 100$, find the two numbers.

   Solutions

   a. Add the 2 numbers and simplify:

$$\begin{aligned} S &= n + n + 2 \\ &= 2n + 2 \end{aligned}$$

   b. Substitute $S = 100$ and solve for n. Then, find $n + 2$.

$$\begin{aligned} S &= 2n + 2 \\ 100 &= 2n + 2 \\ 100 - 2 &= 2n && \text{Subtract 2 from both sides} \\ 98 &= 2n \\ \tfrac{98}{2} &= \tfrac{2n}{2} \\ 49 &= n \end{aligned}$$

   We have 1 number, $n = 49$. Substitute to find the other number:
   $n + 2 = 49 + 2 = 51$

   The two numbers are 49 and 51.

4. Two rectangles have the same area, A. The sides of the first rectangle are 6 cm and $(2x-4)$ cm. The sides of the second rectangle are 4 cm and $(x+2)$ cm. Find the value of x.

   Solution
   Remember that the formula for area of a rectangle is $A = l \times w$.

   The area of the first rectangle is $A = 6(2x-4)$ cm^2 and the area of the second rectangle is $A = 4(x+2)$ cm^2. Set the 2 areas equal and solve for x:

$6(2x-4)$	$=$	$4(x+2)$	$A = A$
$12x-24$	$=$	$4x+8$	Expand both sides
$12x-4x-24$	$=$	8	Subtract $4x$ from both sides
$8x-24$	$=$	8	
$8x$	$=$	$8+24$	Add 24 to both sides
$8x$	$=$	32	
x	$=$	$\frac{32}{8}$	Divide both sides by 8
x	$=$	4 cm	

Practice

1. The length of each side of an equilateral triangle is $2x-3$.
   a. Write an equation for its perimeter, P, in terms of x.
   b. Find the perimeter when $x = 5$ cm.
   c. Find x when the perimeter is 57 cm.
2. Two consecutive odd numbers are n and $n+2$. Their sum is denoted by S.
   a. Write the equation for S in terms of n.
   b. If $S = 68$, find the two numbers.
3. Two friends are the same age, A. One friend says, “You will know my age if you multiply a number by 4 and add 3.” The second friend says, “You will know my age if you multiply the same number as my friend by 6 and subtract 3.” What age are the friends?
4. Two triangles have the same area, A. The base of the first triangle is 4 cm and its height is $(x-2)$ cm. The base of the second triangle is 2 cm and its height is $(x+2)$ cm. Find the value of x.

Lesson Title: Table of Values I	**Theme:** Algebra
Practice Activity: PHM-09-093	**Class:** JSS 3

Learning Outcome
By the end of the lesson, you will be able to create a table of values for a simple linear equation in 2 variables.

Overview

In this lesson, you will create a **table of values** for a simple linear equation in 2 variables. This is the first step in graphing a linear equation. Any linear equation can be graphed, or drawn, on the Cartesian plane. You must know how to make a table of values first.

The linear equation $y = mx + c$ has 2 variables, x and y. The value of y will change depending on the value of x. y is called the **dependent variable**, and x is called the **independent variable**.

You will draw a table of values with several values of x. For example, this is a table of values with x values from -3 to $+3$:

x	-3	-2	-1	0	1	2	3
y							

Each column in the table of values represents 1 solution to the linear equation. To complete a table of values, you will substitute each value of x into the given linear equation and solve for y. This will be one ordered pair. You will do this for each x-value in the table, so that you find several ordered pair solutions. Instead of writing each ordered pair in the form (x, y) you will write it in one column of the table of values.

Solved Examples

1. Create a table of values for the linear equation $y = 7 - 2x$ for values of x from -3 to $+3$.

   Solution
   First, draw the table of values with the given x-values. You will need to find the y-values:

x	-3	-2	-1	0	1	2	3
y							

Substitute each given value of x in the linear equation to get the corresponding values of y:

Given x-value	**Solve for y**
$x = -3,$	$y = 7 - 2(-3) = 13$
$x = -2,$	$y = 7 - 2(-2) = 11$
$x = -1,$	$y = 7 - 2(-1) = 9$
$x = 0,$	$y = 7 - 2(0) = 7$
$x = 1,$	$y = 7 - 2(1) = 5$
$x = 2,$	$y = 7 - 2(2) = 3$
$x = 3,$	$y = 7 - 2(3) = 1$

Write each value of y in the table of values for the linear equation $y = 7 - 2x$:

x	-3	-2	-1	0	1	2	3
y	13	11	9	7	5	3	1

2. Create a table of values for the linear equation $y - 1 = 2x$ for values of x form -3 to $+3$.

Solution

First, make y the subject of the equation:

$$\begin{aligned} y - 1 &= 2x \\ y &= 2x + 1 \end{aligned}$$

Substitute the values of x and evaluate:

When	$x = -3,$	$y = 2(-3) + 1$	$= -5$
	$x = -2,$	$y = 2(-2) + 1$	$= -3$
	$x = -1,$	$y = 2(-1) + 1$	$= -1$
	$x = 0,$	$y = 2(0) + 1$	$= 1$
	$x = 1,$	$y = 2(1) + 1$	$= 3$
	$x = 2,$	$y = 2(2) + 1$	$= 5$
	$x = 3,$	$y = 2(3) + 1$	$= 7$

The table of values for the equation $y = 2x + 1$ is:

x	-3	-2	-1	0	1	2	3
y	-5	-3	-1	1	3	5	7

3. Copy and complete the table of the relation $y + 2x = 0$:

x	-4	-3	-2	-1	0	1	2	3
y		6				-2		

Solution

You are given a table that is partially complete. You only need to find the missing y-values.

First, make y the subject of the equation:

$$\begin{aligned} y + 2x &= 0 \\ y &= 0 - 2x \\ y &= -2x \end{aligned}$$

Substitute values of x and evaluate:

When	$x = -4$	$y = -2(-4)$	$= 8$
	$x = -2$	$y = -2(-2)$	$= 4$
	$x = -1$	$y = -2(-1)$	$= 2$
	$x = 0$	$y = -2(0)$	$= 0$
	$x = 2$	$y = -2(2)$	$= -4$
	$x = 3$	$y = -2(3)$	$= -6$

The table of values for the equation $y + 2x = 0$ is:

x	-4	-3	-2	-1	0	1	2	3
y	8	6	4	2	0	-2	-4	-6

Practice

1. Create a table of values for the equation $y = 2x + 3$ for values of x from -3 to $+3$.
2. Create a table of values for the equation $y + 6 = 3x$ for values of x from -4 to $+2$.
3. Create a table of values for the equation $y = 3 - 2x$ for values of x from -2 to $+4$.
4. Copy and complete the table of values for the relation $y - 5 = 2x$:

x	-3	-2	-1	0	1	2	3
y	-1			5			

Lesson Title: Table of Values II	Theme: Algebra
Practice Activity: PHM-09-094	**Class:** JSS 3

Learning Outcome
By the end of the lesson, you will be able to create a table of values for a more complicated linear equation in 2 variables.

Overview

In this lesson, you will continue to create tables of values. You will follow the same process as you used in the previous lesson. The only difference is that the equations will be a little more complicated. For example, they will involve fractions or decimals.

You may give your answers as either fractions or decimals, unless the problem tells you which to use. If you give your answers as fractions, remember to simplify them and change improper fractions to mixed fractions.

Solved Examples

1. Copy and complete a table of values for the linear equation $y = \frac{1}{2}x + 2$.

x	-3	-2	-1	0	1	2	3
y	0.5			2		3	

Solution
We only need to find the y-values that are missing in the table. Substitute each value of x into the given equation, and find y.
When $x = -2$, $y = \frac{1}{2}(-2) + 2 = -1 + 2 = 1$
When $x = -1$, $y = \frac{1}{2}(-1) + 2 = -0.5 + 2 = 1.5$
When $x = 1$, $y = \frac{1}{2}(1) + 2 = 0.5 + 2 = 2.5$
When $x = 3$, $y = \frac{1}{2}(3) + 2 = 1.5 + 2 = 3.5$

The table of values for $y = \frac{1}{2}x + 2$ is:

x	-3	-2	-1	0	1	2	3
y	0.5	1	1.5	2	2.5	3	3.5

2. Create a table of values for the equation $4y - x = 0$ for values of x from -3 to $+3$.
Solution
First, make y the subject of the equation:

$$
\begin{aligned}
4y - x &= 0 \\
4y &= 0 + x \\
y &= \tfrac{x}{4} = \tfrac{1}{4}x
\end{aligned}
$$

Substitute each value of x into the given equation, and find y.

When $x = -3$, $\quad y = \frac{1}{4}(-3) = -\frac{3}{4}$

When $x = -2$, $\quad y = \frac{1}{4}(-2) = -\frac{2}{4} = -\frac{1}{2}$

When $x = -1$, $\quad y = \frac{1}{4}(-1) = -\frac{1}{4}$

When $x = 0$, $\quad y = \frac{1}{4}(0) = 0$

When $x = 1$, $\quad y = \frac{1}{4}(1) = \frac{1}{4}$

When $x = 2$, $\quad y = \frac{1}{4}(2) = \frac{2}{4} = \frac{1}{2}$

When $x = 3$, $\quad y = \frac{1}{4}(3) = \frac{3}{4}$

The table of values for $y = \frac{1}{4}x$ is:

x	-3	-2	-1	0	1	2	3
y	$-\frac{3}{4}$	$-\frac{1}{2}$	$-\frac{1}{4}$	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$

Practice

1. Complete the table of values for the relation $2y = x + 5$. Give your answers as decimal numbers.

x	-3	-2	-1	0	1	2	3
y		1.5					

2. Create a table of values for the equation $y = \frac{1}{5}x + \frac{3}{5}$ for values of x from -3 to $+3$.
3. Create a table of values for the equation $y = 2x - \frac{1}{2}$ for values of x from -3 to $+3$.
4. Complete the table of values for the relation $2y - 3x = 6$.

x	-3	-2	-1	0	1	2	3
y			1.5			6	

Lesson Title: Review of the Cartesian Plane	**Theme:** Algebra
Practice Activity: PHM-09-095	**Class:** JSS 3

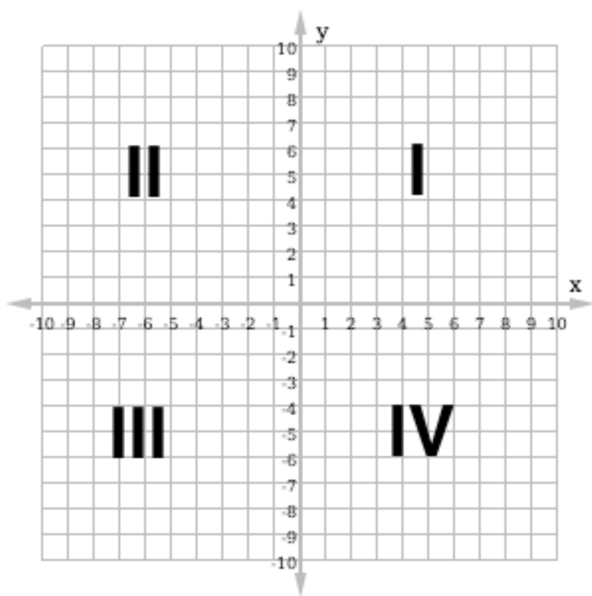
Learning Outcomes
By the end of the lesson, you will be able to:
1. Draw a Cartesian plane.
2. Identify the x- and y-axes and label them with positive and negative values.
3. Identify points in each quadrant of a Cartesian plane and write them in the form (x, y).

Overview

This lesson is on the Cartesian plane. We use the Cartesian plane to draw graphs for equations. In the next lessons, you will use the Cartesian plane to graph linear equations.

The grid system is called a 'Cartesian plane'. A plane is any flat 2-dimensional surface.

We draw the 2 axes on the Cartesian plane, the x-axis and the y-axis. These 2 axes intersect at a right angle.



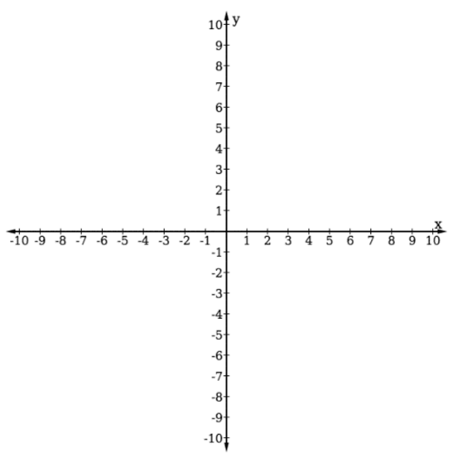
The $\boldsymbol{x}$**-axis** goes from left to right and increases in value as shown by the arrow. Only a small part of the axis is shown, from –10 to +10. Negative values are to the left of the y-axis. Positive values are to the right of the y-axis.

The $\boldsymbol{y}$**-axis** goes from the bottom of the board to the top. It also increases in value in the direction of the arrow. We have shown only the part from –10 to +10. Negative values are below the x-axis. Positive values are above the x-axis.

Both axes go to infinity in both directions. We draw arrows at the end of the axes to show this.

The 2 axes divide the Cartesian plane into 4 **quadrants**, numbered as shown with Roman numerals.

When you draw a Cartesian plane of your own, it is not necessary to draw the entire grid. You may just draw tick marks on each axis, as shown to the right. Make sure all of your tick marks are the same distance apart.

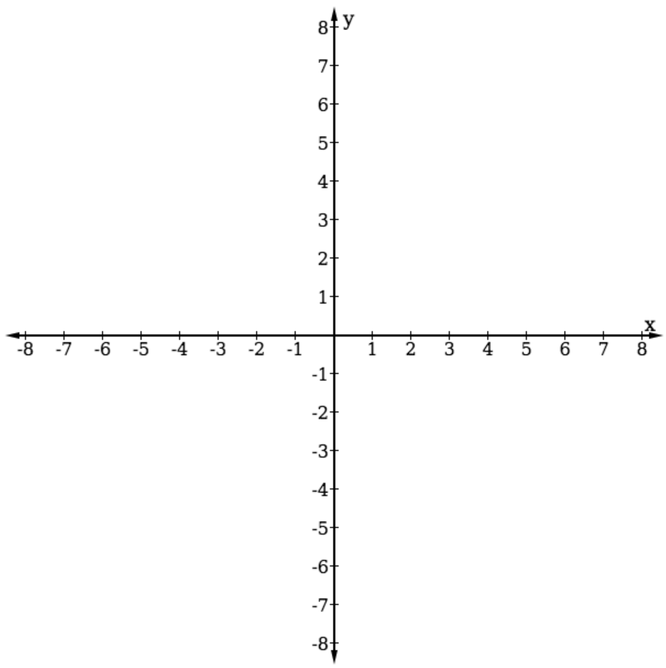


We can identify points on the Cartesian plane by their coordinates, or ordered pair (x, y). The coordinates show their relationships to the x-axis and y-axis. The x-value of a point's coordinates tells how far to move along the x-axis to reach the point. The y-value the point's coordinates tells how far to move along the y-axis to reach the point. See Solved Example 2 for examples of points on the Cartesian plane.

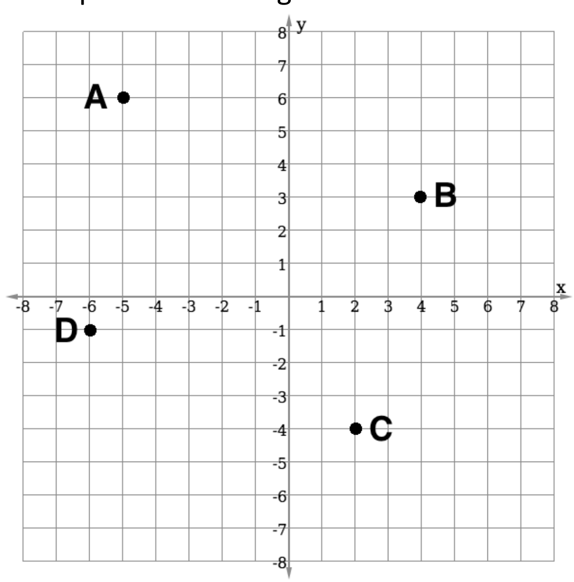
Solved Examples

1. Draw a Cartesian plane with axes from -8 to $+8$:

   Solution
   Your Cartesian plane should look like the one below. Make sure your tick marks are the same distance apart. Label your axes and tick marks.



2. Write the coordinates of each point in the diagram below:



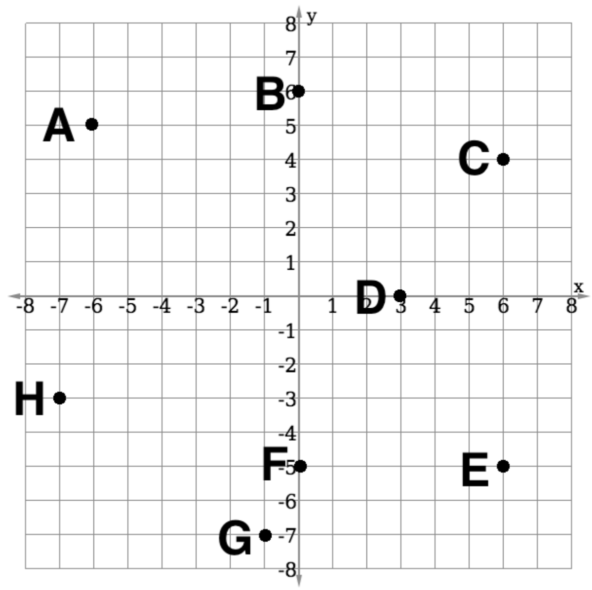
Solution

Remember that each point has an ordered pair (x, y). The x-value tells you its position on the x-axis, and the y-value tells you its position on the y-axis. From each point (A, B, C, D) move along the grid line and find the numbers along the axes that it corresponds to.

$\mathrm{A}(-5,6)$, $\mathrm{B}(4,3)$, $\mathrm{C}(2,-4)$, $\mathrm{D}(-6,-1)$. Note that each point is given by its letter before the ordered pair.

Practice

1. Draw a Cartesian plane with axes from -12 to $+12$.
2. Write the coordinates of each point in the diagram below:



Lesson Title: Plotting Points on the Cartesian Plane	Theme: Algebra
Practice Activity: PHM-09-096	**Class:** JSS 3

Learning Outcome
By the end of the lesson, you will be able to plot given points in any quadrant of the Cartesian plane.

Overview

In the previous lesson, you wrote the coordinates for points that were plotted on the Cartesian plane. In this lesson, you will plot points given their coordinates.

Remember that a set of coordinates is an ordered pair (x, y). The x-value tells you how far to move along the x-axis, and the y-value tells you how far to move along the y-axis. If the x-value is positive, move to the right. If the x-value is negative, move to the left. If the y-value is positive, move up. If the y-value is negative, move down.

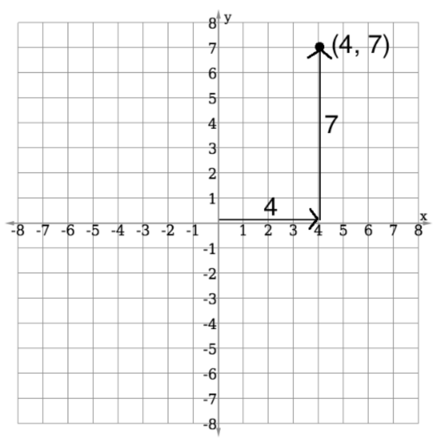
Follow these steps to plot any point:

- Start at the origin $(0, 0)$;
- Move along the x-axis x units from the origin, stop;
- Move y units parallel to the y-axis to the required point;
- Mark the point and write it as the ordered pair, (x, y).

Solved Examples

1. Draw a Cartesian plane and plot the point (4, 7).

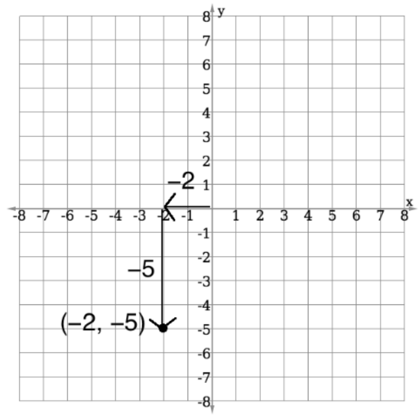
   Solution
   Start at the origin (0, 0). Move 4 units to the right, and 7 units up. Plot the point and write the coordinates as an ordered pair (4, 7).



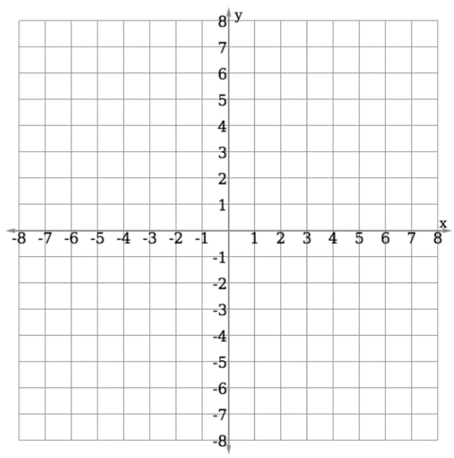
2. Draw a Cartesian plane and plot the point $(-2, -5)$.

   Solution

   Start at the origin (0, 0). Move 2 units to the left, and 5 units down. Plot the point and write the coordinates as an ordered pair $(-2, -5)$.

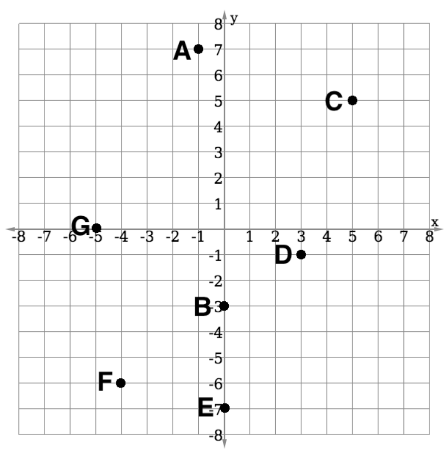


3. Plot the following points on the Cartesian plane below: $\mathrm{A}(-1,7)$, $\mathrm{B}(0,-3)$, $\mathrm{C}(5,5)$, $\mathrm{D}(3,-1)$, $\mathrm{E}(0,-7)$, $\mathrm{F}(-4,-6)$, $\mathrm{G}(-5,0)$.



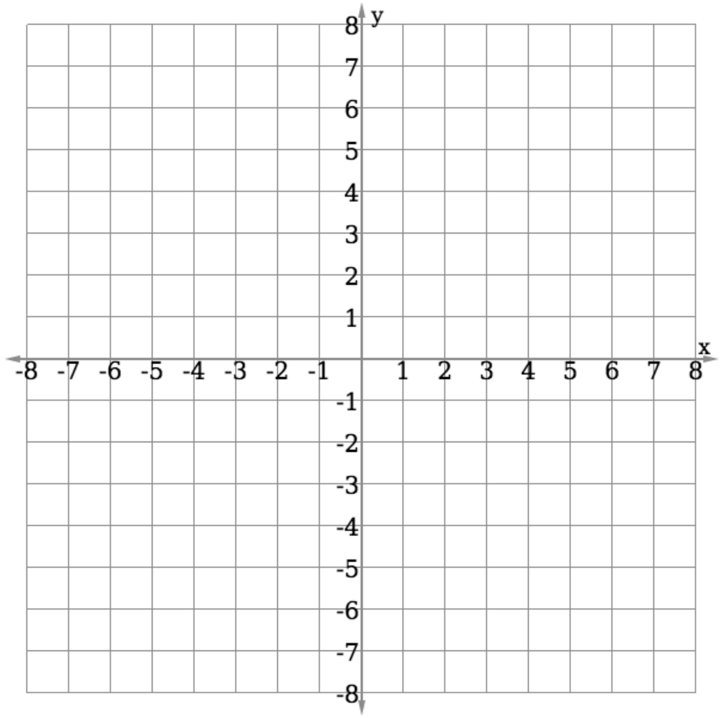
   Solution

   Identify each point on the Cartesian plane and plot it. Write the letter next to each point. It is not necessary to write both the letter and coordinates.

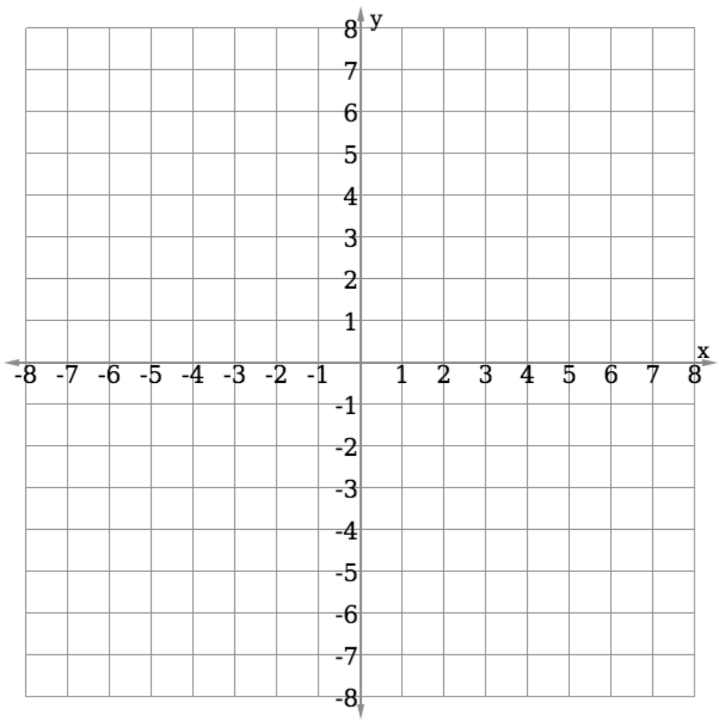


Practice

1. Draw a Cartesian plane and plot the point $(4, -6)$.
2. Draw a Cartesian plane and plot the point $(-3, 7)$.
3. Plot the following points on the Cartesian plane below: $\mathrm{A}(-7,0)$, $\mathrm{B}(0, 6)$, $\mathrm{C}(5, 1)$, $\mathrm{D}(-3, -1)$, $\mathrm{E}(0, -1)$, $\mathrm{F}(7, -6)$, $\mathrm{G}(8, 0)$.



4. Plot the following points on the Cartesian plane below: $\mathrm{T}(-1, -1)$, $\mathrm{U}(7, 7)$, $\mathrm{V}(1, 6)$, $\mathrm{W}(-3, -3)$, $\mathrm{X}(0, -5)$, $\mathrm{Y}(-6, 4)$, $\mathrm{Z}(7, -4)$.



Lesson Title: Plotting Points from a Table of Values	**Theme:** Algebra
Practice Activity: PHM-09-097	**Class:** JSS 3

Learning Outcome
By the end of the lesson, you will be able to plot points from a given table of values on the Cartesian plane.

Overview

Remember that a table of values holds solutions to an equation. For example, the table below holds 7 solutions to the equation $y = x + 2$.

x	-3	-2	-1	0	1	2	3
y	-1	0	1	2	3	4	5

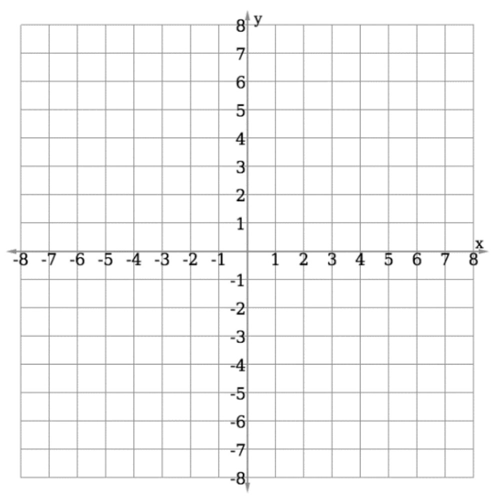
Each column is one ordered pair (x, y). The solutions in this table are $(-3, -1)$, $(-2,0)$, $(-1, 1)$, and so on.

Each ordered pair in a table of values can be plotted on the Cartesian plane. A table of values can be used to graph many different types of equations. After plotting the points, you will be able to see the shape of the graph. You will connect the points with a line or a curve. In this lesson, you will only graph linear equations, which always result in straight lines when graphed.

Solved Examples

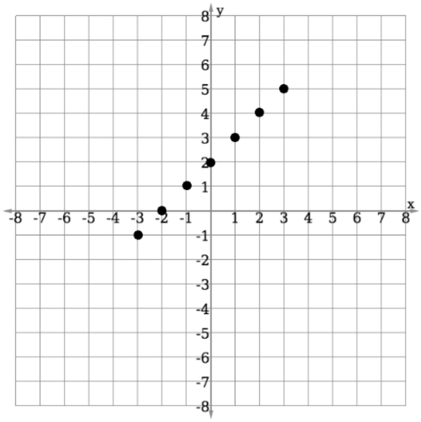
1. The table of values gives solutions to the linear equation $y = x + 2$. Draw the graph using the values in the given Cartesian plane.

x	-3	-2	-1	0	1	2	3
y	-1	0	1	2	3	4	5

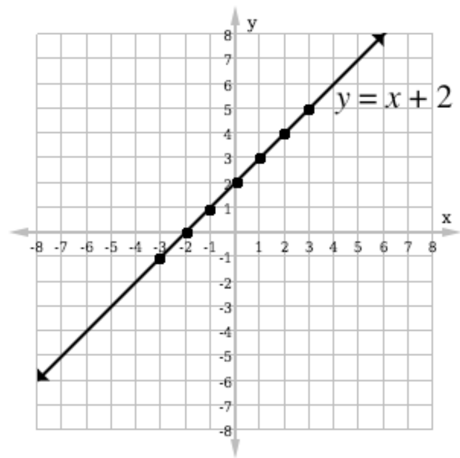


Solution

Step 1. Plot each of the points in the table:



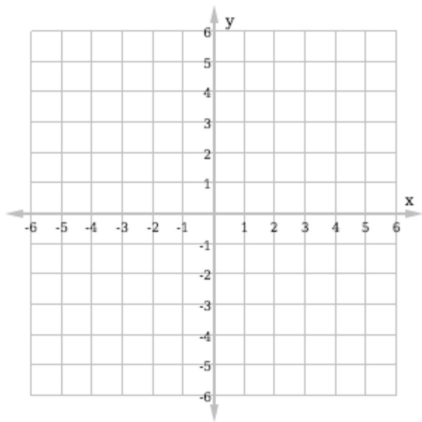
Step 2. Connect the points with a line:



When you graph a line on the Cartesian plane, write the equation next to the line.

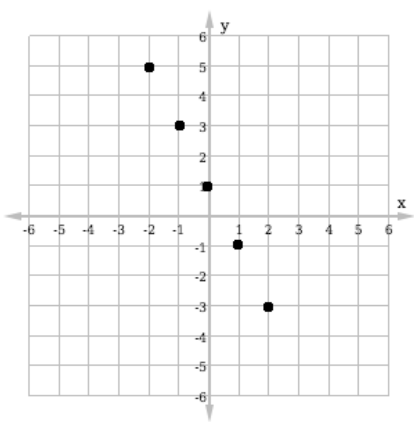
2. The table of values gives solutions to the linear equation $y = -2x + 1$. Draw the graph using the values in the given Cartesian plane.

x	-2	-1	0	1	2
y	5	3	1	-1	-3

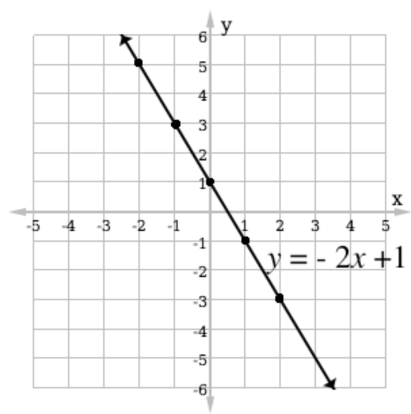


Solution

Step 1. Plot each of the points in the table:



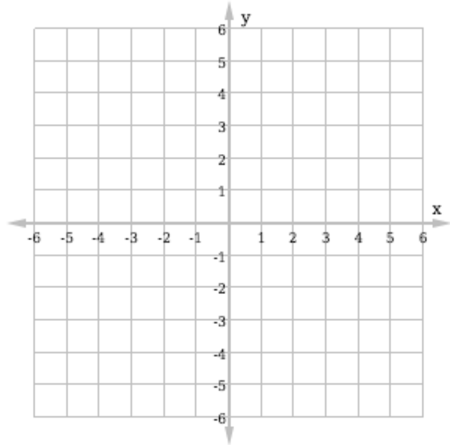
Step 2. Connect the points with a line:



Practice

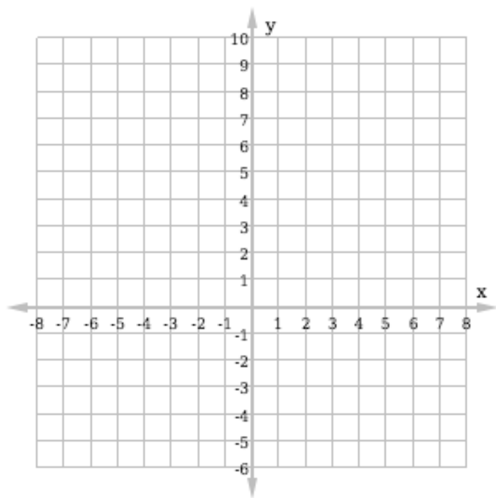
1. The table of values gives solutions to the linear equation $y = -x - 3$. Draw the graph using the values in the given Cartesian plane.

x	-2	-1	0	1	2
y	-1	-2	-3	-4	-5



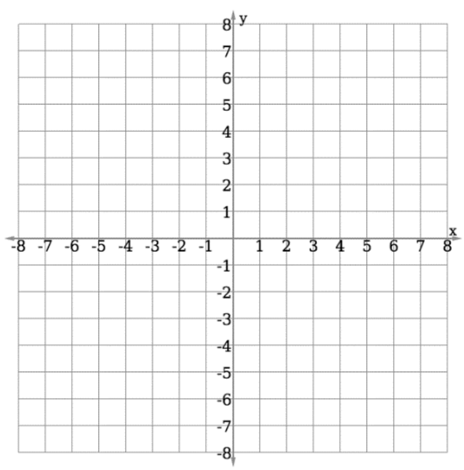
2. The table of values gives solutions to the linear equation $y = 3x - 2$. Draw the graph using the values in the given Cartesian plane.

x	-2	-1	0	1	2
y	-8	-5	-2	1	4



3. The table of values gives solutions to the linear equation $y = 4 - x$. Draw the graph using the values in the given Cartesian plane.

x	-2	-1	0	1	2
y	6	5	4	3	2



Lesson Title: Graphing a Line I	**Theme:** Algebra
Practice Activity: PHM-09-098	**Class:** JSS 3

Learning Outcome
By the end of the lesson, you will be able to create a table of values for a given linear equation in 2 variables and graph it on the Cartesian plane.

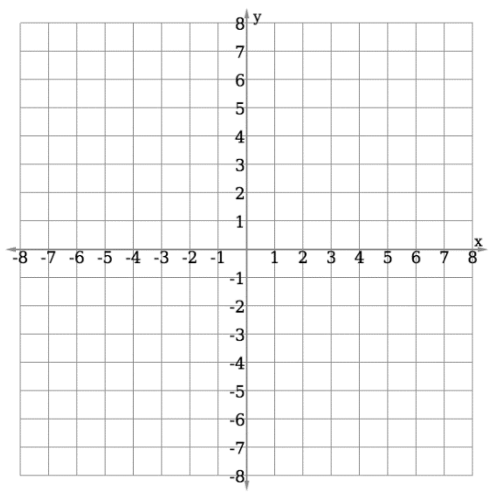
Overview

In PHM-09-093 and PHM-09-094, you learned how to create a table of values for an equation. In PHM-09-097, you learned how to plot the points from a table of values on the Cartesian plane. In this lesson, you will combine these skills. You will create a table of values for a given linear equation in 2 variables and graph it on the Cartesian plane.

Solved Examples

1. Complete the table of values below for the linear equation $y = x - 1$. Graph the relation on the Cartesian plane.

x	-2	-1	0	1	2
y					



Solution

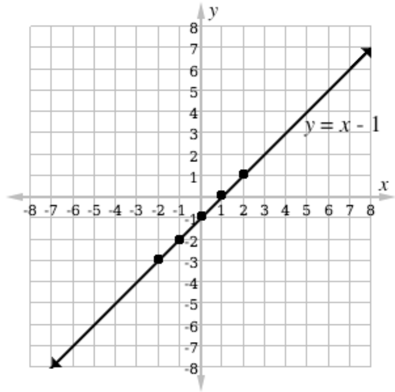
First, complete the table of values. Substitute each value of x into $y = x - 1$, and find y.

When $x = -2$, $\quad y = -2 - 1 = -3$
When $x = -1$, $\quad y = -1 - 1 = -2$
When $x = 0$, $\quad y = 0 - 1 = -1$
When $x = 1$, $\quad y = 1 - 1 = 0$
When $x = 2$, $\quad y = 2 - 1 = 1$

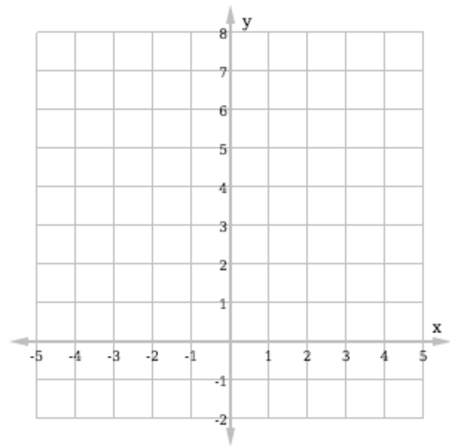
Completed table:

x	-2	-1	0	1	2
y	-3	-2	-1	0	1

Next, plot the points from the table on the Cartesian plane. Connect them with a line, and label the line with the equation $y = x - 1$.



2. Draw a table of values of x from -2 to $+2$ of the linear equation $y = 4 - x$. Use the table of values to graph the equation on the given plane.



Solution

In this problem you are not given the table of values. Draw your own table of values with values of x from -2 to $+2$:

x	-2	-1	0	1	2
y					

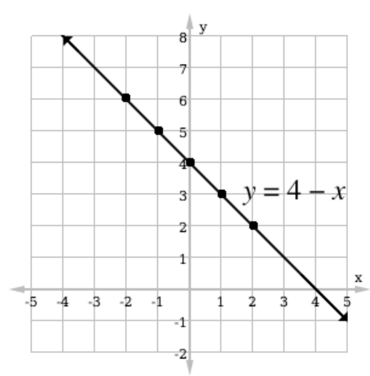
Then, substitute each value of x into the given equation and fill the table.

When $x = -2$, $\quad y = 4 - (-2) = 4 + 2 = 6$
When $x = -1$, $\quad y = 4 - (-1) = 4 + 1 = 5$
When $x = 0$, $\quad y = 4 - (0) = 4 - 0 = 4$
When $x = 1$, $\quad y = 4 - (1) = 4 - 1 = 3$
When $x = 2$, $\quad y = 4 - (2) = 4 - 2 = 2$

Write the results in your table of values:

x	-2	-1	0	1	2
y	6	5	4	3	2

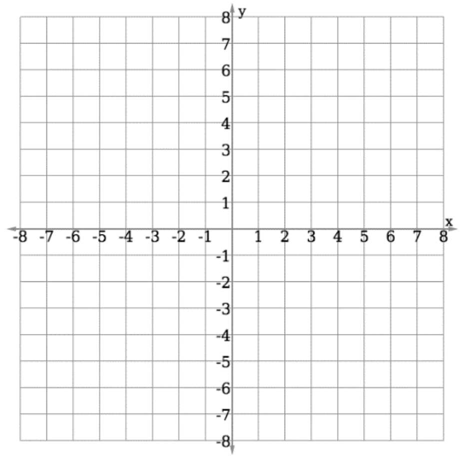
Plot each point on the Cartesian plane, and connect them in a line:



Practice

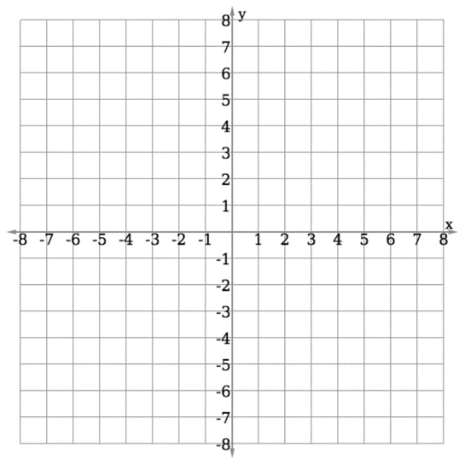
1. Complete the table of values below for the linear equation $y = 2x + 4$. Graph the relation on the Cartesian plane.

x	-3	-2	-1	0
y				

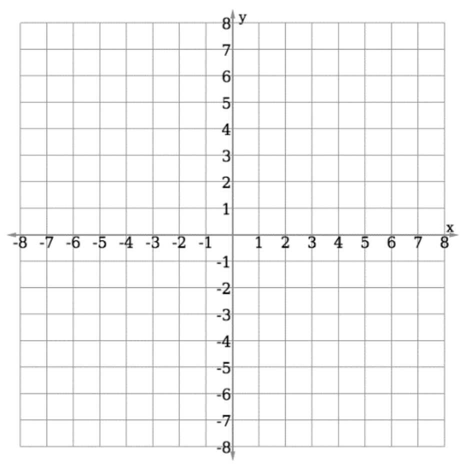


2. Complete the table of values below for the linear equation $y = 3 - 2x$. Graph the relation on the Cartesian plane.

x	-2	-1	0	1	2
y					



3. Create a table of values and draw the graph of the linear equation $y = 2x + 3$ for values of x from -3 to $+2$.



Lesson Title: Graphing a Line II	**Theme:** Algebra
Practice Activity: PHM-09-099	**Class:** JSS 3

Learning Outcome
By the end of the lesson, you will be able to graph more complicated linear equations.

Overview

In this lesson, you will continue to practice graphing a line. You will make your own table of values and draw your own Cartesian plane.

When making your own table of values, use x-values near 0. For example, you may use x-values between -2 and 2.

When drawing a Cartesian plane for an equation, make the axes long enough to contain all of the points in the table of values. For example, if the greatest x-value in the table is 7, the x-axis you draw should extend to at least 8. It is not necessary to draw the full grid each time. You may just draw the axes with tick marks.

Make the tick marks on your Cartesian plane an equal distance apart. In some cases, a scale is given. For example, you may be asked to use 1 cm or 2 cm for every unit on the axes. In that case, use a ruler to measure 1 cm or 2 cm and draw the tick marks on your axes. If you do not have a ruler, estimate the length.

Solved Examples

1. Complete a table of values for $y = 2x$ and graph the line on the Cartesian plane.

   Solution
   Choose reasonable x-values for your table of values. For example, -2 to 2. Substitute each value into $y = 2x$ and complete the table.

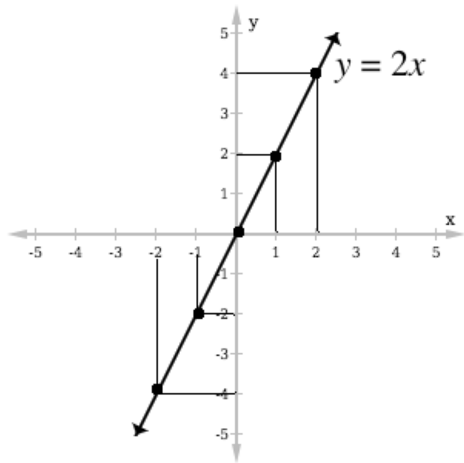
   When $x = -2$, $\quad y = 2(-2) = -4$
   When $x = -1$, $\quad y = 2(-1) = -2$
   When $x = 0$, $\quad y = 2(0) = 0$
   When $x = 1$, $\quad y = 2(1) = 2$
   When $x = 2$, $\quad y = 2(2) = 4$

x	-2	-1	0	1	2
y	-4	-2	0	2	4

   Draw the Cartesian plane. The x-axis should extend beyond -2 and 2. The y-axis should extend beyond -4 and 4. You may draw only the axes and the tick marks, not a full grid.

Plot each point from the table of values on the Cartesian plane. If you did not draw the grid, it is a good idea to use a straight edge to find the location of each point. Line your straight edge up at the x- and y-value of each point and find where the lines meet.

Connect the points and label the line.



2. Use the linear equation $y = 2x - 1$ to complete the following:
   a. Create a table of values.
   b. Draw a Cartesian plane using a scale of 1 cm to 1 unit on both axes.
   c. Graph the equation $y = 2x - 1$ on the Cartesian plane.

Solutions

a. Choose reasonable x-values for your table of values. For example, -2 to 2. Substitute each value into $y = 2x$ and complete the table.

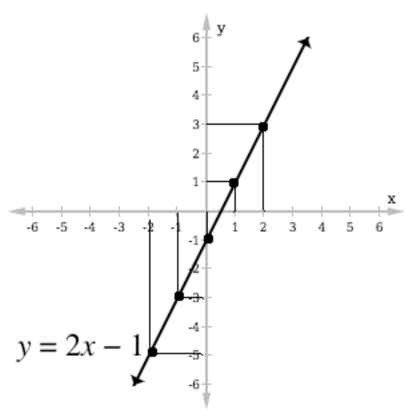
When $x = -2$, $\quad y = 2(-2) - 1 = -4 - 1 = -5$
When $x = -1$, $\quad y = 2(-1) - 1 = -2 - 1 = -3$
When $x = 0$, $\quad y = 2(0) - 1 = 0 - 1 = -1$
When $x = 1$, $\quad y = 2(1) - 1 = 2 - 1 = 1$
When $x = 2$, $\quad y = 2(2) - 1 = 4 - 1 = 3$

Complete the table:

x	-2	-1	0	1	2
y	-5	-3	-1	1	3

b. Your x-axes should extend beyond -2 and 2. Your y-axes should extend beyond -5 and 3.
   Use a ruler to draw your axes with tick marks 1 cm apart. If you do not have a ruler, estimate 1 cm. The Cartesian plane below is printed small and is **not** the correct size.

c. Plot each point from the table of values. Connect the points and label you line.



Practice

1. Complete a table of values for $y = x + 3$ and graph the line on the Cartesian plane.
2. Complete a table of values for $y = 5 - 2x$ and graph the line on the Cartesian plane.
3. Use the linear equation $y = 3x - 1$ to complete the following:
    a. Create a table of values.
    b. Draw a Cartesian plane using a scale of 1 cm to 1 unit on both axes.
    c. Graph the equation $y = 3x - 1$ on the Cartesian plane.

Lesson Title: Graphing a Line III	**Theme:** Algebra
Practice Activity: PHM-09-100	**Class:** JSS 3

Learning Outcome
By the end of the lesson, you will be able to practice graphing a line.

Overview

In this lesson, you will continue to practice graphing lines. You will work with more complicated linear equations. For example, linear equations with fractions.

In some cases, you may need to change the subject so that y is alone on one side of the equation. If you are asked to graph $y + 2x = -6$, solve the equation for y before graphing it. You would graph $y = -2x - 6$.

In other cases, you may be asked to use a different scale on the y-axis and x-axis. Take your time and use a ruler. See Solved Example 2.

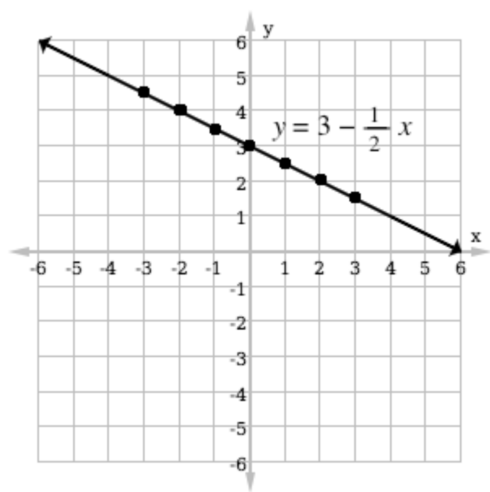
Solved Examples

1. Draw the graph of the relation $y = 3 - \frac{1}{2}x$ using the table of values below. Use a scale of 2 cm to 1 unit on both axes.

x	-3	-2	-1	0	1	2	3
y	4.5			3	2.5	2	

Solution
In this case, part the table of values is given. Find the missing values:
$y = 3 - \frac{1}{2}(-2) = 3 + 1 = 4$
$y = 3 - \frac{1}{2}(-1) = 3 + \frac{1}{2} = 3\frac{1}{2} = 3.5$
$y = 3 - \frac{1}{2}(3) = 3 - \frac{3}{2} = \frac{3}{2} = 1\frac{1}{2} = 1.5$
Fill the table of values:

x	-3	-2	-1	0	1	2	3
y	4.5	4	3.5	3	2.5	2	1.5



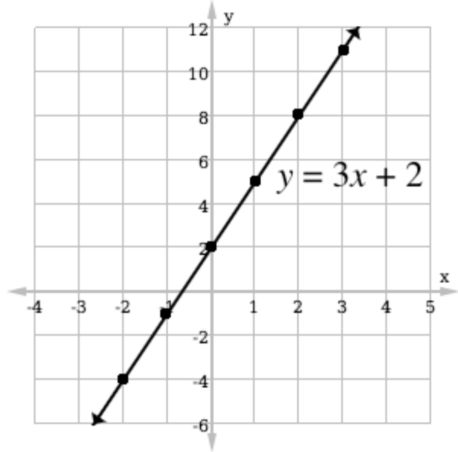
You will need to draw your own Cartesian plane using a ruler and a scale of 2 cm to 1 unit on both axes. Note that the graph above is not drawn to the correct scale (2 cm). Check your plane with a ruler to make sure it is correct.

2. Draw the graph of the relation $y = 3x + 2$ for values of x from -2 to $+3$. Use a scale of 2 cm to 1 unit on the x axis and 2 cm to 2 units on the y axis.

Solution

First, complete the table of values with values of x from -2 to $+3$:

x	-2	-1	0	1	2	3
y	-4	-1	2	5	8	11



Now draw the Cartesian plane. Note that the scales on the 2 axes are different. Draw the x-axis and the y-axis. Mark the x-axis every 2 cm to 1 unit, and mark the y-axis every 2 cm to 2 units. Make sure the axes extend beyond the values in the table.

Note that this Cartesian plane is not drawn to scale. Check your own with a ruler.

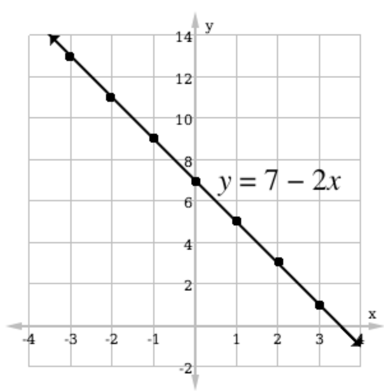
Plot the points on the plane using the values from the table, and connect them with a straight line.

3. Draw the graph of the relation $y = 7 - 2x$ for values of x from -3 to $+3$. Use the table of values as shown below.

x	-3	-2	-1	0	1	2	3
y	13	11	9	7	5	3	1

Solution

The scale is not given, therefore we identify the smallest and the largest values of x and y to create reasonable axes. The values of x are from -3 to $+3$. The smallest value of y is 1 and the largest value is 13. You can come up with your own scale that can absorb all the values in the table.



This example uses 2 cm to 1 unit on the x-axis and 2 cm to 2 units on the y-axis. This scale will absorb the larger values of y.

Mark the x-axis 2 cm to 1 unit from -4 to 4.
Mark the y-axis 2 cm to 2 units from -2 to 14.
Plot the points on the plane using the values from the table and connect them.

4. Graph the equation $8x - 2y = 12$.

Step 1. Change subject and solve for y:

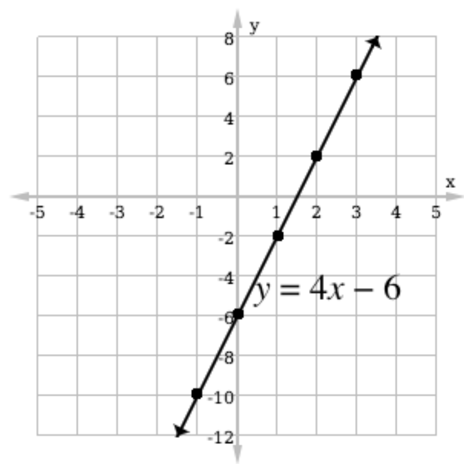
$$\begin{aligned} 8x - 2y &= 12 \\ -2y &= 12 - 8x && \text{Transpose } 8x \\ \frac{-2y}{-2} &= \frac{12}{-2} - \frac{8x}{-2} && \text{Divide throughout by } -2 \\ y &= -6 + 4x \end{aligned}$$

This is the same as $y = 4x - 6$.

Step 2. Graph $y = 4x - 6$. You are not given a table of values, so make your own using several values:

x	-1	0	1	2	3
y	-10	-6	-2	2	6

The y-values have a large range. In this example, we will use a scale of 1 cm to 1 unit on the x-axis, and 1 cm to 2 units on the y-axis.



Practice

1. Draw the graph of the relation $y = 2x + 3$ for values of x from -3 to 3. Use a scale of 2 cm to 1 unit on the x-axis and 2 cm to 2 units on the y-axis. Use the table below.

x	-3	-2	-1	0	1	2	3
y		-1		3			9

2. Draw the graph of the equation $x + 2y = 7$ for values of x from -2 to 2. Use the table below.

x	-2	-1	0	1	2
y			3.5	3	

3. Copy and complete the table of values for the equation $3x - y = 1$ for values of x from -3 to 3. Draw the graph of the equation.

x	-3	-2	-1	0	1	2	3
y		-7		-1			

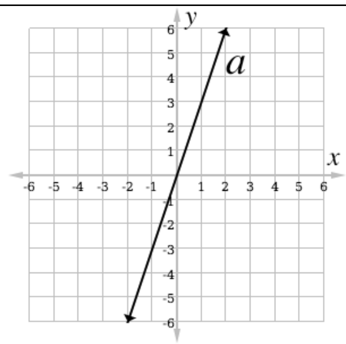
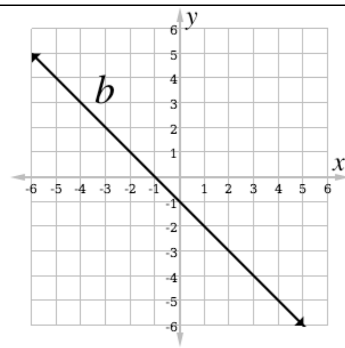
Lesson Title: Introduction to Slope	**Theme:** Algebra
Practice Activity: PHM-09-101	**Class:** JSS 3

Learning Outcomes
By the end of the lesson, you will be able to:
1. Identify that the slope of a line describes its steepness, and is described by the fraction $\frac{\text{rise}}{\text{run}}$.
2. Identify the direction of a positive and negative slope.

Overview

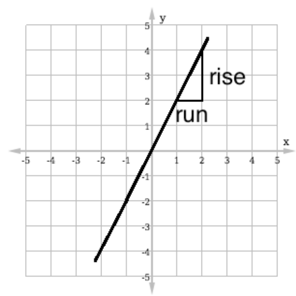
Slope (or **gradient**) is a number that tells us in which direction a line increases, and how steep it is. If a line increases as it goes to the right, or in the positive x-direction, the slope is **positive**. If a line increases as it goes to the left, or in the negative x-direction, the slope is **negative**.

The greater the absolute value of a slope, the steeper the line is. For example, a line with slope 6 is steeper than a line with slope 4. A line with slope -5 is steeper than a line with slope 3.

	
• Line a increases as it goes to the **right**, or the positive x-direction. Line a has a **positive** slope. • Line a is steeper than line b. It has a slope of +3.	• Line b increases as it goes to the **left**, or the negative x-direction. Line b has a **negative** slope. • Line b is not as steep as line a. It has a slope of -1.

The slope of a line can be calculated using any 2 points on the line. It is calculated by dividing the change in y by the change in x between those 2 points.

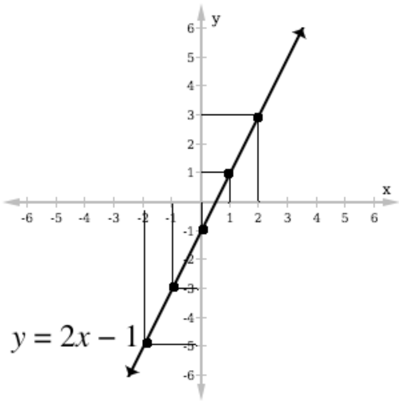
$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in y}}{\text{change in x}}$$



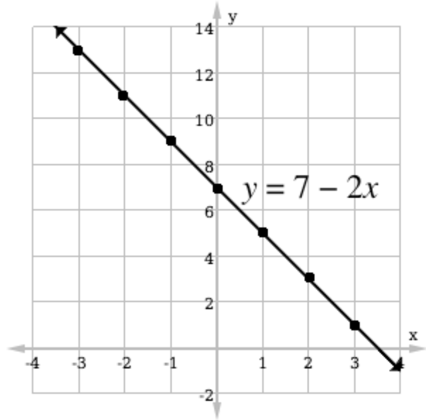
Solved Examples

1. Determine whether each line has a positive slope or a negative slope:

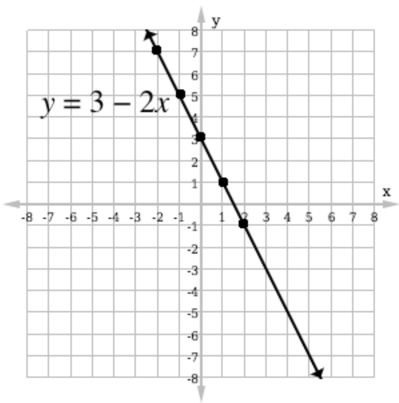
a.



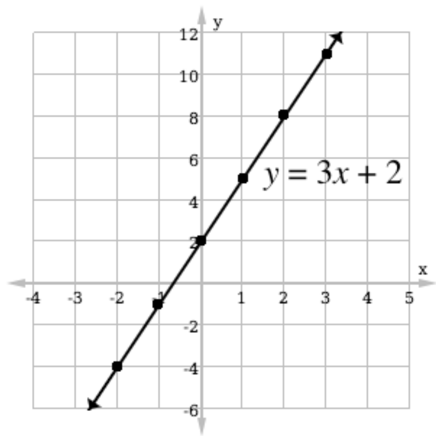
b.



c.



d.

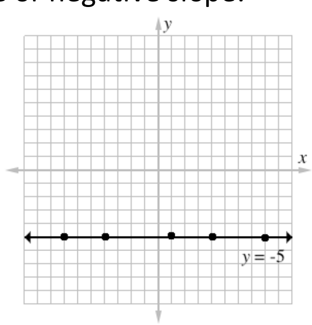


Solutions

Remember that lines that increase in the positive x- direction have a positive slope. Lines that increase in the negative x- direction have a negative slope. The answers are:

a. Positive slope
b. Negative slope
c. Negative slope
d. Positive slope

2. Determine if the line has a positive or negative slope:



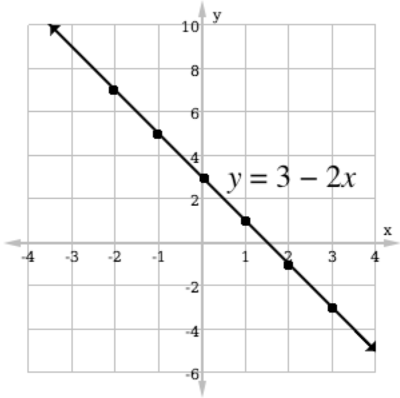
Solution

The slope is neither positive nor negative. A linear equation without the variable x is a horizontal line. The slope is equal to 0.

Practice

Determine whether each line has a positive slope or a negative slope:

1.



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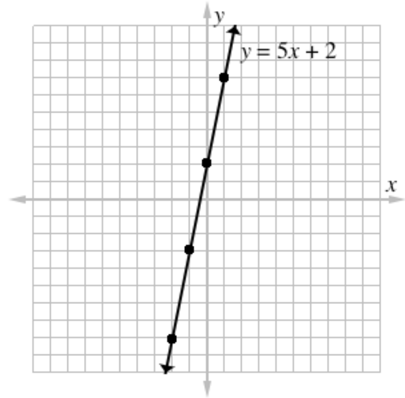
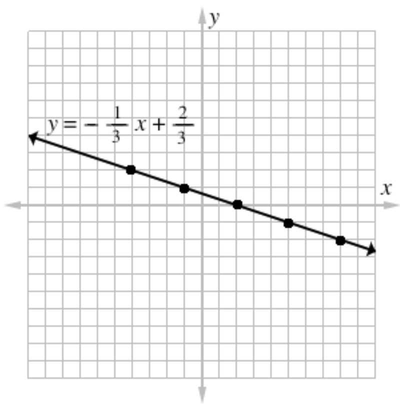


3.



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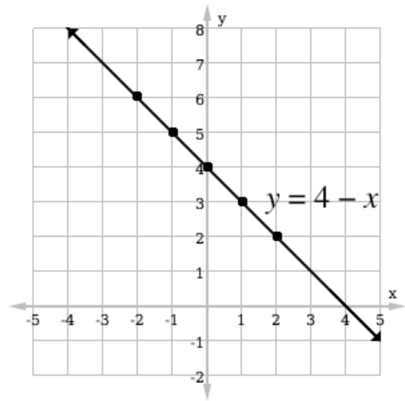

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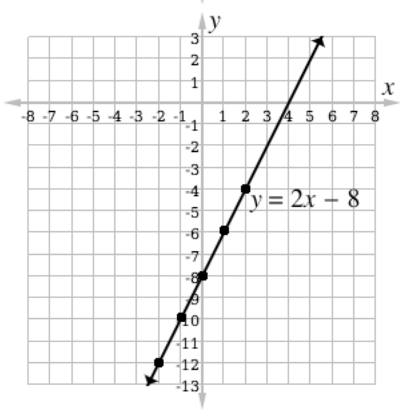

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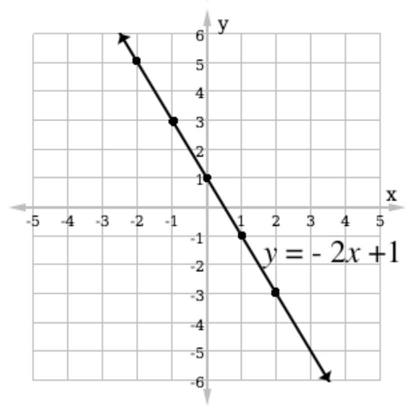


Lesson Title: Finding the Slope of a Line	**Theme:** Algebra
Practice Activity: PHM-09-102	**Class:** JSS 3

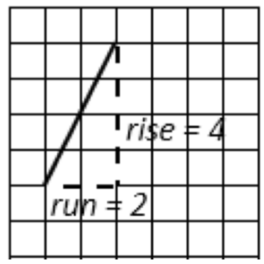
Learning Outcome
By the end of the lesson, you will be able to find the slope of a line by counting and dividing its rise and run.

Overview

The gradient of a line can be calculated using any 2 points on the line. It is calculated by dividing the change in y by the change in x between those 2 points.

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x}$$

To find the change in x and the change in y, choose any 2 points on the line draw a triangle on them, as shown to the right. The line connecting the 2 points is the hypotenuse of the right-angled triangle. The rise and run are the other 2 sides of the triangle.



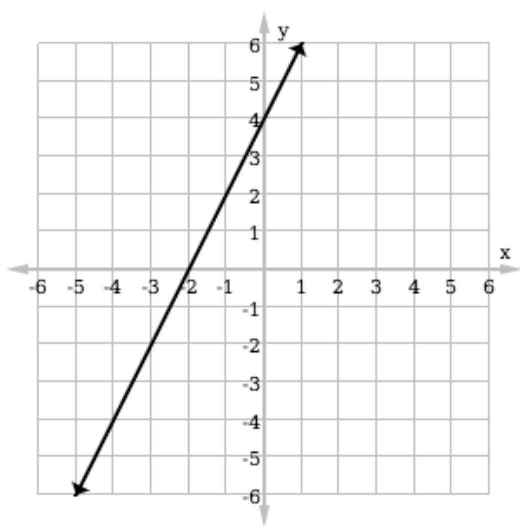
In this example, the rise is 4 and the run is 2. This gives
$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{4}{2} = 2$.

Start at 1 of the 2 points on the line, and count the rise and run. You will use a positive or negative sign for the rise and run depending on the direction in which you move. The directions for counting are similar to when we are plotting points:

- Going across to the right is positive. Going across to the left is negative.
- Going up is positive, and going down is negative.

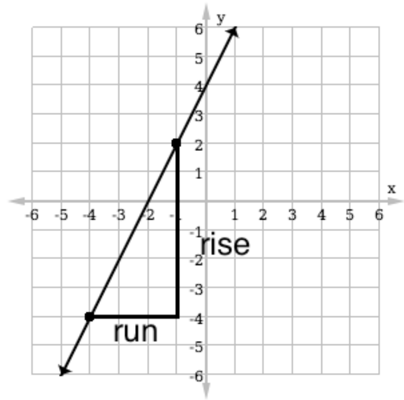
Solved Examples

1. Calculate the slope of the line:



Solution

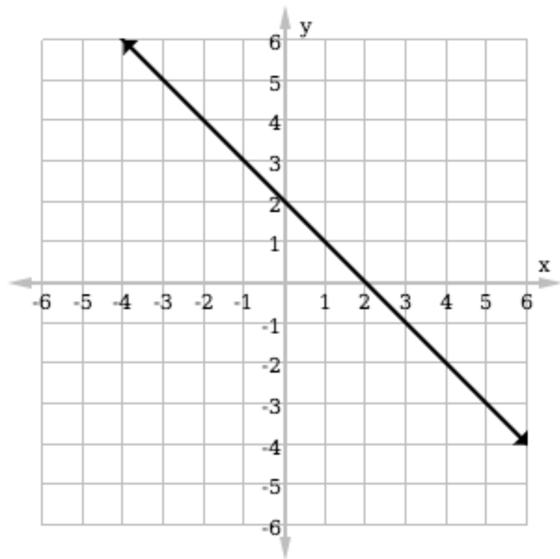
Choose any 2 points on the line. The points $(-4, -4)$ and $(-1, 2)$ are exactly on the line, so we can use those. Draw a triangle on them, and count the rise and run.



Rise = 6, Run = 3

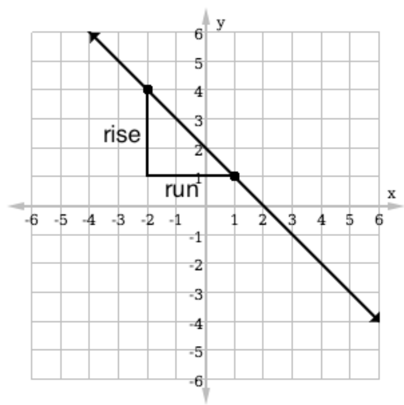
This gives slope $= \frac{\text{rise}}{\text{run}} = \frac{6}{3} = 2$

2. Calculate the gradient of the line:



Solution

Choose any 2 points on the line. The points $(-2, 4)$ and $(1, 1)$ are exactly on the line, so we can use those. Draw a triangle on them, and count the rise and run.



Rise $= -3$, Run $= 3$

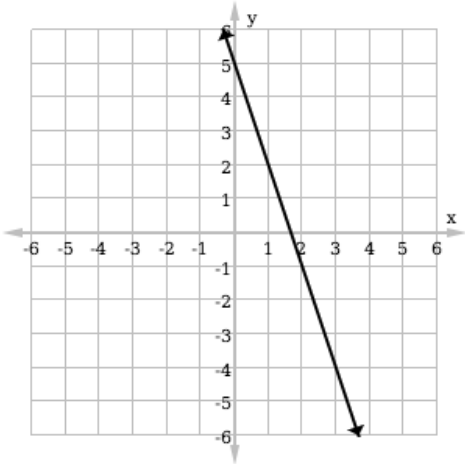
Note that the rise is negative, because we move down to get from $(-2, 4)$ to $(1, 1)$.

This gives slope $= \frac{\text{rise}}{\text{run}} = \frac{-3}{3} = -1$

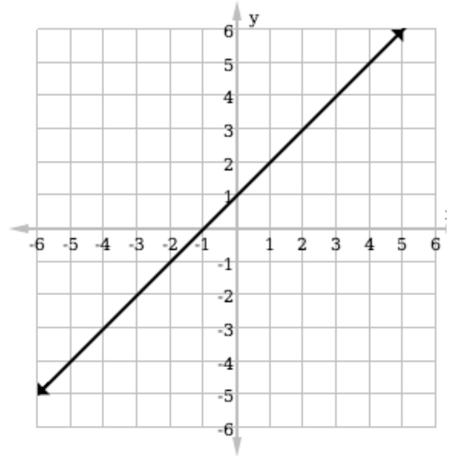
Practice

Calculate the gradient (or slope) of each line:

1.



2.



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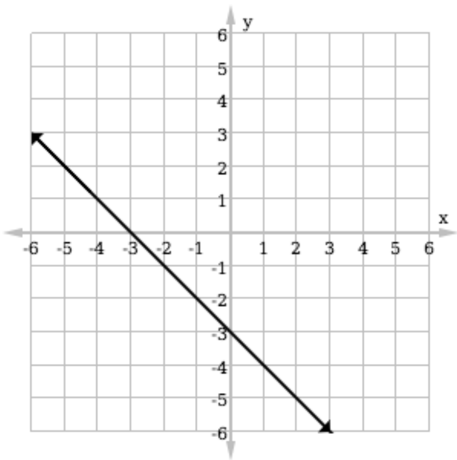

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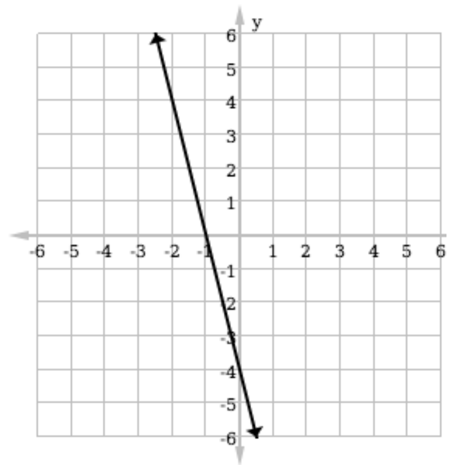

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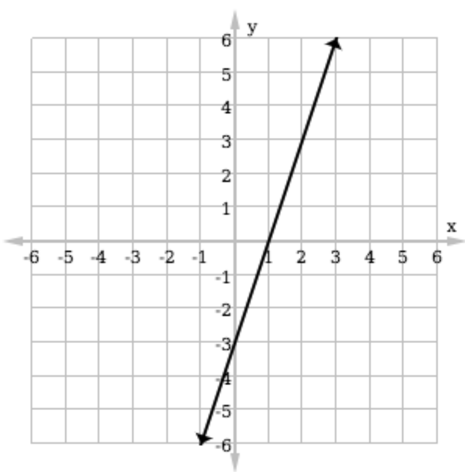

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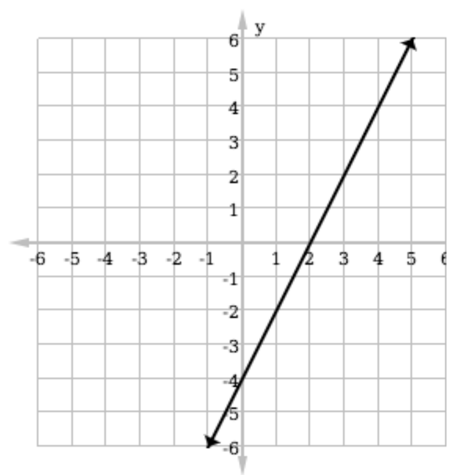


Lesson Title: Slope Formula	**Theme:** Algebra
Practice Activity: PHM-09-103	**Class:** JSS 3

Learning Outcome
By the end of the lesson, you will be able to find the slope of a line using 2 points (x_1, y_1) and (x_2, y_2) on the line, and the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$.

Overview

In the previous lesson, you calculated the slope (or gradient) from 2 points on the line by counting. In this lesson, you will continue to calculate the slope from 2 points, but you will use a formula.

Slope m is given by the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ for any 2 points (x_1, y_1) and (x_2, y_2) on a line. By subtracting the y-values, you find the rise. By subtracting the x-values, you find the run.

Solved Examples

1. Find the gradient of the line passing through $(0, 0)$ and $(1, 3)$.

   Solution
   You are given 2 points. Assign each point values of x and y for the formula. Let $(x_1, y_1) = (0, 0)$ and $(x_2, y_2) = (1, 3)$.

   Substitute the values of x and y into the formula and solve:

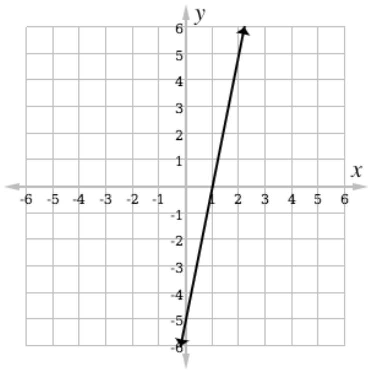
$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} && \text{Formula} \\ &= \frac{3-0}{1-0} && \text{Substitute } x\text{- and } y\text{-values} \\ &= \frac{3}{1} && \text{Simplify} \\ &= 3 \end{aligned}$$

2. Calculate the gradient of the line passing through $(-3, -5)$ and $(5, 11)$.

   Solution
   Substitute $(x_1, y_1) = (-3, -5)$ and $(x_2, y_2) = (5, 11)$ into the formula and solve:

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} && \text{Formula} \\ &= \frac{11-(-5)}{5-(-3)} && \text{Substitute } x\text{- and } y\text{-values} \\ &= \frac{11+5}{5+3} && \text{Simplify} \\ &= \frac{16}{8} \\ &= 2 \end{aligned}$$

3. Calculate the slope of the line:

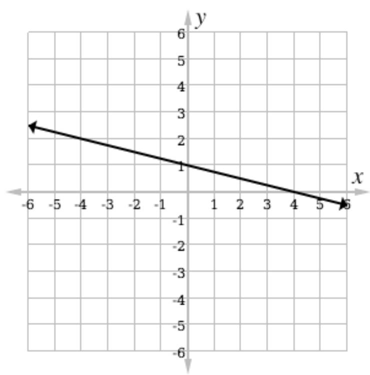


Solution

Choose any 2 points on the line. For example: $(x_1, y_1) = (1, 0)$ and $(x_2, y_2) = (2, 5)$.
Use these points to calculate m:

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} && \text{Formula} \\ &= \frac{5-0}{2-1} && \text{Substitute } x\text{- and } y\text{-values} \\ &= \frac{5}{1} && \text{Simplify} \\ &= 5 \end{aligned}$$

4. Calculate the slope of the line:



Solution

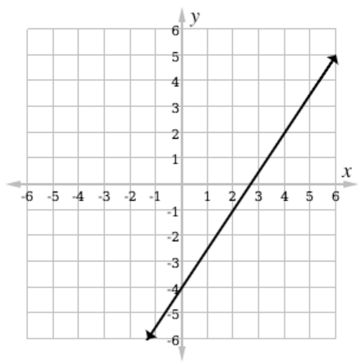
Choose any 2 points on the line. For example: $(x_1, y_1) = (0, 1)$ and $(x_2, y_2) = (4, 0)$.
Use these points to calculate m:

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} && \text{Formula} \\ &= \frac{0-1}{4-0} && \text{Substitute } x\text{- and } y\text{-values} \\ &= \frac{-1}{4} && \text{Simplify} \\ &= -\frac{1}{4} \end{aligned}$$

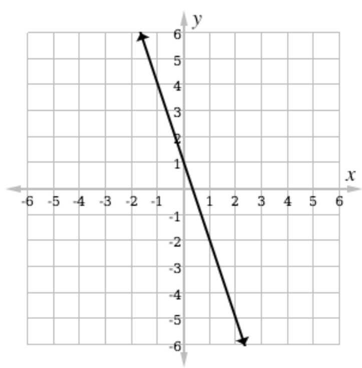
Practice

1. Find the gradient of the line which passes through the points.
    a. $(3, 1)$ and $(6, 10)$
    b. $(7, 5)$ and $(9, 12)$
    c. $(2, 3)$ and $(5, 9)$
    d. $S(2, 3)$ and $T(6, -5)$
    e. $L(0, -4)$ and $M(-3, 0)$

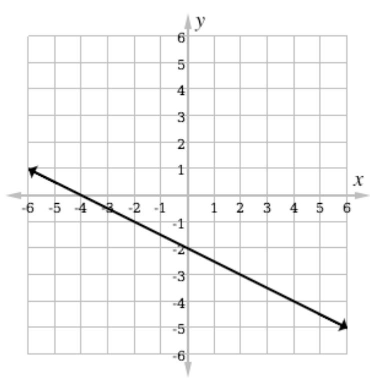
2. Calculate the slope of the line:



3. Calculate the slope of the line:



4. Calculate the slope of the line:



Lesson Title: Slope-intercept Form of Linear Equations	Theme: Algebra
Practice Activity: PHM-09-104	**Class:** JSS 3

Learning Outcomes
By the end of the lesson, you will be able to:
1. Identify the slope (m) and y-intercept (c) of a linear equation in slope-intercept form: $y = mx + c$.
2. Identify the y-intercept of a line on the Cartesian plane.

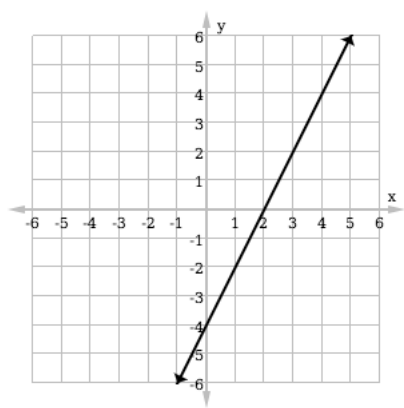
Overview

Recall that linear equations can be written in many different forms. In this lesson, you will work with linear equations written in **slope-intercept form**.

A linear equation in slope-intercept form is $y = mx + c$, where m is the slope and c is the y -intercept. The y-intercept is where the line crosses the y-axis.

For example, consider the line graphed to the right.
The y-intercept is -4.

You can use the methods from previous lessons to find that the slope of the line is 2. The linear equation in slope-intercept form is $y = 2x - 4$.



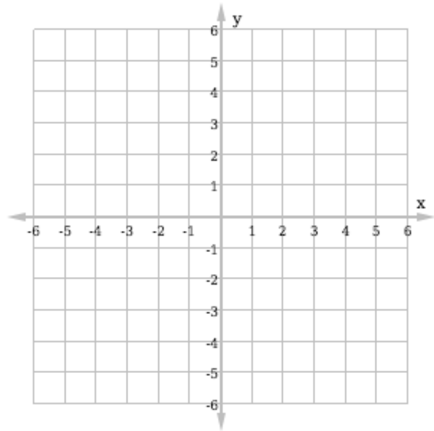
These are more examples of linear equations in slope-intercept form:

$$y = x + 15 \qquad y = 3x + 4 \qquad y = -2x - 1 \qquad y = -x + 4$$

Any linear equation can be written in slope-intercept form using balancing. When an equation is in this form, the coefficient of x is always the slope, and the constant term is always the y-intercept.

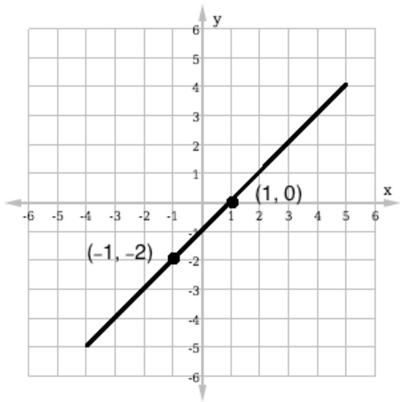
Solved Examples

1. The points with coordinates $(-1, -2)$ and $(1, 0)$ lie on a straight line.
    a. Plot the points and draw the line joining them.
    b. What is the slope of the line?
    c. What is the y-intercept of the line?
    d. Write down the equation of the line.



Solutions

a. Plotted points and line:



b. Calculate the slope by counting or using the formula: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-(-2)}{1-(-1)} = \frac{2}{2} = 1$

c. The y-intercept is -1.

d. Substitute the slope ($m = 1$) and the y-intercept ($c = -1$) into the slope-intercept form of a linear equation:
$y = mx + c$
$y = (1)x + (-1)$
$y = x - 1$

The equation of the line passing through points $(-1, -2)$ and $(1, 0)$ is $y = x - 1$.

2. Complete the table below. Each row has a linear equation in slope-intercept form, its gradient, and its y-intercept.

Equation	Gradient	y-intercept
$y = 5x - 4$	5	-4
$y = -x - 3$		
	-2	1
	12	-30
$y = 2x$		
$y = 7$		

Solution

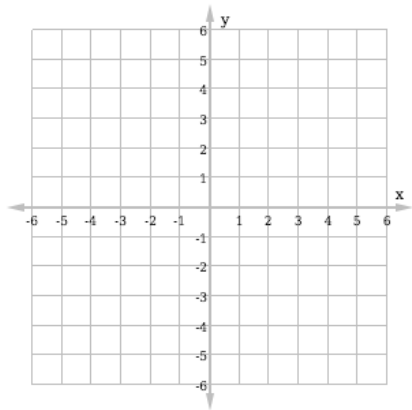
Given the equation of a line in slope-intercept form, you can identify the gradient and y-intercept. Given the gradient and y-intercept, you can write the linear equation. The answers are below.

Note that if the y-intercept is 0, there is no constant term. In this case, the line crosses the y-axis at the origin. Note that if the gradient is 0, there is no term with x. A line with gradient 0 is a horizontal line.

Equation	Gradient	y -intercept
$y = 5x - 4$	5	-4
$y = -x - 3$	-1	-3
$y = -2x + 1$	-2	1
$y = 12x - 30$	12	-30
$y = 2x$	2	0
$y = 7$	0	7

Practice

1. Write the equation of a line with gradient 3 and which passes through the origin, (0, 0).
2. Write the equation of a line with gradient -1 and y-intercept 4.
3. The points with coordinates $(-2,-1)$ and $(1,5)$ lie on a straight line.
    a. Plot the points and draw the line joining them.
    b. What is the slope of the line?
    c. What is the y-intercept of the line?
    d. Write down the equation of the line.



4. Find the equation of a line which passes through points (0, 0) and (3, 6).

Lesson Title: Graphing Lines in Slope-intercept Form	**Theme:** Algebra
Practice Activity: PHM-09-105	**Class:** JSS 3

Learning Outcome
By the end of the lesson, you will be able to graph a linear equation in slope-intercept form using a table of values, and verify its slope and y-intercept.

Overview

The slope-intercept form of a line is given by $y = mx + c$, where m is the gradient and c is the y-intercept of the line. The y-intercept is the point where the line crosses the y-axis.

In previous lessons, you created a table of values to graph a point. However, you only need 2 points to graph a line. It is not necessary to find many points, as you did with a table of values.

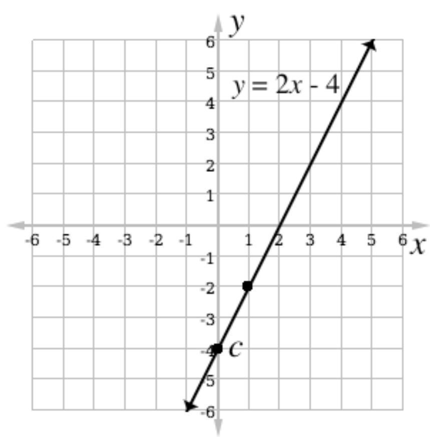
In this lesson, you will graph a line from its slope-intercept form. You will identify 2 points and plot them on the plane. It is easiest to use the y-intercept and 1 other point. Follow these steps:

- Identify and plot the y-intercept.
- Find another point on the line by substituting any value of x and finding the corresponding y-value. Plot that point.
- Draw a line that passes through the y-intercept and the other point you plotted.
- Label the line with its equation.

Solved Examples

1. Graph $y = 2x - 4$ on a Cartesian plane.

Solution
Identify and plot the y-intercept, $c = -4$. It is point $(0, -4)$.



Find any other point on the line. For example, take $x = 1$ and find the corresponding value of y:

$$y = 2(1) - 4 = 2 - 4 = -2$$

This gives point $(1, -2)$. Plot $(1, -2)$ on the plane. Draw a line that passes through both points. Label the line $y = 2x - 4$.

2. Graph $y = -3x + 1$

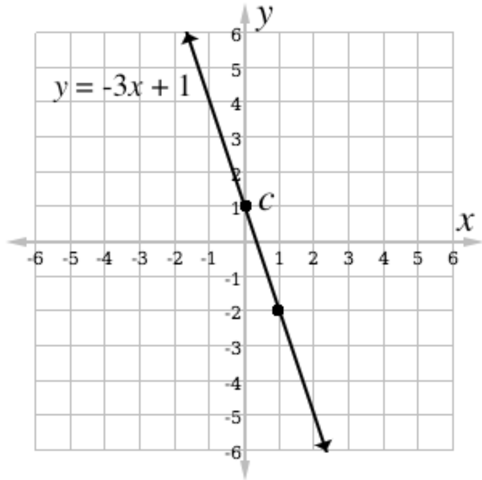
   Solution

   Identify and plot the y-intercept, $c = 1$.

   Find any other point on the line. For example, take $x = 1$ and find the corresponding value of y:

   $y = -3(1) + 1 = -3 + 1 = -2$

   This gives point $(1, -2)$. Plot $(1, -2)$ on the plane. Draw a line that passes through both points. Label the line $y = -3x + 1$.



3. Graph $y = \frac{3}{2}x - 2$

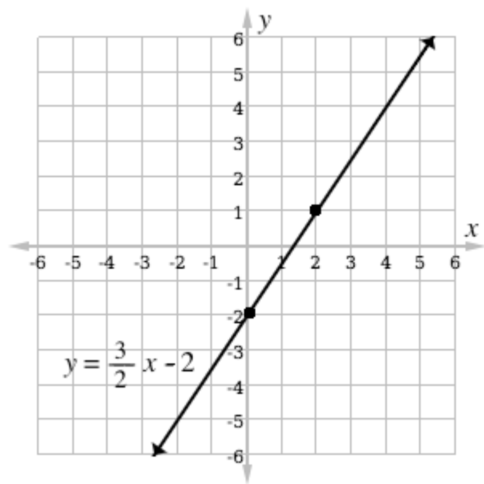
   Solution

   Identify and plot the y-intercept, $c = -2$.

   Find any other point on the line. In this example, we will take $x = 2$ because it will cancel the fraction. Substitute and find the corresponding value of y:

   $y = \frac{3}{2}(2) - 2 = 3 - 2 = 1$

   This gives point $(2, 1)$. Plot $(2, 1)$ on the plane. Draw a line that passes through both points. Label the line $y = \frac{3}{2}x - 2$.



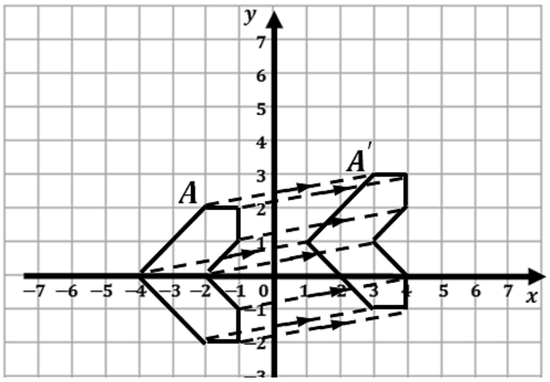
Practice

1. Draw the graph of $y = -2x + 1$
2. Draw the graph of $y = \frac{1}{2}x + 3$
3. Graph $y = 2x - \frac{1}{2}$
4. Draw the graph of $y = \frac{5}{3}x - 2$
5. Graph $y = 4 - x$

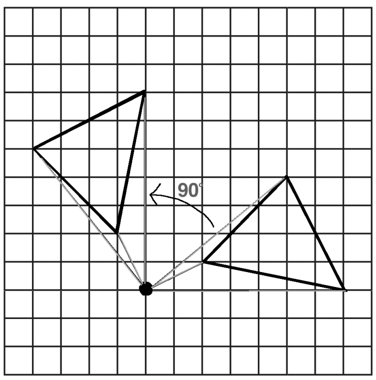
Answer Key – JSS 3 Term 2

Lesson Title: Review of Transformations
Practice Activity: PHM-09-046

1. Translation:

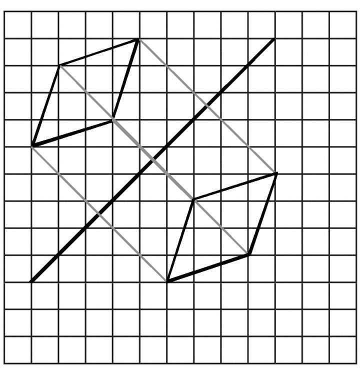


2. Rotation:

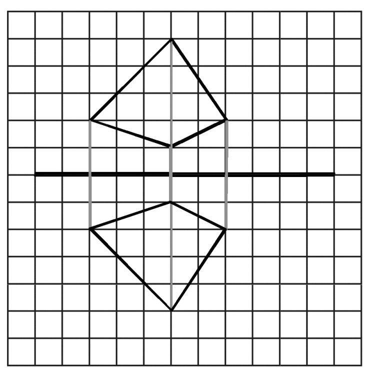


3. Reflection:

a.

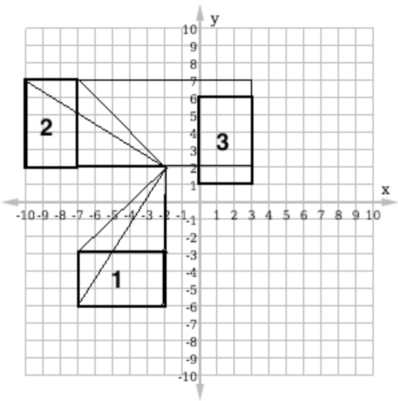


b.

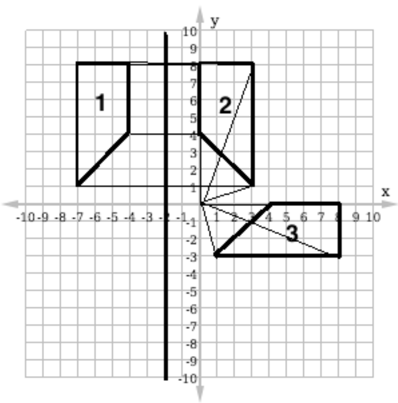


Lesson Title:	Combining Transformations
Practice Activity:	PHM-09-047

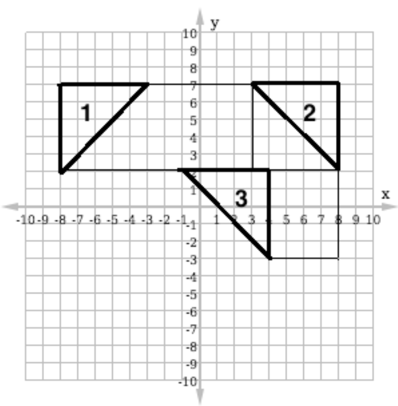
1. See below:



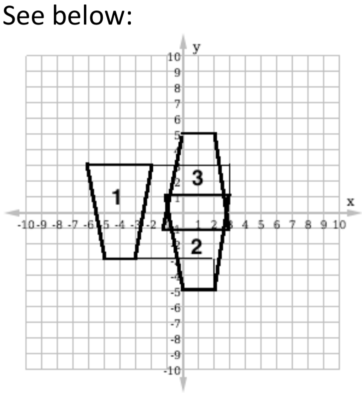
2. See below:



3. See below:



4. See below:

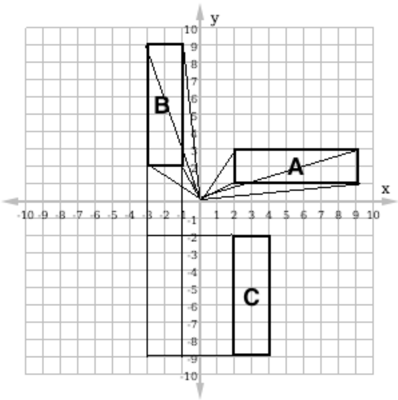


Lesson Title: Congruency	
Practice Activity: PHM-09-048	

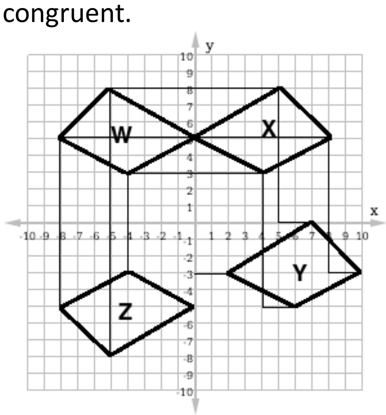
1. C, D and F are congruent to A.
2. A and E are congruent. C, D and G are congruent.
3. a. B is created by reflecting A in the x-axis, C is created by reflecting B in the y-axis, D is created by translating A; b All of the shapes are congruent to A: B, C and D.

Lesson Title: Practice with Congruency	
Practice Activity: PHM-09-049	

1. a. See diagram below; b. See diagram below; c. All shapes are congruent.



2. a. See diagram below; b. See diagram below; c. See diagram below; c. All shapes are congruent.



Lesson Title: Length Measurement of 2 Congruent Shapes	
Practice Activity: PHM-09-050	

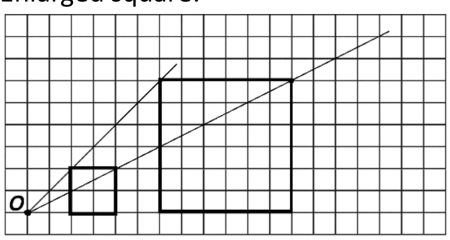
1. Yes, the shapes are congruent. If the lengths of the hypotenuse and a corresponding side of 2 right-angled triangles are equal, then the triangles are congruent.
2. No, the shapes are not congruent. They have the same shape, but the sides have different lengths.
3. Yes, the shapes are congruent. The corresponding sides have the same length.

Lesson Title: Angles of Congruent Shapes
Practice Activity: PHM-09-051

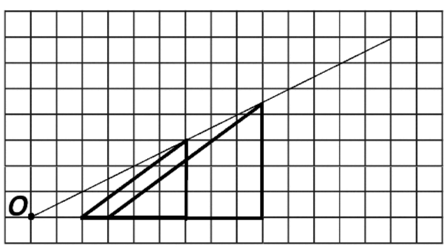
1. a. $|AB| = |XY|$, $|BC| = |YZ|$, $|CA| = |ZX|$; $\angle A = \angle X$, $\angle B = \angle Y$, $\angle C = \angle Z$
   b. $|RS| = |VU|$, $|ST| = |UW|$, $|TR| = |WV|$; $\angle R = \angle V$, $\angle S = \angle U$, $\angle T = \angle W$
2. a. Yes, SAS; b. Yes, ASA; c. there is not enough information; d. Yes, SSS

Lesson Title: Enlargement
Practice Activity: PHM-09-052

1. B is an enlargement of A with scale factor $\frac{1}{2}$. E is an enlargement of A with a scale factor of $1\frac{1}{2}$.
2. Shapes 2 and 5 are not enlargements of 1.
3. Enlarged square:



4. Enlarged triangle:



Lesson Title: Similarity
Practice Activity: PHM-09-053

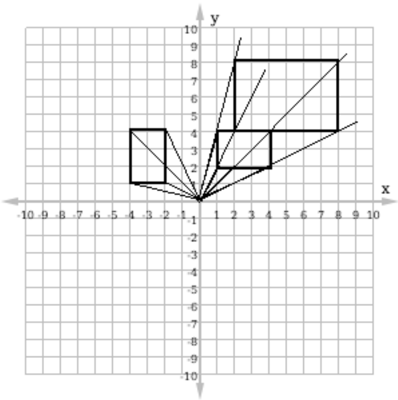
1. The triangles are similar. The corresponding angles are all equal; A=P, B=Q, C=R.
2. a. $\frac{1}{2}$; b. BC = 8 cm, DE = 2.5 cm
3. a. 3; b. NO = 2 cm, RS = 6 cm, ST = 6 cm

Lesson Title:	Comparing Congruent and Similar Shapes
Practice Activity:	PHM-09-054

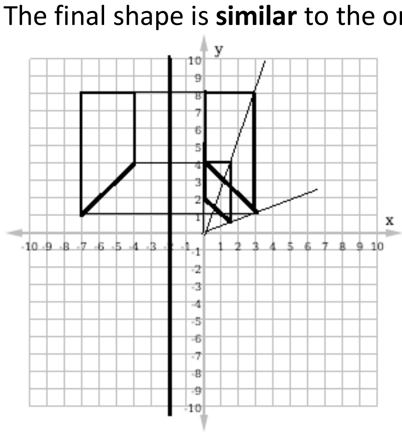
1. A, D, G, H and M are congruent. F, C, J are congruent. The second group (F, C, J) is an enlargement of the first group (A, D, G, H, M) with a scale factor of 2.
2. a. The triangles are similar. The corresponding angles are equal, but the sides have a different length. b. The triangles are similar. The corresponding angles are equal, but the sides have a different length.
3. a. The triangles are similar. The corresponding angles are equal, but the sides have a different length. b. CB = 5 m and CE = 3 m.

Lesson Title:	Transformation Practice
Practice Activity:	PHM-09-055

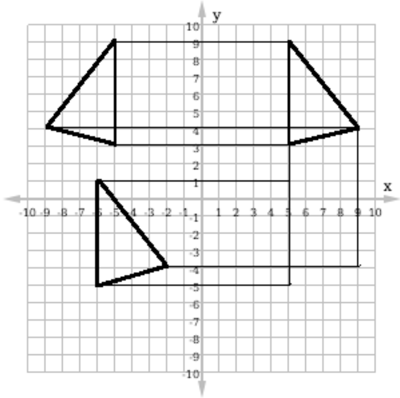
1. The final shape is **similar** to the original. Transformations are shown below:



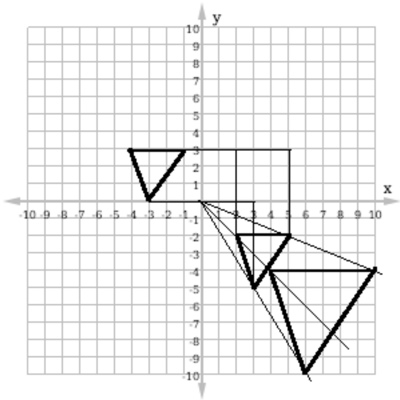
2. The final shape is **similar** to the original. Transformations are shown below:



3. The final shape is **congruent** to the original. Transformations are shown below:



4. The final shape is **similar** to the original. Transformations are shown below:



Lesson Title: Introduction to Trigonometry	
Practice Activity: PHM-09-056	

1. a. $\sin\theta = \frac{1}{2}$; $\cos\theta = \frac{5}{6}$; $\tan\theta = \frac{3}{5}$

b. $\sin\theta = \frac{8}{11}$; $\cos\theta = \frac{5}{11}$; $\tan\theta = 1\frac{3}{5}$

2. $\sin a = \frac{6}{7}$; $\cos a = \frac{4}{7}$; $\tan a = 1\frac{1}{2}$; $\sin b = \frac{4}{7}$; $\cos b = \frac{6}{7}$; $\tan b = \frac{2}{3}$

Lesson Title: Sine	
Practice Activity: PHM-09-057	

1. $b = 10.4$ cm
2. $z = 23.0$ cm
3. $x = 8.0$ m
4. $y = 10.6$ cm

Lesson Title:	Cosine
Practice Activity:	PHM-09-058

1. $u = 17.7$ cm
2. $s = 5.7$ cm
3. $z = 19.3$ cm
4. $y = 13.9$ m

Lesson Title:	Tangent
Practice Activity:	PHM-09-059

1. a. $z = 11.9$ m; b. $x = 24.2$ cm; c. $y = 5.8$ m; d. $b = 5.6$ m
2. $\tan 30° = \frac{\sin 30°}{\cos 30°} = \frac{0.5000}{0.8660} = 0.5774$

Lesson Title:	Applying the Trigonometric Ratios
Practice Activity:	PHM-09-060

1. $x = 14$ cm, $y = 14$ cm
2. $c = 6$ cm, $d = 4$ cm
3. $x = 7$ m, $y = 12$ m
4. $a = 7$ cm, $b = 14$ cm

Lesson Title:	Trigonometric Tables for Sine
Practice Activity:	PHM-09-061

1. a. 0.0872; b. 0.9511
2. 0.1201
3. 0.3592
4. 0.9426
5. a. 0.9884; b. 0.4266; c. 0.9770
6. a. 0.2504; b. 0.4810

Lesson Title:	Trigonometric Tables for Cosine
Practice Activity:	PHM-09-062

1. a. 0.9962; b. 0.3090
2. 0.9928
3. 0.9333
4. 0.3338
5. 0.1516

6. 0.9044
7. 0.2130
8. a. 0.7373; b. 0.6053

Lesson Title:	Trigonometric Tables for Tangent
Practice Activity:	PHM-09-063

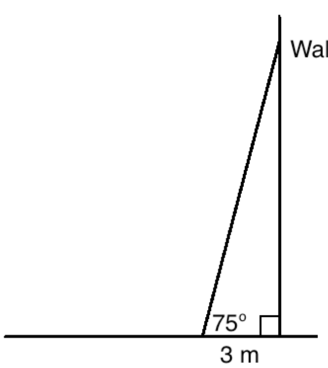
1. 0.1228
2. 0.2290
3. 0.4674
4. 6.314
5. 3.251
6. 0.5839
7. 0.8770
8. 47.74
9. 0.7400
10. 0.9244

Lesson Title:	Trigonometric Practice
Practice Activity:	PHM-09-064

1. 15.5 cm
2. 5.5 cm
3. 17.5 m
4. 12.7 m
5. 11.7 cm
6. 10.2 m

Lesson Title:	Trigonometric Word Problems
Practice Activity:	PHM-09-065

1. 76.1 m^2
2. 1.732
3. a. See diagram below; b. 11.6 m; c. 11.2 m



4. $x = 47°$
5. $y = 20.1°$

Lesson Title:	Changing the Subject of a Formula
Practice Activity:	PHM-09-066

1. a. $b = \frac{a}{13}$; b. $b = a + 3$; c. $b = 4a$; d. $b = \frac{a}{2} + 4$
2. $r = \sqrt{p - 1}$
3. $t = x^2 - u$
4. $a = \frac{p}{2} - b$
5. $F = \frac{9}{5}C + 32$

Lesson Title:	Combining Like Terms
Practice Activity:	PHM-09-067

1. $2x + y$
2. $-2a + 7b$
3. $m + 14n$
4. $5xy - xp$
5. $12mn - m$
6. $2pq + 8qr$
7. $8p^2q - 3pq^2$
8. $9xyz - xz$
9. $2ab + 4a + 4$

Lesson Title:	Solving Linear Equations
Practice Activity:	PHM-09-068

1. $m = 7$
2. $y = 4$
3. $n = 20$
4. $y = 3$
5. $x = 1$
6. $y = 1\frac{1}{4}$ or $y = 1.25$
7. $x = -1$
8. $m = 0$
9. $y = 7$

Lesson Title:	Substitution
Practice Activity:	PHM-09-069

1. a. 10; b. 18; c. 58
2. a. 4; b. 6; c. 2; d. 8; e. -3
3. 6
4. a. 15; b. 35
5. 3
6. 16
7. a. 38 b. $\frac{1}{10}$

Lesson Title:	Practice Solving Algebraic Expressions
Practice Activity:	PHM-09-070

1. a. $5x + 8y$; b. $7ab - 3bc$; c. $x^2 - x + 12$
2. $y = 6$
3. $y = 3$
4. a. 4; b. 16; c. 0
5. 3675

Lesson Title:	Multiplying an Algebraic Expression by an Integer
Practice Activity:	PHM-09-071

1. $5x - 20$
2. $-21y + 28$
3. $-2m - 2n$
4. $6v + 9$
5. $-2x + 4y$
6. $-24m + 16n$
7. $4a + 6$
8. $10a - 30b$

Lesson Title:	Multiplying Variables
Practice Activity:	PHM-09-072

1. y^7
2. y^3x^2
3. $6xy^6$
4. $10x^5y$
5. x^2y^2
6. $5x^3y^3$
7. $64x^3y^3$

8. $36x^2y^6$
9. $5x^2y^2$
10. $9x^6y^{12}$

Lesson Title:	Multiplying an Algebraic Expression by a Variable
Practice Activity:	PHM-09-073

1. $x^2 - 2x$
2. $4x - x^2$
3. $2x^2 - x$
4. $x^3 - 3x^2$
5. $-x^2 - 12x$
6. $-9x + x^2 - xy$
7. $3x^2 - 12x$
8. $-8x^2 - 2x$
9. $4x^3 + 4x^2$
10. $x^2 - x$

Lesson Title:	Algebraic Expression Story Problems
Practice Activity:	PHM-09-074

1. $x^2 + 3x$
2. $5x + 10$
3. $x + 11$
4. $2x + 6$
5. $3x - 2$

Lesson Title:	Introduction to Quadratic Equations
Practice Activity:	PHM-09-075

1. a. Not a quadratic equation;
   b. Not a quadratic equation;
   c. Quadratic equation, $a = 12, b = -7, c = 8$
   d. Quadratic equation, $a = 1, b = 20, c = 35$
2. a. Quadratic equation; $a = 5, b = 5, c = 0$
   b. Quadratic equation; $a = 1, b = 1, c = -8$
   c. Quadratic equation; $a = 1, b = -1, c = 0$
   d. Not a quadratic equation.

Lesson Title:	Multiplying 2 Binomials
Practice Activity:	PHM-09-076

1. $7x^2 + 30x + 8$
2. $p^2 - 11p + 30$
3. $y^2 - 25$
4. $m^2 + m - 6$
5. $8x^2 - 20x + 8$
6. $2y^2 + 8y - 120$
7. $2x^2y^2 + xy - 10$

Lesson Title:	Practice with Multiplying 2 Binomials
Practice Activity:	PHM-09-077

1. $x^2 - 8x + 16 = 0$
2. $n^2 + 6n + 9 = 0$
3. $2x^2 + 14x + 26 = 0$
4. $2y^2 + 5y - 38 = 0$
5. $x^2 + 10x + 39 = 0$

Lesson Title:	Review of Factorisation: Integers
Practice Activity:	PHM-09-078

1. $4(x + 3)$
2. $7(x - 3y)$
3. $2(7 - x)$
4. $10(2x + 3)$
5. $2(2y - 3)$
6. $2(5s + 4t)$
7. $6(2 - 3p)$
8. $3(x^2 + 4x + 10)$
9. $3(3x^2 - 4)$
10. $2(x^3 + 20x^2 + 6x + 12)$

Lesson Title:	Review of Factorisation: Variables
Practice Activity:	PHM-09-079

1. $y(x + 1)$
2. $y(x + z)$

3. $a(2a-1)$
4. $x(3x+8)$
5. $y^2(y+1)$
6. $x(x^2+7x-3)$
7. $3x(x^2+3x-6)$
8. $5x^2(x-3)$
9. $5a(a\ +\ 2)$
10. $7x^2(2x+1)$

Lesson Title:	Factorisation of Quadratic Equations
Practice Activity:	PHM-09-080

1. a. $(x+4)(x+5)$; b.$(p-8)(p-3)$; c. $(y-5)(y-5)$ or $(y-5)^2$; d. $(n-2)(n+3)$
2. $(x-7)$
3. $(u+1)$

Lesson Title:	Practice with Factorisation of Quadratic Equations
Practice Activity:	PHM-09-081

1. $(x+9)(x+2)$
2. $(x+1)(2x-3)$
3. $(x+5)(x+1)$
4. $(x-11)(x-1)$
5. $(x+3)(2x+1)$
6. $(x+11)(x-2)$
7. $(x+1)(x-1)$
8. $(y+6)(y-6)$
9. $(1+x)(1-2x)$
10. $(2+3x)(1-2x)$

Lesson Title:	Factorisation by Completing the Squares Method
Practice Activity:	PHM-09-082

1. $(x-4)^2-14$
2. $(x-5)^2-18$
3. $(x+2)^2-9$
4. $(x+3)^2-3$
5. $(x-2)^2+1$

Lesson Title:	Practice with Completing the Squares Method
Practice Activity:	PHM-09-083

1. $(x+3)^2-8$
2. $(x-1)^2-6$
3. $(x+1)^2-2$
4. $(x+5)^2-13$
5. $(x-3)^2-5$
6. $(x+5)^2$
7. $(x-6)^2$

Lesson Title:	Practice with Factorisation
Practice Activity:	PHM-09-084

1. $3(x+2y-4)$
2. $2y(y+5)$
3. $(x+3)(x+4)$
4. $(2x+1)(x-1)$
5. $6(2x-3)$
6. $(x-1)^2+11$
7. $(x+6)(x-3)$
8. $3x(x-3)$
9. $(1-5x)(2+x)$
10. $(2+y)(2-y)$

Lesson Title:	Story Problems with Quadratic Expressions
Practice Activity:	PHM-09-085

1. $x^2+4x-21$
2. $l=(x+3)$ and $w=(x+2)$
3. x^2+5x
4. $(x+4)$
5. $(x+4)$
6. $(2x-1)$

Lesson Title: Introduction to Linear Equations in 2 Variables	
Practice Activity: PHM-09-086	

1. a. $(-1,5)$; b. $(0,-0.8)$; c. $(0,0)$; d. $\left(\frac{1}{2},-\frac{1}{2}\right)$
2. a. $x - y = 0$ is a linear equation in 2 variables;
   b. $y^2 - 8 = 12$ is **not** a linear equation in 2 variables, it is a quadratic equation in 1 variable;
   c. $3x + 8y = 21$ is a linear equation in 2 variables;
   d. $y = 4x$ is a linear equation in 2 variables;
   e. $x + 1 = 13$ is **not** a linear equation in 2 variables, it is a linear equation in 1 variable.
3. $(-3,6)$
4. $(0,-1)$
5. $(1,3)$
6. $(10,5)$

Lesson Title: Verifying Solutions to Linear Equations	
Practice Activity: PHM-09-087	

1. Yes, $(1,3)$ is a solution. You should find LHS = RHS.
2. No, $(1,-3)$ is not a solution. You should find LHS ≠ RHS.
3. Yes, $(2,-4)$ is a solution. You should find LHS = RHS.
4. a. Yes; b. Yes; c. Yes; d. No
5. a. No; b. Yes; c. Yes

Lesson Title: Finding Solutions to Linear Equations I	
Practice Activity: PHM-09-088	

1. $(2,5)$
2. $(1,6)$
3. a. $(2,3)$; b. $(0,-3)$; c. $(4,9)$; d. $(1,0)$
4. a. $(-1,-7)$; b. $(3,1)$; c. $(5,5)$; d. $(1,-3)$

Lesson Title: Finding Solutions to Linear Equations II	
Practice Activity: PHM-09-089	

1. $y = 3$
2. $x = -2$
3. $x = -8$
4. $(3,1)$
5. $(0,4)$
6. $(4,-1)$
7. $x = -8$
8. $x = 4$
9. $x = 1$

Lesson Title: Practice Solving Linear Equations
Practice Activity: PHM-09-090

1. Yes, $(-1, -1)$ is a solution. You should find LHS = RHS.
2. Yes, $(-1,2)$ is a solution. You should find LHS = RHS.
3. a. $(-3, -2)$; b. $(-17,12)$; c. $(-2, -3)$
4. a. $(-1,2)$; b. $(-\frac{3}{2}, 5)$; c. $(-1, 2)$

Lesson Title: Solving Linear Equation Story Problems I
Practice Activity: PHM-09-091

1. a. $y = x + 8$; b. 17 years old
2. a. $x = 3y - 2$; b. 34 years old
3. a. $y = 4x + 60$; b. $x = 30$
4. a. $P = 6x + 8$; b. 20 cm; c. $x = 10$ cm

Lesson Title: Solving Linear Equation Story Problems II
Practice Activity: PHM-09-092

1. a. $P = 6x - 9$; b. $P = 21$ cm; c. $x = 11$ cm
2. a. $S = 2n + 2$; b. The 2 numbers are 33 and 35.
3. 15 years old
4. $x = 6$

Lesson Title: Table of Values I
Practice Activity: PHM-09-093

1.

x	-3	-2	-1	0	1	2	3
y	-3	-1	1	3	5	7	9

2.

x	-4	-3	-2	-1	0	1	2
y	-18	-15	-12	-9	-6	-3	0

3.

x	-2	-1	0	1	2	3	4
y	7	5	3	1	-1	-3	-5

4.

x	-3	-2	-1	0	1	2	3
y	-1	1	3	5	7	9	11

Lesson Title:	Table of Values II
Practice Activity:	PHM-09-094

1.

x	-3	-2	-1	0	1	2	3
y	1	1.5	2	2.5	3	3.5	4

2. You may give your answers as fractions (row 2) or decimals (row 3):

x	-3	-2	-1	0	1	2	3
y	0	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{3}{5}$	$\frac{4}{5}$	1	$1\frac{1}{5}$
y	0	0.2	0.4	0.6	0.8	1	1.2

3. You may give your answers as fractions (row 2) or decimals (row 3):

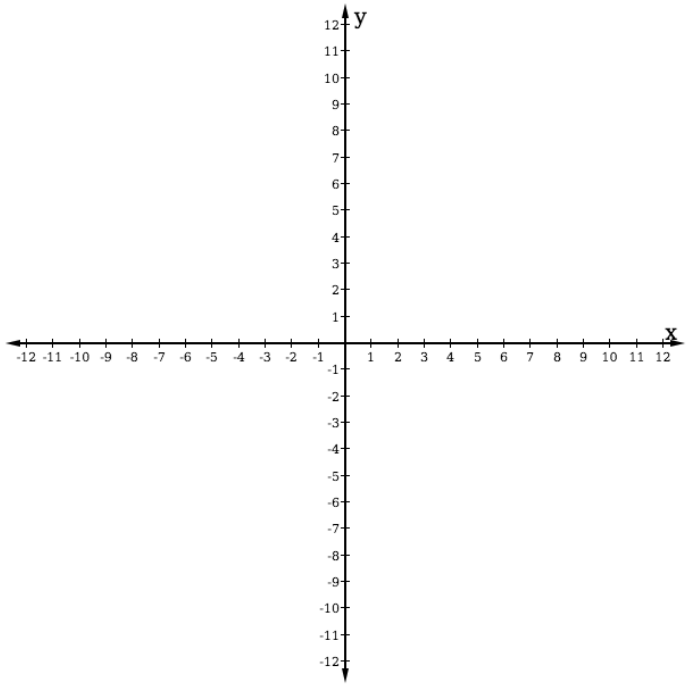
x	-3	-2	-1	0	1	2	3
y	$-6\frac{1}{2}$	$-4\frac{1}{2}$	$-2\frac{1}{2}$	$-\frac{1}{2}$	$1\frac{1}{2}$	$3\frac{1}{2}$	$5\frac{1}{2}$
y	-6.5	-4.5	-2.5	-0.5	1.5	3.5	5.5

4. You may give your answers as fractions (row 2) or decimals (row 3):

x	-3	-2	-1	0	1	2	3
y	$-1\frac{1}{2}$	0	$1\frac{1}{2}$	3	$4\frac{1}{2}$	6	$7\frac{1}{2}$
y	-1.5	0	1.5	3	4.5	6	7.5

Lesson Title: Review of the Cartesian Plane
Practice Activity: PHM-09-095

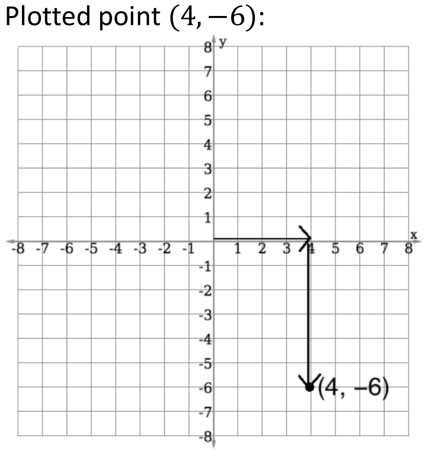
1. Cartesian plane:



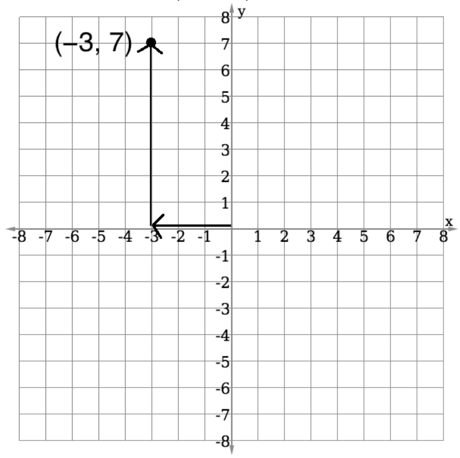
2. $A(-6,5)$, $B(0,6)$, $C(6,4)$, $D(3,0)$, $E(6,-5)$, $F(0,-5)$, $G(-1,-7)$, $H(-7,-3)$

Lesson Title: Plotting Points on the Cartesian Plane
Practice Activity: PHM-09-096

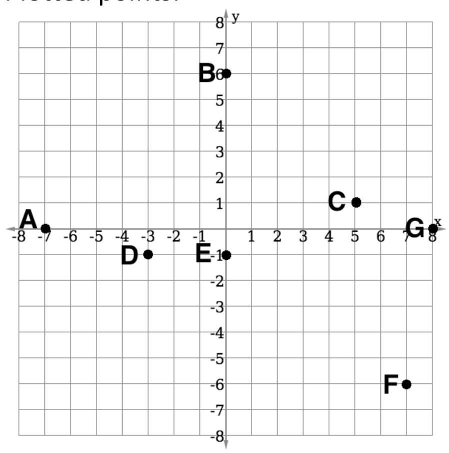
1. Plotted point $(4,-6)$:



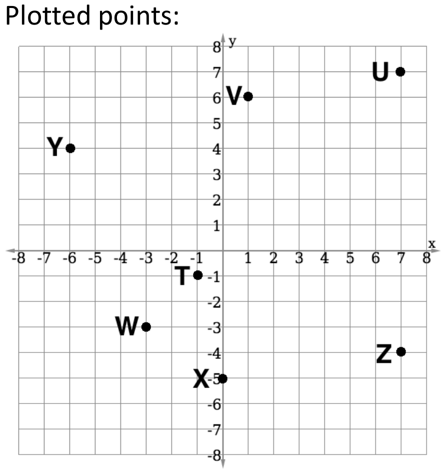
2. Plotted point $(-3, 7)$:



3. Plotted points:

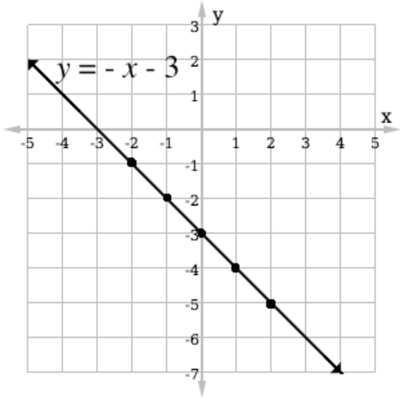


4. Plotted points:

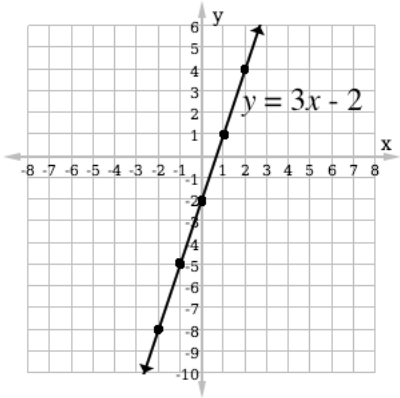


Lesson Title: Plotting Points from a Table of Values	
Practice Activity: PHM-09-097	

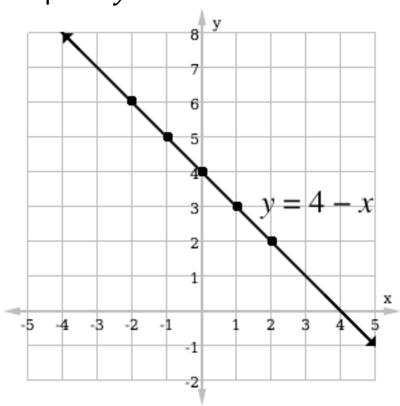
1. Graph of $y = -x - 3$:



2. Graph of $y = 3x - 2$



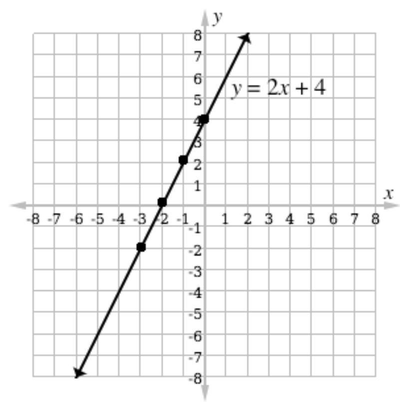
3. Graph of $y = 4 - x$



Lesson Title:	Graphing a Line I
Practice Activity:	PHM-09-098

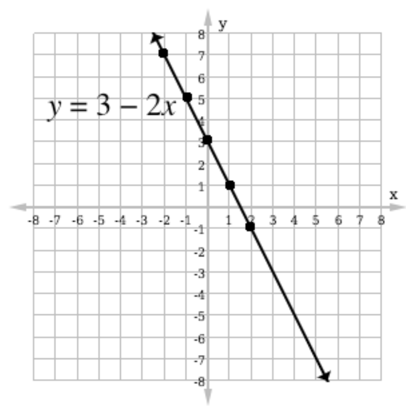
1.

x	-3	-2	-1	0
y	-2	0	2	4



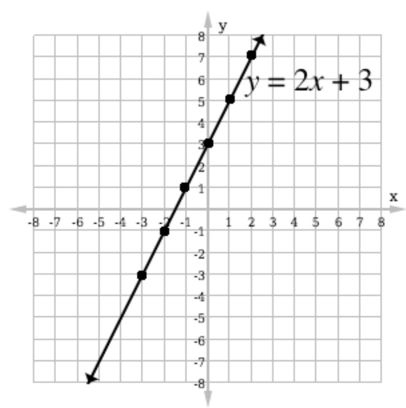
2.

x	-2	-1	0	1	2
y	7	5	3	1	-1



3.

x	-3	-2	-1	0	1	2
y	-3	-1	1	3	5	7

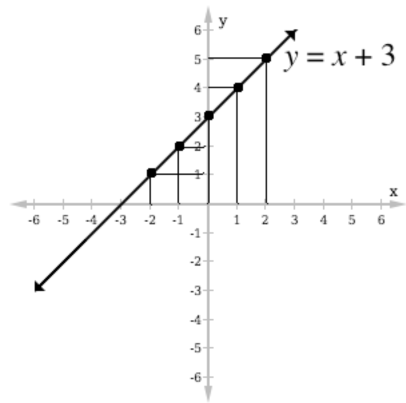


Lesson Title: Graphing a Line II
Practice Activity: PHM-09-099

The tables below give example values. You will create your own table, which may have different values.

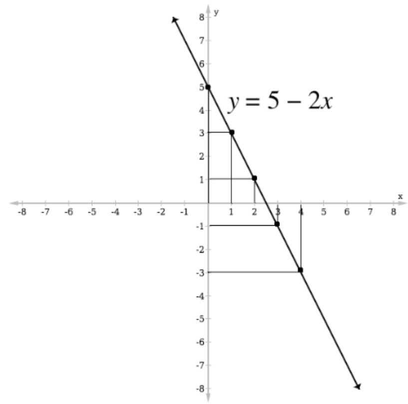
1.

x	-2	-1	0	1	2
y	1	2	3	4	5



2.

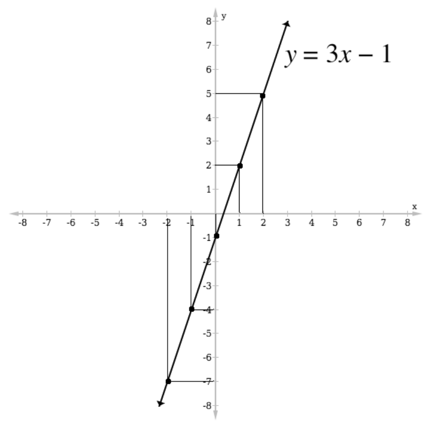
x	0	1	2	3	4
y	5	3	1	-1	-3



3.

x	-2	-1	0	1	2
y	-7	-4	-1	2	5

The graph is not the correct size. Make sure your own axes have a scale of 1 cm to each unit.

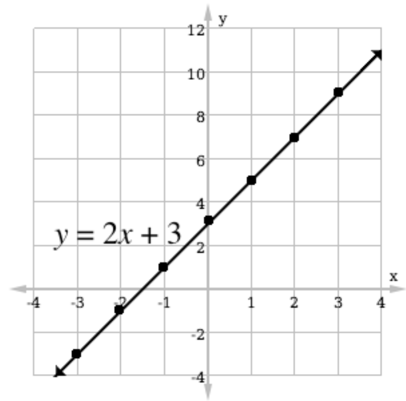


Lesson Title: Graphing a Line III
Practice Activity: PHM-09-100

1.

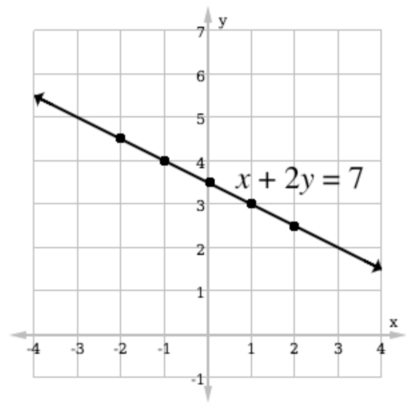
x	-3	-2	-1	0	1	2	3
y	-3	-1	1	3	5	7	9

The graph is not the correct size. Make sure your own axes have a scale of 2 cm to 1 unit on the x axis and 2 cm to 2 units on the y axis.



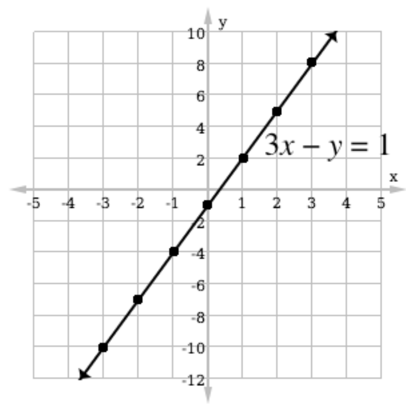
2.

x	-2	-1	0	1	2
y	4.5	4	3.5	3	2.5



3.

x	-3	-2	-1	0	1	2	3
y	-10	-7	-4	-1	2	5	8



Lesson Title: Introduction to Slope
Practice Activity: PHM-09-101

1. Negative slope
2. Positive slope
3. Negative slope
4. Negative slope
5. Positive slope
6. Negative slope

Lesson Title:	Finding the Slope of a Line
Practice Activity:	PHM-09-102

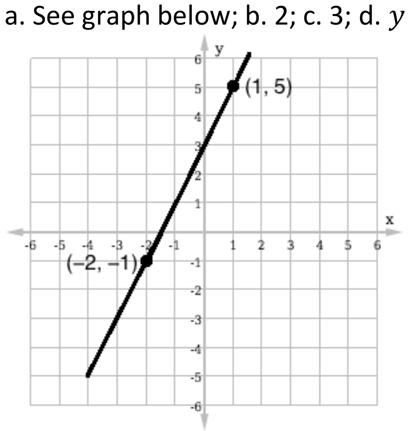
1. -3
2. 1
3. -1
4. -4
5. 3
6. 2

Lesson Title:	Slope Formula
Practice Activity:	PHM-09-103

1. a. $m = 3$; b. $m = 3\frac{1}{2}$; c. $m = 2$; d. $m = -2$; e. $m = -1\frac{1}{3}$
2. $m = 1\frac{1}{2}$
3. $m = -3$
4. $m = -\frac{1}{2}$

Lesson Title:	Slope-intercept Form of Linear Equations
Practice Activity:	PHM-09-104

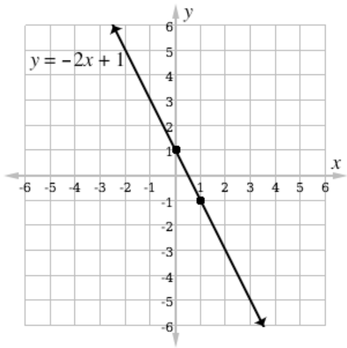
1. $y = 3x$
2. $y = -x + 4$
3. a. See graph below; b. 2; c. 3; d. $y = 2x + 3$



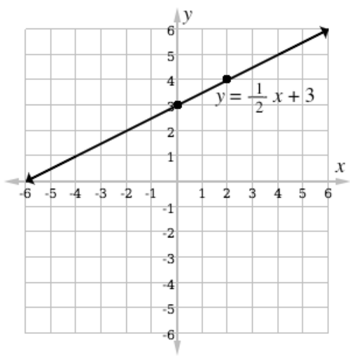
4. $y = 2x$

Lesson Title:	Graphing Lines in Slope-intercept Form
Practice Activity:	PHM-09-105

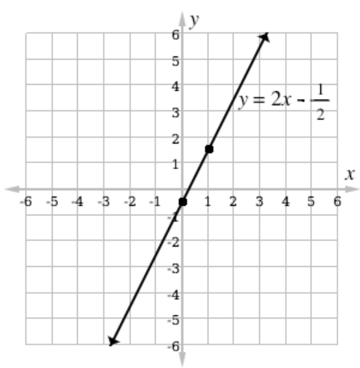
1.



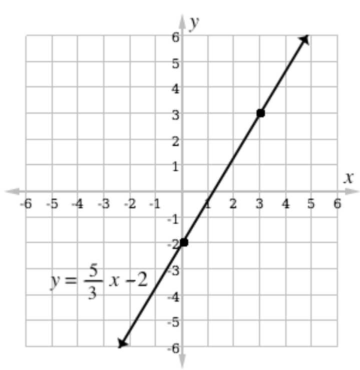
2.



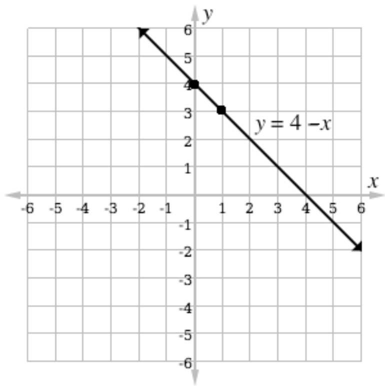
3.



4.



5.



Appendix I: Sines of Angles

Sines of Angles (x in degrees)

$x \to \sin$

x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	ADD Differences								
											1	2	3	4	5	6	7	8	9
0	0·0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	2	3	5	7	9	10	12	14	16
1	0·0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	2	3	5	7	9	10	12	14	16
2	0·0349	0366	0384	0401	0419	0436	0454	0471	0488	0506	2	3	5	7	9	10	12	14	16
3	0·0523	0541	0558	0576	0593	0610	0628	0645	0663	0680	2	3	5	7	9	10	12	14	16
4	0·0698	0715	0732	0750	0767	0785	0802	0819	0837	0854	2	3	5	7	9	10	12	14	16
5	0·0872	0889	0906	0924	0941	0958	0976	0993	1011	1028	2	3	5	7	9	10	12	14	16
6	0·1045	1063	1080	1097	1115	1132	1149	1167	1184	1201	2	3	5	7	9	10	12	14	16
7	0·1219	1236	1253	1271	1288	1305	1323	1340	1357	1374	2	3	5	7	9	10	12	14	16
8	0·1392	1409	1426	1444	1461	1478	1495	1513	1530	1547	2	3	5	7	9	10	12	14	16
9	0·1564	1582	1599	1616	1633	1650	1668	1685	1702	1719	2	3	5	7	9	10	12	14	15
10	0·1736	1754	1771	1788	1805	1822	1840	1857	1874	1891	2	3	5	7	9	10	12	14	15
11	0·1908	1925	1942	1959	1977	1994	2011	2028	2045	2062	2	3	5	7	9	10	12	14	15
12	0·2079	2096	2113	2130	2147	2164	2181	2198	2215	2233	2	3	5	7	9	10	12	14	15
13	0·2250	2267	2284	2300	2317	2334	2351	2368	2385	2402	2	3	5	7	8	10	12	14	15
14	0·2419	2436	2453	2470	2487	2504	2521	2538	2554	2571	2	3	5	7	8	10	12	14	15
15	0·2588	2605	2622	2639	2656	2672	2689	2706	2723	2740	2	3	5	7	8	10	12	13	15
16	0·2756	2773	2790	2807	2823	2840	2857	2874	2890	2907	2	3	5	7	8	10	12	13	15
17	0·2924	2940	2957	2974	2990	3007	3024	3040	3057	3074	2	3	5	7	8	10	12	13	15
18	0·3090	3107	3123	3140	3156	3173	3190	3206	3223	3239	2	3	5	7	8	10	12	13	15
19	0·3256	3272	3289	3305	3322	3338	3355	3371	3387	3404	2	3	5	7	8	10	12	13	15
20	0·3420	3437	3453	3469	3486	3502	3518	3535	3551	3567	2	3	5	7	8	10	11	13	15
21	0·3584	3600	3616	3633	3649	3665	3681	3697	3714	3730	2	3	5	6	8	10	11	13	15
22	0·3746	3762	3778	3795	3811	3827	3843	3859	3875	3891	2	3	5	6	8	10	11	13	15
23	0·3907	3923	3939	3955	3971	3987	4003	4019	4035	4051	2	3	5	6	8	10	11	13	14
24	0·4067	4083	4099	4115	4131	4147	4163	4179	4195	4210	2	3	5	6	8	10	11	13	14
25	0·4226	4242	4258	4274	4289	4305	4321	4337	4352	4368	2	3	5	6	8	9	11	13	14
26	0·4384	4399	4415	4431	4446	4462	4478	4493	4509	4524	2	3	5	5	8	9	11	12	14
27	0·4540	4555	4571	4586	4602	4617	4633	4648	4664	4679	2	3	5	6	8	9	11	12	14
28	0·4695	4710	4726	4741	4756	4772	4787	4802	4818	4833	2	3	5	6	8	9	11	12	14
29	0·4848	4863	4879	4894	4909	4924	4939	4955	4970	4985	2	3	5	6	8	9	11	12	14
30	0·5000	5015	5030	5045	5060	5075	5090	5105	5120	5135	2	3	5	6	8	9	11	12	14
31	0·5150	5165	5180	5195	5210	5225	5240	5255	5270	5284	1	3	4	6	7	9	10	12	13
32	0·5299	5314	5329	5344	5358	5373	5388	5402	5417	5432	1	3	4	6	7	9	10	12	13
33	0·5446	5461	5476	5490	5505	5519	5534	5548	5563	5577	1	3	4	6	7	9	10	12	13
34	0·5592	5606	5621	5635	5650	5664	5678	5693	5707	5721	1	3	4	6	7	9	10	12	13
35	0·5736	5750	5764	5779	5793	5807	5821	5835	5850	5864	1	3	4	6	7	9	10	11	13
36	0·5878	5892	5906	5920	5934	5948	5962	5976	5990	6004	1	3	4	6	7	8	10	11	13
37	0·6018	6032	6046	6060	6074	6088	6101	6115	6129	6143	1	3	4	6	7	8	10	11	12
38	0·6157	6170	6184	6198	6211	6225	6239	6252	6266	6280	1	3	4	5	7	8	10	11	12
39	0·6293	6307	6320	6334	6347	6361	6374	6388	6401	6414	1	3	4	5	7	8	9	11	12
40	0·6428	6441	6455	6468	6481	6494	6508	6521	6534	6547	1	3	4	5	7	8	9	11	12
41	0·6561	6574	6587	6600	6613	6626	6639	6652	6665	6678	1	3	4	5	7	8	9	10	12
42	0·6691	6704	6717	6730	6743	6756	6769	6782	6794	6807	1	3	4	5	6	8	9	10	12
43	0·6820	6833	6845	6858	6871	6884	6896	6909	6921	6934	1	3	4	5	6	8	9	10	11
44	0·6947	6959	6972	6984	6997	7009	7022	7034	7046	7059	1	2	4	5	6	7	9	10	11
45	0·7071	7083	7096	7108	7120	7133	7145	7157	7169	7181	1	2	4	5	6	7	9	10	11
46	0·7193	7206	7218	7230	7242	7254	7266	7278	7290	7302	1	2	4	5	6	7	8	10	11
47	0·7314	7325	7337	7349	7361	7373	7385	7396	7408	7420	1	2	4	5	6	7	8	9	11
48	0·7431	7443	7455	7466	7478	7490	7501	7513	7524	7536	1	2	3	5	6	7	8	9	10
49	0·7547	7559	7570	7581	7593	7604	7615	7627	7638	7649	1	2	3	5	6	7	8	9	10
50	0·7660	7672	7683	7694	7705	7716	7727	7738	7749	7760	1	2	3	4	6	7	8	9	10
51	0·7771	7782	7793	7804	7815	7826	7837	7848	7859	7869	1	2	3	4	5	7	8	9	10
52	0·7880	7891	7902	7912	7923	7934	7944	7955	7965	7976	1	2	3	4	5	6	7	8	10
53	0·7986	7997	8007	8018	8028	8039	8049	8059	8070	8080	1	2	3	4	5	6	7	8	9
54	0·8090	8100	8111	8121	8131	8141	8151	8161	8171	8181	1	2	3	4	5	6	7	8	9
55	0·8192	8202	8211	8221	8231	8241	8251	8261	8271	8281	1	2	3	4	5	6	7	8	9
56	0·8290	8300	8310	8320	8329	8339	8348	8358	8368	8377	1	2	3	4	5	6	7	8	9
57	0·8387	8396	8406	8415	8425	8434	8443	8453	8462	8471	1	2	3	4	5	6	7	8	8
58	0·8480	8490	8499	8508	8517	8526	8536	8545	8554	8563	1	2	3	4	5	5	6	7	8
59	0·8572	8581	8590	8599	8607	8616	8625	8634	8643	8652	1	2	3	4	4	5	6	7	8
60	0·8660	8669	8678	8686	8695	8704	8712	8721	8729	8738	1	2	3	3	4	5	6	7	8
61	0·8746	8755	8763	8771	8780	8788	8796	8805	8813	8821	1	2	2	3	4	5	6	7	7
62	0·8829	8838	8846	8854	8862	8870	8878	8886	8894	8902	1	2	2	3	4	5	6	6	7
63	0·8910	8918	8926	8934	8942	8949	8957	8965	8973	8980	1	2	2	3	4	5	5	6	7
64	0·8988	8996	9003	9011	9018	9026	9033	9041	9048	9056	1	2	2	3	4	5	5	6	7
65	0·9063	9070	9078	9085	9092	9100	9107	9114	9121	9128	1	1	2	3	4	4	5	6	7
66	0·9135	9143	9150	9157	9164	9171	9178	9184	9191	9198	1	1	2	3	3	4	5	6	6
67	0·9205	9212	9219	9225	9232	9239	9245	9252	9259	9265	1	1	2	3	3	4	5	5	6
68	0·9272	9278	9285	9291	9298	9304	9311	9317	9323	9330	1	1	2	3	3	4	4	5	6
69	0·9336	9342	9348	9354	9361	9367	9373	9379	9385	9391	1	1	2	2	3	4	4	5	6
70	0·9397	9403	9409	9415	9421	9426	9432	9438	9444	9449	1	1	2	2	3	3	4	5	5
71	0·9455	9461	9466	9472	9478	9483	9489	9494	9500	9505	1	1	2	2	3	3	4	4	5
72	0·9511	9516	9521	9527	9532	9537	9542	9548	9553	9558	1	1	2	2	3	3	4	4	5
73	0·9563	9568	9573	9578	9583	9588	9593	9598	9603	9608	0	1	1	2	2	3	3	4	4
74	0·9613	9617	9622	9627	9632	9636	9641	9646	9650	9655	0	1	1	2	2	3	3	4	4
75	0·9659	9664	9668	9673	9677	9681	9686	9690	9694	9699	0	1	1	2	2	3	3	3	4
76	0·9703	9707	9711	9715	9720	9724	9728	9732	9736	9740	0	1	1	2	2	2	3	3	4
77	0·9744	9748	9751	9755	9759	9763	9767	9770	9774	9778	0	1	1	2	2	2	3	3	3
78	0·9781	9785	9789	9792	9796	9799	9803	9806	9810	9813	0	1	1	1	2	2	2	3	3
79	0·9816	9820	9823	9826	9829	9833	9836	9839	9842	9845	0	1	1	1	2	2	2	3	3
80	0·9848	9851	9854	9857	9860	9863	9866	9869	9871	9874	0	1	1	1	1	2	2	2	3
81	0·9877	9880	9882	9885	9888	9890	9893	9895	9898	9900	0	1	1	1	1	2	2	2	2
82	0·9903	9905	9907	9910	9912	9914	9917	9919	9921	9923	0	0	1	1	1	1	2	2	2
83	0·9925	9928	9930	9932	9934	9936	9938	9940	9942	9943	0	0	1	1	1	1	1	2	2
84	0·9945	9947	9949	9951	9952	9954	9956	9957	9959	9960	0	0	1	1	1	1	1	1	2
85	0·9962	9963	9965	9966	9968	9969	9971	9972	9973	9974	0	0	0	1	1	1	1	1	1
86	0·9976	9977	9978	9979	9980	9981	9982	9983	9984	9985	0	0	0	0	1	1	1	1	1
87	0·9986	9987	9988	9989	9990	9990	9991	9992	9993	9993	0	0	0	0	0	0	1	1	1
88	0·9994	9995	9995	9996	9996	9997	9997	9997	9998	9998									
89	0·9998	9999	9999	9999	9999	1·0000	1·0000	1·0000	1·0000	1·0000									
x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9

Appendix II: Cosines of Angles

Cosines of Angles (*x* in degrees)

x → cos *x*

x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9
											SUBTRACT Differences								
0	1·0000	1·000	1·000	1·000	1·000	1·000	·9999	·9999	·9999	·9999									
1	0·9998	·9998	·9998	·9997	·9997	·9997	·9996	·9996	·9995	·9995									
2	0·9994	·9993	·9993	·9992	·9991	·9990	·9990	·9989	·9988	·9987	0	0	0	0	0	0	1	1	1
3	0·9986	·9985	·9984	·9983	·9982	·9981	·9980	·9979	·9978	·9977	0	0	0	0	1	1	1	1	1
4	0·9976	·9974	·9973	·9972	·9971	·9969	·9968	·9966	·9965	·9963	0	0	0	1	1	1	1	1	1
5	0·9962	·9960	·9959	·9957	·9956	·9954	·9952	·9951	·9949	·9947	0	0	1	1	1	1	1	1	2
6	0·9945	·9943	·9942	·9940	·9938	·9936	·9934	·9932	·9930	·9928	0	0	1	1	1	1	1	2	2
7	0·9925	·9923	·9921	·9919	·9917	·9914	·9912	·9910	·9907	·9905	0	0	1	1	1	1	2	2	2
8	0·9903	·9900	·9898	·9895	·9893	·9890	·9888	·9885	·9882	·9880	0	1	1	1	1	2	2	2	2
9	0·9877	·9874	·9871	·9869	·9866	·9863	·9860	·9857	·9854	·9851	0	1	1	1	1	2	2	2	3
10	0·9848	·9845	·9842	·9839	·9836	·9833	·9829	·9826	·9823	·9820	0	1	1	1	2	2	2	3	3
11	0·9816	·9813	·9810	·9806	·9803	·9799	·9796	·9792	·9789	·9785	0	1	1	1	2	2	2	3	3
12	0·9781	·9778	·9774	·9770	·9767	·9763	·9759	·9755	·9751	·9748	0	1	1	2	2	2	3	3	3
13	0·9744	·9740	·9736	·9732	·9728	·9724	·9720	·9715	·9711	·9707	0	1	1	2	2	2	3	3	4
14	0·9703	·9699	·9694	·9690	·9686	·9681	·9677	·9673	·9668	·9664	0	1	1	2	2	3	3	3	4
15	0·9659	·9655	·9650	·9646	·9641	·9636	·9632	·9627	·9622	·9617	0	1	1	2	2	3	3	4	4
16	0·9613	·9608	·9603	·9598	·9593	·9588	·9583	·9578	·9573	·9568	0	1	1	2	2	3	3	4	4
17	0·9563	·9558	·9553	·9548	·9542	·9537	·9532	·9527	·9521	·9516	1	1	2	2	3	3	4	4	5
18	0·9511	·9505	·9500	·9494	·9489	·9483	·9478	·9472	·9466	·9461	1	1	2	2	3	3	4	4	5
19	0·9455	·9449	·9444	·9438	·9432	·9426	·9421	·9415	·9409	·9403	1	1	2	2	3	3	4	5	5
20	0·9397	·9391	·9385	·9379	·9373	·9367	·9361	·9354	·9348	·9342	1	1	2	2	3	4	4	5	6
21	0·9336	·9330	·9323	·9317	·9311	·9304	·9298	·9291	·9285	·9278	1	1	2	3	3	4	4	5	6
22	0·9272	·9265	·9259	·9252	·9245	·9239	·9232	·9225	·9219	·9212	1	1	2	3	3	4	5	5	6
23	0·9205	·9198	·9191	·9184	·9178	·9171	·9164	·9157	·9150	·9143	1	1	2	3	3	4	5	6	6
24	0·9135	·9128	·9121	·9114	·9107	·9100	·9092	·9085	·9078	·9070	1	1	2	3	4	4	5	6	7
25	0·9063	·9056	·9048	·9041	·9033	·9026	·9018	·9011	·9003	·8996	1	2	2	3	4	5	5	6	7
26	0·8988	·8980	·8973	·8965	·8957	·8949	·8942	·8934	·8926	·8918	1	2	2	3	4	5	5	6	7
27	0·8910	·8902	·8894	·8886	·8878	·8870	·8862	·8854	·8846	·8838	1	2	2	3	4	5	6	6	7
28	0·8829	·8821	·8813	·8805	·8796	·8788	·8780	·8771	·8763	·8755	1	2	2	3	4	5	6	7	7
29	0·8746	·8738	·8729	·8721	·8712	·8704	·8695	·8686	·8678	·8669	1	2	3	3	4	5	6	7	8
30	0·8660	·8652	·8643	·8634	·8625	·8616	·8607	·8599	·8590	·8581	1	2	3	4	4	5	6	7	8
31	0·8572	·8563	·8554	·8545	·8536	·8526	·8517	·8508	·8499	·8490	1	2	3	4	5	5	6	7	8
32	0·8480	·8471	·8462	·8453	·8443	·8434	·8425	·8415	·8406	·8396	1	2	3	4	5	6	7	8	8
33	0·8387	·8377	·8368	·8358	·8348	·8339	·8329	·8320	·8310	·8300	1	2	3	4	5	6	7	8	9
34	0·8290	·8281	·8271	·8261	·8251	·8241	·8231	·8221	·8211	·8202	1	2	3	4	5	6	7	8	9
35	0·8192	·8181	·8171	·8161	·8151	·8141	·8131	·8121	·8111	·8100	1	2	3	4	5	6	7	8	9
36	0·8090	·8080	·8070	·8059	·8049	·8039	·8028	·8018	·8007	·7997	1	2	3	4	5	6	7	8	9
37	0·7986	·7976	·7965	·7955	·7944	·7934	·7923	·7912	·7902	·7891	1	2	3	4	5	6	7	8	10
38	0·7880	·7869	·7859	·7848	·7837	·7826	·7815	·7804	·7793	·7782	1	2	3	4	5	6	7	9	10
39	0·7771	·7760	·7749	·7738	·7727	·7716	·7705	·7694	·7683	·7672	1	2	3	4	6	7	8	9	10
40	0·7660	·7649	·7638	·7627	·7615	·7604	·7593	·7581	·7570	·7559	1	2	3	5	6	7	8	9	10
41	0·7547	·7536	·7524	·7513	·7501	·7490	·7478	·7466	·7455	·7443	1	2	3	5	6	7	8	9	10
42	0·7431	·7420	·7408	·7396	·7385	·7373	·7361	·7349	·7337	·7325	1	2	4	5	6	7	8	9	11
43	0·7314	·7302	·7290	·7278	·7266	·7254	·7242	·7230	·7218	·7206	1	2	4	5	6	7	8	10	11
44	0·7193	·7181	·7169	·7157	·7145	·7133	·7120	·7108	·7096	·7083	1	2	4	5	6	7	9	10	11
x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9

x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9
											SUBTRACT Differences								
45	0·7071	·7059	·7046	·7034	·7022	·7009	·6997	·6984	·6972	·6959	1	2	4	5	6	7	9	10	11
46	0·6947	·6934	·6921	·6909	·6896	·6884	·6871	·6858	·6845	·6833	1	3	4	5	6	8	9	10	11
47	0·6820	·6807	·6794	·6782	·6769	·6756	·6743	·6730	·6717	·6704	1	3	4	5	6	8	9	10	12
48	0·6691	·6678	·6665	·6652	·6639	·6626	·6613	·6600	·6587	·6574	1	3	4	5	7	8	9	10	12
49	0·6561	·6547	·6534	·6521	·6508	·6494	·6481	·6468	·6455	·6441	1	3	4	5	7	8	9	11	12
50	0·6428	·6414	·6401	·6388	·6374	·6361	·6347	·6334	·6320	·6307	1	3	4	5	7	8	9	11	12
51	0·6293	·6280	·6266	·6252	·6239	·6225	·6211	·6198	·6184	·6170	1	3	4	5	7	8	10	11	12
52	0·6157	·6143	·6129	·6115	·6101	·6088	·6074	·6060	·6046	·6032	1	3	4	5	7	8	10	11	12
53	0·6018	·6004	·5990	·5976	·5962	·5948	·5934	·5920	·5906	·5892	1	3	4	6	7	8	10	11	13
54	0·5878	·5864	·5850	·5835	·5821	·5807	·5793	·5779	·5764	·5750	1	3	4	6	7	9	10	11	13
55	0·5736	·5721	·5707	·5693	·5678	·5664	·5650	·5635	·5621	·5606	1	3	4	6	7	9	10	12	13
56	0·5592	·5577	·5563	·5548	·5534	·5519	·5505	·5490	·5476	·5461	1	3	4	6	7	9	10	12	13
57	0·5446	·5432	·5417	·5402	·5388	·5373	·5358	·5344	·5329	·5314	1	3	4	6	7	9	10	12	13
58	0·5299	·5284	·5270	·5255	·5240	·5225	·5210	·5195	·5180	·5165	1	3	4	6	7	9	10	12	13
59	0·5150	·5135	·5120	·5105	·5090	·5075	·5060	·5045	·5030	·5015	2	3	5	6	8	9	11	12	14
60	0·5000	·4985	·4970	·4955	·4939	·4924	·4909	·4894	·4879	·4863	2	3	5	6	8	9	11	12	14
61	0·4848	·4833	·4818	·4802	·4787	·4772	·4756	·4741	·4726	·4710	2	3	5	6	8	9	11	12	14
62	0·4695	·4679	·4664	·4648	·4633	·4617	·4602	·4586	·4571	·4555	2	3	5	6	8	9	11	12	14
63	0·4540	·4524	·4509	·4493	·4478	·4462	·4446	·4431	·4415	·4399	2	3	5	6	8	9	11	12	14
64	0·4384	·4368	·4352	·4337	·4321	·4305	·4289	·4274	·4258	·4242	2	3	5	6	8	9	11	13	14
65	0·4226	·4210	·4195	·4179	·4163	·4147	·4131	·4115	·4099	·4083	2	3	5	6	8	10	11	13	14
66	0·4067	·4051	·4035	·4019	·4003	·3987	·3971	·3955	·3939	·3923	2	3	5	6	8	10	11	13	14
67	0·3907	·3891	·3875	·3859	·3843	·3827	·3811	·3795	·3778	·3762	2	3	5	6	8	10	11	13	15
68	0·3746	·3730	·3714	·3697	·3681	·3665	·3649	·3633	·3616	·3600	2	3	5	6	8	10	11	13	15
69	0·3584	·3567	·3551	·3535	·3518	·3502	·3486	·3469	·3453	·3437	2	3	5	6	8	10	11	13	15
70	0·3420	·3404	·3387	·3371	·3355	·3338	·3322	·3305	·3289	·3272	2	3	5	7	8	10	12	13	15
71	0·3256	·3239	·3223	·3206	·3190	·3173	·3156	·3140	·3123	·3107	2	3	5	7	8	10	12	13	15
72	0·3090	·3074	·3057	·3040	·3024	·3007	·2990	·2974	·2957	·2940	2	3	5	7	8	10	12	13	15
73	0·2924	·2907	·2890	·2874	·2857	·2840	·2823	·2807	·2790	·2773	2	3	5	7	8	10	12	13	15
74	0·2756	·2740	·2723	·2706	·2689	·2672	·2656	·2639	·2622	·2605	2	3	5	7	8	10	12	13	15
75	0·2588	·2571	·2554	·2538	·2521	·2504	·2487	·2470	·2453	·2436	2	3	5	7	8	10	12	14	15
76	0·2419	·2402	·2385	·2368	·2351	·2334	·2317	·2300	·2284	·2267	2	3	5	7	8	10	12	14	15
77	0·2250	·2233	·2215	·2198	·2181	·2164	·2147	·2130	·2113	·2096	2	3	5	7	9	10	12	13	15
78	0·2079	·2062	·2045	·2028	·2011	·1994	·1977	·1959	·1942	·1925	2	3	5	7	9	10	12	14	15
79	0·1908	·1891	·1874	·1857	·1840	·1822	·1805	·1788	·1771	·1754	2	3	5	7	9	10	12	14	15
80	0·1736	·1719	·1702	·1685	·1668	·1650	·1633	·1616	·1599	·1582	2	3	5	7	9	10	12	14	15
81	0·1564	·1547	·1530	·1513	·1495	·1478	·1461	·1444	·1426	·1409	2	3	5	7	9	10	12	14	16
82	0·1392	·1374	·1357	·1340	·1323	·1305	·1288	·1271	·1253	·1236	2	3	5	7	9	10	12	14	16
83	0·1219	·1201	·1184	·1167	·1149	·1132	·1115	·1097	·1080	·1063	2	3	5	7	9	10	12	14	16
84	0·1045	·1028	·1011	·0993	·0976	·0958	·0941	·0924	·0906	·0889	2	3	5	7	9	10	12	14	16
85	0·0872	·0854	·0837	·0819	·0802	·0785	·0767	·0750	·0732	·0715	2	3	5	7	9	10	12	14	16
86	0·0698	·0680	·0663	·0645	·0628	·0610	·0593	·0576	·0558	·0541	2	3	5	7	9	10	12	14	16
87	0·0523	·0506	·0488	·0471	·0454	·0436	·0419	·0401	·0384	·0366	2	3	5	7	9	10	12	14	16
88	0·0349	·0332	·0314	·0297	·0279	·0262	·0244	·0227	·0209	·0192	2	3	5	7	9	10	12	14	16
89	0·0175	·0157	·0140	·0122	·0105	·0087	·0070	·0052	·0035	·0017	2	3	5	7	9	10	12	14	16
x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9

Appendix III: Tangents of Angles

Tangents of Angles (*x* in degrees)

$x \rightarrow \tan x$

											ADD Differences								
x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9
0	0·0000	·0017	·0035	·0052	·0070	·0087	·0105	·0122	·0140	·0157	2	3	5	7	9	10	12	14	16
1	0·0175	·0192	·0209	·0227	·0244	·0262	·0279	·0297	·0314	·0332	2	3	5	7	9	10	12	14	16
2	0·0349	·0367	·0384	·0402	·0419	·0437	·0454	·0472	·0489	·0507	2	3	5	7	9	10	12	14	16
3	0·0524	·0542	·0559	·0577	·0594	·0612	·0629	·0647	·0664	·0682	2	4	5	7	9	11	12	14	16
4	0·0699	·0717	·0734	·0752	·0769	·0787	·0805	·0822	·0840	·0857	2	4	5	7	9	11	12	14	16
5	0·0875	·0892	·0910	·0928	·0945	·0963	·0981	·0998	·1016	·1033	2	4	5	7	9	11	12	14	16
6	0·1051	·1069	·1086	·1104	·1122	·1139	·1157	·1175	·1192	·1210	2	4	5	7	9	11	12	14	16
7	0·1228	·1246	·1263	·1281	·1299	·1317	·1334	·1352	·1370	·1388	2	4	5	7	9	11	12	14	16
8	0·1405	·1423	·1441	·1459	·1477	·1495	·1512	·1530	·1548	·1566	2	4	5	7	9	11	12	14	16
9	0·1584	·1602	·1620	·1638	·1655	·1673	·1691	·1709	·1727	·1745	2	4	5	7	9	11	13	14	16
10	0·1763	·1781	·1799	·1817	·1835	·1853	·1871	·1890	·1908	·1926	2	4	5	7	9	11	13	14	16
11	0·1944	·1962	·1980	·1998	·2016	·2035	·2053	·2071	·2089	·2107	2	4	5	7	9	11	13	15	16
12	0·2126	·2144	·2162	·2180	·2199	·2217	·2235	·2254	·2272	·2290	2	4	5	7	9	11	13	15	16
13	0·2309	·2327	·2345	·2364	·2382	·2401	·2419	·2438	·2456	·2475	2	4	6	7	9	11	13	15	17
14	0·2493	·2512	·2530	·2549	·2568	·2586	·2605	·2623	·2642	·2661	2	4	6	7	9	11	13	15	17
15	0·2679	·2698	·2717	·2736	·2754	·2773	·2792	·2811	·2830	·2849	2	4	6	8	9	11	13	15	17
16	0·2867	·2886	·2905	·2924	·2943	·2962	·2981	·3000	·3019	·3038	2	4	6	8	9	11	13	15	17
17	0·3057	·3076	·3096	·3115	·3134	·3153	·3172	·3191	·3211	·3230	2	4	6	8	10	12	13	15	17
18	0·3249	·3269	·3288	·3307	·3327	·3346	·3365	·3385	·3404	·3424	2	4	6	8	10	12	14	16	17
19	0·3443	·3463	·3482	·3502	·3522	·3541	·3561	·3581	·3600	·3620	2	4	6	8	10	12	14	16	18
20	0·3640	·3659	·3679	·3699	·3719	·3739	·3759	·3779	·3799	·3819	2	4	6	8	10	12	14	16	18
21	0·3839	·3859	·3879	·3899	·3919	·3939	·3959	·3979	·4000	·4020	2	4	6	8	10	12	14	16	18
22	0·4040	·4061	·4081	·4101	·4122	·4142	·4163	·4183	·4204	·4224	2	4	6	8	10	12	14	16	18
23	0·4245	·4265	·4286	·4307	·4327	·4348	·4369	·4390	·4411	·4431	2	4	6	8	10	12	15	17	19
24	0·4452	·4473	·4494	·4515	·4536	·4557	·4578	·4599	·4621	·4642	2	4	6	8	11	13	15	17	19
25	0·4663	·4684	·4706	·4727	·4748	·4770	·4791	·4813	·4834	·4856	2	4	6	9	11	13	15	17	19
26	0·4877	·4899	·4921	·4942	·4964	·4986	·5008	·5029	·5051	·5073	2	4	7	9	11	13	15	17	20
27	0·5095	·5117	·5139	·5161	·5184	·5206	·5228	·5250	·5272	·5295	2	4	7	9	11	13	16	18	20
28	0·5317	·5340	·5362	·5384	·5407	·5430	·5452	·5475	·5498	·5520	2	5	7	9	11	14	16	18	20
29	0·5543	·5566	·5589	·5612	·5635	·5658	·5681	·5704	·5727	·5750	2	5	7	9	12	14	16	18	21
30	0·5774	·5797	·5820	·5844	·5867	·5890	·5914	·5938	·5961	·5985	2	5	7	9	12	14	16	19	21
31	0·6009	·6032	·6056	·6080	·6104	·6128	·6152	·6176	·6200	·6224	2	5	7	10	12	14	17	19	22
32	0·6249	·6273	·6297	·6322	·6346	·6371	·6395	·6420	·6445	·6469	2	5	7	10	12	15	17	20	22
33	0·6494	·6519	·6544	·6569	·6594	·6619	·6644	·6669	·6694	·6720	3	5	8	10	13	15	18	20	23
34	0·6745	·6771	·6796	·6822	·6847	·6873	·6899	·6924	·6950	·6976	3	5	8	10	13	15	18	21	23
35	0·7002	·7028	·7054	·7080	·7107	·7133	·7159	·7186	·7212	·7239	3	5	8	11	13	16	18	21	24
36	0·7265	·7292	·7319	·7346	·7373	·7400	·7427	·7454	·7481	·7508	3	5	8	11	14	16	19	22	24
37	0·7536	·7563	·7590	·7618	·7646	·7673	·7701	·7729	·7757	·7785	3	6	8	11	14	17	19	22	25
38	0·7813	·7841	·7869	·7898	·7926	·7954	·7983	·8012	·8040	·8069	3	6	9	11	14	17	20	23	26
39	0·8098	·8127	·8156	·8185	·8214	·8243	·8273	·8302	·8332	·8361	3	6	9	12	15	18	21	23	26
40	0·8391	·8421	·8451	·8481	·8511	·8541	·8571	·8601	·8632	·8662	3	6	9	12	15	18	21	24	27
41	0·8693	·8724	·8754	·8785	·8816	·8847	·8878	·8910	·8941	·8972	3	6	9	12	16	19	22	25	28
42	0·9004	·9036	·9067	·9099	·9131	·9163	·9195	·9228	·9260	·9293	3	6	10	13	16	19	22	26	29
43	0·9325	·9358	·9391	·9424	·9457	·9490	·9523	·9556	·9590	·9623	3	7	10	13	17	20	23	27	30
44	0·9657	·9691	·9725	·9759	·9793	·9827	·9861	·9896	·9930	·9965	3	7	10	14	17	21	24	27	31
x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9

											ADD Differences								
x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9
45	1·000	1·003	1·007	1·011	1·014	1·018	1·021	1·025	1·028	1·032	0	1	1	1	2	2	2	3	3
46	1·036	1·039	1·043	1·046	1·050	1·054	1·057	1·061	1·065	1·069	0	1	1	1	2	2	3	3	3
47	1·072	1·076	1·080	1·084	1·087	1·091	1·095	1·099	1·103	1·107	0	1	1	2	2	2	3	3	3
48	1·111	1·115	1·118	1·122	1·126	1·130	1·134	1·138	1·142	1·146	0	1	1	2	2	2	3	3	4
49	1·150	1·154	1·159	1·163	1·167	1·171	1·175	1·179	1·183	1·188	0	1	1	2	2	2	3	3	4
50	1·192	1·196	1·200	1·205	1·209	1·213	1·217	1·222	1·226	1·230	0	1	1	2	2	3	3	4	4
51	1·235	1·239	1·244	1·248	1·253	1·257	1·262	1·266	1·271	1·275	0	1	1	2	2	3	3	4	4
52	1·280	1·285	1·289	1·294	1·299	1·303	1·308	1·313	1·317	1·322	0	1	1	2	2	3	3	4	4
53	1·327	1·332	1·337	1·342	1·347	1·351	1·356	1·361	1·366	1·371	0	1	1	2	2	3	3	4	4
54	1·376	1·381	1·387	1·392	1·397	1·402	1·407	1·412	1·418	1·423	1	1	2	2	3	3	4	4	5
55	1·428	1·433	1·439	1·444	1·450	1·455	1·460	1·466	1·471	1·477	1	1	2	2	3	3	4	4	5
56	1·483	1·488	1·494	1·499	1·505	1·511	1·517	1·522	1·528	1·534	1	1	2	2	3	3	4	5	5
57	1·540	1·546	1·552	1·558	1·564	1·570	1·576	1·582	1·588	1·594	1	1	2	2	3	4	4	5	5
58	1·600	1·607	1·613	1·619	1·625	1·632	1·638	1·645	1·651	1·658	1	1	2	3	3	4	4	5	6
59	1·664	1·671	1·678	1·684	1·691	1·698	1·704	1·711	1·718	1·725	1	1	2	3	3	4	5	5	6
60	1·732	1·739	1·746	1·753	1·760	1·767	1·775	1·782	1·789	1·797	1	1	2	3	4	4	5	6	6
61	1·804	1·811	1·819	1·827	1·834	1·842	1·849	1·857	1·865	1·873	1	2	2	3	4	5	5	6	7
62	1·881	1·889	1·897	1·905	1·913	1·921	1·929	1·937	1·946	1·954	1	2	2	3	4	5	6	7	7
63	1·963	1·971	1·980	1·988	1·997	2·006	2·014	2·023	2·032	2·041	1	2	3	4	4	5	6	7	8
64	2·050	2·059	2·069	2·078	2·087	2·097	2·106	2·116	2·125	2·135	1	2	3	4	5	6	7	8	8
65	2·145	2·154	2·164	2·174	2·184	2·194	2·204	2·215	2·225	2·236	1	2	3	4	5	6	7	8	9
66	2·246	2·257	2·267	2·278	2·289	2·300	2·311	2·322	2·333	2·344	1	2	3	4	5	7	8	9	10
67	2·356	2·367	2·379	2·391	2·402	2·414	2·426	2·438	2·450	2·463	1	2	4	5	6	7	8	10	11
68	2·475	2·488	2·500	2·513	2·526	2·539	2·552	2·565	2·578	2·592	1	3	4	5	7	8	9	10	12
69	2·605	2·619	2·633	2·646	2·660	2·675	2·689	2·703	2·718	2·733	1	3	4	6	7	9	10	11	13
70	2·747	2·762	2·778	2·793	2·808	2·824	2·840	2·856	2·872	2·888	2	3	5	6	8	9	11	13	14
71	2·904	2·921	2·937	2·954	2·971	2·989	3·006	3·024	3·042	3·060	2	3	5	7	9	10	12	14	16
72	3·078	3·096	3·115	3·133	3·152	3·172	3·191	3·211	3·230	3·251	2	4	6	8	10	12	14	15	17
73	3·271	3·291	3·312	3·333	3·354	3·376	3·398	3·420	3·442	3·465	2	4	6	9	11	13	15	17	19
74	3·487	3·511	3·534	3·558	3·582	3·606	3·630	3·655	3·681	3·706	2	5	7	10	12	15	17	20	22
75	3·732	3·758	3·785	3·812	3·839	3·867	3·895	3·923	3·952	3·981	3	6	8	11	14	17	20	22	25
76	4·011	4·041	4·071	4·102	4·134	4·165	4·198	4·230	4·264	4·297	3	6	10	13	16	19	22	26	29
77	4·331	4·366	4·402	4·437	4·474	4·511	4·548	4·586	4·625	4·666									
78	4·705	4·745	4·787	4·829	4·872	4·915	4·959	5·005	5·050	5·097									
79	5·145	5·193	5·242	5·292	5·343	5·396	5·449	5·503	5·558	5·614									
80	5·671	5·730	5·789	5·850	5·912	5·976	6·041	6·107	6·174	6·243	differences unreliable:								
81	6·314	6·386	6·460	6·535	6·612	6·691	6·772	6·855	6·940	7·026									
82	7·115	7·207	7·300	7·396	7·495	7·596	7·700	7·806	7·916	8·028									
83	8·144	8·264	8·386	8·513	8·643	8·777	8·915	9·058	9·205	9·357									
84	9·514	9·677	9·845	10·02	10·20	10·39	10·58	10·78	10·99	11·20									
85	11·43	11·66	11·91	12·16	12·43	12·71	13·00	13·30	13·62	13·95									
86	14·30	14·67	15·06	15·46	15·89	16·35	16·83	17·34	17·89	18·46									
87	19·08	19·74	20·45	21·20	22·02	22·90	23·86	24·90	26·03	27·27									
88	28·64	30·14	31·82	33·69	35·80	38·19	40·92	44·07	47·74	52·08									
89	57·29	63·66	71·62	81·85	95·46	114·6	143·2	191·0	286·5	573·0									
x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9

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