Free Quality School Education

Ministry of Basic and Senior Secondary Education

Pupils' Handbook for

Senior Secondary *Mathematics*

SSS II

Term I

STRICTLY NOT FOR SALE

DOI: 10.5281/zenodo.3745368
Please see final page for further information.

Foreword

These Lesson Plans and the accompanying Pupils' Handbooks are essential educational resources for the promotion of quality education in senior secondary schools in Sierra Leone. As Minister of Basic and Senior Secondary Education, I am pleased with the professional competencies demonstrated by the writers of these educational materials in English Language and Mathematics.

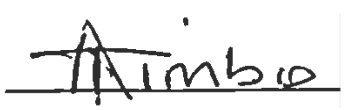
The Lesson Plans give teachers the support they need to cover each element of the national curriculum, as well as prepare pupils for the West African Examinations Council's (WAEC) examinations. The practice activities in the Pupils' Handbooks are designed to support self-study by pupils, and to give them additional opportunities to learn independently. In total, we have produced 516 lesson plans and 516 practice activities – one for each lesson, in each term, in each year, for each class. The production of these materials in a matter of months is a remarkable achievement.

These plans have been written by experienced Sierra Leoneans together with international educators. They have been reviewed by officials of my Ministry to ensure that they meet the specific needs of the Sierra Leonean population. They provide step-by-step guidance for each learning outcome, using a range of recognized techniques to deliver the best teaching.

I call on all teachers and heads of schools across the country to make the best use of these materials. We are supporting our teachers through a detailed training programme designed specifically for these new lesson plans. It is really important that the Lesson Plans and Pupils' Handbooks are used, together with any other materials they may have.

This is just the start of educational transformation in Sierra Leone as pronounced by His Excellency, the President of the Republic of Sierra Leone, Brigadier Rtd Julius Maada Bio. I am committed to continue to strive for the changes that will make our country stronger and better.

I do thank our partners for their continued support. Finally, I also thank the teachers of our country for their hard work in securing our future



Mr. Alpha Osman Timbo

Minister of Basic and Senior Secondary Education

The policy of the Ministry of Basic and Senior Secondary Education, Sierra Leone, on textbooks stipulates that every printed book should have a lifespan of three years.

To achieve thus, DO NOT WRITE IN THE BOOKS.

Table of Contents

Introduction

to the Pupils' Handbook

These practice activities are aligned to the Lesson Plans, and are based on the National Curriculum and the West Africa Examination Council syllabus guidelines. They meet the requirements established by the Ministry of Basic and Senior Secondary Education.

1. The practice activities will not take the whole term, so use any extra time to revise material or re-do activities where you made mistakes.
2. Use other textbooks or resources to help you learn better and practise what you have learned in the lessons.
3. Read the questions carefully before answering them. After completing the practice activities, check your answers using the answer key at the end of the book.
4. Make sure you understand the learning outcomes for the practice activities and check to see that you have achieved them. Each lesson plan shows these using the symbol to the right.

   Learning Outcomes
5. Organise yourself so that you have enough time to complete all of the practice activities. If there is time, quickly revise what you learned in the lesson before starting the practice activities. If it is taking you too long to complete the activities, you may need more practice on that particular topic.
6. Seek help from your teacher or your peers if you are having trouble completing the practice activities independently.
7. Make sure you write the answers in your exercise book in a clear and systematic way so that your teacher can check your work and you can refer back to it when you prepare for examinations.
8. Congratulate yourself when you get questions right! Do not worry if you do not get the right answer – ask for help and continue practising!

KEY TAKEAWAYS FROM SIERRA LEONE'S PERFORMANCE IN WEST AFRICAN SENIOR SCHOOL CERTIFICATE EXAMINATION – GENERAL MATHEMATICS[1]

This section, seeks to outline key takeaways from assessing Sierra Leonean pupils' responses on the West African Senior School Certificate Examination. The common errors pupils make are highlighted below with the intention of giving teachers an insight into areas to focus on, to improve pupil performance on the examination. Suggestions are provided for addressing these issues.

Common errors

1. Errors in applying principles of BODMAS
2. Mistakes in simplifying fractions
3. Errors in application of Maths learned in class to real-life situations, and vis-a-versa.
4. Errors in solving geometric constructions.
5. Mistakes in solving problems on circle theorems.
6. Proofs are often left out from solutions, derivations are often missing from quadratic equations.

Suggested solutions

1. Practice answering questions to the detail requested
2. Practice re-reading questions to make sure all the components are answered.
3. If possible, procure as many geometry sets to practice geometry construction.
4. Check that depth and level of the lesson taught is appropriate for the grade level.

[1] This information is derived from an evaluation of WAEC Examiner Reports, as well as input from WAEC examiners and Sierra Leonean teachers.

Lesson Title: Review of Number Bases and Indices	**Theme:** Review of SSS 1
Practice Activity: PHM2-L001	**Class:** SSS 2

Learning Outcomes
By the end of the lesson, you will be able to:
1. Convert between number bases.
2. Apply the laws of indices to simplify expressions.

Overview

Number bases:

Number bases are written with digits, and a smaller number in subscript that can be a digit or spelled (example: 423_{three} or 423_3, which is read as "423 base three"). Base ten has ten digits (0-9), base nine has nine digits (0-8), base eight has eight digits (0-7), and so on. When we see numbers without a base given, they are in base ten. Numbers are usually written in base ten.

How to convert between number bases:

- **To convert from another base to base 10:** Each digit of the number must be converted using powers of the base you are converting from.
- **To convert from base 10 to another base:** Successive division is used. The number in base 10 is divided repeatedly by the base number you are converting to. The answer is obtained by reading the remainders upwards.
- **To convert between two bases that are not 10:** First convert to base ten, then convert the result to the required base.

Laws of indices:
The laws of indices can be used to simplify expressions involving numbers and variables. These are the four laws of indices:

1. $x^a \times x^b = x^{a+b}$
2. $x^a \div x^b = x^{a-b}$
3. $x^0 = 1, x \neq 0$
4. $(x^a)^b = x^{ab}$

Solved Examples

Conversion of number bases:

1. Convert 243_{five} to base ten:

$$\begin{aligned} 243_{five} &= (2 \times 5^2) + (4 \times 5^1) + (3 \times 5^0) \\ &= (2 \times 25) + (4 \times 5) + (3 \times 1) \end{aligned}$$

$$= 73_{ten}$$
$$= 73$$

2. Convert to 44_{ten} to base five:

$$\begin{array}{c|l} 5 & 44 \\ \hline & 8\ rem\ 4 \\ \hline & 1\ rem\ 3 \\ \hline & 0\ rem\ 1 \end{array}\ \uparrow$$

$$44_{ten} = 134_{five}$$

3. Convert 236_{four} to base six:

Step 1. Change 236_{four} to base ten:

$$\begin{aligned} 236_{four} &= (2 \times 4^2) + (3 \times 4^1) + (6 \times 4^0) \\ &= (2 \times 16) + (3 \times 4) + (6 \times 1) \\ &= 50_{ten} \\ &= 50 \end{aligned}$$

Step 2. Change 50_{ten} to base six:

$$\begin{array}{c|l} 6 & 50 \\ \hline & 8\ rem\ 2 \\ \hline & 1\ rem\ 2 \\ \hline & 0\ rem\ 1 \end{array}\ \uparrow$$

$$50_{ten} = 122_{six}$$

Therefore, $236_{four} = 122_{six}$

Applying the laws of indices to simplify expressions:

4. Simplify: $2a^2 \times 8a^3$

$$\begin{aligned} 2a^2 \times 8a^3 &= (2 \times 8)a^{2+3} && \text{Apply the first law of indices} \\ &= 16a^5 \end{aligned}$$

5. Simplify: $\frac{(y^2)^3}{y^4}$

$$\begin{aligned} \frac{(y^2)^3}{y^4} &= \frac{y^{2\times3}}{y^4} && \text{Apply the fourth law of indices to the numerator} \\ &= \frac{y^6}{y^4} \\ &= y^{6-4} && \text{Apply the second law of indices} \end{aligned}$$

$= \quad y^2$

6. Simplify: $22n^7 \div 2n^3$

$22n^7 \div 2n^3 \quad = \quad \frac{22n^7}{2n^3}$ Apply the second law of indices

$= \quad 11n^{7-3}$

$= \quad 11n^4$

7. Simplify: $(a^2b^3)^2$

$(a^2b^3)^2 \quad = \quad a^{2\times2} \times b^{3\times2}$ Apply the fourth law of indices

$= \quad a^4 \times b^6$

$= \quad a^4b^6$

Practice

1. Simplify the following:
   a. $(-4g^5)^3$
   b. $(2x^4y^{-3})(4x^{-2}y^6)$
   c. $\frac{c^2 \times c^4}{c^5}$
   d. $144p^8 \div 12p^7$
   e. $(x^{-5}y^2)^3$
2. Express 437_{ten} in base five.
3. Change 625_{seven} to base ten.
4. Convert 134_{five} to base two.
5. Convert 213_4 to base three.

Lesson Title: Review of Linear Equations	**Theme:** Review of SSS 1
Practice Activity: PHM2-L002	**Class:** SSS 2

Learning Outcomes
By the end of the lesson, you will be able to:
1. Solve linear equations algebraically.
2. Graph linear functions.

Overview

Solutions to linear equations can be written as ordered pairs: (x, y). To find a solution to a linear equation, substitute any value of x and solve for y.

To graph a linear equation, fill a table of values with multiple solutions to the equation. Graph each solution on the Cartesian plane, and connect them with a straight line.

Solved Examples

1. Graph the equation $y = -x - 3$

$$\begin{aligned} y &= -(-2) - 3 \\ &= 2 - 3 \\ &= -1 \end{aligned}$$

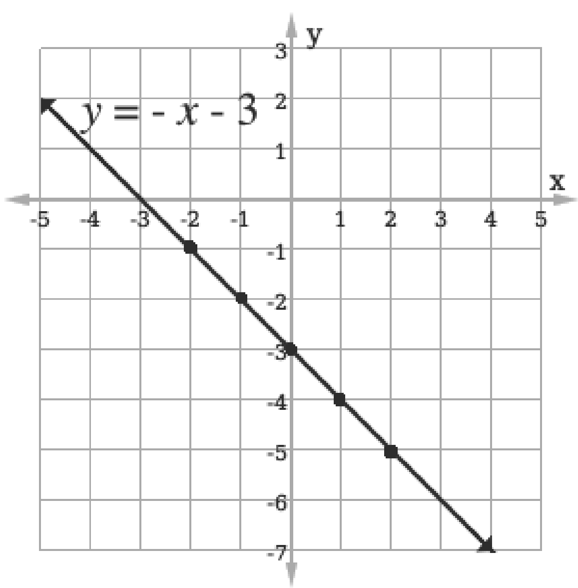
x	-2	-1	0	1	2
y	-1	-2	-3	-4	-5

$$\begin{aligned} y &= -(-1) - 3 \\ &= 1 - 3 \\ &= -2 \end{aligned}$$

$$\begin{aligned} y &= -(0) - 3 \\ &= 0 - 3 \\ &= -3 \end{aligned}$$

$$\begin{aligned} y &= -(1) - 3 \\ &= -1 - 3 \\ &= -4 \end{aligned}$$

$$\begin{aligned} y &= -(2) - 3 \\ &= -2 - 3 \\ &= -5 \end{aligned}$$



2. Graph the equation $8x - 2y = 12$

Step 1. Change subject and solve for y:

$$\begin{aligned} 8x - 2y &= 12 \\ -2y &= 12 - 8x && \text{Transpose } 8x \\ \frac{-2y}{-2} &= \frac{12}{-2} - \frac{8x}{-2} && \text{Divide throughout by } -2 \\ y &= -6 + 4x \end{aligned}$$

This is the same as $y = 4x - 6$.

Step 2. Substitute values of x into the equation and graph.

$$\begin{aligned} y &= 4(-1) - 6 \\ &= -4 - 6 \\ &= -10 \end{aligned}$$

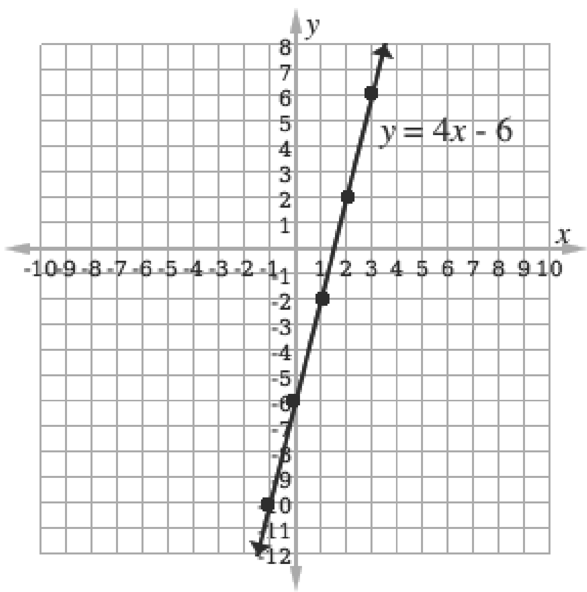
x	-1	0	1	2	3
y	-10	-6	-2	2	6

$$\begin{aligned} y &= 4(0) - 6 \\ &= 0 - 6 \\ &= -6 \end{aligned}$$

$$\begin{aligned} y &= 4(1) - 6 \\ &= 4 - 6 \\ &= -2 \end{aligned}$$

$$\begin{aligned} y &= 4(2) - 6 \\ &= 8 - 6 \\ &= 2 \end{aligned}$$

$$\begin{aligned} y &= 4(3) - 6 \\ &= 12 - 6 \\ &= 6 \end{aligned}$$



3. Complete a table of values for relation the $y = x - 1$ for values of x from -2 to 2. Graph the relation on the Cartesian plane.

$$\begin{aligned} y &= (-2) - 1 \\ &= -3 \end{aligned}$$

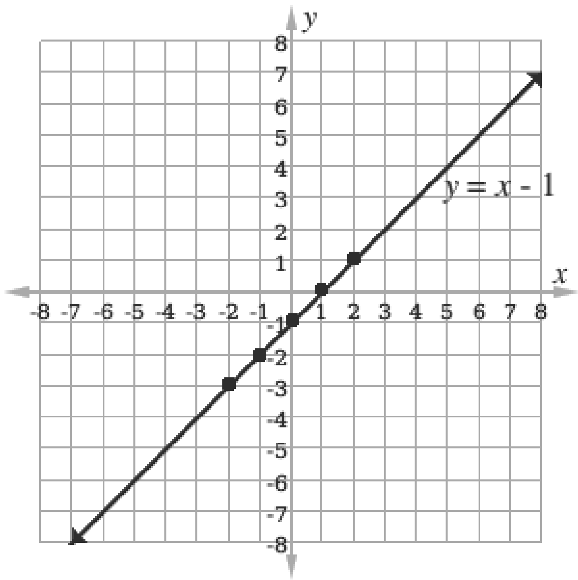
x	-2	-1	0	1	2
y	-3	-2	-1	0	1

$$\begin{aligned} y &= (-1) - 1 \\ &= -2 \end{aligned}$$

$$\begin{aligned} y &= (0) - 1 \\ &= -1 \end{aligned}$$

$$y = (1) - 1$$
$$= 0$$

$$y = (2) - 1$$
$$= 1$$



Practice

1. Graph the equation $y = 2x - 3$ for values of x from -3 to 3.
2. Graph the equation $y = 2x + 4$ for values of x from -3 to 0.
3. Graph the equation $y + 3x = 4$ using the values of $-2 \leq x \leq 2$.

Lesson Title: Review of Quadratic Equations	**Theme:** Review of SSS 1
Practice Activity: PHM2-L003	**Class:** SSS 2

Learning Outcomes
By the end of the lesson, you will be able to:
1. Solve quadratic equations algebraically.
2. Graph and interpret quadratic functions.

Overview

A “quadratic equation” is given in a way that allows you to solve for a variable and does not have a y-value (Example: $x^2 + 4x + 3 = 0$). A “quadratic function” has a y-value and can be graphed (Example: $y = x^2 + 4x + 3$). Quadratic equations and functions have a term with x^2.
We can find the solutions to quadratic equations in several ways:

- By graphing the related function
- Using factorisation
- Completing the square
- Using the quadratic formula

The graph of a quadratic function is a parabola. The solutions (or roots) of a quadratic equation are the points where the parabola crosses the x-axis.

Solved Examples

1. Solve the quadratic equation $x^2 + 2x - 3 = 0$ graphically, using factorization, and using the quadratic formula.

Graphically:

x	-3	-2	-1	0	1
y	0	-3	-4	-3	0

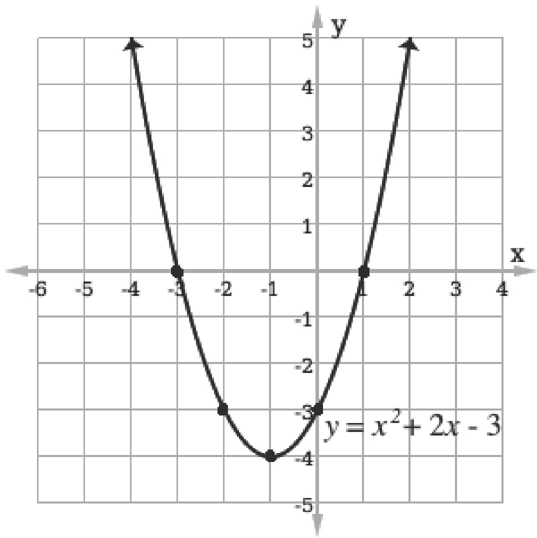
$$\begin{aligned} y &= (-3)^2 + 2(-3) - 3 \\ &= 9 - 6 - 3 \\ &= 0 \end{aligned}$$

$$\begin{aligned} y &= (-2)^2 + 2(-2) - 3 \\ &= 4 - 4 - 3 \\ &= -3 \end{aligned}$$

$$\begin{aligned} y &= (-1)^2 + 2(-1) - 3 \\ &= 1 - 2 - 3 \\ &= -4 \end{aligned}$$

$$\begin{aligned} y &= (0)^2 + 2(0) - 3 \\ &= 0 - 0 - 3 \\ &= -3 \end{aligned}$$

$$\begin{aligned} y &= (1)^2 + 2(1) - 3 \\ &= 1 + 2 - 3 \\ &= 0 \end{aligned}$$



Because the x-intercepts occur at $(-3, 0)$ and $(1, 0)$, the roots of the quadratic equation are $x = -3$ and $x = 1$.

Factorisation:

- Factorisation involves rewriting the quadratic equation in the form $x^2 + 2x - 3 = (x + a)(x + b)$, where a and b are numbers.
- a and b should sum to the coefficient of the second term of the equation, and multiply to get the third term.
- After finding a and b, set each binomial equal to 0 and solve to find the value of x. These values are the roots.

$x^2 + 2x - 3 \quad = \quad (x + a)(x + b)$		Set up the equation
$a + b = 2$	$a \times b = -3$	Note that the values of a and b must be -3 and 1 to satisfy these equations
$x^2 + 4x + 3 \quad = \quad (x + 3)(x - 1)$		Substitute values of a and b into the equation
$x + 3 = 0$ $x = -3$	$x - 1 = 0$ $x = 1$	Set each binomial equal to 0 and solve for x. These are the roots.

Quadratic formula:

Given a quadratic equation $ax^2 + bx + c = 0$, the quadratic formula can be applied to find its roots: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

x	$=$	$\frac{-2\pm\sqrt{2^2-4(1)(-3)}}{2(1)}$	Substitute the values of a, b, and c
	$=$	$\frac{-2\pm\sqrt{16}}{2}$	Simplify
	$=$	$\frac{-2\pm 4}{2}$	Note: The $\pm$ symbol tells us that we can find 2 solutions: one by adding, and the other by subtracting
	$=$	$\frac{-2+4}{2}$ and $\frac{-2-4}{2}$	
	$=$	1 and -3	Identify the 2 roots

Practice

Solve the following quadratic equations graphically, using factorisation, and using the quadratic formula. Check that you arrive at the same answer using each method.

1. $x^2 - 4x - 5 = 0$
2. $x^2 - 2x - 8 = 0$
3. $x^2 + 2x + 1 = 0$

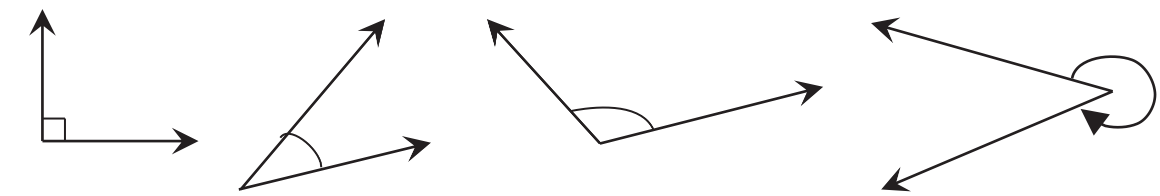
Lesson Title: Review of Angles and Triangles	**Theme:** Review of SSS 1
Practice Activity: PHM2-L004	**Class:** SSS 2

Learning Outcomes
By the end of the lesson, you will be able to:
1. Identify types of angles and triangles.
2. Solve triangles by finding angle and side measures.

Overview

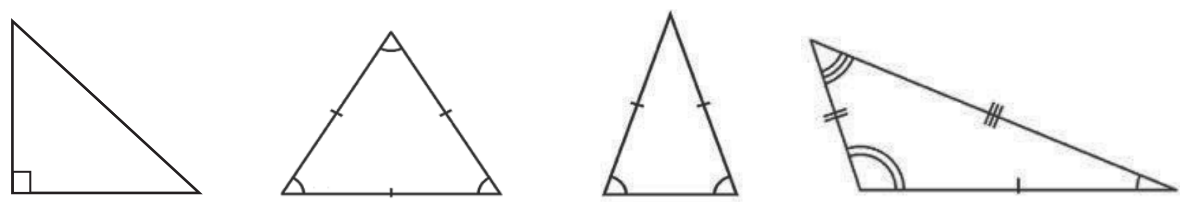
This lesson reviews angle and triangle types. It then reviews solving for angle and side measures in triangles.

Angle types:



Right angle	**Acute angle**	**Obtuse angle**	**Reflex angle**
90°	less than 90°	greater than 90° and less than 180°	greater than 180° and less than 360°

Triangle types:

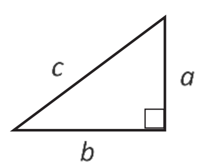


Right-angled	**Equilateral**	**Isosceles**	**Scalene**
one right angle	3 equal sides 3 equal angles, 60°	2 equal sides 2 equal angles	no equal sides or angles

Finding missing angles and sides:

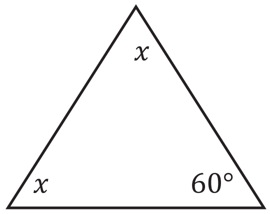
- The angles of a triangle always add up to 180 degrees. Subtract the known angles from 180 to find the measurement of a missing angle.
- To find the missing side of a right-angled triangle, apply Pythagoras' Theorem:

$$a^2 + b^2 = c^2$$



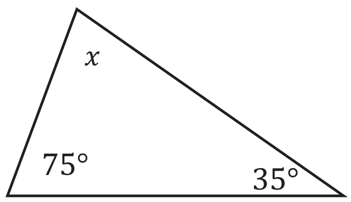
Solved Examples

1. Find the measure of angle x in the figure below:



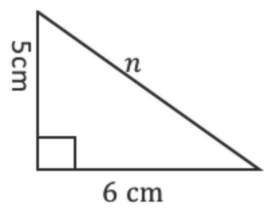
$x + x + 60^0$	$=$	180^o	Sum of angles is 180^o
$2x + 60^o$	$=$	180^o	
$2x$	$=$	$180^o - 60^o$	Transpose 60^o
$2x$	$=$	120^o	
$\frac{2x}{2}$	$=$	$\frac{120^o}{2}$	Divide throughout by 2
x	$=$	60^o	

2. Find the value of the angle in the diagram below



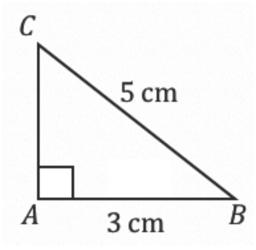
$35^0 + 75^o + x^o$	$=$	180^o	Sum of angles is 180^o
$110^o + x^o$	$=$	180^o	
x	$=$	$180^o - 110^o$	Transpose 110^o
x	$=$	70^o	

3. If the two sides shown below have lengths of 5 cm. and 6 cm., calculate the length of the hypotenuse.
   Solution:



n^2	$=$	$5^2 + 6^2$	Apply Pythagoras theorem
n^2	$=$	$25 + 36$	
n^2	$=$	61	
n	$=$	$\sqrt{61}$	Take the square root of both sides
n	$=$	7.81 cm.	

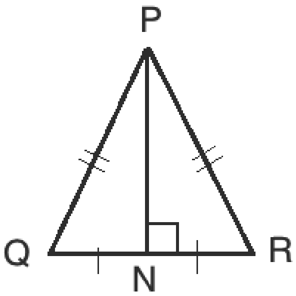
4. Find $\overline{AC}$ in the figure below



$\overline{BC}^2$	$=$	$\overline{AB}^2 + \overline{AC}^2$	Apply Pythagoras' theorem
5^2	$=$	$3^2 + \overline{AC}^2$	
25	$=$	$9 + \overline{AC}^2$	Transpose 9
$25 - 9$	$=$	$\overline{AC}^2$	
$\overline{AC}^2$	$=$	16	
$\overline{AC}$	$=$	$\sqrt{16}$	Take the square root of both sides
$\overline{AC}$	$=$	4 cm.	

5. In the diagram below, $\overline{PQ} = 13$ cm and $\overline{PN} = 12$ cm. Calculate the length $\overline{QR}$.

Step 1. Apply Pythagoras' theorem to find $\overline{QN}$:



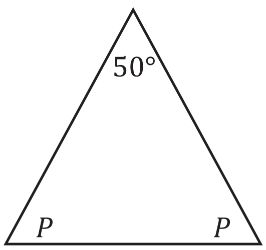
$$
\begin{aligned}
\overline{PQ}^2 &= \overline{QN}^2 + \overline{PN}^2 && \text{Apply Pythagoras theorem} \\
13^2 &= \overline{QN}^2 + 12^2 \\
169 &= \overline{QN}^2 + 144 \\
\overline{QN}^2 &= 169 - 144 && \text{Transpose } 144 \\
\overline{QN}^2 &= 25 \\
\overline{QN} &= \sqrt{25} && \text{Take the square root of both sides} \\
\overline{QN} &= 5 \text{ cm}
\end{aligned}
$$

Step 2. Multiply by 2 to find $\overline{QR}$: $\overline{QR} = 2 \times \overline{QN} = 2 \times 5 \text{ cm} = 10 \text{ cm}$

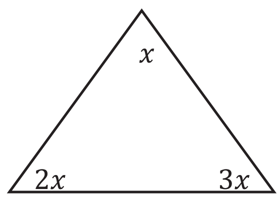
Practice

1. Find the values of the variables in the diagrams below:

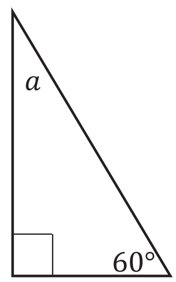
a.



b.

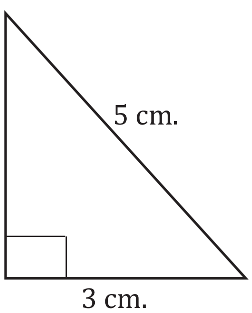


c.

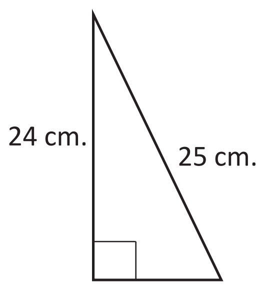


2. Find the missing lengths in the triangles below:

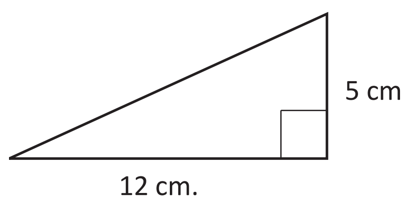
a.



b.



c.



Lesson Title: Significant Figures	**Theme:** Numbers and Numeration
Practice Activity: PHM2-L005	**Class:** SSS 2

Learning Outcome

By the end of the lesson, you will be able to round numbers to a given number of significant figures.

Overview

Rules for identifying significant figures (s.f.):

Rules	Examples	Number of s.f.
1. All non-zero digits are significant.	123	(3 s.f.)
2. Zeros between non-zero digits are significant.	12.507 304	(5 s.f.) (3 s.f.)
3. Zeros to the left of the first non-zero digit are not significant.	1.02 0.12 0.012	(3 s.f.) (2 s.f.) (2 s.f.)
4. If a number ends in zeros to the right of the decimal point, those zeros are significant.	2.0 2.00	(2 s.f.) (3 s.f.)
5. Zeros at the end of a whole number are not significant, unless there is a decimal point.	4300 4300.0	(2 s.f.) (5 s.f.)

Rounding to a stated number of significant figures:

a. Find the last significant figure you want to round to.
b. Look at the next significant figure immediately to the right.
c. If the next significant figure is less than 5, leave the last significant figure you want as it is. If the next significant figure is 5 or more, add 1 to the last significant figure you want.

Solved Examples

1. Round 287540 to a. 4 s.f.; b. 3 s.f.; c. 2 s.f.
   Answers: a. 287500; b. 288000; c. 290000

2. Round 0.0397 to 2 s.f.
   Answers: 0.040

3. Approximate the following numbers to 3 s.f.:
   a. 0.032847 b. 4.29984 c. 31.2511

Answers:

a. 0.0328 b. 4.30 c. 31.3

4. Correct each of these numbers to 2 s.f.:

   a. 12,304 b. 23.104 c. 0.29467

   Answers:

   a. 12,000 b. 23 c. 0.29

Practice

1. Round 587,257 to:

   a. 4 s.f. b. 3 s.f. c. 2 s.f.

2. Round 0.04662578 to 5 s.f.

3. Approximate the following numbers to 2 s.f.:

   a. 0.032847 b. 4.29984 c. 31.2511

4. Correct each of these numbers to 1 s.f.:

   a. 23,000 b. 539 c. 0.583

5. Mrs. Bangura owns a shop where she sells goods. In one day, her profit was 315,700. Round her profit to 2 significant figures.

6. According to the national census, the population of Freetown was 1,050,301 in 2015. Correct this number to 3 s.f.

Lesson Title: Estimation	Theme: Numbers and Numeration
Practice Activity: PHM2-L006	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to make a rough estimate of a calculation.

Overview

When we estimate an amount, we find the approximate value or quantity. We find something **close** to the answer, but not the exact answer. We can often make a rough estimate of a calculation by rounding numbers first, then applying operations. Estimation is very useful in everyday life.

Solved Examples

1. Sia spends Le 29,500.00 per month on transportation. How much does she spend per year? Find a rough estimate.
   Solution:
   By rounding to 1 s.f., we find that Sia spends around Le 30,000.00 per month on transportation. Using this estimate, she spends $30{,}000.00 \times 12 = Le\ 360{,}000.00$ per year on transportation.

2. A budget for local councils was organized in 2015. A council requested $3{,}825{,}000.00$ Leones for a radio broadcast. Give at least three ways in which a newspaper might have reported this amount.
   Solution:
   Possibilities include 4 million, 3.8 million, and 3.83 million Leones

3. The monthly salary of a director of studies is $Le\ 764{,}306.00$ per month.
   a. Calculate the director's yearly salary.
   b. Estimate the director's monthly salary, and calculate for the year.
   Solution:
   a. $Le\ 764{,}306.00 \times 12 = Le\ 9{,}171{,}672.00$
   b. By rounding to 1 s.f., we find that his monthly salary is $800{,}000.00$ per month. Therefore, the yearly estimate is $Le\ 800{,}000.00 \times 12 = Le\ 9{,}600{,}000.00$.

4. a. Use a calculator to find the value of $\frac{3.967 \times 0.0992}{2.06}$.
   b. Check your result by making a rough estimate.
   Solution:
   a. $\frac{3.967 \times 0.0992}{2.06} = \frac{0.3935264}{2.06}$ Multiply the numerator
   $= 0.191032$ Divide (rounded to 6 decimal places)

b. $\frac{3.967 \times 0.0992}{2.06} = \frac{4 \times 0.1}{2}$ Use rough estimates of the numbers

$= \frac{0.4}{2}$ Multiply and simplify

$= 0.2$

The answer to b. is near the answer to a. It is a good estimate.

Practice

1. The table below gives the estimated population of 5 districts in Sierra Leone in 2012.

District	Population	Estimated Population
Kailahun	525,372	
Kenema	609,873	
Kono	505,767	
Kambia	343,686	
Koinadugu	408,097	

   a. Round the populations to the nearest hundred thousand and complete the table. Use the rounded figures to answer the following questions.
   b. What is the approximate population difference between the districts with the highest and lowest populations?
   c. What is the approximate total population of the 5 districts?

2. A pupil writes 47 words in five lines of writing. His exercise book has 28 lines in each page.
   a. Approximately how many words does he write in one line?
   b. Approximately how many words does he write on one page?
   c. Approximately how many pages will a 600 words essay take up?

Lesson Title: Percentage Error	**Theme:** Numbers and Numeration
Practice Activity: PHM2-L007	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to calculate the percentage error when using rounded values.

Overview

When we make an estimation, percentage error tells us how close to the exact value our estimated value is. A smaller percentage error means our estimate is **more** accurate, and a larger percentage error means our estimate is **less** accurate.

When using measurements, estimating with smaller units of measure gives us a smaller percentage error. For example, estimating with centimetres is more accurate than estimating with metres. The percentage error will be smaller.

Percentage error is calculated from another amount called simply "error". We calculate error first, then percentage error.

How to find error:

- Find the range of numbers that a rounded quantity could fall between.
  - Example: If an estimated measurement is 2.5 metres to 2 s.f., then the actual measurement could lie in the range 2.45 m to 2.55 m.
- Error is the difference between the estimated amount, and the minimum and maximum possible amounts that the actual quantity could be.
  - Example: In the case above, error is $2.55 - 2.5 = +0.05 \text{ m}$ or $2.45 - 2.5 = -0.05 \text{ m}$. This is written as $\text{error} = \pm 0.05 \text{ m}$.
- Error is usually written with a $\pm$ symbol.

How to find percentage error:

- Percentage error is found by finding an error as a percentage of the estimated measurement.
- The formula is:

$$\text{Percentage error} = \frac{\text{error}}{\text{measurement}} \times 100\%$$

Solved Examples

1. The width of a room is measured, and is found to be 2.5 metres to 2 s.f. What is the percentage error?

$$\text{error} = 2.55 - 2.5 = +0.05 \text{ m}$$

$$\text{or error} = 2.45 - 2.5 = -0.05 \text{ m}$$

$$\begin{aligned} \text{Percentage error} &= \frac{\text{error}}{\text{measurement}} \times 100\% \\ &= \frac{\pm 0.05 \text{ m}}{2.5 \text{ m}} \times 100\% \\ &= \pm 2\% \end{aligned}$$

2. If 5.34 is approximated to 5. Calculate the percentage error.

   Solution:

$$\begin{aligned} \text{error} &= 5.34 - 5.00 = 0.34 \\ \text{Percentage error} &= \frac{\text{error}}{\text{measurement}} \times 100\% \\ &= \frac{0.34}{5.34} \times 100\% \\ &= 6.37\% \end{aligned}$$

3. The side of a triangle is measured at 6 cm to the nearest cm.
   a. What is the range of its actual length?
   b. What is the percentage error?

   Solutions:

   a. Numbers in the range 5.5 cm to 6.5 cm would round to 6 cm. This is the range of the actual length of the side of the triangle.
   b. Percentage error:

$$\begin{aligned} \text{error} &= 6.5 - 6.0 = 0.5 \text{ m} \\ \text{or error} &= 5.5 - 6.0 = -0.5 \text{ m} \end{aligned}$$

$$\begin{aligned} \text{Percentage error} &= \frac{\text{error}}{\text{measurement}} \times 100\% \\ \text{Percentage error} &= \frac{\pm 0.5 \text{ m.}}{6 \text{ m.}} \times 100\% \\ &= \pm \frac{50}{6}\% = \pm 8\frac{2}{6}\% = \pm 8\frac{1}{3}\% \end{aligned}$$

4. The length of a fence is given at 250 m, accurate to the nearest 10 m.
   a. What is the range of its actual length?
   b. What is the error?
   c. What is the percentage error?

   Solutions:

   a. Numbers in the range 245 to 255 would round to 250 m when rounded to the nearest 10 m. This is the range of the actual length of the fence.
   b. Error: $245 - 250 = -5 \text{ m}$ or $255 - 250 = 5 \text{ m}$

      Error = $\pm 5 \text{ m}$
   c. Percentage error:

$$\text{Percentage error} = \frac{\pm 5 \text{ m.}}{250 \text{ m.}} \times 100\%$$

$$= \pm\frac{500}{250}\% = \pm 2\%$$

5. The length of a rectangle is 10 cm. A pupil measures it as 9.9 cm. What is the percentage error?

   Solution:

$$\text{error} = 9.9 \text{ cm} - 10\text{cm} = -0.1 \text{ cm}$$

$$\begin{aligned}\text{Percentage error} &= \frac{\text{error}}{\text{measurement}} \times 100\% \\ &= \frac{-0.1 \text{ cm.}}{10} \times 100\% \\ &= -1\%\end{aligned}$$

Practice

1. A length was measured by a pupil to be 2.5 cm to the nearest 1 decimal place. Calculate the percentage error.
2. The length of a house is measured as 8 metres to the nearest metre.
   a. What is the range of its actual length?
   b. Calculate the percentage error.
3. A child's height was measured at 0.5 metres, correct to 1 decimal place. What is the percentage error?
4. The school yard was measured to be 200 metres long, rounded to the nearest 10 m. What is the percentage error?
5. The height of a tree is 7.5 metres to 2 s.f.
   a. What is the range of its actual length?
   b. Calculate the percentage error.
6. The weight of a bag of rice is 20 kg. An old scale weighed the bag as 20.4 kg. What is the percentage error?

Lesson Title: Degree of accuracy	**Theme:** Numbers and Numeration
Practice Activity: PHM2-L008	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to decide on the degree of accuracy that is appropriate for given data which may have been rounded.

Overview

"Degree of accuracy" refers to how many significant figures or decimal places a number has. Generally, using more digits makes a number more accurate.
For example, consider rounding the number 23547:

23550	23500	24000	20000
4 s.f.	3 s.f.	2 s.f.	1 s.f.

more accurate → *less accurate*

The degree of accuracy that you should give in an answer depends on the degree of accuracy in the problem. The answer should not be given to more significant figures than the numbers requested in the problem.
It is not necessary to round to the given number of significant figures at each step, or in the middle of a calculation. Numbers in a middle step of your calculation may have more significant figures than those in the problem and answer.

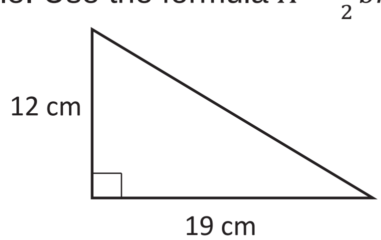
Solved Examples

1. Calculate the area of a circle with radius 6.5 m. Take π to be 3.14 and use $A = \pi r^2$.
   Solution:

$$\begin{aligned} A &= \pi r^2 \\ &= (3.14)(6.5\text{ m})^2 \\ &= 132.665\text{ m}^2 \\ &= 130\text{ m}^2 \text{ to 2 s.f.} \end{aligned}$$

   Note: The answer is given to 2 s.f. because the data in the problem ($r = 6.5\text{ m}$) is given to 2 s.f.

2. In the triangle below, the lengths of two sides are given to the nearest centimetre. Find the area of the triangle. Use the formula $A = \frac{1}{2}bh$ and give your answer to 2 s.f.



Solution:

$$
\begin{aligned}
A &= \tfrac{1}{2}(19\text{ cm})(12\text{ cm}) \\
&= (6\text{ cm})(19\text{ cm}) \\
&= 114\text{ cm}^2 \\
&= 110\text{ cm}^2 \quad \text{to 2 s.f.}
\end{aligned}
$$

3. A carpenter wants to make a table top from a piece of wood that is 30.1 cm long and 25 cm wide. What is the area of the table top, correct to 2 s.f.? Use the formula $A = l \times w$.

   Solution:

$$
\begin{aligned}
A &= (30.1\text{ cm})(25\text{ cm}) \\
&= 752.5\text{ cm}^2 \\
&= 750\text{ cm}^2 \quad \text{to 2 s.f.}
\end{aligned}
$$

4. A bus going from Kailahun to Freetown travels 400 km in 9 hours. Estimate its speed in km/hour.

   Solution:

$$
\begin{aligned}
\text{speed} &= \frac{400\text{ km}}{9\text{ hours}} \\
&= 44.\overline{4}\text{ km/hour} \\
&= 40\text{ km/hour} \quad \text{to 1 s.f.}
\end{aligned}
$$

Practice

1. The population of a village is 25,500 people. If exactly half of them are female, how many females live in the village? Give your answer to 3 s.f.
2. Fatu wants to find the area of her house, which is in the shape of a rectangle. She measures the width as 12.5 m, and the length as 18.5 m. What is the area of her house? Use the formula $A = l \times w$ and give your answer to 3 s.f.
3. A circle has radius 25 m. Find the circumference of the circle. Take π to be 3.14, and use $C = 2\pi r$. Give your answer to 2 s.f.
4. Foday uses geometry construction to draw a rectangle that is 7.5 cm long and 5.0 cm wide. What is the area of the rectangle? Give your answer to 2 sf.
5. A man walks 12 km in 150 minutes. How long does it take him to walk 1 kilometre? Give your answer to 2 s.f.
6. A painter will paint a wall and he wants to know how much paint to buy. He measures a wall to be 4.1 m wide by 2.3 m tall. What is the area of the wall? Use the formula $A = l \times w$ and give your answer to 2 s.f.

Lesson Title: Simultaneous linear equations using elimination	Theme: Algebraic Processes
Practice Activity: PHM2-L009	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to solve simultaneous linear equations using elimination.

Overview

Simultaneous linear equations are 2 equations that are solved at the same time (or simultaneously) and have the same answer. We must solve for 2 unknown variables (usually x and y) and the two results should satisfy both equations.

For example, this is a set of simultaneous equations:

$$\begin{aligned} 2x + 3y &= 10 \quad (1) \\ 2x + y &= 2 \quad (2) \end{aligned}$$

This is the answer to the simultaneous equations: $x = -1, y = 4$, or $(-1, 4)$
There are 3 ways to solve simultaneous equations. This lesson covers the method of elimination.

To solve by elimination:

- Subtract one equation from the other so that one of the variables (x or y) is eliminated.
- If one of the variables has the same coefficient in the two equations, it will easily be canceled by subtraction.
- If neither of the variables has the same coefficient, you will need to multiply the equations throughout so that the coefficients are the same before subtracting.
- Check your answers by substituting the values for x and y into the original equations.

Solved Examples

1. Solve:

$$\begin{aligned} 2x + 3y &= 10 \\ 2x + y &= 2 \end{aligned}$$

Solution:

$$\begin{aligned} 2x + 3y &= 10 \quad (1) \\ -(2x + y &= 2) \quad (2) \end{aligned}$$

Subtract equation (2) from equation (1).

$$\begin{aligned} 0 + 2y &= 8 \\ 2y &= 8 \\ \frac{2y}{2} &= \frac{8}{2} \\ y &= 4 \end{aligned}$$

Note: The negative sign will change all the signs across the bracket

Divide both sides of the equation by the co-efficient of y (which is 2)

Substitute the value of $y = 4$ in either equation 1 or 2 to find the value of x:

$2x + 3y$	$=$	10	(1)
$2x + 3(4)$	$=$	10	Substitute $y = 4$ in equation (1)
$2x + 12$	$=$	10	
$2x$	$=$	$10 - 12$	Transpose 12
$2x$	$=$	-2	
$\frac{2x}{2}$	$=$	$\frac{-2}{2}$	Divide throughout by 2
x	$=$	-1	

The solution is therefore $x = -1, y = 4$, or $(-1, 4)$
Check the answer by substituting $x = -1, y = 4$ into equation 2. The left-hand side (LHS) should equal the right-hand side (RHS):

$2x + y$	$=$	2	(2)
$2(-1) + 4$	$=$	2	Substitute $x = -1, y = 4$ in equation (2)
$-2 + 4$	$=$	2	
2	$=$	2	
LHS	$=$	RHS	

2. Solve:

$2x + 5y$	$=$	2	(1)
$3x + 2y$	$=$	25	(2)

Note: None of the variables have the same co-efficient. We need to make two coefficients the same for one of the variables. Let's choose x.

Make the coefficients of x the same by multiplying equation (1) throughout by the co-efficient of x in equation (2), and multiplying equation (2) throughout by the co-efficient of x in equation (1).

Solution:

$6x + 15y$	$=$	6	(3)	Multiply equation (1) $\times\ 3$
$6x\ +\ 4y$	$=$	50	(4)	Multiply equation (2) $\times 2$
$0 + 11y$	$=$	-44		Subtract equation (4) from equation (3)
$\frac{11y}{11}$	$=$	$\frac{-44}{11}$		Divide throughout by 11
y	$=$	-4		
$2x + 5y$	$=$	2	(1)	
$2x + 5(-4)$	$=$	2		Substitute $y = -4$ in equation (1)
$2x - 20$	$=$	2		
$2x$	$=$	$2 + 20$		Transpose -20
$2x$	$=$	22		
$\frac{2x}{2}$	$=$	$\frac{22}{2}$		Divide throughout by 5
x	$=$	11		

The solution is therefore $x = 11, y = -4$, or the ordered pair $(11, -4)$. This can be checked by substituting these values into the original equations.

Practice

Use elimination to solve the simultaneous equations:

1. $p + 2q = 5$
   $p + q = 3$

2. $3n + 2p = 3{,}800$
   $n + 2p = 1{,}400$

3. $3x + 2y = 8$
   $4x - 3y = 5$

4. $5x + 3y = 10$
   $3x + 2y = 6$

Lesson Title: Simultaneous linear equations using substitution	Theme: Algebraic Processes
Practice Activity: PHM2-L010	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to solve simultaneous linear equations using substitution.

Overview

There are 3 ways to solve simultaneous equations. This lesson covers the method of substitution.

To solve by substitution:

- We must change the subject.
- We should choose one of the given equations and make one of the variables the subject of the other one.
- If one of the variables does not have a coefficient, it is easiest to solve for that variable.
- After changing the subject, we substitute the expression into the other linear equation.

Solved Examples

1. Solve:

$$\begin{aligned} a + 2b &= 13 \\ 2a - 3b &= 5 \end{aligned}$$

Solution:

Note: The first equation has a variable without a coefficient (a). It will be easiest to solve the first equation for a, and substitute the result into the second equation.

$a + 2b$	$=$	13	(1)	
a	$=$	$13 - 2b$		Change the subject of equation (1) by transposing $2b$
$2(13 - 2b) - 3b$	$=$	5	(2)	Substitute equation (1) into equation (2)
$26 - 4b - 3b$	$=$	5		Simplify the left-hand side
$26 - 7b$	$=$	5		
$-7b$	$=$	$5 - 26$		Transpose 26
$-7b$	$=$	-21		
$\frac{-7b}{-7}$	$=$	$\frac{-21}{-7}$		Divide throughout by -7

$$b = 3$$

Substitute the value of $b = 3$ into the formula $a = 13 - 2b$ to find the value of a.

$$a = 13 - 2(3) \qquad \text{Substitute } b = 3 \text{ in the formula for } a$$
$$a = 13 - 6$$
$$a = 7$$

The solution is therefore $a = 7, b = 3$.

Check the solution by substituting the values for a and b in equation (2):

$$2a - 3b = 5$$
$$2(7) - 3(3) = 5$$
$$14 - 9 = 5$$
$$5 = 5$$
$$\text{LHS} = \text{RHS}$$

2. Solve:

$$2x - y = 1$$
$$x + 2y = 3$$

Solution:

$$x + 2y = 3 \qquad (2)$$
$$x = 3 - 2y \qquad \text{Change the subject of equation (2)}$$

$$2(3 - 2y) - y = 1 \qquad (1) \qquad \text{Substitute equation (2) into equation (1)}$$
$$6 - 4y - y = 1 \qquad \text{Simplify the left-hand side}$$
$$6 - 5y = 1$$
$$-5y = 1 - 6 \qquad \text{Transpose 6}$$
$$-5y = -5$$
$$\frac{-5y}{-5} = \frac{-5}{-5} \qquad \text{Divide throughout by } -5$$
$$y = 1$$

Substitute the value $y = 1$ into the formula for x, $x = 3 - 2y$.

$$x = 3 - 2(1) \qquad \text{Substitute } y = 1 \text{ in the formula for } x$$
$$x = 3 - 2$$
$$x = 1$$

Thus, the solution is $x = 1$, $y = 1$.

Practice

Solve the following simultaneous equations by the method of substitution:

1. $5x + 3y = 18$
   $y = 2x - 5$

2. $x + 2y = -4$
   $x = y + 5$

3. $2x + 3y = 16$
   $y - x = 2$

4. $2s - 5t = 1$
   $s - 3t = -1$

Lesson Title: Simultaneous linear equations using graphical methods – Part 1	**Theme:** Algebraic Processes
Practice Activity: PHM2-L011	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to solve simultaneous linear equations using graphical methods.

Overview

There are 3 ways to solve simultaneous equations. This lesson and the next lesson cover the graphical method.

We can solve a set of simultaneous equations by graphing both lines. The solution is the point where the lines intersect. At this point, the x-value and y-value satisfy both of the equations.

After graphing, remember to check your answer by substituting the values into the original equations.

When graphing, the answers we get may be only approximate. The more accurately we draw our Cartesian planes, the more accurate the solution will be. It is important to draw your plane precisely, with each point on the axes the same distance apart.

Solved Examples

Solve the simultaneous equations by using the given tables of values, and graphing them on the Cartesian plane:

$$-x + y = 1 \quad (1)$$
$$2x + y = 4 \quad (2)$$

Equation (1)			
x	0	1	2
y			

Equation (2)			
x	0	1	2
y			

Solution:
Fill the table of values for equation (1).

$$-x + y = 1 \quad (1)$$
$$y = x + 1$$ Change the subject

$$y = (0) + 1$$ Substitute each value of x and solve for y

$$y = 1$$

$$y = (1) + 1$$
$$y = 2$$

$$y = (2) + 1$$
$$y = 3$$

Equation (1)			
x	0	1	2
y	1	2	3

Fill the table of values for equation (2).

$2x + y = 4$ (2)

$y = -2x + 4$ Change the subject

$y = -2(0) + 4$ Substitute each value of x and solve for y

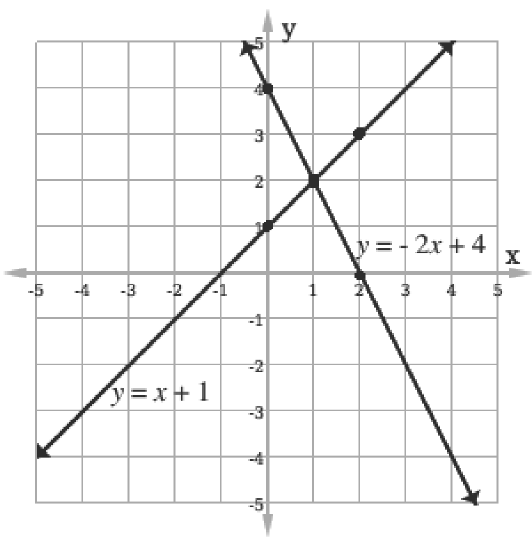
$y = 4$

$$y = -2(1) + 4$$
$$y = 2$$

$$y = -2(2) + 4$$
$$y = 0$$

Equation (2)			
x	0	1	2
y	4	2	0

Graph both lines on a Cartesian plane:



Observe the point of intersection (where the 2 lines cross). This is the solution to the simultaneous equations.

The solution is $(1, 2)$.

Practice

Solve the simultaneous equation by using the given tables of values and graphing on the Cartesian plane.

1. $x + y = 8$ (1)
   $x - y = 4$ (2)

Equation 1				
x	2	4	6	8
y				

Equation 2				
x	2	4	6	8
y				

2. $y = x + 1$ (1)
   $2y = 5 - x$ (2)

Equation 1				
x	1	2	3	4
y				

Equation 2				
x	1	2	3	4
y				

3. $y + 2x = 1$ (1)
   $y - 3x = 11$ (2)

Equation 1			
x	-3	-2	-1
y			

Equation 2			
x	-3	-2	-1
y			

4. $4x + 2y = 12$ (1)
   $y - x = 3$ (2)

Equation 1			
x	0	2	4
y			

Equation 2			
x	0	2	4
y			

Lesson Title: Simultaneous linear equations using graphical methods – Part 2	**Theme:** Algebraic Processes
Practice Activity: PHM2-L012	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to solve simultaneous linear equations using graphical methods.

Overview

This lesson follows the previous lesson, which was also on solving simultaneous equations using graphical methods. In the previous lesson, you were given a table of values to fill. In this lesson, the problems are more challenging. You must make your own table of values. You may have to look for the point of intersection, because it may not be as obvious.

Solved Examples

1. Using graphical methods, find the solution of the simultaneous equations:

$$-2x + y = 2 \quad (1)$$
$$-x + y = -1 \quad (2)$$

Solution:

First, create tables of values and fill them. For your table, it is better to choose x-values near zero (for example, between -5 and 5). These will be easier to graph. For example, here are 2 tables of values we can fill:

Equation (1)

x	-2	0	2
y			

Equation (2)

x	-2	0	2
y			

Fill the table of values for equation (1).

$$-2x + y = 2 \quad (1)$$
$$y = 2x + 2 \quad \text{Change the subject}$$

$$y = 2(-2) + 2$$
$$y = -2$$

Substitute each value of x and solve for y

$$y = 2(0) + 2$$
$$y = 2$$

$$y = 2(2) + 2$$
$$y = 6$$

Equation (1)

x	-2	0	2
y	-2	2	6

Fill the table of values for equation (2).

$$-x + y = -1 \qquad (1)$$

$$y = x - 1$$ Change the subject

$$y = (-2) - 1$$
$$y = -3$$

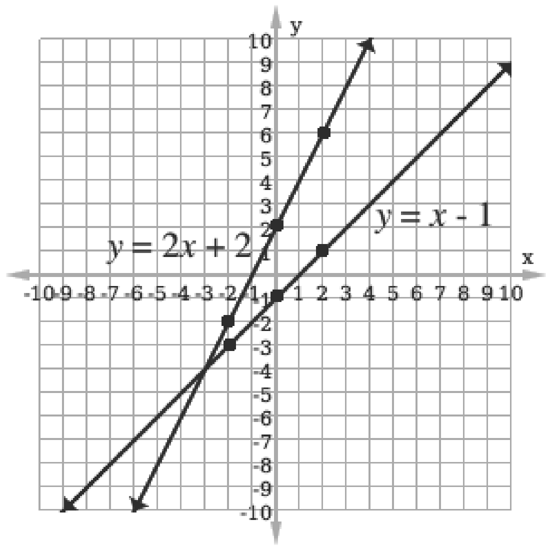
Substitute each value of x and solve for y

$$y = (0) - 1$$
$$y = -1$$

Equation (2)			
x	-2	0	2
y	-3	-1	1

$$y = (2) - 1$$
$$y = 1$$

Graph both lines on a Cartesian plane:



Observe the point of intersection (where the 2 lines cross). You may need to extend your lines using a straight edge to see the intersection point. This is the solution to the simultaneous equations.
The solution is $(-3, -4)$

Practice

1. On the same axes, draw graphs of the relations $x + y = 2$ and $y = 3x - 2$ in the range $-1 \leq x \leq 2$. Find values (x, y) which satisfy both relations at the same time.
2. Using graphical methods, find the solution of the simultaneous equations $y + 2x = 4$ and $y = 2x$. Use the range $-1 \leq x \leq 3$.
3. On the same axes, draw graphs of the relations $2x + y = 5$ and $x + 2y = 7$ in the range $0 \leq x \leq 3$. Find the solution of these simultaneous equations.

Lesson Title: Words problems on simultaneous linear equations	**Theme:** Algebraic Processes
Practice Activity: PHM2-L013	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to solve word problems leading to simultaneous linear equations.

Overview

Algebra is often used to solve word problems. We often need to identify the unknown variables in word problems, and use letters to represent them. In some cases, a word problem will give two equations with the same variables. These are simultaneous equations, and they can be solved using either elimination or substitution.

Follow this process to solve simultaneous equation word problems:

Step 1. Identify the unknown variables
Step 2. Create simultaneous linear equations
Step 3. Solve using the method you prefer

Solved Examples

1. One pen and 2 exercise books cost Le 4,500.00. Two pens and 3 exercise books cost Le 7,000.00. How much does each pen and exercise book cost?

Solution:

Step 1. Identify the unknown variables:

p: The cost of a pen
e: The cost of an exercise book

Step 2. Create the linear equations:

$p + 2e = 4{,}500.00$ (1)

$2p + 3e = 7{,}000.00$ (2)

Step 3. Solve the equations using the method you prefer (choose one):

Elimination:

$2(p + 2e = 4{,}500)$ (1)x2
$-(2p + 3e = 7{,}000)$ (2)

$\downarrow$ $\downarrow$

$2p + 4e = 9{,}000$ (1)x2

Substitution:

$p + 2e = 4{,}500$ (1)
$p = 4{,}500 - 2e$

Substitute $p = 4{,}500 - 2e$ into equation 2:

$2(4{,}500 - 2e) + 3e = 7{,}000$ (2)

$$\begin{aligned} -(2p + 3e &= 7{,}000) \quad (2) \\ \hline 0 + e &= 2{,}000 \\ e &= 2{,}000 \end{aligned}$$

$$\begin{aligned} p + 2(2{,}000) &= 4{,}500 \quad (1) \\ p + 4{,}000 &= 4{,}500 \\ p &= 500 \end{aligned}$$

$$\begin{aligned} 9{,}000 - 4e + 3e &= 7{,}000 \\ 9{,}000 - e &= 7{,}000 \\ 2{,}000 &= e \end{aligned}$$

$$\begin{aligned} p &= 4{,}500 - 2(2{,}000) \quad (1) \\ p &= 500 \end{aligned}$$

Answer: $e = \text{Le } 2{,}000.00$, $p = \text{Le } 500.00$

2. The sum of two numbers is 8, and the product is 15. Find the numbers.
Solution:

Step 1. Identify the two variables. Let them be x and y.

Step 2. Create the linear equations:

$$x + y = 8 \quad (1)$$

$$xy = 15 \quad (2)$$

Step 3. Solve the equations using the method you prefer. Substitution is shown:

Change the subject of (1): $x = 8 - y$

Substitute for x in equation (2):

$$\begin{aligned} y(8 - y) &= 15 \\ 8y - y^2 &= 15 && \text{Clear the bracket} \\ y^2 - 8y + 15 &= 0 \\ (y - 3)(y - 5) &= 0 && \text{Factorising} \\ y - 3 = 0 \quad &\text{or} \quad y - 5 = 0 \\ y = 3 \quad &\text{or} \quad y = 5 \end{aligned}$$

Substitute for y in equation (1):

$$\begin{aligned} x &= 8 - 3 \\ x &= 5 \end{aligned}$$

And

$$\begin{aligned} x &= 8 - 5 \\ x &= 3 \end{aligned}$$

Therefore:

When $y = 3, \quad x = 5$

When $y = 5, \quad x = 3$

The two numbers are 3 and 5.

3. A number is made of 2 digits. The sum of the digits is 12. If the digits are interchanged, the original number is increased by 18. Find the number.
   Solution:
   Step 1. Identify the two variables. Let them be a and b.

   Step 2. Create the linear equations:
   $$a + b = 12 \qquad (1)$$
   For the second equation, consider that the number with a in the tens place and b in the ones place is reversed. The result is a number that is 18 more than the original number. This can be written:
   $$10a + b + 18 = 10b + a \qquad (2)$$
   Combine like terms:
   $$9a - 9b = -18 \qquad (2)$$
   Step 3. Solve the equations. Elimination is shown:
   $$\begin{array}{rcll} 9(a + b & = & 12) & (1)\text{x}9 \\ -(9a - 9b & = & -18) & (2) \\ \hline \downarrow & & \downarrow & \\ 9a + 9b & = & 108 & (1) \\ -(9a - 9b & = & -18) & (2) \\ \hline 0 + 18b & = & 126 & \\ b & = & 7 & \\ & & & \\ a + 7 & = & 12 & (1) \\ a & = & 5 & \end{array}$$
   Answer: The original number is 57.

Practice

1. Williams and Kamara ran against each other in a constituency election. Williams was elected with 240 more votes than Kamara. The total number of votes cast was 1,210. How many votes did each of the two candidates received?
2. A woman bought four knives and six forks at the cost of Le 12,000.00. At another time, she bought six knives and five forks at the cost of Le 14,000.00. Find the cost of a knife and fork.
3. The sum two numbers is 64 and their difference is 30. Find the numbers.
4. A number is made up of two digits. The sum of the digits is 11. If the digits are interchanged, the original number is increased by 9. Find the number.

Lesson Title: Simultaneous linear and quadratic equations using substitution	**Theme:** Algebraic Processes
Practice Activity: PHM2-L014	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to solve simultaneous linear and quadratic equations using substitution.

Overview

Linear and quadratic equations can be simultaneous. As with simultaneous linear equations, they are solved for variables that satisfy both equations.

These are simultaneous linear and quadratic equations:

$$y = x + 2 \quad (1) \quad \text{Linear}$$
$$y = x^2 \quad (2) \quad \text{Quadratic}$$

Simultaneous linear and quadratic equations can be solved using substitution or graphing. They can have 0, 1 or 2 solutions. Most of the problems you are asked to solve have 2 solutions. The solutions are ordered pairs, (x, y).

To solve simultaneous linear and quadratic equations using substitution:

- Make y the subject of one equation (solve for y), then substitute it into the other equation.
- Simplify until it has the form of a standard quadratic equation ($ax^2 + bx + c = 0$).
- Solve the quadratic equation using any method. This practice activity uses factorisation in the examples.
- Each solution to the quadratic equation is an x-value. Substitute these into one of the original equations to find each corresponding y-value.

Solved Examples

1. Solve:

$$y = x + 2 \quad (1)$$
$$y = x^2 \quad (2)$$

Solution:

$$x^2 = x + 2 \quad (1) \qquad \text{Substitute } y = x^2 \text{ for } y \text{ in equation (1)}$$
$$x^2 - x - 2 = 0 \qquad \text{Transpose } x \text{ and } 2$$
$$(x - 2)(x + 1) = 0 \qquad \text{Factorise the quadratic equation}$$

$$x - 2 = 0 \text{ or } x + 1 = 0 \qquad \text{Set each binomial equal to 0}$$
$$x = 2 \text{ or } x = -1 \qquad \text{Transpose } -2 \text{ and } 1$$

$$y = (2) + 2 \quad \text{Substitute } x = 2 \text{ into equation (2)}$$
$$y = 4$$

$$y = (-1) + 2 \quad \text{Substitute } x = -1 \text{ into equation (2)}$$
$$y = 1$$

Solutions: $(2, 4)$ and $(-1, 1)$

2. Solve:

$$y = x^2 - 5x + 7 \qquad (1)$$
$$y - 2x = 1 \qquad (2)$$

Solution:

$(x^2 - 5x + 7) - 2x$	$=$	1	(2)	Substitute equation (1) into equation (2)
$x^2 - 7x + 7$	$=$	1		Simplify
$x^2 - 7x + 7 - 1$	$=$	0		Transpose 1
$x^2 - 7x + 6$	$=$	0		
$(x - 6)(x - 1)$	$=$	0		Factorise the quadratic equation
$x - 6 = 0$	or	$x - 1 = 0$		Set each binomial equal to 0
$x = 6$	or	$x = 1$		Transpose -6 and -1
$y - 2(6)$	$=$	1		Substitute $x = 6$ into equation (2)
$y - 12$	$=$	1		
y	$=$	$1 + 12$		Transpose -12
y	$=$	13		
$y - 2(1)$	$=$	1		Substitute $x = 1$ into equation (2)
$y - 2$	$=$	1		
y	$=$	$1 + 2$		Transpose -2
y	$=$	3		

Solutions: $(6, 13)$ and $(1, 3)$

Practice

Solve the following sets of simultaneous equations:

1. $x^2 + y = 3$
   $x + y = 1$

2. $y = x^2 + 3x + 1$
   $y = -2x - 5$

3. $y = 4x - 2$
   $2x^2 + y = -4$

4. $y - x^2 = 9$
   $10x + y = 0$

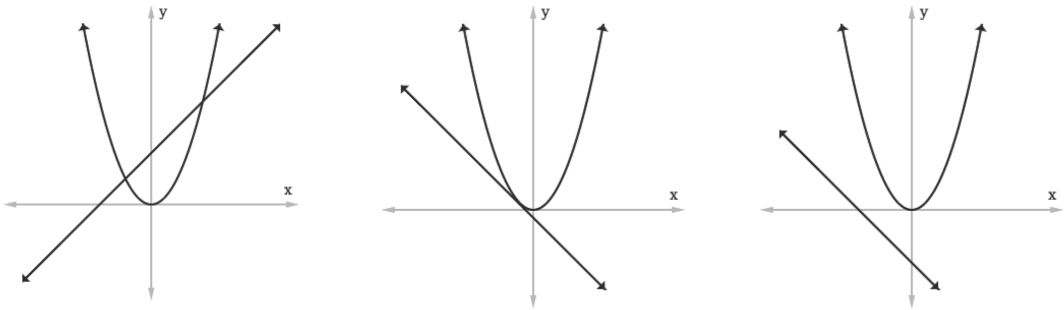
5. $y = x^2 - 9x + 3$
   $y + x = -4$

Lesson Title: Simultaneous linear and quadratic equations using graphical methods - Part 1	**Theme:** Algebraic Processes
Practice Activity: PHM2-L015	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to solve simultaneous linear and quadratic equations using graphical methods.

Overview

When we graph simultaneous linear and quadratic equations, the intersection points of the curve and line are the solutions to the simultaneous equations.
Simultaneous linear and quadratic equations can have 2 solutions, 1 solution, or no solutions:



2 solutions **1 solution** **0 solutions**

Note that when the simultaneous equations have 1 solution, the line touches the parabola at one point. This is also called a "tangent line". When there are 0 solutions, the line and parabola will never touch at any point.

Solved Examples

1. Solve by graphing:

$$y = x + 2 \quad (1)$$
$$y = x^2 \quad (2)$$

Solution:

Fill the table of values for $y = x + 2$:

x	-2	-1	0	1	2
y	0	1	2	3	4

$$\begin{aligned} y &= (-2) + 2 \\ &= 0 \end{aligned}$$

$$\begin{aligned} y &= (-1) + 2 \\ &= 1 \end{aligned}$$

$$\begin{aligned} y &= (0) + 2 \\ &= 2 \end{aligned}$$

$$\begin{aligned} y &= (1) + 2 \\ &= 3 \end{aligned}$$

$$\begin{aligned} y &= (2) + 2 \\ &= 4 \end{aligned}$$

Fill the table of values for $y = x^2$:

x	-2	-1	0	1	2
y	0	1	2	3	4

$$y = (-2)^2 = 4$$

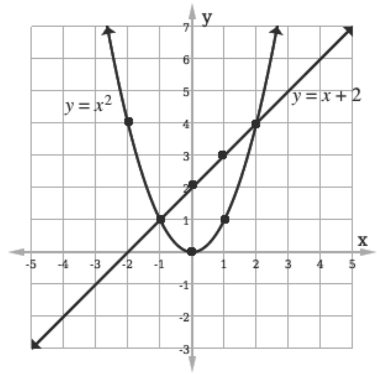
$$y = (-1)^2 = 1$$

$$y = (1)^2 = 1$$

$$y = (0)^2 = 0$$

$$y = (2)^2 = 4$$

Plot the points from both tables. Draw the line and curve:



Solutions: $(-1, 1)$, $(2, 4)$

2. Solve by graphing:

$$y = -x^2 - 2 \quad (1)$$
$$y = -2x + 1 \quad (2)$$

Solution:

Fill the table of values for $y = -x^2 - 2$:

x	-2	-1	0	1	2
y	-6	-3	-2	-3	-6

$$y = -(-2)^2 - 2 = -6$$

$$y = -(1)^2 - 2 = -3$$

$$y = -(-1)^2 - 2 = -3$$

$$y = -(2)^2 - 2 = -6$$

$$y = -(0)^2 - 2 = -2$$

Fill the table of values for $y = -2x + 1$:

x	-2	-1	0	1	2
y	5	3	1	-1	-3

$$y = -2(-2) + 1 = 5$$

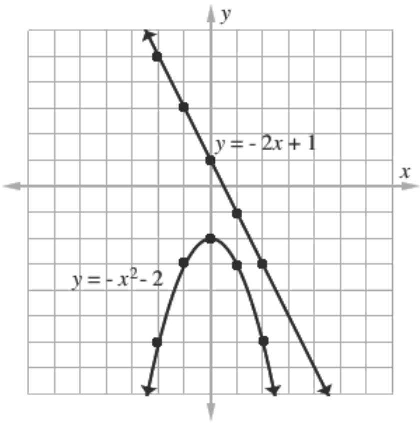
$$y = -2(-1) + 1 = 3$$

$$y = -2(1) + 1 = -1$$

$$y = -2(0) + 1$$
$$= 1$$

$$y = -2(2) + 1$$
$$= -3$$

Plot the points from both tables. Draw the line and curve:



No solution

Practice

Solve the simultaneous equations by filling the given tables of values and graphing:

1. $y = x + 3$
   $y = x^2 - 2x + 3$

x	-1	0	1	2	3
y					

x	-1	0	1	2	3
y					

2. $y = x^2 - 3x + 5$
   $y = x - 1$

x	0	1	2	3	4
y					

x	0	1	2	3	4
y					

3. $y = -x^2 + x + 2$
   $y + 2 = x$

x	-2	-1	0	1	2	3
y						

x	-2	-1	0	1	2	3
y						

4. $y = x^2 - 3x + 2$
   $y = x - 2$

x	0	1	2	3
y				

x	0	1	2	3
y				

Lesson Title: Simultaneous linear and quadratic equations using graphical methods - Part 2	**Theme:** Algebraic Processes
Practice Activity: PHM2-L016	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to solve simultaneous linear and quadratic equations using graphical methods.

Overview

This lesson follows the previous lesson, which was also on solving simultaneous linear and quadratic equations using graphical methods. In the previous lesson, you were given a table of values to fill. In this lesson, the problems are more challenging. You must make your own table of values. You may have to look for the point of intersection, because it may not be obvious.

If you plot the points and the parabola is not clear, add more columns to the table of values and plot more points. If you cannot see the point of intersection, it may be outside of the points you graphed. Plot more points until the intersection becomes clear. Remember that there could be 0, 1, or 2 points of intersection.

Solved Examples

1. Locate the points where $y = x^2 - 2x - 3$ intersects $y = x - 5$.
   Solution:
   First, create tables of values and fill them. For your table, it is better to choose x-values near zero (for example, between -5 and 5). These will be easier to graph. For example, here are 2 tables of values we can fill:

$y = x^2 - 2x - 3$

x	-1	0	1	2	3
y					

$y = x - 5$

x	-1	0	1	2	3

Fill the table of values for $y = x^2 - 2x - 3$:

$$y = (-1)^2 - 2(-1) - 3$$
$$y = 0$$

$$y = (0)^2 - 2(0) - 3$$
$$y = -3$$

$$y = (1)^2 - 2(1) - 3$$
$$y = -4$$

$$y = (2)^2 - 2(2) - 3$$
$$y = -3$$

$$y = (3)^2 - 2(3) - 3$$
$$y = 0$$

x	-1	0	1	2	3
y	0	-3	-4	-3	0

Fill the table of values for $y = x - 5$:

$y = (-1) - 5$
$y = -6$

$y = (0) - 5$
$y = -5$

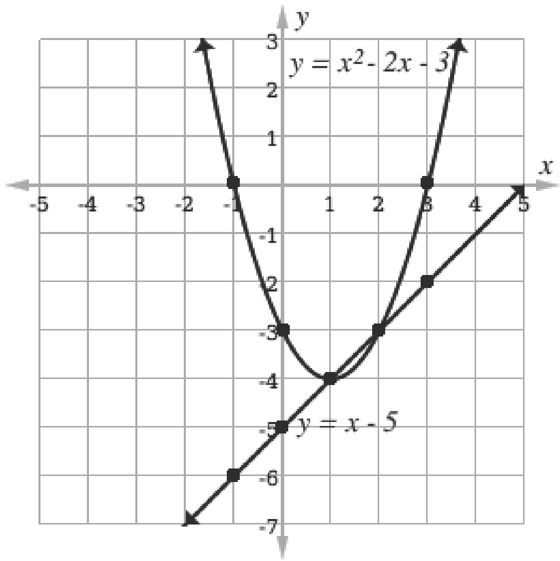
$y = (1) - 5$
$y = -4$

$y = (2) - 5$
$y = -3$

$y = (3) - 5$
$y = -2$

x	-1	0	1	2	3
y	-6	-5	-4	-3	-2

Graph both lines on a Cartesian plane →



Observe the point of intersection (where the parabola and line cross). These are the solutions to the simultaneous equations.

Solutions: $(1, -4)$ and $(2, -3)$

Practice

Use the graphical method to solve the following equations simultaneously.

1. $y = -x$
   $y = 2 - x^2$

2. $y = x$
   $y = x^2 - 6$

3. $y = -x - 4$
   $y = 2 - x^2$

4. $y = x - 4$
   $y = x^2 + x - 4$

Lesson Title: Direct variation	**Theme:** Algebraic Processes
Practice Activity: PHM2-L017	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to solve numerical and word problems involving direct variation.

Overview

This lesson uses direct variation to solve numerical and word problems. Direct variation means that two quantities x and y are related such that an increase in one results in an increase in the other in the same ratio. At the same time, a decrease in one results in a decrease in the other in the same ratio. We can also say that x and y are "directly proportional".

Direct variation can be shown with the symbol $\propto$. The statement $x \propto y$ means that x and y are directly proportional.

Direct variation can be shown with the equation $x = ky$, where k is a constant, and x and y represent two quantities that are directly proportional.

Solved Examples

1. If $c \propto n$ and $c = 5$ when $n = 20$, find the formula connecting c and n.
   Solution:

$c \propto n$		Identify the relationship between c and n
$c = kn$		
$5 = k(20)$		Substitute known values for c and n
$k = \frac{5}{20}$		Solve for the constant, k
$k = \frac{1}{4}$		
$c = \frac{1}{4}n$		Write the formula

2. If y varies directly as x and $y = 9$ when $x = 3$, find the value of y when $x = 4$.
   Solution:

$y \propto x$	Identify the relationship between y and x
$y = kx$	
$9 = k(3)$	Substitute known values for y and x
$k = \frac{9}{3}$	Solve for the constant, k
$k = 3$	

$$
\begin{aligned}
y &= 3x && \text{Write the formula} \\
y &= 3(4) && \text{Find } y \text{ when } x = 4 \\
y &= 12
\end{aligned}
$$

3. The extension of a sketched spring is directly proportional to its tension ($T \propto E$). When it is extended 2 cm, the tension is 8 Newtons. When it is extended again, it produces a tension of 12 Newtons. How far was it extended the second time?
   Solution:
   You may see problems that are scientific, and use terms that you do not know. It is not necessary to understand tension and Newtons to solve this problem. Use the information you know about variation to set up the equation and solve.

   Let T represent tension and E extension.

$$
\begin{aligned}
T &= kE \\
8 &= k(2) && \text{Substitute known values for } T \text{ and } E \\
k &= \tfrac{8}{2} && \text{Solve for the constant} \\
k &= 4 \\
T &= 4E && \text{Write the formula} \\
12 &= 4E && \text{Find } E \text{ when } T = 8 \\
\tfrac{12}{4} &= E \\
E &= 3 \text{ cm} && \text{The extension that produced a tension of 12 Newtons}
\end{aligned}
$$

4. The cost of electroplating a square tray varies as the square of its length. The cost of a tray 5 cm square is Le 15,000.00. Find the cost for a tray that is 12 cm square.
   Solution:
   Let C be the cost in Leones and L represent the length of tray.

$$
\begin{aligned}
C &\propto L^2 && \text{Identify the relationship between } C \text{ and } L \\
C &= kL^2 \\
15{,}000 &= k5^2 && \text{Substitute known values for } C \text{ and } L \\
15{,}000 &= \text{k}25 && \text{Solve for the constant} \\
k &= \tfrac{15{,}000}{25} \\
k &= 600 \\
C &= 600L^2 && \text{The formula connecting the 2 variables} \\
C &= 600 \times 12^2 && \text{Substitute } L = 12 \text{ cm} \\
\text{C} &= 600 \times 144 \\
C &= Le\ 86{,}400 && \text{The cost for a tray of 12 cm}
\end{aligned}
$$

Practice

1. If $a \propto b$ and $a = 7$ when $b = 35$, find the formula connecting a and b.
2. If y varies directly as x and $y = 24$ when $x = 8$, find the value of y when $x = 12$.
3. Given that y varies directly as x, and that $x = 3$ when $y = 6$, calculate:
    a. y when $x = 7.5$
    b. x when $y = 16.5$
4. The amount of rice consumed by customers of a hotel varies directly based on their number. If 900 kg of rice are needed in a month by 50 customers, how many customers are there if the hotel needs 2,610 kg of rice?
5. The resistance R to the motion of a moving vehicle varies directly as the velocity (V). When $R = 600$ Newtons, the velocity is $V = 20$ km/hr. Find:
    a. The relationship between R and V.
    b. The velocity when the resistance is 330 Newtons.
    c. The resistance when the velocity is 3 km/hr.

Lesson Title: Inverse Variation	**Theme:** Algebraic Processes
Practice Activity: PHM2-L018	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to solve numerical and word problems involving inverse variation.

Overview

This lesson uses inverse variation to solve numerical and word problems. Direct variation means that two quantities x and y are related such that an increase in one results in a decrease in the other. At the same time, a decrease in one results in an increase in the other. We can also say that x and y are "inversely proportional".

Inverse variation uses the same symbol as direct variation: ∝. The statement $x \propto \frac{1}{y}$ means that x and y are inversely proportional.

Inverse variation can be shown with the equation $x = k\frac{1}{y}$ or $x = \frac{k}{y}$, where k is a constant, and x and y represent two quantities that are inversely proportional.

Solved Examples

1. If $y \propto \frac{1}{x}$, and $y = 4$ when $x = 2$, find:
   a. The formula that connects x and y
   b. The value of y when $x = 16$

   Solution:

   a.

$y \propto \frac{1}{x}$	Identify the relationship between y and x
$y = \frac{k}{x}$	
$4 = \frac{k}{2}$	Substitute known values for y and x
$k = 4 \times 2$	Solve for the constant, k
$k = 8$	
$y = \frac{8}{x}$	Write the formula

   b.

$y = \frac{8}{16}$	Substitute $x = 16$ into the formula
$y = \frac{1}{2}$	Simplify

2. Michael is traveling to Bo. If he drives at the rate of 60 kph it will take him 4 hours. How long will it take him to reach Bo if he drives at a rate of 80 kph?

Solution:
Speed is inversely proportional to time. If Michael drives faster, it will take less time to reach Freetown. If he drives slower, it will take more time.

s	$\propto$	$\frac{1}{t}$	The relationship between speed and time
s	$=$	$\frac{k}{t}$	
60	$=$	$\frac{k}{4}$	Substitute known values for s and t
k	$=$	60×4	Solve for the constant, k
k	$=$	240	
s	$=$	$\frac{240}{t}$	Write the formula
80	$=$	$\frac{240}{t}$	Substitute $s = 80$
t	$=$	$\frac{240}{80}$	Solve for t
t	$=$	3 hr	

3. The electrical resistance of a wire varies inversely as the square of its radius. The resistance is 0.4 ohms when the radius is 0.3 cm. Find the resistance when the radius is 0.6 cm.
   Solution:
   Let R represent the resistance in ohms and r the radius in cm

R	$\propto$	$\frac{1}{r^2}$	Identify the relationship between R and r
R	$=$	$\frac{k}{r^2}$	
0.4	$=$	$\frac{k}{(0.3)^2}$	Substitute known values for R and r
0.4	$=$	$\frac{k}{0.09}$	solve for the constant
k	$=$	0.036	
R	$=$	$\frac{0.036}{r^2}$	Write the formula
R	$=$	$\frac{0.036}{(0.6)^2}$	Substitute $r = 0.6$
R	$=$	$\frac{0.036}{0.36}$	Solve for R
R	$=$	$0.1\ ohm$	

Practice

1. If y varies inversely as x, and $y = 4$ when $x = 24$, find:
   a. The formula that connects x and y.

b. The value of y when $x = 32$.

2. If y varies inversely as x^2, and $y = 12$ when $x = 3$, find:
    a. The formula that connects x and y.
    b. The value of y when $x = 2$.
3. y varies inversely as x. If $y = 10$ when $x = 2,$ find:
    a. The relationship between x and y.
    b. The value of x when $y = 4$
    c. The value of y when $x = 100$
4. The speed with which a pipe can pump water from a well to an overhead tank varies inversely as the distance between the well and the tank. With a distance of 1.5 m, the speed of the water pumped is 40 metres per second. Find:
    a. The relationship connecting the speed and the distance.
    b. The speed when the distance is 20 m.
    c. The distance when the speed is 120 metres per second.

Lesson Title: Joint Variation	**Theme:** Algebraic Processes
Practice Activity: PHM2-L019	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to solve numerical and word problems involving joint variation.

Overview

This lesson uses joint variation to solve numerical and word problems. Joint variation occurs when a variable varies directly or inversely with multiple variables. For example:

- If x varies directly with both y and z, we have $x \propto yz$ or $x = kyz$.
- If x varies directly with y and inversely with z, we have $x \propto \frac{y}{z}$ or $x = \frac{ky}{z}$.

Joint variation problems can be solved using the same process as other variation problems: Set up the equation and substitute the given set of 3 values to find the constant k. Write the equation with constant k, and substitute the other 2 given values to find the answer.

Solved Examples

1. z varies directly as the product of x and y. When $x = 3$, $y = 8$, and $z = 6$. Find z when $x = 6$ and $y = 4$.

   Solution:

z	$\propto$	xy	Identify the relationship between z, x and y
z	$=$	kxy	
6	$=$	$k(3)(8)$	Substitute known values for z, x and y
6	$=$	$24k$	Solve for the constant, k
k	$=$	$\frac{6}{24}$	
k	$=$	$\frac{1}{4}$	
z	$=$	$\frac{1}{4}xy$	Write the formula
z	$=$	$\frac{1}{4}(6)(4)$	Substitute $x = 6$ and $y = 4$ into the formula
z	$=$	$\frac{1}{4}(24)$	Simplify
z	$=$	6	

2. x varies directly with y and inversely with z. If $x = 32$ when $y = 8$ and $z = 4$, find x when $y = 3$ and $z = 2$.

Solution:

From the problem, we have $x \propto y$ and $x \propto \frac{1}{z}$. Together, this gives $x \propto \frac{y}{z}$, or $x = \frac{ky}{z}$.

x	$=$	$\frac{ky}{z}$	Relationship between x, y and z
32	$=$	$\frac{k(8)}{(4)}$	Substitute known values for x, y and z
$32(4)$	$=$	$8k$	Solve for the constant, k
k	$=$	16	
x	$=$	$\frac{16y}{z}$	Write the formula
x	$=$	$\frac{16(3)}{(2)}$	Substitute $y = 3$ and $z = 2$ into the formula
x	$=$	24	Simplify

3. The volume (V) of a given mass of gas varies directly as the temperature (T) and inversely the pressure (P). When $T = 240^{\circ}$; $P = 600 \text{ mm}$ and $V = 420 \text{ cm}^3$.
   a. Find the relation between V, P and T.
   b. Calculate the volume when the temperature is 300° and the pressure is 900 mm.

   Solution:

   a. We know that $V \propto T$ and $V \propto \frac{1}{P}$. Together, these give us $V \propto \frac{T}{P}$. Find k to find the relationship between the 3 variables:

V	$=$	$\frac{kT}{P}$	
420	$=$	$\frac{240k}{600}$	Substitute known values for v, t and p
420×600	$=$	$240k$	Solve for k
k	$=$	$\frac{420 \times 600}{240}$	
k	$=$	$1{,}050$	

   The relationship is: $V = \frac{1050T}{P}$

   b. Use the relationship to calculate the volume:

V	$=$	$\frac{1{,}050 \times 300^{\circ}}{900}$	Substitute $T = 300^{\circ}$ and $P = 900 \text{ mm}$
V	$=$	350 cm^3	Evaluate

Practice

1. The variable x, y, and z are related by joint variation. When $x = 3$ and $y = 4, z = 36$. Find z when $x = 4$ and $y = 6$ if:
   a. $z \propto xy$
   b. $z \propto xy^2$
   c. $z \propto \frac{x}{y}$.

2. The profit which a bookshop makes varies jointly based on the percentage profit on each book and the number of books sold. A bookshop made $Le60{,}000.00$ profit when it sold $10{,}000$ books at a profit of 20% each. How much profit would it make by selling $15{,}000$ books at a profit of 15% each?
3. The volume of gas (V cm^3) with a given mass varies directly as its temperature (T K) and inversely as it pressure (p $\mathrm{N/m^2}$). A volume of $195\ \mathrm{cm}^3$ of gas was collected at a pressure of $100\ \mathrm{N/m^2}$ and a temperature of $300\ \mathrm{K}$.
    a. Find the formula that connects V, T, and p.
    b. Find the volume of gas if the temperature is $200\ K$ and the pressure is $250\ \mathrm{N/m^2}$.

Lesson Title: Partial Variation	**Theme:** Algebraic Processes
Practice Activity: PHM2-L020	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to solve numerical and word problems involving partial variation.

Overview

This lesson uses partial variation to solve numerical and word problems. Partial variation occurs when a variable is related to two or more other variables added together. This lesson focuses on the case where a variable (such as x) is partly a constant, and partly varies directly with another variable (such as y).

For example, $x = k_1 + ky$ states that x is partly related to the constant k_1, and varies partly as y.

In many cases, you will be asked to determine the relationship between two variables (such as x and y) that are related by partial variation. This involves finding two constants, k_1 and k. These can be found by forming two simultaneous linear equations (see Solved Example 2).

Solved Examples

1. Fatu hired Mohamed to take her to the market with her goods. His base rate is Le 15,000.00, and he charges an additional Le 1,000.00 per kilometre. If she traveled 7 kilometres, how much did she pay him?
   Solution:
   Set up the equation: $C = k_1 + kd$, which says that cost is partly a constant k_1, and partially varies with distance, d.

   From the problem, we have $k_1 = 15{,}000.00$, $k = 1{,}000.00$, and $d = 7$.

   $$\begin{aligned} C &= k_1 + kd \\ &= 15{,}000 + (1{,}000)(7) && \text{Substitute } k_1, k, \text{ and } d \\ &= 15{,}000 + 7{,}000 && \text{Simplify} \\ &= 22{,}000 \end{aligned}$$

   Answer: Fatu paid him Le 22,000.00.

2. x is partly constant and partly varies as y. When $y = 2$, $x = 20$, and when $y = 5$, $x = 35$.
   a. Find the relationship between x and y.
   b. Find x when $y = 4$.

Solution:

a. Write the relationship with variables: $x = k_1 + ky$

Substitute the given values of x and y into $x = k_1 + ky$, which gives two equations:

$$x = k_1 + ky$$

$$20 = k_1 + 2k \quad (1) \qquad \text{Substitute } y = 2 \text{ and } x = 20$$

$$35 = k_1 + 5k \quad (2) \qquad \text{Substitute } y = 5 \text{ and } x = 35$$

Solve the simultaneous equations by subtracting (1) from (2):

$$35 = k_1 + 5k \quad (2)$$

$$-(\,20 = k_1 + 2k) \quad (1) \qquad \text{Subtract each term}$$

$$15 = 3k$$

$$\frac{15}{3} = \frac{3k}{3} \qquad \text{Divide throughout by 3}$$

$$5 = k$$

$$20 = k_1 + 2(5) \quad (1) \qquad \text{Substitute } k = 5 \text{ into (1)}$$

$$20 = k_1 + 10 \qquad \text{Simplify}$$

$$20 - 10 = k_1 \qquad \text{Transpose 10}$$

$$10 = k_1$$

Write the relationship: $x = 10 + 5y$

b.

$$x = 10 + 5y \qquad \text{Relationship}$$

$$x = 10 + 5(4) \qquad \text{Substitute } y = 4$$

$$x = 10 + 20 \qquad \text{Simplify}$$

$$x = 30$$

3. The cost of catering per head in a senior secondary school is partly constant and partly varies inversely as the number of pupils present. If the cost per head for 10 pupils is Le 1,400.00 and the cost per head for 60 pupils is Le 1,200.00, find the cost per head for 80 pupils.

   Solution:

   Step 1. Find the equation connecting the cost with the number of pupils:
   Let c be the cost per head when there are n pupils. Then we have the relationship: $c = k_1 + \frac{k}{n}$.

   Substitute the given values of c and n into the formula, which gives two equations:

$$c = k_1 + \frac{k}{n}$$

$$1{,}400 = k_1 + \frac{k}{10}$$

$14{,}000$	$=$	$10k_1 + k$	(1)	Multiply throughout by 10
$1{,}200$	$=$	$k_1 + \frac{k}{60}$		
$72{,}000$	$=$	$60k_1 + k$	(2)	Multiply throughout by 60

Solve the simultaneous equations by subtracting (1) from (2):

$72{,}000$	$=$	$60k_1 + k$	(2)	
$-(\ 14{,}000$	$=$	$10k_1 + k)$	(1)	Subtract each term
$58{,}000$	$=$	$50k_1$		
$\frac{58{,}000}{50}$	$=$	$\frac{50k_1}{50}$		Divide throughout by 50
$1{,}160$	$=$	k_1		
$14{,}000$	$=$	$10(1{,}160) + k$	(1)	Substitute $k_1 = 1{,}160$ into (1)
$14{,}000$	$=$	$11{,}600 + k$		Simplify
$14{,}000 - 11{,}600$	$=$	k		Transpose 10
$2{,}400$	$=$	k		

Write the relationship: $c = 1{,}160 + \frac{2{,}400}{n}$

Step 2. Use the equation to find the cost of 80 pupils:

c	$=$	$1{,}160 + \frac{2{,}400}{n}$	Relationship
	$=$	$1{,}160 + \frac{2{,}400}{80}$	Substitute $n = 80$
	$=$	$1{,}160 + 30$	Simplify
	$=$	$1{,}190$	

The cost per head for 80 pupils would be Le 1,190.00.

Practice

1. The cost of using mobile internet is partially constant, and varies partially based on the number of gigabytes (GB) used. The company charges Le 20,000.00 for the first 2 GB, and 12,000.00 for each GB after that.
   a. Write a formula for the relationship between internet usage and cost.
   b. If Michael uses 12 GB internet, how much will he pay?
2. M is partly constant and partly varies as N. $M = 20$ when $N = 10$ and $M = 30$ when $N = 5$. Find M when $N = 8$.
3. Fatu wants to hire a DJ to play music at her birthday party. The cost is partially constant, and varies partially as the number of hours the DJ works. For 3 hours, he charges Le 124,000.00. For 8 hours, he charges Le 164,000.00.
   a. Find the formula connecting hours worked with the DJ's cost.
   b. How much will Fatu pay if she hires him for 5 hours?

Lesson Title: Inequalities on a number line	**Theme:** Algebraic Processes
Practice Activity: PHM2-L021	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to represent inequalities in one variable on a number line.

Overview

Inequalities in one variable are any inequalities that include a variable, inequality symbol ($<$, $>$, $\geq$ or $\leq$), and one or more numbers.

For example, these are inequalities in one variable:

$x \geq -6$ $\quad$ $y < -6$ $\quad$ $x + 3 > 4$ $\quad$ $4 \leq b - 9$

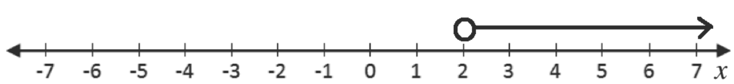
Inequalities in one variable can be shown on a number line using circles and arrows. These are the arrows that show each inequality symbol:

Less than ($<$) | Greater than ($>$) | Less than or equal to ($\leq$) | Greater than or equal to ($\geq$)

For expressions with two inequalities (example: $-4 \leq x < 2$, there is no arrow. The two end points are shown with circles. For example:

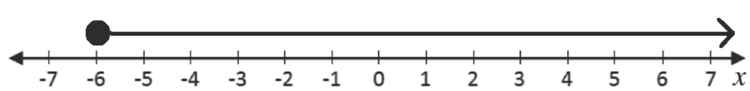
Solved Examples

1. Show $x > 2$ on a number line.
   Solution:



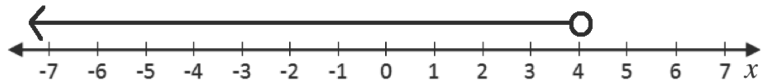
2. Show the truth set of $x \leq -1$ on a number line.
   Solution:

3. Illustrate $x \geq -6$ on a number line.
   Solution:



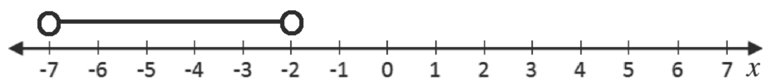
4. Illustrate on a number line: $x < 4$

Solution:



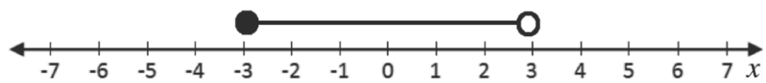
5. Show the truth set on a number line: $-7 < x < -2$

Solution:



6. Illustrate on a number line: $3 > x \geq -3$

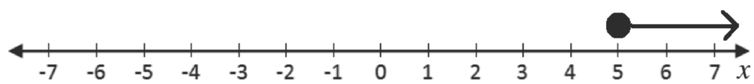
Solution:



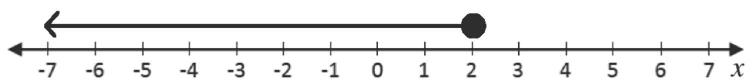
Practice

Write the inequality for each number line:

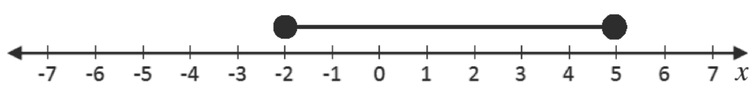
1.



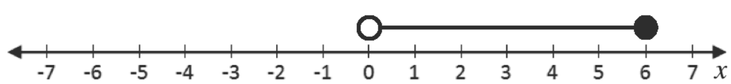
2.



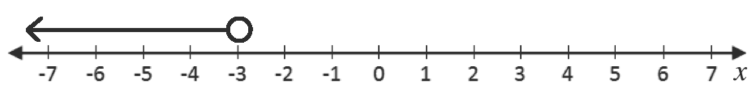
3.



4.



5.



Draw the truth set of each inequality on a number line:

6. $x > 4$
7. $x \geq -4$
8. $-7 \leq x < -5$
9. $x < -2$
10. $-4 < x < 4$

Lesson Title: Solutions of inequalities	**Theme:** Algebraic Processes
Practice Activity: PHM2-L022	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to solve inequalities in one variable.

Overview

Inequalities can be solved using nearly the same process that is used for equations. Here are some steps you can take to solve an inequality:

- Add or subtract a term from both sides
- Divide both sides by a positive number
- Divide both sides by a negative number (the direction of the inequality is **reversed**)

As with solving an equation, we want to get the variable alone on one side of the inequality. Add or subtract numbers from both sides first, then multiply or divide.

Solved Examples

1. Solve: $x + 3 > 5$
   Solution:

$$\begin{aligned} x + 3 &> 5 \\ x &> 5 - 3 && \text{Transpose 3} \\ x &> 2 \end{aligned}$$

2. Solve: $b - 8 \geq 7$
   Solution:

$$\begin{aligned} b - 8 &\geq 7 \\ b &\geq 7 + 8 && \text{Transpose } -8 \\ y &\geq 15 \end{aligned}$$

3. Solve: $4x > 8$
   Solution:

$$\begin{aligned} 4x &> 8 \\ \tfrac{4x}{4} &> \tfrac{8}{4} && \text{Divide throughout by 3} \\ x &> 2 \end{aligned}$$

4. Solve: $-4x > 8$
   Solution:

$$-4x > 8$$

$$\frac{-4x}{-4} < \frac{8}{-4}$$ Divide throughout by -4 (reverse the direction of the inequality)

$$x < -2$$

5. Solve: $-5x + 3 \leq 18$

   Solution:

$-5x + 3$	$\leq$	18	
$-5x$	$\leq$	$18 - 3$	Transpose 3
$-5x$	$\leq$	15	
$\frac{-5x}{-5}$	$\geq$	$\frac{15}{-5}$	Divide throughout by -5 (reverse the direction of the inequality)
x	$\geq$	-3	

6. Solve: $3(x + 2) < 2(13 - x)$

   Solution:

$3(x + 2)$	$<$	$2(13 - x)$	
$3x + 6$	$<$	$26 - 2x$	Multiply through each side
$3x + 2x + 6$	$<$	26	Transpose $-2x$
$5x + 6$	$<$	26	
$5x$	$<$	$26 - 6$	Transpose 6
$5x$	$<$	20	
$\frac{5x}{5}$	$<$	$\frac{20}{5}$	Divide throughout by 5
x	$<$	4	

Practice

Solve the following inequalities:

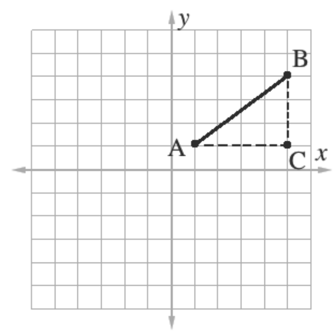
1. $y + 7 < 12$
2. $3x + 2 \geq 11$
3. $2 - 5x \leq x - 4$
4. $18 < -6y$
5. $5x - 5 \geq 6 - 6x$
6. $2y + 3 \leq 4y - 3$
7. $2(2x - 3) < 2x - 5$
8. $3x - 2 > 2x - 1$
9. $3(x - 2) < 2(x - 1)$
10. $5(3 - 2x) < 3(4 - 3x)$

Lesson Title: Distance formula	**Theme:** Algebraic Processes
Practice Activity: PHM2-L023	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to apply the distance formula to find the distance from one point to another on a line.

Overview

The distance formula is related to Pythagoras' Theorem. For any points A(x_1, y_1) and B(x_2, y_2), we can use the distance formula below to calculate the distance between them using their coordinates.



Distance formula:

$$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

for any points A(x_1, y_1) and B(x_2, y_2)

Solved Examples

1. Find the length of the line joining C$(0, -2)$ and D$(5, 10)$
   Solution:

$$\begin{aligned} |CD| &= \sqrt{(5-0)^2 + (10-(-2))^2} && \text{Substitute } x\text{- and } y\text{-values} \\ &= \sqrt{5^2 + (10+2)^2} && \text{Simplify} \\ &= \sqrt{5^2 + 12^2} \\ &= \sqrt{25 + 144} \\ &= \sqrt{169} \\ &= 13 \end{aligned}$$

2. Find the length of the line joining $A(-1, 3)$ and $B(5, 2)$.
   Solution:

$$\begin{aligned} |AB| &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} && \text{Distance formula} \\ &= \sqrt{(5-(-1))^2 + (2-3)^2} && \text{Substitute } x\text{- and } y\text{-values} \\ &= \sqrt{(5+1)^2 + (-1)^2} && \text{Simplify} \\ &= \sqrt{6^2 + 1} \\ &= \sqrt{37} \end{aligned}$$

3. Find the distance between the coordinates $E(3, 4)$ and $F(7, 1)$.

Solution:

$\lvert EF \rvert$	=	$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	Distance formula
	=	$\sqrt{(7-3)^2 + (1-4)^2}$	Substitute x- and y-values
	=	$\sqrt{4^2 + (-3)^2}$	Simplify
	=	$\sqrt{16+9}$	
	=	$\sqrt{25}$	
	=	5	

4. Find the length of the line joining $v(1,2)$ and $w(5,2)$.
   Solution:

$\lvert vw \rvert$	=	$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	Distance formula
	=	$\sqrt{(5-1)^2 + (2-2)^2}$	Substitute x- and y-values
	=	$\sqrt{(4)^2 + (0)^2}$	Simplify
	=	$\sqrt{16+0}$	
	=	$\sqrt{16}$	
	=	4	

5. Find the length of the line joining $a(-1, 4)$ and $b(3, 6)$.
   Solution:

$\lvert ab \rvert$	=	$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	Distance formula
	=	$\sqrt{(3-(-1))^2 + (6-4)^2}$	Substitute x- and y-values
	=	$\sqrt{(4)^2 + (2)^2}$	Simplify
	=	$\sqrt{16+4}$	
	=	$\sqrt{20}$	
	=	$\sqrt{4 \times 5}$	
	=	$\sqrt{4} \times \sqrt{5}$	
	=	$2\sqrt{5}$	

Practice

Find the distance between the points with the following coordinates:

1. $C(-2, 3)$, $D(4, 3)$
2. $P(2, 8)$, $Q(5, 7)$
3. $S(-6,1)$, $T(6, 6)$
4. $A(11, 9)$, $B(18, 16)$
5. $G(-2, -4)$, $H(-10, -10)$

Lesson Title: Mid-point formula	Theme: Algebraic Processes
Practice Activity: PHM2-L024	**Class:** SSS 2

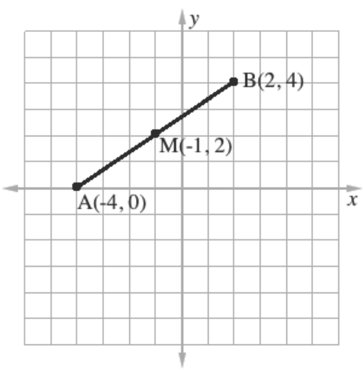
Learning Outcome
By the end of the lesson, you will be able to apply the mid-point formula to find the mid-point of a line.

Overview

The mid-point is the point that is **exactly** midway, or in the middle, of two other points. The mid-point of two points (x_1, y_1) and (x_2, y_2) is the point M found by the following formula:

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

In the graph below, M is the mid-point of line AB.



Solved Examples

1. Find the mid-point of line ST if S is $(1, 0)$ and T is $(3, 4)$.
   Solution:

$M = \left(\frac{1+3}{2}, \frac{0+4}{2}\right)$ Substitute x- and y-values

$= \left(\frac{4}{2}, \frac{4}{2}\right)$ Simplify

$= (2, 2)$

2. Find the mid-point M between $(-1, 4)$ and $(3, 6)$.
   Solution:

$M = \left(\frac{-1+3}{2}, \frac{4+6}{2}\right)$ Substitute x- and y-values

$= \left(\frac{2}{2}, \frac{10}{2}\right)$ Simplify

$= (1, 5)$

3. Find the value of p such that $(-3, 6)$ is the mid-point between A$(-2, p)$ and B$(-4, 4)$.

Solution:

$$M = (-3, 6) \quad \leftarrow \text{Use this fact}$$

$$(-3, 6) = \left(\frac{-2+(-4)}{2}, \frac{p+4}{2}\right) \quad \text{Apply the mid-point formula}$$

$$(-3, 6) = \left(\frac{-6}{2}, \frac{p+4}{2}\right) \quad \text{Simplify}$$

$$(-3, 6) = \left(-3, \frac{p+4}{2}\right)$$

Note that the x-coordinates already match. We know the y-coordinates are equal to each other too. Set them equal and solve for p:

$$6 = \frac{p+4}{2}$$

$$2(6) = 2\left(\frac{p+4}{2}\right) \quad \text{Multiply both sides by 2}$$

$$12 = p + 4 \quad \text{Transpose 4}$$

$$12 - 4 = p$$

$$p = 8$$

4. $M(4, 7)$ is the mid-point of $A(x, y)$ and $B(6, 11)$. Find the values of x and y.

Solution:

$$M = (4, 7) \quad \leftarrow \text{Use this fact}$$

$$(4, 7) = \left(\frac{x+6}{2}, \frac{y+11}{2}\right) \quad \text{Apply the mid-point formula}$$

Note: Note that both the x and y coordinates are equal to each other. Set them equal and solve for x and y.

Solve for x:

$$4 = \frac{x+6}{2}$$

$$2(4) = 2\left(\frac{x+6}{2}\right) \quad \text{Multiply both sides by 2}$$

$$8 = x + 6$$

$$8 - 6 = x \quad \text{Transpose 4}$$

$$x = 2$$

Solve for y:

$$7 = \frac{y+11}{2}$$

$$2(7) = 2\left(\frac{x+11}{2}\right) \quad \text{Multiply both sides by 2}$$

$$14 = y + 11$$

$$14 - 11 = y \quad \text{Transpose 11}$$

$$y = 3$$

The answer is $x = 2$ and $y = 3$.

5. Find the value of g such that $(-6, 9)$ is the midpoint between $C(-4, 10)$ and $D(-8, g)$.

Solution:

$$M = (-6,9)$$ ← Use this for mid-point

$$(-6,9) = \left(\frac{-4+(-8)}{2}, \frac{10+g}{2}\right)$$ Apply the mid-point formula

$$(-6,9) = \left(\frac{-12}{2}, \frac{10+g}{2}\right)$$ Simplify

$$(-6,9) = \left(-6, \frac{10+g}{2}\right)$$

Note that the x-coordinates already match. We know the y-coordinates are equal to each other too. Set them equal and solve for g:

$$9 = \frac{10+g}{2}$$

$$2(9) = 2\left(\frac{10+g}{2}\right)$$ Multiply both sides by 2

$$18 = 10 + g$$

$$18 - 10 = g$$ Transpose 10

$$g = 8$$

Practice

1. Find the mid-point of $A(-2,3)$ and $B(4,5)$.
2. Find the mid-point of the line joining the points $C(-2, 3)$ and $B(-8, 5)$.
3. M is the mid-point of LN. If the coordinates of L is $(-5, 4)$ and M is $(-2,1)$, find the coordinates of N.
4. $M(6,9)$ is the mid-point of $A(x, y)$ and $B(8,13)$. Determine the values of x and y.

Lesson Title: Gradient of a straight line	**Theme:** Algebraic Processes
Practice Activity: PHM2-L025	**Class:** SSS 2

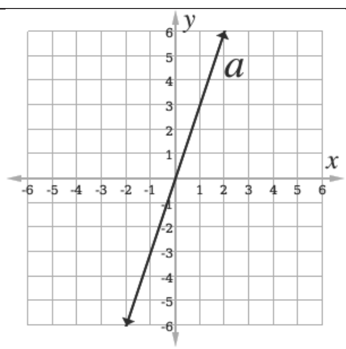
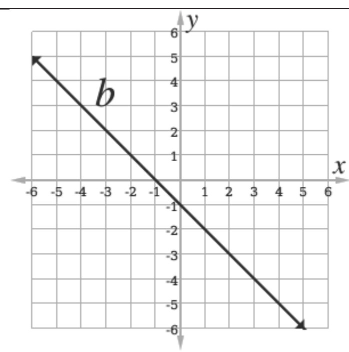
Learning Outcome
By the end of the lesson, you will be able to find the gradient of a line using two points, and the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$.

Overview

Gradient is a number that tells us in which direction a line increases, and how steep it is. If a line increases as it goes to the right, or in the positive x-direction, the gradient is positive. If a line increases as it goes to the left, or in the negative x-direction, the gradient is negative.

The greater the absolute value of a gradient, the steeper the line is. For example, a line with gradient 6 is steeper than a line with gradient 4. A line with gradient -5 is steeper than a line with gradient 3.

Examples:

 • Line a increases as it goes to the **right**, or the positive x-direction. Line a has a **positive** gradient. • Line a is steeper than line b. It has a gradient of +3.	 • Line b increases as it goes to the **left**, or the negative x-direction. Line b has a **negative** gradient. • Line b is not as steep as line a. It has a gradient of -1.

The gradient of a line can be calculated using any 2 points on the line. It is calculated by dividing the change in y by the change in x between those 2 points.

$$gradient = \frac{rise}{run} = \frac{change\ in\ y}{change\ in\ x}$$

Gradient m is given by the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ for any 2 points (x_1, y_1) and (x_2, y_2) on a line.

Solved Examples

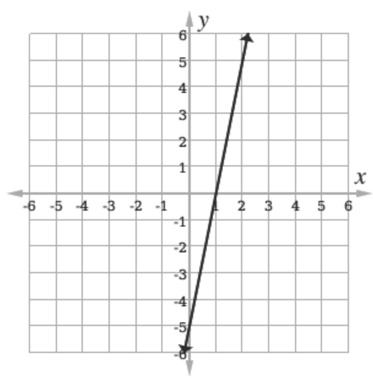
1. Find the gradient of the line passing through $(0, 0)$ and $(1, 3)$.
   Solution:

$$m = \frac{3-0}{1-0} \qquad \text{Substitute } x\text{- and } y\text{-values}$$
$$= \frac{3}{1} \qquad \text{Simplify}$$
$$= 3$$

2. Calculate the gradient of the line passing through $(-3, -5)$ and $(5, 11)$.
   Solution:

$$m = \frac{11-(-5)}{5-(-3)} \qquad \text{Substitute } x\text{- and } y\text{-values}$$
$$= \frac{11+5}{5+3} \qquad \text{Simplify}$$
$$= \frac{16}{8}$$
$$= 2$$

3. Calculate the slope of the line:



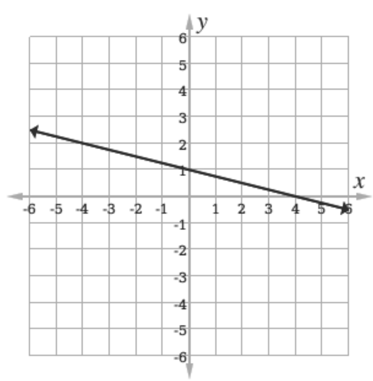
Solution:

Choose any 2 points on the line. For example: $(1, 0)$ and $(2, 5)$.

Use these points to calculate m:

$$m = \frac{5-0}{2-1} \qquad \text{Substitute } x\text{- and } y\text{-values}$$
$$= \frac{5}{1} \qquad \text{Simplify}$$
$$= 4$$

4. Calculate the slope of the line:



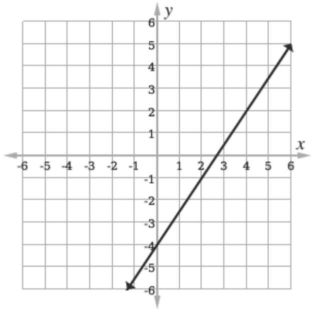
Solution:
Choose any 2 points on the line. For example: $(0, 1)$ and $(4, 0)$.

Use these points to calculate m:

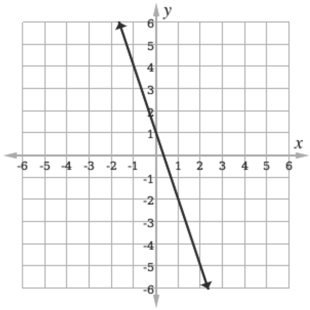
m	$=$	$\frac{0-1}{4-0}$	Substitute x- and y-values
	$=$	$\frac{-1}{4}$	Simplify
	$=$	$-\frac{1}{4}$	

Practice

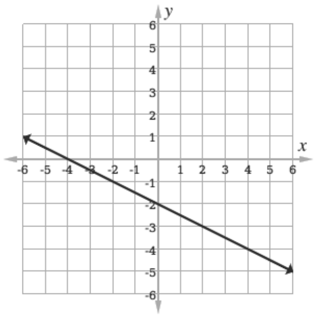
1. Find the gradient of the line which passes through the points.
    a. $A(3, 1)$ and $B(6, 10)$
    b. $P(7, 5)$ and $Q(9, 12)$
    c. $C(2, 3)$ and $D(5, 9)$
    d. $S(2, 3)$ and $T(6, -5)$
    e. $L(0, -4)$ and $M(-3, 0)$
2. Calculate the slope of the line:



3. Calculate the slope of the line:



4. Calculate the slope of the line:



Lesson Title: Sketching graphs of straight lines	**Theme:** Algebraic Processes
Practice Activity: PHM2-L026	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to sketch the graph of a straight line whose equation is $y = mx + c$ on the Cartesian plane, where m is the gradient of the line and c is the y-intercept.

Overview

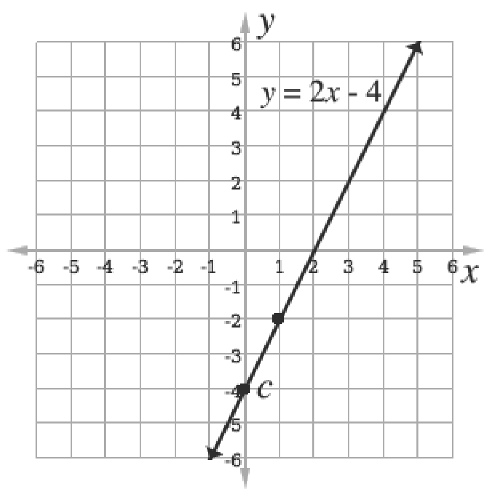
The slope-intercept form of a line is given by $y = mx + c$, where m is the gradient and c is the y-intercept of the line. The y-intercept is the point where the line crosses the y-axis.

To graph a line based on its slope-intercept form:

- Identify and plot the y-intercept.
- Remember that $m = \frac{rise}{run}$. From the y-intercept, count up or down (rise) the correct number of units in the y-direction. Then, count to the right or left (run) the correct number of units in the x-direction. Plot this point.
- Draw a line that passes through the y-intercept and the other point you plotted. Label the line with its equation.

Solved Examples

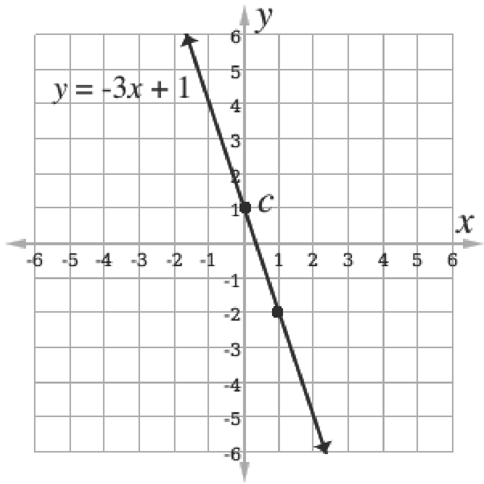
1. Graph $y = 2x - 4$ on a Cartesian plane.
   Solution:
   - Identify and plot the y-intercept, $c = -4$.
   - In this case, $m = 2 = \frac{2}{1}$. From the y-intercept, count **up** 2 units in the y-direction. Count to the right 1 unit in the x-direction. Plot this point $(1, -2)$.
   - Draw a line that passes through $(1, -2)$ and the y-intercept.
   - Label the line $y = 2x - 4$.



2. Graph $y = -3x + 1$

Solution:

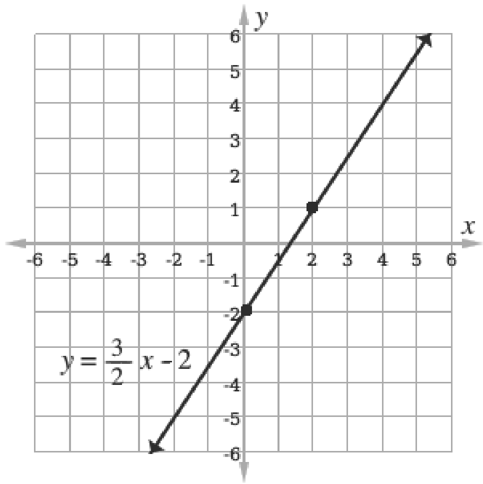
- Identify and plot the y-intercept, $c = 1$.
- Using $m = -3 = \frac{-3}{1}$, find another point. From the y-intercept, count **down** 3 units in the y-direction. Count to the right 1 unit in the x-direction. Plot this point $(1, -2)$.
- Draw a line that passes through $(1, -2)$ and the y-intercept.
- Label the line $y = -3x + 1$.



3. Graph $y = \frac{3}{2}x - 2$

Solution:

- Identify and plot the y-intercept, $c = -2$.
- Using $m = \frac{3}{2}$, find another point. From the y-intercept, count **up** 3 units in the y-direction. Count to the right 2 units in the x-direction. Plot this point $(2, 1)$.
- Draw a line that passes through $(2, 1)$ and the y-intercept.
- Label the line $y = \frac{3}{2}x - 2$.



Practice

1. Draw the graph of $y = -2x + 1$
2. Draw the graph of $y = \frac{1}{2}x + 3$
3. Graph $y = 2x - \frac{1}{2}$
4. Draw the graph of $y = \frac{5}{3}x - 2$
5. Graph $y = 4 - x$

Lesson Title: Equation of a straight line	**Theme:** Algebraic Processes
Practice Activity: PHM2-L027	**Class:** SSS 2

Learning Outcomes
By the end of the lesson, you will be able to:
1. Determine the equation of a straight line from the gradient and a given point.
2. Determine the equation of a straight line from two given points.

Overview

Linear equations can be given by the formula $y - y_1 = m(x - x_1)$ where m is the gradient and (x_1, y_1) is a specific point on the line. (x, y) is a general point on the line.

Using this formula, we can find the equation of a line if we are given **either** of the following:

- The gradient m and a given point (x_1, y_1)
- Two given points (x_1, y_1) and (x_2, y_2)

To find the equation of a line given the gradient and a point, substitute the values of m, x_1, and y_1 into the formula $y - y_1 = m(x - x_1)$. Simplify and write the equation in slope-intercept form.

To find the equation of a line given two points, use the formula for gradient ($m = \frac{y_2 - y_1}{x_2 - x_1}$) to find the gradient. Then, follow the process above, using the gradient and **one** of the points on the line.

Solved Examples

1. Determine the equation of a straight line whose gradient is -3 and that passes through the point $(1, 4)$.
   Solution:
   Let $m = -3$, and $(x_1, y_1) = (1, 4)$

$y - 4$	$=$	$-3(x - 1)$	Substitute values for m, x_1, and y_1
$y - 4$	$=$	$-3x + 3$	Simplify
y	$=$	$-3x + 3 + 4$	Transpose -4
y	$=$	$-3x + 7$	

2. Find the equation of the straight line passing through the points $(-1,-1)$ and $(3,7)$.
Solution:
Let $(x_1, y_1) = (-1,-1)$ and $(x_2, y_2) = (3,7)$
Step 1. Find the gradient:

m	$=$	$\frac{7-(-1)}{3-(-1)}$	Substitute into the formula for m
	$=$	$\frac{7+1}{3+1}$	Simplify
	$=$	$\frac{8}{4}$	
m	$=$	2	

Step 2. Find the equation. You may use either point, $(-1,-1)$ or $(3,7)$.

$y - y_1$	$=$	$m(x - x_1)$	Equation of a straight line
$y - 7$	$=$	$2(x - 3)$	Substitute $m = 2$ and one point, $(3,7)$
$y - 7$	$=$	$2x - 6$	
y	$=$	$2x - 6 + 7$	Transpose -7
y	$=$	$2x + 1$	Equation of the line

3. Determine the equation of a straight line whose gradient is 3 and that passes through the point $(2,8)$.
Solution:
Let $m = 3$, and $(x_1, y_1) = (2,8)$

$y - 8$	$=$	$3(x - 2)$	Substitute values for m, x_1, and y_1
$y - 8$	$=$	$3x - 6$	Simplify
y	$=$	$3x - 6 + 8$	Transpose -8
y	$=$	$3x + 2$	

4. Find the equation of the straight line passing through the point $(1,3)$ and $(-2,5)$.
Solution:
Let $(x_1, y_1) = (1,3)$ and $(x_2, y_2) = (-2,5)$
Step 1. Find the gradient:

m	$=$	$\frac{5-3}{-2-1}$	Substitute into the formula for m
	$=$	$-\frac{2}{3}$	Simplify

Step 2. Find the equation:

$y - y_1$	$=$	$m(x - x_1)$	Equation of a straight line
$y - 3$	$=$	$-\frac{2}{3}(x - 1)$	Substitute $m = \frac{2}{3}$ and one point $(1,3)$
$3(y - 3)$	$=$	$-2(x - 1)$	Multiply throughout by 3
$3y - 9$	$=$	$-2x + 2$	

$3y$	$=$	$-2x+2+9$	Transpose -9
$3y$	$=$	$-2x+11$	Simplify
y	$=$	$-\frac{2}{3}x+\frac{11}{3}$	Divide throughout by 3

Practice

1. Write the equation (in slope-intercept form) of the lines with:
    a. Gradient -2 and passing through the point $(-1,-3)$
    b. Gradient 2 and passing through the point $(2,3)$
    c. Gradient 1 and passing through the point $(7,-1)$
2. Find the equation (in slope-intercept form) of the line passing through the point:
    a. $A(1,2)$ and $B(3,4)$
    b. $C(-2,7)$ and $D(2,-3)$
    c. $P(-3,0)$ and $Q(0,-2)$

Lesson Title: Practice with straight lines	**Theme:** Algebraic Processes
Practice Activity: PHM2-L028	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to determine the equation of a straight line and graph it on the Cartesian plane.

Overview

This lesson revises facts from the previous 3 lessons. These facts are used to find the equation of a line, and graph it on the Cartesian plane. These facts are:

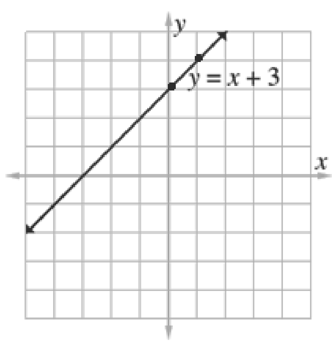
- The equation of a line can be found with either: a. The gradient and a point on the line, or b. Any two points on the line.
- The formula used to find the equation of a line is given by $y - y_1 = m(x - x_1)$.
- The formula for gradient is given by $m = \frac{y_2 - y_1}{x_2 - x_1}$.
- A line should be written in slope-intercept form ($y = mx + c$) before graphing it.

Solved Examples

1. If the gradient of a line is 1 and $(-2, 1)$ is a point on the line, find its equation. Draw the line on the Cartesian plane.
   Solution:
   Step 1. Find the equation:
   Let $m = 1$, and $(x_1, y_1) = (-2, 1)$.

$y - y_1$	$=$	$m(x - x_1)$	Equation of a straight line
$y - 1$	$=$	$1(x - (-2))$	Substitute values
$y - 1$	$=$	$x + 2$	Simplify
y	$=$	$x + 2 + 1$	Transpose -1
y	$=$	$x + 3$	

   Step 2. Graph the linear equation:



2. Determine the equation of the straight line passing through the points $(3, 4)$ and $(1, -2)$. Graph it on the Cartesian plane.

   Solution:

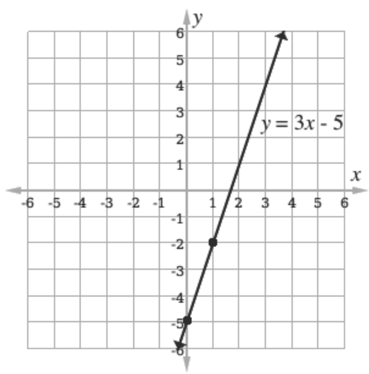
   Step 1. Find the gradient:

m	$=$	$\frac{y_2 - y_1}{x_2 - x_1}$	Gradient of a line
m	$=$	$\frac{-2-4}{1-3}$	Substitute into the formula of m
m	$=$	$\frac{-6}{-2}$	Simplify
m	$=$	3	

   Step 2. Find the equation:

$y - y_1$	$=$	$m(x - x_1)$	Equation of a straight line
$y - 4$	$=$	$3(x - 3)$	Substitute the values
$y - 4$	$=$	$3x - 9$	Simplify
y	$=$	$3x - 9 + 4$	Transpose -4
y	$=$	$3x - 5$	

   Step 3. Graph the linear equation:



Practice

1. Find the equation of the line passing through the points $(1, 2)$ and $(-1, -4)$. Graph the line on the Cartesian plane.
2. Find the equation of the line with gradient 2 and passing through the point $(2, 3)$. Then, graph the line on the Cartesian plane.
3. Find the equation of the line passing through the points $(0, 3)$ and $(4, 5)$, and graph it on the Cartesian plane.
4. Find the equation of the line passing through the point $(-3, -3)$ with gradient -2. Graph it on the Cartesian plane.

Lesson Title: Gradient of a curve – Part 1	**Theme:** Algebraic Processes
Practice Activity: PHM2-L029	**Class:** SSS 2

Learning Outcomes
By the end of the lesson, you will be able to:
1. Draw the tangent to a curve at a given point.
2. Use the tangent to find an appropriate value for the gradient of a curve at a given point.

Overview

This lesson introduces gradient of a curve. The gradient of a curve changes from point to point. The gradient at any point on a curve is the same as the gradient of the tangent line at that exact point.
To draw a tangent line:

- A tangent line touches the curve at only one point.
- A tangent to a curve at point P can be drawn by placing a straight edge on the curve at P, then drawing a line.
- The "angles" between the curve and line should be nearly equal.
- The parabola and tangent line must be drawn very accurately and clearly to find the correct gradient. Use a very exact scale on the x- and y-axes (draw all of the gridlines the same distance apart).

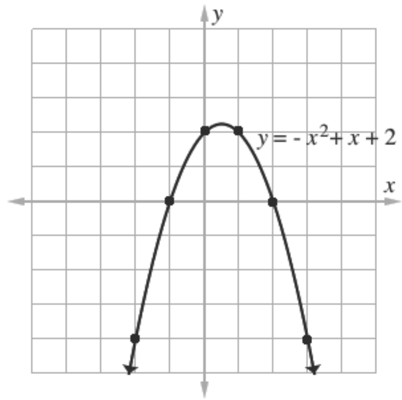
To find the gradient of a curve at point P:

- Draw the tangent at point P.
- Find any 2 points on the tangent line and use them in the gradient formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$.

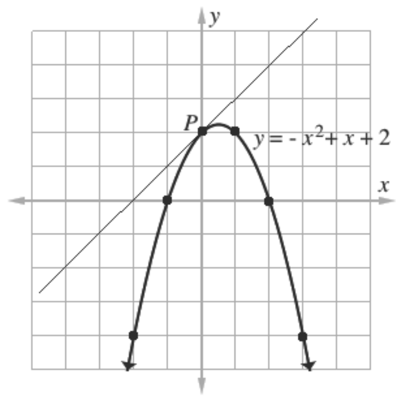
Solved Examples

1. Graph $y = -x^2 + x + 2$ on the Cartesian plane. Find the gradient at point $P(0, 2)$ by drawing a tangent line.
   Solution:
   Step 1. Graph the parabola by creating a table of values:

x	-2	-1	0	1	2	3
y	-4	0	2	2	0	-4



Step 2. Draw the tangent at point $P(0, 2)$:



Step 3. Identify any two points on the tangent line.

Example: $(x_1, y_1) = (-2, 0)$ and $(x_2, y_2) = (0, 2)$

Step 4. Substitute these coordinates into the gradient formula and solve:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{2-0}{0-(-2)}$$ Substitute x- and y-values.

$$= \frac{2}{2}$$ Simplify

$$m = 1$$

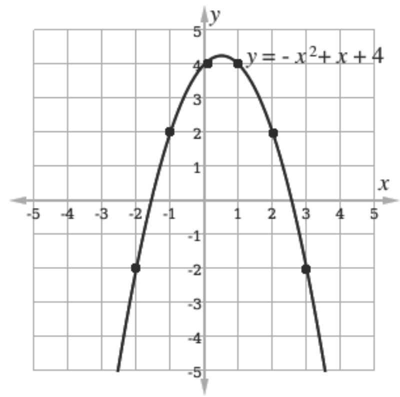
By finding the gradient of the tangent line at point P we have also found that the gradient of the curve $y = -x^2 + x + 2$ at P is $m = 1$.

2. Graph $y = 4 + x - x^2$ for $-2 \leq x \leq 3$ on the Cartesian plane. Find the gradient at $x = 0$.

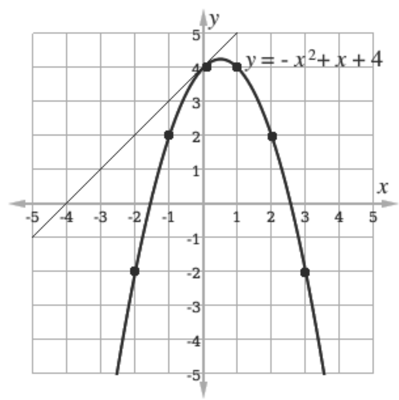
   Solution:

   Step 1. Graph the parabola by creating a table of values:

x	-2	-1	0	1	2	3
y	-2	2	4	4	2	-2



Step 2. Draw the tangent at $x = 0$:



Step 3. Identify two points on the tangent line.

Example: $(x_1, y_1) = (-1, 3)$ and $(x_2, y_2) = (0, 4)$.

Step 4. Substitute these coordinates into the gradient formula and solve:

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{4-3}{0-(-1)} && \text{Substitute } x\text{- and } y\text{-values.} \\ &= \frac{1}{1} && \text{Simplify} \\ m &= 1 \end{aligned}$$

By finding the gradient of the tangent line at $x = 0$, the gradient of the curve $y = 4 + x - x^2$ at $x = 0$ is also $m = 1$.

Practice

1. Draw the graph of $y = x^2 - 2x + 1$ on the Cartesian plane. Find gradient of the curve at $x = 3$.
2. Graph $y = -2x^2 + 5x + 1$, and estimate the gradient of the curve at $x = 2$.
3. Draw the graph of $y = x^2 + x - 3$ for $-3 \leq x \leq 2$ on the Cartesian plane. Find the gradient of the curve at $x = -1$.

Lesson Title: Gradient of a curve – Part 2	**Theme:** Algebraic Processes
Practice Activity: PHM2-L030	**Class:** SSS 2

Learning Outcomes
By the end of the lesson, you will be able to:
1. Draw the tangent to a curve at a given point.
2. Use the tangent to find an appropriate value for the gradient of a curve at a given point.

Overview

This lesson continues work with the gradient of a curve. It introduces questions that involve multiple steps, similar to those that are featured in the WASSCE exam. These questions combine different topics related to quadratic functions. You will use previous knowledge from other lessons on quadratic functions.

Solved Examples

1. Evaluate:
   a. Complete the table of values for the relation $y = x^2 + x - 2$.

x	-3	-2	-1	0	1	2
y						

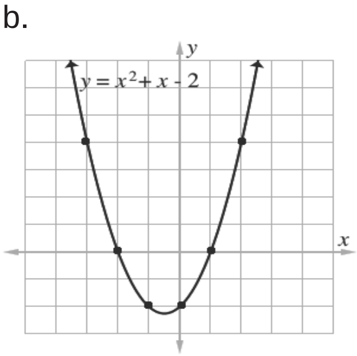
   b. Draw the graph of y for $-3 \leq x \leq 2$.
   c. From your graph:
      i. Find the roots of the equation $x^2 + x - 2 = 0$
      ii. Estimate the minimum value of y.
      iii. Calculate the gradient of the curve at the point $x = 0$.

Solutions:

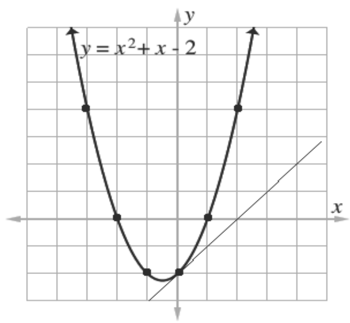
a.

x	-3	-2	-1	0	1	2
y	4	0	-2	-2	0	4

b.



c.
i. The roots are the x-intercepts: $x = -2$ and $x = 1$
ii. The minimum value of y is slightly less than -2. Estimate: $y = -2.2$
iii. Draw the tangent line at $x = 0$, and find its gradient:



2. Identify any two points on the tangent. Example: $(x_1, y_1) = (0, -2)$ and $(x_2, y_2) = (2, 0)$). Use them to find the gradient of the curve:

$$
\begin{aligned}
m &= \frac{y_2 - y_1}{x_2 - x_1} \\
&= \frac{0-(-2)}{2-0} && \text{Substitute } x\text{- and } y\text{-values.} \\
&= \frac{2}{2} && \text{Simplify} \\
m &= 1
\end{aligned}
$$

3. Evaluate:
   a. Complete the following table of values for the relation: $y = 2x^2 - 7x - 3$

x	-2	-1	0	1	2	3	4	5
y	19		-3		-9			

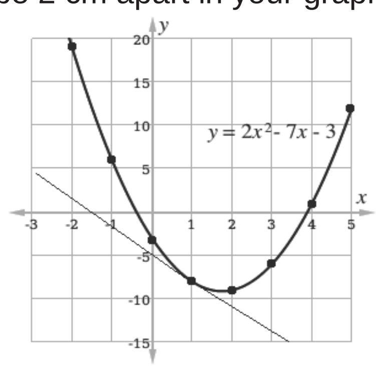
   b. Using 2 cm to 1 unit on the x-axis and 2 cm to 5 units on the y-axis, draw the graph of $y = 2x^2 - 7x - 3$.
   c. From your graph find the following:
      i. Minimum value of y.
      ii. Gradient of the curve at $x = 1$.
   d. By drawing a suitable straight line, find the values of x for which $2x^2 - 7x - 3 = 3x - 11$.

Solutions:

a. Table:

x	-2	-1	0	1	2	3	4	5
y	19	6	-3	-8	-9	-6	1	12

b. Draw the graph. Note that the graph below is not printed to scale. The marks on the x- and y-axes should be 2 cm apart in your graph.



c. i. Minimum value of $y = -9$

ii. Choose 2 points on the tangent line (Example: $(0, -5)$ and $(1, -8)$). Calculate the gradient of the curve at $x = 1$:

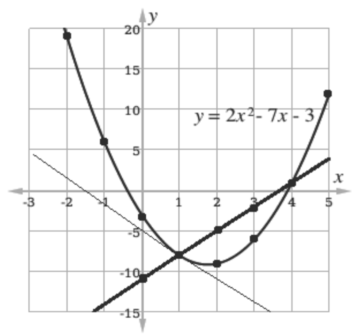
$$m = \frac{-8-(-5)}{1-0}$$
$$= \frac{-8+5}{1}$$
$$m = -3$$

d. Note that the values of x for which $2x^2 - 7x - 3 = 3x - 11$ are the solutions to the simultaneous equations $y = 2x^2 - 7x - 3$ and $y = 3x - 11$. Graph them on the same Cartesian plane and find their points of intersection.

Table of values for line $y = 3x - 11$:

x	0	1	2	3	4
y	-11	-8	-5	-2	1

Graph the line:



The answer is $x = 1$ and $x = 4$.

Practice

1. Complete the following for the relation $y = -x^2 + x + 6$
   a. Complete the table of values:

x	-2	-1	0	1	2	3
y						

   b. Draw the graph of the relation.
   c. Draw the tangent line at $x = 1$ and find the gradient of the curve.
2. Complete the following for the relation $y = x^2 - 2x + 3$.
   a. Complete the table of values:

x	-2	-1	0	1	2	3	4
y	11		3			6	

   b. Using the scale of 2 cm to 1 unit on the x-axis and 2 cm to 2 units on the y-axis, draw the graph of the relation $y = x^2 - 2x + 3$.
   c. What is the minimum value of y?
   d. Draw on the same Cartesian plane the line $y = 2x - 1$.
   e. What do you notice about the graph of the line $y = 2x - 1$?
   f. What is the gradient of the curve at $x = 2$?

Lesson Title: Simplification of algebraic fractions – Part 1	**Theme:** Algebraic Processes
Practice Activity: PHM2-L031	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to use factorisation to simplify algebraic fractions by reducing them to their lowest terms.

Overview

This is the first of two lessons on simplifying algebraic fractions. Algebraic fractions are fractions with variables in them.

These are the steps for simplifying algebraic fractions:

1. Factor the numerator and denominator if possible.
2. Find the highest common factor (HCF) of the numerator and denominator. Remember that the HCF is simply all of the common factors multiplied together.
3. Divide (cancel) the HCF from the numerator and denominator.

Solved Examples

1. Simplify: $\frac{wxy}{xyz}$

   Solution:

   Divide the numerator and denominator by xy: $\frac{wxy \div xy}{xyz \div xy} = \frac{w}{z}$

   Note:

   - The numerator and denominator have common factors x and y. The HCF is $x \times y = xy$.
   - It is not necessary to write the division ($\div\, xy$) each time you solve a problem. When dividing by variables, you can simply cancel those variables in the fraction.

2. Reduce $\frac{8x^3}{2x}$ to its lowest terms.

   Solution:

   Divide the numerator and denominator by $2x$ (the HCF), and simplify.

   $$\frac{8x^3}{2x} = \frac{8x^3 \div 2x}{2x \div 2x} = \frac{4x^{3-1}}{1} = 4x^2$$

3. Simplify: $\frac{24x^6}{8x^4}$

   Solution:

   Divide the denominator and numerator by $8x$ (the HCF), and simplify.

$$\frac{24x^6}{8x^4} = \frac{24x^6 \div 8x}{8x^4 \div 8x} = \frac{3x^{6-1}}{x^{4-1}} = \frac{3x^5}{x^3} = 3x^{5-3} = 3x^2$$

4. Simplify: $\frac{a^3b^2c^3}{ab^4c^2}$
   Solution:
   Remember that you do not need to show division each time. In this case, it is simple to cancel the variables in the fraction. Cancel the same number of factors in the numerator and denominator. In this example, if you cancel a from the denominator, the numerator is reduced by 1 factor of a, from a^3 to a^2.
   $$\frac{a^3b^2c^3}{ab^4c^2} = \frac{a^2c}{b^2}$$

Practice

Simplify the following:

1. $\frac{36y^8}{6y^2}$
2. $\frac{22v^6w^8}{2v^3w^6}$
3. $\frac{75y^{10}}{15y^7}$
4. $\frac{16x^2y^4}{8xy^2}$
5. $\frac{25wxy}{5xyz}$
6. $\frac{56x^4}{14x^8}$
7. $\frac{64v^4w^3}{8vw^2}$

Lesson Title: Simplification of algebraic fractions – Part 2	**Theme:** Algebraic Processes
Practice Activity: PHM2-L032	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to use factorisation to simplify more complex algebraic fractions by reducing them to their lowest terms.

Overview

This is the second of two lessons on simplifying algebraic fractions. In this lesson, algebraic fractions have polynomials that must be factored, including quadratic expressions.

These are the steps for simplifying algebraic fractions:

1. Factor the numerator and denominator if possible.
2. Find the highest common factor (HCF) of the numerator and denominator. Remember that the HCF is simply all of the common factors multiplied together.
3. Divide (cancel) the HCF from the numerator and denominator.

Solved Examples

1. Simplify: $\frac{2x+xy}{2y+y^2}$

   Solution:

   Step 1. Factor the numerator and denominator:
   $$\frac{2x+xy}{2y+y^2} = \frac{x(2+y)}{y(2+y)}$$

   Step 2. Identify that the GCF in the numerator and denominator is $(2+y)$.
   Step 3. Divide (cancel) the numerator and denominator by the GCF:
   $$\frac{2x+xy}{2y+y^2} = \frac{x(\cancel{2+y})}{y(\cancel{2+y})} = \frac{x}{y}$$

2. Reduce $\frac{x^2+6x+5}{x^2+5x}$ to its lowest terms.

   Solution:
   Step 1. Factor the numerator and denominator:
   $$\frac{x^2+6x+5}{x^2+5x} = \frac{(x+5)(x+1)}{x(x+5)}$$
   Step 2. Identify that the GCF in the numerator and denominator is $(x+5)$.
   Step 3. Divide (cancel) the numerator and denominator by the GCF:
   $$\frac{(\cancel{x+5})(x+1)}{x(\cancel{x+5})} = \frac{x+1}{x}$$

3. Simplify: $\frac{x^2-y^2}{(x-y)^2}$

Solution:

Step 1. Factor the numerator and denominator:

$$\frac{x^2-y^2}{(x-y)^2} = \frac{(x-y)(x+y)}{(x-y)(x-y)}$$

Step 2. Identify that GCF in the numerator and denominator is $(x - y)$.

Step 3. Divide (cancel) the numerator and denominator by the GCF.

$$\frac{x^2-y^2}{(x-y)^2} = \frac{\cancel{(x-y)}(x+y)}{\cancel{(x-y)}(x-y)} = \frac{x+y}{x-y}$$

4. Simplify: $\frac{4-x^2}{2x-x^2}$

Solution:

Step 1. Factor the numerator and denominator

$$\frac{4-x^2}{2x-x^2} = \frac{2^2-x^2}{2x-x^2} = \frac{(2+x)\cancel{(2-x)}}{x\cancel{(2-x)}}$$

Step 2. Identify that GCF in the numerator and denominator is $(2 - x)$.

Step 3. Divide (cancel) the numerator and denominator by the GCF.

$$\frac{4-x^2}{2x-x^2} = \frac{2+x}{x}$$

Practice

Simplify the following expressions:

1. $\frac{x^2-16}{3x+12}$
2. $\frac{2x^2-4x}{x^2(x-2)}$
3. $\frac{4x-12}{x^2-3x}$
4. $\frac{2x-16}{x^2-64}$
5. $\frac{54k^2-6}{3k+1}$
6. $\frac{x^2-y^2}{3x+3y}$
7. $\frac{x^2-8x+16}{x^2-7x+12}$
8. $\frac{2x^2-5x-12}{4x^2-9}$

Lesson Title: Multiplication of algebraic fractions	**Theme:** Algebraic Processes
Practice Activity: PHM2-L033	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to multiply algebraic fractions, reducing them to their lowest terms.

Overview

This lesson focuses on multiplying algebraic fractions. These are the steps for multiplying algebraic fractions:

1. Factor the numerators and denominators.
2. Identify the common factors in the numerators and denominators.
3. Divide the numerators and denominators by the factors they have in common. In other words, cancel factors that are in both the numerator and denominator.
4. Multiply the result, and leave it as your answer. If there are brackets, you do not need to multiply them out.

Solved Examples

1. Simplify: $\frac{x^2+x}{x+4} \times \frac{xy+4y}{x+1}$

   Solution:

   Step 1. Factor the numerators and denominators:

   $$\frac{x^2+x}{x+4} \times \frac{xy+4y}{x+1} = \frac{x(x+1)}{x+4} \times \frac{y(x+4)}{x+1}$$

   Step 2. Identify that common factors in the numerator and denominator are $(x+1)$ and $(x+4)$

   Step 3. Divide (cancel) the numerator and denominator by the factors:

   $$\frac{x\cancel{(x+1)}}{\cancel{x+4}} \times \frac{y\cancel{(x+4)}}{\cancel{x+1}}$$

   Step 4. Multiply the result: xy

2. Simplify $\left(\frac{x^2-2x+1}{2}\right) \times \frac{1}{(x-1)}$

   Solution:

   $\frac{x^2-2x+1}{2} \times \frac{1}{x-1} = \frac{(x-1)(x-1)}{2} \times \frac{1}{x-1}$ Factorise the numerator

   $= \frac{(x-1)\cancel{(x-1)}}{2} \times \frac{1}{\cancel{x-1}}$ Cancel the common factor, $x-1$

   $= \frac{(x-1)}{2}$ Multiply

Note that problems can often be solved in different ways. For example, multiplication can be applied to $\frac{x^2-2x+1}{2} \times \frac{1}{x-1}$ before factoring and simplifying it.

3. Simplify $\frac{3x^2}{4y^2} \times \frac{2y^3}{9x}$

Solution:

At times it may be easier to cancel (divide) more than once. In this problem, you can cancel the numbers that are common factors before canceling the variables that are common factors.

$\frac{3x^2}{4y^2} \times \frac{2y^3}{9x}$	$= \frac{\cancel{3}x^2}{\cancel{4}y^2} \times \frac{\cancel{2}y^3}{\cancel{9}x}$	Divide by common number factors
	$= \frac{x^2}{2y^2} \times \frac{y^3}{3x}$	Rewrite
	$= \frac{x}{2} \times \frac{y}{3}$	Divide by common variable factors
	$= \frac{xy}{6}$	Multiply

4. Simplify $\frac{18x^2+6xy}{7z} \times \frac{21z^3}{6x+2y}$

Solution:

In complicated problems like this, you may cancel common factors, and then notice there are more to be canceled. Cancel as many times as needed.

$\frac{18x^2+6xy}{7z} \times \frac{21z^3}{6x+2y}$	$= \frac{6x(3x+y)}{7z} \times \frac{21z^3}{2(3x+y)}$	Factor the numerator and denominator
	$= \frac{6x}{z} \times \frac{3z^3}{2}$	Cancel any common factors you see
	$= \frac{3x}{1} \times \frac{3z^2}{1}$	Cancel more common factors
	$= 9xz^2$	Multiply

Practice

Simplify the following:

1. $\frac{4xy}{6a^2b^3} \times \frac{4b^4}{12x^2}$
2. $\frac{x^2-4}{x^2+x} \times \frac{x+1}{x^2-4x+4}$
3. $\frac{6x^2y^2}{13p^2q} \times \frac{39qr}{18y^2z}$
4. $\frac{x^2-4}{3y^2+y} \times \frac{3y^2-2y-1}{x^2+x-6}$
5. $\frac{(x-1)^2}{x^2+3x-4} \times \frac{x^2+4x}{x^2-8x+7}$

Lesson Title: Division of algebraic fractions	**Theme:** Algebraic Processes
Practice Activity: PHM2-L034	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to divide algebraic fractions, reducing them to their lowest terms.

Overview

This lesson focuses on dividing algebraic fractions. These are the steps for dividing algebraic fractions:

1. Multiply by the reciprocal (inverse) of the second fraction. This is the same step we take when dividing fractions with numbers.
2. Follow the steps for multiplication of algebraic fractions from the previous lesson:
    a. Factor the numerators and denominators.
    b. Identify the common factors in the numerators and denominators.
    c. Divide the numerators and denominators by the factors they have in common. In other words, cancel factors that are in both the numerator and denominator.
    d. Multiply the result, and leave it as your answer. If there are brackets, you do not need to multiply them out.

Solved Examples

1. Simplify: $\frac{x^2+2x+1}{x+3} \div \frac{x+1}{5x+15}$

   Solution:

   Step 1. Multiply by the reciprocal of the second fraction:

   $$\frac{x^2+2x+1}{x+3} \times \frac{5x+15}{x+1}$$

   Step 2. Factor the numerators and denominators:

   $$\frac{x^2+2x+1}{x+3} \times \frac{5x+15}{x+1} = \frac{(x+1)(x+1)}{x+3} \times \frac{5(x+3)}{x+1}$$

   Step 3. Identify that common factors in the numerator and denominator are $(x+1)$ and $(x+3)$

   Step 4. Divide (cancel) the numerator and denominator by the factors:

   $$\frac{(x+1)\cancel{(x+1)}}{\cancel{x+3}} \times \frac{5\cancel{(x+3)}}{\cancel{x+1}}$$

   Step 5. Multiply: $5(x+1)$

2. Simplify: $\frac{x^2-2x+1}{x^2+3x-4} \div \frac{x^2-8x+7}{x^2+4x}$

$\frac{x^2-2x+1}{x^2+3x-4} \div \frac{x^2-8x+7}{x^2+4x} = \frac{x^2-2x+1}{x^2+3x-4} \times \frac{x^2+4x}{x^2-8x+7}$ Multiply by the reciprocal

$= \frac{(x-1)(x-1)}{(x+4)(x-1)} \times \frac{x(x+4)}{(x-1)(x-7)}$ Factor

$= \frac{\cancel{(x-1)}\cancel{(x-1)}}{\cancel{(x+4)}\cancel{(x-1)}} \times \frac{x\cancel{(x+4)}}{\cancel{(x-1)}(x-7)}$ Cancel common factors

$= \frac{x}{x-7}$

3. Simplify: $\frac{10x}{x^2+3x} \div \frac{15x}{x^2-x-12}$

$\frac{10x}{x^2+3x} \div \frac{15x}{x^2-x-12} = \frac{10x}{x^2+3x} \times \frac{x^2-x-12}{15x}$ Multiply by the reciprocal

$= \frac{10x}{x(x+3)} \times \frac{(x-4)(x+3)}{15x}$ Factor

$= \frac{\cancel{10x}}{\cancel{x}\cancel{(x+3)}} \times \frac{(x-4)\cancel{(x+3)}}{\cancel{15x}}$ Cancel common factors

$= \frac{2(x-4)}{3x}$

Practice

1. $\frac{x}{7} \div \frac{x^2}{x+1}$

2. $\frac{4a^2+8ab}{3} \div \frac{5ab+10b^2}{9}$

3. $\frac{(a+2)(a-5)}{a+7} \div \frac{(a-2)(a+2)}{(a+7)(a-7)}$

4. $\frac{x}{x^2-x} \div \frac{10x}{x^2+x-2}$

5. $\frac{5(x-6)}{x+5} \div \frac{x-6}{12}$

6. $\frac{x^2-4}{x^2+x} \div \frac{x^2+4x+4}{x+1}$

Lesson Title: Addition and subtraction of algebraic fractions – Part 1	**Theme:** Algebraic Processes
Practice Activity: PHM2-L035	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to add and subtract algebraic fractions to give a single algebraic fraction.

Overview

This lesson focuses on adding and subtracting algebraic fractions. The steps are similar to those for adding numerical fractions. That is, only **like fractions** (those with the same denominator) can be added.
These are the steps for adding or subtracting algebraic fractions:

1. Factor the algebraic fractions if possible.
2. To add or subtract algebraic fractions, the denominators should be the same. Express each fraction with a denominator that is the LCM of the denominators in the problem.
3. Once the fractions have the same denominator, they are like fractions and can be added or subtracted.
4. Combine like terms if possible.
5. If the answer can be simplified, simplify it. This requires factoring the answer if possible.

Solved Examples

1. Simplify $\frac{5}{x}+\frac{2}{3y}$

   Solution:

   Note that the LCM of x and $3y$ is found by multiplying them because they do not have common factors. Thus, the LCM of the denominators is $3xy$.

$\frac{5}{x}+\frac{2}{3y}$	$=\frac{5\times 3y}{x\times 3y}+\frac{2\times x}{3y\times x}$	Change both denominators to the LCM
	$=\frac{15y}{3xy}+\frac{2x}{3xy}$	Simplify
	$=\frac{15y+2x}{3xy}$	Add the numerators

2. Simplify $\frac{3x+2}{2}-\frac{x+6}{3}$

   Solution:

$\frac{3x+2}{2}-\frac{x+6}{3}$	$=\frac{(3x+2)\times 3}{2\times 3}-\frac{(x+6)\times 2}{3\times 2}$	Change both denominators to the LCM

$$= \frac{9x+6}{6} - \frac{2x+12}{6}$$ Simplify

$$= \frac{9x+6-(2x+12)}{6}$$ Subtract the numerators

$$= \frac{9x+6-2x-12}{6}$$

$$= \frac{7x-6}{6}$$ Combine like terms

3. Simplify: $\frac{2}{3xy} - \frac{3}{4yz}$

Solution:

$$\frac{2}{3xy} - \frac{3}{4yz} = \frac{2\times 4z}{3xy\times 4z} - \frac{3\times 3x}{4yz\times 3x}$$ Change both denominators to the LCM, $12xyz$

$$= \frac{8z}{12xyz} - \frac{9x}{12xyz}$$ Simplify

$$= \frac{8z-9x}{12xyz}$$ Subtract the numerators

Cannot be factored or simplified.

4. Simplify: $\frac{6}{7d} + \frac{9}{5c}$

Solution:

Note that the LCM of $7d$ and $5c$ is found by multiplying them because they do not have common factors.

$$\frac{6}{7d} + \frac{9}{5c} = \frac{6\times 5c}{7d\times 5c} + \frac{9\times 7d}{5c\times 7d}$$ Change both denominators to the LCM

$$= \frac{30c}{35cd} + \frac{63d}{35cd}$$ Simplify

$$= \frac{30c+63d}{35cd}$$ Add the numerators

Practice

Simplify the following:

1. $\frac{a+4}{a} - \frac{1}{5ab}$
2. $\frac{a+b}{ab} - \frac{b+c}{bc}$
3. $\frac{7}{a} - \frac{4}{3b}$
4. $\frac{3a+b}{3ab} + \frac{b-a}{2ab}$
5. $\frac{2x-1}{3} - \frac{x+3}{2}$

Lesson Title: Addition and subtraction of algebraic fractions – Part 2	**Theme:** Algebraic Processes
Practice Activity: PHM2-L036	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to add and subtract algebraic fractions to give a single algebraic fraction.

Overview

This is the second lesson on adding and subtracting algebraic fractions. This lesson uses the same process as the previous lesson, but handles more complex problems.

Solved Examples

1. Simplify: $\frac{x}{x+1}+\frac{x^2-4}{3x+3}$

   Solution:

   Note that the denominators can be factored to $x+1$ and $3(x+1)$, which makes their LCM $3(x+1)$.

$\frac{x}{x+1}+\frac{x^2-4}{3x+3}$	$= \frac{x}{x+1}+\frac{(x+2)(x-2)}{3(x+1)}$	Factor the denominators
	$= \frac{3x}{3(x+1)}+\frac{(x+2)(x-2)}{3(x+1)}$	Change denominators to the LCM
	$= \frac{3x+(x+2)(x-2)}{3(x+1)}$	Add the numerators
	$= \frac{3x+x^2+2x-2x-4}{3(x+1)}$	Multiply out the brackets
	$= \frac{x^2+3x-4}{3(x+1)}$	Combine like terms
	$= \frac{(x+4)(x-1)}{3(x+1)}$	Factor

2. Simplify: $\frac{x+2}{x^2-2x}-\frac{x+1}{x^2-x-2}$

   Solution:

   Note that the denominators can be factored to $x(x-2)$ and $(x-2)(x+1)$, which makes their LCM $x(x-2)(x+1)$.

$\frac{x+2}{x^2-2x}-\frac{x+1}{x^2-x-2}$	$= \frac{x+2}{x(x-2)}-\frac{x+1}{(x-2)(x+1)}$	Factor the denominators
	$= \frac{(x+2)(x+1)}{x(x-2)(x+1)}-\frac{x(x+1)}{x(x-2)(x+1)}$	Change denominators to the LCM
	$= \frac{(x+2)(x+1)-x(x+1)}{x(x-2)(x+1)}$	Subtract the numerators
	$= \frac{x^2+3x+2-x^2-x}{x(x-2)(x+1)}$	Multiply out the brackets
	$= \frac{2x+2}{x(x-2)(x+1)}$	Combine like terms

$= \frac{2(x+1)}{x(x-2)(x+1)}$ Cancel $(x+1)$

$= \frac{2}{x(x-2)}$

3. Simplify: $\frac{x+2}{x-2} - \frac{x+3}{x-1}$

   Solution:

   Note that we cannot factor the denominators, therefore the LCM is the two binomials multiplied: $(x-2)(x-1)$.

   $\frac{x+2}{x-2} - \frac{x+3}{x-1} = \frac{(x-1)(x+2)}{(x-2)(x-1)} - \frac{(x-2)(x+3)}{(x-2)(x-1)}$ Change denominators to the LCM

   $= \frac{(x-1)(x+2)-(x-2)(x+3)}{(x-2)(x-1)}$ Subtract the numerators

   $= \frac{(x^2+x-2)-(x^2+x-6)}{(x-2)(x-1)}$ Remove the brackets

   $= \frac{x^2+x-2-x^2-x+6}{(x-2)(x-1)}$

   $= \frac{4}{(x-2)(x-1)}$ Combine like terms

4. Simplify: $\frac{4x^3}{4x^2-8xy} - \frac{4y^3}{xy-2y^2}$

   Solution:

   Note that the denominator can be factored to $4x(x-2y)$ and $y(2y-x)$. Then, we can cancel to make the denominators the same.

   $\frac{4x^3}{4x^2-8xy} - \frac{4y^3}{xy-2y^2} = \frac{4x^3}{4x(x-2y)} - \frac{4y^3}{y(x-2y)}$

   $= \frac{x^2}{x-2y} - \frac{4y^2}{x-2y}$ Cancel $4x$ in the first and y in the second fraction.

   $= \frac{x^2-4y^2}{x-2y}$ Subtract the numerators

   $= \frac{(x+2y)(x-2y)}{x-2y}$ Factor the numerator

   $= x+2y$ Cancel $(x-2y)$

5. Simplify: $\frac{1}{x-1} - \frac{1}{x+1} + \frac{2x}{x^2-1}$

   Solution:

   Note that when there are 3 terms the problem is solved following the same steps. The denominators are changed to the LCM, and the operations are applied. Note that the LCM of these denominators is $x^2-1 = (x+1)(x-1)$.

   $\frac{1}{x-1} - \frac{1}{x+1} + \frac{2x}{x^2-1} = \frac{1}{x-1} - \frac{1}{x+1} + \frac{2x}{(x+1)(x-1)}$

   $= \frac{x+1}{(x+1)(x-1)} - \frac{x-1}{(x+1)(x-1)} + \frac{2x}{(x+1)(x-1)}$

   $= \frac{(x+1)-(x-1)+2x}{(x+1)(x-1)}$

   $= \frac{x+1-x+1+2x}{(x+1)(x-1)}$

$$= \frac{2x+2}{(x+1)(x-1)}$$
$$= \frac{2(x+1)}{(x+1)(x-1)}$$
$$= \frac{2}{x-1}$$

Practice

Simplify the following:

1. $\frac{2}{x-3} - \frac{3}{x-2}$

2. $\frac{(x-y)^2}{(x-y)} + \frac{x^2-y^2}{(3x+3y)}$

3. $\frac{5x+8}{x^2-9} - \frac{3x+2}{x^2-9}$

4. $\frac{1}{x-2} - \frac{1}{x+2} + \frac{2x}{x^2-4}$

5. $1 + \frac{3}{2x-1} + \frac{x+1}{6x^2-3x}$

Lesson Title: Substitution in algebraic fractions	**Theme:** Algebraic Processes
Practice Activity: PHM2-L037	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to use substitution of numerical values or algebraic terms to simplify given algebraic fractions.

Overview

This lesson handles 2 different types of problems involving substitution in algebraic fractions.

The first type has the form: If $\frac{x}{y} = \frac{1}{2}$, evaluate $\frac{4x-y}{2x+3y}$

- The value of $\frac{x}{y}$ is given. However, there is no $\frac{x}{y}$ in the given formula.
- Divide the numerator and denominator of the formula by y. That is, divide each term in the numerator and denominator by y. Each x term will become a fraction with $\frac{x}{y}$, and each y will be eliminated.
- Note that sometimes the fraction is given as a ratio. If $x: y = 1: 2$ is given in the problem, recall that this is the same as $\frac{x}{y} = \frac{1}{2}$.

The second type has the form: If $x = \frac{z+1}{z-1}$, express $\frac{x+1}{x-1}$ in terms of z

- When you see a problem with "in terms of", you want to get find the result with only the given variable (in this case z).
- Substitute $x = \frac{z+1}{z-1}$ into the formula $\frac{x+1}{x-1}$ for each x, and simplify the result.

Solved Examples

1. If $\frac{x}{y} = \frac{1}{2}$, evaluate $\frac{4x-y}{2x+3y}$.

Solution:

$\frac{4x-y}{2x+3y} = \frac{4\left(\frac{x}{y}\right)-1}{2\left(\frac{x}{y}\right)+3}$ Divide throughout by y

$= \frac{4\left(\frac{1}{2}\right)-1}{2\left(\frac{1}{2}\right)+3}$ Substitute $\frac{x}{y} = \frac{1}{2}$

$= \frac{\frac{4}{2}-1}{\frac{2}{2}+3}$ Simplify

$= \frac{2-1}{1+3}$

$= \frac{1}{4}$

2. If $x = \frac{z+1}{z-1}$, express $\frac{x+1}{x-1}$ in terms of z.

$\frac{x+1}{x-1}$	$=$	$\frac{\left(\frac{z+1}{z-1}\right)+1}{\left(\frac{z+1}{z-1}\right)-1}$	Substitute $x = \frac{z+1}{z-1}$
	$=$	$\frac{(z+1)+(z-1)}{(z+1)-(z-1)}$	Multiply throughout by $z-1$
	$=$	$\frac{z+1+z-1}{z+1-z+1}$	Remove brackets
	$=$	$\frac{2z}{2}$	Combine like terms
	$=$	z	Simplify

3. Given $p:q = 9:5$, calculate $\frac{15p-2q}{5p+16q}$

Solution:

Note that $p:q = 9:5$ is the same as $\frac{p}{q} = \frac{9}{5}$

$\frac{15p-2q}{5p+16q}$	$=$	$\frac{15\left(\frac{p}{q}\right)-2}{5\left(\frac{p}{q}\right)+16}$	Divide through by q
	$=$	$\frac{15\left(\frac{9}{5}\right)-2}{5\left(\frac{9}{5}\right)+16}$	Substitute $\frac{9}{5}$ for $\frac{p}{q}$ in the expression
	$=$	$\frac{3\times9-2}{1\times9+16}$	Simplify
	$=$	$\frac{27-2}{9+16}$	
	$=$	$\frac{25}{25}$	
	$=$	1	

4. If $h = \frac{m+1}{m-1}$, express $\frac{2h-1}{2h+1}$ in terms of m.

Solution:

$\frac{2h-1}{2h+1}$	$=$	$\frac{2\times\frac{m+1}{m-1}-1}{2\times\frac{m+1}{m-1}+1}$	Substitute $h = \frac{m+1}{m-1}$
	$=$	$\frac{\frac{2m+2}{m-1}-1}{\frac{2m+2}{m-1}+1}$	Simplify
	$=$	$\frac{(2m+2)-(m-1)}{(2m+2)+(m-1)}$	Multiply throughout by $(m-1)$
	$=$	$\frac{2m+2-m+1}{2m+2+m-1}$	Remove brackets
	$=$	$\frac{m+3}{3m+1}$	Combine like terms

Practice

1. If $x = \frac{2a+3}{3a-2}$, express $\frac{x-1}{2x+1}$ in terms of a
2. If $a:b = 5:3$, evaluate $\frac{6a+b}{a-\frac{1}{3}b}$
3. Given that $\frac{x}{y} = \frac{2}{7}$, evaluate $\frac{7x+y}{x-\frac{1}{7}y}$

4. If $x = \frac{3w-1}{w+2}$, express $\frac{2x-3}{3x-1}$ in terms of w
5. If $a = \frac{d+1}{d-1}$, express $\frac{a+1}{a-1}$ in terms of d

Lesson Title: Equations with algebraic fractions	**Theme:** Algebraic Processes
Practice Activity: PHM2-L038	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to solve equations that contain algebraic fractions.

Overview

This lesson focuses on solving equations with algebraic fractions. Equations may have 1 or more algebraic fractions. The process for solving is the same for all such problems:

- Multiply throughout by the LCM of the denominators of any fractions in the equation. This eliminates the fractions.
- Balance the equation and solve for the variable.

Solved Examples

1. Solve: $\frac{2}{x} = x - 1$

   Solution:

   Note that there is only one fraction, thus the LCM of the denominators is x.

$$\begin{array}{rcll} \frac{2}{x} & = & x - 1 & \\ x\left(\frac{2}{x}\right) & = & x(x - 1) & \text{Multiply throughout by the LCM} \\ 2 & = & x^2 - x & \text{Cancel } x \text{ and remove brackets} \\ x^2 - x - 2 & = & 0 & \text{Balance the equation} \\ (x - 2)(x + 1) & = & 0 & \text{Factor} \\ x - 2 = 0 \text{ or } x + 1 = 0 & & & \text{Solve for } x \\ \textbf{Answer: } x = 2, -1 & & & \end{array}$$

2. Solve: $x = \frac{2}{x+1}$

   Solution:

   Note that the LCM of the denominators is $x + 1$.

$$\begin{array}{rcll} x & = & \frac{2}{x+1} & \\ x(x + 1) & = & 2 & \text{Multiply throughout by the LCM} \\ x^2 + x & = & 2 & \text{Remove brackets} \\ x^2 + x - 2 & = & 0 & \text{Balance the equation} \\ (x + 2)(x - 1) & = & 0 & \text{Factor} \\ x + 2 = 0 \text{ or } x - 1 = 0 & & & \text{Solve for } x \\ \textbf{Answer: } x = -2, 1 & & & \end{array}$$

3. Solve: $\frac{4}{x+3} - \frac{3}{x+2} = 0$

Solution:

Note that the LCM of the denominators is $(x+3)(x+2)$

$\frac{4}{x+3} - \frac{3}{x+2}$	$=$	0	
$\frac{4(x+3)(x+2)}{x+3} - \frac{3(x+3)(x+2)}{x+2}$	$=$	0	Multiply throughout by the LCM
$4(x+2) - 3(x+3)$	$=$	0	Cancel $(x+3)$ and $(x+2)$
$4x + 8 - 3x - 9$	$=$	0	Remove brackets
$x - 1$	$=$	0	Combine like terms
x	$=$	1	Solve for x

4. Solve: $\frac{x}{2x-3} + \frac{4}{x+1} = 1$

Solution:

Note that the LCM of the denominators is $(2x-3)(x-1)$

$\frac{x}{2x-3} + \frac{4}{x+1}$	$=$	1	
$\frac{x(2x-3)(x+1)}{2x-3} + \frac{4(2x-3)(x+1)}{x+1}$	$=$	$1(2x-3)(x+1)$	Multiply throughout by the LCM
$x(x+1) + 4(2x-3)$	$=$	$(2x-3)(x+1)$	Cancel $(2x-3)$ and $(x+1)$
$x^2 + x + 8x - 12$	$=$	$2x^2 + 2x - 3x - 3$	Remove brackets
$x^2 + 9x - 12$	$=$	$2x^2 - x - 3$	Combine like terms
0	$=$	$x^2 - 10x + 9$	Transpose terms
$(x-9)(x-1)$	$=$	0	
$x - 9 = 0$ or $x - 1 = 0$			Solve for x
Answer: $x = 9$ or $x = 1$			

5. Solve the equation: $\frac{2}{y-1} + \frac{3}{y+1} = \frac{5}{y}$

Solution:

$\frac{2}{y-1} + \frac{3}{y+1}$	$=$	$\frac{5}{y}$	
$\frac{2y(y-1)(y+1)}{y-1} + \frac{3y(y-1)(y+1)}{y+1}$	$=$	$\frac{5y(y-1)(y+1)}{y}$	Multiply throughout by the LCM
$2y(y+1) + 3y(y-1)$	$=$	$5(y-1)(y+1)$	Cancel
$2y^2 + 2y + 3y^2 - 3y$	$=$	$5y^2 - 5$	Remove brackets
$5y^2 - y$	$=$	$5y^2 - 5$	Combine like terms
$-y$	$=$	-5	Transpose terms
y	$=$	5	Solve for y

6. Solve: $\frac{3}{2x+1} + \frac{4}{5x-1} = 2$

Solution:

$\frac{3}{2x+1} + \frac{4}{5x-1}$	$=$	2	
$\frac{3(2x+1)(5x-1)}{2x+1} + \frac{4(2x+1)(5x-1)}{5x-1}$	$=$	$2(2x+1)(5x-1)$	Multiply throughout by the LCM
$3(5x-1) + 4(2x+1)$	$=$	$2(2x+1)(5x-1)$	Cancel

$$\begin{aligned} 15x - 3 + 8x + 4 &= 20x^2 + 6x - 2 && \text{Remove brackets} \\ 0 &= 20x^2 - 17x - 3 && \text{Transpose and combine terms} \\ (20x + 3)(x - 1) &= 0 && \text{Factor} \\ 20x + 3 = 0 \text{ or } x - 1 &= 0 && \text{Solve for } x \end{aligned}$$

Answer: $x = -\frac{3}{20}$ or $x = 1$

Practice

Solve the following equations:

1. $x - 2 = \frac{x}{3}$
2. $\frac{18}{y} + 2 = 0$
3. $\frac{2}{t} - \frac{3}{t+1} = 0$
4. $\frac{x+3}{x+2} = \frac{x+4}{x+1}$
5. $\frac{3}{q} + \frac{4}{q+1} = 2$
6. $\frac{1}{y-1} + \frac{2}{y+1} = \frac{3}{\mathrm{y}}$

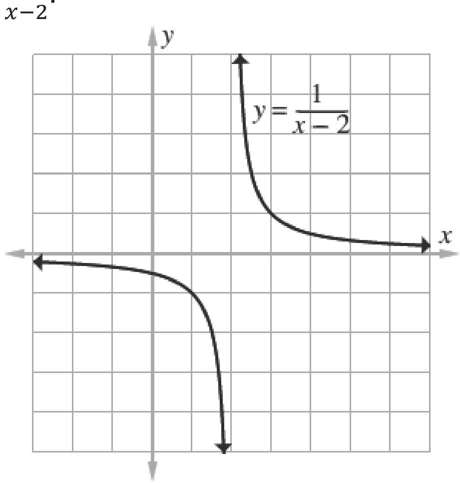
Lesson Title: Undefined algebraic fractions	**Theme:** Algebraic Processes
Practice Activity: PHM2-L039	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to determine the values that make an algebraic fraction undefined.

Overview

An algebraic fraction is undefined when the denominator is equal to zero. This is because it is impossible to divide by zero.

Consider the graph of $y = \frac{1}{x-2}$:



The function y approaches $x = 2$ from the left and from the right. It never touches $x = 2$. The function is said to be **undefined** at $x = 2$ because it has no value there.
An algebraic fraction can be undefined at one or more values. To find the value or values at which a fraction is undefined, set the denominator equal to zero and solve for the variable.

Solved Examples

1. Find the value of x for which $\frac{x-3}{x+4}$ is not defined.

   Solution:

   Step 1. Set the denominator equal to zero:

   $x + 4 = 0$

   Step 2. Solve for x by transposing 4:

   $x = -4$

   The fraction is undefined at $x = -4$.

2. Find the values of x for which $\frac{x^2+2x+1}{x^2-4x-5}$ is undefined.

Solution:

Step 1. Set the denominator equal to zero:

$$x^2 - 4x - 5 = 0$$

Step 2. Factor the denominator:

$$(x - 5)(x + 1) = 0$$

Step 3. Solve for x in each factor:

$$x - 5 = 0 \rightarrow x = 5$$

$$x + 1 = 0 \rightarrow x = -1$$

The fraction is undefined at $x = 5$ and $x = -1$.

3. Find the values of x for which the fraction $\frac{1-x}{x^2-3x+2}$ is not defined.

Solution:

Step 1. Set the denominator equal to zero:

$$x^2 - 3x + 2 = 0$$

Step 2. Factor the denominator:

$$(x - 1)(x - 2) = 0$$

Step 3. Solve for x in each factor:

$$x - 1 = 0 \rightarrow x = 1$$

$$x - 2 = 0 \rightarrow x = 2$$

The fraction is undefined at $x = 1$ and $x = 2$.

4. Find the values of x which make the fraction $\frac{1}{r^2-5r+6}$ undefined.

Solution:

Step 1. Set the denominator equal to zero:

$$r^2 - 5r + 6 = 0$$

Step 2. Factor the denominator:

$$(r - 3)(r - 2) = 0$$

Step 3. Solve for x in each factor:

$$r - 3 = 0 \rightarrow r = 3$$

$$r - 2 = 0 \rightarrow r = 2$$

The fraction is undefined at $r = 3$ and $r = 2$.

5. Find the values of x which makes the fraction $\frac{x+2}{x^2-3x}$.

Solution:

Step 1. Set the denominator equal to zero:

$$x^2 - 3x = 0$$

Step 2. Factor the denominator:

$$x(x - 3) = 0$$

Step 3. Solve for x in each factor:

$$x = 0$$

$$x - 3 = 0 \rightarrow x = 3$$

The fraction is undefined at $x = 0$ and $x = 3$.

Practice

Find the value(s) of the variables for which the following expressions are undefined:

1. $\frac{4x-8}{x-4}$
2. $\frac{3}{x^2-x}$
3. $\frac{6x-1}{x^2-8x-20}$
4. $\frac{2x+11}{x^2-x-20}$
5. $\frac{x^2-3x-10}{x^2+12x+36}$
6. $\frac{2x^3}{3x+4}$
7. $\frac{p}{p^2-5p+4}$
8. $\frac{2r-5}{r^2-4}$

Lesson Title: Algebraic fraction word problems	**Theme:** Algebraic Processes
Practice Activity: PHM2-L040	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to solve word problems that contain algebraic fractions.

Overview

This lesson is practice on solving WASSCE-style questions on algebraic fractions. These problems require knowledge from the previous 9 lessons.

Solved Examples

1. If $\frac{5}{x-1}-\frac{6}{x-2}$ is equal to $\frac{P}{(x-1)(x-2)}$, find P.

   Solution:

   Note that the LCM of the denominators is $(x-1)(x-2)$.

$$
\begin{aligned}
\frac{5}{x-1}-\frac{6}{x-2} &= \frac{5(x-2)}{(x-1)(x-2)}-\frac{6(x-1)}{(x-1)(x-2)} && \text{Change denominators to the LCM}\\
&= \frac{5(x-2)-6(x-1)}{(x-1)(x-2)} && \text{Subtract the numerators}\\
&= \frac{5x-10-6x+6}{(x-1)(x-2)} && \text{Multiply out the brackets}\\
&= \frac{-x-4}{(x-1)(x-2)} && \text{Combine like terms}
\end{aligned}
$$

   The numerator is $P=-x-4$.

2. Simplify: $\frac{1}{x-3}-\frac{3(x-1)}{x^2-9}$

   Solution:

$$
\begin{aligned}
\frac{1}{x-3}-\frac{3(x-1)}{x^2-9} &= \frac{1}{x-3}-\frac{3(x-1)}{(x-3)(x+3)} && \text{Factor the second denominator}\\
&= \frac{x+3}{(x-3)(x+3)}-\frac{3(x-1)}{(x-3)(x+3)} && \text{Change denominators to the LCM}\\
&= \frac{x+3-3x+3}{(x-3)(x+3)} && \text{Subtract}\\
&= \frac{-2x+6}{(x-3)(x+3)} && \text{Simplify}\\
&= \frac{-2(x-3)}{(x-3)(x+3)} && \text{Factor the numerator}\\
&= \frac{-2}{x+3} && \text{Simplify}
\end{aligned}
$$

3. Given that $\frac{5y-x}{8y+3x}=\frac{1}{5}$, find the value of $\frac{x}{y}$ to 2 decimal places.

Solution:

$\frac{5y-x}{8y+3x}$	$=$	$\frac{1}{5}$	
$5(5y-x)$	$=$	$8y+3x$	Cross multiply
$25y-5x$	$=$	$8y+3x$	Clear bracket
$25y-8y$	$=$	$3x+5x$	Transpose
$17y$	$=$	$8x$	Combine like terms
$8x$	$=$	$17y$	
$\frac{x}{y}$	$=$	$\frac{17}{8}$	
	$=$	2.125	
	$=$	2.13	To 2 decimal places

4. Simplify: $\left(\frac{x^2}{2}-x+\frac{1}{2}\right)\left(\frac{1}{x-1}\right)$

Solution:

$\left(\frac{x^2}{2}-x+\frac{1}{2}\right)\left(\frac{1}{x-1}\right)$	$=$	$\left(\frac{x^2-2x+1}{2}\right)\left(\frac{1}{x-1}\right)$	Add and subtract in brackets
	$=$	$\frac{x^2-2x+1}{2(x-1)}$	Multiply
	$=$	$\frac{(x-1)(x-1)}{2(x-1)}$	Simplify
	$=$	$\frac{x-1}{2}$	

5. Simplify: $\frac{\frac{2}{3}cd-\frac{1}{3}c^2}{\frac{1}{2}d^2-\frac{1}{4}cd}$

Solution:

$\frac{\frac{2}{3}cd-\frac{1}{3}c^2}{\frac{1}{2}d^2-\frac{1}{4}cd}$	$=$	$\frac{\frac{2cd}{3}-\frac{c^2}{3}}{\frac{d^2}{2}-\frac{cd}{4}}$	Add and subtract within the numerator and denominator
	$=$	$\frac{\frac{2cd-c^2}{3}}{\frac{2d^2-cd}{4}}$	
	$=$	$\frac{2cd-c^2}{3}\div\frac{2d^2-cd}{4}$	Write division horizontally
	$=$	$\frac{c(2d-c)}{3}\div\frac{d(2d-c)}{4}$	Factor both numerators
	$=$	$\frac{c(2d-c)}{3}\times\frac{4}{d(2d-c)}$	Multiply by the reciprocal of the second fraction
	$=$	$\frac{4c}{3d}$	Simplify

Practice

1. Simplify: $\left(\frac{3}{x} - \frac{15}{2y}\right) \div \frac{6}{xy}$
2. Simplify: $\frac{3}{m+2n} - \frac{2}{m-3n}$
3. Find the values of x for which the expression is undefined: $\frac{6x-1}{x^2+4x-5}$
4. If $p = \frac{2u}{1-u}$ and $q = \frac{1+u}{1-u}$, express $\frac{p+q}{p-q}$ in terms of u.
5. Simplify: $\frac{m}{n} + \frac{(m-1)}{5n} - \frac{(m-2)}{10n}$, where $n \neq 0$
6. Divide: $\frac{x^2-4}{x^2+x}$ by $\frac{x^2-4x+4}{x+1}$
7. Simplify: $\frac{3x-y}{xy} - \frac{2x+3y}{2xy} + \frac{1}{2}$
8. For what values of x is the expression $\frac{3x-2}{4x^2+9x-9}$ undefined?
9. Solve: $\frac{4r-3}{6r+1} = \frac{2r-1}{3r+4}$
10. Simplify: $1 - \frac{2x}{4x-3}$

Lesson Title: Simple statements	**Theme:** Logical Reasoning
Practice Activity: PHM2-L041	**Class:** SSS 2

Learning Outcomes
By the end of the lesson, you will be able to:
1. Identify and form open and closed simple statements.
2. Deduce the truth or otherwise from simple statements.

Overview

This lesson handles two types of simple statements, open and closed. You should be able to determine whether a statement is open or closed using the following facts:

Open	It is not known if the statement is true or false. It can be either true or false, but cannot be both true and false.
Closed	The statement is always true, or always false

Solved Examples

1. Identify whether the following statements are open or closed simple statements:

	Statement	Open	Closed
$a.$	Obtuse angles measure less than 90 degrees.		✓
$b.$	-2 is greater than -7.		✓
$c.$	She is a skilled engineer.	✓	
$d.$	x is greater than 9.	✓	
$e.$	Triangles have 3 sides.		✓
$f.$	Freetown is far from his village.	✓	

2. Identify whether the following statements are true or false. If it is impossible to determine, check “unknown”.

	Statement	True	False	Unknown
$a.$	Angle x is obtuse.			✓
$b.$	5 is greater than -2.	✓		
$c.$	She walks a far distance to school.			✓
$d.$	Hexagons have 6 angles.	✓		
$e.$	x is greater than 9.			✓
$f.$	4 is an odd number.		✓	

Practice

1. Identify whether the following statements are open or closed simple statements.

	Statement	Open	Closed
$a.$	All Sierra Leoneans are Africans.		
$b.$	She can solve algebra problems very well.		
$c.$	The formula to find the volume of a sphere is $\frac{4}{3}\pi r^3$.		
$d.$	A complete revolution is equal to 360^o.		
$e.$	The principal said they must go home for a week.		
$f.$	Her exercise book is full.		
$g.$	All solids have faces, and most solids have edges.		

2. Identify whether the following statements are true or false. If it is impossible to determine, check “unknown”.

	Statement	True	False	Unknown
$a.$	All people are cows.			
$b.$	$2 < 3$			
$c.$	Every African is a Sierra Leonean.			
$d.$	He will be going to the football field after school.			
$e.$	Her favourite shape is a pentagon.			
$f.$	The best fruit in Africa is cassava.			
$g.$	The angles of a triangle sum to 180°.			

Lesson Title: Negation	**Theme:** Logical Reasoning
Practice Activity: PHM2-L042	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to form the negation of a simple statement.

Overview

This lesson focuses on forming the negation of both open and closed simple statements. The negation of a statement has an **opposite** meaning to the original statement. The negation of a statement is formed by adding the word "not".
The negation of a statement p is written as $\sim p$. For example:

p: Francis is a tailor.
$\sim p$: Francis is **not** a tailor.

The negation can also be written with a different letter. Consider two statements:

m: Bingo is a good dog.
n: Bingo is not a good dog.

Although different letters are used for the two statements, one is still the negation of the other. We can say that "n is the negation of m". If n is true, then m must be false. If n is false, then m must be true.

At times, the sentence you get by adding the word "not" is not grammatically correct. You may need to change words to make the negation grammatically correct.

For example, consider:
b: "He ate 3 bananas."

The negation is:
$\sim b$: "He **did not eat** 3 bananas."

Solved Examples

1. Write the negation of each of the following statements:

q: Monkeys are the smartest animals.
r: There are many birds in Sierra Leone.
s: She went to the market.
t: Triangles have 4 angles.
u: Angle x is greater than 100 degrees.
v: He likes the color orange.

Answers:

$\sim q$: Monkeys are **not** the smartest animals.
$\sim r$: There are **not** many birds in Sierra Leone.
$\sim s$: She **did not go** to the market.
$\sim t$: Triangles **do not** have 4 angles.
$\sim u$: Angle x is **not** greater than 100 degrees.
$\sim v$: He **does not like** the colour orange.

2. Write the negation of each of the following statements:
a: They like eating ice cream.
b: We can survive without food.
c: She went to church.
d: Mike is a bad boy.
e: Idrissa called John's line.
f: He loves plantains.

Answers:

$\sim a$: They **do not** like eating ice-cream
$\sim b$: We **cannot** survive without food.
$\sim c$: She **did not go** to church.
$\sim d$: Mike is **not** a bad boy.
$\sim e$: Idriss **did not call** John's line.
$\sim f$: He **does not love** plantains.

Practice

1. Write the negation of each of the following statements:

g: Peter is one of the pupils.
h: The sum of the angles in a triangle is 360°.
i: Multiplying 4.6×10^6 gives $46{,}000{,}000$.
j: If x represents a number, then 5 more than the number is given by $x - 5$.
k: In probability, the multiplication rule for $p(A\ and\ B)$ is $p(A) + p(B)$.
l: Two angles are supplementary if their sum is equal to 90°.

Lesson Title: Compound statements	**Theme:** Logical Reasoning
Practice Activity: PHM2-L043	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to distinguish between simple and compound statements.

Overview

Compound statements are made from simple statements and connecting words. Common connecting words are: and, but, or, if and only if, “either…or”, “if…then”. Consider the compound statement “Sami is a cat and he likes to eat fish”. It has two parts: “Sami is a cat” and “he likes to eat fish”. The two parts of the sentence are connected by the word “and”.

Solved Examples

1. Circle the connecting word or words in each compound statement. Write two simple statements from each one:

   a: She loves geometry but she does not like algebra.
   b: Octagons are shaxxpes with 8 angles and 8 sides.
   c: If it is not raining then he goes to the market.

   Answers:

   a: She loves geometry but she does not like algebra.
   Simple statements:
   She loves geometry.
   She does not like algebra.

   b: Octagons are shapes with 8 angles and 8 sides.
   Simple statements:
   Octagons are shapes with 8 angles.
   Octagons are shapes with 8 sides.

   c: If it is not raining then he goes to the market.
   Simple statements:
   It is not raining.
   He goes to the market.

Make compound statements by connecting the simple statements:

1.	Shape B is a hexagon.	+	It has 6 sides.
2.	Susu is a monkey.	+	He swings from trees.
3.	The doctor will help her.	+	She goes to the hospital.
4.	x is greater than -9.	+	x is less than 0.
5.	You may go to the market.	+	You may stay home.
6.	He will pass Maths.	+	He studies hard.

Answers:
(Note that some of the simple statements may be connected using different connecting words. These are example answers.)

1. Shape B is a hexagon if and only if it has 6 sides.
2. Susu is a monkey and he swings from trees.
3. The doctor will help her if she goes to the hospital.
4. x is greater than -9 and less than 0.
5. You may go to the market or stay home.
6. He will pass Maths if he studies hard.

Practice

1. Circle the connecting word or words in each compound statement. White two simple statements from each one.
   a. A square has four sides and all its sides are equal.
   b. The squares of 2 and 8 are 4 and 64, respectively.
   c. The sum of two integers can be positive or negative.
   d. He is either a fool or a madman.
2. Make the connection using different connection words in the simple statements below.
   a. He is eligible for university. + He doesn't have school fees.
   b. He doesn't drink. + He doesn't smoke.
   c. She will spend her money. + She will invest her money.
   d. You must apologise. + You will be punished.
   e. You can have tea. + You can have coffee.

Lesson Title: Implication	**Theme:** Logical Reasoning
Practice Activity: PHM2-L044	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to draw conclusions from a given implication.

Overview

Implications are compound statements that can be written with the connecting words "if… then". The first statement implies that the second is true.
Consider the implication "If Hawa lives in Freetown, then she lives in Sierra Leone." There are 2 simple statements in this implication:

A: Hawa lives in Freetown.
B: She lives in Sierra Leone.

This implication can be written with symbols: $A \Rightarrow B$
Note that if A is true (Hawa lives in Freetown) then B is definitely true. She must live in Sierra Leone because that's where Freetown is located.

Solved Examples

1. Connect the statements in an implication. Write the implication using words and symbols.

   E: Sam passes the exam.
   G: He will graduate.
   Answer: If Sam passes the exam, then he will graduate. $E \Rightarrow G$

2. Connect the statements in an implication. Write the implication using words and symbols.

   W: I wash my dishes.
   C: My dishes are clean.
   Answer: If I wash my dishes, then my dishes are clean. $W \Rightarrow C$

3. Consider the following statements:
   S: Bentu studies hard.
   P: Bentu passes Maths exams.

   If $S \Rightarrow P$, write the implication with words.
   Answer: If Bentu studies hard, then she passes Maths exams.

4. Consider the following statements:
   S: A shape has 3 sides.

T: It is a triangle.

If $S \Rightarrow T$, write the implication with words.
Answer: If a shape has 3 sides, then it is a triangle.

Practice

Connect the following statements in implications. Write the implications using words and symbols.

1. E: John has measles.
   F: John is in the hospital.
2. A: Amid plays football.
   B: Amid scores many goals.
3. C: Amie did her Maths assignment well.
   D: Amie got the answers right.
4. E: Alie goes for lunch.
   F: It is lunch time.
5. C: Peter wants some textbooks.
   D: Peter goes to the bookshop.
6. A: James scores very good marks in the interview.
   D: He will get the job.
7. X: Victor swims very well.
   Y: He will be able to cross to the other side of the river.
8. G: Paul enters the university to study engineering.
   H: He passed with six credits in WASSCE.
9. V: Joseph can interpret graphs very well.
   W: He can answer questions with graphs.
10. S: Francis knows the quadratic formula.
    T: He can solve quadratic equation problems.

Lesson Title: Conjunction and disjunction	**Theme:** Logical Reasoning
Practice Activity: PHM2-L045	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to distinguish between conjunction and disjunction, representing them on truth tables.

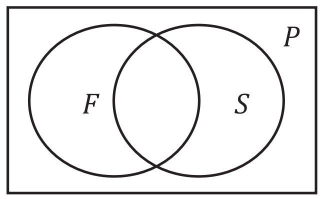
Overview

This lesson introduces conjunction and disjunction. A conjunction is a compound statement that uses “and”. A disjunction is a compound statement that uses “or”.

Consider 3 sets of people:

P: People who live in Freetown
F: Females
S: Students

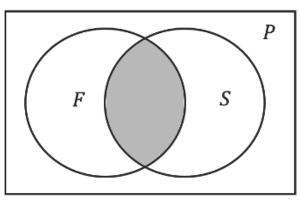
These can be visualised with a Venn diagram:



Consider the compound statements, A_1 and A_2:

A_1: people who are female and are students
A_2: people who are either female, or are students, or both

A_1 is a conjunction. The two statements “people who are female” and “people who are pupils” are linked by the word **“and”**. In the Venn diagram, this is where the two circles intersect.



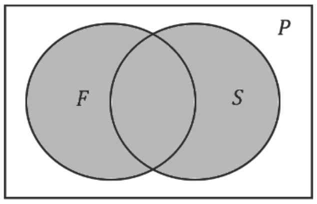
The conjunction can be written in symbols: $A_1 = F \cap S$ or $A_1 = F \wedge S$

A_1 can be represented in a truth table:

F	S	$F \wedge S$
T	T	T
T	F	F
F	T	F
F	F	F

This truth table says that statement A_1 is only true if both of the sub-statements are true ("people who are female" and "people who are students"). If either of the sub-statements is false, then statement A_1 is also false.

A_2 is a disjunction. The two statements "people who are female" and "people who are students" are linked by the words **"either – or – or both"**. In the diagram, this is all of the space inside of the two circles, including where they intersect.



The disjunction can be written in symbols: $A_2 = F \cup S$ or $A_2 = F \vee S$

A_2 can be represented in a truth table:

F	S	$F \vee S$
T	T	T
T	F	T
F	T	T
F	F	F

This truth table says that A_2 is true if either or both of the sub-statements are true. A_2 is false only if both sub-statements are false.

Solved Examples

1. Consider the following statements about some people living in a village:
   C: Some people are children.
   M: Some people are male.

   Based on this information, prepare truth tables that describe:

   a. A person in the village who is a male child.
   b. A person in the village who is either a child, a male, or both.

   Solutions:

   a.

C	M	$C \wedge M$
T	T	T
T	F	F
F	T	F
F	F	F

   b.

C	M	$C \vee M$
T	T	T
T	F	T
F	T	T
F	F	F

2. Consider the following statement:

   S: The teacher entered and took out her book.

Use appropriate symbols and truth tables to describe the conditions for S to be true.

Solution:
Determine two sub-statements of S:

E: The teacher entered.
B: The teacher took out her book.

If S is true, then both sub-statements must be true. In other words, $E \wedge B$ must be true. If one of the sub-statements is false, then S will be false.
Symbols: $S = E \wedge B$

Truth table:

E	B	$E \wedge B$
T	T	T
T	F	F
F	T	F
F	F	F

Practice

1. Consider the following statements:
   Y: David studied very hard and passed his Mathematics exam.
   Use appropriate symbols and truth tables to describe the conditions for Y to be true.
2. Consider the following statements about some pupils in Freetown.
   A: Some of the pupils are in the science faculty.
   B: Some of the pupils are members of the science society.
   Based on this information, prepare a truth table that describes:
   a. A student who is either in the science faculty or a member of the science society.
   b. A student who is both in the science faculty and a member of the science society.
3. Consider the following statements, T and S.
   T: People who are teachers.
   S: People who sell goods.
   Prepare a truth table and the related Venn diagram to describe each of the statements below.
   a. a person who is either a teacher or sells goods.
   b. a person who is both a teacher and sells goods.

Lesson Title: Equivalence and the chain rule	**Theme:** Logical Reasoning
Practice Activity: PHM2-L046	**Class:** SSS 2

Learning Outcomes
By the end of the lesson, you will be able to:
1. Recognise equivalent statements and apply them to arguments.
2. Recognise the chain rule and apply it to arguments.

Overview

This lesson introduces equivalence and the chain rule.
Equivalence:

- If $X \Rightarrow Y$ and $Y \Rightarrow X$, then X and Y are equivalent and we write $X \Leftrightarrow Y$. The implication $Y \Rightarrow X$ is the converse of $X \Rightarrow Y$.
  - For example, consider $x^2 = 16 \Rightarrow x = \pm 4$. Its converse is $x = \pm 4 \Rightarrow x^2 = 16$. This is true, and we can also write $x^2 = 16 \Leftrightarrow x = \pm 4$.
- It is important to note that if $X \Rightarrow Y$, then $\sim Y \Rightarrow \sim X$ is an equivalent statement.
  - For example, consider the implication: "If Fatu lives in Freetown, then she lives in Sierra Leone". This is an equivalent statement: "If Fatu does not live in Sierra Leone, then she does not live in Freetown".

Chain rule:

- If X, Y and Z are 3 statements such that $X \Rightarrow Y$ and $Y \Rightarrow Z$, then $X \Rightarrow Z$.
  - For example, consider the statements, where $X \Rightarrow Y$ and $Y \Rightarrow Z$:
    X: Hawa studies hard.
    Y: Hawa passes exams.
    Z: Hawa graduates secondary school.
  - If $X \Rightarrow Y$ and $Y \Rightarrow Z$, then $X \Rightarrow Z$. In other words, these statements are true:
    $X \Rightarrow Y$: If Hawa studies hard then she passes exams.
    $Y \Rightarrow Z$: If Hawa passes exams then she graduates secondary school.
    $X \Rightarrow Z$: If Hawa studies hard then she graduates secondary school.

Solved Examples

1. Write the converse of the following statements:
   a. If a shape has 3 sides, then it is a triangle.
   b. If you are living, then your heart is beating.
   c. If it is a cat, then it meows.

   Solutions:
   a. If a shape is a triangle, then it has 3 sides.
   b. If your heart is beating, then you are living.

c. If it meows, then it is a cat.

2. Identify whether the following sets of statements are equivalent:
   a. A: $x = \pm 3$ B: $x^2 = 9$
   b. S: $x = 4$ T: $x = 3$
   c. X: $x = 5$ Y: $x^2 = 25$

Solutions:

a. The statements are equivalent, because $A \Rightarrow B$ and $B \Rightarrow A$.
b. The statements are not equivalent. Neither implication that we can write is true.
c. The statements are not equivalent. Although $X \Rightarrow Y$, it is **not true** to say that $Y \Rightarrow X$. If we know that $x^2 = 25$, then the implication is that $x = \pm 5$, not $x = 5$.

3. For the statements below, $A \Rightarrow B$ and $B \Rightarrow C$. Write as many statements as possible with this information.
   A: Foday injures his leg.
   B: Foday doesn't exercise.
   C: Foday gains weight.

Solutions:

$A \Rightarrow B$	If Foday injures his leg, then he doesn't exercise.
$B \Rightarrow C$	If Foday doesn't exercise, then he gains weight.
$A \Rightarrow C$	If Foday injures his leg, then he gains weight.
$\sim B \Rightarrow \sim A$	If Foday exercises, then he does not injure his leg.
$\sim C \Rightarrow \sim B$	If Foday does not gain weight, then he does exercise.
$\sim C \Rightarrow \sim A$	If Foday does not gain weight, then he does not injure his leg.

4. Consider the following statements about some female teachers in a secondary school. For these statements, $A \Rightarrow B$ and $B \Rightarrow C$.
   A: The female teachers teach Mathematics.
   B: The female teachers studied Mathematics in university.
   C: The female teachers are skilled in Mathematics.
   Write as many statements as possible with this information.

Solutions:

$A \Rightarrow B$	If the female teachers teach Mathematics, then they studied Mathematics in university.
$B \Rightarrow C$	If the female teachers studied Mathematics in university, then they are skilled in Mathematics.
$A \Rightarrow C$	If the female teachers teach Mathematics, then they are skilled in Mathematics.

$\sim B \Rightarrow \sim A$	If the female teachers did not study Mathematics in university, then they do not teach Mathematics.
$\sim C \Rightarrow \sim B$	If the female teachers are not skilled in Mathematics, then they did not study Mathematics in university.
$\sim C \Rightarrow \sim A$	If the female teachers are not skilled in Mathematics, then they do not teach Mathematics.

5. Consider the true statement below, and write another true statement with this information:

   If Hawa does not sell goods, then she does not make money.

   Solution:

   Recall that if $X \Rightarrow Y$, then $\sim Y \Rightarrow \sim X$. In this statement, we have:

   X: Hawa does not sell goods.

   Y: Hawa does not make money.

   Therefore, we can write the statement $\sim Y \Rightarrow \sim X$**:**

   If Hawa makes money, then she sells goods.

Practice

1. Consider the true statements below, and write other true statements with this information:
   a. If Foday harvests his farm, then he has a lot to eat.
   b. If Sia lives in Bo, then she lives in Sierra Leone.
   c. If Issa is from Sierra Leone, then he is African.

2. For the statements below, $X \Rightarrow Y$ and $Y \Rightarrow Z$. Write as many statements as possible with this information.

   X: John practices solving Mathematics problems every day.

   Y: John is a very good mathematician.

   Z: John can solve every problem in Mathematics.

3. Identify whether the following sets of statements are equivalent:
   a. A: $x = \pm 10$ B: $x^2 = 100$
   b. S: A shape is a square. T: A shape has 4 sides.
   c. X: A shape is a pentagon. Y: A shape has 5 sides.

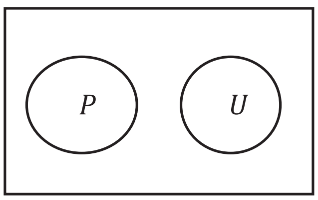
Lesson Title: Venn diagrams	**Theme:** Logical Reasoning
Practice Activity: PHM2-L047	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to use Venn diagrams to demonstrate connections between statements.

Overview

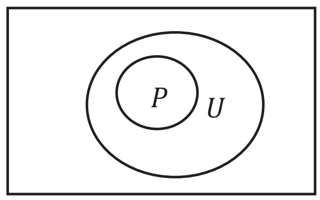
Recall from SSS 1 how relationships between sets are shown with Venn diagrams:

X:



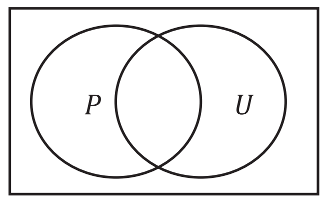
$P \cap U = \emptyset$
P and U are separate.

Y:



$P \subset U$
P is a subset of U.

Z:



$P \cap U$
P intersects with U.

This lesson uses Venn diagrams to represent statements. For example, the Venn diagrams above show the following statements:

X: No police officers wear uniforms.
Y: All police officers wear uniforms.
Z: Some police officers wear uniforms.

Where P ={police officers} and U ={people who wear uniforms}

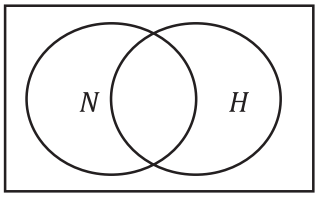
Key words in statements tell you which type of Venn diagram to draw. See examples below:

Key Words	**Type**	**Examples**
"no", "never", "all…do not"	Separate sets	No sick pupils come to class. Sick pupils never come to class. All sick pupils do not come to class. (S ={sick pupils} and P ={pupils who come to class} are disjoint)
"all", "no…not", "if…then"	Subset	All football players exercise. There is no football player who does not exercise. If someone is a football player, then they exercise. (F ={football players} is a subset of E ={people who exercise})

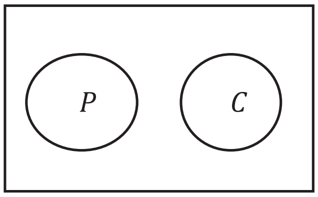
"some", "most", "not all"	Intersection	Some pupils use computers. Most pupils use computers. Not all pupils use computers. (P ={pupils} intersects with C ={people who use computers})

Solved Examples

1. Draw a Venn diagram for the statement: A: Some nurses work at the hospital. Let N={nurses} and H={people who work at the hospital}.
   Solution:
   Note that the key word "some" means this is an intersection.

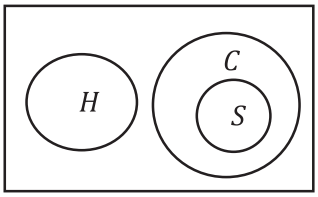


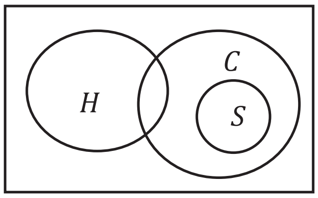
2. Draw a Venn diagram for the statement: No pupils drive cars.
   Solution:
   There are no letters assigned to the sets in the problem, so assign them: P={pupils} and C={people who drive cars}. Note that the key word "no" means these are separate sets.



3. The following statements are true for a certain country.
   A: There is **no** civil servant that has a small house.
   B: **All** civil servants have private cars.

   Draw one Venn diagram to show the statements. Let S={Civil servants} $H =$ {People with small houses}, and C= {People with private cars}.
   Solution:
   "Civil servants" is a subset of "people with private cars". "Civil servants" does not intersect with "people with small houses". They are separate. We do not know about the relationship between "People with small houses" and "people with private cars". Therefore, either Venn diagram is correct:





Practice

1. Draw a Venn diagram for each statement:
   a. Some teachers are male. Let T={teachers } and M={males}.
   b. If someone is a pupil, then they are a child. Let P={pupils} and C={children}.
   c. Not all pupils have phones. Let P={pupils} and H={people with phones}.
   d. None of the pupils are badly behaved. Let P={pupils} and B={badly behaved people}.
2. Consider the following statements:

   X: All school inspectors wear uniforms.

   Y: No minister wears a uniform.

   If I ={Inspectors}, U ={People who wear uniforms} and M ={Ministers}, draw a Venn diagram to illustrate the above statements.

Lesson Title: Validity	**Theme:** Logical Reasoning
Practice Activity: PHM2-L048	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to determine the validity of an argument.

Overview

This lesson focuses on determining the validity of statements. An argument is valid if and only if the conclusion can be drawn from other statements.
The validity of an argument can be determined using information you already know. This includes:

- A Venn diagram.
- The fact that if $p \Rightarrow q$, then $\sim q \Rightarrow \sim p$ is an equivalent statement.
- The chain rule.

This lesson focuses primarily on Venn diagrams, but you may use the other two methods as well.
Note that the actual **truth does not matter** when determining whether a statement is valid. For example, consider:

A: Monrovia is in Sierra Leone.
B: Sierra Leone is in Asia.

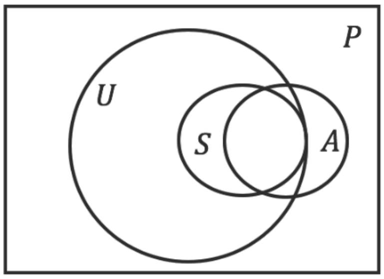
Although these statements are false, a valid conclusion can be drawn. Based on A and B and using the chain rule, the following is valid: C: Monrovia is in Asia.

Solved Examples

1. Consider the following statements and answer the questions:
   X: All senior secondary pupils wear uniforms.
   Y: Most senior secondary pupils have high attendance.
   a. Draw one Venn diagram to illustrate both statements.
   b. Determine which of the following implications are valid based on X and Y:
      i. Fatu wears a uniform ⇒ Fatu is a senior secondary pupil.
      ii. Michael is a senior secondary pupil ⇒ He has high attendance.
      iii. Hawa does not wear a uniform ⇒ She is not a senior secondary pupil.

Solutions:

a. Draw one Venn diagram to illustrate both statements.
   $P =$ {all pupils}, $U =$ {pupils who were uniforms}, $S =$ {senior secondary pupils}, $A =$ {pupils with high attendance}



b. Determine which of the following implications are valid based on X and Y:
   i. **Not valid**. All senior secondary pupils wear uniforms. However, all pupils who wear uniforms are not necessarily senior secondary pupils. Fatu may be in U, but outside of S.
   ii. **Not valid.** Although most senior secondary pupils have high attendance, not all of them do. Michael might be in set S, but outside of set A.
   iii. **Valid.** If Hawa does not wear a uniform, she is outside of set U. She must also be outside of set S, which is a subset of U.

2. Consider the following statements:

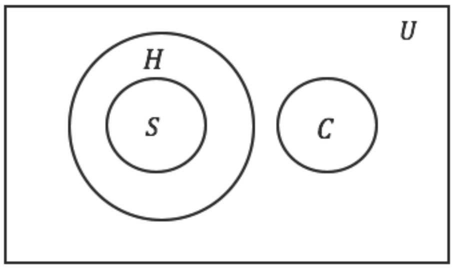
   X: **All** pupils are hardworking.
   Y: **No** hardworking person is careless.

   a. Draw a Venn diagram to illustrate the above statements.
   b. Determine which of the following are valid conclusions from the pupils X and Y.
      i. Mariama is a student. ⟹ Mariam is not careless.
      ii. Medo is hardworking. ⟹ Medo is a pupil.
      iii. Battu is careless. ⟹ Battu is not a pupil.

Solutions:

a. The statements are about students, hardworking persons, and careless people, therefore:

   Let U ={all persons}, S ={students}, H ={hardworking persons} and C ={careless persons}.
   From statements X and Y, we know that S is a subset of H, and that H and C are separate sets.
   Below is the Venn diagram illustrating the statements X and Y.



b. i. **Valid**. "Mariama is a student" means she is in region S and therefore cannot be in region C. Therefore, the statement "Mariama is a student ⇒ Mariama is not careless" is valid.
ii. **Not valid**. "Medo is hardworking" means he is in region H and therefore can be within region S or outside S. Therefore, the statement is not always true and not a valid conclusion.
iii. **Valid**. "Battu is careless" means she is region C and therefore cannot be in region S. Therefore the statement is valid.

Practice

1. Consider the following statements:

   X: **All** lazy pupils are careless.

   Y: **Some** strong pupils are lazy.

   a. Illustrate the information above on a Venn diagram.

   b. Using the Venn diagram or otherwise, determine whether or not each of the following statements is valid.

      i. Muriel is careless ⇒ Muriel is lazy
      ii. Michaela is lazy ⟹ Michaela is careless
      iii. Nanday is strong ⟹ Nanday is lazy
      iv. Daddy-kay is lazy ⟹ Daddy-kay is strong
      v. Ann-Marie is lazy and strong ⟹ Ann-Marie is careless
      vi. Angus is careless and strong ⟹ Angus is lazy

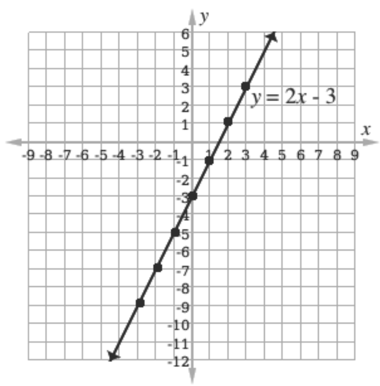
Answer Key – Term 1

Lesson Title:	Review of Number Bases and Indices
Practice Activity:	PHM2-L001

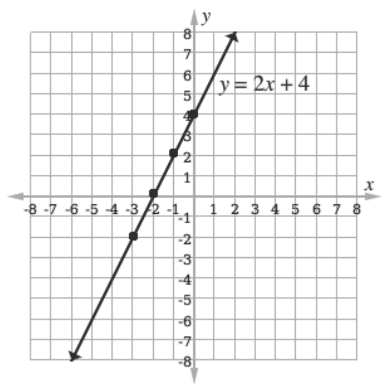
1.
   a. $-64g^{15}$
   b. $8x^2y^3$
   c. c
   d. $12p$
   e. $x^{-15}y^6$ *or* $\frac{y^6}{x^{15}}$
2. $437_{ten} = 3{,}222_{five}$
3. $625_{\text{seven}} = 313_{\text{ten}} = 313$
4. $134_{five} = 101{,}100_{two}$
5. $1{,}110_3$

Lesson Title:	Review of Linear Equations
Practice Activity:	PHM2-L002

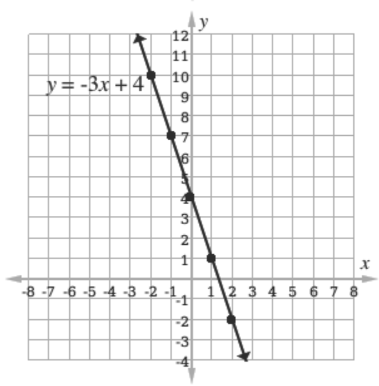
1.



2.

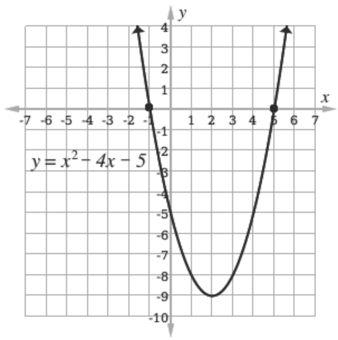


3.

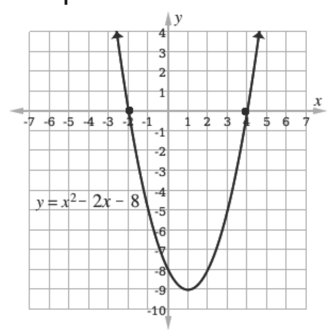


Lesson Title:	Review of Quadratic Equations
Practice Activity:	PHM2-L003

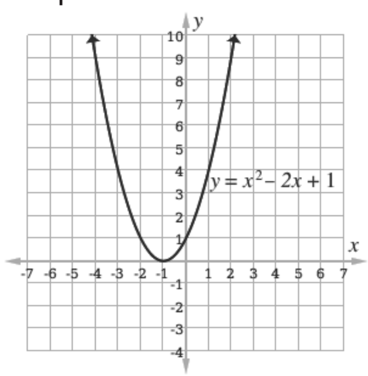
1. Solutions (roots): $x = -1$ and $x = 5$.
   Graph:



2. Solutions (roots): $x = -2$ and $x = 4$.
   Graph:



3. Solution (root): $x = -1$.
   Graph:



Lesson Title:	Review of Angles and Triangles
Practice Activity:	PHM2-L004

1. a. $P = 65°$, b. $x = 30°$, c. $a = 30°$
2. a. 4 cm, b. 7 cm, c. 13 cm

Lesson Title:	Significant Figures
Practice Activity:	PHM2-L005

1. a. 587,300, b. 587,000, c. 590,000
2. 0.046626
3. a. 0.033, b. 4.3, c. 31
4. a. 20,000, b. 500, c. 0.6
5. Le 320,000
6. 1,050,000

Lesson Title:	Estimation
Practice Activity:	PHM2-L006

1.
   a. Completed table:

District	Population	Estimated Population
Kailahun	525,372	500,000
Kenema	609,873	600,000
Kono	505,767	500,000
Kambia	343,686	300,000
Koinadugu	408,097	400,000

   b. Population difference: 300,000
   c. Total population: 2,300,000
2.
   a. 10 words
   b. 300 words
   c. 2 pages

Lesson Title:	Percentage Error
Practice Activity:	PHM2-L007

1. 2%
2. a. Range: 7.5 m – 8.5 m. b. Percentage error: $6\frac{1}{4}\%$ or : 6.25%
3. 10%
4. $2\frac{1}{2}\%$ or 2.5%
5. a. Range: 7.45 – 7.55 m. b. Percentage error: $\frac{2}{3}\%$
6. 2%

Lesson Title:	Degree of accuracy
Practice Activity:	PHM2-L008

1. 12,800 females
2. 231 m^2
3. 160 m
4. 38 cm^2
5. 13 minutes
6. 9.4 m^2

Lesson Title:	Simultaneous linear equations using elimination
Practice Activity:	PHM2-L009

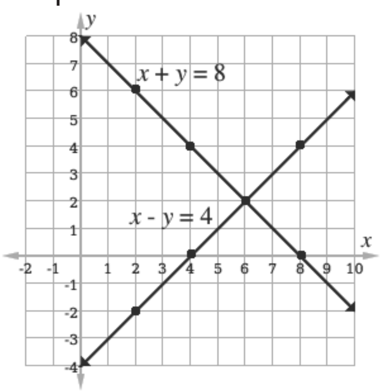
1. $p = 1, q = 2$ or $(1, 2)$
2. $n = 1{,}200, p = 100$ or $(1200, 100)$
3. $x = 2, y = 1$ or $(2, 1)$
4. $x = 2, y = 0$ or $(2, 0)$

Lesson Title:	Simultaneous linear equations using substitution
Practice Activity:	PHM2-L010

1. $x = 3, y = 1$ or $(3, 1)$
2. $x = 2, y = -3$ or $(2, -3)$
3. $x = 2, y = 4$ or $(2, 4)$
4. $s = 8, t = 3$ or $(8, 3)$

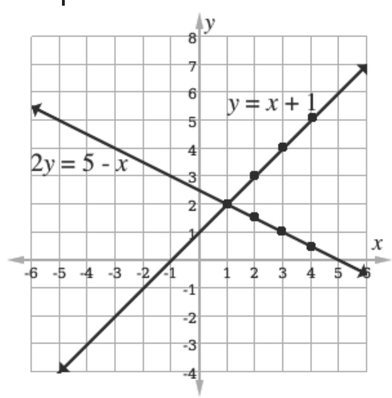
Lesson Title:	Simultaneous linear equations using graphical methods – Part 1
Practice Activity:	PHM2-L011

1. Graph:



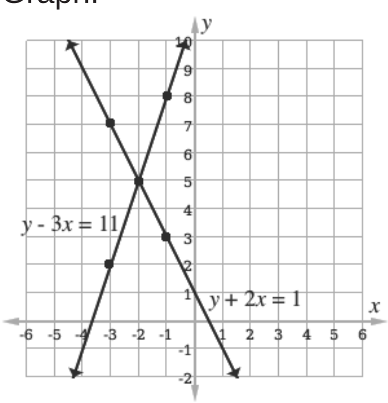
Solution: $(6, 2)$

2. Graph:



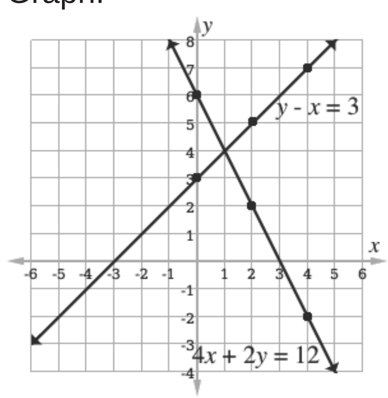
Solution: $(1, 2)$

3. Graph:



Solution: $(-2, 5)$

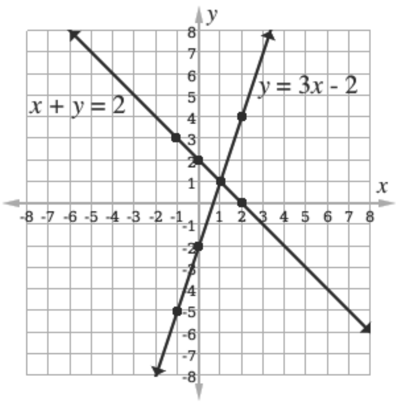
4. Graph:



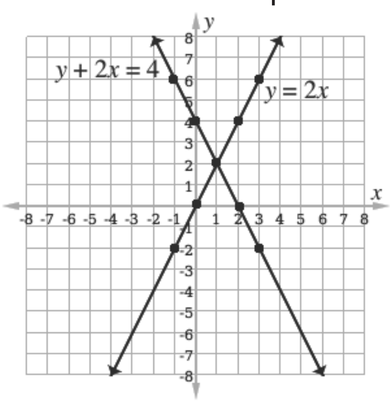
Solution: $(1, 4)$

Lesson Title:	Simultaneous linear equations using graphical methods – Part 2
Practice Activity:	PHM2-L012

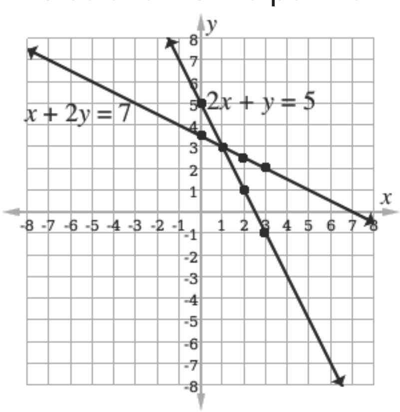
1. The solution is the point of intersection, (1, 1).



2. The solution is the point of intersection, (1, 2).



3. The solution is the point of intersection, (1, 3).



Lesson Title:	Words problems on simultaneous linear equations
Practice Activity:	PHM2-L013

1. Williams received 725 votes, and Kamara received 485 votes.
2. A knife costs Le1,500.00 and a fork costs Le1,000.00
3. The two numbers are 17 and 47
4. The number is 65

Lesson Title:	Simultaneous linear and quadratic equations using substitution
Practice Activity:	PHM2-L014

1. $(2,-1)(-1,2)$
2. $(-3,1)(-2,-1)$
3. $(-1,-6)$
4. $(-9,90)$, $(-1,10)$
5. $(7,11)$, $(1,-5)$

Lesson Title: Simultaneous linear and quadratic equations using graphical methods - Part 1	
Practice Activity:	PHM2-L015

1. Tables of values:

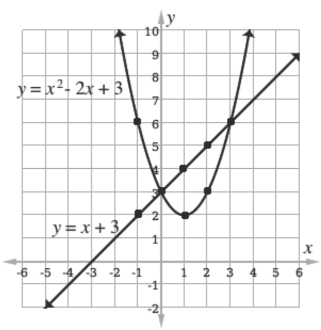
$y = x + 3$

x	-1	0	1	2	3
y	2	3	4	5	6

$y = x^2 - 2x + 3$

x	-1	0	1	2	3
y	6	3	2	3	6

Graph:



Solutions: $(0,3)$ and $(3,6)$

2. Tables of values:

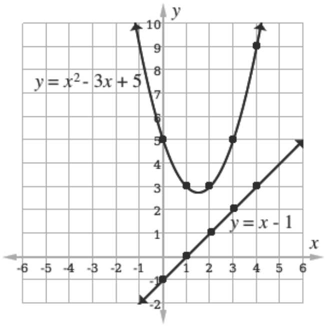
$y = x - 1$

x	0	1	2	3	4
y	-1	0	1	2	3

$y = x^2 - 3x + 5$

x	0	1	2	3	4
y	5	3	3	5	9

Graph:



No solution

3. Tables of values:

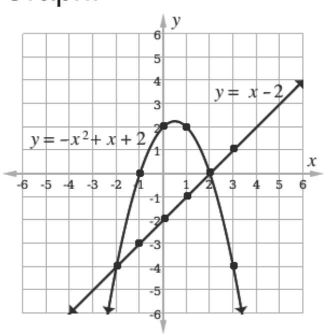
$y = x - 2$

x	-2	-1	0	1	2	3
y	-4	-3	-2	-1	0	1

$y = -x^2 + x + 2$

x	-2	-1	0	1	2	3
y	-4	0	2	2	0	-4

Graph:



Solutions: $(-2, -4)$ and $(2, 0)$

4. Tables of values:

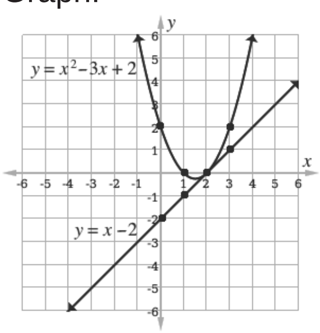
$y = x - 2$

x	0	1	2	3
y	-2	-1	0	1

$y = x^2 - 3x + 2$

x	0	1	2	3
y	2	0	0	2

Graph:



Solution: $(2, 0)$

Lesson Title: Simultaneous linear and quadratic equations using graphical methods - Part 2
Practice Activity: PHM2-L016

Note that your tables of values and graphs could have different plotted points, based on the points you choose.

1. Tables of values:

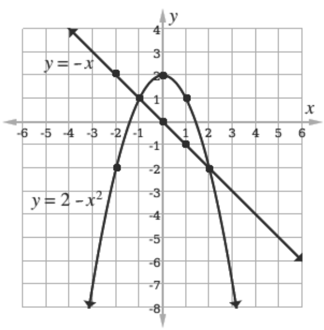
$y = x$

x	-2	-1	0	1	2
y	2	1	0	-1	-2

$y = 2 - x^2$

x	-2	-1	0	1	2
y	-2	1	2	1	-2

Graph:



Solutions: $(-1, 1)$ and $(2, -2)$

2. Tables of values:

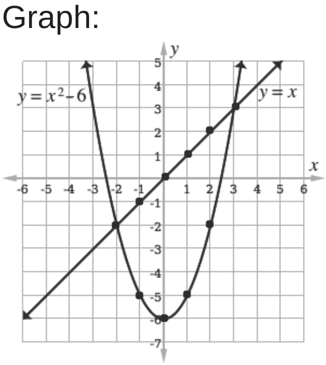
$y = x$

x	-2	-1	0	1	2	3
y	-2	-1	0	1	2	3

$y = x^2 - 6$

x	-2	-1	0	1	2	3
y	-2	-5	-6	-5	-2	3

Graph:



Solutions: $(-2, -2)$ and $(3, 3)$

3. Tables of values:

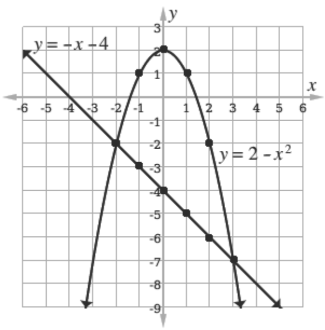
$y = -x - 4$

x	-2	-1	0	1	2	3
y	-2	-3	-4	-5	-6	-7

$y = 2 - x^2$

x	-2	-1	0	1	2	3
y	-2	1	2	1	-2	-7

Graph:



Solutions: $(-2, -2)$ and $(3, -7)$

4. Tables of values:

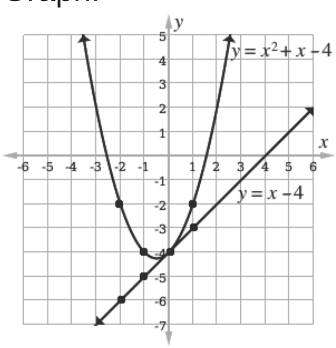
$y = x - 4$

x	-2	-1	0	1
y	-6	-5	-4	-3

$y = x^2 + x - 4$

x	-2	-1	0	1
y	-2	-4	-4	-2

Graph:



Solution: $(0, -4)$

Lesson Title:	Direct variation
Practice Activity:	PHM2-L017

1. $a = \frac{1}{5}b$
2. $y = 36$
3. a. $y = 15$
   b. $x = 8.25$
4. 145 customers
5. a. $R = 30V$
   b. $V = 11$ km/hr
   c. $R = 90$ Newtons

Lesson Title:	Indirect variation
Practice Activity:	PHM2-L018

1. a. $y = \frac{96}{x}$
   b. $y = 3$
2. a. $y = \frac{108}{x^2}$
   b. $y = 27$
3. a. $y = \frac{20}{x}$
   b. $x = 5$
   c. $y = \frac{1}{5}$
4. a. $s = \frac{60}{d}$
   b. $s = 3 \text{ m/second}$
   c. $d = 0.5 \text{ m}$

Lesson Title:	Joint Variation
Practice Activity:	PHM2-L019

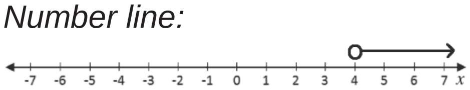
1. a. 72 b. 108 c. 32
2. Le $67{,}500.00$
3. a. $V = \frac{65T}{P}$ b. $V = 52 \text{ cm}^3$

Lesson Title:	Partial Variation
Practice Activity:	PHM2-L020

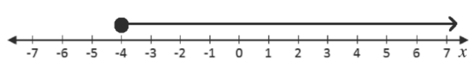
1. a. $c = 20{,}000 + 12{,}000u$, where c is cost and u is internet usage.
   b. He will pay Le 164,000.00
2. $M = 24$
3. a. $c = 100{,}000 + 8{,}000t$, where c is cost, and t is hours worked.
   b. Le 140,000.00

Lesson Title:	Inequalities on a number line
Practice Activity:	PHM2-L021

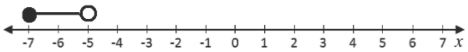
1. $x \geq 5$
2. $x \leq 2$
3. $-2 \leq x \leq 5$
4. $0 < x \leq 6$
5. $x < -3$
6. *Number line:*



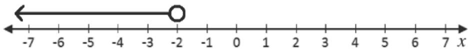
7. *Number line:*



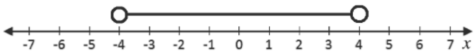
8. *Number line:*



9. *Number line:*



10. *Number line:*



Lesson Title:	Solutions of inequalities
Practice Activity:	PHM2-L022

1. $y < 5$
2. $x \geq 3$
3. $x \geq 1$
4. $y < -3$
5. $x \geq 1$
6. $y \geq 3$
7. $x < \frac{1}{2}$
8. $x > 1$
9. $x < 4$
10. $x > 3$

Lesson Title:	Distance formula
Practice Activity:	PHM2-L023

1. $|CD| = 6$
2. $|PQ| = \sqrt{10}$
3. $|ST| = 13$
4. $|AB| = 7\sqrt{2}$
5. $|GH| = 10$

Lesson Title:	Midpoint formula
Practice Activity:	PHM2-L024

1. $M = (1,4)$
2. $M = (-5,4)$
3. N is $(1,-2)$
4. $(x, y) = (4,5)$

Lesson Title:	Gradient of a Line
Practice Activity:	PHM2-L025

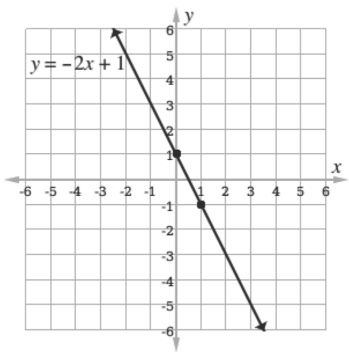
1.
   a. $m = 3$
   b. $m = 3\frac{1}{2}$

c. $m = 2$
d. $m = -2$
e. $m = -1\frac{1}{3}$

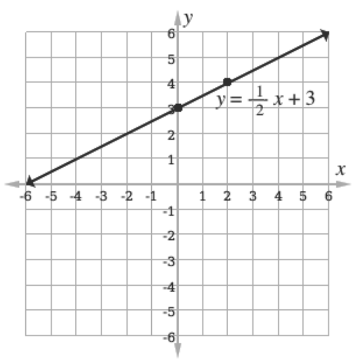
2. $m = 1\frac{1}{2}$
3. $m = -3$
4. $m = -\frac{1}{2}$

Lesson Title:	Sketching graphs of straight lines
Practice Activity:	PHM2-L026

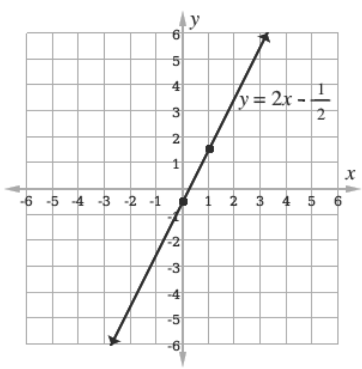
1.



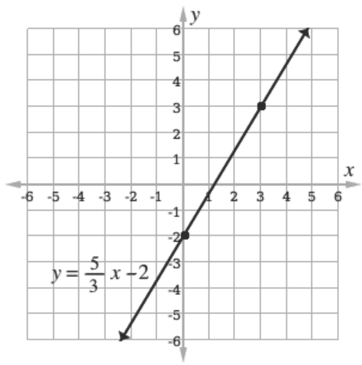
2.



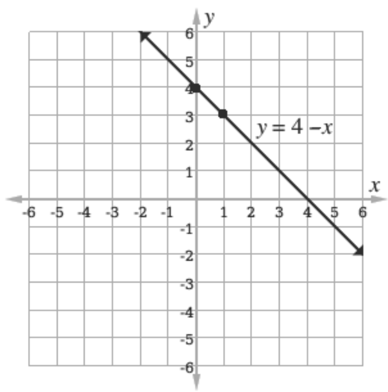
3.



4.



5.



Lesson Title:	Equation of a straight line
Practice Activity:	PHM2-L027

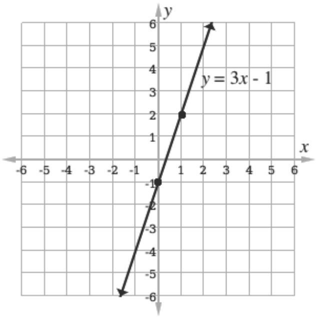
1. a. $y = -2x - 5$
   b. $y = 2x - 1$
   c. $y = x - 8$

2. a. $y = x + 1$
   b. $y = -\frac{5}{2}x + 2$
   c. $y = -\frac{2}{3}x - 2$

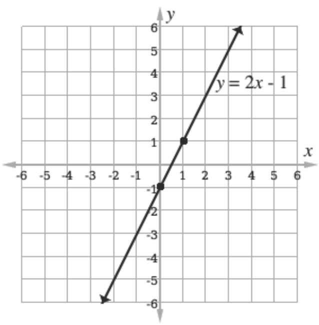
Lesson Title:	Practice with straight lines
Practice Activity:	PHM2-L028

1. Equation: $y = 3x - 1$

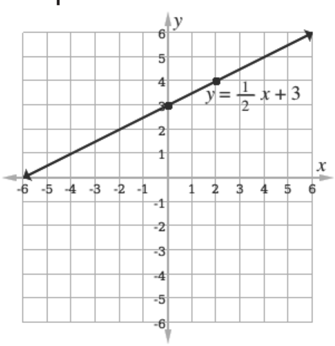
Graph:



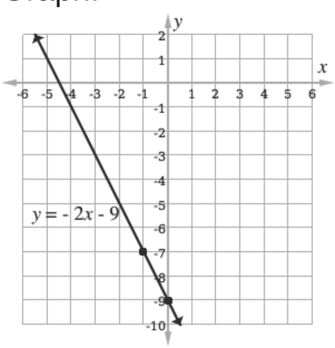
2. Equation: $y = 2x - 1$
Graph:



3. Equation: $y = \frac{1}{2}x + 3$
Graph:

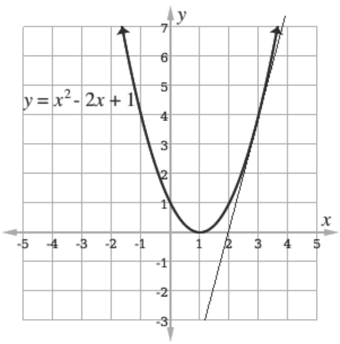


4. Equation: $y = -2x - 9$
Graph:



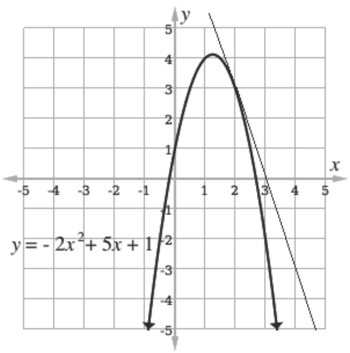
Lesson Title: Gradient of a curve – Part 1
Practice Activity: PHM2-L029

1. Graph:



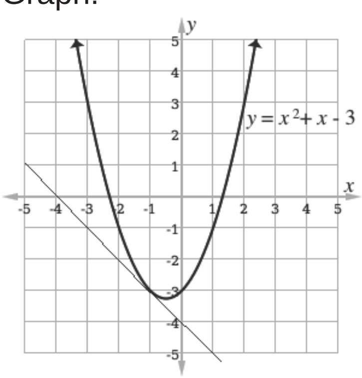
Using points $(3, 4)$ and $(2, 0)$ on the tangent line, we can find the gradient fo the curve at $x = 3$ is $m = 4$.

2. Graph:



Using points $(2, 3)$ and $(3, 0)$ on the tangent line, we can find the gradient of the curve at $x = 2$ is $m = -3$.

3. Graph:



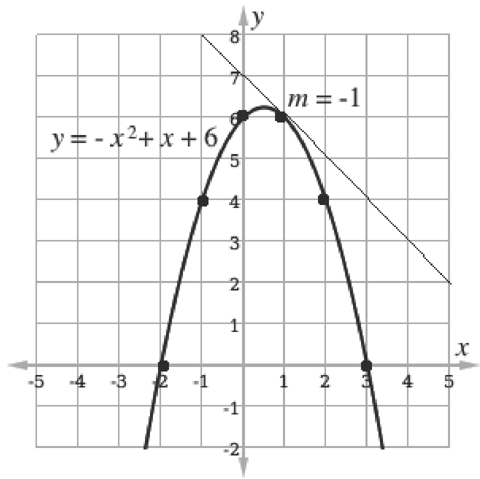
Using points $(-1, -3)$ and $(0, -4)$ on the tangent line, we can find the gradient of the curve at $x = -1$ is $m = -1$.

Lesson Title: Gradient of a curve – Part 1	
Practice Activity: PHM2-L030	

1. a. Completed table:

x	-2	-1	0	1	2	3
y	0	4	6	6	4	0

b. Graph:

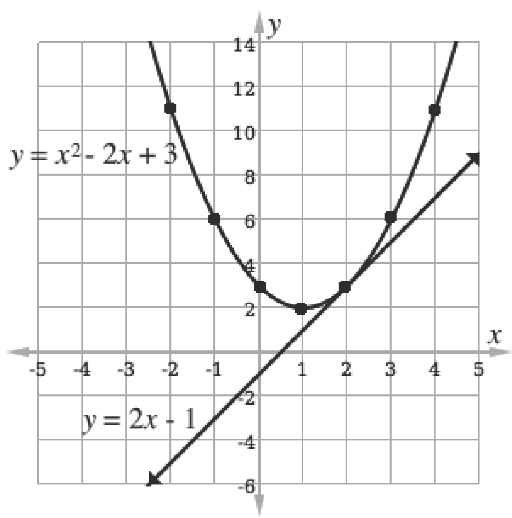


c. See above for graph of the tangent line; the slope is $m = -1$.

2.
a. Completed table:

x	-2	-1	0	1	2	3	4
y	11	6	3	2	3	6	11

b. Graph:



c. $y = 2$

d. See the graph above. The line can be graphed using any 2 points or the slope-intercept form.
e. It is the tangent of the curve at $x = 2$.
f. $m = 2$

Lesson Title: Simplification of algebraic fractions – Part 1	
Practice Activity: PHM2-L031	

1. $6y^6$
2. $11v^3w^2$
3. $5y^3$
4. $2xy^2$
5. $\frac{5w}{z}$
6. $\frac{4}{x^4}$
7. $8v^3w$

Lesson Title: Simplification of algebraic fractions – Part 2	
Practice Activity: PHM2-L032	

1. $\frac{x-4}{3}$
2. $\frac{2}{x}$
3. $\frac{4}{x}$
4. $\frac{2}{(x+8)}$
5. $6(3\mathrm{k}-1)$
6. $\frac{x-y}{3}$
7. $\frac{x-4}{x-3}$
8. $\frac{x-4}{2x-3}$

Lesson Title: Multiplication of algebraic fractions	
Practice Activity: PHM2-L033	

1. $\frac{2\mathrm{by}}{9\mathrm{a}^2x}$
2. $\frac{x+2}{x(x-2)}$
3. $\frac{x^2\mathrm{r}}{\mathrm{p}^2z}$
4. $\frac{(x+2)(y-1)}{\mathrm{y}(x+3)}$
5. $\frac{x}{x-7}$

Lesson Title Division of algebraic fractions	
Practice Activity: PHM2-L034	

1. $\frac{x+1}{7x}$
2. $\frac{12a}{5\mathrm{b}}$

3. $\frac{(a-5)(a-7)}{\mathrm{a}-2}$
4. $\frac{x+2}{10x}$
5. $\frac{60}{x+5}$
6. $\frac{x-2}{x(x+2)}$

Lesson Title: Addition and subtraction of algebraic fractions – Part 1
Practice Activity: PHM2-L035

1. $\frac{5ab+20b-1}{5ab}$
2. $\frac{c-a}{ac}$
3. $\frac{21b-4a}{3ab}$
4. $\frac{3a+5b}{6ab}$
5. $\frac{x-11}{6}$

Lesson Title: Addition and subtraction of algebraic fractions – Part 2
Practice Activity: PHM2-L036

1. $\frac{-x+5}{(x-2)(x-3)}$
2. $\frac{4(x-y)}{3}$
3. $\frac{2}{x-3}$
4. $\frac{2}{x-2}$
5. $\frac{(6x+1)(x+1)}{3x(2x-1)}$

Lesson Title: Substitution in algebraic fractions
Practice Activity: PHM2-L037

1. $\frac{5-\mathrm{a}}{7\mathrm{a}+4}$
2. $8\frac{1}{4}$
3. 21
4. $\frac{3\mathrm{w}-8}{8\mathrm{w}-5}$
5. d

Lesson Title: Equations with algebraic fractions
Practice Activity: PHM2-L038

1. $x = 3$
2. $y = -9$
3. $t = 2$
4. $x = -2\frac{1}{2}$
5. $q = 3, -\frac{1}{2}$
6. $y = 3$

Lesson Title: Undefined algebraic fractions	
Practice Activity: PHM2-L039	

1. $x = 4$
2. $x = 0, 1$
3. $x = -2, 10$
4. $x = -4, 5$
5. $x = -6$
6. $x = -\frac{4}{3}$
7. $p = 1, 4$
8. $r = -2, 2$

Lesson Title: Algebraic fraction problem solving	
Practice Activity: PHM2-L040	

1. $\frac{2y-5x}{4}$
2. $\frac{m-13n}{(m+2n)(m-3n)}$
3. $x = -5, 1$
4. $\frac{3u+1}{u-1}$
5. $\frac{11m}{10n}$
6. $\frac{x+2}{x(x-2)}$
7. $\frac{4x-5y+xy}{2xy}$
8. $x = \frac{3}{4}, -3$
9. $r = 1$
10. $\frac{2x-3}{4x-3}$

Lesson Title: Simple Statements
Practice Activity: PHM2-L041

1.

	Open	Closed
a:		✓
b:	✓	
c:		✓
d:		✓
e:	✓	
f:	✓	
g:		✓

2.

2.)	True	False	Unknown
a:		✓	
b:	✓		
c:		✓	
d:			✓
e:			✓
f:		✓	
g:	✓		

Lesson Title: Negation
Practice Activity: PHM2-L042

1. The negations are:

   $\sim g$: Peter is **not** one of the pupils.
   $\sim h$: The sum of the angles in a triangle is **not** 360°.
   $\sim i$: Multiplying 4.6×10^6 **does not give** $46{,}000{,}000$.
   $\sim j$: If x represents a number, then 5 more than the number is **not** given by $x - 5$.
   $\sim k$: In probability, the multiplication rule for $p(A\ and\ B)$ is **not** $p(A) + p(B)$
   $\sim l$: Two angles are **not** supplementary if their sum is equal to 90°.

Lesson Title: Compound statements	
Practice Activity: PHM2-L043	

1.

   a. A square has four sides and all its sides are equal.
      Simple statements:
      A square has four sides.
      A square's sides are equal.
   b. The squares of 2 and 8 are 4 and 64, respectively.
      Simple statements:
      The square of 2 is 4.
      The square of 8 is 64.
   c. The sum of two integers can be positive or negative.
      Simple statements:
      The sum of two integers can be positive.
      The sum of two integers can be negative.
   d. He is either a fool or a madman.
      Simple statements:
      He is a fool.
      He is a madman.

2. Note that some of the simple statements may be connected using different connecting words. These are example answers.
   a. He is eligible for university, but he doesn't have school fees.
   b. He doesn't drink or smoke.
   c. She will spend and invest her money.
   d. You must apologise or you will be punished.
   e. You can have tea or coffee.

Lesson Title: Implications	
Practice Activity: PHM2-L044	

1. If John has measles, then he is in the hospital. $(E \Longrightarrow F)$
2. If Amid plays football, then he scores many goals. $(A \Longrightarrow B)$
3. If Amie did her Maths assignment well, then she got the answers right. $(C \Longrightarrow D)$
4. If Alie goes for lunch, then it is lunch time. $(E \Longrightarrow F)$
5. If Peter wants some textbooks, then he goes to the bookshop. $(C \Longrightarrow D)$
6. If James scores very good marks in the interview, then he will get the job. $(A \Longrightarrow D)$

7. If Victor swims very well, then he will be able to cross to the other side of the river. $(X \Longrightarrow Y)$
8. If Paul enters the university to study engineering, then he passed with six credits in WASSCE. $(G \Longrightarrow H)$
9. If Joseph can interpret graphs very well, then he can answer questions with graphs. $(V \Longrightarrow W)$
10. If Francis knows the quadratic formula, then he can solve quadratic equation problems. $(S \Longrightarrow T)$

Lesson Title:	Conjunction and Disjunction
Practice Activity:	PHM2-L045

1. For two sub-statements of **Y**:
   S: David studied very hard.
   P: David passed his Mathematics exam.
   Symbols: $\mathrm{Y} = \mathrm{S} \wedge \mathrm{P}$
   Truth table:

S	P	$S \wedge P$
T	T	T
T	F	F
F	T	F
F	F	F

2. a. The truth table for the disjunction:

A	B	$A \vee B$
T	T	T
T	F	T
F	T	T
F	F	F

   b. The truth table for the conjunction:

A	B	$A \wedge B$
T	T	T
T	F	F
F	T	F
F	F	F

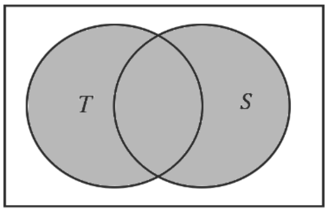
3.

a.

Truth Table:

T	S	$T \vee S$
T	T	T
T	F	T
F	T	T
F	F	F

Venn Diagram:

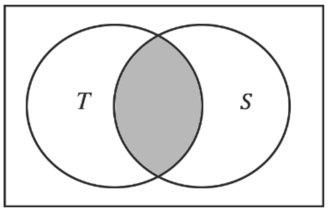


b.

Truth Table:

T	S	$T \wedge S$
T	T	T
T	F	F
F	T	F
F	F	F

Venn Diagram:



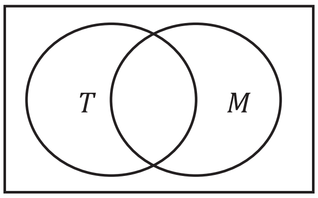
Lesson Title:	Equivalence and the Chain rule
Practice Activity:	PHM2-L046

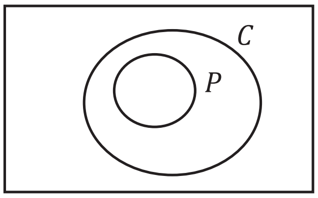
1. a. If Foday does not have a lot to eat, then he does not harvest his farm.
   b. If Sia does not live in Sierra Leone, then she does not live in Bo.
   c. If Issa is not African, then he is not from Sierra Leone.
2.

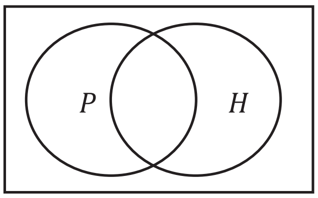
$X \Rightarrow Y$	If John practices solving Mathematics problems every day, then he is very good in Mathematics.
$Y \Rightarrow Z$	If John is a very good mathematician, then he can solve every problem in Mathematics.
$X \Rightarrow Z$	If John practices solving Mathematics problems every day, then he can solve every problem in Mathematics.
$\sim Y \Rightarrow \sim X$	If John is not a very good mathematician, then he does not practice solving Mathematics problems every day.
$\sim Z \Rightarrow \sim Y$	If John cannot solve every problem in Mathematics, then he is not a very good mathematician.
$\sim Z \Rightarrow \sim X$	If John cannot solve every problem in Mathematics, then he does not practice solving Mathematics problems every day.

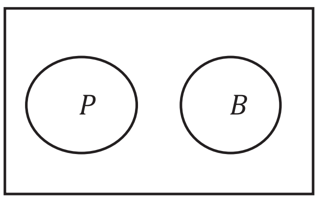
3. a. The statements are equivalent. $A \Leftrightarrow B$
   b. The statements are not equivalent. $S \Rightarrow T$, but the converse is not true because 'a shape has 4 sides' does not imply that the shape is a square. It could be another type of quadrilateral.
   c. The statements are equivalent. All shapes with 5 sides are pentagons, and all pentagons have 5 sides. $X \Leftrightarrow Y$

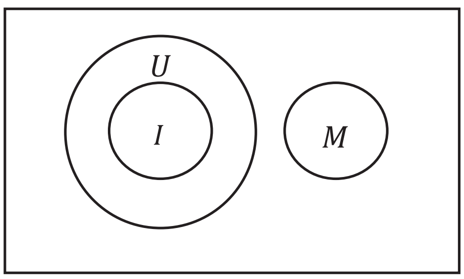
Lesson Title:	Venn diagrams
Practice Activity:	PHM2-L047

1. Venn diagrams:
   a.


   b.


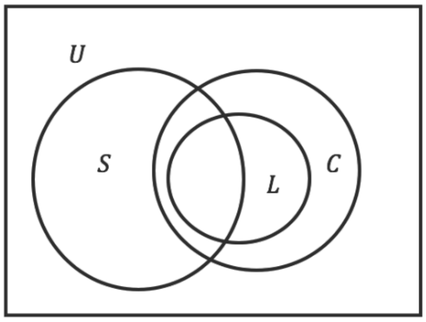
   c.


   d.


2. Venn diagram:


Lesson Title:	Validity
Practice Activity:	PHM2-L048

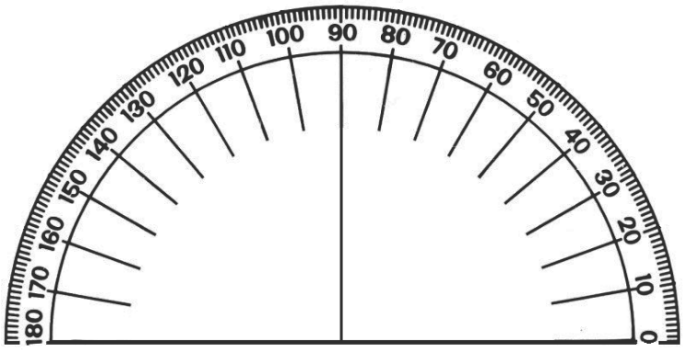
1.
   a. Illustration on a Venn diagram. Let U={all students}, S={strong students}, L={lazy students} and C={careless students}



   b. i. Not valid
      ii. Valid
      iii. Not valid
      iv. Not valid
      v. Valid
      vi. Not valid

Appendix I: Protractor

You can use a protractor to measure angles. If you do not have a protractor, you can make one with paper. Trace this protractor with a pen onto another piece of paper. Then, cut out the semi-circle using scissors.



GOVERNMENT OF SIERRA LEONE

FUNDED BY

UKaid
from the British people

IN PARTNERSHIP WITH

INTERNATIONAL RESCUE COMMITTEE

CAMBRIDGE EDUCATION

STRICTLY NOT FOR SALE