Ministry of Basic and Senior Secondary Education

Lesson Plans for

Senior Secondary *Mathematics*

SSS III

TERM I

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NOT FOR SALE

Please see final page for further information.

Foreword

These Lesson Plans and the accompanying Pupils' Handbooks are essential educational resources for the promotion of quality education in senior secondary schools in Sierra Leone. As Minister of Basic and Senior Secondary Education, I am pleased with the professional competencies demonstrated by the writers of these educational materials in English Language and Mathematics.

The Lesson Plans give teachers the support they need to cover each element of the national curriculum, as well as prepare pupils for the West African Examinations Council's (WAEC) examinations. The practice activities in the Pupils' Handbooks are designed to support self-study by pupils, and to give them additional opportunities to learn independently. In total, we have produced 516 lesson plans and 516 practice activities – one for each lesson, in each term, in each year, for each class. The production of these materials in a matter of months is a remarkable achievement.

These plans have been written by experienced Sierra Leoneans together with international educators. They have been reviewed by officials of my Ministry to ensure that they meet the specific needs of the Sierra Leonean population. They provide step-by-step guidance for each learning outcome, using a range of recognized techniques to deliver the best teaching.

I call on all teachers and heads of schools across the country to make the best use of these materials. We are supporting our teachers through a detailed training programme designed specifically for these new lesson plans. It is really important that the Lesson Plans and Pupils' Handbooks are used, together with any other materials they may have.

This is just the start of educational transformation in Sierra Leone as pronounced by His Excellency, the President of the Republic of Sierra Leone, Brigadier Rtd Julius Maada Bio. I am committed to continue to strive for the changes that will make our country stronger and better.

I do thank our partners for their continued support. Finally, I also thank the teachers of our country for their hard work in securing our future.

Mr. Alpha Osman Timbo

Minister of Basic and Senior Secondary Education

The policy of the Ministry of Basic and Senior Secondary Education, Sierra Leone, on textbooks stipulates that every printed book should have a lifespan of three years.

To achieve thus, DO NOT WRITE IN THE BOOKS.

Table of Contents

Introduction
to the Lesson Plans

These lesson plans are based on the National Curriculum and the West Africa Examination Council syllabus guidelines, and meet the requirements established by the Ministry of Basic and Senior Secondary Education.

1. The lesson plans will not take the whole term, so use spare time to review material or prepare for examinations.
2. Teachers can use other textbooks alongside or instead of these lesson plans.
3. Read the lesson plan before you start the lesson. Look ahead to the next lesson, and see if you need to tell pupils to bring materials for next time.
4. Make sure you understand the learning outcomes, and have teaching aids and other preparation ready – each lesson plan shows these using the symbols on the right.
5. If there is time, quickly review what you taught last time before starting each lesson.
6. Follow the suggested time allocations for each part of the lesson. If time permits, extend practice with additional work.
7. Lesson plans have a mix of activities for the whole class and for individuals or in pairs.
8. Use the board and other visual aids as you teach.
9. Interact with all pupils in the class – including the quiet ones.
10. Congratulate pupils when they get questions right! Offer solutions when they don't, and thank them for trying.

Learning outcomes

Preparation

KEY TAKEAWAYS FROM SIERRA LEONE’S PERFORMANCE IN WEST AFRICAN SENIOR SCHOOL CERTIFICATE EXAMINATION – GENERAL MATHEMATICS[1]

This section, seeks to outline key takeaways from assessing Sierra Leonean pupils’ responses on the West African Senior School Certificate Examination. The common errors pupils make are highlighted below with the intention of giving teachers an insight into areas to focus on, to improve pupil performance on the examination. Suggestions are provided for addressing these issues.

Common errors

1. Errors in applying principles of BODMAS
2. Mistakes in simplifying fractions
3. Errors in application of Maths learned in class to real-life situations, and vis-a-versa.
4. Errors in solving geometric constructions.
5. Mistakes in solving problems on circle theorems.
6. Proofs are often left out from solutions, derivations are often missing from quadratic equations.

Suggested solutions

1. Practice answering questions to the detail requested
2. Practice re-reading questions to make sure all the components are answered.
3. If possible, procure as many geometry sets to practice geometry construction.
4. Check that depth and level of the lesson taught is appropriate for the grade level.

[1] This information is derived from an evaluation of WAEC Examiners’ Reports, as well as input from their examiners and Sierra Leonean teachers.

FACILITATION STRATEGIES

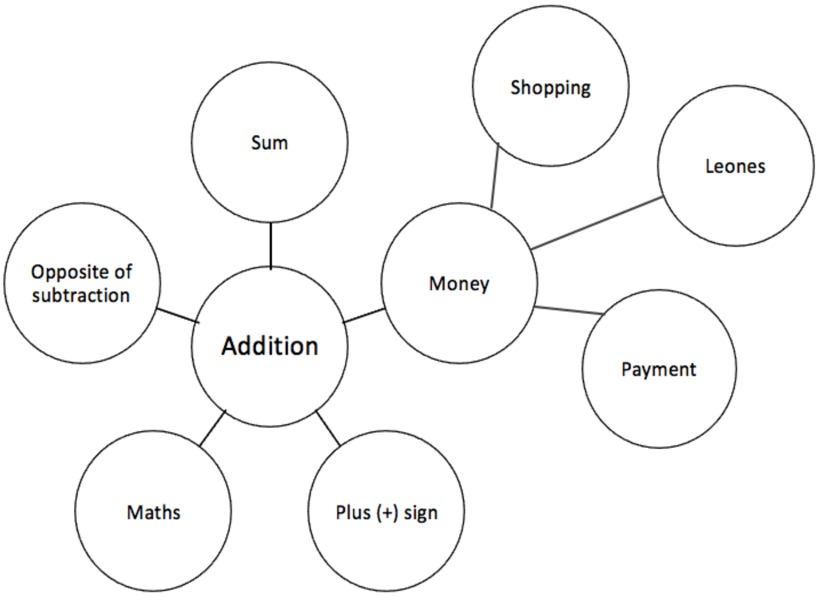
This section includes a list of suggested strategies for facilitating specific classroom and evaluation activities. These strategies were developed with input from national experts and international consultants during the materials development process for the Lesson Plans and Pupils' Handbooks for Senior Secondary Schools in Sierra Leone.

Strategies for introducing a new concept

- **Unpack prior knowledge:** Find out what pupils know about the topic before introducing new concepts, through questions and discussion. This will activate the relevant information in pupils' minds and give the teacher a good starting point for teaching, based on pupils' knowledge of the topic.
- **Relate to real-life experiences:** Ask questions or discuss real-life situations where the topic of the lesson can be applied. This will make the lesson relevant for pupils.
- **K-W-L:** Briefly tell pupils about the topic of the lesson, and ask them to discuss 'What I know' and 'What I want to know' about the topic. At the end of the lesson have pupils share 'What I learned' about the topic. This strategy activates prior knowledge, gives the teacher a sense of what pupils already know and gets pupils to think about how the lesson is relevant to what they want to learn.
- **Use teaching aids from the environment:** Use everyday objects available in the classroom or home as examples or tools to explain a concept. Being able to relate concepts to tangible examples will aid pupils' understanding and retention.
- **Brainstorming:** Freestyle brainstorming, where the teacher writes the topic on the board and pupils call out words or phrases related that topic, can be used to activate prior knowledge and engage pupils in the content which is going to be taught in the lesson.

Strategies for reviewing a concept in 3-5 minutes

- **Mind-mapping:** Write the name of the topic on the board. Ask pupils to identify words or phrases related to the topic. Draw lines from the topic to other related words. This will create a 'mind-map', showing pupils how the topic of the lesson can be mapped out to relate to other themes. Example below:



- **Ask questions:** Ask short questions to review key concepts. Questions that ask pupils to summarise the main idea or recall what was taught is an effective way to review a concept quickly. Remember to pick volunteers from all parts of the classroom to answer the questions.
- **Brainstorming:** Freestyle brainstorming, where the teacher writes the topic on the board and pupils call out words or phrases related that topic, is an effective way to review concepts as a whole group.
- **Matching:** Write the main concepts in one column and a word or a phrase related to each concept in the second column, in a jumbled order. Ask pupils to match the concept in the first column with the words or phrases that relate to in the second column.

Strategies for assessing learning without writing

- **Raise your hand:** Ask a question with multiple-choice answers. Give pupils time to think about the answer and then go through the multiple-choice options one by one, asking pupils to raise their hand if they agree with the option being presented. Then give the correct answer and explain why the other answers are incorrect.
- **Ask questions:** Ask short questions about the core concepts. Questions which require pupils to recall concepts and key information from the lesson are an effective way to assess understanding. Remember to pick volunteers from all parts of the classroom to answer the questions.
- **Think-pair-share:** Give pupils a question or topic and ask them to turn to seatmates to discuss it. Then, have pupils volunteer to share their ideas with the rest of the class.
- **Oral evaluation:** Invite volunteers to share their answers with the class to assess their work.

Strategies for assessing learning with writing

- **Exit ticket:** At the end of the lesson, assign a short 2-3 minute task to assess how much pupils have understood from the lesson. Pupils must hand in their answers on a sheet of paper before the end of the lesson.
- **Answer on the board:** Ask pupils to volunteer to come up to the board and answer a question. In order to keep all pupils engaged, the rest of the class can also answer the question in their exercise books. Check the answers together. If needed, correct the answer on the board and ask pupils to correct their own work.
- **Continuous assessment of written work:** Collect a set number of exercise books per day/per week to review pupils' written work in order to get a sense of their level of understanding. This is a useful way to review all the exercise books in a class which may have a large number of pupils.
- **Write and share:** Have pupils answer a question in their exercise books and then invite volunteers to read their answers aloud. Answer the question on the board at the end for the benefit of all pupils.
- **Paired check:** After pupils have completed a given activity, ask them to exchange their exercise books with someone sitting near them. Provide the answers, and ask pupils to check their partner's work.
- **Move around:** If there is enough space, move around the classroom and check pupils' work as they are working on a given task or after they have completed a given task and are working on a different activity.

Strategies for engaging different kinds of learners

- For pupils who progress faster than others:
  - Plan extension activities in the lesson.
  - Plan a small writing project which they can work on independently.
  - Plan more challenging tasks than the ones assigned to the rest of the class.
  - Pair them with pupils who need more support.

- For pupils who need more time or support:
  - Pair them with pupils who are progressing faster, and have the latter support the former.
  - Set aside time to revise previously taught concepts while other pupils are working independently.
  - Organise extra lessons or private meetings to learn more about their progress and provide support.
  - Plan revision activities to be completed in the class or for homework.
  - Pay special attention to them in class, to observe their participation and engagement.

<table>
<tr><td>Lesson Title: Algebraic Processes</td><td colspan="2">Theme: Review of SSS 2</td></tr>
<tr><td>Lesson Number: M3-L001</td><td>Class: SSS 3</td><td>Time: 40 minutes</td></tr>
<tr><td>Learning Outcome
By the end of the lesson, pupils will be able to solve simultaneous linear equations using elimination, substitution, or graphing.</td><td colspan="2">Preparation
1. Read note at the end of this lesson plan
2. Write on the board:
$5x + 2y = 16, 3x + y = 9$</td></tr>
</table>

Opening *(3 minutes)*

1. Ask pupils to raise their hands if they know what type of equation is on the board.
2. Select a pupil with raised hand to answer. (Example answers: linear equations, equations of straight lines, simultaneous linear equations)
3. Ask pupils to write the general form of the linear equations shown on the board.
4. Ask volunteers to give their answers. (Answer: $ax + by = c$)
5. Tell pupils that today they will learn to solve simultaneous linear equations using elimination, substitution, or graphing.

Teaching and Learning *(20 minutes)*

1. Explain that in the general equation:
   - x and y are variables;
   - x is the independent variable, y is the dependent variable.
   - The letters a and b are the co-efficients of x and y respectively.
   - c is a constant term.
2. Explain that the solution for 2 linear equations with 2 unknown variables are the values of the variables that satisfy both equations at the same time, i.e. simultaneously.
3. Start by solving the linear equations by **elimination.**
4. Explain that the first step is to label the equations (1) and (2).
5. Write on the board:

$$5x + 2y = 16 \quad (1)$$
$$3x + y = 9 \quad (2)$$

6. Ask the pupils what they notice about the coefficients of the variables in the two equations. (Example answers: they are all different; none of them have the same value)
7. Explain:
   - Eliminate one of the variables, say x, by making its co-efficients the same in both equations.
   - Multiply equation (1) throughout by the co-efficient of x in equation (2).
   - Multiply equation (2) throughout by the co-efficient of x in equation (1).
8. Ask pupils to follow the steps and copy the solution into their exercise books.
   Solution by elimination:

$15x + 6y$	$=$	48	(3)	Multiply equation (1) by 3 to get equation (3)
$15x + 5y$	$=$	45	(4)	Multiply equation (2) by 5 to get equation (4)
$0 + 1y$	$=$	3		Subtract equation (4) from equation (3) to
y	$=$	3		
$5x + 2y$	$=$	16	(1)	Re-write equation (1)
$5x + 2(3)$	$=$	16		Substitute $y = 3$ in equation (1)
$5x + 6$	$=$	16		
$5x$	$=$	$16 - 6$		Transpose 6
$5x$	$=$	10		
$\frac{5x}{5}$	$=$	$\frac{10}{5}$		Divide throughout by 5
x	$=$	2		

The solution is therefore $x = 2, y = 3$.

9. Explain:
   - The same procedure is used to eliminate y first, before solving for x.
   - The solution can be checked by substituting the values for x and y in **both** equations.
10. Allow 1 minute for pupils to check the solution. Show check on the board.

Check:

$5x + 2y$	$=$	16	(1)	$3x + y$	$=$	9	(2)
$5(2) + 2(3)$	$=$	16		$3(2) + 3$	$=$	9	
$10 + 6$	$=$	16		$6 + 3$	$=$	9	
16	$=$	16		9	$=$	9	
LHS	$=$	RHS		LHS	$=$	RHS	

11. We will now use the method of **substitution** to solve for x and y.
12. Explain: Make one variable the subject of the equation. If there is a variable with a coefficient of 1 (no written coefficient), choose that equation and make that variable the subject. Refer to the result as the changed equation.

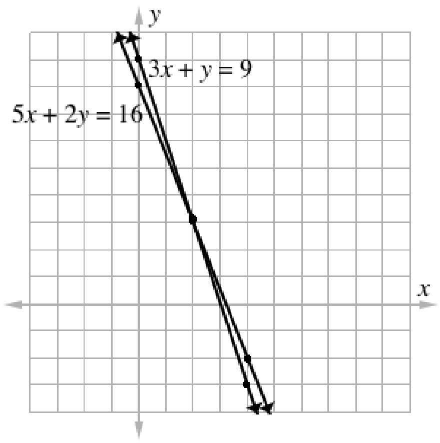
Solution by substitution:

$5x + 2y$	$=$	16	(1)	
$3x + y$	$=$	9	(2)	
y	$=$	$9 - 3x$	(3)	Make y the subject of equation (2)
$5x + 2(9 - 3x)$	$=$	16		Substitute $y = 9 - 3x$ in equation (1)
$5x + 18 - 6x$	$=$	16		
$5x - 6x$	$=$	$16 - 18$		
$-x$	$=$	-2		
x	$=$	2		
y	$=$	$9 - 3x$	(3)	Re-write equation (3)
y	$=$	$9 - 3(2)$		Substitute $x = 2$ in equation (3)
y	$=$	$9 - 6$		
y	$=$	3		

The solution is, as before, $x = 2, y = 3$.

13. We will now use a **graphical method** to solve for x and y.
14. Explain that we will create a table of values for the 2 graphs. Create each table of values by substituting the x-value into each equation and solving for the y-value.

Solution by graphing:



$5x + 2y = 16$

x	0	2	4
y	8	3	-2

$3x + y = 9$

x	0	2	4
y	9	3	-3

15. Explain that the point of intersection of the two linear equations is the solution of the simultaneous linear equations. The solution is, as before, $x = 2, y = 3$.

Practice *(15 minutes)*

1. Write the questions below on the board. Ask pupils to work individually to answer the questions.
2. Solve the simultaneous linear equations using a different method each time. Check by substituting the answers in both equations
   a. $2x + y = 5$, $3x - 2y = 4$
   b. $2x - y = -1$, $x + 2y = -3$
   c. $3x - y = 20$, $x + 6y = -1$
3. Ask volunteers to come to the board to show their solutions.
   The rest of the class should check their solutions and correct any mistakes.

Solutions:

a. $2x + y = 5, 3x - 2y = 4$

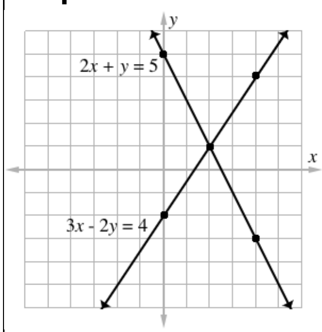
Elimination

$2x + y = 5$ (1)
$3x - 2y = 4$ (2)
$4x + 2y = 10$ (3) (1) × 2
$3x - 2y = 4$ (4) (2) × 1
$7x + 0 = 14$ Add
$x = 2$
$2x + y = 5$ (1)
$2(2) + y = 5$
$4 + y = 5$
$y = 5 - 4$
$y = 1$
$x = 2, y = 1$

Substitution

$2x + y = 5$ (1)
$3x - 2y = 4$ (2)
$y = 5 - 2x$ (3)
$3x - 2(5 - 2x) = 4$
$3x - 10 + 4x = 4$
$7x = 14$
$x = 2$
$y = 5 - 2x$ (3)
$= 5 - 2(2)$
$= 5 - 4$
$y = 1$
$x = 2, y = 1$

Graph



The point of intersection gives the solution: $x = 2, y = 1$.

b. $2x - y = -1$, $x + 2y = -3$

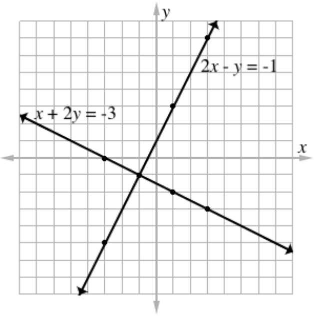
Elimination

$2x - y = -1$ (1)
$x + 2y = -3$ (2)
$2x - y = -1$ (3) (1) × 1
$2x + 4y = -6$ (4) (2) × 2
$0x - 5y = 5$ Subtract
$x = -1$
$2x - y = -1$ (1)
$2(-1) - y = -1$
$-2 - y = -1$
$-y = -1 + 2$
$-y = 1$
$y = -1$
$x = -1, y = -1$

Substitution

$2x - y = -1$ (1)
$x + 2y = -3$ (2)
$y = 2x + 1$ (3)
$x + 2(2x + 1) = -3$
$x + 4x + 2 = -3$
$5x = -5$
$x = -1$
$y = 2x + 1$ (3)
$= 2(-1) + 1$
$= -2 + 1$
$y = -1$
$x = -1, y = -1$

Graph



The point of the intersection gives the solution:
$x = -1, y = -1$

c. $3x - y = 20$, $x + 6y = -1$

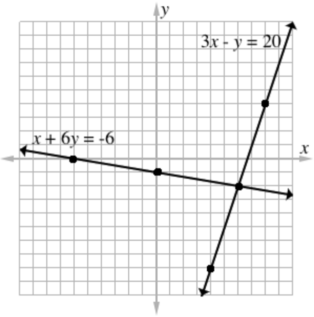
Elimination

$3x - y = 20$ (1)
$x + 6y = -6$ (2)
$3x - y = 20$ (3) (1) × 1
$3x + 18y = -18$ (4) (2) × 3
$0 - 19y = 38$ Subtract
$y = -2$
$3x - y = 20$ (1)
$3x - (-2) = 20$
$3x + 2 = 20$
$3x = 20 - 2$
$3x = 18$
$x = 6$
$x = 6, y = -2$

Substitution

$3x - y = 20$ (1)
$x + 6y = -6$ (2)
$x = -6 - 6y$ (3)
$3(-6 - 6y) - y = 20$
$-18 - 18y - y = 20$
$-19y = 38$
$y = -2$
$x = -6 - 6y$ (3)
$= -6 - 6(-2)$
$= -6 + 12$
$x = 6$
$x = 6, y = -2$

Graph



The point of the intersection gives the solution:
$x = 6, y = -2$

Closing *(2 minutes)*

1. Ask pupils to think about all three methods and choose the method they prefer to solve for simultaneous linear equations. Ask for volunteers to tell the class their favourite method and why they like it the best.
2. Tell pupils the next lesson will be finding the equation of a line given 2 points and drawing its graph

[NOTE]

The first week of SSS 3 is spent revising selected topics from SSS 2. This is the first revision lesson. You may find additional explanations and examples of these topics in the SSS 2 Lesson Plans if needed.

Lesson Title: Algebraic Processes	**Theme:** Review of SSS 2	
Lesson Number: M3-L002	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to find the equation of a line given two points and graph it on the Cartesian plane.	**Preparation** Write on the board: $y = 4x - 3$	

Opening *(3 minutes)*

1. Ask pupils to raise their hands if they know what type of equation is on the board.
2. Select a pupil with a raised hand to answer. (Example answers: linear equation, equation of a straight line)
3. Ask a volunteer to give the general form of the equation. (Answer: $y = mx + c$)
4. Ask volunteers to explain what each letter represents. (Answer: y is the dependent variable; x is the independent variable; m is the gradient of the line; c is a constant term indicating the y-intercept where the line cuts the y-axis.)
5. Tell pupils that they will learn to find the equation of a line given two points and graph it on the Cartesian plane.

Teaching and Learning *(20 minutes)*

1. Explain:
   - For any 2 points given by (x_1, y_1), (x_2, y_2), the equation of the straight line through the 2 points is given by $y - y_1 = m(x - x_1)$ where $m = \frac{y_2 - y_1}{x_2 - x_1}$.
2. Write on the board: Find the equation of the line through the points with co-ordinates $(2,5)$ and $(3,7)$. Draw the graph of the straight line.
3. Find the equation of the line, explaining each step to pupils:
   Solution:

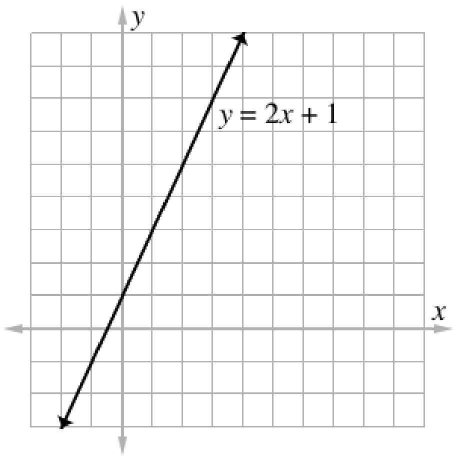
$m = \frac{y_2 - y_1}{x_2 - x_1}$	
$x_1 = 2;\ y_1 = 5;\ x_2 = 3;\ y_2 = 7$	Assign variables to the co-ordinates of points $(2,5)$ and $(3,7)$.
$m = \frac{7-5}{3-2}$	Substitute
$= \frac{2}{1}$	Simplify
$m = 2$	
$y - y_1 = m(x - x_1)$	Equation of a straight line
$y - 5 = 2(x - 2)$	Substitute co-ordinates of point $(2,5)$
$y - 5 = 2x - 4$	
$y = 2x - 4 + 5$	Transpose -5
$y = 2x + 1$	Equation of the line

Check:

7	$=$	$2(3)+1$	Substitute co-ordinates of the second point $(3,7)$
7	$=$	$6+1$	
7	$=$	7	
LHS	$=$	RHS	

Therefore $y = 2x + 1$ is the line through the points with co-ordinates $(2,5)$ and $(3,7)$.

4. Draw the graph of the equation $y = 2x + 1$ on the board, using $m = 2$ and $c = 1$:
   - Identify and plot the y-intercept, $c = 1$.
   - Remind pupils that $m = \frac{rise}{run}$. In this case, $m = 2 = \frac{2}{1}$. From the y-intercept, count up (rise) 2 points in the y-direction. Count to the right (run) 1 point in the x-direction. Plot this point (1, 3) and draw a line that passes through this and the y-intercept.



5. Solve another example problem on the board.
6. Involve pupils by asking for volunteers to tell the class the next step of the solution. Politely acknowledge all answers, even incorrect ones.
7. Write on the board: Find the equation of the line through the points with co-ordinates $(-1,-5)$ and $(5,4)$. Draw the graph of the straight line.
8. Find the equation of the line, explaining each step to pupils:
   Solution:

m	$=$	$\frac{y_2-y_1}{x_2-x_1}$	
$x_1 = -1;\ y_1$	$=$	$-5;\ x_2 = 5;\ y_2 = 4$	Assign variables to the co-ordinates of points $(-1,-5)$ and $(5,4)$
m	$=$	$\frac{4-(-5)}{5-(-1)}$	Substitute
	$=$	$\frac{4+5}{5+1} = \frac{9}{6} = \frac{3}{2}$	Simplify
m	$=$	$\frac{3}{2}$	
$y - y_1$	$=$	$m(x - x_1)$	Equation of a straight line
$y - (-5)$	$=$	$\frac{3}{2}(x - (-1))$	Substitute co-ordinates of point $(-1,-5)$

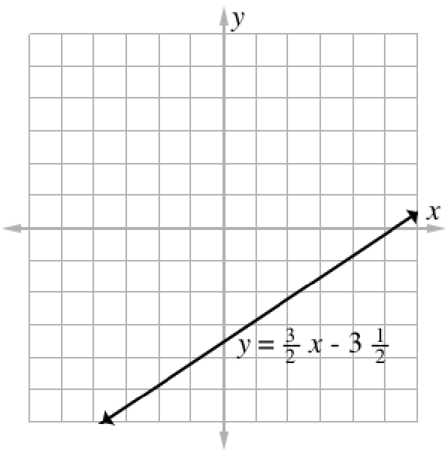
$$
\begin{aligned}
y+5 &= \tfrac{3}{2}(x+1) \\
y &= \tfrac{3}{2}x+\tfrac{3}{2}-5 && \text{Transpose 5} \\
&= \tfrac{3}{2}x-\tfrac{7}{2} && \text{Simplify } \left(\tfrac{3}{2}-5=\tfrac{3-10}{2}=\tfrac{-7}{2}=-\tfrac{7}{2}\right) \\
y &= \tfrac{3}{2}x-3\tfrac{1}{2} && \text{Equation of the line}
\end{aligned}
$$

Check:

$$
\begin{aligned}
4 &= \tfrac{3}{2}\times 5-3\tfrac{1}{2} && \text{Substitute co-ordinates of point } (5,4) \\
4 &= \tfrac{3\times 5}{2}-\tfrac{7}{2}=4 && \text{Simplify } \left(\tfrac{(3\times 5)-7}{2}=\tfrac{15-7}{2}=\tfrac{8}{2}=4\right) \\
4 &= 4 \\
\text{LHS} &= \text{RHS}
\end{aligned}
$$

Therefore $y=\frac{3}{2}x-3\frac{1}{2}$ is the line through the points with co-ordinates $(-1,-5)$ and $(5,4)$.

9. Draw the graph of $y=\frac{3}{2}x-3\frac{1}{2}$ on the board, using $m=\frac{3}{2}$ and $c=-3\frac{1}{2}$:



Practice *(15 minutes)*

1. Write the questions below on the board. Ask pupils to work individually to answer the questions.
2. Find the equation of the line through the points with the co-ordinates. Check your solutions. Draw the graph of the straight line.
   d. $(2,3)$ and $(3,5)$ e. $(-4,0)$ and $(1,3\frac{3}{4})$
3. Ask volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.
   Solutions:

a. $(2,3)$ and $(3,5)$

$m=\frac{y_2-y_1}{x_2-x_1}$

$x_1=2;\ y_1=3;$ $x_2=3;\ y_2=5$ Assign variables

$m=\frac{5-3}{3-2}$ Substitute

b. $(-4,0)$ and $(1,3\frac{3}{4})$

$m=\frac{y_2-y_1}{x_2-x_1}$

$x_1=-4;\ y_1=0;$ $x_2=1;\ y_2=3\frac{3}{4}$ Assign variables

$= \frac{2}{1}$ Simplify

$m = 2$

$y - y_1 = m(x - x_1)$

$y - 3 = 2(x - 2)$ Substitute $(2,3)$

$y - 3 = 2x - 4$

$y = 2x - 4 + 3$ Transpose -3

$y = 2x - 1$ Equation of the

Check:

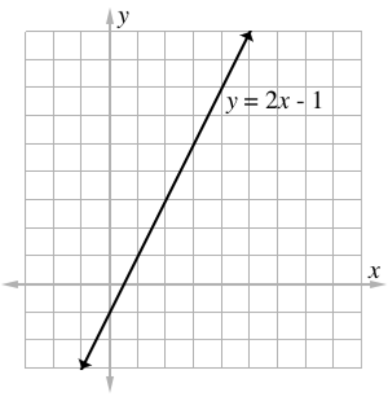
$5 = 2(3) - 1$ Substitute $(3,5)$

$5 = 6 - 1 = 5$

LHS $=$ RHS

The equation of the line is $y = 2x - 1$

Graph of line $y = 2x - 1$:



$m = \frac{\frac{15}{4} - 0}{1 - (-4)}$ Substitute $\left(3\frac{3}{4} = \frac{15}{4}\right)$

$= \frac{\frac{15}{4}}{5} = \frac{15}{4\times5} = \frac{15}{20}$ Simplify

$m = \frac{3}{4}$

$y - y_1 = m(x - x_1)$

$y - 0 = \frac{3}{4}(x - (-4))$ Substitute $(-4,0)$

$y = \frac{3}{4}(x + 4)$

$y = \frac{3}{4}x + 3$ Equation of the line

Check:

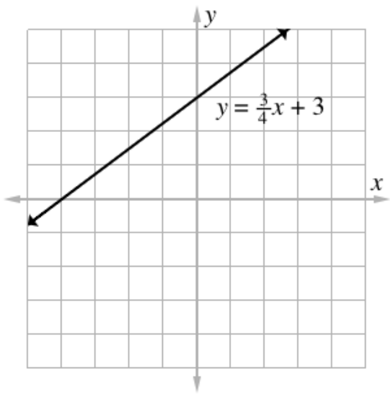
$3\frac{3}{4} = \frac{3}{4}(1) + 3$ Substitute $(1, 3\frac{3}{4})$

$3\frac{3}{4} = 3\frac{3}{4}$

LHS $=$ RHS

The equation of the line is $y = \frac{3}{4}x + 3$

Graph of line $y = \frac{3}{4}x + 3$:



Closing *(2 minutes)*

1. Ask for volunteers to tell the class one thing they understand better after this lesson.
2. Tell pupils the next lesson will be on angles, triangles, quadrilaterals and other polygons.

<table>
<tr><td>Lesson Title: Geometry</td><td colspan="2">Theme: SSS 2 Review</td></tr>
<tr><td>Lesson Number: M3-L003</td><td>Class: SSS 3</td><td>Time: 40 minutes</td></tr>
<tr><td>Learning Outcomes
By the end of the lesson, pupils will be able to:
1. Calculate missing angle measures and side lengths of triangles.
2. Calculate interior and exterior angles of triangles, quadrilaterals and polygons.</td><td colspan="2">Preparation
Write the problems found at the end of this lesson plan under QUESTIONS FOR TEACHING AND LEARNING on the board.</td></tr>
</table>

Opening *(4 minutes)*

1. Ask pupils to identify the hypotenuse on the right angled triangles on the board,
2. Invite volunteers to answer. (Answer: Line BC in triangle X; AC in triangle Y).
3. Remind pupils that in a right-angled triangle, the hypotenuse is always the line opposite the right-angle.
4. Explain:
   - In this lesson, they will review 2 very important concepts in Mathematics.
   - They will calculate the missing angle measures and side lengths of triangles.
   - They will also calculate interior and external angles of triangles and polygon.

Teaching and Learning *(25 minutes)*

1. Explain that missing angles and side lengths of right-angled triangles can be solved using two methods.
2. Ask pupils if they can recall what these methods are.
3. Invite a volunteer to answer. (Answer: Pythagoras' Theorem and Trigonometry)
4. Ask pupils to look at the 2 right-angled triangles and the information provided. How can we decide which method to use to find the missing value?
5. Allow 1 minute for pupils to discuss and share ideas.
6. Select volunteers to answer: (Example answers: if the problem is to find a missing angle, use Trigonometry; if it is to find a missing side and an angle is provided, use Trigonometry, if there is no angle use Pythagoras)
7. Explain:
   - This is just a general guide. Always read the question properly before deciding which method is the most suitable.
   - In complex diagrams, we may need to use both methods to find all the information requested.
8. Solve problems a and b from the end of this lesson on the board, explaining each step:

a. Find missing length when hypotenuse and 1 side is given – triangle X:

$$a^2 = b^2 + c^2$$
$$b^2 = a^2 - c^2$$
$$b^2 = 7^2 - 3^2$$
$$= 49 - 9 = 40$$
$$b = \sqrt{40}$$
b = 6.32 cm to 2 decimal places

b. Find missing angle when hypotenuse and 1 side is given – triangle Y

$$\sin a = \frac{\text{opposite}}{\text{hypotenuse}}$$
$$= \frac{6}{8} = 0.75$$
a = 49 48.59
a = 49 49° to the nearest degree

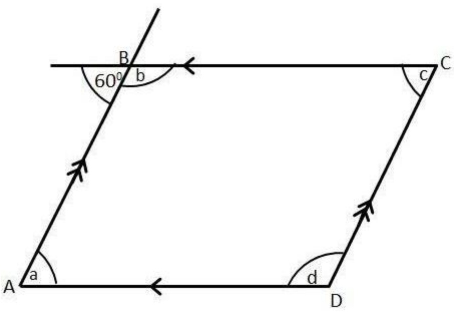
9. Ask pupils to work together to answer the next question:
ABC is a right-angled triangle. ∠ABC = 90°. $|\mathrm{AC}| = 10$ cm. $|\mathrm{BC}| = 6$ cm. Find: ∠ACB to the nearest degree; the length of side AB.
10. Write the solution on the board.
Solution:
Start by drawing a diagram of the situation.
(You can modify triangle Y on the board.)

$$\cos \angle \text{ACB} = \frac{\text{adjacent}}{\text{hypotenuse}}$$
$$= \frac{6}{10}$$
$$= 0.6$$
$$\angle \text{ACB} = 53.13$$
∠ACB = 53° to the nearest degree

Let $a = |\mathrm{AC}|, b = |\mathrm{AB}|, c = |\mathrm{BC}|$

$$a^2 = b^2 + c^2$$
$$b^2 = a^2 - c^2$$
$$b^2 = 10^2 - 6^2$$
$$= 100 - 36 = 64$$
$$b = \sqrt{64}$$
b = 8 cm

11. Draw the shape shown on the right on the board.
12. Write on the board: ABCD is a parallelogram. Find the size of each angle of the parallelogram.
13. Tell pupils we will use angle properties of polygons to find the missing angles.



Solution:

$a = 60°$ Alternate angles on parallel lines BC and AD are equal

$b + 60° = 180°$
$b = 180° - 60°$ Angles in a straight line add up to 180°
$b = 120°$

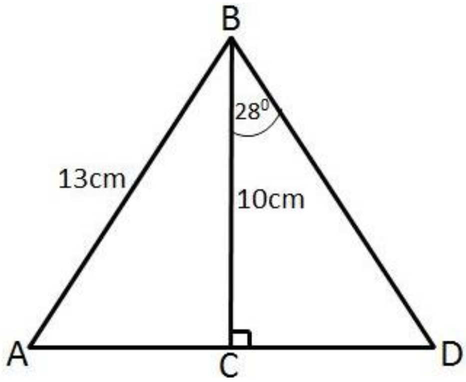
c	$=$	$60°$	Corresponding angles on parallel lines BA and CD are equal
$a+b+c+d$	$=$	$360°$	Angle sum of a quadrilateral is $360°$
$60°+120°+60°+d$	$=$	$360°$	
$240°+d$	$=$	$360°$	
d	$=$	$360°-240°$	
d	$=$	$120°$	

Practice *(10 minutes)*

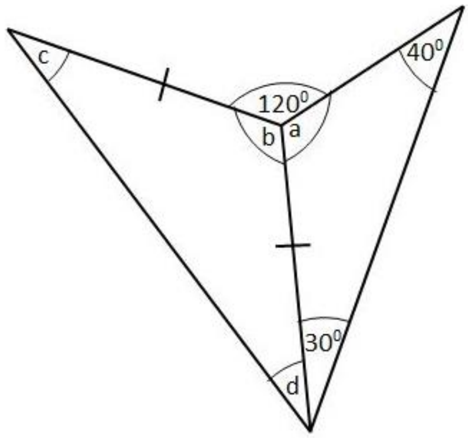
1. Write the questions below on the board.
   Ask pupils to work individually to answer the questions.
2. Solve the following problems.
   a. In the given triangle, ACD is a straight line and ∠BCD = 90°.

      Calculate:
      i. Length of AC
      ii. Length of CD

      Give answers to 1 decimal place.



   b. Find the angles a, b, c and d in the given diagram. Give reasons for your answers.



3. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.
4. **Solutions:**

a. i.

$\|AB\|^2$	$=$	$\|BC\|^2+\|AC\|^2$	Use Pythagoras' Theorem
$\|AC\|^2$	$=$	$\|AB\|^2-\|BC\|^2$	
$\|AC\|^2$	$=$	13^2-10^2	Substitute $\|AB\|=13, \|BC\|=10$
	$=$	$169-100$	
	$=$	69	
$\|AC\|$	$=$	$\sqrt{69}$	
	$=$	8.3066	
$\|AC\|$	$=$	8.3 cm to 1 decimal place	

ii. $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$

$\tan 28 = \frac{|CD|}{|BC|}$

$|CD| = |BC| \times \tan 28$

$= 10 \times 0.5317$ Substitute $|BC| = 10, \tan 28 = 0.5317$

$= 5.317$

$= 5.3$ cm to 1 d.p.

b. $40° + 30° + a = 180°$ Angle sum of a triangle is $180°$

$70° + a = 180°$

$a = 180° - 70°$

$a = 110°$

$120° + a + b = 360°$ Angle sum around a point is $360°$

$120° + 110° + b = 360°$

$230° + b = 360°$

$b = 360° - 230°$

$b = 130°$

$c + b + d = 180°$ Angle sum of a triangle is $180°$

$c + 130° + c = 180°$ Base angles in an isosceles triangle are equal $\therefore c = d$

$2c + 130 = 180°$

$2c = 180° - 130°$

$2c = 50°$

$c = 25°$

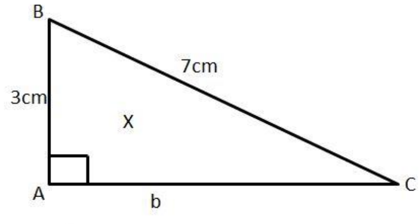
$\therefore d = 25°$

Closing *(1 minute)*

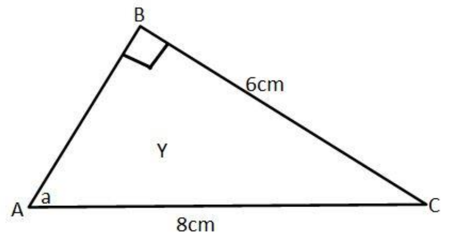
1. For homework, have pupils do the practice activity PHM3-L003 in the Pupil Handbook.

[QUESTIONS FOR TEACHING AND LEARNING]

a. Find the length of AC correct to 2 decimal places in the following right-angled triangle.



b. Find angle a in the following right-angled triangle. Give your answer to the nearest degree.



Lesson Title: Statistics	Theme: SS 2 Review	
Lesson Number: M3-L004	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to: 1. Present and interpret data. 2. Calculate measures of central tendency.	**Preparation** Write the problems found at the end of this lesson plan on the board.	

Opening *(4 minutes)*

1. Explain:
   - Two types of data can be represented using graphs and diagrams.
   - One type of data is called qualitative data.
2. Invite a volunteer to tell you the other type of data. (Answer: quantitative data)
3. Explain: One type of diagram used to represent data is called a bar chart.
4. Invite volunteers to give other diagrams used to represent data. (Example answers: pie charts, frequency tables, histograms, frequency polygons, cumulative frequency graphs)
5. Tell pupils that during this lesson, we will review two ways of representing data. We will also review the ways of calculating measures of central tendency.

Teaching and Learning *(20 minutes)*

1. Explain that we will:
   - Represent grouped data in frequency tables and histograms.
   - Calculate the mean, median and mode of the data.
2. Read question a. on the board.
3. Explain that these are all the different types of questions that are commonly asked so we are going to work through the solution step-by-step.
   Solution:
   [NOTE: Prompts have been given to invite volunteers to answer questions to complete the frequency table and other values. Write the answers in the table as they are answered by pupils.]
4. **Step 1.** i. Draw the frequency table.
   - Draw the columns for Height, Tally and Frequency only.
   - Fill in the class intervals in the Height column.
5. **Step 2.** Tally the number of pupils in each class interval.
   - Invite volunteers to tell you in which class interval a particular height belongs.
   - Make a tally mark for each height as shown in the table. The 5^{th} tally mark crosses the others for ease of counting.
6. **Step 3.** Write down the total number of pupils or frequency for each class interval**.**

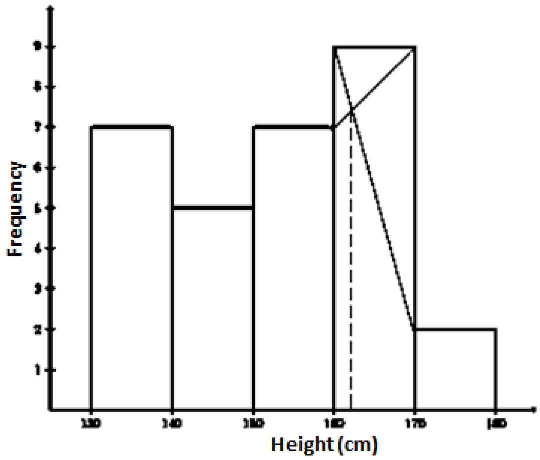
- Invite volunteers to tell you the total number of pupils or frequency for each class interval.

7. **Step 4.** Add the frequencies together.
   - Invite a volunteer to add the frequencies.. Make sure $\sum f$ =30.

The first 3 columns of the table below should now be completely filled in.

Height (cm)	**Tally**	**Frequency f**	**Mid-point x**	fx
130 – 139	~~IIII~~ II	7	134.5	941.5
140 – 149	~~IIII~~	5	144.5	722.5
150 – 159	~~IIII~~ II	7	154.5	1081.5
160 – 169	~~IIII~~ IIII	9	164.5	1480.5
170 – 179	II	2	174.5	349
		$\sum f$ =**30**		$\sum fx =$ **4575**

8. **Step 5.** ii. Draw the histogram shown at right**.**
   Do not draw the lines for estimating the modal mark.



9. **Step 6.** Extend the frequency table. Find the mid-point of each class interval.
   - To calculate the mean, add columns for the mid-point x and the product fx to the frequency table.
   - Invite volunteers to give the mid-point x for each class interval – see example below.

$$\text{Mid-point } x \text{ for class interval } 130 - 139 = \frac{130 + 139}{2} = 134.5$$

10. **Step 7.** Calculate the product $f \times x$ for each class interval. Calculate the sum of the products.
    - Invite volunteers to give the product $f \times x$ for each class interval.
    - Invite a volunteer to give the sum of the products.

11. **Step 8.** iii. Calculate the mean height.
    - Invite a volunteer to give the formula to calculate the mean height. (Answer: $\frac{\sum fx}{\sum f}$)
    - Ask pupils to calculate the mean height in their exercise books.
    - Invite a volunteer to show their answer on the board. The rest of the class should check their solutions and correct any mistakes.

$$\text{Mean} = \frac{\sum fx}{\sum f} = \frac{4,575}{30}$$

$= \ 152.5$ cm

12. **Step 9.** iv. Estimate the median class of the distribution**.**
    - Explain:
      - For grouped data, the median is given by the data value for $\frac{\sum f}{2}$.
      - $\sum f = 30$ for our distribution. So, $\frac{\sum f}{2} = 15$. The median is given by the class interval, which contains the 15th pupil.
    - Invite a volunteer to say which class interval this is. (Answer: class interval of heights150 – 159 cm. Since there are 12 pupils in the first 2 class intervals, we have to go to the 3rd class interval before we find the 15th pupil.)
13. **Step 10.** v. Estimate the median height for the distribution.
    - Explain:
      - Since the median class was found to be 150 – 159, we can estimate the median height as the mid-point of the class interval. This is 154.5 cm.
14. **Step 11.** vi. Estimate the modal class of the distribution.
    - Ask pupils to raise their hand when they have found the class interval in the histogram with the most number of pupils.
    - Invite a volunteer to give the class interval and the number of pupils in that group. (Answer: Height of 160 – 169 cm with 9 pupils)
15. **Step 12.** vii. Estimate the modal height**.**
    1. Draw on the modal class of the histogram lines to the neighbouring classes as shown.
    2. Ask pupils to read off the histogram where the two lines intersect.
    3. Invite a volunteer to give the answer. (Answer: 162 cm).

Practice *(15 minutes)*

1. Ask pupils to work independently to answer question b. on the board.
2. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.
   Solution:
   See solution at the end of this lesson plan.

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM3-L004 in the Pupil Handbook.

[QUESTIONS]

a. The heights in centimetres of 30 pupils are given below.

142	163	169	132	139	140	152	168	139	150
161	132	162	172	146	152	150	132	157	133
141	170	156	155	169	138	142	160	164	168

i. Using class intervals of 130 – 139, 140 – 149, 150 – 159, …, construct a frequency table.
ii. Draw a histogram to represent the distribution.
iii. Calculate the mean of the distribution.
iv. Estimate the median class of the distribution.
v. Estimate the median height for the distribution.
vi. Estimate the modal class for the distribution.
vii. Using your histogram, estimate the modal height.

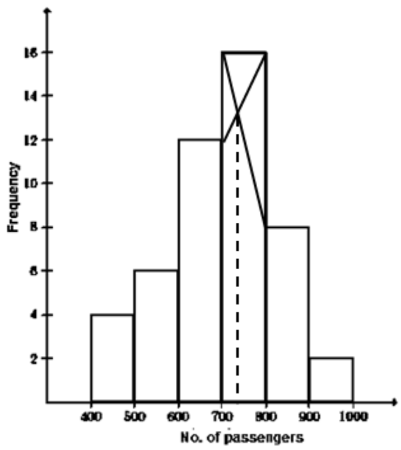
b. The table shows the distribution of passengers per week for 48 weeks who travel in a mini-van taxi.

No. of passengers	Frequency f
400 – 499	4
500 – 599	6
600 – 699	12
700 – 799	16
800 – 899	8
900 – 999	2

i. Draw a histogram to represent the distribution.
ii. Calculate the mean of the distribution. Round your answer to the nearest whole number.
iii. Estimate the median class of the distribution.
iv. Estimate the median number of passengers for the distribution.
v. Estimate the modal class for the distribution.
vi. Using your histogram, estimate the modal number of passengers.

Solution:

i.



ii.

No. of passengers	Frequency f	Mid-point x	fx
400 – 499	4	449.5	1798
500 – 599	6	549.5	3297
600 – 699	12	649.5	7794
700 – 799	16	749.5	11992
800 – 899	8	849.5	6796
900 – 999	2	949.5	1899
	$\sum f = 48$		$\sum fx = 33576$

Mean $= \frac{\sum f}{\sum fx}$

$= \frac{33576}{48} = 699.5$

$\approx$ 700 passengers

iii. Median class interval is given by: $\frac{\sum f}{2} = \frac{48}{2} = 24$
The class interval which contains the 24th week is 700 – 799.

iv. The median value is estimated at the mid-point of the median class interval of 700 – 799. This is 749.5 or 750 passengers to the nearest whole number.

v The modal class is for the number of weeks the taxi had the highest number of passengers:
700 – 799 passengers for 16 weeks.

vi. From the histogram the modal number of passengers is estimated to be: 740 passengers.

Lesson Title: Review of perimeter of regular shapes	**Theme:** Mensuration	
Lesson Number: M3-L005	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to describe and use the correct formula to calculate the perimeter of a specified shape.	**Preparation** Write the question at the end of this lesson plan under **Preparation** on the board.	

Opening *(4 minutes)*

1. Ask a volunteer to remind the class what the perimeter of a shape is. (Example answers: the distance around the shape; the total length around a shape)
2. Ask pupils to look at the information on the board giving the names and formulas for perimeters of different shapes.
3. Ask pupils to match the name of each shape with its formula and write the answer in their exercise books. Allow 2 minutes for pupils to answer.
4. Ask volunteers to give each shape and its formula. (Answers: see the end of this lesson plan)
5. Tell pupils that after today's lesson, they will be able to describe and use the correct formula to calculate the perimeter of a specified shape.

Teaching and Learning *(20 minutes)*

1. Tell pupils we will be using the formulas to answer questions throughout the lesson. We will use P to represent the perimeter.
2. Write on the board: The length of a rectangle is 2 cm more than the width of the rectangle. The perimeter of the rectangle is 20 cm. Find the length and the width of the rectangle.

   Solution:

   Let l denote length, w the width, and P the perimeter of a rectangle.

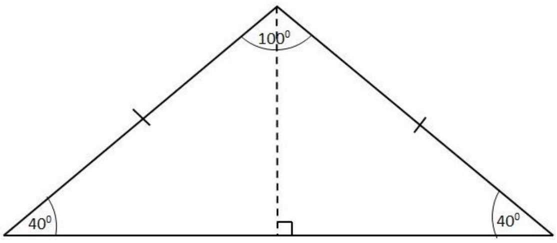
$$
\begin{aligned}
P &= 2l + 2w = 2(l + w) & \\
P &= 2(w + 2 + w) & \text{Note that } l = w + 2\text{. Substitute for } l \\
&= 2w + 4 + 2w & \\
P &= 4w + 4 & \\
20 &= 4w + 4 & \text{Substitute } P = 20 \\
20 - 4 &= 4w & \text{Transpose } 4 \\
16 &= 4w & \\
w &= \frac{16}{4} = 4 & \\
l &= w + 2 = 4 + 2 = 6 & \text{Substitute } w = 4 \text{ into } l = w + 2
\end{aligned}
$$

   Answer: width, $w = 4$ cm, length, $l = 6$ cm
3. Solve another example problem on the board.

4. Write on the board: A triangle with angles 40°, 40° and 100° has a perimeter of 20 cm. Find the length of each side of the triangle. Give your answer to 2 decimal places.

Solution:

Explain:

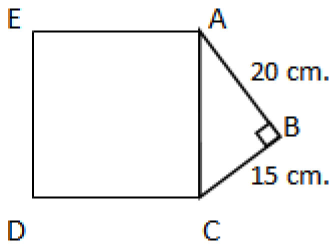


- Start by drawing a sketch of the problem.
- Draw an isosceles triangle (2 equal angles).
- Draw a perpendicular from the apex of the triangle to the base b.
- Mark the right angle (shown on the right).
- We now have 2 right-angled triangles and we can use either of them to find the lengths of sides a and b.
- Ask pupils to think what concept can be used to find the missing lengths. Answer: (trigonometric ratios)

P	$=$	$2a+b$	Construct a formula for the perimeter of the triangle
20	$=$	$2a+b$	Substitute $P=20$
b	$=$	$20-2a$	Transpose $2a$ and re-arrange
b	$=$	$2(10-a)$	Write b in terms of a
$\cos 40^0$	$=$	$\frac{\frac{b}{2}}{a}$	Use the cosine ratio
0.766	$=$	$\frac{\frac{2(10-a)}{2}}{a}$	$\cos 40^0 = 0.7660$ from the cosine table
$0.766a$	$=$	$10-a$	
$1.766a$	$=$	10	Transpose a and simplify
a	$=$	$\frac{10}{1.766} = 5.662$	
b	$=$	$2(10-5.662) = 8.676$	

The length of the sides are $a = 5.66$ cm (twice), $b = 8.68$ cm

5. Ask pupils to work with seatmates to solve the next problem. Allow 5 minutes for pupils to discuss and share ideas.
6. Write on the board: In the diagram shown, **ACDE** is a square. $|AB| = 20$ cm and $|BC| = 15$ cm. Find the perimeter of shape **ABCDE**.



7. Invite a volunteer to come to the board to show their solution to the problem.

Solution:

a^2	$=$	b^2+c^2	Use Pythagoras' Theorem to find the length of AC
a^2	$=$	20^2+15^2	Let $b=20,\ c=15$
	$=$	$400+225=625$	
a	$=$	$\sqrt{625}$	
a	$=$	25 cm $\quad = \lvert AC\rvert$	

$P = |AB| + |BC| + |CD| + |DE| + |EA|$ Note that the 3 sides of the square are all equal to $|AC|$

$P = |AB| + |BC| + 3|AC|$

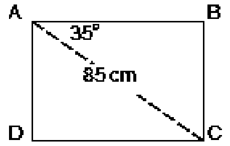
$= 20 + 15 + (3 \times 25)$

$20 + 15 + 75 = 110$

$P = 110$ cm

Practice *(15 minutes)*

1. Write the questions below on the board. Ask pupils to work individually to answer the questions.
2. Find the perimeter of the given shapes.
   a. The base of a parallelogram is three times the length of the other side. If the perimeter is 248 cm, find the base and side length of the parallelogram.
   b. Find the perimeter of the rectangle ABCD. Give your answer correct to 1 decimal place.



3. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.

Solutions:

a.

$P = 2b + 2s$ Note that $b = 3s$.

$P = 2(3s) + 2s$ Substitute for b

$= 6s + 2s$

$P = 8s$

$248 = 8s$ Substitute $P = 248$

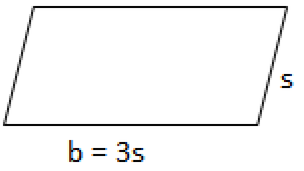
$\frac{248}{8} = s$

$s = 31$ cm

$b = 3s = 3 \times 31$

$b = 93$ cm base, $b = 93$ cm, side, $s = 31$ cm

Sketch of the shape:



b. We can use the trigonometric ratios to find the missing lengths.

$\cos 35^0 = \frac{|AB|}{|AC|}$ Use the cosine ratio to find $|AB|$

$|AB| = |AC| \cos 35^0$

$= 85 \cos 35^0$ $(85 \times 0.8192$; $\cos 35^0 = 0.8192$ from the cosine table)

$|AB| = 69.63$ cm (allow for any rounding errors)

$\sin 35^0 = \frac{|BC|}{|AC|}$ Use the sine ratio to find $|BC|$

$|BC| = |AC| \sin 35^0$ $(85 \times 0.5736$; $\sin 35^0 = 0.5736$ from the sine table)

$|BC| = 48.75$ cm (allow for any rounding errors)

$P = 2l + 2w = 2(l + w)$

$= 2(69.63 + 48.75)$

$= 2 \times 118.38 = 236.76$

$P = 236.8$ cm

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM3-L005 in the Pupil Handbook.

Preparation

Write the information below on the board:

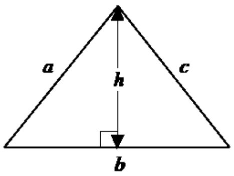
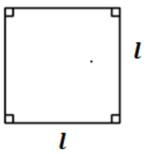
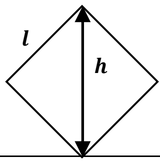
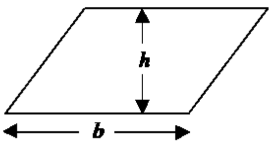
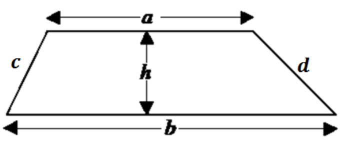
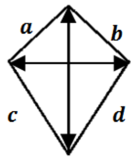
square		rectangle		circle
	triangle		parallelogram	
rhombus		$a+b+c+d$		$a+b+c$
	$2b+2s$		trapezium	
$2\pi r$		$2l+2w$		$4l$
	kite			

The letters in all the formulas above represent the lengths of sides of the particular shape.

Solution

The table below does not have to be written on the board.
It will be reproduced in the pupils' handbook.

If possible, draw quick sketches of the shapes and use the information to check pupils' answers and for discussion.

Shape	Additional information	Perimeter
triangle		$a+b+c$
square, rhombus	 	$4l$
rectangle		$2l+2w$
parallelogram		$2b+2s$
trapezium, kite	 	$a+b+c+d$

Lesson Title: Review of area of regular shapes	**Theme:** Mensuration	
Lesson Number: M3-L006	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to determine and use the correct formula to calculate the area of a specified shape.	**Preparation** None	

Opening *(2 minutes)*

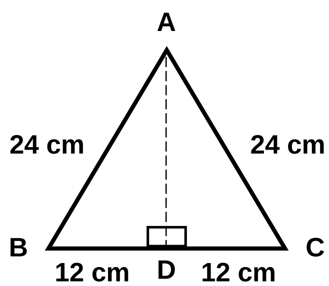
1. Ask a volunteer to remind the class what the area of a shape is. (Example answer: the **amount of space** enclosed by the sides of the shape)
2. Select 3 to 5 shapes from the table at the end of this lesson plan and ask volunteers to give the formula for the area of the shape. (Answers: see table)
3. Tell pupils that after today's lesson, they will be able to determine and use the correct formula to calculate the area of a specified shape.

Teaching and Learning *(20 minutes)*

1. Tell pupils we will be using the formulas to answer questions throughout the lesson. We will use A to represent the area.
2. Write on the board: The length of a rectangle is twice its width. The area of the rectangle is 18 cm^2. Find the length and the width of the rectangle.
   Solution:

$$\begin{array}{rcll} A &=& lw & \text{Formula for the area of a rectangle} \\ A &=& (2w)w & \text{Substitute } l = 2w \\ &=& 2w^2 & \\ 18 &=& 2w^2 & \text{Substitute } A = 18 \\ w^2 &=& 9 & \text{Simplify and re-arrange} \\ w &=& \sqrt{9} & \\ &=& \pm 3 & \\ w &=& 3 \text{ cm} & \text{Take the positive square root since lengths are always positive} \\ l &=& 2w = 2 \times 3 = 6 \text{ cm} & \end{array}$$

   The width, w of the rectangle $= 3$ cm, length, $l = 6$ cm
3. Write another example problem on the board:
   The length of the sides of an equilateral triangle is 24 cm. Find the area of the triangle to 1 decimal place.
   Solution:



   Explain:

   - Start by drawing a sketch of the problem (shown right).

- We now have 2 right-angled triangles and we can use either of them to find the perpendicular height.
- Ask pupils to think what method can be used to find the missing height. (Answer: Pythagoras' Theorem)

Solution:

a^2	$=$	$b^2 + c^2$	Use Pythagoras' Theorem for $\lvert\text{AD}\rvert$
24^2	$=$	$12^2 + \lvert\text{AD}\rvert^2$	Let $a = 24,\ b = 12,\ c = \lvert\text{AD}\rvert$
$\lvert\text{AD}\rvert^2$	$=$	$24^2 - 12^2$	Make c^2 the subject of the formula
	$=$	$576 - 144 = 432$	
$\lvert\text{AD}\rvert^2$	$=$	$\sqrt{432}$	
$\lvert\text{AD}\rvert$	$=$	20.785 cm	Take the positive square root since lengths are always positive
A	$=$	$\frac{1}{2} \times \text{base} \times \text{height}$	
A	$=$	$\frac{1}{2}(\lvert\text{BC}\rvert \times \lvert\text{AD}\rvert)$	
	$=$	$\frac{1}{2}(24 \times 20.785)$	
		$\frac{1}{2} \times 498.84$	
	$=$	249.42	
A	$=$	249.4 cm^2 to 1 d.p.	

The area of the triangle is 249.4 cm^2.

4. Ask pupils to work with seatmates to solve the next problem.
5. Allow 3 minutes for pupils to discuss and share ideas.
6. Write on the board: A rhombus has perimeter of 36 cm and a height of 5 cm. Find the area of the rhombus.
7. Invite a volunteer to come to the board to show their solution to the problem.

Solution:

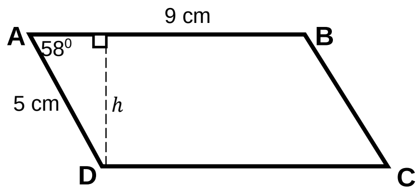
P	$=$	$4l$	
36	$=$	$4l$	Substitute $P = 36$
l	$=$	$\frac{36}{4} = 9$	
A	$=$	lh	
	$=$	9×5	Substitute $h = 5$
	$=$	45	
A	$=$	45 cm^2	

The area of the rhombus is 45 cm^2.

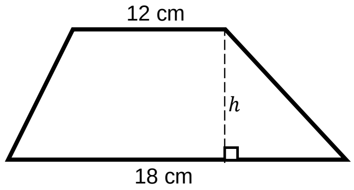
Practice *(15 minutes)*

1. Write the questions below on the board.
2. Ask pupils to work individually to answer the questions.
3. Solve the following problems.

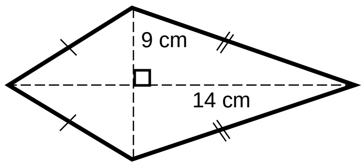
f. Find the area of parallelogram **ABCD**. Give your answer correct to 2 significant figures.



g. The given trapezium has an area of 60 cm^2. Find its height.



h. The diagonals of a kite are 14 cm and 9 cm long. Find the area of the kite.



4. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.

Solutions:

a.

$A = \frac{1}{2}bh$

$\sin 58^0 = \frac{h}{|\mathrm{AD}|}$ Use sine ratio to find h

$h = |\mathrm{AD}| \sin 58^0$

$= 5 \sin 58^0$ $(5 \times 0.8480;\ \sin 58^0 = 0.8480$ from the sine table)

$h = 4.24$ cm (allow for any rounding errors)

$A = \frac{1}{2} \times |\mathrm{DC}| \times 4.24$

$= \frac{1}{2} \times 9 \times 4.24$ Since $|\mathrm{DC}| = |\mathrm{AD}|$

$= 19.08$

$A = 19$ cm^2 to 2 s.f.

The area of the parallelogram is 19 cm^2.

b.

$A = \frac{1}{2}(a + b)h$

$60 = \frac{1}{2}(12 + 18)h$ Substitute $A = 60, a = 12, b = 18$

$= \left(\frac{1}{2} \times 30\right) h$

$60 = 15h$

$h = \frac{60}{15}$

$h = 4$ cm

The height of the trapezium is 4 cm.

c.

$A = \frac{1}{2} \times \text{the product of the diagonals}$

$= \frac{1}{2}(14 \times 9) = \frac{1}{2} \times 126$

$= 63$ cm

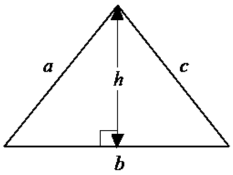
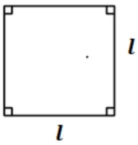
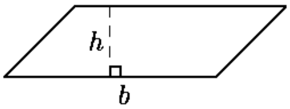
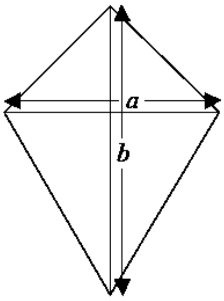
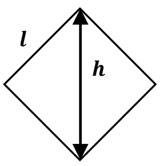
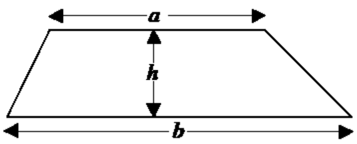
The area of the kite is 63 cm^2.

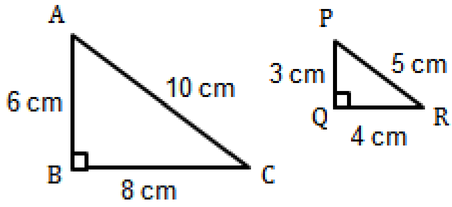
Closing *(3 minutes)*

1. Invite a volunteer to say anything they notice about the formulas for area of a parallelogram and a rhombus. (Example answer: Both shapes have the same basic formula, $\text{area} = \text{base} \times \text{perpendicular height}$)
2. For homework, have pupils do the practice activity PHM3-L006 in the Pupil Handbook.

Solution for Opening Activity

[Note: This table is reproduced in the pupil handbook]

Name	Shape	Area
triangle		$\frac{1}{2}bh$
square		l^2
rectangle		lw
parallelogram		bh
kite		$\frac{1}{2} \times \text{the product}$ $\text{of the diagonals} = \frac{1}{2}ab$
rhombus		lh OR $\frac{1}{2} \times \text{the product}$ of the diagonals
trapezium		$\frac{1}{2}(a+b)h$

Lesson Title: Area of similar shapes	**Theme:** Mensuration	
Lesson Number: M3-L007	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to calculate the area of similar shapes using the appropriate formulae.	**Preparation** Draw the triangles shown below on the board: 	

Opening *(3 minutes)*

1. Tell pupils that the triangles on the board are similar shapes.
2. Ask pupils to use the shapes on the board to write down any properties they notice of similar shapes.
3. Allow 1 minute for pupils to complete their answers.
4. Ask 2-3 volunteers to read out what they have written. (Example answers: see below for **Properties of similar shapes**)
5. Tell pupils that after today's lesson, they will be able to determine and use the correct formula to calculate the area of similar shapes.

Teaching and Learning *(25 minutes)*

1. Write the **Properties of similar shapes** on the board:
   - They are the same shape but are almost always different sizes. (They are the same shape **and** size for congruent shapes).
   - The corresponding lengths of the sides are in the same ratio.
   - The corresponding interior angles are the same.
2. Ask volunteers to say which pairs of angles in triangles ABC and PQR are equal. (Answer: $\angle ABC = \angle PQR = 90^\circ$; $\angle ACB = \angle PRQ$; $\angle BAC = \angle QPR$)
3. Remind pupils that:
   - The ratio of corresponding lengths is called the scale factor.
   - Scale factor is used to find any missing lengths in the triangle.
4. Write on the board:

$$\frac{|AB|}{|PQ|} = \frac{|BC|}{|QR|} = \frac{|CA|}{|RP|}$$

$$\frac{6}{3} = \frac{8}{4} = \frac{10}{5} = 2$$

5. Explain that the scale factor of the 2 triangles is 2 which means that all the lengths of sides in triangle ABC are twice the corresponding lengths in triangle PQR.
6. Explain that we want to find out the relationship between the ratio of the lengths and the ratio of the areas.

7. Ask pupils to use the area formula for triangles to calculate the areas of triangles ABC and PQR.
8. Allow pupils 3 minutes to complete the answers for the areas.
9. Invite 2 volunteers to write the answers on the board.
   Answers:

Triangle ABC

$$A = \frac{1}{2}(|BC| \times |AB|)$$
$$A = \frac{1}{2}(8 \times 6) = \frac{1}{2} \times 48 = 24$$

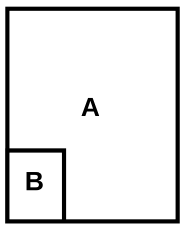
The area of triangle ABC is 24 cm^2.

Triangle PQR

$$A = \frac{1}{2}(|QR| \times |PQ|)$$
$$A = \frac{1}{2}(4 \times 3) = \frac{1}{2} \times 12 = 6$$

The area of triangle PQR is 6 cm^2.

10. The ratio of the 2 areas is given by: $\frac{24}{6} = 4$
11. So, when the ratio of lengths is 2, the ratio of areas is 4.
12. Write another problem on the board:
    The diagram shows 2 similar rectangles A and B with widths of 12 cm and 4 cm respectively. If A has length 15 cm, find: a. The ratio of their widths; b. The length of B; c. the ratio of their areas.



13. Solve the problem on the board, explaining each step.
    Solution:

a. $\frac{\text{width of } A}{\text{width of } B} = \frac{12}{4} = 3$

b. We know that the ratio for lengths is the same as the ratio for widths (3):

$$\frac{\text{length of } A}{\text{length of } B} = 3$$
$$\text{length of } B = \frac{\text{height of } A}{3}$$
$$= \frac{15}{3} = 5 \text{ cm}$$

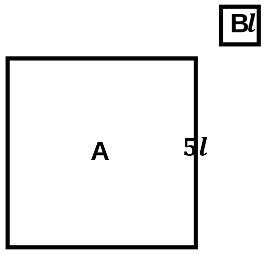
c. $\frac{\text{area of } A}{\text{area of } B} = \frac{12 \times 15}{4 \times 5}$

$$\frac{\text{area of } A}{\text{area of } B} = \frac{180}{20} = 9$$

14. So, when the ratio of widths is 3, the ratio of areas is 9.
15. Write another problem on the board: Two similar squares A and B have sides of length $5l$ and l respectively. Find: a. The ratio of their lengths; b. The ratio of their areas.
16. Ask pupils to think of the previous 2 examples and give suggestions of what they expect the ratio of the areas will be.
17. Invite 2-3 volunteers to give their expectations.
18. Ask pupils to work with seatmates to solve the problem.
19. Invite volunteers to come to the board to show their solutions for a. and b.

Solution:

Start by drawing the 2 squares (shown right):



a. $\frac{\text{length of } A}{\text{length of } B} = \frac{5l}{l} = 5$

b. $\frac{\text{area of } A}{\text{area of } B} = \frac{5l \times 5l}{l \times l}$

$\frac{\text{area of } A}{\text{area of } B} = \frac{25l^2}{l^2} = 25$

20. So, when the ratio of lengths is 5, the ratio of areas is 25.
21. Write on the board:

Ratio of lengths	Ratio of areas
2	4
3	9
5	25

22. Discuss: What pattern do pupils see that they can use to explain the relationship between lengths and areas of similar shapes? (Example answer: The ratios of areas of similar shapes are squares of the ratio of lengths.)
23. Explain:
    - The ratios of areas of similar shapes (e.g. A and B) are squares of the ratio of lengths:

$$\frac{\text{area of } A}{\text{area of } B} = \left(\frac{\text{length of A}}{\text{length of B}}\right)^2$$

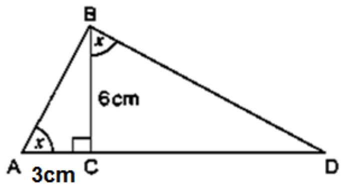
    - The relationship will allow us to predict the ratio of the areas of similar shapes if we know the ratio of the lengths.
    - Tell pupils that we can predict the ratio of lengths of similar shapes in the same way if we know the ratio of areas.
    - From this, we can find any missing lengths of the shapes.

Practice *(10 minutes)*

1. Write the questions below on the board.
2. Ask pupils to work individually to answer the questions.
3. Solve the following:
    a. The ratio of the base lengths of 2 squares is 6.
        i. What is the ratio of their areas?
        ii. If the area of the larger square is 432 cm^2, what is the area of the smaller parallelogram?

    b. Two similar trapeziums P and Q have base lengths of 3 cm and 12 cm respectively. If the area of P is 10 cm^2, what is the area of Q?

c. Triangles ABC and BCD are similar. Calculate the length of CD.



4. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.

Solutions:

a. i. Let A = larger square, B = smaller square

$$\frac{\text{area of A}}{\text{area of B}} = \left(\frac{\text{length of A}}{\text{length of B}}\right)^2$$
$$= 6^2$$
$$\frac{\text{area of A}}{\text{area of B}} = 36$$

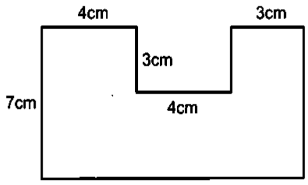
ii.
$$\text{area of B} = \frac{432}{36}$$
$$= 12 \text{ cm}^2$$

b.
$$\frac{\text{length of Q}}{\text{length of P}} = \frac{12}{3}$$
$$= 4$$
$$\frac{\text{area of Q}}{\text{area of P}} = 4^2$$
$$= 16$$
$$\text{area of Q} = 16 \times \text{ area of P}$$
$$= 16 \times 10$$
$$= 160 \text{ cm}^2$$

c.
$$\frac{\text{BC}}{\text{AC}} = \frac{6}{3}$$
$$= 2$$
$$\frac{\text{CD}}{\text{BC}} = 2$$
$$\frac{\text{CD}}{6} = 2$$
$$\text{CD} = 2 \times 6$$
$$= 12 \text{ cm}$$

Closing *(2 minutes)*

1. Ask pupils to write on a piece of paper the ratio of the lengths of a trapezium if the ratio of its areas is 16.
2. Ask pupils hold their answers up so you can check pupils' understanding of today's topic. (Answer: 4)
3. For homework, have pupils do the practice activity PHM3-L007 in the Pupil Handbook.

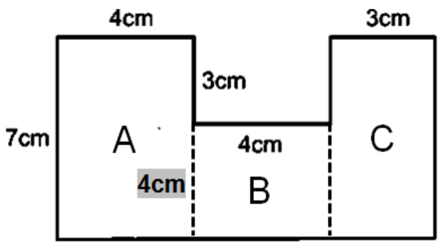
Lesson Title: Area of compound shapes	**Theme:** Mensuration	
Lesson Number: M3-L008	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to calculate the area of compound shapes using the appropriate formulae.	**Preparation** Draw the shape shown on the board: 	

Opening *(1 minute)*

1. Tell pupils that after today's lesson, they will be able to calculate the area of compound shapes using the appropriate formulas.

Teaching and Learning *(20 minutes)*

1. Explain to pupils that compound shapes are shapes made up of one or more other types of shapes.
2. Ask pupils if they can see the different shapes which make up the shape on the board.
3. Invite volunteers to answer. (Example answers: rectangle; square).
4. Explain:
   - To find the area of the shape, first split it into rectangles or squares.
   - Find any missing lengths of sides.
   - Find the individual areas using the appropriate formulas.
   - Finally, add the areas together.

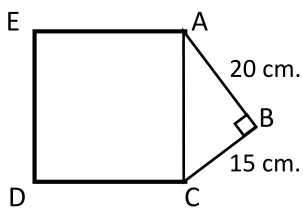


5. Tell pupils we will now find the area of the shape.
   Solution:
   Split the shape into 3 rectangles A, B and C.
   Find the length of the broken line: $7 - 3 = 4$ cm

   area of shape = area of A + area of B + area of C
   $= (7 \times 4) + (4 \times 4) + (7 \times 3)$
   $= 28 + 16 + 21$
   $= 65 \text{ cm}^2$

   The area of the shape is 65 cm^2.

6. Explain to pupils that we sometimes have to find the missing lengths using either Pythagoras' Theorem or Trigonometry.
7. Write on the board:
   In the diagram shown, ACDE is a square. $|\text{AB}| = 20$ cm and $|\text{BC}| = 15$ cm. Find the area of shape ABCDE.

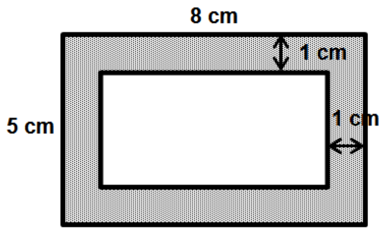


Solution:

$$c^2 = a^2 + b^2 \quad \text{Use Pythagoras' Theorem to find the length of AC}$$
$$c^2 = 20^2 + 15^2 \quad \text{Let } a = 20,\ b = 15$$
$$c^2 = 400 + 225 = 625$$
$$c = \sqrt{625}$$
$$c = 25 \text{ cm} = |\text{AC}|$$
$$A = \text{area of square + area of triangle}$$
$$= |\text{AC}|^2 + \tfrac{1}{2} \times |\text{BC}| \times |\text{AB}|$$
$$= 25^2 + \tfrac{1}{2} \times (15 \times 20)$$
$$= 625 + 150$$
$$A = 775 \text{ cm}^2$$

The area of ABCDE is 775 cm^2.

8. Ask pupils to work with seatmates to solve the next problem. Allow 5 minutes for pupils to discuss and share ideas.
9. Draw the shape shown on the board. Ask pupils to find the area of the shaded part.



10. Invite a volunteer to come to the board to show their solution to the problem.

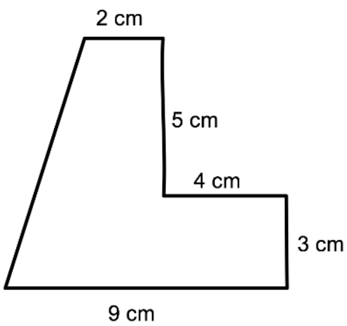
Solution:

Note that the smaller rectangle is 1 cm smaller on each side. Therefore, its width and length are both 2 cm shorter than the large triangle. For the small triangle: $l = 8 - 2 = 6$ cm; $w = 5 - 2 = 3$ cm.

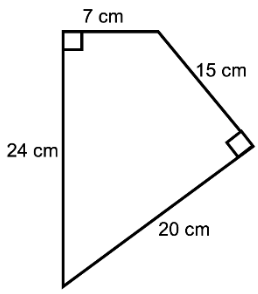
$$\text{area of shaded part} = (8 \times 5) - (6 \times 3)$$
$$= 40 - 18$$
$$= 22 \text{ cm}^2$$

Practice *(17 minutes)*

1. Write the questions below on the board.
2. Ask pupils to work individually to answer the questions.
3. Find the area of the given shapes.

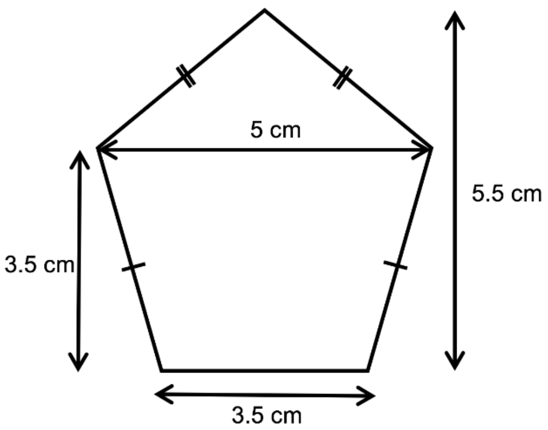


a. a.



b.

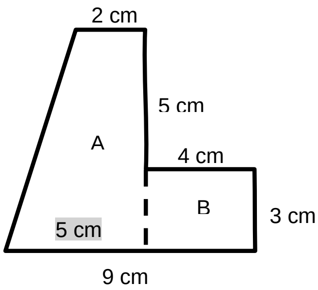
c.



4. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.

Solutions:

a. Divide the shape as shown:



area of shape = area of trapezium A + area of rectangle B

$= \frac{1}{2}(a+b)h + (l \times w)$

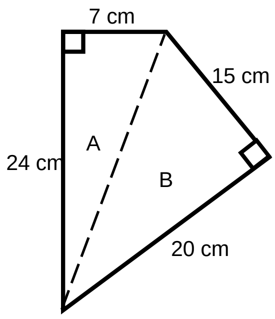
$= \frac{1}{2}(2+5) \times 8 + (4 \times 3)$

$= 28 + 12$

area of shape = 40 cm²

The area of the shape is 40 cm².

b. Divide the shape into 2 triangles A and B



area of shape = area of A + area of B

$= \frac{1}{2}(7 \times 24) + \frac{1}{2}(20 \times 15)$

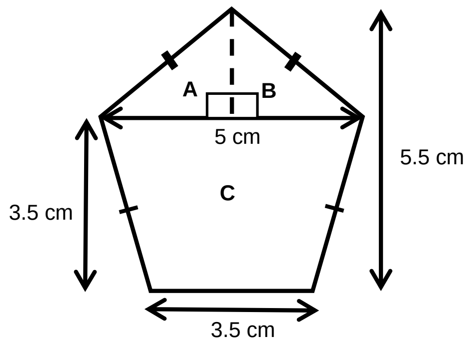
$= \frac{1}{2} \times 168 + \frac{1}{2} \times 300 = \frac{1}{2} \times (168 + 300)$

$= \frac{1}{2} \times 468$

area of shape = 234 cm²

The area of the shape is 234 cm².

c. Divide the shape as shown



height of triangle A = $5.5 - 3.5 = 2$

area of shape = area of A + area of B + area of C

area of A = area of B

$$\begin{aligned} \text{area of shape} &= 2 \times \left(\frac{1}{2} \times 2.5 \times 2\right) + \frac{1}{2}(5 + 3.5) \times 3.5 \\ &= (2.5 \times 2) + \frac{1}{2} \times 8.5 \times 3.5 \\ &= 5 + 14.875 \\ &= 19.875 \end{aligned}$$

The area of the shape is 19.9 cm^2.

Closing *(2 minutes)*

1. Ask 3 to 5 volunteers to tell the class one new thing they learned during the lesson.
2. For homework, have pupils do the practice activity PHM3-L008 in the Pupil Handbook.

<table>
<tr><td>Lesson Title: Review of circles</td><td colspan="2">Theme: Geometry</td></tr>
<tr><td>Lesson Number: M3-L009</td><td>Class: SSS 3</td><td>Time: 40 minutes</td></tr>
<tr><td>Learning Outcomes
By the end of the lesson, pupils will be able to:
1. Identify parts of a circle.
2. Calculate the circumference of a circle using the formula $C = 2\pi r$.</td><td colspan="2">Preparation
Draw the diagram of the parts of a circle found at the end of this lesson plan on the board.</td></tr>
</table>

Opening *(4 minutes)*

1. Ask pupils to look at the diagram of the parts of a circle on the board.
2. Ask pupils to write down a description of the diameter of a circle in their exercise books.
3. Give pupils 1 minute to write the description down.
4. Select a volunteer to read out their description to the class.
5. Tell pupils that after today's lesson, they will be able to identify the parts of a circle. They will also be able to find the circumference of a circle.

Teaching and Learning *(20 minutes)*

1. Explain the instructions below to pupils:
   - Draw the circle on the board in their exercise books.
   - Listen to the description of one of the parts of a circle.
   - Match the description of the circle part with one of the names on the board.
   - Write the name of the part on the circle.
2. Allow time for pupils to draw the circle in their exercise books.
3. Read the descriptions found at the end of the lesson plan one at a time.
4. Give pupils time (a few moments only) to select the part of the circle from the list found on the board and write it on the circle in their exercise books.
5. Continue reading each description, giving pupils time to find and write the name on their circle until all the descriptions have been read out.
6. Point to a part of the circle and ask a volunteer to give you the name of the part.
7. Write the name of the part on the circle.
8. Continue in this way until all the parts of the circle have been filled in.
9. Ask pupils to correct their work, using the labelled circle, in their exercise books.
10. Ask a volunteer to remind the class of the formula for the circumference of a circle. (Answer: $C = 2\pi r$ where r is the radius of the circle; accept $C = \pi d$ where d is the diameter of the circle)
11. Solve the following problems on the board.
    a. Find the circumference of a circle whose radius is 21 cm. Take $\pi = \frac{22}{7}$.

       Solution:

       $$C \quad = \quad 2\pi r$$

$$= \quad 2 \times \frac{22}{7} \times 21$$
$$C \quad = \quad 132 \text{ cm}$$

The circumference of the circle is 132 cm.

b. The circumference of a circle is 88 cm. Find its radius. Take $\pi = \frac{22}{7}$.

Solution:

$$C \quad = \quad 2\pi r$$
$$88 \quad = \quad 2 \times \frac{22}{7} \times r$$
$$= \quad \frac{2 \times 22}{7} \times r$$
$$88 \quad = \quad \frac{44}{7} \times r$$
$$\frac{88 \times 7}{44} \quad = \quad r$$
$$r \quad = \quad 14 \text{ cm}$$

The radius of the circle is 14 cm.

c. The distance around a circular playing field is 200 m. Find the radius of the field to the nearest metre. Take $\pi = 3.14$. Give your answer to 2 decimal places.

Solution:

$$C \quad = \quad 2\pi r$$
$$200 \quad = \quad 2 \times 3.14 \times r$$
$$= \quad 6.28 \times r$$
$$r \quad = \quad \frac{200}{6.28}$$
$$= \quad 31.847$$
$$r \quad = \quad 31.85$$

The radius of the playing field is 31.85 m to 2 decimal places.

12. Ask pupils to solve the next problem with seatmates:

d. The wheel of a motorbike has a diameter of 35 cm. How far will it travel in one complete revolution? Take $\pi = \frac{22}{7}$

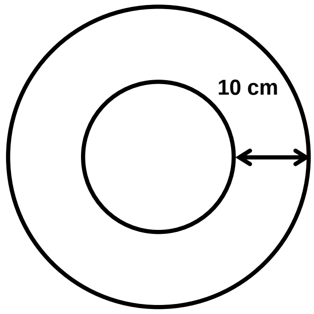
Solution:

$$r \quad = \quad \frac{d}{2}$$
$$= \quad \frac{35}{2} \qquad \text{Substitute } d = 35$$
$$C \quad = \quad 2\pi r$$
$$= \quad 2 \times \frac{22}{7} \times \frac{35}{2}$$
$$C \quad = \quad 110 \text{ cm}$$

The distance travelled in one complete revolution is 110 cm.

Practice *(15 minutes)*

1. Write the questions below on the board.
   Ask pupils to work individually to answer the questions.
2. Solve the following circle problems. Take $\pi = \frac{22}{7}$.
   a. Find the circumference of a circle whose radius is: a. 56 cm; b. 6.37 m.
   b. Find the radius of a circle whose circumference is: a. 220 m; b. 101.2 mm.
   c. The radius of the outer circle shown is 25.4 cm.
   Find:
      i. The radius of the inner circle.
      ii. The circumference of the inner circle.
   Take $\pi = 3.14$. Give your answer to the nearest centimetre.



3. Invite volunteers to come to the board to show their solutions.
   The rest of the class should check their solutions and correct any mistakes.
   Solutions:

a. i.

$$C = 2\pi r$$
$$C = 2 \times \frac{22}{7} \times 56$$
$$C = 352 \text{ cm}$$

The circumference is 352 cm.

ii.

$$C = 2\pi r$$
$$= 2 \times \frac{22}{7} \times 6.37$$
$$= \frac{280.28}{7}$$
$$C = 40.04 \text{ cm}$$

The circumference is 40.04 cm.

b. i.

$$C = 2\pi r$$
$$220 = 2 \times \frac{22}{7} \times r$$
$$220 = \frac{44r}{7}$$
$$r = \frac{220 \times 7}{44}$$
$$r \quad 35\ m$$

The radius is 35 m.

ii.

$$C = 2\pi r$$
$$101.2 = 2 \times \frac{22}{7} \times r$$
$$101.2 = \frac{44r}{7}$$
$$r = \frac{101.2 \times 7}{44}$$
$$r = 16.1\ m$$

The radius is 16.1 m.

c. i.

$R =$ outer circle, $r =$ inner circle

$$r \quad 25.4 - 10$$
$$r \quad 15.4\ cm$$
$$r \quad 15.4\ cm$$

The radius is 15.4 cm.

ii.

$$C = 2\pi r$$
$$= 2 \times 3.14 \times 15.4$$
$$= 96.712$$
$$= 97 \text{ m}$$

The circumference is 97 m to the nearest centimetre.

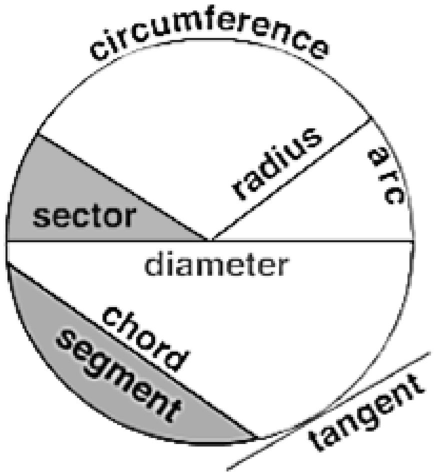
Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM3-L009 in the Pupil Handbook.

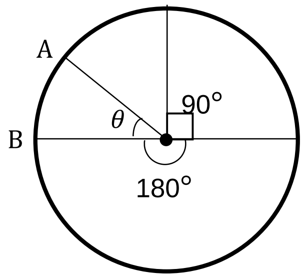
[DIAGRAM FOR OPENING, TEACHING AND LEARNING ACTIVITIES]

Draw the diagram of parts of a circle on the board.

Use the descriptions given for pupils to match with a part of the circle.



Parts of circle	Description
Circumference	The distance around a circle
Radius	The distance from the centre of the circle to any point on the circumference. It is also half of the diameter.
Diameter	A straight line passing through the centre of the circle to touch both sides of the circumference. It is also twice the length of the radius.
Chord	A straight line joining two points on the circumference of a circle. The diameter is a special kind of chord which passes through the centre of the circle.
Arc	A section of the circumference; part of the circumference of a circle
Sector	A section of a circle bounded by two radii and an arc.
Segment	A section of a circle bounded by a chord and an arc.
Tangent	A straight line touching the circumference at a given point.
Semi-circle	Half of a circle.
Quadrant	Quarter of a circle.

Lesson Title: Length of an arc	**Theme:** Geometry	
Lesson Number: M3-L010	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to calculate the length of an arc.	**Preparation** Draw the diagram below on the board: 	

Opening *(3 minutes)*

1. Ask a volunteer to remind the class what an arc is. (Example answers: A section of the circumference; Part of the circumference of a circle.)
2. Tell pupils that after today's lesson, they will be able to calculate the length of an arc.

Teaching and Learning *(20 minutes)*

Angle subtended	Length of arc
360°	$2\pi r$
180°	πr
90°	$\frac{\pi r}{2}$

1. Draw the table shown at right, below the circle on the board.
   Do not complete any of the values in the table.
2. Ask a volunteer to remind the class what the formula is for the circumference of a circle. (Answer: $C = 2\pi r$)
3. Ask a volunteer to remind the class how many degrees altogether in a circle. (Answer: 360°)
4. Explain that the whole circumference is an arc which subtends 360° at the centre.
5. Complete the first line of the table.
6. Ask a volunteer to read from the circle how many degrees there are in a semi-circle. (Answer: 180°)
7. Explain that the semi-circle is half of a circle and it subtends 180° at the centre.
8. Ask pupils to discuss for a moment what they think the length of the semi-circle might be if the full circle is $2\pi r$.
9. Invite a volunteer to answer. (Answer: half of the circumference of the circle; $\frac{2\pi r}{2} = \pi r$)
10. Enter this information in the table.
11. Now ask about what proportion of the circle makes 90° at the centre. (Answer: $\frac{1}{4}$)
12. Invite a volunteer to give the answer for what the arc length would be for a 90° angle. (Answer: a quarter of the circumference of the circle; $\frac{2\pi r}{4} = \frac{\pi r}{2}$)

13. Explain:
    - From the table we can see that the length of the arc is proportional to the angle it subtends at the centre of the circle.
    - The circumference subtends 360° and all other lengths of arcs are in proportion to the angles they subtend.
    - **This means that if we take the ratio of the length of an arc to the circumference, it will be equal to the ratio of the angles subtended at the centre.**
14. Write on the board:

$$\frac{\text{length of arc}}{\text{circumference}} = \frac{\text{angle subtended by arc}}{360}$$

e.g. $\frac{\pi r}{2\pi r} = \frac{180}{360} = \frac{1}{2}$ Divide LHS by πr

And, $\frac{\frac{\pi r}{2}}{2\pi r} = \frac{90}{360} = \frac{1}{4}$

Similarly, $\frac{|\text{AB}|}{2\pi r} = \frac{\theta}{360}$ Where θ is the angle subtended by the arc AB

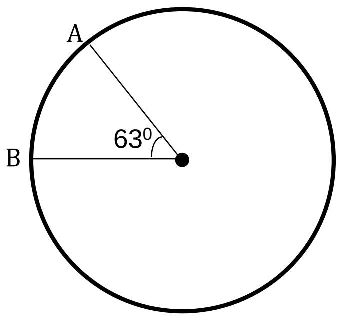
Therefore: $|\text{AB}| = \frac{\theta}{360} \times 2\pi r$

Length of arc $= \frac{\theta}{360} \times 2\pi r$

15. Use the formula to solve problems.
16. Write on the board: An arc subtends an angle of 63° at the centre of a circle of radius 12 cm. Find the length of the arc. Take $\pi = \frac{22}{7}$.

    Solution:

    First draw a diagram of the problem (shown below).



$|\text{AB}| = \frac{\theta}{360} \times 2\pi r$

$= \frac{63}{360} \times 2 \times \frac{22}{7} \times 12$ Substitute $\theta = 63, r = 12$

$= \frac{63 \times 2 \times 22 \times 12}{360 \times 7}$ Simplify

$|\text{AB}| = = 13.2$ cm

The length of the arc AB $= 13.2$ cm.

17. Solve another problem on the board.
18. The arc XY is 55 cm long. What angle does it subtend at the centre of a circle of radius 21 cm? Take $\pi = \frac{22}{7}$.

    Solution:

$$|XY| = \frac{\theta}{360} \times 2\pi r$$

$$55 = \frac{\theta}{360} \times 2 \times \frac{22}{7} \times 21$$ Substitute $|XY| = 55, r = 21$

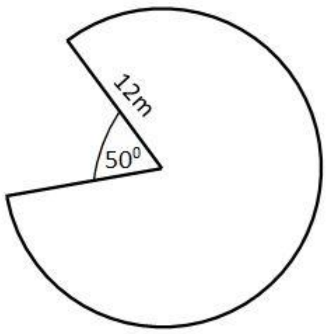
$$\theta = \frac{55 \times 360 \times 7}{2 \times 22 \times 21}$$ Simplify

$$\theta = 150^{\circ}$$

The angle, θ, subtended by the arc XY $= 150^{\circ}$

Practice *(15 minutes)*

1. Write the questions below on the board.
   Ask pupils to work independently to answer the questions.
   Ask pupils to draw a sketch to help them solve the problem.
2. Solve the following problems.
   c. An arc subtends an angle of 28° at the centre of a circle with radius 18 cm. Find the length of the arc. Take $\pi = \frac{22}{7}$
   d. A circle with a radius of 7 cm has an arc length of 11 cm. Find its angle. Take $\pi = \frac{22}{7}$
   e. Calculate the arc length of the given shape. Give your answer to 3 significant figures. Take $\pi = 3.14$.



   f. An arc has a length of 35 cm. Find the angle subtended by the arc at the centre of the circle with radius 8 cm. Take $\pi = 3.14$
3. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.
   Solutions:

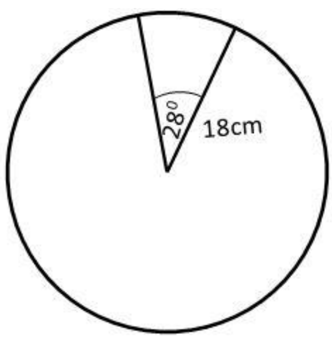
a. length of arc $= \frac{\theta}{360} \times 2\pi r$

$$= \frac{28}{360} \times 2 \times \frac{22}{7} \times 18$$

$$\frac{28 \times 2 \times 22 \times 18}{360 \times 7}$$

$$= 8.8$$

length of arc $= 8.8$ cm



The length of the arc is 8.8 cm

b. length of arc $= \frac{\theta}{360} \times 2\pi r$

$\theta = \frac{\text{length of arc} \times 360}{2\pi r}$

$= \frac{11 \times 360}{2 \times \frac{22}{7} \times 7}$

$= \frac{11 \times 360 \times 7}{2 \times 22 \times 7}$

$\theta = 90°$

The angle subtended by the arc is $90°$.

c. length of arc $= \frac{\theta}{360} \times 2\pi r$

$= \frac{310}{360} \times 2 \times 3.14 \times 12$ Since $\theta = 360 - 50 = 310$

$= \frac{310 \times 2 \times 3.14 \times 12}{360}$

$= 64.89$

length of arc $= 64.9$ m

The arc has a length of 64.9 m to 3 s.f.

d. length of arc $= \frac{\theta}{360} \times 2\pi r$

$35 = \frac{\theta}{360} \times 2 \times 3.14 \times 8$

$\theta = \frac{35 \times 360}{2 \times 3.14 \times 8}$

$\theta = 250.79$

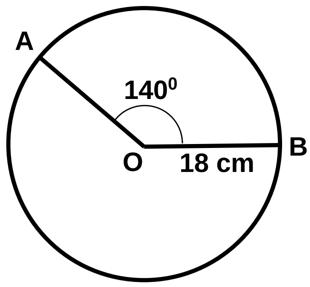
The angle subtended by the arc = $280.8°$ to 1 decimal place.

Closing *(2 minutes)*

1. Tell pupils they will use the relationships to find the lengths of arcs in other areas of circle geometry.
2. For homework, have pupils do the practice activity PHM3-L010 in the Pupil Handbook.

Lesson Title: Perimeter of a sector	**Theme:** Geometry	
Lesson Number: M3-L011	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to calculate the perimeter of the sector of a circle.	**Preparation** Write on the board: Find the perimeter of a sector of a circle with a radius of 18 cm. The angle of the sector is 140°. Take $\pi = \frac{22}{7}$.	

Opening *(4 minutes)*



1. Ask pupils to draw a freehand circle with radius 18 cm in their exercise books.
2. Ask pupils to draw a sector of 140° and label it AOB.
3. Allow 2 minutes for pupils to do the sketch as requested.
4. Draw the diagram shown right and ask pupils to check their diagram and correct any errors.
5. Tell pupils that after today's lesson, they will be able to calculate the perimeter of the sector of a circle.

Teaching and Learning *(15 minutes)*

1. Explain:
   - A sector of a circle is the region between two radii and an arc.
   - AOB is a sector in the circle shown with radius 18 cm.
   - The perimeter, P of the sector is the distance around the sector.
   - It is found by adding together the lengths of AO, OB and AB.

   $$P = |\text{AO}| + |\text{OB}| + |\text{AB}|$$
   $$= 2 \times |\text{OB}| + |\text{AB}| \qquad \text{Since } |\text{OB}| = |\text{AO}| = 18$$

   We need to first find the length of arc AB

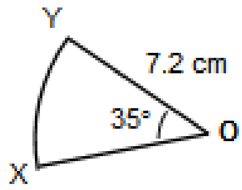
   $$|\text{AB}| = \frac{\theta}{360} \times 2\pi r$$
   $$= \frac{140}{360} \times 2 \times \frac{22}{7} \times 18 \qquad \text{Substitute } \theta = 140, \text{r} = 18$$
   $$= \frac{140 \times 2 \times 22 \times 18}{360 \times 7}$$
   $$= 44 \text{ cm}$$

   We can now find the perimeter of AOB

   $$P = (2 \times 18) + 44 \qquad \text{Substitute } |\text{OB}| = 18, |\text{AB}| = 44$$
   $$= 36 + 44$$
   $$= 80 \text{ cm}$$

   The perimeter of arc AOB $= 80$ cm
2. Write another problem on the board:
   The diagram shows sector XOY of a circle with a radius of 7.2 m. An arc XY subtends an angel of 35° at the centre of the circle.

Find a. the length of the arc XY; b. the perimeter, P, of the sector XOY. Give your answer to 1 decimal place. Take $\pi = 3.14$.



Solution:

a. $|XY| = \frac{\theta}{360} \times 2\pi r$

$= \frac{35}{360} \times 2 \times 3.14 \times 7.2$ Substitute $\theta = 35, r = 7.2$

$= \frac{35 \times 2 \times 3.14 \times 7.2}{360}$ Simplify

$|XY| = = 4.396$ cm

The length of the arc XY $= 4.4$ cm to 1 decimal place.

b. $P = 2 \times |OX| + |XY|$

$= (2 \times 7.2) + 4.4|$

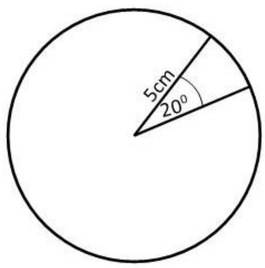
$= 14.4 + 4.4$

$P = 18.8$

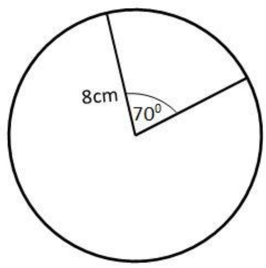
The perimeter of XOY $= 18.8$ cm. to 1 decimal place.

Practice *(20 minutes)*

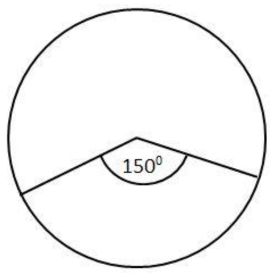
1. Write the questions below on the board.
   Ask pupils to work individually to answer the questions.
2. Solve the following problems. Take $\pi = 3.14$ unless told otherwise.
   a. An arc subtends an angle 20° at the centre of a circle of radius 5 cm.
      Calculate: i. The length of the arc; ii. The perimeter of the sector.
      Give your answers to the nearest centimetre.



   b. The angle of the sector of a circle is 70°. Calculate the perimeter of the sector if the radius 6 cm. Give your answers to the nearest centimetre.



   c. An arc subtends an angle 150° at the centre of a circle. If the perimeter of the sector is 42 cm and the length of the arc is 24 cm, calculate the radius of the sector.



d. The perimeter of a sector of a circle is 72 cm. If the radius is 20 cm, find: i. the length of the arc of the sector; ii. The angle at the centre of the circle subtended by the arc to the nearest degree.

3. Invite volunteers to come to the board to show their solutions.
The rest of the class should check their solutions and correct any mistakes.
Solutions:

a. i.

$$\text{arc length} = \frac{\theta}{360} \times 2\pi r$$
$$\frac{20}{360} \times 2 \times 3.14 \times 5$$
$$= 1.744$$
$$\text{Length of arc} = 2 \text{ cm}$$

The length of the arc is 2 cm to the nearest cm.

ii.

$$\text{Perimeter} = \text{arc length} + 2r$$
$$= 2 + (2 \times 5)$$
$$= 2 + 10$$
$$= 12 \text{ cm}$$

The perimeter of the sector is 12 cm

b. First find the arc length

$$\text{Length of arc} = \frac{\theta}{360} \times 2\pi r$$
$$\frac{70}{360} \times 2 \times 3.14 \times 6$$
$$= 7.326$$
$$\text{Length of arc} = 7 \text{ cm to the nearest cm}$$

$$\text{Perimeter} = \text{arc length} + 2r$$
$$= 7 + (2 \times 6)$$
$$= 7 + 12$$
$$= 19 \text{ cm}$$

The perimeter of the sector is 19 cm.

c.

$$\text{Perimeter} = \text{arc length} + 2r$$
$$42 = 24 + (2r)$$
$$2r = 42 - 24$$
$$r = \frac{42 - 24}{2}$$
$$r = 9 \text{ cm}$$

The radius of the sector is 9 cm

d. i.

$$\text{Perimeter} = \text{arc length} + 2r$$
$$72 = \text{arc length} + (2 \times 20)$$

arc length $= 72 - 40$

arc length $= 32$ cm

The length of the arc is 32 cm.

ii. arc length $= \frac{\theta}{360} \times 2\pi r$

$$\theta = \frac{\text{arc length} \times 360}{2\pi r}$$

$$= \frac{32 \times 360}{2 \times 3.14 \times 20}$$

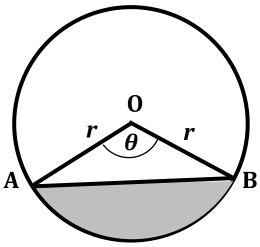
$$= 91.7°$$

$$\theta = 92°$$

The angle of the sector is $92°$ to the nearest degree.

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM3-L011 in the Pupil Handbook.

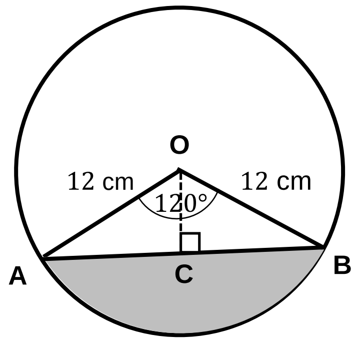
Lesson Title: Perimeter of a segment	**Theme:** Geometry	
Lesson Number: M3-L012	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to calculate the perimeter of a segment of a circle.	**Preparation** Draw the diagram on the board: 	

Opening *(2 minutes)*

1. Ask pupils to give the name to the line AB on the diagram on the board.
2. Invite a volunteer to answer (Answer: chord)
3. Tell pupils that after today's lesson, they will be able to calculate the perimeter of the segment of a circle.

Teaching and Learning *(20 minutes)*

1. Ask pupils to look again at the diagram on the board. How can we find the perimeter of the shaded segment?
2. Allow pupils to discuss and share ideas for a few moments.
   Invite a volunteer to answer: (Example answer: By adding together the length of the arc AB and the length of the chord AB.)
3. Explain that we can find the length of the arc using the formula we used previously.
4. Invite a volunteer to remind the class of the formula.
   (Answer: length of arc $= \frac{\theta}{360} \times 2\pi r$)
5. Explain: There are times we need to calculate the length of the chord before we can find the perimeter of the segment.
6. Modify the circle on the board. Let $r = 12$ cm and $\theta = 120°$. Find: a. The length of chord AB; b. The length of the arc AB; c. The perimeter of segment AB. Give your answers to 1 decimal place. Take $\pi = 3.14$.



7. Explain:
   - Draw a perpendicular line from the centre of the circle to the chord AB.
   - This bisects (divides in half) both the angle at the centre and the chord giving 2 equal right-angled triangles, OCA and OCB.

a. $\sin 60° = \frac{|\text{AC}|}{12}$

Use the sine ratio to find the length of AC
Note: $\angle\text{AOC} = \frac{\angle\text{AOB}}{2} = \frac{120}{2} = 60°$

$$|AC| = 12 \times \sin 60°$$
$$= 12 \times 0.8660 \qquad \sin 60^{\circ} = 0.8660 \text{ from the sine table}$$
$$= 10.392$$
$$|AC| = 10.392 \text{ cm}$$
$$|AC| \times 2 = |AB|$$
$$10.392 \times 2 = 20.78$$

The length of the chord $|AB|$ = 20.8 cm to 1 decimal place.

Note: For a given angle at the centre, the length of the chord will always be radius $\times \sin(\frac{\text{angle at centre}}{2})$

b. arc $|AB| = \frac{\angle AOC}{360} \times 2\pi r$

$= \frac{120}{360} \times 2 \times 3.14 \times 12$ Substitute $\angle AOC = 120, r = 12$

$= 25.12$ Simplify

arc $|AB| = 25.1$ cm

The length of arc $|AB|$ = 25.1 cm to 1 decimal place.

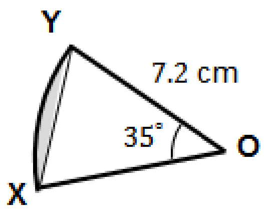
c. Let P = perimeter of sector AB

$P = |OC| + \text{arc } |AB|$

$= 20.8 + 25.1 = 45.9$ cm

The perimeter of sector AB = 35.5 cm to 1 decimal place.

8. Ask pupils to work with seatmates to solve the next problem.
9. Allow 3 minutes for them to discuss and share ideas.
   The diagram shows sector XOY of a circle with a radius of 7.2 cm. If the chord XY is 4 cm, find the perimeter of the shaded segment. Take $\pi = \frac{22}{7}$.



10. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.

Solution:

Let P = perimeter of sector

P = chord $|XY|$ + arc $|XY|$

chord $|XY|$ = 4 cm

arc $|XY| = \frac{\angle XOY}{360} \times 2\pi r$

$= \frac{35}{360} \times 2 \times \frac{22}{7} \times 7.2$ Substitute $\angle XOY = 35,\ r = 12$

$= \frac{35 \times 2 \times 22 \times 7.2}{360 \times 7}$ Simplify

$= 4.4$ cm

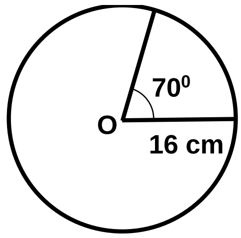
$P = 4 + 4.4 = 8.4$ cm

The perimeter of shaded sector = 8.4 cm.

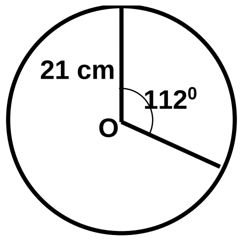
Practice *(15 minutes)*

1. Write the questions below on the board. Ask pupils to work individually to answer the questions.
2. Solve the following questions. Take $\pi = 3.14$ unless told otherwise. Give your answers to 1 decimal place.
   a. Find the perimeter of the segments for the following circles.
   i. radius = 6 cm, chord = 9 cm, $\theta = 85°$ ii. radius $= 10$ cm, chord $= 7$ cm, $\theta = 120°$.

   b. An arc subtends an angle of $70°$ at the centre of a circle with a diameter of 16 cm. If the length of the arc is 10 cm, find: i. The length of the chord; ii. The perimeter of the segment.



   c. The radius of a circle is 21 cm. An arc subtends an angle of $112°$ at the centre of the circle. Find: i. The length of chord; ii. The length of the arc; iii. The perimeter of the segment.



3. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.
   Solutions:

a. i.

$$\text{Let } P = \text{perimeter of the segment}$$
$$P = \text{chord length } + \text{arc length}$$
$$\text{chord length} = 9 \text{ cm}$$
$$\text{arc length} = \frac{\theta}{360} \times 2\pi r$$
$$= \frac{85}{360} \times 2 \times 3.14 \times 6$$
$$= 8.8966$$
$$\text{arc length} = 8.9 \text{ cm}$$
$$P = 8.9 + 9$$
$$P = 17.9 \text{ cm}$$

The perimeter of the segment is 17.9 cm to 1 d.p.

ii.

$$\text{Let } P = \text{perimeter of segment}$$
$$P = \text{chord length } + \text{arc length}$$
$$\text{chord length} = 7 \text{ cm}$$
$$\text{arc length} = \frac{\theta}{360} \times 2\pi r$$
$$= \frac{120}{360} \times 2 \times 3.14 \times 10$$
$$= 20.933$$

$$\text{arc length} = 20.9 \text{ cm}$$
$$P = 20.9 + 7$$
$$P = 27.9 \text{ cm}$$

The perimeter of the segment is 27.9 cm to 1 d.p.

b. i. $\frac{1}{2}$ chord length $= r \times \sin 35$
$= 8 \times 0.5736$
$\frac{1}{2}$ chord length $= 4.588$ cm

The length of the chord is 9.2 cm to 1 d.p.

ii. Let P = perimeter of segment
P = chord length + arc length
$= 9.2 + 10$
$P = 19.2$ cm

The perimeter of the segment is 19.2 cm.

c. i. $\frac{1}{2}$ chord length $= r \times \sin 56$
$= 21 \times 0.8290$
$= 17.409$
$\frac{1}{2}$ chord length $= 17.4$ cm

The length of the chord is 34.8 cm to 1 d.p.

ii.
$$\text{arc length} = \frac{\theta}{360} \times 2\pi r$$
$$= \frac{112}{360} \times 2 \times 3.14 \times 21$$
$$= 41.029$$
$$= 41.0 \text{ cm}$$
$$\text{arc length} = 41.0 \text{ cm}$$

The length of the arc is 41.0 cm to 1 d.p.

iii. Let P = perimeter of segment
P = chord length + arc length
$= 34.8 + 41.0$
$P = 75.8$ cm

The perimeter of the segment is 75.8 cm.

Closing *(3 minutes)*

1. Ask pupils to write down either one new thing they learned this lesson or one thing they understand better after this lesson.
2. Invite a volunteer to share their answer with the class. (Answer: Various)
3. For homework, have pupils do the practice activity PHM3-L012 in the Pupil Handbook.

Lesson Title: Area of a circle	**Theme:** Geometry	
Lesson Number: M3-L013	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcomes By the end of the lesson, pupils will be able to calculate the area of a circle using the formula $A = \pi r^2$.	**Preparation** Write on the board: A circle has a diameter of 4.2 m. Find the area of the circle. Give your answer to 2 decimal places. Take $\pi = 3.14$.	

Opening *(2 minutes)*

1. Invite a volunteer to remind the class the formula to calculate the area of a circle. (Answer: Area $A = \pi r^2$)
2. Tell pupils that in today's lesson, we will review how to calculate the area of a circle.

Teaching and Learning *(20 minutes)*

1. Ask pupils what the relationship is between the diameter of a circle and its radius.
2. Select a volunteer to answer. (Answer: diameter, $d = 2r$, where r is the radius.)
   Solution:

$$A = \pi r^2$$
$$d = 2r$$
$$r = \frac{d}{2}$$
$$\frac{4.2}{2} = 2.1 \qquad \text{Substitute } d = 4.2$$

We can now find the area of the circle

$$A = \pi r^2$$
$$= 3.14 \times 2.1^2 \qquad \text{Substitute r} = 2.1$$
$$= 13.847 \text{ cm}^2$$

The area of the circle $= 13.85$ cm^2 to 2 d.p.

3. Write another problem on the board: The area of a circle is 616 m^2. Find: a. The radius; b. The circumference, C, of the circle. Take $\pi = \frac{22}{7}$.
   Solution:

a.
$$A = \pi r^2$$
$$616 = \frac{22}{7} \times r^2 \qquad \text{Substitute } A = 616$$
$$\frac{616 \times 7}{22} = r^2$$
$$r^2 = 196$$
$$r = \sqrt{196}$$
$$r = 14$$

The radius the circle 14 cm.

b. $$\begin{aligned} P &= 2\pi r \\ &= 2 \times \frac{22}{7} \times 14 \quad \text{Substitute } r = 14 \\ &= 88 \\ P &= 88 \text{ cm} \end{aligned}$$

The perimeter of XOY $= 88$ cm.

4. Ask pupils to work with seatmates to solve the next problem.
5. Allow 5 minutes for them to discuss and share ideas.
6. The perimeter of a circular field is 440 m. Find its area. Take $\pi = \frac{22}{7}$.
7. Invite a volunteer to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.

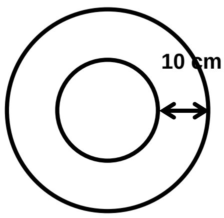
Solution:

$$\begin{aligned} P &= 2\pi r \\ 440 &= 2 \times \frac{22}{7} \times \text{r} \quad \text{Substitute } P = 440 \\ r &= \frac{440 \times 7}{2 \times 22} \\ r &= 70 \text{ m} \\ A &= \pi r^2 \\ &= \frac{22}{7} \times 70^2 \\ &= 15{,}400 \end{aligned}$$

The area of the field $= 15{,}400$ m^2.

Practice *(17 minutes)*

1. Write the questions below on the board.
2. Ask pupils to work individually to answer the questions.
3. Unless otherwise stated, take $\pi = \frac{22}{7}$.
   a. Find the area of a circle whose radius is: i. 7 cm; ii. 17.5 m.
   b. Find the radius of a circle whose area is 66 m^2. Give your answer to 2 decimal places.
   c. The radius of the outer circle shown is 35 cm. Find:
      iii. The radius of the inner circle.
      iv. The area of the inner circle.
   Take $\pi = 3.142$.



4. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.

Solutions:

Let $A =$ area of the circle

1 a. $A = \pi r^2$

$= \frac{22}{7} \times 7^2$

$A = 154$ cm^2

The area of the circle is 154 cm^2.

b. $A = \pi r^2$

$= \frac{22}{7} \times 17.5^2$

$A = 962.5$ m^2

The area of the circle is 962.5 m^2.

2. $A = \pi r^2$

$r^2 = \frac{A}{\pi}$

$= \frac{66}{3.14}$ Take $\pi = 3.14$

$= 21.019$

$= \sqrt{21.019} = 4.5846$

$r = 4.58$ cm

The radius of the circle is 4.58 cm to 2 d.p.

3. a. Let r = radius of small circle, R = radius of big circle

$r = R - 10$ distance apart = 10 cm

$= 35 - 10$

$= 25$ cm

The radius of the small circle is 25 cm to the nearest cm.

b. $A = \pi r^2$

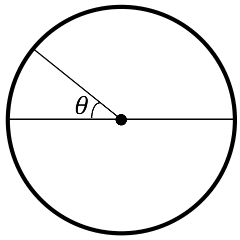
$= 3.14 \times 25^2$

$A = 1{,}962.5$ cm^2

The area of the smaller circle is 1963.cm^2 to the nearest cm^2.

Closing *(1 minute)*

1. Tell pupils the topic for the next lesson is calculating the perimeters of segments of circles.
2. For homework, have pupils do the practice activity PHM3-L013 in the Pupil Handbook.

Lesson Title: Area of a sector	**Theme:** Geometry	
Lesson Number: M3-L014	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to calculate the area of a sector of a circle.	**Preparation** Draw the diagram below on the board:  	

Opening *(3 minutes)*

1. Ask pupils to look back in their exercise books to the lesson on finding lengths of arcs.
2. Ask a volunteer to remind the class of the relationship between the length of an arc in a circle with the angle it subtends at the centre of the circle. (Example Answer: The length of an arc **is proportional** to the angle it subtends at the centre of the circle.)
3. Tell pupils that after today's lesson, they will be able to calculate the area of a sector in a circle.

Teaching and Learning *(20 minutes)*

1. Explain:
   - We know that the lengths of the arc in a circle are proportional to the angles they subtend at the centre of the circle.
   - In the same way, the area of a sector of a circle is proportional to the angle of the sector.
   - The circle is a sector with an angle of $360°$ and area πr^2.
   - All other sectors have areas in proportion to the angle of the sector.
   - **This means that if we take the ratio of the area of a sector to the area of the circle, it will be equal to the ratio of the angles of the sector.**
2. Write on the board:

$$\frac{\text{area of sector}}{\text{area of circle}} = \frac{\text{angle of sector}}{360}$$

Let $A =$ area of sector, θ=angle of sector

$$\frac{A}{\pi r^2} = \frac{\theta}{360}$$

$$A = \frac{\theta}{360} \times \pi r^2$$

$$\text{area of sector} = \frac{\theta}{360} \times \pi r^2$$

3. Use the formula to solve the given problems.
4. Write on the board: An arc subtends an angle of 63° at the centre of a circle of radius 12 cm. Find the area of the sector correct to the nearest cm². Take $\pi = 3.14$.

   Solution:

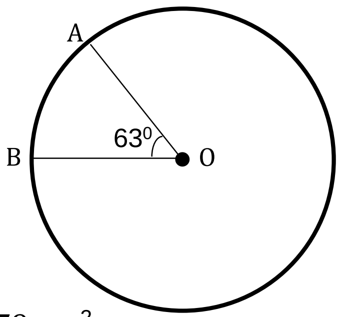
   First draw a diagram of the problem (shown below).

$$A = \frac{\theta}{360} \times \pi r^2$$

$$= \frac{63}{360} \times 3.14 \times 12^2 \quad \text{Substitute } \theta = 63, r = 12$$

$$= 79.128 \quad \text{Simplify}$$

$$A = = 79 \text{ cm}^2$$



The area of the sector to the nearest cm² = 79 cm².

5. Solve another problem on the board.
6. The area of a circle is 12 cm². Find the area of a sector AOB which has an angle of 140°. Take $\pi = \frac{22}{7}$.

   Solution:

$$\frac{\text{area of sector}}{\text{area of circle}} = \frac{\text{angle of sector}}{360}$$

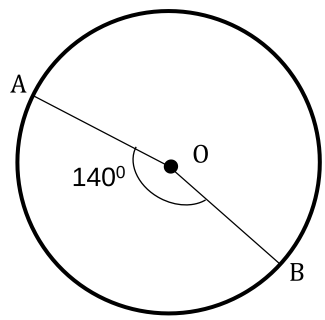
$$\frac{A}{\pi r^2} = \frac{140}{360}$$

$$A = \frac{140}{360} \times \pi r^2$$

$$= \frac{140}{360} \times \frac{22}{7} \times 12^2$$

$$= \frac{140 \times 22 \times 144}{360 \times 7} \quad \text{Simplify}$$

$$A = 176$$



The area of sector AOB = 176 cm².

Practice *(15 minutes)*

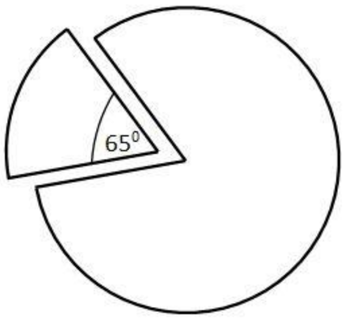
1. Write the questions below on the board.
   Ask pupils to work individually to answer the questions.
2. Solve the following problems. Give all answers correct to 1 decimal place. Take $\pi = 3.14$ unless told otherwise.

a. An arc subtends an angle of $125°$ at the centre of a circle of radius 9 cm. Find the area of the sector.

b. The area of a sector is 690 cm^2. If the radius of the circle is 0.45 m, find the angle of the sector to the nearest degree.

c. A circle has a radius of 8 cm. The length of an arc of the circle is 6.5cm. Calculate the area of the sector.

d. A sector of $65°$ was removed from a circle of radius 15 cm. What is the area of the circle left?



3. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.
Solutions:

a.
$$\begin{aligned} A &= \frac{\theta}{360} \times \pi r^2 \\ &= \frac{125}{360} \times 3.14 \times 9^2 \\ &= 88.3125 \\ A &= 88.31 \text{ cm}^2 \end{aligned}$$

The area of the sector is 88.3 cm^2 to 1 d.p.

b.
$$\begin{aligned} A &= \frac{\theta}{360} \times \pi r^2 \\ 690 &= \frac{\theta}{360} \times 3.14 \times 45^2 \qquad 0.45 \text{ m} = 45 \text{ cm} \\ \theta &= \frac{690 \times 360}{3.14 \times 45^2} \\ &= 39.07 \end{aligned}$$

The angle subtended by the sector is $39°$ to the nearest degree.

c.
$$\begin{aligned} A &= \frac{\theta}{360} \times \pi r^2 \\ \text{Length of arc} &= \frac{\theta}{360} \times 2\pi r \qquad \text{from a previous lesson} \\ 6.5 &= \frac{\theta}{360} \times 2 \times 3.14 \times 8 \\ \theta &= \frac{6.5 \times 360}{2 \times 3.14 \times 8} \\ &= 46.576 \end{aligned}$$

Angle of sector, θ = $46.576° = 46.6°$

Area of sector $= \frac{\theta}{360} \times \pi r^2$

$= \frac{46.6}{360} \times 3.14 \times 8^2$

$= 26.013$

Area of sector $= 26.0$ cm^2

The area of the sector is 26.0 cm^2 to 1 d.p.

d. Area of sector $= \frac{\theta}{360} \times \pi r^2$

$\theta = 360 - 65$ From given diagram.

$\theta = 295°$

Area of sector $= \frac{295}{360} \times 3.14 \times 15^2$

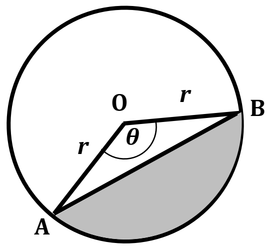
$= 578.9375$

Area of sector $= 579.4$ cm^2

The area of the remaining circle is 579.4 cm^2 to 1 d.p.

Closing *(2 minutes)*

1. Tell pupils that the relationships to find lengths of arcs and areas of sectors are very important in understand other topics they will experience on circle geometry.
2. For homework, have pupils do the practice activity PHM3-L014 in the Pupil Handbook.

<table>
<tr><td>Lesson Title: Area of a segment</td><td colspan="2">Theme: Geometry</td></tr>
<tr><td>Lesson Number: M3-L015</td><td>Class: SSS 3</td><td>Time: 40 minutes</td></tr>
<tr><td>Learning Outcome
By the end of the lesson, pupils will be able to calculate the area of the segment of a circle.</td><td colspan="2">Preparation
Draw the diagram on the board:
</td></tr>
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Opening *(2 minutes)*

1. Ask pupils to write down the formula for the area of a sector in their exercise books.
2. Ask a volunteer to read their answer. (Answer: $A = \frac{\theta}{360} \times \pi r^2$)
3. Tell pupils that after today's lesson, they will be able to calculate the area of the segment of a circle.

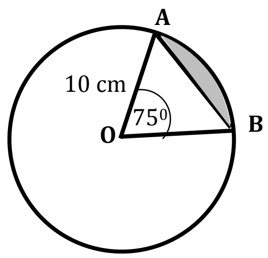
Teaching and Learning *(25 minutes)*

1. Remind pupils that the region between the chord AB and the arc AB is called the segment of the circle.
2. Tell pupils: The area of the segment of a circle is given by:

area of segment AB = area of sector OAB – area of triangle OAB

area of segment AB = $\frac{\theta}{360} \times \pi r^2 - \frac{1}{2} r^2 \sin\theta$

3. Write on the board: The circle shown has a radius of 10 cm. An arc subtends an angle of 75° at the centre of the circle. Find the area of the segment AB. Take $\pi = 3.14$. Give your answer to 2 significant figures.



Solution:

area of segment AB = $\frac{\theta}{360} \times \pi r^2 - \frac{1}{2} r^2 \sin\theta$

area of sector OAB = $\frac{75}{360} \times 3.14 \times 10^2$ Substitute $\theta = 75, r = 10$

= 65.42 cm^2 Simplify

area of triangle OAB = $\frac{1}{2} 10^2 \times \sin 75$ Substitute $\theta = 75, r = 10$

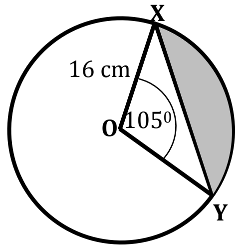
= 48.30 cm^2 Simplify

area of segment $\mathrm{AB} = 65.42 - 48.30$

$= 17.12$

The area of the segment $\mathrm{AB} = 17$ cm^2 to 2 s.f.

4. Solve another problem on the board:
The diagram shows sector OXY of a circle with a radius of 16 cm. An arc XY subtends an angle of 105° at the centre of the circle. Find: a. The length of the arc XY; b. The area of the sector OXY; c. The area of the shaded segment XY. Take $\pi = 3.14$. Give your answers to 2 decimal places.



Solution:

a. length of arc $= \frac{\theta}{360} \times 2\pi r$

$= \frac{105}{360} \times 2 \times 3.14 \times 16$ Substitute $\theta = 105, r = 16$

$= 20.307$ Simplify

The length of the arc $\mathrm{XY} = 20.31$ cm to 2 d.p.

b. area of sector OXY $= \frac{\theta}{360} \times \pi r^2$

$= \frac{105}{360} \times 3.14 \times 16^2$ Substitute $\theta = 105, r = 16$

$= 234.453$ Simplify

The area of sector $\mathrm{OXY} = 234.45$ cm^2 to 2 d.p..

c. area of segment XY $=$ area of sector OXY $-$ area of triangle OXY

$= \frac{\theta}{360} \times \pi r^2 - \frac{1}{2} r^2 \sin\theta$

area of sector OXY $= 234.45$ cm^2 From part a.

area of triangle OXY $= \frac{1}{2} r^2 \sin\theta$

$= \frac{1}{2} \times 16^2 \times \sin 105$

$= 123.639$

$= 123.64$ cm^2

$= 234.45 - 123.64$

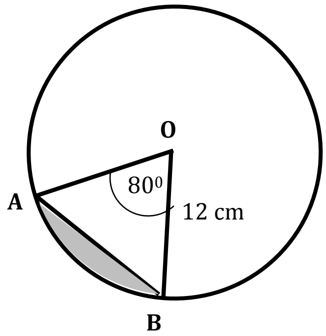
area of segment XY $= 110.81$ cm^2

The area of the segment $\mathrm{XY} = 110.81$ cm^2 to 2 d.p.

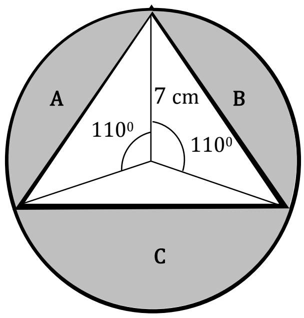
Practice *(12 minutes)*

1. Write the questions below on the board.
   Ask pupils to work individually to answer the questions.
2. Solve the following problems:

   a. The given circle has a radius of 12 cm. The sector AB has an angle of 80°. Find the area of the segment. Give your answer to 3 significant figures. Take $\pi = 3.14$.



   b. The given circle has a radius of 7 cm. The sectors shown each have an angle of 110°. Find to 2 decimal places: a. The area of the 3 shaded segments; b. The total area shaded in the circle. Take $\pi = 3.142$.



3. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.
   Solutions:

a.

$$\begin{aligned}\text{area of segment} &= \left(\frac{\theta}{360} \times \pi r^2\right) - \left(\frac{1}{2} r^2 \sin\theta\right)\\ &= \left(\frac{80}{360} \times 3.14 \times 12^2\right) - \left(\frac{1}{2} \times 12^2 \times \sin 80\right)\\ &= 100.48 - 70.906\\ \text{area of segment} &= 29.574\end{aligned}$$

The area of the segment is 29.6 cm^2 to 3 s.f.

b.
i.

$$\begin{aligned}\text{area of segment A} &= \left(\frac{\theta}{360} \times \pi r^2\right) - \left(\frac{1}{2} r^2 \sin\theta\right)\\ &= \left(\frac{110}{360} \times 3.142 \times 7^2\right) - \left(\frac{1}{2} \times 7^2 \times \sin 110\right)\\ &= 47.043 - 23.022\\ \text{area of segment A} &= 24.021 \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{angle of sector A} &= \text{angle of sector B}\\ \therefore \text{area of segment A} &= \text{area of segment B}\\ \text{area of segment B} &= 24.021 \text{ cm}^2\end{aligned}$$

area of segment C $= \left(\frac{\theta}{360} \times \pi r^2\right) - \left(\frac{1}{2} r^2 \sin\theta\right)$

$= \left(\frac{140}{360} \times 3.142 \times 7^2\right) - \left(\frac{1}{2} \times 7^2 \times \sin 140\right)$

$= 59.873 - 15.748$

area of segment C $= 44.125$ cm^2

ii. Total shaded area = area of segment A +area of segment B +area of s

$= 24.021 + 24.021 + 44.125$

$= 92.167$

$= 92.17$ cm^2

The total shaded area is 92.17 cm^2.

Closing *(1 minute)*

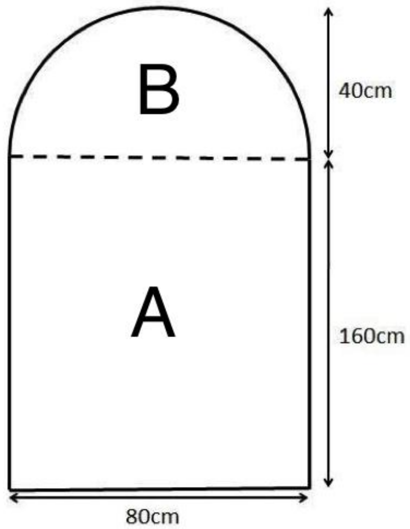
1. For homework, have pupils do the practice activity PHM3-L015 in the Pupil Handbook.

Lesson Title: Area and perimeter of composite shapes	**Theme:** Geometry	
Lesson Number: M3-L0016	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to solve problems involving areas and perimeters of composite shapes.	**Preparation** None	

Opening *(1 minute)*

1. Tell pupils that after today's lesson, they will be able to solve problems involving areas and perimeters of composite shapes.

Teaching and Learning *(25 minutes)*



1. Draw the shape at right shown on the board.
2. Explain to pupils that composite shapes are shapes made up of one or more types of shapes.
3. Ask pupils if they can see the different shapes which make up the shape on the board.
4. Select a volunteer to answer. (Example answers: rectangle; semi-circle).
5. Explain:
   - To find the area or perimeter of the shape, first divide it into its individual parts.
   - Find any missing lengths of sides.
   - Find the individual areas or perimeters using the appropriate formula. For perimeter, only include those sides on the outside of the shape. Do not include shared sides that are inside the shape.
   - Finally, add the areas or perimeters together.
6. Find the area and perimeter of the shape on the board. Round answers to the nearest cm or cm^2.

   Solution:

   Divide the shape into a rectangle A and a semi-circle B.

perimeter of shape	=	perimeter of A + perimeter of B	
perimeter of A		$2l + w$	Since the top line of the rectangle $w = 80$ cm is not counted
	=	$(2 \times 160) + 80$	
	=	$320 + 80$	
	=	400 cm	
perimeter of B	=	πr	B is a semi-circle, take half of the perimeter of a circle

$= 3.14 \times 40$ Take $\pi = 3.14$

$= 125.6$ cm

perimeter of shape $= 400 + 125.6$

$= 525.6$

$= 526$ cm to the nearest cm

The perimeter of the shape is 526 cm.

area shape = area of A + area of B

area of A $= lw$

$= 160 \times 80$

$= 12{,}800$ cm^2

area of B $= \frac{1}{2} \times \pi r^2$ B is a semi-circle

$= \frac{1}{2} \times 3.14 \times 40^2$

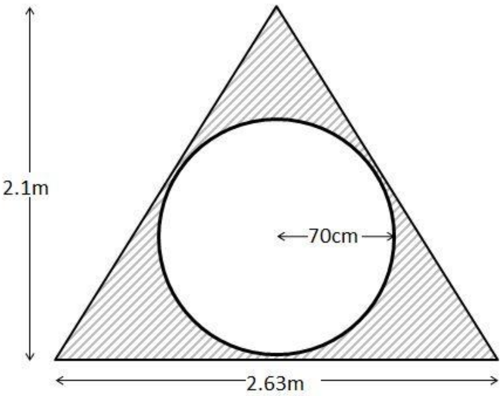
$= 2512$ cm^2

area of shape $= 12{,}800 + 2512$

$= 15{,}312$

The area of the shape is 15,312 cm^2.

7. Write on the board:
   The diagram shows a circular hole punched into a triangular piece of wood. Calculate the area of the shaded section. Give your answer to the 1 decimal place. Take $\pi = 3.14$.



   Solution:
   Let A be the triangle, B the circle

area of shaded section = area of A – area of B

area of A $= \frac{1}{2}bh$

$= \frac{1}{2} \times 2.63 \times 2.1$

$= 2.7615$ m^2

area of B $= \pi r^2$

$= 3.14 \times 0.7^2$ 70 cm = 0.7 m

$= 1.5386$ m^2

area of shaded section $= 2.7615 - 1.5386$

$= 1.2229$

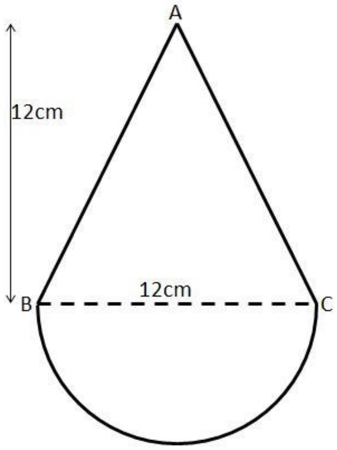
The area of the shaded section is 1.2 m^2 to 1 decimal place.

Practice *(10 minutes)*

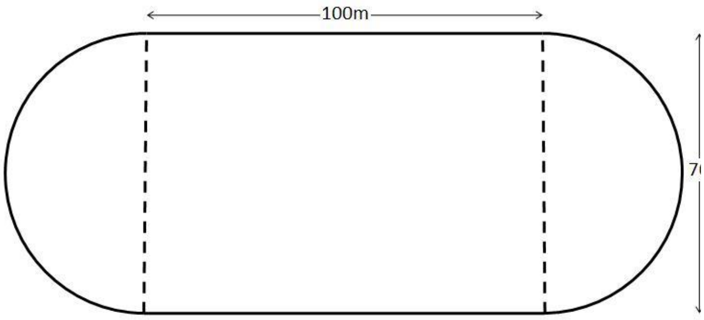
1. Write the questions below on the board.

2. Ask pupils to work individually to answer the questions.
3. Solve the following problems. Find the area and perimeter around the outer edges of the given shapes. Give answers to 1 decimal place. Take $\pi = 3.14$.

a.



b.



4. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.

Solutions:

a. Let T be the triangle, S the semi-circle

perimeter of shape = perimeter of T + perimeter of S

To calculate the perimeter of T, first find the length of side AB. In the perimeter formula, only include sides on the outside of the shape, AB and AC. Draw a perpendicular line from A and mark point D on BC

$|AB|^2 = |AD|^2 + |BD|^2$

$= 12^2 + 6^2$ Since $|BD| = \frac{1}{2} \times |BC| = 6$ cm

$= 144 + 36$

$|AB|^2 = 180$

$|AB| = \sqrt{180}$

$= 13.416$ cm

perimeter of T $= |AB| + |AC|$

$= (2 \times |AB|)$ Since $|AC| = |AB|$

$= 2 \times 13.416$

perimeter of T $= 26.832$ cm

perimeter of S $= \pi r$ S is a semi-circle, take half of the perimeter of a circle

$= 3.14 \times 6 = 18.84$ Take $\pi = 3.14$

perimeter of shape $= 26.832 + 18.84$

$= 45.672$ cm

The perimeter of the shape is 45.7 cm to 1 d.p.

area shape = area of T + area of S

area of T = $\frac{1}{2}bh$

= $\frac{1}{2} \times 12 \times 12$

= $\frac{1}{2} \times 144$

= 72 cm^2

area of S = $\frac{1}{2} \times \pi r^2$ B is a semi-circle

= $\frac{1}{2} \times 3.14 \times 6^2$

= 56.52 cm^2

area of shape = $72 + 56.52$

= 128.52 cm^2

The area of the shape is 128.5 cm^2 to 1 d.p.

b. The shape is made up of 2 semi-circles A and B and a rectangle C.

perimeter of shape = perimeter of A + perimeter of B + perimeter of C

= 2 × (perimeter of A) + perimeter of C

2 × perimeter of A = $2\pi r$ The 2 semi-circles make a complete circle of radius $r = \frac{70}{2}$.

= $2 \times 3.14 \times 35$

= 219.8 cm

perimeter of C = $2l$ Outer edges of shape only

= 2×100

= $200\ m$

perimeter of shape = $219.8 + 200$

= 419.8

The perimeter of the shape is 419.8 cm to 1 d.p.

area shape = area of A + area of B + area of C

= 2 × (area of A) + area of C

= $\pi r^2 + lw$

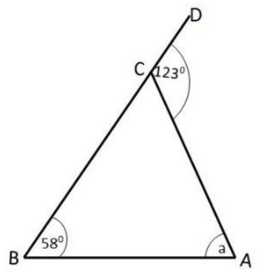
= $3.14 \times 35^2 + (100 \times 70)$

area of shape = $10{,}846.5$ cm^2

The area of the shape is 10,846.5 cm^2.

Closing *(4 minutes)*

1. Ask 3 to 5 volunteers to tell the class one new thing they learned during the lesson. (Answers: various)
2. For homework, have pupils do the practice activity PHM3-L016 in the Pupil Handbook.

Lesson Title: Circle Theorem 1	**Theme:** Geometry	
Lesson Number: M3-L017	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to identify and demonstrate that: A straight line from the centre of a circle that bisects a chord is at right angles to the chord.	**Preparation** Draw the diagram on the right on the board: 	

Opening *(4 minutes)*

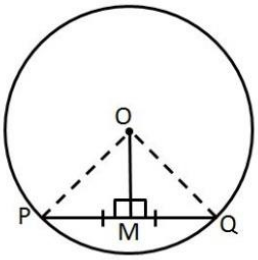
1. Ask pupils to look at the triangle on the board and find the missing angle a. Ask them to give the reason for their answer.
2. Invite a volunteer to answer. (Answer: $a = 65°$. Since $\angle ACD = 123 = 58 + a$; exterior angle equals the sum of the interior angle in the other 2 vertices.)
3. Tell pupils that after today's lesson, they will be able to show that a straight line from the centre of a circle that bisects a chord is at right angles to the chord.

Teaching and Learning *(20 minutes)*

1. Explain:
   - As we saw in the example we just did, we use rules to determine lengths and angles in triangles.
   - In the same way, rules are used to determine lengths and angles in circles.
   - These rules are called **theorems**. They are true for every circle regardless of the size of the circle.
   - We will be spending the next few weeks looking at circle theorems.
   - We will be using the rules we already know about triangles and other polygons to help us find missing sides and angles in circles,
   - We have numbered them in the order we will be working with them. They may be numbered differently in other textbooks.
2. Write on the board:

   Circle Theorem 1: A straight line from the centre of a circle that bisects a chord is at right angles to the chord
3. Ask pupils to raise their hand if they understand what the word "bisect" means.

4. Invite a volunteer to answer. (Example answer: to cut in half; to divide into 2 equal parts.)
5. Draw the diagram at right on the board.
6. Write on the board:



Given: Circle with the centre O and line OM to mid-point M on chord PQ such that $|PM| = |QM|$.
To prove: OM ⊥ PQ

Proof:

In both Δ OMP and Δ OMQ

$\|OP\|$	=	$\|OQ\|$	equal radii
$\|PM\|$	=	$\|QM\|$	given
$\|OM\|$	=	$\|OM\|$	common side
∴ ΔOMP	=	ΔOMQ	SSS
∴ ∠OMP	=	∠OMQ	
∠OMP + ∠OMQ	=	$180°$	angles on a straight line
∴ ∠OMP = ∠OMQ	=	$90°$	
Therefore $OM \perp PQ$			line from centre to mid-point ⊥, (a)

7. Explain:
   - The proof shows step by step how we use previous knowledge from triangles to find the required result.
   - We give reasons for the mathematical statement we make by using a known fact, e.g. the equal radii OA and OB, or angles on a straight line $= 180°$.
   - When we solve problems we are often asked to give reasons for our answers.
   - Use the statement **"line from centre to mid-point ⊥"** when using this theorem to solve circle problems.
   - The ⊥ means "is perpendicular" or "is at right angles" to the chord.
8. Refer to line OM to demonstrate the converse theorems. Write them on the board.
   - If a line is drawn from the centre of a circle perpendicular to a chord, then it bisects the chord. The statement is **"⊥ from centre bisects chord"**, (b).
   - If the perpendicular bisector of a chord is drawn, then the line will pass through the centre of the circle, **"⊥ bisector passes through centre"**, (c).
9. Tell pupils to use statements (a), (b) and (c) when referring to these theorems in solving problems.
10. Write the following problem on the board: The radius of a circle is 12 cm. The length of a chord of the circle is 18 cm. Calculate the distance of the mid-point of the chord from the centre of the circle. Give your answer to the nearest cm.
11. Tell pupils the first step is to assess the problem and write down all the given information.

    Solution:
    Using the previous circle:

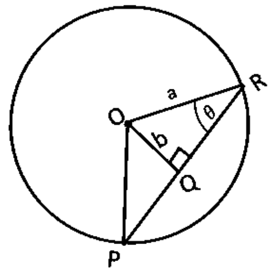
    M = mid-point of PQ
    |MQ| = 9 cm $\quad \frac{1}{2}\times|PQ| = \frac{18}{2} = 9$
12. Ask pupils what facts can we use to calculate the length of OQ?

13. Invite a volunteer to answer. (Example answers: Line from O is perpendicular to the mid-point of chord PQ; $\angle OMQ = 90°$; Pythagoras' Theorem for Δ OMQ.)
14. Write the solution on the board, giving reasons for the mathematical statements:

$\angle OMQ$	=	$90°$	line from centre to mid-point $\perp$
$\lvert OQ\rvert^2$	=	$\lvert OM\rvert^2 + \lvert MQ\rvert^2$	Pythagoras' Theorem
12^2	=	$\lvert OM\rvert^2 + 9^2$	substitute $\lvert OQ\rvert = 12$ cm, $\lvert MQ\rvert = 9$ cm
$\lvert OM\rvert^2$	=	$12^2 - 9^2$	
	=	$144 - 81$	
$\lvert OM\rvert$	=	$\sqrt{63} = 7.94$	
$\lvert OM\rvert$	=	8 cm	

The distance from the mid-point of the chord to the centre of the circle is 8 cm to the nearest cm.

15. Write on the board: PR is a chord of a circle with centre O and mid-point Q. Calculate $\angle POR$ if $\angle OPQ$ is a) 27°; b) 36°.



16. Tell pupils we will solve the problems step by step giving reasons for the mathematical statements we make.

Solutions:

Using the diagram shown:

a.

$\angle OPQ$	=	$27°$	given
$\angle OQP$	=	$90°$	line from centre to mid-point $\perp$
$\angle POQ$	=	$180 - 27 - 90$	sum of <s in a $\Delta = 180°$
$\angle POQ$	=	$63°$	
$\angle ROQ$	=	$63°$	ΔPOQ=ΔROQ
$\angle POR$	=	$\angle POQ + \angle ROQ$	
	=	$63 + 63$	
$\angle POR$	=	$126°$	

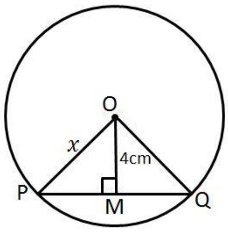
b.

$\angle OPQ$	=	$36°$	given
$\angle PQO$	=	$90°$	line from centre to mid-point $\perp$
$\angle POQ$	=	$180 - 36 - 90$	sum of <s in a $\Delta = 180°$
$\angle POQ$	=	$54°$	
$\angle ROQ$	=	$54°$	ΔPOQ=ΔROQ
$\angle POR$	=	$\angle POQ + \angle ROQ$	
	=	$54 + 54$	
$\angle POR$	=	$108°$	

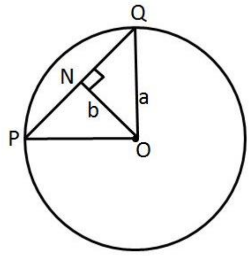
Practice *(15 minutes)*

1. Write the questions below on the board.
2. Ask pupils to work individually to answer the questions.

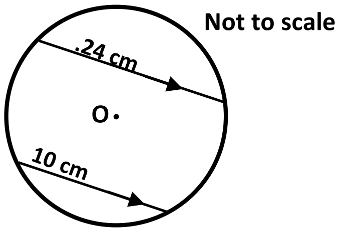
3. Solve the following circle problems. Give your answers to the nearest cm.
   a. The distance of the chord of a circle from the centre of the circle is 4 cm if the radius of the circle is 8 cm, calculate the length of the chord.



   b. A chord PQ of length 12 cm is 15 cm from the centre of the circle. Calculate the radius of the circle.



   c. Two parallel chords lie on opposite sides of the centre of a circle of radius 13 cm. Their lengths are 10 cm and 24 cm. What is the distance between the chords?



4. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.

Solutions:

a.	M	=	mid-point of |PQ|	
	$\angle OMQ$	=	$90°$	line from centre to mid-point ⊥
	$\|OQ\|^2$	=	$\|OM\|^2 + \|MQ\|^2$	Pythagoras' Theorem
	8^2	=	$4^2 + \|MQ\|^2$	
	$\|MQ\|^2$	=	$8^2 - 4^2$	substitute $\|OB\| = 8$ cm, $\|OM\| = 4$ cm
		=	$64 - 16$	
	$\|MQ\|$	=	$\sqrt{48}$	
		=	6.928	
	$\|MQ\|$	=	7 cm	
	$\|MQ\|$	=	$\|PM\|$	equal radii
	$\|PQ\|$	=	$\|PM\| + \|MQ\|$	
		=	$7 + 7$	
	$\|PQ\|$	=	14 cm	

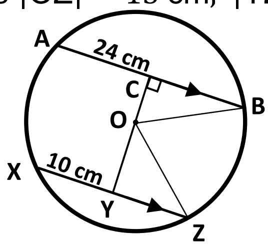
The length of the chord is 14 cm to the nearest cm.

b.	N	=	mid-point of |PQ|	
	$\angle ONQ$	=	$90°$	line from centre to mid-point ⊥
	$\|OQ\|^2$	=	$\|ON\|^2 + \|NQ\|^2$	Pythagoras' Theorem
	$\|OQ\|^2$	=	$\|15\|^2 + 6^2$	
	$\|OQ\|^2$	=	$225 + 36 = 261$	
	$\|OQ\|$	=	$\sqrt{261}$	
		=	16.155	
	$\|OQ\|$	=	16 cm	

The radius of the circle is 16 cm to the nearest cm.

c.	$\|XZ\|$	=	10 cm	given

Y	$=$	mid-point of $\|XZ\|$	$\perp$ from centre bisects chord
$\|YZ\|$	$=$	5 cm	
$\|OZ\|^2$	$=$	$\|OY\|^2 + \|YZ\|^2$	Pythagoras' Theorem
13^2	$=$	$\|OY\|^2 + 5^2$	substitute $\|OZ\| = 13$ cm, $\|YZ\| = 5$ cm
$\|OY\|^2$	$=$	$13^2 - 5^2$	
	$=$	$169 - 25$	
	$=$	144	
$\|OY\|$	$=$	$\sqrt{144}$	
	$=$	12 cm	
$\|AB\|$	$=$	24cm	
C	$=$	mid-point of $\|AB\|$	$\perp$ from centre bisects chord
$\|CB\|$	$=$	12 cm	
$\|OB\|^2$	$=$	$\|OC\|^2 + \|CB\|^2$	Pythagoras' Theorem
13^2	$=$	$\|OC\|^2 + 12^2$	substitute $\|OZ\| = 13$ cm, $\|CB\| = 12$ cm
$\|OC\|^2$	$=$	$13^2 - 12^2$	
	$=$	$169 - 144$	
	$=$	25	
$\|OC\|$	$=$	$\sqrt{25}$	
	$=$	5 cm	
$\|CY\|$	$=$	$\|OC\| + \|OY\|$	
	$=$	$12 + 5$	
$\|CY\|$	$=$	17 cm	



The distance between the two chords is 17 cm to the nearest cm.

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM3-L017 in the Pupil Handbook.

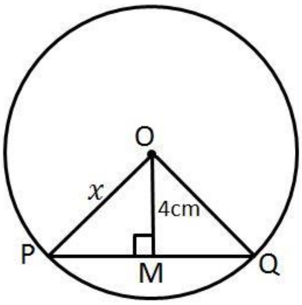
Lesson Title: Applications of Circle Theorem 1	**Theme:** Geometry	
Lesson Number: M3-L018	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to solve problems using Circle Theorem 1.	**Preparation** None	

Opening *(2 minutes)*

1. Ask for a volunteer to remind the class what the statement for Circle Theorem 1 says. (Answer: A straight line from the centre of a circle that bisects a chord is at right angles to the chord.)
2. Tell pupils that after today's lesson, they will be able to solve problems using Circle Theorem 1.

Teaching and Learning *(15 minutes)*

1. Write the following problem on the board.
   In the circle with centre O, OM ⊥ PQ, OM = 4 cm and PQ = 10 cm. Find x to 1 decimal place.

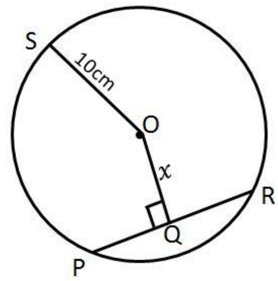


2. Ask pupils what facts we are given in the problem.
3. Invite volunteers to answer. (Example answers: That the line from the centre of the circle is perpendicular to chord PR; The lengths of line OQ and PR.)
4. Ask pupils how we can use this fact.
5. Invite different volunteers to answer. (Example answers: The line from the centre bisects the chord so the length of line PQ is half that of PR; PR is 5 cm.)
6. Write the solution on the board, giving reasons for the mathematical statements:
   Solution:

$\|PQ\|$	=	10 cm	
$\|MP\|$	=	5 cm	⊥ from the centre bisects the chord
$\|OP\|^2$	=	$\|OM\|^2 + \|MP\|^2$	Pythagoras' Theorem
	=	$4^2 + 5^2$	substitute $\|OM\| = 4$ cm , $\|PM\| = 5$ cm
	=	$16 + 25$	
$\|OP\|$	=	$\sqrt{41}$	
	=	6.403	
$\|OP\|$	=	6.4 cm	

x is 6.4 cm to 1 d.p.

7. Ask pupils to solve the next problem with seatmates. They must give reasons for the mathematical statements they make.
   In the circle with centre O and radius = 10 cm, OQ ⊥ PR and PR = 8 cm. Find x to the nearest cm.

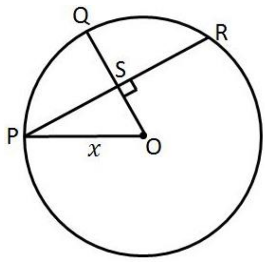


$\|PR\|$	$=$	8 cm	given
$\|PQ\|$	$=$	4 cm	⊥ from the centre bisects the chord
$\|OP\|$	$=$	10 cm	
$\|OP\|^2$	$=$	$\|OQ\|^2 + \|QP\|^2$	Pythagoras' theorem
$\|OQ\|^2$	$=$	$\|OP\|^2 - \|QP\|^2$	substitute $\|OP\| = 10$ cm, $\|QP\| = 4$ cm
$\|OQ\|^2$	$=$	$10^2 - 4^2$	
	$=$	$100 - 16$	
$\|OQ\|$	$=$	$\sqrt{84}$	
	$=$	9.165	
$\|OQ\|$	$=$	9 cm	

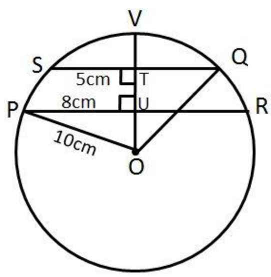
x is 9 cm to the nearest cm.

Practice *(20 minutes)*

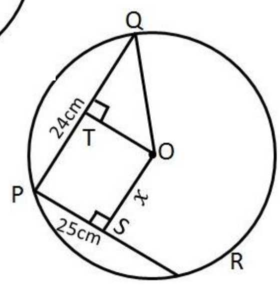
1. Write the questions below on the board.
2. Ask pupils to work individually to answer the questions.
3. Solve the following circle problems:
   a. In the circle with centre O, OQ ⊥ PR, PR = 12 cm and SQ = 2 cm. Find x.



   b. In the circle with centre O, OT ⊥ SQ, OT ⊥ PR, OP = 10 cm, ST = 5 cm and PU = 8 cm. Find the length of TU.



   c. In the circle with centre O, OT ⊥ QP, OS ⊥ PR, OT = 5 cm, PQ = 24 cm and PR = 25 units. Find the length of OS = x.



d. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.
   Solutions:

   a.

$\|PR\|$	$=$	12cm	
$\|SP\|$	$=$	6 cm	⊥ from the centre bisects the chord

|OS|	=	$x - 2$ cm	
$\|OP^2\|$	=	$\|OS\|^2 + \|SP\|^2$	Pythagoras' theorem
x^2	=	$(x-2)^2 + 6^2$	substitute $\|OP\| = x, \|OS\| = x - 2$
x^2	=	$x^2 - 4x + 4 + 36$	
x^2	=	$x^2 - 4x + 40$	
$4x$	=	40	
x	=	10	

x is 10 cm

b.

|TU|	=	|OT| – |OU|	
$\|OQ\|^2$	=	$\|OT\|^2 + \|TQ\|^2$	Pythagoras' theorem
$\|OT\|^2$	=	$\|OQ\|^2 - \|TQ\|^2$	since $\|TQ\| = \|ST\| = 5$ cm
	=	$10^2 - 5^2$	since $\|OQ\| = \|OP\| = 10$ cm
	=	$100 - 25 = 75$	
|OT|	=	$\sqrt{75}$	
|OT|	=	8.66 cm	
$\|OP\|^2$	=	$\|OU\|^2 + \|UP\|^2$	Pythagoras' theorem
$\|OU\|^2$	=	$\|OP\|^2 - \|UP\|^2$	
	=	$10^2 - 8^2$	substitute $\|UP\| = 8$ cm
	=	$100 - 64 = 36$	
|OU|	=	$\sqrt{36}$	
|OU|	=	6 cm	
|TU|	=	$8.66 - 6$	
|TU|	=	2.66 cm	

|TU| is 2.66 cm

c.

$\|OP\|^2$	=	$\|OS\|^2 + \|SP\|^2$	Pythagoras' theorem
$\|OS\|^2$	=	$\|OP\|^2 - \|SP\|^2$	
$\|OP\|^2 = \|OQ\|^2$	=	$\|OT\|^2 + \|TQ\|^2$	Pythagoras' theorem, |OP| = |OQ|
	=	$5^2 + 12^2$	|OT| – 5 cm, |TQ| = |PQ| = 12 cm
	=	$25 + 144$	
|OP|	=	$\sqrt{169}$	
|OP|	=	13 cm	
$\|OS\|^2$	=	$\|OP\|^2 - \|SP\|^2$	Pythagoras' theorem
	=	$13^2 - 12.5^2$	|SP| = |SR| = 12.5 cm
	=	$169 - 156.25$	
x	=	$\sqrt{12.75}$	
	=	3.57	
x	=	3.6 cm	

x is 3.6 cm

Closing *(3 minutes)*

1. Ask pupils to write down in their exercise books one thing they will need more practise with.
2. Allow pupils 2 minutes to write this down.
3. For homework, have pupils do the practice activity PHM3-L018 in the Pupil Handbook.

Lesson Title: Circle Theorem 2	**Theme:** Geometry	
Lesson Number: M3-L019	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to identify and demonstrate: The angle subtended at the centre of a circle is twice that subtended at the circumference.	**Preparation** Write on the board: **Circle Theorem 2** **The angle subtended at the centre of a circle is twice that subtended at the circumference.**	

Opening *(1 minute)*

1. Tell pupils that after today's lesson, they will be able to identify and demonstrate: Circle Theorem 2 which states that the angle subtended at the centre of a circle is twice that subtended at the circumference.

Teaching and Learning *(22 minutes)*

1. Draw the 3 circles shown below on the board (Figures a., b. and c.).
2. Explain and demonstrate on each of the circles drawn:
   - Arc AB subtends ∠ AOB at the centre of the circle.
   - Depending on the position of point P on the circle, we can have 3 different ways of how ∠ APB is formed at the circumference of the circle.
   - The 3 ways are shown on the circles on the board
3. Write on the board:
   Given: Circle with centre O, arc AB subtending ∠AOB at the centre of the circle, and ∠APB at the circumference.
   To prove: ∠AOB=2 × ∠APB

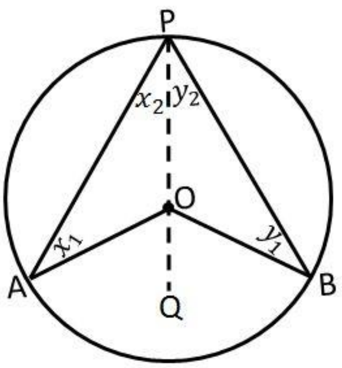


Figure a

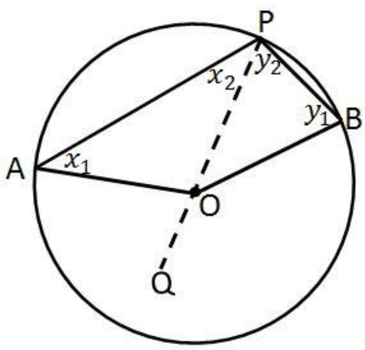


Figure b

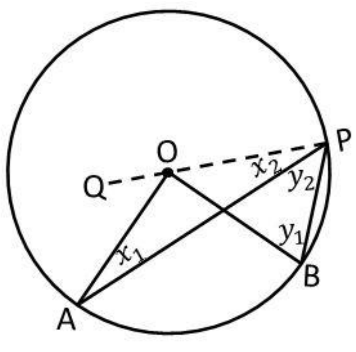


Figure c

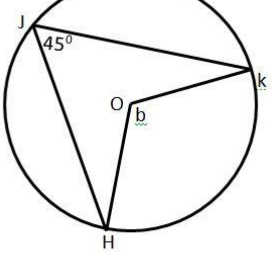
4. Write on the board, explaining the fact or theorem for each mathematical statement.
5. Tell pupils to pay particular attention to the explanation for Figure b.
   Proof:

$\lvert OA \rvert$	$=$	$\lvert OP \rvert$	equal radii
x_1	$=$	x_2	equal base angles of isosceles Δ

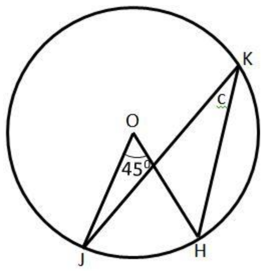
	∠AOQ	=	$x_1 + x_2$	exterior ∠ = sum of interior opposite ∠s
	∠AOQ	=	$2x_2$	$(x_1 = x_2)$
Similarly,	∠BOQ	=	$2y_2$	
In Figure a	∠AOB	=	∠AOQ +∠BOQ	
In Figure b	reflex ∠AOB			
		=	$2x_2 + 2y_2$	
		=	$2(x_2 + y_2)$	
		=	2 × ∠APB	
In Figure c	∠AOB	=	∠BOQ − ∠AOQ	
		=	$2y_2 - 2x_2$	
		=	$2(y_2 - x_2)$	
		=	2 × ∠APB	
In every case,	∠AOB	=	2 × ∠APB	

6. Use the statement **∠ at centre = 2∠ at circumference** when referring to this theorem in solving problems.
7. Write the following problems on the board:
   Given O is the centre of the circle, determine the unknown angle in each of the circles below. Mark the arc subtended by the angles.

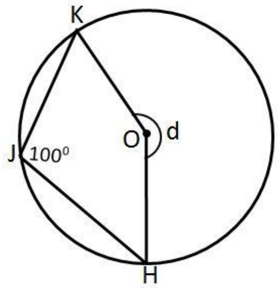
a.



b.

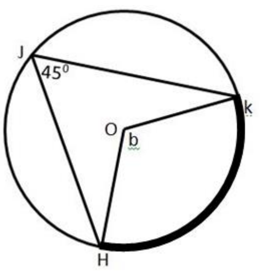


c.



Solutions:

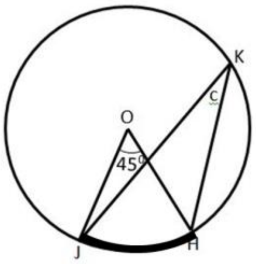
a.



$b = 2 \times 45°$
$b = 90°$

∠ at the centre = 2∠ at the circumference

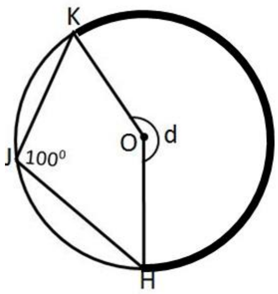
b.



$c = \frac{1}{2} \times 45°$
$c = 22.5°$

∠ at the centre = 2∠ at the circumference

c.



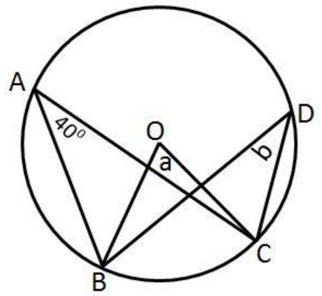
$$d = 2 \times 100°$$
$$d = 200°$$

∠ at the centre = 2∠ at the circumference

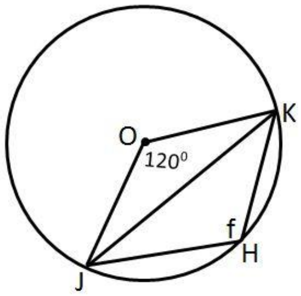
Practice *(15 minutes)*

1. Write the questions below on the board.
2. Ask pupils to work individually to answer the questions.
3. Given O is the centre of the circle, determine the unknown angle in each of the circles below. Mark the arc subtended by the angles.

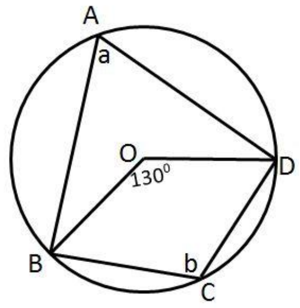
a.



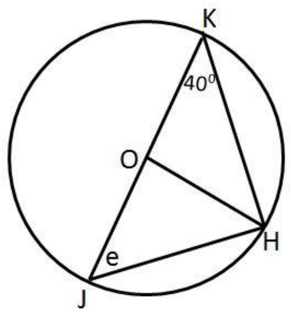
b.



c.

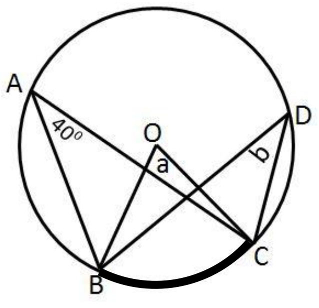


d.



Solutions:

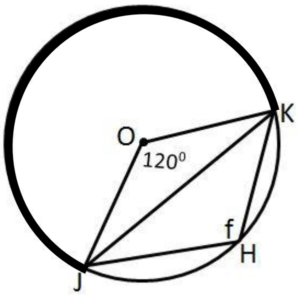
a.



$$a = 2 \times 40°$$
$$a = 80°$$
$$b = \frac{1}{2} \times 80°$$
$$b = 40°$$

∠ at the centre = 2∠ at the circumference

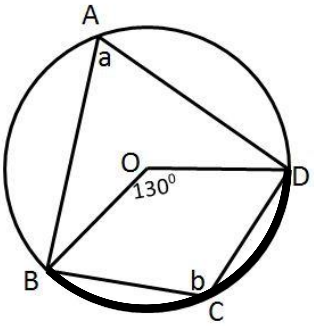
b.



$$\begin{aligned} \text{reflex } \angle JOK &= 360 - 120 \\ &= 240° \\ f &= \tfrac{1}{2} \times 240° \\ f &= 120° \end{aligned}$$

$\angle$ at the centre = 2$\angle$ at the circumference

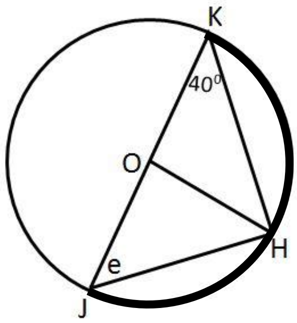
c.



$$\begin{aligned} a &= \tfrac{1}{2} \times 130 \\ a &= 65° \\ \text{reflex } \angle BOD &= 360 - 130 \\ &= 230° \\ b &= \tfrac{1}{2} \times 230 \\ b &= 115° \end{aligned}$$

$\angle$ at the centre = 2$\angle$ at the circumference

d.



$$\begin{aligned} \angle HJK + \angle JKH + \angle KHJ &= 180° \\ \angle HJK &= 180 - \angle KHJ - \angle JKH \end{aligned}$$

since $\angle JOK$	=	180°	$\angle$ straight line=180°
$\angle KHJ$	=	90°	$\angle$ at the centre = 2$\angle$ at the circumference
e	=	$180 - 90 - 40$	
e	=	50°	

4. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.

Closing *(2 minutes)*

1. Tell pupils that we now have 2 circle theorems for solving for lines and angles in circles.
2. Tell pupils that the questions will get more complex and we will need to use more than one theorem to solve a particular problem.
3. For homework, have pupils do the practice activity PHM3-L019 in the Pupil Handbook.

Lesson Title: Applications of Circle Theorem 2	Theme: Geometry	
Lesson Number: M3-L020	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to solve problems using Circle Theorem 2.	**Preparation** Write the Teaching and Learning questions (a., b. and c.) found at the end of this lesson plan on the board.	

Opening *(2 minutes)*

1. Tell pupils to raise their hands if they can answer this question: A $110°$ angle is subtended at the centre of a circle. What angle is subtended at the circumference?
2. Invite a volunteer to answer: (Answer: $55°$)
3. Ask for a volunteer to remind the class what the statement for Circle Theorem 2 says. (Answer: The angle subtended at the centre of a circle is twice that subtended at the circumference.)
4. Tell pupils that after today's lesson, they will be able to solve problems using this theorem.

Teaching and Learning *(15 minutes)*

1. Explain:
   - When solving circle problems, we first assess and extract from the problem the information we are given.
   - Next, we use theorems and the given information to find all equal angles and sides on the diagram, giving reasons for our answers.
   - Lastly we solve for the unknown value.
2. Solve question a. on the board.

   Solutions:

   a. **Step 1.** Assess and extract the given information from the problem.

   $\angle POR = 210°$

   Step 2. Use theorems and the given information to find all equal angles and sides on the diagram.

   $a = \frac{1}{2} \times 210$ ∠ at centre = 2∠ at circumference

   $a = 105°$

   reflex$\angle POR = 360 - 210$ sum of ∠s in a circle = $360°$

   $= 150°$

   Step 3. Solve for b.

   $b = \frac{1}{2} \times 150$ ∠ at centre = 2∠ at circumference

   $b = 75°$

3. Solve question b. on the board, giving reasons for the answers.

b.

$$\begin{aligned} \angle LMO &= 26° \\ d &= 180-90-26 && \text{sum of } \angle\text{s in } \Delta = 180° \\ d &= 64° \\ \angle LON &= 52° && \angle \text{ at centre} = 2\angle \text{ at circumference} \\ e &= 180-90-52 && \text{sum of } \angle\text{s in } \Delta = 180° \\ e &= 38° \end{aligned}$$

4. Ask pupils to solve question c. with seatmates.

c.

$$\begin{aligned} a &= 27 && \text{equal base } \angle\text{s of isosceles } \Delta\text{s} \\ b &= 180-27-27 && \text{sum of } \angle\text{s in } \Delta = 180° \\ b &= 126 \\ c &= \frac{1}{2}\times 126 && \angle \text{ at centre} = 2\angle \text{ at circumference} \\ c &= 63° \end{aligned}$$

Practice *(20 minutes)*

1. Write questions d. through g. found at the end of this lesson plan on the board.
2. Ask pupils to work individually to answer the questions.
3. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.

Solutions:

d.

Given angle $= 32°$

$$\begin{aligned} y &= 2\times 32 && \angle \text{ at centre} = 2\angle \text{ at circumference} \\ y &= 64° \\ w &= \frac{1}{2}\times 64 && \angle \text{ at centre} = 2\angle \text{ at circumference} \\ w &= 32° \end{aligned}$$

e.

Given angle $= 21°$

$$\begin{aligned} w &= 180-90-21 && \text{sum of } \angle\text{s in } \Delta = 180° \\ w &= 69° \\ x &= 2\times 69 && \angle \text{ at centre} = 2\angle \text{ at circumference} \\ x &= 138° \\ y &= 360-138 && \text{sum of } \angle\text{s in a circle} = 360° \\ y &= 222° \\ z &= \frac{1}{2}\times 222 && \angle \text{ at centre} = 2\angle \text{ at circumference} \\ z &= 111° \end{aligned}$$

f. Given angle $= 134°$

$$
\begin{aligned}
e &= 180 - 134 && \text{sum of } \angle\text{s in a straight line} = 180° \\
e &= 46° \\
f &= 2 \times 46 && \angle \text{ at centre} = 2\angle \text{ at circumference} \\
f &= 92° \\
92 + g + h &= 180° && \text{sum of } \angle\text{s in } \Delta = 180° \\
g &= h && \text{equal base } \angle\text{s of isosceles } \Delta\text{s} \\
92 + 2g &= 180° \\
g &= \frac{180 - 92}{2} \\
g &= 44°
\end{aligned}
$$

g. Given angle $= 43°$

$$
\begin{aligned}
a &= 43° && \text{vertically opposite } \angle\text{s} \\
a &= c \\
b &= 180 - 43 - 43 && \text{sum of } \angle\text{s in } \Delta = 180° \\
b &= 94 \\
d &= \frac{1}{2} \times 94 && \angle \text{ at centre} = 2\angle \text{ at circumference} \\
d &= 47°
\end{aligned}
$$

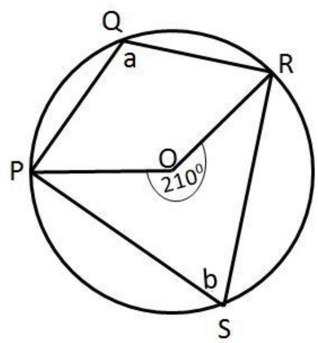
Closing *(3 minutes)*

1. Ask pupils to write down in their exercise books one thing they will need more practise with.
2. Allow pupils 2 minutes to write this down.
3. For homework, have pupils do the practice activity PHM3-L020 in the Pupil Handbook.

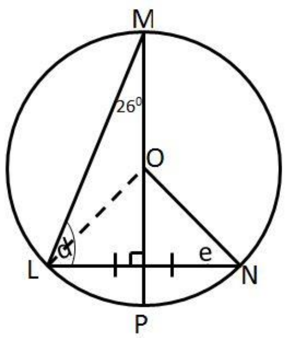
[QUESTIONS FOR TEACHING AND LEARNING]

Given O is the centre of the circle, find the unknown angles in each of the circles below.

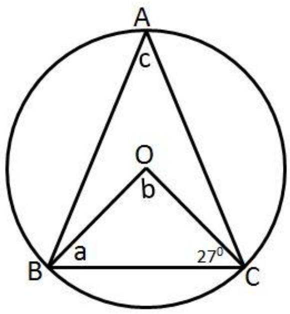
a.



b.



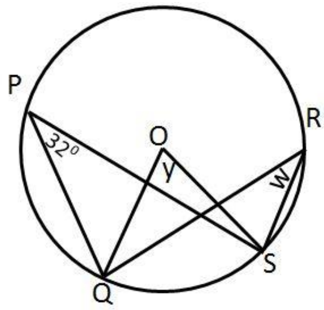
c.



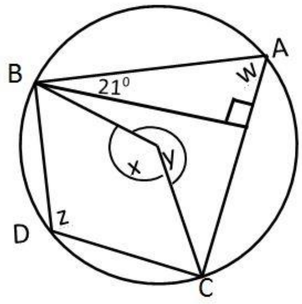
[QUESTIONS FOR PRACTICE]

Given O is the centre of the circle, find the unknown angles in each of the circles below.

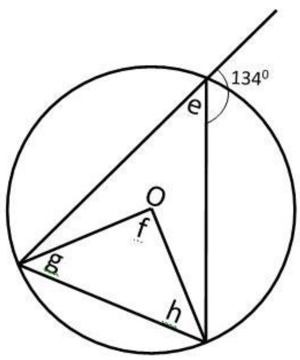
d.



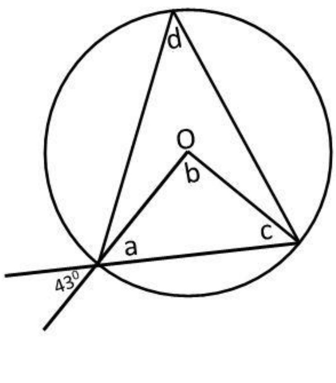
e.



f.



g.



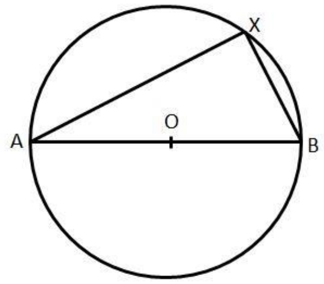
Lesson Title: Circle Theorems 3 and 4	**Theme:** Geometry	
Lesson Number: M3-L021	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcomes By the end of the lesson, pupils will be able to identify and demonstrate: 1. The angle in a semi-circle is a right angle. 2. Angles in the same segment are equal.	**Preparation** Write the statements for Circle Theorems 3 and 4 on the board.	

Opening *(1 minute)*

1. Invite volunteers to read the theorems written on the board out loud.
2. Tell pupils that after today's lesson, they will be able to identify and demonstrate these 2 theorems.

Teaching and Learning *(20 minutes)*

1. Draw the circle shown at right on the board.
2. Write on the board:
   Given: Circle with centre O and diameter AB. X is any point on the circumference of the circle.
   To prove: $\angle AXB$=90°

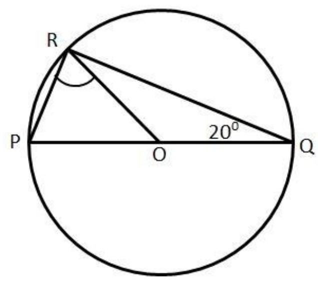


   Proof (Circle Theorem 3)

$$\begin{aligned} \angle AOB &= 2\angle AXB && \angle \text{ at centre} = 2\angle \text{ at circumference} \\ \angle AOB &= 180° && \angle \text{ straight line} = 180° \\ 2\angle AXB &= 180° \\ \angle AXB &= \frac{180°}{2} \\ \angle AXB &= 90° \end{aligned}$$

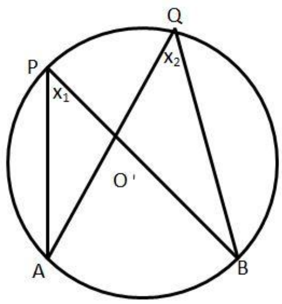
3. Ask pupils what this theorem tells us about the angle of the diameter of a circle in relation to its circumference.
4. Invite a volunteer to answer. (Answer: It shows that the diameter of a circle subtends a right angle at the circumference.)
5. Tell pupils to use the statement **∠ in semi-circle** when referring to this theorem in solving problems.
6. Tell pupils that this theorem is a special case of ∠ at centre = 2∠ at circumference

7. Solve the following problem on the board:
   P, Q and R are points on a circle, centre O. If $\angle RQO$=20°, what is the size of $\angle PRO$?
   Solution:



$$\begin{aligned} \angle QRO &= \angle RQO = 20° && \text{base } \angle\text{s of isosceles } \Delta \\ \angle PRO &= \angle PRQ - \angle QRO && \\ \angle PRO &= 90 - 20 && \angle \text{ in semi-circle} \\ \angle PRO &= 70° && \end{aligned}$$

8. Ask pupils to look at the solution to the problem. What other fact did we use to solve the problem?
9. Invite a volunteer to answer. (Answer: The base angles of an isosceles triangle are equal.)
10. Remind pupils that when solving circle problems, we will also use facts and rules from triangles and other polygons.
11. Tell pupils we will now look at Circle Theorem 4.
12. Draw the circle shown on the board.
13. Write on the board:
    Given: Circle with centre O with points P and Q on the circumference of the circle. Arc AB subtends $\angle APB$ and $\angle AQB$ in the same segment of the circle.
    To prove: $\angle APB$=$\angle AQB$

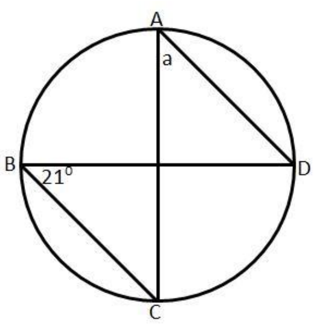


Proof (Circle Theorem 4)

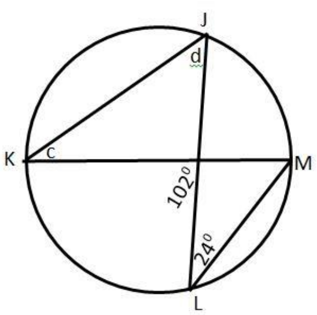
$$\begin{aligned} \angle AOB &= 2x_1 && \angle \text{ at centre} = 2\angle \text{ at circumference} \\ \angle AOB &= 2x_2 && \angle \text{ at centre} = 2\angle \text{ at circumference} \\ x_1 &= x_2 && \\ \angle APB &= \angle AQB && \end{aligned}$$

14. Tell pupils to use the statement **∠s in same segment** when referring to this theorem in solving problems.
15. Draw the circles shown below on the board.

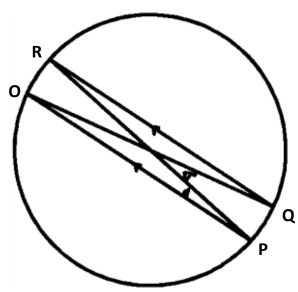
a.



b.



c.



16. Explain:
- We are going to use the theorems we have learned so far to find the values of the unknown angles on the board.
- We will also use any other facts we need from triangles and other polygons.
- We will give reasons for our answers.

Solutions:

a. $a = 21°$ ∠s in same segment

b. $c = 24°$ ∠s in same segment
$d = 102 - 24$ exterior ∠ of Δ = sum of interior opposite ∠s
$d = 78°$

17. Ask pupils to work with seatmates to find the unknown angle d in question c., giving reasons for our answers.
18. Give pupils a hint to get them started: Use the fact that PO is parallel to QR to find what angle d is equal to.
19. Allow 4 minutes for pupils to answer.
20. Invite a volunteer to show their solution on the board.

Solution:

$d = \angle PRQ$ alternate ∠s, PO ∥ QR

$\angle POQ = \angle PRQ$ ∠s in same segment

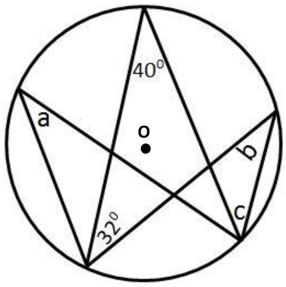
$\angle PRQ = \frac{1}{2} \times 17$ ∠ at centre = 2∠ at circumference

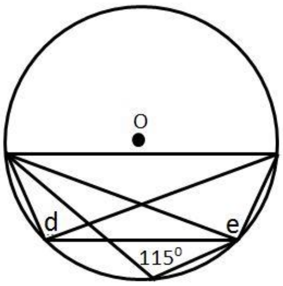
$= 8.5°$

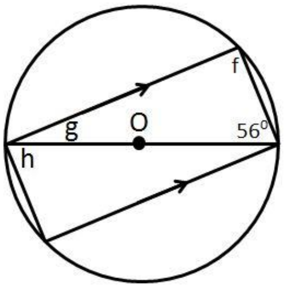
$\therefore d = 8.5°$

Practice *(15 minutes)*

1. Write the questions below on the board.
2. Ask pupils to work individually to answer the questions.
3. Find the unknown angles for each of the circles shown below. Point O is the centre of the circle. Give reasons for your answers.

a.


b.


c.


4. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.
   Solutions:

a.	a	$=$	$40°$	∠s in same segment
	a	$=$	b	∠s in same segment
	b	$=$	$40°$	
	c	$=$	$32°$	∠s in same segment
b.	d	$=$	$115°$	∠s in same segment
	e	$=$	$90°$	∠ in a semi-circle
c.	f	$=$	$90°$	∠ in a semi-circle
	g	$=$	$180-90-56$	∠ in a triangle
	g	$=$	$34°$	
	h	$=$	$56°$	alternate ∠s

Closing *(4 minutes)*

1. Write on the board: **Happy, Unhappy, Half-n-half**.
2. Ask pupils to use one of the words to describe how they feel about today's lesson.
3. Explain that pupils should write:
   - **Happy** if they understand the theorems and can answer questions easily.
   - **Unhappy** if they do not understand the theorems and cannot answer questions at all.
   - **Half-n-half** if they understand some of it and can attempt to answer questions.
4. Ask pupils to write the word on a blank piece of paper. They should not write their name on the paper.
5. Ask pupils to hand the piece of paper to you.
6. Use the responses received to see how much pupils understood of the theorems covered in this lesson.
7. Use the responses to plan the level of assistance required in the next lesson.
8. For homework, have pupils do the practice activity PHM3-L021 in the Pupil Handbook.

Lesson Title: Applications of Circle Theorems 3 and 4	**Theme:** Geometry	
Lesson Number: M3-L022	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to solve problems using Circle Theorems 3 and 4.	**Preparation** Write the Teaching and Learning questions (a., b. and c) found at the end of this lesson plan on the board.	

Opening *(4 minutes)*

1. Ask pupils who can remind the class what angle the diameter of a circle subtends at the circumference.
2. Invite a volunteer to answer. (Answer: The diameter of a circle subtends a right angle at the circumference.)
3. Ask pupils what theorem represents the answer.
4. Invite a volunteer to answer. (Answer: Angle in a semi-circle.)
5. Ask pupils what other theorem we looked at in the last lesson.
6. Invite a volunteer to answer. (Answer: Angles in the same segment (are equal.))
7. Tell pupils that after today's lesson, they will be able to solve problems with these 2 theorems.

Teaching and Learning *(20 minutes)*

1. Explain:
   - When solving circle problems, we first assess and extract from the problem the information we are given.
   - Next, we use theorems and the given information to find all equal angles and sides on the diagram, giving reasons for our answers.
   - Lastly we solve for the unknown value.
2. Solve questions a. on the board:

   Solution:

   Step 1. Assess and extract the given information from the problem.
   Given angle $= 65°$

   Step 2. Use theorems and the given information to find all equal angles and sides on the diagram.

   $i = 65°$ ∠s in same segment
   $i + j = 90$ ∠ in a semi-circle
   $65 + j = 90$

   Step 3. Solve for j.

   $j = 90 - 65$
   $j = 25°$

3. Ask pupils to look at question b. and identify which theorem they need to use to find angle k.
4. Invite a volunteer to answer. (Answer: Angle in a semi-circle).

5. Solve question b. on the board.
   Solution:

   Given angle $= 53°$

$$\begin{aligned} k + 53 &= 90 && \angle \text{ in a semi-circle} \\ k &= 90 - 53 \\ k &= 37° \\ 90 + 37 + l &= 180 && \angle \text{ in a triangle} \\ l &= 180 - 90 - 37 \\ l &= 53° \end{aligned}$$

6. Ask pupils to solve question c. with seatmates.
   Solution:

   Given angle $= 20°$

$$\begin{aligned} m &= 20° && \text{base } \angle\text{s of isosceles } \Delta \\ p &= m && \angle\text{s in same segment} \\ p &= 20° \\ n &= 90 - p && \angle \text{ in a semi-circle} \\ n &= 90 - 20 \\ n &= 70° \end{aligned}$$

7. Ask pupils to think back on the solutions so far. What comment can they make on the procedure to solve problems on circle theorems?
8. Invite volunteers to answer. (Example answers: Various but may include statements such as: You need to be systematic in the order to solve for the unknown angles; Know the rules for triangles, e.g. base angles of isosceles triangles are equal or sum of angles in a triangle $= 180°$).
9. Tell pupils as we continue to solve problems they will be able to identify the rules from triangles and polygons that are useful in solving circle problems.

Practice *(15 minutes)*

1. Write the questions d. through f. found at the end of this lesson plan on the board.
2. Ask pupils to work individually to answer the questions.
3. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.
   Solutions:

   d. Angles as given

$$\begin{aligned} q &= 180 - 80 - 40 && \angle\text{s in a triangle} \\ q &= 60° \\ r &= 180 - 80 - 10 - 40 && \angle\text{s in a triangle} \\ r &= 50° \\ s &= r && \angle\text{s in same segment} \\ s &= 50° \\ t &= 10° && \angle\text{s in a triangle} \\ u &= 80° && \angle\text{s in a triangle} \\ v &= 40° && \angle\text{s in a semi-circle} \end{aligned}$$

e. Given angle = $126°$

w	$=$	2×126	∠ at the centre=2∠ at the circumference
w	$=$	$252°$	
x	$=$	$360 - 252$	∠s in a circle
x	$=$	$108°$	
y	$=$	$\frac{1}{2} \times 108$	∠ at the centre=2∠ at the circumference
y	$=$	$54°$	
z	$=$	$180 - 54$	∠s in a straight line
z	$=$	$126°$	

f. Angles as given

a	$=$	$50°$	base ∠s of isosceles Δ
b	$=$	$180 - 50 - 50$	∠s in a triangle
b	$=$	$80°$	
c	$=$	$90 - 20 - 50$	∠ in a semi-circle
c	$=$	$20°$	
d	$=$	$50°$	∠s in same segment
e	$=$	$\frac{1}{2} \times b$	∠ at the centre =2∠ at the circumference
e	$=$	$\frac{1}{2} \times 80$	
e	$=$	$40°$	

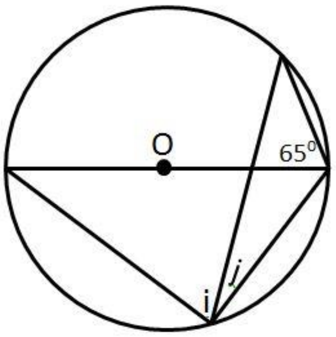
Closing *(1 minute)*

1. Tell pupils to do questions from practice activity PHM3-L022 in the Pupil Handbook for homework.

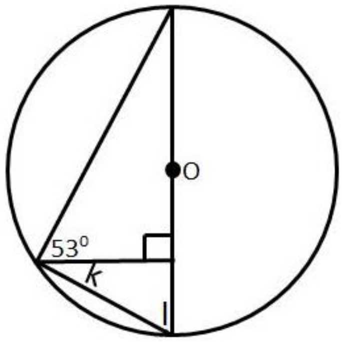
[QUESTIONS FOR TEACHING AND LEARNING]

Given O is the centre of the circle, determine the unknown angles in each of the circles below.

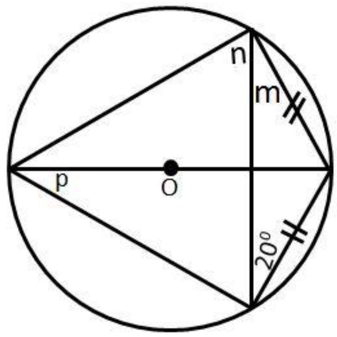
a.



b.



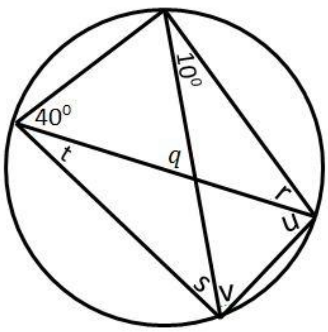
c.



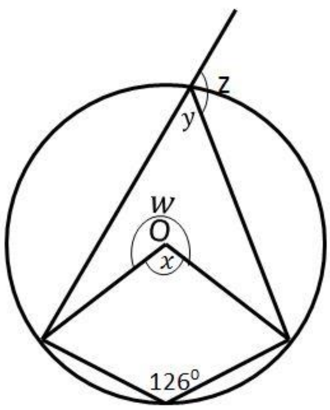
[QUESTIONS FOR PRACTICE]

Given O is the centre of the circle, determine the unknown angles in each of the circles below.

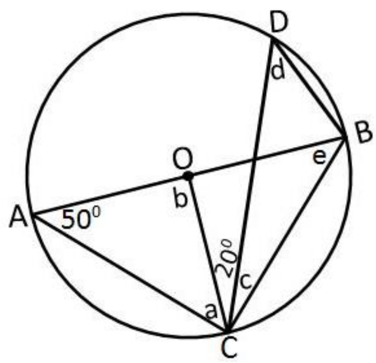
d.



e.



f.



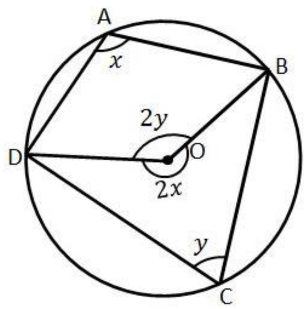
Lesson Title: Circle Theorem 5	**Theme:** Geometry	
Lesson Number: M3-L023	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to identify and demonstrate: Opposite angles of a cyclic quadrilateral are supplementary.	**Preparation** 1. Write the statements for Circle Theorem 5 on the board. 2. Draw the circle shown in the Proof on the board.	

Opening *(3 minutes)*

1. Ask pupils to give some properties they know as quadrilaterals.
2. Invite volunteers to answer. (Example answers: A closed shape with 4 sides; Includes squares, rectangles, trapeziums; some have special properties, e.g. a square has 4 equal sides and 4 equal $90°$ angles)
3. Tell pupils that after today's lesson, they will be able to identify and demonstrate that opposite angles of a cyclic quadrilateral are supplementary.

Teaching and Learning *(20 minutes)*

1. Invite a volunteer to remind the class what supplementary angles are. (Answer: angles which add up to $180°$.)
2. Show pupils the example of a cyclic quadrilateral on the board.
3. Ask pupils what they notice about the shape. (Example answer: All 4 vertices lie on the circumference of the circle.)
4. Write on the board:
   Given: Cyclic quadrilateral $ABCD$, with A, B, C and D on the circumference of the circle.
   To prove: $\angle BAD + \angle BCD = 180°$
   Proof:



$\angle BOD$	$= 2y$	∠ at the centre=2∠ at circumference
reflex $\angle BOD$	$= 2x$	∠ at the centre=2∠ at circumference
$\therefore 2x + 2y$	$= 360°$	$\angle s$ at a point
$\therefore x + y$	$= 180°$	
$\angle BAD + \angle BCD$	$= 180°$	

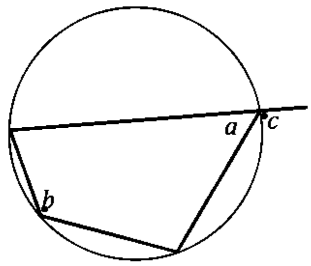
5. Explain:
   - We can show in the same way that: $\angle ABC + \angle ADC = 180°$.
   - Tell pupils to use the statement **opposite ∠s of cyclic quadrilateral** when referring to this theorem in solving problems.
6. Draw the circle for question a. found at the end of this lesson plan under QUESTIONS FOR TEACHING AND LEARNING on the board.
   Tell pupils we want to find the missing angle *a*.
   Solution:
   a. **Step 1.** Assess and extract the given information from the problem.
   given $\angle WXY = 106°, \angle XYZ = 87°$

Step 2. Use theorems and the given information to find all equal angles on the diagram.

Step 3. Solve for a and b

$$\begin{aligned} a + 87 &= 180 && \text{opposite } \angle\text{s of cyclic quadrilateral} \\ a &= 180 - 87 \\ a &= 93° \\ b + 106 &= 180 && \text{opposite } \angle\text{s of cyclic quadrilateral} \\ b &= 180 - 106 \\ b &= 74° \end{aligned}$$

7. Draw the circle shown on the right on the board.
8. Ask pupils to look at the circle and observe the angles marked *b* and *c*.
9. Tell pupils that these 2 angles are equal. Ask them if they can explain why.
10. Invite volunteers to answer. (Example answers: They are both supplementary angles to *a;* $b = c = 180 - a$.)



11. Explain:
    - The exterior angle of a cyclic quadrilateral is equal to the opposite interior angle.
    - Use the statement **exterior cyclic quadrilateral = opposite interior angle** when using this theorem to solve problems.
12. Draw the circle for question b. found at the end of this lesson plan on the board. Explain that we want to find the missing angle *a*.

    Solution:

given angle $= 114°$

$$\begin{aligned} a &= \angle HIJ && \angle \text{ exterior cyclic quadrilateral = opposite interior angle} \\ a &= 114° \end{aligned}$$

Practice *(16 minutes)*

1. Draw the circles in QUESTIONS FOR PRACTICE found at the end of this lesson plan on the board.
2. Ask pupils to work independently to find the unknown angles.
3. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.

    Solutions:

$$\begin{aligned} a &= 180 - 80 && \angle\text{s in a cyclic quadrilateral} \\ a &= 100° \\ b &= 180 - 110 && \angle\text{s in a cyclic quadrilateral} \\ b &= 70° \end{aligned}$$

$$\begin{aligned} a &= 180 - 60 && \angle\text{s in a cyclic quadrilateral} \\ a &= 120° \end{aligned}$$

$b = 180 - 105$ ∠s in a cyclic quadrilateral

$b = 75°$

$c = 180 - 31$ ∠s in a cyclic quadrilateral

$c = 149°$

$d = 180 - 57$ ∠s in a cyclic quadrilateral

$d = 123°$

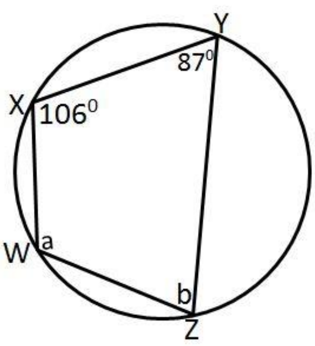
Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM3-L023 in the Pupil Handbook.

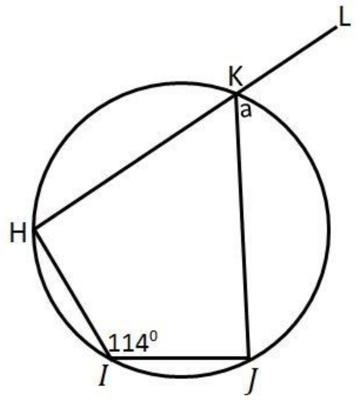
[QUESTIONS FOR TEACHING AND LEARNING]

Given O is the centre of the circle, determine the unknown angles in each of the circles below.

a.



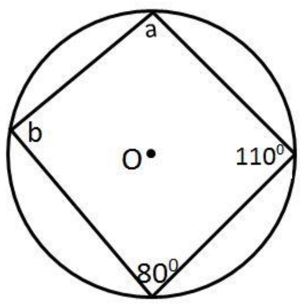
b.



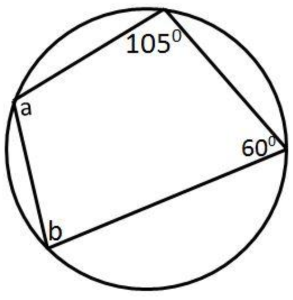
[QUESTIONS FOR PRACTICE]

Given O is the centre of the circle, determine the unknown angles in each of the circles below.

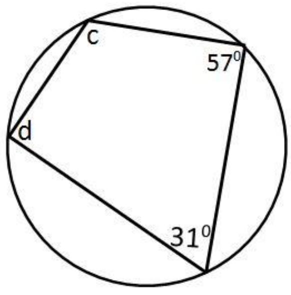
c.



d.



e.



Lesson Title: Applications of Circle Theorem 5	**Theme:** Geometry	
Lesson Number: M3-L024	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to solve problems using Circle Theorem 5.	**Preparation** Draw the circles (a. through f.) and write the directions found at the end of this lesson plan.	

Opening (*4 minutes)*

1. Invite a volunteer to remind the class what a cyclic quadrilateral is. (Answer: A quadrilateral with all 4 vertices on the circumference of a circle.)
2. Invite a volunteer to explain Circle Theorem 5 in their own words. (Example answers: Opposite angles of a cyclic quadrilateral are supplementary; opposite angles add up to $180°$.)
3. Tell pupils that after today's lesson, they will be able to solve problems involving cyclic quadrilaterals.

Teaching and Learning *(15 minutes)*

1. Tell pupils we will be solving problems in this lesson that will require us to use more than one theorem.
2. Refer to question a. on the board.
   Solution:
   Step 1. Assess and extract the given information from the problem.
   Write on the board: given $\angle ROP = 102°$
3. Explain:
   - We are required to find $\angle RQP$.
   - We can see from the diagram that we have an angle at the centre and 2 angles at the circumference.
   - This should lead us to the theorem ∠ at the centre = 2∠ at circumference
   - We also have a cyclic quadrilateral $PQRS$ that may be of use in finding the missing angle because opposite ∠s of cyclic quadrilateral $= 180°$.

   Step 2. Use theorems and the given information to find all equal angles on the diagram.

$$\begin{aligned} \angle RSP &= \frac{1}{2} \times 102 && \text{∠ at the centre = 2∠ at circumference} \\ \angle RSP &= 51° \\ \angle RQP + 51 &= 180 && \text{opposite ∠s of cyclic quadrilateral} \end{aligned}$$

   Step 3. Solve for $\angle RQP$.

$$\begin{aligned} \angle RQP &= 180 - 51 \\ \angle RQP &= 129° \end{aligned}$$

4. Refer to question b. on the board. Find the missing angle.

5. Invite a volunteer to tell the class what should be the first step. (Answer: Assess the problem and write down the given information.)

   Solution:

   b. Given $\angle VWX = 70°$

$\angle XYV + 70$	$=$	180	opposite ∠s of the cyclic quadrilateral
$\angle XYV$	$=$	$180 - 70$	
$\angle XYV$	$=$	$110°$	

6. Invite a volunteer to suggest what we should do next. (Example Answer: Find *j* using the isosceles triangle $\angle XYV$.

$2j + 110$	$=$	$180°$	sum of ∠s in an isosceles. ∆
j	$=$	$\frac{180 - 110}{2}$	
j	$=$	$35°$	

7. Ask pupils to solve problem c. with seatmates:

   Solution:

   c. Given $\angle SPQ = 34°$

a	$=$	$90°$	∠ in a semi-circle
$b + 90 + 34$	$=$	180	∠s in a triangle
b	$=$	$180 - 90 - 34$	
b	$=$	$56°$	
$c + 34$	$=$	180	opposite ∠s of the cyclic quadrilateral
c	$=$	$180 - 34$	
c	$=$	$146°$	

Practice *(20 minutes)*

1. Draw the circles in QUESTIONS FOR PRACTICE found at the end of this lesson plan on the board.
2. Ask pupils to work independently to find the unknown angles.
3. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.

   Solutions:

   Given $\angle XUV = 86°,\ \angle WVX = 57°$

$\angle VWX + 86$	$=$	180	opposite ∠s of cyclic quadrilateral
$\angle VWX$	$=$	$180 - 86$	
$\angle VWX$	$=$	$94°$	
$a + \angle VWX + 57$	$=$	$180°$	∠s in a triangle
$a + 94 + 57$	$=$	$180°$	
a	$=$	$180 - 94 - 57$	
a	$=$	$29°$	

   Given $\angle BOD = 210°,\ \angle ODC = 20°$

$2a$	$=$	210	∠ at the centre = 2∠ at circumference

$$
\begin{aligned}
a &= \frac{1}{2} \times 210 \\
a &= 105° \\
c + 105 &= 180 && \text{opposite ∠s of cyclic quadrilateral} \\
c &= 180 - 105 \\
c &= 75° \\
b + 20 + 75 + 210 &= 360 && \text{sum of ∠s in a quadrilateral} \\
b &= 360 - 20 - 75 - 210 \\
b &= 55°
\end{aligned}
$$

$$
\begin{aligned}
2x + 30 + x &= 180° && \text{opposite ∠s of cyclic quadrilateral} \\
3x &= 180 - 30 = 150 \\
x &= \frac{150}{3} \\
x &= 50°
\end{aligned}
$$

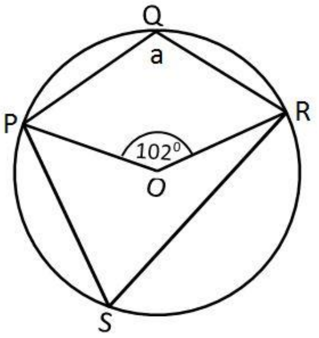
Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM3-L024 in the Pupil Handbook.

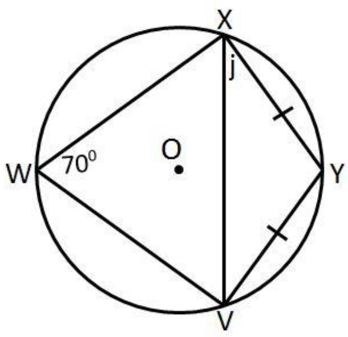
[QUESTIONS FOR TEACHING AND LEARNING]

Given O is the centre of the circle, determine the unknown angles in each of the circles below.

a.



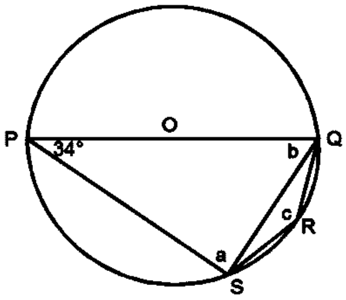
b.



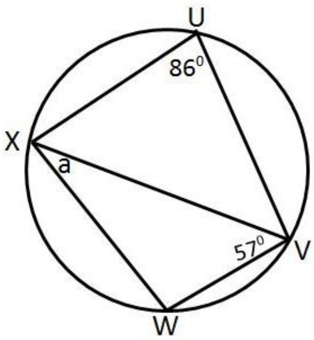
[QUESTIONS FOR PRACTICE]

Given O is the centre of the circle, determine the unknown angles in each of the circles below.

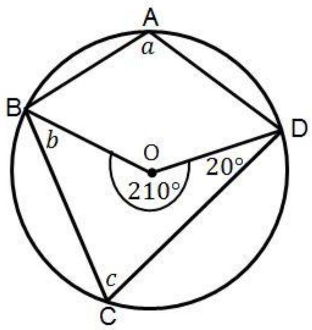
c.



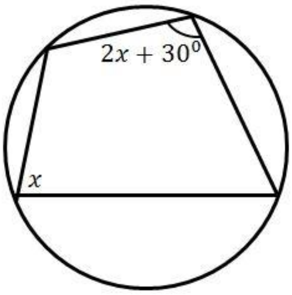
d.



e.



f.

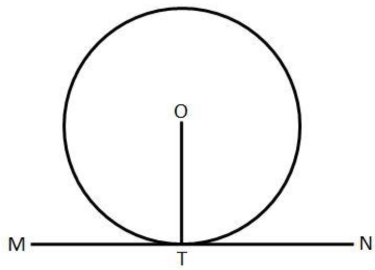


Lesson Title: Circle Theorems 6 and 7	**Theme:** Geometry	
Lesson Number: M3-L025	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcomes By the end of the lesson, pupils will be able to identify and draw the tangent to a circle, and identify and demonstrate: 1. The angle between a tangent and a radius in a circle is equal to $90°$. 2. The lengths of the two tangents from a point to a circle are equal.	**Preparation** Write the statements for Circle Theorems 6 and 7 on the board: The angle between a tangent and a radius is equal to $90°$. The lengths of the two tangents from a point to a circle are equal.	

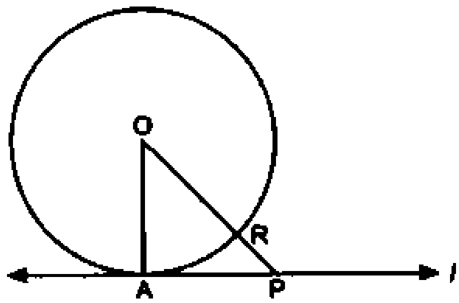
Opening *(2 minutes)*

1. Invite a volunteer to remind the class what a tangent is. (Example answers: A line which touches a circle at one point without cutting across the circle; a straight line that makes contact with a circle at only one point on the circumference.)
2. Tell pupils that after today's lesson, they will be able to identify and demonstrate: the 2 tangent theorems.

Teaching and Learning *(22 minutes)*



1. Tell pupils to do the following:
   - Draw a circle in your exercise books.
   - Mark a point, T, on the circle as shown.
   - Draw a straight line through T, making sure you just touch the circle with the line.
   - Explain they have now drawn a tangent to a circle.
2. Ask pupils to draw 3 more circles and tangents at different positions on the circle. (Allow 1 minute for this.)
3. Tell pupils we are now going to prove Theorem 6 – The angle between a tangent and a radius is equal to $90°$
4. Draw the diagram at right on the board.
5. Write on the board:



   Given: Circle with centre O, line l is a tangent to the circle at A.
   To prove: OA $\perp l$
   Proof:
   We know, by definition, that the tangent to the circle touches the circle at one point only.
   No other points on the tangent touch the circle.

$|\text{OP}| > |\text{OR}|$ $\quad |\text{OR}|$ = radius of circle
$\Rightarrow |\text{OP}| > |\text{OA}|$ $\quad$ since $|\text{OR}| = |\text{OA}|$
$\Rightarrow |\text{OA}| < |\text{OP}|$
$\Rightarrow \text{OA}$ is the shortest line from O to a point on the tangent.

The shortest line from the centre of a circle to a tangent is a perpendicular line.

∴ OA $\perp l$ **radius ⊥ tangent**

6. Tell pupils to use the statement **"radius ⊥ tangent"** whenever this theorem is used in solving problems.
7. Draw the circle for question a. found at the end of this lesson plan.
8. Find the missing angle in the given circle.

   Solution:

   Step 1. Assess and extract the given information from the problem.

   given angle $= 65°$

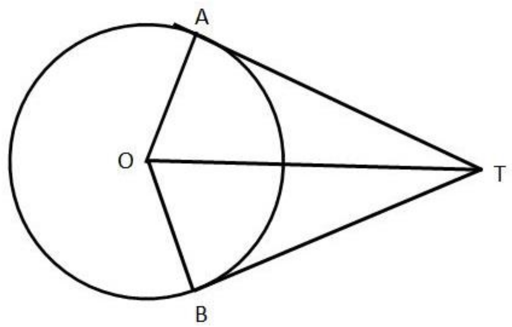
   Step 2. Use theorems and the given information to find all equal angles on the diagram.

   $a + 90 + 65 = 180$ ∠s in a triangle

   $a = 180 - 90 - 65$

   Step 3. Solve for a

   $a = 25°$
9. Explain to pupils that we will now look at Theorem 7 – The lengths of the two tangents from a point to a circle are equal.
10. Draw the diagram to the right on the board.



11. Write on the board:

    Given: A point T outside a circle with centre O. TA and TB are tangents to the circle at A and B respectively.

    To prove: |TA| = |TB|

    Proof:

∠OAT	=	∠OBT = $90°$	radius ⊥ tangent
|OA|	=	|OB|	equal radii
|OT|	=	|OT|	common side
∴ ΔOAT	=	ΔOBT	RHS
∴ |TA|	=	|TB|	equal tangents from same point

12. Explain:
    - Use the statement **"equal tangents from same point"** when referring to this theorem when solving problems,
    - Since ∠AOT=∠BOT and ∠ATO=∠BTO, it means that line TO bisects the angles at O and T.
    - TO is the line of symmetry for the diagram.
13. Draw the circle for question b. found at the end of this lesson plan
14. Find the missing angle in the given circle.

∠POT	=	$65°$	given
∠PTO	=	$90°$	radius ⊥ tangent
a	=	∠TPO	symmetry
$a + 65 + 90$	=	180	∠s in a triangle
a	=	$180 - 65 - 90$	
a	=	$25°$	

Practice *(12 minutes)*

1. Draw the circles for the QUESTIONS FOR PRACTICE found at the end of this lesson plan on the board.
2. Ask pupils to work independently to find the unknown angles and sides.
3. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.

   Solutions:

c.

$\angle OBA$	$=$	$20°$	given
$\angle OAB$	$=$	$90°$	radius $\perp$ tangent
$90+20+a$	$=$	180	$\angle$s in a triangle
a	$=$	$180-110$	
a	$=$	$70°$	
b	$=$	c	base $\angle$s of isosceles Δ
$2b+70$	$=$	180	
$2b$	$=$	$180-70=110$	
b	$=$	$\frac{110}{2}$	
b	$=$	$55°$	
c	$=$	$55°$	

$\lvert OI\rvert$	$=$	5 cm	
$\lvert IH\rvert$	$=$	8 cm	
$\angle HOJ = \angle HOI$	$=$	$72°$	symmetry
$\angle HIO$	$=$	$90°$	radius $\perp$ tangent
$b+\angle HIO+\angle HOI$	$=$	180	$\angle$s in a triangle
$b+90+72$	$=$	180	
b	$=$	$180-90-72$	
b	$=$	$18°$	
$\lvert OH\rvert^2$	$=$	$\lvert OI\rvert^2+\lvert IH\rvert^2$	Pythagoras theorem
a^2	$=$	5^2+8^2	substitute given values
a^2	$=$	$25+64$	
a^2	$=$	89	
a	$=$	$\sqrt{89}$	
a	$=$	9.4 cm	

Closing *(4 minutes)*

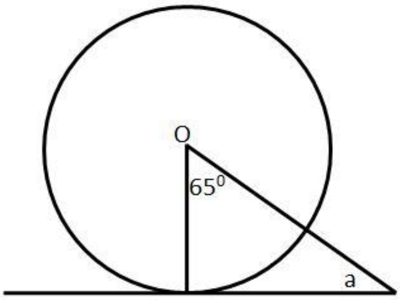
1. Write on the board: **Happy, Unhappy, Half-n-half**.
2. Ask pupils to:
   - Choose one of the words to say how they feel about the lesson.
   - Write the word down on a piece of paper without writing down their names.
   - Hand it in before the end of the lesson.
3. Use the responses as a guide for planning the level of assistance for the next lesson.

4. For homework, have pupils do the practice activity PHM3-L025 in the Pupil Handbook.

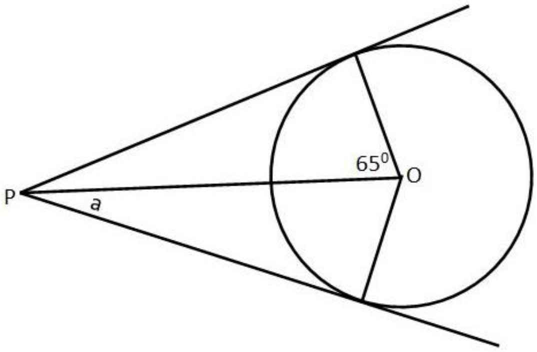
[QUESTIONS FOR TEACHING AND LEARNING]

Given O is the centre of the circle, determine the unknown angles in each of the circles below.

a.



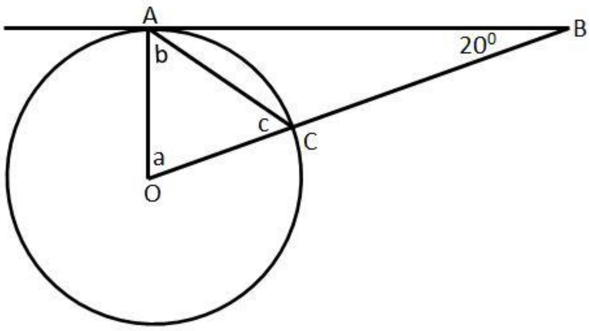
b.



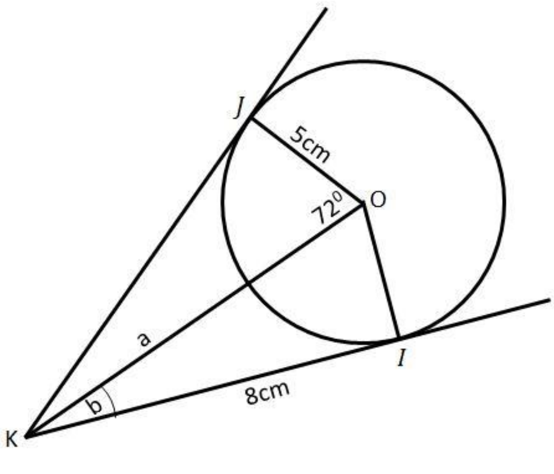
[QUESTIONS FOR PRACTICE]

Given O is the centre of the circle, determine the unknown angles in each of the circles below.

c.



d.



Lesson Title: Applications of Circle Theorems 6 and 7	**Theme:** Geometry	
Lesson Number: M3-L026	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to solve problems using Circle Theorems 6 and 7.	**Preparation** Draw the circles (a. through f.) and write the directions found at the end of this lesson plan on the board.	

Opening *(4 minutes)*

1. Ask pupils to write down 2 things they learned about in the last lesson concerning tangents.
2. Invite 2-3 volunteers to share what they wrote down with the class. (Example answers: Various but may include, “The angle between a tangent and a radius is $90°$”; “The lengths of 2 tangents from the same point to a circle are equal”.)
3. Tell pupils that after today’s lesson, they will be able to solve problems using the 2 theorems from the last lesson.

Teaching and Learning *(18 minutes)*

1. Refer to question a. on the board.
2. Invite volunteers to assess the diagram. What information can they see? (Example answers: $\angle ADO = 36°$; $DA \perp OA$; ΔOAD is a right-angled triangle.)
3. Explain to pupils we will write and use the information step-by-step to solve the problem.
   Solution:

   a. **Step 1.** Assess and extract the given information from the problem.
   given $\angle ADO = 36°$
   Step 2. Use theorems and the given information to find all equal angles on the diagram.

$$\begin{aligned} \angle OAD &= 90° && \text{radius} \perp \text{tangent} \\ \angle AOD + 36 + 90 &= 180 && \angle\text{s in a triangle} \\ \angle AOD &= 180 - 36 - 90 \\ &= 54 \end{aligned}$$

   Step 3. Solve for $\angle ABC$

$$\begin{aligned} \angle ABC &= \frac{1}{2} \times 54° && \angle \text{ at the centre} = 2\angle \text{ at the circumference} \\ \angle ABC &= 27° \end{aligned}$$

4. Refer to question b. on the board.
5. Invite volunteers to tell the class what they notice about the tangents in the diagram. (Example answers: There is only 1 tangent; the length of the tangent is given as 4 cm)
6. Ask pupils to think about the tangent theorems. Which ones can be used to solve this problem?
7. Invite volunteers to answer. (Answer: radius $\perp$ tangent)

8. Invite volunteers to say why they think that is the one to use. (Example answer: We have information for only 1 tangent.)
9. Explain:
   - For tangent questions, a good guide that we only need the "radius ⊥ tangent" theorem is when we only have one tangent in the question.
   - When we have 2 tangents in the question, we may have to use the "equal tangents from same point" as well.
10. Ask pupils to work with seatmates to find the answer for the angle b in the diagram.
11. Invite a volunteer to come to the board to show their answer. The rest of the class should check their solution and correct any mistakes.

    Solution:

    b. given angle $= 53°$

$b + 90 + 53$	$=$	180	∠s in a triangle
b	$=$	$180 - 90 - 53$	
b	$=$	$37°$	

12. Ask pupils to work with seatmates to find the answer for the length r in the diagram.
13. Invite a volunteer to come to the board to show their answer. The rest of the class should check their solution and correct any mistakes.

    Solution:

a^2	$=$	$b^2 + c^2$	Pythagoras' Theorem
$a = 5$ cm, $b = 4$ cm, $c = r$			given
5^2	$=$	$4^2 + r^2$	substitute given values
r^2	$=$	$5^2 - 4^2$	
r^2	$=$	$25 - 16$	
r	$=$	$\sqrt{9}$	
r	$=$	3 cm	

Practice *(15 minutes)*

1. Ask pupils to work independently to answer questions c., d. and e. on the board.
2. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.

   Solutions:

|LM| $= 6$ cm, |OK| $= 2\ cm,$ |LN| $= 7.5$ cm — given

|LM|	$=$	|LK|	equal tangents from same point
|MN|	$=$	|LN| – |LM|	
	$=$	$7.5 - 6$	
|MN|	$=$	1.5 cm	
|OM|	$=$	2 cm	equal radii

From Δ OMN;

e^2	$=$	$\|OM\|^2 + \|MN\|^2$	Pythagoras theorem
e^2	$=$	$2^2 + 1.5^2$	

$$
\begin{aligned}
&= 4 + 2.25 \\
e^2 &= 6.25 \\
e &= \sqrt{6.25} \\
e &= 2.5 \text{ cm}
\end{aligned}
$$

$\angle$BAC	$=$	$37°$	given
$\angle$BOC	$=$	2×37	$\angle$ at the centre=2$\angle$ at circumference
	$=$	$74°$	
$\angle$TOC	$=$	$\angle$TOB	symmetry
$\angle$TOC	$=$	$\frac{74}{2}$	
	$=$	$37°$	
$\angle$CTO $+ 90 + 37$	$=$	180	$\angle$s in a triangle
$\angle$CTO	$=$	$180 - 90 - 37$	
	$=$	$53°$	
$\angle$BTC	$=$	$53 + 53$	
$\angle$BTC	$=$	$106°$	

$\angle$AOT	$=$	$67°$	given
$\angle$ATO $+ 90 + 67$	$=$	180	$\angle$s in a triangle
$\angle$ATO	$=$	$180 - 90 - 67$	
	$=$	$23°$	
$\angle$ATO	$=$	$\angle$BTO	symmetry
$\angle$ATB	$=$	$23 + 23$	
	$=$	$46°$	
$\angle$ATB $+$ $\angle$ATC	$=$	180	
$46 +$ $\angle$ATC	$=$	180	$\angle$s in a straight line
$\angle$ATC	$=$	$180 - 46$	
$\angle$ATC	$=$	$134°$	

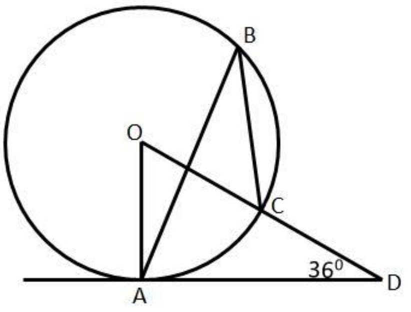
Closing *(3 minutes)*

1. Ask pupils to write in their exercise books how they can tell when they need only the “radius ⊥ tangent” theorem.
2. Invite a volunteer to give the answer. (Answer: When there is only one tangent in the diagram.)
3. Explain that if there are 2 tangents and we may need **both** theorems to answer the question.
4. For homework, have pupils do the practice activity PHM1-L026 in the Pupil Handbook.

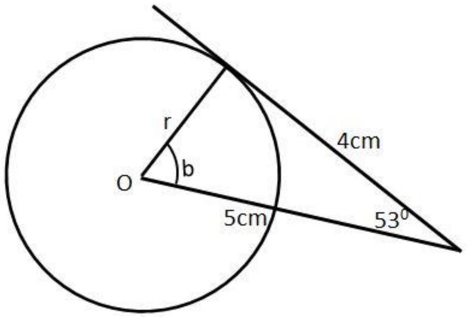
[QUESTIONS FOR TEACHING AND LEARNING]

Given O is the centre of the circle, determine the unknown angles in each of the circles below.

a.



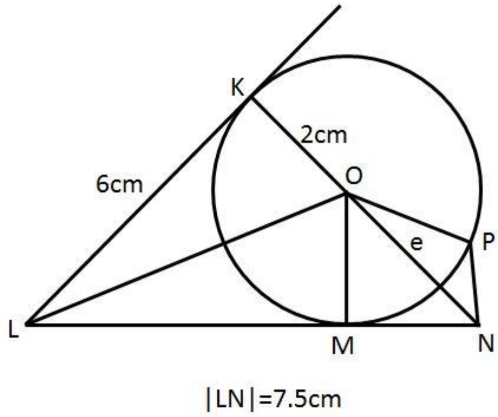
b.



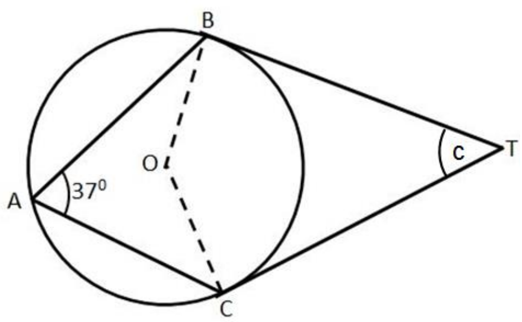
[QUESTIONS FOR PRACTICE]

Given O is the centre of the circle, determine the unknown angle or side in each of the circles below.

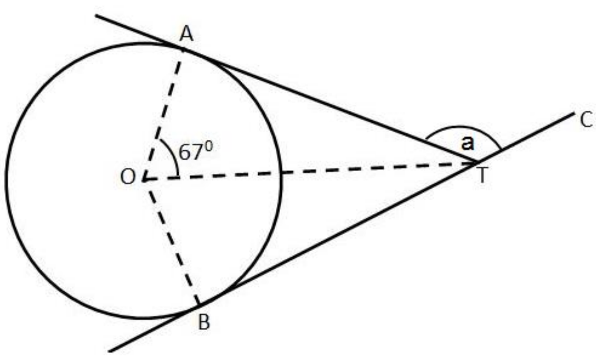
c.



d.



e.



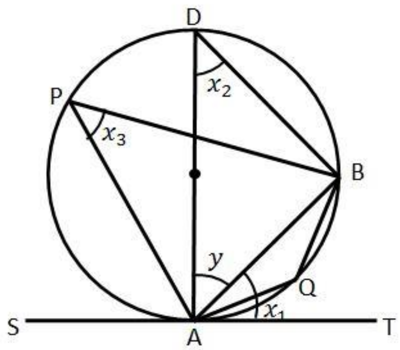
Lesson Title: Circle Theorem 8 – the alternate segment theorem	**Theme:** Geometry	
Lesson Number: M3-L027	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to identify and demonstrate: The alternate segment theorem.	**Preparation** Write the statement for the alternate segment on the board: The angle between a tangent to a circle and a chord drawn at the point of contact is equal to the angle which the chord subtends in the alternate segment.	

Opening *(1 minute)*

1. Tell pupils that after today's lesson, they will be able to identify and demonstrate the alternate segment theorem.

Teaching and Learning *(20 minutes)*

1. Draw the diagram to the right on the board.
2. Write on the board:
   Given: Circle with centre O and tangent ST touching the circle at A. Chord PA subtends ∠APB and∠ADB.
   To prove:
   - ∠TAB = ∠APB
   - ∠SAB = ∠AQB



   Proof:

$x_1 + y$	$=$	$90°$	tangent ⊥ radius
∠ ABD	$=$	$90°$	$\angle s$ in a semi-circle
$\therefore x_2 + y$	$=$	$90°$	$\angle s$ in a triangle
$\therefore x_1$	$=$	$x_2 = x_3$	$\angle s$ in the same segment
∴ ∠ TAB	$=$	∠ APB	
also ∠ SAB	$=$	$180 - x_1$	$\angle s$ on a straight line
	$=$	$180 - x_3$	$x_1 = x_3$ Proved
∠ SAB	$=$	∠ AQB	Opposite $\angle s$ of a cyclic quadrilateral

3. Tell pupils to use the statement **"$\angle s$ in alternate segment"** whenever this theorem is used in solving problems.
4. Draw the circle for question a. found at the end of this lesson plan.
5. Find the missing angles in the given circle.
   Solution:
   Step 1. Assess and extract the given information from the problem.
   $\angle PQR = 33°$ given
   Step 2. Use theorems and the given information to find all equal angles on the diagram.
   Step 3. Solve for a and b.
   i. $a = 33°$ $\angle s$ in alternate segment

$$b = 33° \quad \text{alternate angles, OP} \parallel \text{SR}$$

6. Draw the circle for question b. found at the end of this lesson plan.
7. Ask pupils to work with seatmates to find the answer for the angle d in the diagram.
8. Invite a volunteer to come to the board to show their solution, giving reasons for their answers. The rest of the class should check their solution and correct any mistakes.

   Solution:

   b.
   $$\begin{aligned} \angle PQR &= 72° && \text{given} \\ c &= 72° && \angle\text{s in alternate segment} \\ d &= \frac{180-72}{2} && \text{isosceles triangle} \\ d &= 54° \end{aligned}$$

9. Draw the circle for question c. found at the end of this lesson plan.
10. Explain: When we have to find several angles, we want to be sure that we are correctly matching the tangent / chord in one segment with the angle subtended by the chord in the alternate segment.
11. Ask pupils to raise their hands when they can see what tangent and chord makes $47°$ between each other in diagram c.
12. Invite a volunteer to tell the class what they are: (Answer: The tangent is OQ, chord SQ.)
13. Tell pupils to discuss with seatmates what angle is subtended by chord SQ in the alternate segment.
14. Ask pupils to write this angle down in their exercise books.
15. Invite a volunteer to share their answer with the class. (Answer: ∠SRQ or g.)
16. Write on the board: $g = 47°$.
17. Ask pupils to work with seatmates to do the same for angle $38°$.
18. Invite a volunteer to give their answer. (Answer: $f = 38°$.)
19. Explain to pupils that it is very important once they have found the angle the tangent makes with the chord that they look in the alternate segment for the angle subtended by the same chord.

Practice *(18 minutes)*

1. Draw the circles in the QUESTIONS FOR PRACTICE section on the board.
2. Ask pupils to work independently to find the unknown angles.
3. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.

   Solutions:

   d.
   $$\begin{aligned} \angle OPQ &= 66° && \text{given} \\ \angle ROQ = \angle RQO = &= 66° && \text{isosceles triangle, } \angle s \text{ in alternate segment} \\ 66+66+l &= = 180 && \angle\text{s in a triangle} \\ l &= 180-66-66 \\ l &= 48° \end{aligned}$$

e. $101 + 39 + i = 180$ ∠s in a straight line
$i = 180 - 101 - 39$
$i = 40°$
$j = 101°$ ∠s in alternate segment
$k = i$ ∠s in alternate segment
$k = 40°$

f. $n = 34°$ ∠s in alternate segment
$90 + 34 + o = 180$ ∠s in a triangle
$o = 180 - 90 - 34$
$o = 56°$
$o = m$ ∠s in alternate segment
$m = 56°$

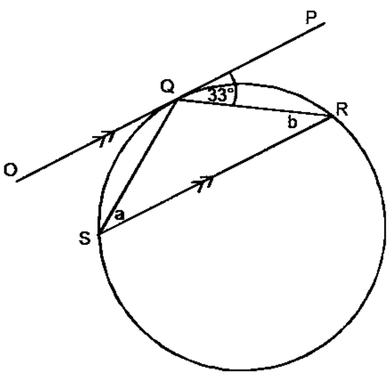
Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM3-L027 in the Pupil Handbook.

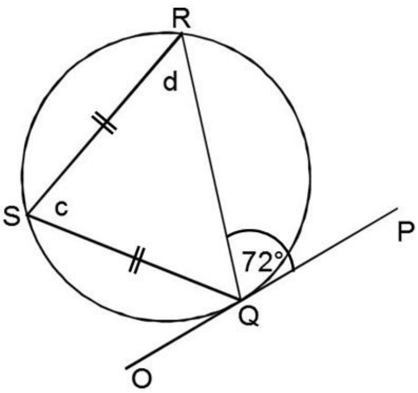
[QUESTIONS FOR TEACHING AND LEARNING]

Given the tangents shown, find the unknown angles in each of the circles below.

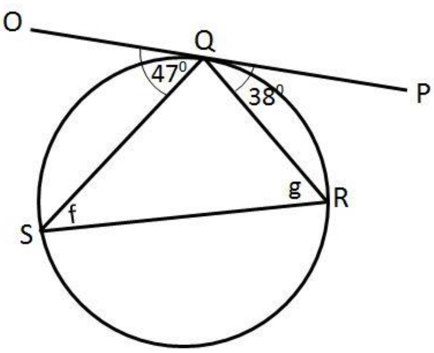
a.



b.



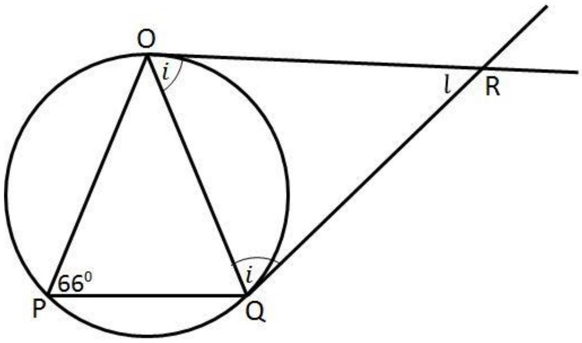
c.



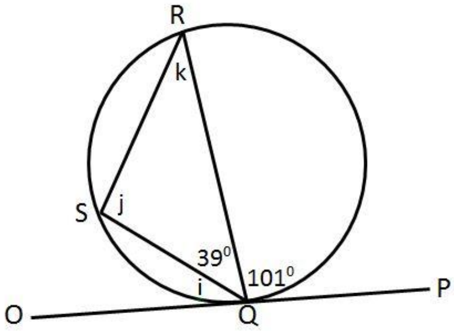
[QUESTIONS FOR PRACTICE]

Given O is the centre of the circle, determine the unknown angles in each of the circles below.

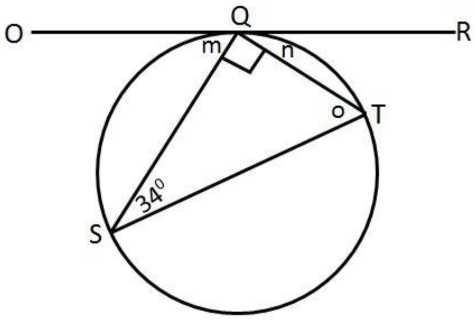
d.



e.



f.



Lesson Title: Apply the alternate segment theorem	**Theme:** Geometry	
Lesson Number: M3-L028	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to solve problems using the alternate segment theorem.	**Preparation** Draw the circles (a. through f.) found at the end of this lesson plan on the board.	

Opening *(1 minute)*

1. Tell pupils that after today's lesson, they will be able to solve problems using the alternate segment theorem.

Teaching and Learning *(13 minutes)*

1. Refer to question a. on the board.
2. Invite volunteers to assess the diagram. What information can they see? (Example answers: ∠ RQT $= 52°$
3. Explain to pupils we will use the information step by step to solve the problem.
   Solution:

   a. **Step 1.** Assess and extract the given information from the problem.
      given ∠ RQT $= 52°$
      Step 2. Use theorems and the given information to find all equal angles on the diagram.
      Step 3. Solve for unknown angles.

$$
\begin{array}{rcll}
q & = & 52° & \angle s \text{ in alternate segment} \\
\angle \text{OTS} = r & = & 90° & \angle \text{ in a semi-circle} \\
p + 52 & = & 90 & \text{radius} \perp \text{tangent} \\
p & = & 90 - 52 & \\
p & = & 38° &
\end{array}
$$

4. Refer to question b. on the board.
5. Ask pupils to work with seatmates to find the answer for the angle b in the diagram.
6. Invite a volunteer to come to the board to show their answer. The rest of the class should check their solution and correct any mistakes.
   Solution:
   b. Given angle $= 64°$

$$
\begin{array}{rcll}
\angle \text{OPT} & = & 90 & \text{radius} \perp \text{tangent} \\
90 & = & a + 64 & \\
a & = & 90 - 64 & \\
a & = & 26° & \\
b & = & 64° & \angle s \text{ in alternate segment} \\
c & = & 180 - 2a & \\
c & = & 180 - 2 \times 26 & \text{isosceles triangle} \\
c & = & 128° &
\end{array}
$$

Practice *(25 minutes)*

1. Ask pupils to work independently to answer questions c., d. and e. on the board.
2. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.

   Solutions:

   c. Given angles $= 74°, 36°$

a	$=$	$74°$	$\angle s$ in alternate segment
b	$=$	$36°$	$\angle s$ in alternate segment
$c + 74 + 74$	$=$	180	isosceles triangle
c	$=$	$180 - 74 - 74$	
c	$=$	$32°$	

   d. Given angle $= 65°$

x	$=$	$65°$	$\angle s$ in alternate segment
y	$=$	2×65	$\angle$ at the centre=2$\angle$ at circumference
	$=$	$130°$	
$\angle$ TAB	$=$	$\angle$ TBA	isosceles triangle
$z + 65 + 65$	$=$	180	$\angle s$ in a triangle
z	$=$	$180 - 65 - 65$	
z	$=$	$50°$	

   e. Given angle $= 62°$

x	$=$	$62°$	$\angle s$ in alternate segment
$\angle$ TAB	$=$	$\angle$ TBA	isosceles triangle
$Y + 62 + 62$	$=$	180	$\angle s$ in a triangle
Y	$=$	$180 - 62 - 62$	
Y	$=$	$56°$	

   f. Given angle $= x°$

i.	$\angle$ TAB	$=$	$\angle$ ABT	isosceles triangle
	$2\angle$ ABT $+ x$	$=$	180	$\angle s$ in a triangle
	$2\angle$ ABT	$=$	$180 - x$	
	$\angle$ ABT	$=$	$\frac{180 - x}{2}$	
	$\angle$ ABT	$=$	$90 - \frac{x}{2}$	
ii.	$\angle$ OAT	$=$	90	radius $\perp$ tangent
		$=$	$\angle$ OBA $+\angle$ ABT	
	$\therefore$ $\angle$ OBA	$=$	$\angle$ OAT $-\angle$ ABT	
		$=$	$90 -(90 - \frac{x}{2})$	
	$\angle$ OBA	$=$	$\frac{x}{2}$	
iii.	$\angle$ AOB	$=$	$2\angle$ ACB	$\angle$ at the centre=2$\angle$ at circumference
		$=$	$180 - \angle$ OBA$-\angle$ OAB	$\angle s$ in a triangle
		$=$	$180 - 2\angle$ OBA	isosceles triangle

$$
\begin{aligned}
&= 180 - 2 \times \tfrac{x}{2} \\
\angle AOB &= 180 - x \\
\therefore\ 2\angle ACB &= 180 - x \\
\angle ACB &= \frac{180 - x}{2} \\
\angle C &= 90 - \frac{x}{2}
\end{aligned}
$$

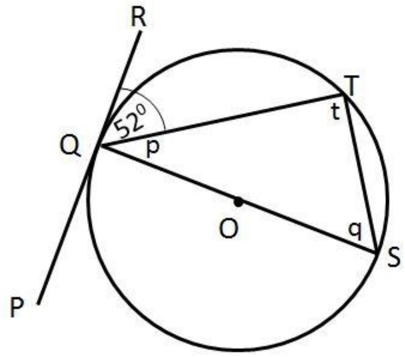
Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM1-L028 in the Pupil Handbook.

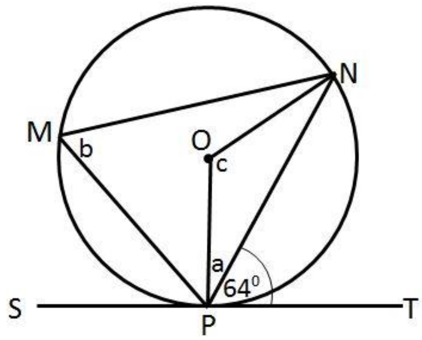
[QUESTIONS FOR TEACHING AND LEARNING]

Given O is the centre of the circle, determine the unknown angles in each of the circles below.

a. PR is a tangent to the circle shown below.



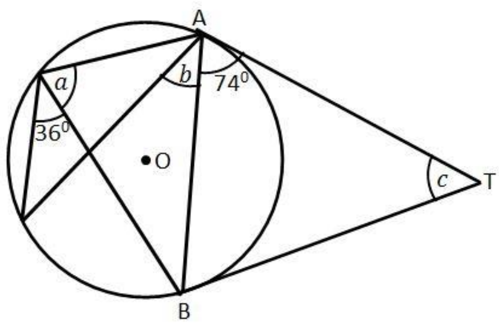
b. ST is a tangent to the circle shown below.



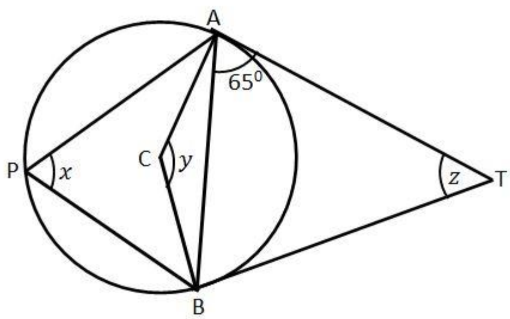
[QUESTIONS FOR PRACTICE]

Given O is the centre of the circle, determine the unknown angles in each of the circles below.

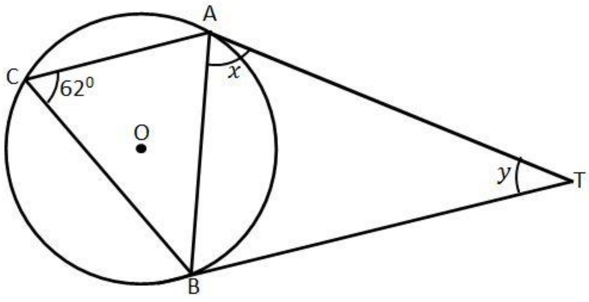
c.



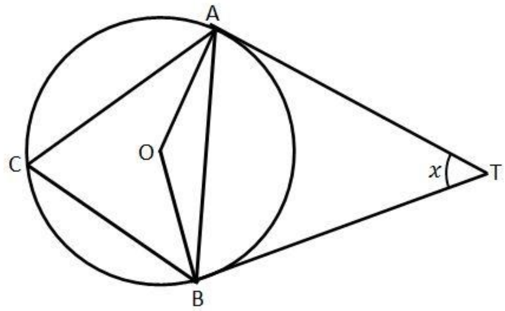
d.



e.



f.



Express the following in terms of x, giving reasons for your answers:

i. $\angle$ ABT
ii. $\angle$ OBA
iii. $\angle$ C

Lesson Title: Solving problems on circles	**Theme:** Mensuration	
Lesson Number: M3-L029	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to apply circle theorems and other properties to find missing angles in various circle diagrams.	**Preparation** Write the questions for the lesson found at the end of this lesson plan on the board.	

Opening *(2 minutes)*

1. Ask pupils to write down 2 things they would like to learn in this lesson.
2. Invite 2-3 volunteers to share what they have written down with the class. (Example answers: Various)
3. Tell pupils that after today's lesson, they will be able to apply circle theorems and other properties to find missing angles in various circle diagrams.

Teaching and Learning *(15 minutes)*

1. Explain to pupils that now we know all the circle theorems, we need to be able to identify which ones we can apply to a particular problem.
2. Refer to question a. on the board.
3. Explain:
   - We will use a step-by-step approach to solving the problem.
   - We will give reasons for our answers at each step.
4. Write the solution on the board. Explain the reasoning for each mathematical statement.
   Solution:

   a. **Step 1.** Assess and extract the given information from the problem.
   Given: $\text{BT is a tangent; BC bisects the angle ABT}$
   Step 2. Use theorems and the given information to find all equal angles on the diagram.
   $\angle CBT = \angle CBA$ BC bisects $\angle ABT$
   $\angle CBT = \angle CAB$ $\angle$s in alternate segment
   $\angle CBA = \angle CAB$
   Step 3. Write the answer.
   $\therefore CA = CB$ since Δ CAB is isosceles
5. Refer to question b. on the board.
6. Ask pupils to assess the question.
   - Invite volunteers to say what information is given just by the question. (Example answers: in a circle with centre O, RST is a tangent at S; $\angle SOP = 96°$)
   - Invite volunteers to say if there is any additional information given by the diagram. (Example answers: equal radii OS = OP, $\angle OST = 90°$)
   - Invite a volunteer to say what we are required to find. (Answer: $\angle PST$)

7. Explain to pupils that there are at least 2 different methods they can use to solve this problem.
8. Ask pupils to work with seatmates to give a step-by-step solution for finding ∠PST, making sure they give reasons for their answers
9. Invite a volunteer to come to the board to show one solution.
10. Invite another volunteer to show a different method from the one on the board. In both cases, the rest of the class should check their solution and correct any mistakes.

Solutions:

b. Given: RST is a tangent at S; ∠SOP = 96°)

Method 1.

∠PST	=	∠SOP	∠s in alternate segment
∠SOP	=	$\frac{1}{2}\times$ ∠SQP	∠ at the centre=2∠ at circumference
∠SOP	=	$\frac{1}{2}\times 96$	
	=	$48°$	
∴ ∠PST	=	$48°$	

Method 2.

OS	=	OP	equal radii
$2\times$ ∠OSP $+96$	=	180	isosceles triangle
∠OSP	=	$\frac{180-96}{2}$	
	=	$42°$	
∠OST	=	$90°$	radius ⊥ tangent
∠PST $+42$	=	$90°$	
∠PST	=	$90-42$	
	=	$48°$	

11. Ask pupils if anyone has a different method than these 2. (Example answer: Yes. However, their solutions may not be identical but they would have used the same theorems; the above are the 2 methods which can be used for this problem)
12. Explain:
    - In many circle theorem problems, there can be more than one method to solve a problem.
    - It does not matter which method is used as long as it is logical, they can explain the reasons for their answers, and they get the correct result.

Practice *(20 minutes)*

1. Write questions c., d., e. and f. on the board.
2. Ask pupils to work independently to answer the questions.
3. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.

Solutions:

c. $\angle$ XPY = $56°$ and $\angle$ PXY = $80°$ — given

$\angle$ XYP $+56+80 = 180$ — $\angle s$ in a triangle

$\angle$ XYP $= 180-56-80$

$= 44°$

$\angle$ YPQ $= 44°$ — alternate $\angle s$

$\angle$ PQY $= 180-56-44$ — $\angle s$ in a triangle

$\angle$ PQY $= 80°$

d. $\angle$ SOR $=64°$ $\angle$ PSO $=36°$, — given

$\angle$ OSR $= \frac{180-64}{2}$ — isosceles triangle

$= 58°$

$\angle$ PSR $= 58+36$

$= 94°$

$\angle$ PQR $+ 94$ $\quad 180$ — $\angle$s in a cyclic quadrilateral

$\angle$ PQR $= 180-94$

$\angle$ PQR $= 88°$

e. $|PQ| = z$ — given

$|RP| = \frac{1}{2}z$

$x^2 = y^2 + \left(\frac{1}{2}z\right)^2$ — Pythagoras' Theorem

$\left(\frac{1}{2}z\right)^2 = x^2 - y^2$

$\frac{1}{2}z = \sqrt{x^2-y^2}$

$Z = 2\sqrt{x^2-y^2}$

f. $\angle$ BCA = $40°$ and $\angle$ DAT = $52°$ — given

$\angle$ ACD $= 52°$ — $\angle s$ in alternate segment

$\angle$ BCD $= 40+52$

$= 92°$

$\angle$ BAD $= 180-92$ — $\angle$s in a cyclic quadrilateral

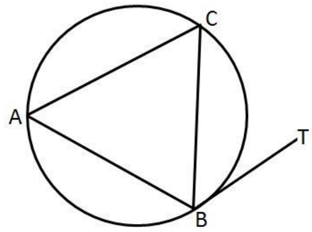
$\angle$ BAD $= 88°$

Closing (3 minutes)

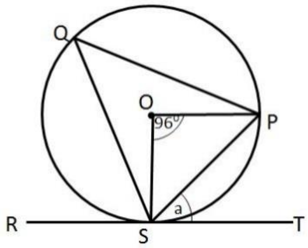
1. Ask pupils to look back to what they wrote at the start of the lesson.
2. Invite volunteers to share with the class something they learnt in the lesson they did not know before. (Answer: various.)
3. For homework, have pupils do the practice activity PHM3-L029 in the Pupil Handbook.

[QUESTIONS FOR TEACHING AND LEARNING]

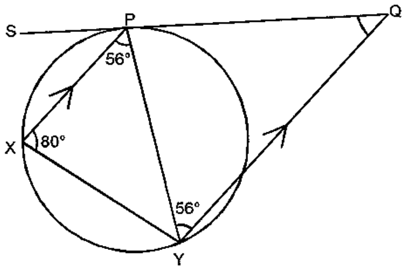
a. A, B, and C are points on the circumference of the circle. BT is a tangent on the circle. BC bisects angle ABT. Prove that CA = CB



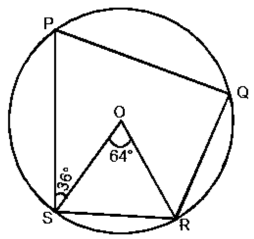
b. PQS is a circle with centre O. RST is a tangent at S and ∠ SOP = $96°$. Find ∠ PST.



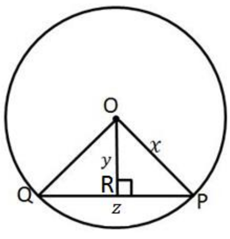
c. In the diagram, SQ is a tangent to the circle at P, XP ∥ YQ, ∠ XPY = $56°$ and ∠ PXY = $80°$. Find angle PQY.



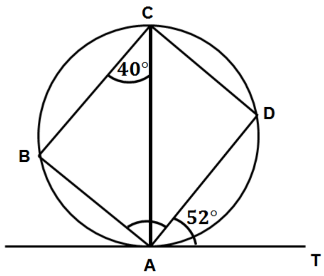
d. A circle has a centre at O. If ∠ SOR =$64°$ and ∠ PSO =$36°$, calculate ∠ PQR.



e. In the diagram, O is the centre of the circle with radius x. |PQ| $= z$, |OR| $= y$ and ∠ ORP $= 90°$. Find the value of z in terms of x and y.



f. TA is a tangent to the given circle at A. if ∠ BCA = $40°$ and ∠ DAT = $52°$, find ∠ BAD.



Lesson Title: Surface area of a cube	**Theme:** Mensuration	
Lesson Number: M3-L030	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to calculate the surface area of a cube using the appropriate formula.	**Preparation** Write question a. found at the end of this lesson plan on the board.	

Opening *(2 minutes)*

1. Ask pupils to calculate the area of one of the faces of the cube.
2. Ask pupils to write their answer on a piece of paper.
3. Ask pupils to show their answer to seatmates to check.
4. Invite a volunteer to confirm the answer. (Answer: 49 cm^2)
5. Tell pupils that after today's lesson, they will be able to calculate the surface area of a cube using the appropriate formula.

Teaching and Learning *(20 minutes)*

1. Explain:
   - The shape on the board is the **net** of a cube.
   - A *net* is a two-dimensional flat shape that can be folded to make a three-dimensional object.
   - There are 11 different nets of a cube. Two of them are shown on the board.
2. Ask pupils to draw in their exercise books a different net of a cube from the ones on the board.
3. Tell pupils their net must be completely different. They should not just turn the ones on the board back to front or upside down.
4. After 2 minutes, invite a volunteer to draw their net on the board.
5. Invite other volunteers to confirm that the net is correct and not like one already on the board.
6. Tell pupils they can compare their nets with seatmates.
7. Explain:
   - The net of a solid shape like a cube shows what a 3-dimensional shape looks like when it is opened flat into a 2-dimensional shape.
   - It can be used to find the **surface area** of the shape.
   - The surface area of a shape is the **sum of the areas** of the faces of the three-dimensional shape.
8. Invite a volunteer to remind the class the area of the face of the cube with a length of 7 cm. (Answer: 49 cm^2)
9. Invite a volunteer to say how many faces there are on a cube. (Answer: 6)
10. Ask pupils to work with seatmates to work out the surface area of the cube.
11. Invite a volunteer to answer, giving reasons. (Answer: 294 cm^2; because we have 6 faces each with area 49 cm; $6 \times 49 = 294$ cm^2)

12. Ask pupils to work with seatmates to come up with a formula for the surface area of a cube. The surface area should work for all cubes no matter what the length of its sides.
13. Invite a volunteer to write their answer on the board. (Answer: surface area of cube $= 6 \times l^2$, where l is the length of side of one of the faces of the cube.)
14. Check that the formula has been written correctly on the board before pupils copy it in their exercise books.
15. Solve question b. i. on the board:

Solution:

b. i. **Step 1.** Assess and extract the given information from the problem.

Given: cube of side length = 7.5 cm

Step 2. Substitute into the appropriate formula.

$$\begin{aligned} \text{surface area} &= 6l^2 \\ &= 6 \times 7.5^2 \\ &= 6 \times 56.25 \\ &= 337.5 \end{aligned}$$

Step 3. Write the answer.

surface area $=$ 338 cm^2 to 3 s.f.

16. Ask pupils to work with seatmates to answer question b. ii.
17. Invite a volunteer to come to the board to show their solution. The rest of the class should check their solution and correct any mistakes.

Solution:

b. ii. Given: cube of side length = 18.3 cm

$$\begin{aligned} \text{surface area} &= 6l^2 \\ &= 6 \times 18.3^2 \\ &= 6 \times 334.89 \\ &= 2{,}009.34 \end{aligned}$$

surface area $=$ $2{,}010$ cm^2 to 3 s.f.

18. Explain that sometimes we are given the surface area of the cube and we are asked to find the side length.
19. Invite a volunteer to read question c. on the board.
20. Ask pupils to work with seatmates to answer the question
21. Invite a volunteer to come to the board to show their solution. The rest of the class should check their solution and correct any mistakes.

Solution:

c. Given: cube with surface area = 57.2 cm^2

$$\begin{aligned} \text{surface area} &= 6l^2 \\ 57.2 &= 6 \times l^2 \\ \frac{57.2}{6} &= l^2 \\ l^2 &= 9.5333 \\ l &= 3.083 \end{aligned}$$

l = 3 cm to the nearest cm

Practice *(15 minutes)*

1. Write questions d., e. and f. on the board.
2. Ask pupils to work independently to answer questions d., e. and f.
3. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.

   Solutions:

   d. Given: cube of side length = 12 cm

   surface area $= 5l^2$ open box has only 5 faces

   $= 5 \times 12^2$

   surface area $= 720$ cm^2

   The surface area of the cube is 720 cm^2

   e. i Given: base: area $= 112.8$ cm^2

   base area $= l^2$

   $112.8 = l^2$

   $l = \sqrt{112.8}$

   $l = 10.62$ cm

   The length of the cube is 10.62 cm.

   ii surface area $= 6l^2$

   $= 6 \times 10.62^2$

   $= 676.706$

   Surface area $= 676.71$ cm^2

   The surface area of the cube is 676.71 cm^2

   f. Given: gift box size 12 cm by 12 cm by 12 cm

   surface area $= 6l^2$ exact amount of wrapping paper

   $= 6 \times 12^2$

   $= 864$

   The surface area of the cube is 864 cm^2.

   She will need more wrapping paper than given by the surface area.

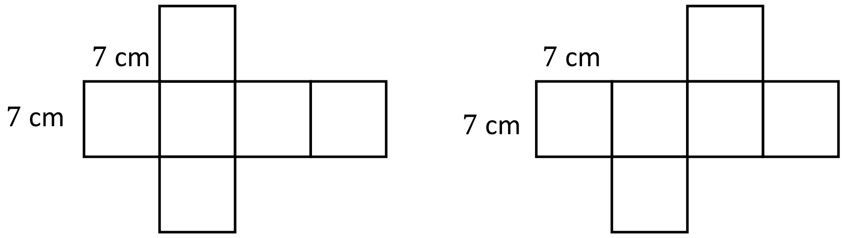
   She needs extra to overlap the paper when wrapping.

Closing *(3 minutes)*

1. For homework, have pupils do the practice activity PHM3-L030 in the Pupil Handbook.

[QUESTIONS]

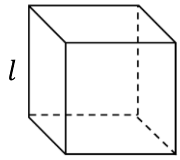
a. The nets below are made from squares with a side length of 7 cm. What is the total surface area of the resulting cube?



b. Find the surface area of a cube of side length:

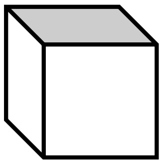
i. $l = 7.5$ cm   ii. $l = 18.3$ cm

Give your answer to 3 significant figures.



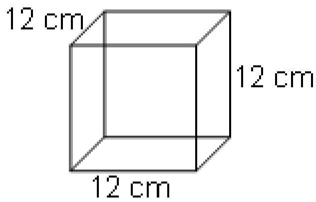
c. A cube has a surface area of 57.2 cm^2. What is the length of the side of the cube? Give your answer to the nearest cm.

d. The length of an open cube is 12 cm. What is its total surface area?



e. The area of the base of a cube is 112.8 cm^2.
Calculate: i. The side length of the cube; ii. The total surface area of the cube. Give your answer to 2 decimal places.

f. Isata bought a gift for her mother’s birthday. The gift was put in the box shown. She wants to wrap the gift with wrapping paper. How much wrapping paper does she need? Discuss your answer with seatmates.



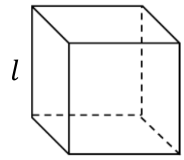
Lesson Title: Volume of a cube	**Theme:** Mensuration	
Lesson Number: M3-L031	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to calculate the volume of a cube using the appropriate formula.	**Preparation** Write question a. found at the end of this lesson plan on the board.	

Opening *(3 minutes)*

1. Ask pupils to calculate the surface area of a cube with a side length of 7 cm.
2. Ask pupils to show their answer to seatmates to check.
3. Invite a volunteer to give the answer. (Answer: 294 cm^2)
4. Tell pupils that after today's lesson, they will be able to calculate the volume of a cube using the appropriate formula.

Teaching and Learning *(15 minutes)*

1. Explain:
   a. The volume of a three-dimensional solid is a measurement of the space occupied by the shape.
   b. The volume of a cube is given by the formula $V = l^3$ where l is the side length of the cube.
   c. If we know the area, A of the face of the cube, then the volume $V = Al$.



2. Solve question a. i. on the board:
   Solution:

   a. i. **Step 1.** Assess and extract the given information from the problem.
   Given: cube of side length = 7.5 cm
   Step 2. Substitute into the appropriate formula.

$$\begin{aligned} \text{volume } V &= l^3 \\ &= 7.5^3 \\ &= 421.875 \end{aligned}$$

   Step 3. Write the answer.
   The volume of the cube $=$ 422 cm^3 to 3 s.f.
3. Ask pupils to work with seatmates to answer question a. ii.
4. Invite a volunteer to come to the board to show their solution. The rest of the class should check their solution and correct any mistakes.
   Solution:
   a. ii. Given: cube of side length = 18.3 cm

$$\begin{aligned} \text{volume } V &= l^3 \\ &= 18.3^3 \\ &= 6{,}128.487 \\ &= 6{,}128.49 \\ \text{volume } V &= 6{,}128 \text{ cm}^3 \text{ to 4 s.f.} \end{aligned}$$

5. Explain that sometimes we are given the volume of the cube and we are asked to find its side length.
6. Invite a volunteer to read question b. on the board.
7. Ask pupils to work with seatmates to answer the question
8. Invite a volunteer to come to the board to show their solution. The rest of the class should check their work and correct any mistakes.

Solution:

b. Given: cube with volume = 592.7 cm^3

$$\begin{aligned} \text{volume } V &= l^3 \\ 592.7 &= l^3 \\ l &= \sqrt[3]{592.7} \\ &= 8.39998 \\ l &= 8 \text{ cm} \end{aligned}$$

The length of the cube is 8 cm to the nearest cm.

Practice *(20 minutes)*

1. Write questions c. through f. on the board.
2. Ask pupils to work independently to answer questions c. through f.
3. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.

Solutions:

c. Given: surface area = 235 cm^2

$$\begin{aligned} \text{surface area} &= 6l^2 \\ 235 &= 6l^2 \\ l^2 &= \frac{235}{6} \\ &= 39.167 \\ l &= \sqrt{39.167} \\ l &= 6.258 \\ \text{Volume } V &= l^3 \\ &= (6.258)^3 \\ &= 245.1119 \\ &= 245.12 \text{ cm}^3 \end{aligned}$$

The volume of the cube is 245 cm^3.

d. Given: base: area = 22 cm^2 = l^2

i
$$\begin{aligned} \text{surface area} &= 6l^2 \\ &= 6 \times 22 \\ \text{surface area} &= 132 \text{ cm}^2 \end{aligned}$$

The surface area of the cube is 132 cm^2.

ii
$$l^2 = 22$$

$$l = \sqrt{22}$$
$$l = 4.690 \text{ cm}$$
$$\text{volume} = l^3$$
$$= (4.690)^3$$
$$= 103.16$$
$$\text{volume} = 103.16 \text{ cm}^3$$

The volume of the cube is 103.16 cm^3.

e Given: cube shaped room with volume = 59 m^3

$$\text{volume} = 59 \text{ m}^3$$
$$l^3 = 59$$
$$l = \sqrt[3]{59}$$
$$l = 3.893 \text{ cm}$$
$$\text{surface area} = 6l^2$$
$$= 6 \times 3.893^2$$
$$= 90.93$$
$$\text{surface area} = 90.9 \text{ cm}^2$$

The surface area of the room is 91 cm^2.

f. Given: cubes of side length $l = 4$ cm, box of side length $L = 12$ cm

$$\text{number of cubes} = \frac{\text{volume of box}}{\text{volume of cube}}$$
$$\text{volume of box} = L^3$$
$$= 12^3$$
$$= 1{,}728 \text{ cm}^3$$
$$\text{volume of cube} = l^3$$
$$= 4^3$$
$$= 64$$
$$\text{number of cubes} = \frac{1{,}728}{64}$$

The number of cubes that will fit into the box $= 27$.

Closing *(2 minutes)*

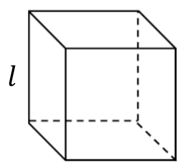
1. For homework, have pupils do the practice activity PHM3-L031 in the Pupil Handbook.

[QUESTIONS]

a. Find the volume of a cube of side length.

i. $l = 7.5$ cm ii. $l = 18.3$ cm

Give your answer to 3 significant figures.



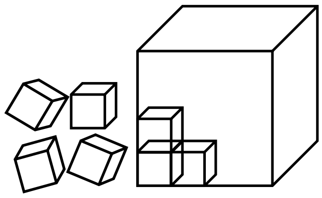
b. A cube has volume of 592.7 cm^3. What is the length of the side of the cube? Give your answer to the nearest cm.

c. The surface area of a cube is 235 cm^2. What is its volume to the nearest cm^3?

d. The area of the base of a cube is 22 cm^2. Calculate: i. The surface area of the cube; ii. The total volume of the cube. Give your answer to 2 decimal places.

e. A room in the shape of a cube has a volume of 59 m^3. What is its surface area to the nearest cm^2?

f. How many cubes with a side length of 4 cm can fit into a box with a side length of 12 cm?



Lesson Title: Surface area of a cuboid	Theme: Mensuration	
Lesson Number: M3-L032	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to calculate the surface area of a cuboid using the appropriate formula.	**Preparation** Draw the diagrams for Teaching and Learning found at the end of this lesson plan on the board.	

Opening *(2 minutes)*

1. Ask pupils to find the area of a rectangle with length $l = 7$ cm and width $w = 3$ cm.
2. Invite a volunteer to tell the class the area is (Answer: $A = lw = 7 \times 3 = 21$ cm^2)
3. Ask a different volunteer what formula they used to find the area of the rectangle. (Answer: $A = lw$)
4. Tell pupils that after today's lesson, they will be able to calculate the surface area of a cuboid using the appropriate formula.

Teaching and Learning *(20 minutes)*

1. Refer to the cuboid and its net on the board.
2. Invite a volunteer to say how many faces the cuboid is made up of. (Answer: 6)
3. Ask a different volunteer to say what shape the faces are. (Answer: rectangular)
4. Explain
   - The same letter appears in several places in the net.
   - Use them to match which sides will meet with each other to re-make the cuboid.
   - To find the surface area of the cuboid, we need to find the area of each of the faces of the cuboid.
   - The surface area is the **sum of the areas** of the faces of the cuboid.
5. Invite a volunteer to say what the area of face ABCD is. (Answer: lh)
6. Write the area of face ABCD on the net of the cuboid.
7. Invite a different volunteer to say what the area of face EFGH is. (Answer: lh)
8. Write the area of face EFGH on the net of the cuboid.
9. Invite different volunteers from around the room to give the areas of faces EADH, BEHC, EFBA and DCGH. (Answers: area EADH $= hw$, area BEHC $= hw$, area EFBA $= lw$, and area DCGH $= lw$).
10. Write the areas inside the appropriate face of the net.
11. Invite a volunteer to write down the surface area of the cuboid on the board.
12. Discuss with the class if they agree with the formula written on the board.
13. Ask volunteers for their own idea of what the formula should be.
    Write all the suggestions on the board. (Example answers: Various; Pupils may find areas of composite shapes instead of individual faces. The end result is the same.)
14. Explain:

- The surface area is the sum of the areas of the faces of the cuboid.

surface area = area of ABCD + area of EFGH + area of EADH + area of BEHC + area of EFBA + area of DCGH

$= lh + lh + hw + hw + lw + lw$

surface area $= 2(lh + hw + lw)$

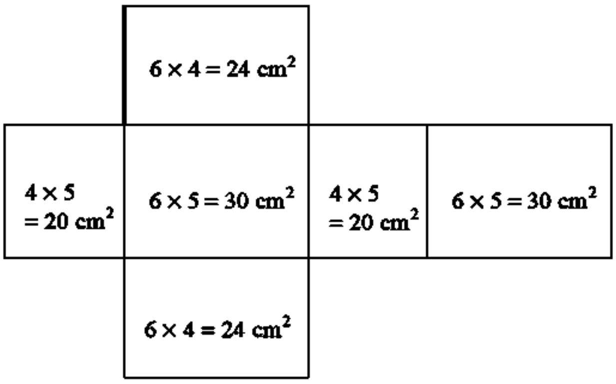
15. Write question a. i. found at the end of this lesson plan on the board.
16. Complete each of the steps below on the board as you go through the solution.
17. Invite a volunteer to tell the class what information we have from the diagram. (Answer: cuboid with $l = 6$ cm, $h = 4$ cm, $w = 5$ cm)
18. Invite a volunteer to come to the board to draw a net of the cuboid. (Answer: Step 2).
19. Ask pupils to work with seatmates to find the area of each face of the cuboid.
20. Invite volunteers to come to the board and write down the area of each face. They must show their working. (Answer: Step 3)

Solution:

a. i. **Step 1.** Assess and extract the given information from the problem.
given: cuboid with $l = 6$ cm, $h = 4$ cm, $w = 5$ cm

Step 2. Draw the net of the cuboid. There are at least 50 different ways.

Step 3. Find the individual areas for each rectangular face of the cuboid.



Step 4. Find the sum of the areas of the faces of the net.

surface area $= 2(lh + hw + lw)$

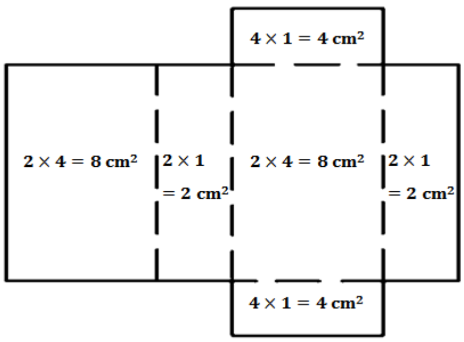
$= 2 \times (24 + 20 + 30)$

surface area $= 2 \times 74 = 148$

Step 5. Write the final answer.
The surface area of the cuboid $= 148$ cm^2.

21. Ask pupils to work with seatmates to answer question a. ii.
22. Invite volunteers to come to the board to show different parts of the solution. The rest of the class should check their solution and correct any mistakes.

Solution:



a. ii. Given: cuboid with $l = 2$ cm, $h = 4$ cm, $w = 1$ cm
net of cuboid shown right

surface area $= 2(lh + hw + lw)$

$= 2 \times (8 + 2 + 4)$

surface area $= 2 \times 14 = 28$

The surface area of the cuboid = 28 cm^2.

23. Explain:
    - Once we are familiar with the process we do not need to draw the net of the cuboid to calculate the surface area.
    - We can simply substitute the given values into the formula and find the surface area.
24. Invite a volunteer to read question b. on the board.
25. Ask pupils to work with seatmates to answer the question.
26. Invite a volunteer to come to the board to show their solution. The rest of the class should check their solution and correct any mistakes.

Solution:

b. Given: cuboid with $l = 7$ cm, $h = 4$ cm, $w = 6$ cm

surface area $= 2(lh + hw + lw)$

$= 2((7 \times 4) + (4 \times 6) + (7 \times 6))$

$= 2 \times (28 + 24 + 42)$

$= 2 \times 94$

surface area $= 188$

The surface area of the cuboid = 188 cm^2.

Practice *(15 minutes)*

1. Write questions c., d., and e. on the board.
2. Ask pupils to work independently to answer questions c, d. and e.
3. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.

Solutions:

c. Given: cuboid with $l = 8$ cm, $h = 6$ cm, $w = 5$ cm

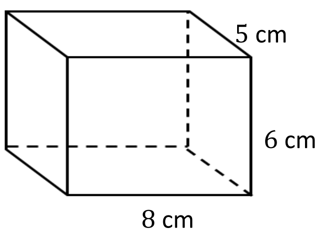
surface area $= 2(lh + hw + lw)$

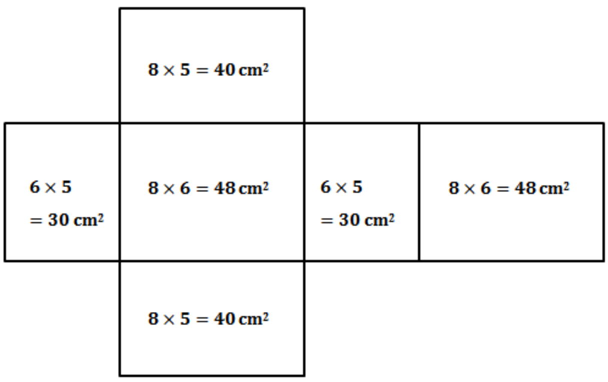
$= 2 \times (48 + 30 + 40)$

$= 2 \times 118$

surface area $= 236$

The surface area of the cuboid = 236cm^2.





d. Given: cuboid with $l = 10$ cm, $w = 8$ cm, $h = 7$ cm

$$\begin{aligned} \text{surface area} &= 2(lh + hw + lw) \\ &= 2 \times ((10 \times 7) + (7 \times 8) + (10 \times 8)) \\ &= 2 \times (70 + 56 + 80) \\ \text{surface area} &= 2 \times 206 = 412 \end{aligned}$$

The surface area of the cuboid $= 412$ cm^2.

e. Given: cuboid 1 with $l = 7$ cm, $w = 4$ cm, $h = 2$ cm,
cuboid 2 with $l =$ y cm, $w = 1$ cm, $h = 2$ cm

$$\begin{aligned} \text{surface area} &= 2(lh + hw + lw) \\ \text{surface area of cuboid 1} &= \text{surface area of cuboid 2} \\ 2 \times ((7 \times 2) + (2 \times 4) + (7 \times 4)) &= 2 \times ((\text{y} \times 2) + (2 \times 1) + (y \times 1)) \\ 2 \times (14 + 8 + 28) &= 2 \times (2\text{y} + 2 + \text{y}) \\ 2 \times 50 &= 2 \times (3\text{y} + 2) \\ 100 &= 6\text{y} + 4 \\ 6\text{y} &= 100 - 4 \\ 6\text{y} &= 96 \\ \text{y} & \quad \frac{96}{6} = 16 \end{aligned}$$

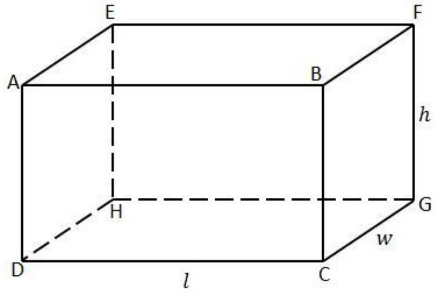
The missing length y $= 16$cm.

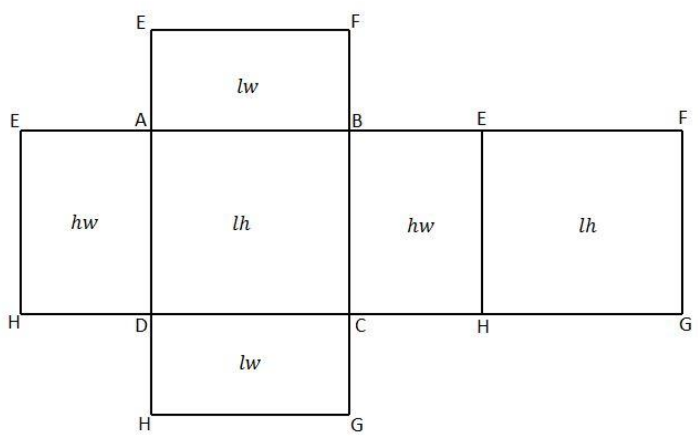
Closing *(3 minutes)*

1. Ask pupils to use their net in question c. to cut out and make a solid cuboid.
2. For homework have pupils:
   - Cut up the outline of the net.
   - Fold and paste together to make a solid cuboid
   - Do the practice activity PHM3-L032 in the Pupil Handbook.

[DIAGRAMS FOR TEACHING AND LEARNING]

Draw the diagrams shown below on the board.
[NOTE] Do not write the individual areas in the net until during the lesson.

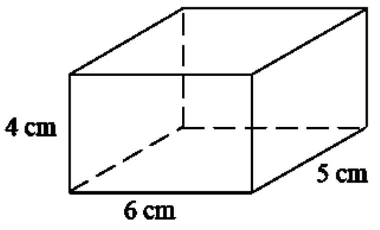




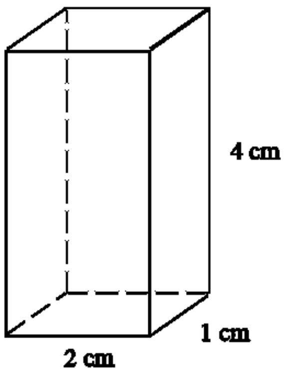
[QUESTIONS]

a. Draw a net of the cuboid shown below.
Use the net to find the surface area of the cuboid.

i.



ii.

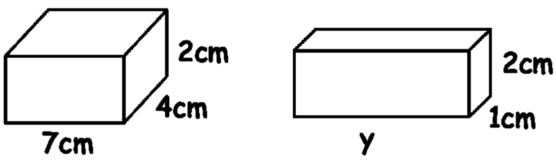


b. The length, width and height of a cuboid are 7 cm, 6 cm and 4 cm respectively. Find the surface area of the cuboid.

c. Draw the cuboid and net with dimensions 8 cm by 6 cm by 5 cm. What is its surface area?

d. The length, width and height of a cuboid are 10 cm, 8 cm and 7 cm respectively. What is its surface area?

e. The 2 cuboids shown have the same surface area. Find the missing length y.



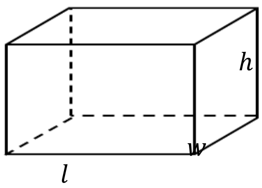
Lesson Title: Volume of a cuboid	Theme: Mensuration	
Lesson Number: M3-L033	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to calculate the volume of a cuboid using the appropriate formula.	**Preparation** Write question a. found at the end of this lesson plan on the board.	

Opening *(3 minutes)*

1. Ask pupils to find the surface area of a cuboid with length $l = 7$ cm, width $w = 3$ cm and height $h = 2$ cm.
2. Invite a volunteer to tell the class the value calculated for the surface area. (Answer: $2\big((7 \times 2) + (2 \times 3) + (7 \times 3)\big) = 2(14 + 6 + 21) = 2 \times 41 = 82$ cm^2)
3. Tell pupils that after today's lesson, they will be able to calculate the volume of a cuboid using the appropriate formula.

Teaching and Learning *(15 minutes)*

1. Explain:
   d. The cuboid shown has length l, width w and height h.
   e. The volume of the cuboid is given by the formula $V =$ length $\times$ width $\times$ height $= lwh$.
   f. If we know the area A of the base of the cube then the volume $V = Ah$.
   g. The formula $V = A \times$ side length can be used to find volume given the area and the length of any of the sides.



2. Ask pupils to write down the volume of the cuboid given at the start of the lesson.
3. Invite a volunteer to give the answer. (Answer: $V = lwh = 7 \times 3 \times 2 = 42$ cm^3.
4. Solve question a. i. on the board:
   Solution:

   a. i. **Step 1.** Assess and extract the given information from the problem.
   Given: cuboid with $l = 6$ cm$, w = 5$ cm$, h = 4$ cm
   Step 2. Substitute into the appropriate formula.

   $$\begin{aligned} \text{volume} &= lwh \\ &= 6 \times 5 \times 4 \\ &= 120 \end{aligned}$$

   Step 3. Write the answer.

   $$\text{volume} = 120 \text{ cm}^3$$

5. Ask pupils to work with seatmates to answer question a. ii.
6. Invite a volunteer to come to the board to show their solution. The rest of the class should check their solution and correct any mistakes.
   Solution:
   a. ii. Given: cuboid with $l = 2$ cm$, w = 1$ cm$, h = 4$ cm

   $$\begin{aligned} \text{volume} &= lwh \\ &= 2 \times 1 \times 4 \end{aligned}$$

$$= 8$$
volume $= 8$ cm^3

7. Explain that sometimes we are given the volume of the cube and we are asked to find its side length.
8. Invite a volunteer to read question b. on the board.
9. Ask pupils to work with seatmates to answer the question
10. Invite a volunteer to come to the board to show their solution. The rest of the class should check their solution and correct any mistakes.

Solution:

b. Given: cuboid A with $L = 7$ cm, $W = 4$ cm, $H = 2$ cm,
cuboid B with $l =$ y cm, $w = 1$ cm, $h = 2$ cm

$$\begin{aligned}
\text{volume of A} &= \text{volume of B} \\
L \times W \times H &= l \times w \times h \\
7 \times 4 \times 2 &= y \times 1 \times 2 \\
56 &= 2y \\
y &= \frac{56}{2} \\
& 28 \text{ cm}
\end{aligned}$$

The value of y is 28 cm.

Practice *(20 minutes)*

1. Write questions c. through f. on the board.
2. Ask pupils to work independently to answer questions c. through f.
3. Ask volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.

Solutions:

c. Given: cuboid with $l = 7$ cm, $w = 6$ cm, $h = 4$ cm

$$\begin{aligned}
\text{volume} &= l \times w \times h \\
&= 7 \times 6 \times 4 \\
\text{volume} &= 168 \text{ cm}^3
\end{aligned}$$

The volume of the cuboid is 168 cm^3.

d. Given: volume of cuboid = 150 cm^3, width = 3 cm, height = 10 cm

$$\begin{aligned}
\text{volume} &= l \times w \times h \\
150 &= l \times 3 \times 10 \\
150 &= 30l \\
l &= \frac{150}{30} \\
l &= 5 \text{ cm}
\end{aligned}$$

The length of the cuboid is 5 cm.

e. Given: cuboid A with $l = 5$ cm, $w = 3$ cm, $h = 2$ cm,
cuboid B with $L = 25$ cm, $W = 30$ cm, $H = 12$ cm

$$\begin{aligned}
\text{number that can fit into carton} &= \frac{\text{volume of carton}}{\text{volume a match box}} \\
\text{volume of carton} &= L \times W \times H
\end{aligned}$$

	=	$25 \times 30 \times 12$
volume of carton	=	$9{,}000$ cm^3
volume of match box	=	$l \times w \times h$
	=	$5 \times 3 \times 2$
	=	30 cm^3
Amount that can fit	=	$\frac{9{,}000}{30}$
	=	300

300 matchboxes can fit into the carton.

f. i. Given: cuboid with $l = 1.2$ m, $w = 2.3$ m, $h = 1.4$ m

Volume of tank	=	$l \times w \times h$
	=	$1.2 \times 2.3 \times 1.4$
	=	3.864
	=	3.86 cm^3

The volume of the tank is 3.9 cm^3 to 1 d.p.

ii. Given: depth of water $= 1.2$ m

volume of water	=	$l \times w \times h$
	=	$2.3 \times 1.2 \times 1.2$
	=	3.312
	=	3.31 cm^3

The volume of water in the tank is 3.3 cm^3.

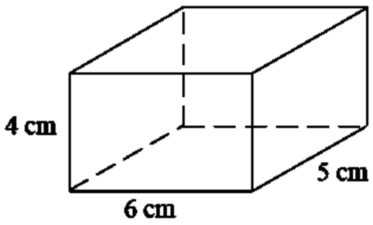
Closing *(2 minutes)*

1. For homework, have pupils do the practice activity PHM3-L033 in the Pupil Handbook.

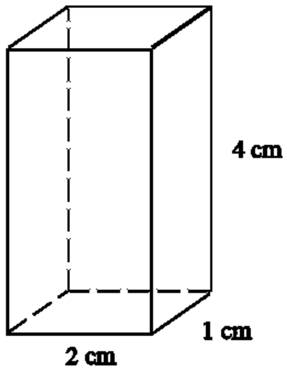
[QUESTIONS]

a. Find the volume of the following cuboids:

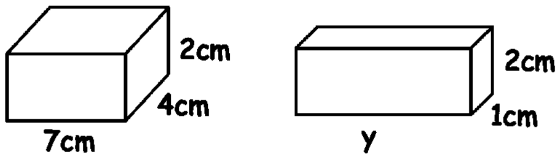
i.



ii.



b. The 2 cuboids shown have the same volume. Find the missing length y.



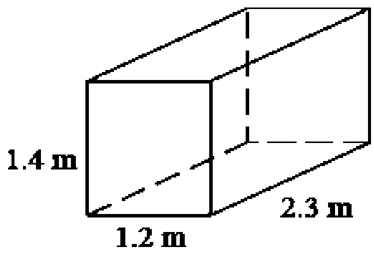
c. The length, width and height of a cuboid are 7 cm, 6 cm, and 4 cm respectively. Find the volume of the cuboid.

d. A cuboid has volume of 150 cm^3. Its width is 3 cm and its height is 10 cm. Find the length of the cuboid.

e. Matchboxes are in the shape of a cuboid of dimensions 5 cm by 3 cm by 2 cm. How many can be packed into a carton of dimension 25 cm by 30 cm by 12 cm?

f. A water tank has the dimensions shown in the diagram.
   i. Find the volume of the tank.
   ii. If the depth of the water is 1.2 m, find the volume of water in the tank.

   Give your answers to 1 decimal place.



Lesson Title: Nets of Prisms	Theme: Mensuration	
Lesson Number: M3-L034	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to draw nets of prisms.	**Preparation** Draw the diagrams for Teaching and Learning found at the end of this lesson plan on the board.	

Opening *(1 minute)*

1. Invite a volunteer to tell the class what the net of a solid is. (Example answer: A two-dimensional flat shape that can be folded to make a three-dimensional object.)
2. Tell pupils that after today's lesson, they will be able to draw nets of prisms.

Teaching and Learning *(30 minutes)*

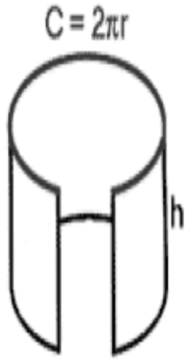
1. Invite a volunteer to identify the shapes that make up the cuboid on the board.
2. Write the suggestions underneath the shape on the board. (Example answers: rectangles, squares)
3. Ask pupils to write in their exercise books the names of the shapes that make up the triangular prism and the cylinder.
4. Invite volunteers to give their answers. (Example answers: Triangular prism – triangle, rectangle, parallelogram. Cylinder – circle, rectangle)
5. Refer to the prisms on the board during the following explanation.
6. Explain
   - A prism is a solid object with 2 identical ends and flat sides.
   - The cross-section of the prism is the same all along its length.
   - For each of the 3 prisms shown on the board, a part of the cross-section is shown halfway along the length of the prism.
   - This highlights that the end faces run right through the full length of the prism.
   - The shape of the ends of the prism is sometimes used to name the prism.
   - In order to draw nets of prisms, we need to know the number and type of the individual shapes that they are made of.
7. Draw the diagram for question a. on the board.
8. Invite a volunteer to make a guess at what name is sometimes given to a cuboid. (Answer: rectangular prism; its cross-section is rectangular)
9. Invite a volunteer to say how many faces there are in a cuboid. (Answer: 6 rectangular faces.)
10. Ask pupils to draw an accurate net of the cuboid using a ruler and pencil
11. Allow pupils to work with seatmates to compare and share ideas.
12. Invite a volunteer to show the diagram of their net on the board.

    Solution:

    a. **Step 1.** Assess and extract the given information from the problem.
       Given: cuboid with $l = 4$ cm, $h = 1$ cm, $w = 3$ cm
       Step 2. Draw the net of the solid.

See the solution next to the question at the end of this lesson plan.

13. Invite a volunteer describe the end faces of the triangular prism on the board (Answer: triangles)
14. Invite another volunteer to tell the class how many triangles there are in the triangular prism (Answer: 2)
15. Invite another volunteer to say what other faces there are. (Answer: 3 rectangles; accept parallelograms for rectangles)
16. Draw an accurate net of the triangular prism in question b. on the board.
17. Ask pupils to copy the net of the triangular prism in their exercise books.
    Solution: See the solution next to the question at the end of this lesson plan.
18. Explain: A cylinder is sometimes referred to as a circular prism even though its sides are round not flat.
19. Invite a volunteer to say what type of shape is at the ends of the cylinder. (Answer: 2 circles)
20. Invite a volunteer to say the best way to draw the length of the cylinder on paper. (Example answer: as a rectangle)
    - If no one volunteers, ask pupils to think of how the cylinder opens up to make a flat shape.
    - Show them, using a piece of rolled up paper, how a cylinder with radius r flattens to a rectangle with length $2\pi r$ (see right).



21. Ask pupils to **sketch** the net of the cylinder in question c, with radius $r = 3$ cm and height $h = 10$ cm. They can show the length in terms of π.
22. Allow pupils to work with seatmates to compare and share ideas.
23. Invite a volunteer to come to the board to show their solution.
    The rest of the class should check their solution and correct any mistakes.
    Solution: See the solution next to the question at the end of this lesson plan.

Practice *(8 minutes)*

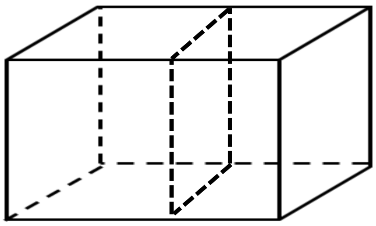
1. Ask pupils to work independently to draw the net for question d. given the information below.
2. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.
   Solution: See the solution next to the question at the end of this lesson plan.

Closing *(1 minute)*

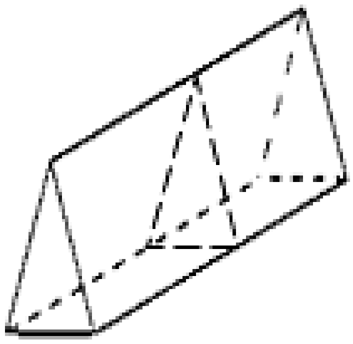
1. For homework have pupils do the practice activity PHM3-L034 in the Pupil Handbook.

[DIAGRAMS FOR TEACHING AND LEARNING]

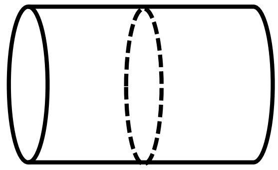
Draw the diagrams shown below on the board.



cuboid



triangular prism



cylinder

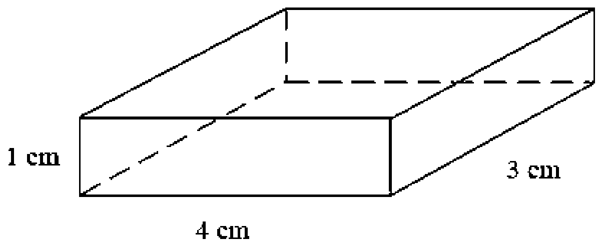
[QUESTIONS]

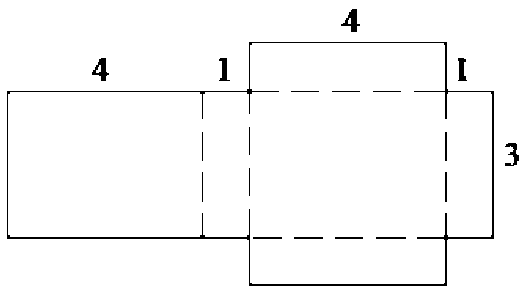
[NOTE: Most solids have more than one net but they will all fold up to make the same shape. We will only consider one net per solid in this lesson. To verify their nets, have pupils cut out and make the solid shapes from their nets.]

Draw an accurate net of each of the prisms shown below.

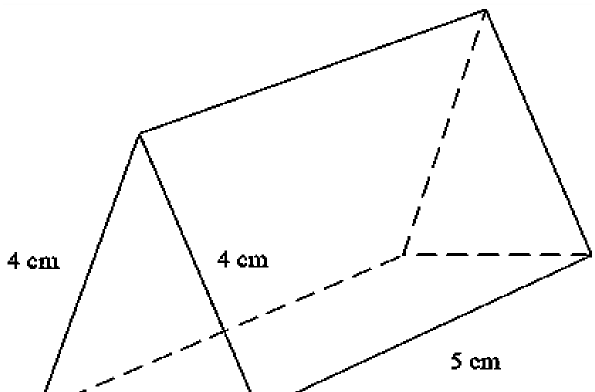
Solutions:

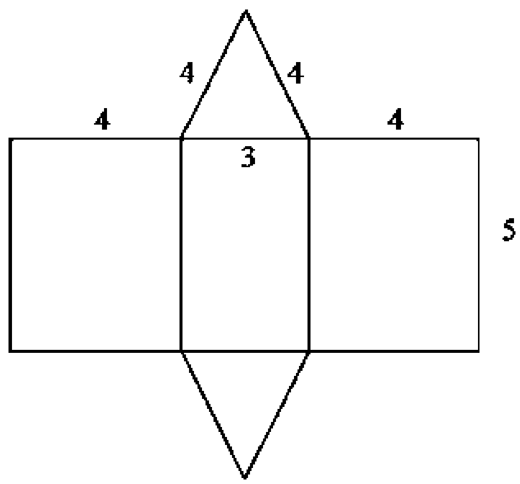
a.



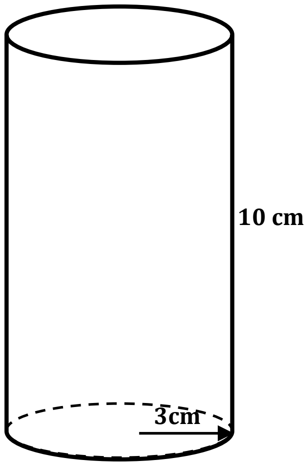


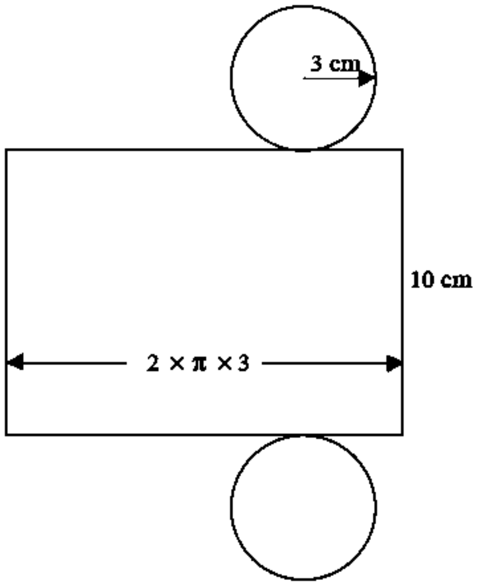
b.



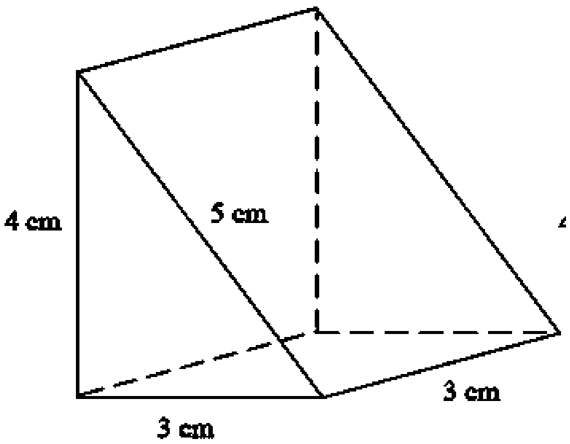


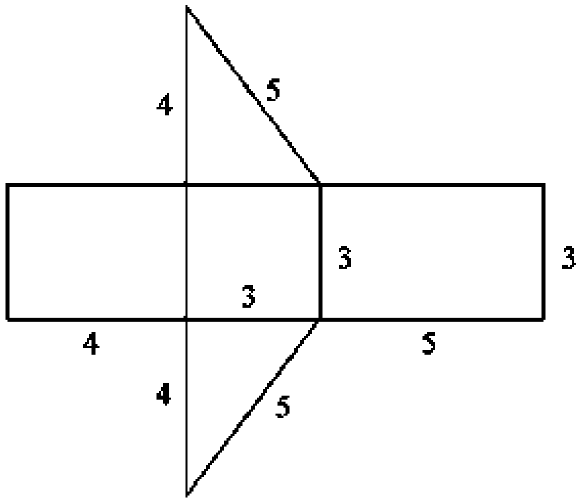
c.





d.





Lesson Title: Surface Area of a triangular prism	**Theme:** Mensuration	
Lesson Number: M3-L035	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to calculate the surface area of a triangular prism using the appropriate formula.	**Preparation** Draw the diagrams for Teaching and Learning found at the end of this lesson plan on the board.	

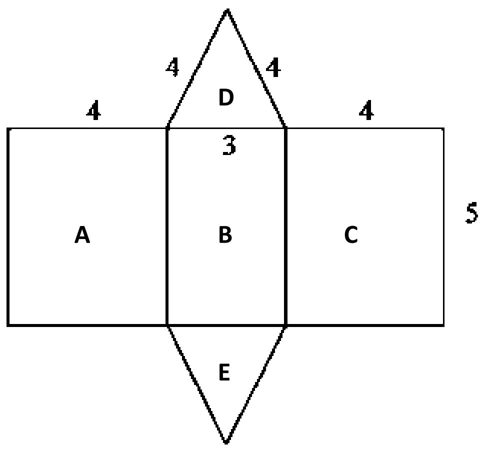
Opening *(4 minutes)*

1. Ask pupils to find the area of a triangle with base $b = 5$ cm and height $h = 8$ cm.
2. Invite a volunteer to tell the class what the area of the triangle is. (Answer: $A = \frac{1}{2}bh = \frac{1}{2}(8 \times 5) = \frac{1}{2} \times 40 = 20$ cm^2)
3. Ask a different volunteer what formula they used to find the area of the rectangle. (Answer: $A = \frac{1}{2}bh$)
4. Tell pupils that after today's lesson, they will be able to calculate the surface area of a triangular prism using the appropriate formula.

Teaching and Learning *(20 minutes)*

1. Invite a volunteer to give the basic method of finding surface areas of solids. (Example answer: Find the area of the individual faces, add up the areas.)
2. Explain that we will go through a step by step procedure for finding the surface area of the triangular prism in question a. i. on the board.
   Solution:

   Step 1. Assess and extract the given information from the problem.
   given: triangular prism with side lengths as shown
   Step 2. Draw the net of the triangular prism.



3. Invite a volunteer to draw the net of the triangular prism from the last lesson on the board given the measurements at right. (Answer: shown right)
4. Invite a volunteer to say what type of triangular the face is. (Answer: isosceles triangle)
5. Explain that this face gives the cross-section of the prism.
6. Ask pupils to copy the net in their exercise books, with each shape labelled as shown.

   Step 3. Find the areas for each face of the triangular prism.

7. Ask pupils to work with seatmates to write down a formula that we can use to find the surface area of the triangular prism.

8. Invite a volunteer to write down the surface area of the triangular prism on the board. Example answer:

surface area = area of rectangle A + area of rectangle B + area of rectangle C + area of triangle D + area of triangle E

9. Discuss with the class if they agree with the formula written on the board.
10. Ask volunteers for their own idea of what the formula should be. Write the suggestions on the board. (Example answers: Various; Pupils may find the area of the composite shape (ABC) instead of individual faces.)
11. Explain that we will look at 2 methods. Pupils can use other methods if they wish.
12. Invite volunteers to give answers to the areas of the individual shapes before writing them on the board.

Method 1.

area of rectangle A = area of rectangle C

= $4 \times 5 = 20$ cm^2

area of rectangle B = $3 \times 5 = 15$ cm^2

area of triangle D = area of triangle E

= $\frac{1}{2} \times 3 \times h$ where h is the height of the triangle

Find h: $4^2 = h^2 + 1.5^2$ Pythagoras' Theorem

$h^2 = 16 - 2.25 = 13.75$

$h = \sqrt{13.75}$

= 3.708 cm

area of triangle D = $\frac{1}{2} \times 3 \times 3.708$

= 5.562 cm^2

Step 4. Find the sum of the areas of the faces of the net.

surface area = $20 + 15 + 20 + 5.562 + 5.562$

= 66.124

Step 5. Write the final answer.

The surface area of the triangular prism = 66 cm^2 to the nearest cm^2.

13. Explain that we will now look at another method to find the surface area of the triangular prism.

Method 2.

surface area = area of large rectangle formed by A, B and C + area of triangle D + area of triangle E

= area of large rectangle formed by A, B and C + (2 × area of triangle D)

area of large rectangle formed by A, B and C = $(4 + 3 + 4) \times 5$ add the lengths for side ABC

= $11 \times 5 = 55$

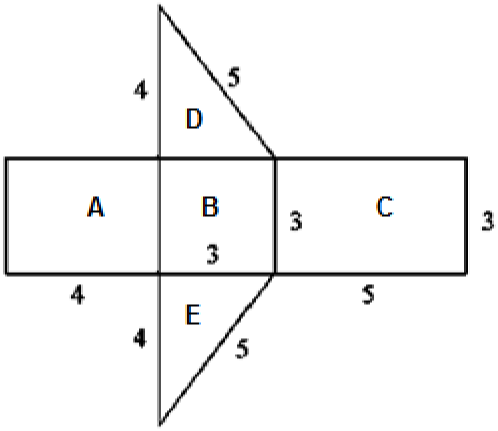
2 × area of triangle D = $3 \times 3.708 = 11.124$ since $2 \times \frac{1}{2}bh = bh$

surface area = $55 + 11.124 = 66.124$

= 66 cm^2 as before

14. Ask pupils to now look at the trapezium in question a. ii. What type of triangle is the end face or cross-section of the prism? (Answer right-angled triangle)

15. Invite a volunteer to come to the board to draw the net of the triangular prism from last lesson given the measurements at right.
(Answer: shown below)



16. Ask pupils to work with seatmates to find the surface area of the triangular prism.
17. Invite a volunteer to come to the board to show their solution.
18. Check the method follows logically to the final result.
19. The rest of the class should check their solution and correct any mistakes.

Solution:

Given: triangular prism with side lengths as shown

surface area = area of rectangle A + area of rectangle B + area of rectangle C + area of triangle D + area of triangle E

= area of rectangle A + area of rectangle B + area of rectangle C + $2\times$ area of triangle D

area of rectangle A = $4\times 3=12$ cm^2

area of rectangle B = $3\times 3=9$ cm^2

area of rectangle C = $5\times 3=15$ cm^2

this is the same as: $(4+3+5)\times 3=12\times 3=36$ cm^2

$2\times$ area of triangle D = $3\times 4=12$ cm^2

surface area = $12+9+15+12=48$

The surface area of the triangular prism = 48 cm^2

20. Explain that in the case of a triangular prism it is advisable to draw the net of the prism to help with calculating the surface area.

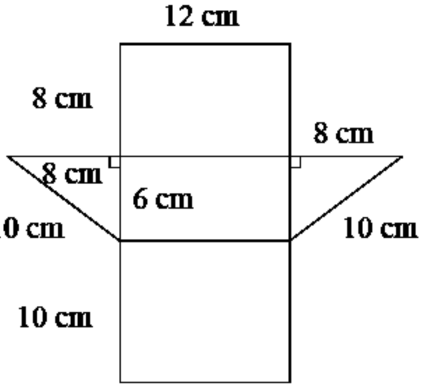
Practice *(15 minutes)*

1. Write question b. on the board.
2. Ask pupils to work independently to answer question b.
3. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.

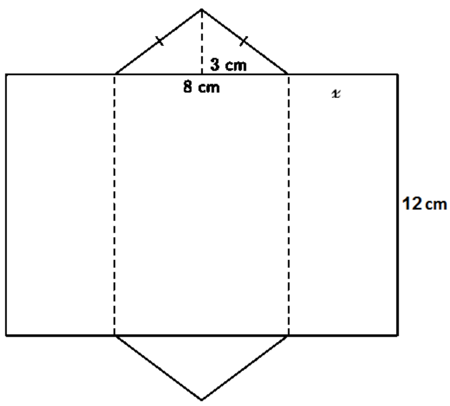
Solutions:

b. Given: triangular prism with side lengths as shown

i



ii.



i. surface area $= $ area of large rectangle + 2 × area of (right-angled) triangle

$= ((8 + 6 + 10) \times 12) + (6 \times 8)$ since $2 \times \frac{1}{2} \times 6 \times 8 = 6 \times 8$

$= (24 \times 12) + 48$

surface area $= 336$ cm^2

ii. surface area $=$ area of large rectangle + 2 × area of (isosceles) triangle

length of rectangle side $x =$ slant height of triangle

$x^2 = 3^2 + 4^2$

$x^2 = 9 + 16 = 25$

$x = \sqrt{25} = 5$ cm

surface area $= ((5 + 8 + 5) \times 12) + (3 \times 8)$

$= (18 \times 12) + 24$

surface area $= 240$ cm^2

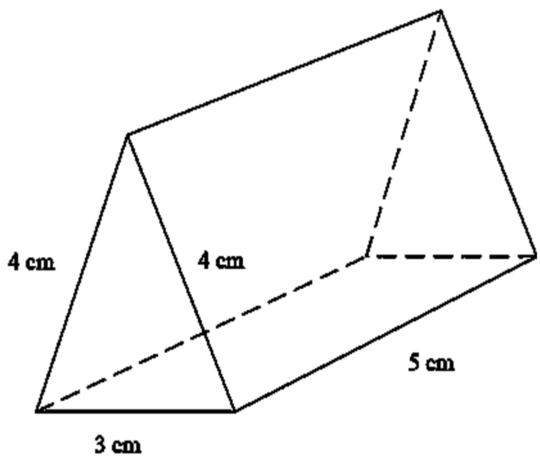
Closing *(1 minute)*

1. For homework have pupils do the practice activity PHM3-L035 in the Pupil Handbook.

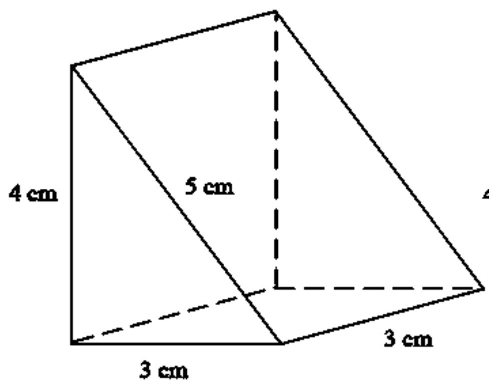
[QUESTIONS]

a. Use the nets of the triangular prisms from the previous lesson .to find the surface area of the given prisms. Give answers to the nearest cm^2

i.

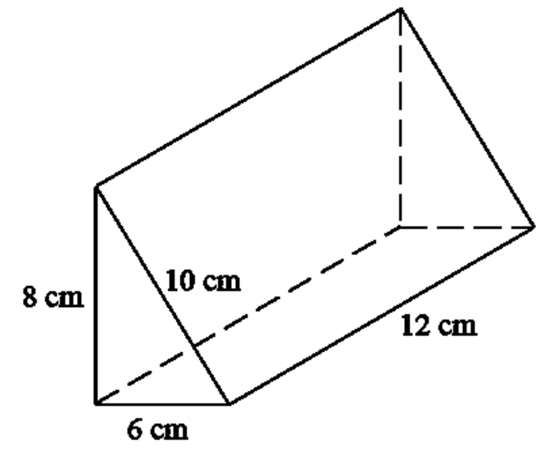


ii.



b. Find the surface area of the prisms below.

i.



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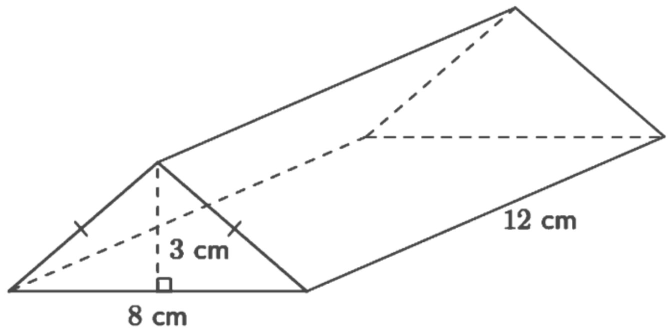
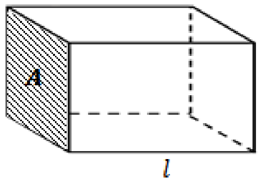


<table>
<tr><td>Lesson Title: Volume of a triangular P-prism</td><td colspan="2">Theme: Mensuration</td></tr>
<tr><td>Lesson Number: M3-L036</td><td>Class: SSS 3</td><td>Time: 40 minutes</td></tr>
<tr><td>Learning Outcome
By the end of the lesson, pupils will be able to calculate the volume of a triangular prism using the appropriate formula.</td><td colspan="2">Preparation
Sketch the cuboid shown at right on the board.
</td></tr>
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Opening *(4 minutes)*

1. Ask pupils to find the volume of the cuboid shown if the area A of the cross-section (end face) is 25 cm^2 and the side length l is 4 cm.
2. Invite a volunteer to say the formula they used to calculate the volume. (Answer: $V = Al$)
3. Invite another volunteer to give the answer. (Answer: 100 cm^3)
4. Tell pupils that after today's lesson, they will be able to calculate the volume of a triangular prism using the appropriate formula.

Teaching and Learning *(15 minutes)*

1. Explain:
   h. The formula for the volume of a cuboid or rectangular prism can be used to find the volume of all prisms.
   i. If we know the area A of the cross-section of the prism then the volume $V = Al$ where l is the length of the prism.
   j. We will use the prisms from last lesson to practise how to find volumes of triangular prisms.
   [NOTE: They have been reproduced at the end of this lesson plan for convenience.]
2. Invite a volunteer to draw the triangular prism for question a. i. from last lesson.
3. Explain the solution to find the volume of a triangular prism step by step.
   Solution:

a. i. **Step 1.** Assess and extract the given information from the problem.
Given: triangular prism with side lengths as shown

Step 2. Find the area A of the cross-section using the appropriate formula

area of cross-section $A = \frac{1}{2}bh$ where b is the base and h is the height of the triangle

$= \frac{1}{2} \times 3 \times h$

Find the height h $4^2 = h^2 + 1.5^2$ Pythagoras' Theorem

$h^2 = 16 - 2.25 = 13.75$

$h = \sqrt{13.75} = 3.708$

$$A = \frac{1}{2} \times 3 \times 3.708$$
$$A = 5.562 \text{ cm}^2$$

Step 3. Substitute into the appropriate formula.

$$\begin{aligned} \text{volume, } V &= Al \\ &= 5.562 \times 5 \\ V &= 27.81 \end{aligned}$$

Step 4. Write the answer.

The volume of the triangular prism $= 27.8$ cm^3 to 1 decimal place.

4. Invite a volunteer to draw the triangular prism for question a. ii. on the board.
5. Ask pupils to work with seatmates to find the area of the cross-section.
6. Invite a volunteer to say what the area is. (Answer: $A = \frac{1}{2}bh = \frac{1}{2} \times 3 \times 4 = 6$ cm^2)
7. Ask pupils to work with seatmates to find the volume of the triangular prism.
8. Invite another volunteer to say what the volume. (Answer: $V = 6 \times 3 = 18$ cm^3)
9. Ask pupils to write up the solution neatly in their exercise books.

   Solution:

   a. ii. Given: triangular prism with side lengths as shown

   $$\begin{aligned} \text{volume, } V &= Al \\ &= \left(\frac{1}{2} \times 3 \times 4\right) \times 3 \\ &= 6 \times 3 \\ V &= 18 \text{ cm}^3 \end{aligned}$$

   The volume of the triangular prism is 18 cm^3.

10. Explain that sometimes we are given the volume of the prism and we are asked to find the cross-sectional area or side length.
11. Write question b. on the board.
12. Ask pupils to work with seatmates to answer the question
13. Invite a volunteer to come to the board to show their solution. The rest of the class should check their solution and correct any mistakes.

    Solution:

    b. Given: triangular prism with Volume $V = 112.5$ cm^3 and length $l = 15$ cm

    $$\begin{aligned} \text{Volume } V &= Al \\ 112.5 &= 15A \\ \frac{112.5}{15} &= A \\ A &= 7.5 \text{ cm}^2 \end{aligned}$$

    The cross-sectional area of the triangular prism is 7.5 cm^2.

Practice *(20 minutes)*

1. Write the questions c. and d. on the board.
2. Ask pupils to work independently to answer questions c. and d.
3. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.

   Solutions:

c. i Given: triangular prism with side lengths as shown

$$\begin{aligned} \text{volume, } V &= Al \\ &= \left(\frac{1}{2} \times 6 \times 8\right) \times 12 \\ &= 24 \times 12 \\ & 288 \text{ cm}^3 \end{aligned}$$

The volume of the triangular prism is 288 cm^3.

ii. Given: triangular prism with side lengths as shown

$$\begin{aligned} \text{volume, } V &= Al \\ &= \left(\frac{1}{2} \times 8 \times 3\right) \times 12 \\ &= 12 \times 12 \\ & 144 \text{ cm}^3 \end{aligned}$$

The volume of the triangular prism is 144 cm^3.

d Given: triangular prism roof with cross-section $b = 1.2$ m, $h = 80$ cm = 0.8 m, $l = 8$ m,

$$\begin{aligned} \text{volume, } V &= Al \\ &= \left(\frac{1}{2} \times 1.2 \times 0.8\right) \times 8 \\ &= 0.48 \times 8 \\ & 3.84 \text{ m}^3 \end{aligned}$$

The volume of the roof is 3.84 m^3.

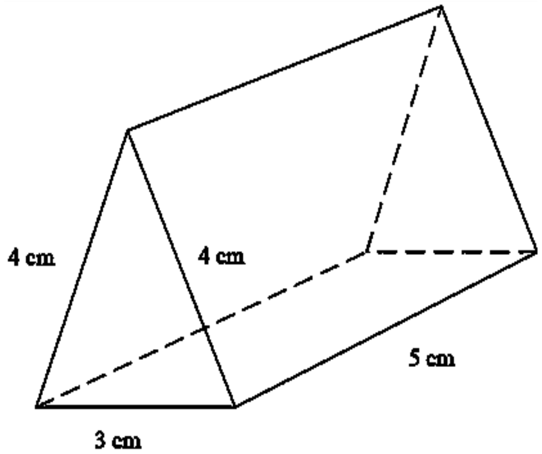
Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM3-L036 in the Pupil Handbook.

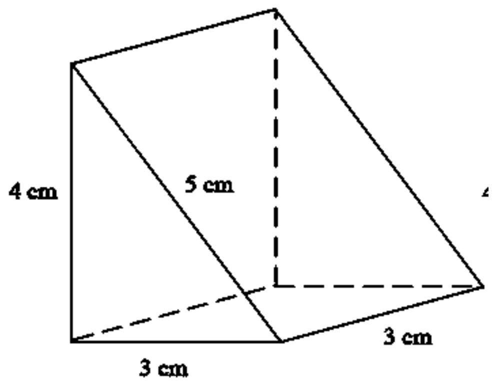
[QUESTIONS]

- Find the volume of the given prisms. Give answers to a reasonable accuracy.

i.



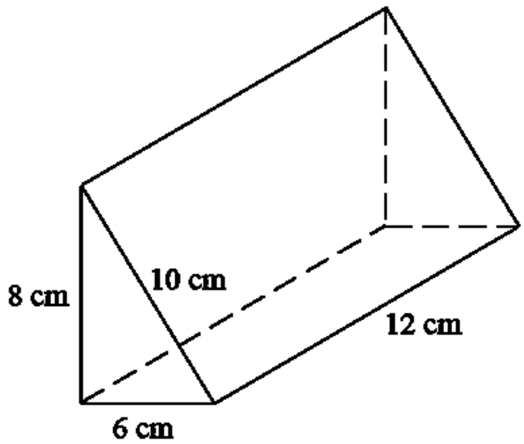
ii.



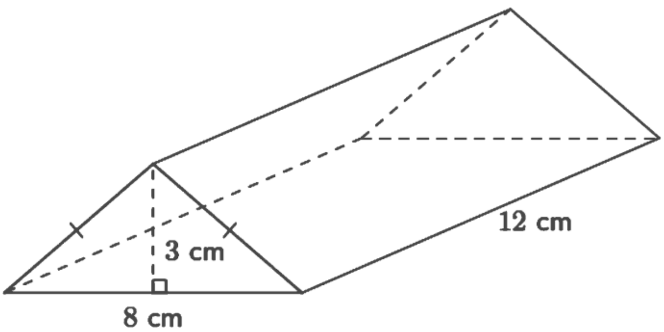
b. The volume of a triangular prism of length 15 cm is 112.5 cm^3. Find its cross-sectional area.

c. Find the surface area of the prisms below.

i.



ii.



d. The roof of a hut is in the shape of a triangular prism. The cross-section of the roof is an isosceles triangle with base 1.2 m and height 80 cm. If the roof is 8 m long what is the volume of the roof?

Lesson Title: Surface area of a cylinder	**Theme:** Mensuration	
Lesson Number: M3-L037	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to calculate the surface area of a cylinder using the appropriate formula.	**Preparation** Draw the diagrams for Teaching and Learning found at the end of this lesson plan on the board.	

Opening *(4 minutes)*

1. Ask pupils to find the circumference of a circle with radius $r = 7$ cm. Take $\pi = \frac{22}{7}$.
2. Invite a volunteer to tell the class what the circumference of the circle is. (Answer: $C = 2\pi r = 2 \times \frac{22}{7} \times 7 = 44$ cm)
3. Tell pupils that after today's lesson, they will be able to calculate the surface area of a cylinder using the appropriate formula.

Teaching and Learning *(20 minutes)*

1. Refer to the diagram on the board.
2. Point out on the diagram the different properties of the cylinder. Make sure each is clear before continuing.
3. Explain:
   - A cylinder is a prism with a circular cross-section. If we cut across the cylinder, the end faces will always be circular.
   - We can cut down the height of the cylinder. If we flatten the resulting shape, we get a rectangle with a width equal to the curved surface.
     [NOTE: This can be shown practically using a rolled up piece of paper]
   - The width of the curved surface equals the circumference of the circle.
4. Invite a volunteer to say what pieces make up the cylinder. (Answer: 3 pieces; 2 circular end faces and a curved surface.)
5. Explain:
   - A **solid** cylinder will always have these 3 pieces.
   - A **hollow** cylinder with one end open will have 2 pieces – one circular end face and the curved surface.
   - Assume a cylinder is solid unless otherwise stated.
6. Invite a volunteer to say how many pieces a hollow cylinder with both ends open will have. (Answer: 1 – the curved surface only.)
7. Invite a volunteer to give the basic method of finding surface areas of solids. (Example answer: Find the area of the individual faces, add up the areas.)
8. Explain that to find the surface area of a solid cylinder, we need to find the individual areas.

surface area of solid cylinder = area of circular end faces + area of curved surface

area of circular end faces = $2 \times \pi r^2$ since there are 2 end faces

$= 2\pi r^2$

area of curved surface $= 2\pi r \times h$ — area of rectangle with width $2\pi r$ and height, h

$= 2\pi rh$

surface area of solid cylinder $= 2\pi r^2 + 2\pi rh$

$= 2\pi r(r + h)$

surface area of hollow cylinder with one end open $= \pi r^2 + 2\pi rh$ — since there is only 1 end face

$= \pi r(r + 2h)$

surface area of hollow cylinder with both ends open $= 2\pi rh$ — since there are no end faces

9. Write question a. i. on the board:

Solution:

Step 1. Assess and extract the given information from the problem.

Given: cylinder with radius $r = 7$ cm and height $h = 10$ cm

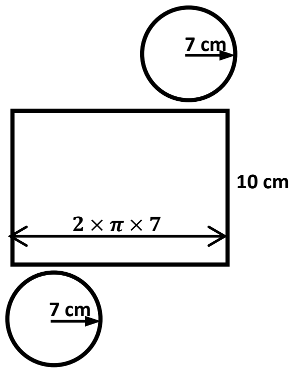
Step 2. Draw the net of the cylinder.

10. Invite a volunteer to draw the net of the cylinder on the board.
(Answer: shown below right)

Step 3. Find the sum of the areas of the faces of the net.

surface area = area of circular end faces + area of curved surface

Step 4. Find the surface area of the cylinder.



$$\begin{aligned} \text{surface area} &= 2\pi r^2 + 2\pi rh \\ &= 2\pi r(r + h) \\ &= 2 \times \frac{22}{7} \times 7 \times (7 + 10) \\ &= 2 \times 22 \times 17 \\ \text{surface area} &= 748 \text{ cm}^2 \end{aligned}$$

Step 5. Write the final answer.

The surface area of the cylinder is 748 cm^2.

11. Write question a. ii. on the board.
12. Ask pupils to work with seatmates to find the surface area of the cylinder in question a. ii.
13. Invite a volunteer to come to the board to show their solution. The rest of the class should check their solution and correct any mistakes.

Solution:

a. ii. Given: cylinder with radius $d = 14$ cm and height $h = 25$ cm

$$\begin{aligned} \text{surface area} &= 2\pi r^2 + 2\pi rh \\ &= 2\pi r(r + h) \\ &= 2 \times \frac{22}{7} \times 7 \times (7 + 25) \qquad \text{since } r = \tfrac{14}{2} = 7 \text{ cm} \\ &= 2 \times 22 \times 32 \\ \text{surface area} &= 1{,}408 \text{ cm}^2 \end{aligned}$$

The surface area of the cylinder is $1{,}408$ cm^2.

14. Ask pupils to work out the surface area of the cylinder if it was hollow at one end.
15. Invite a volunteer to come to the board to show their solution.

Given: hollow cylinder with radius $d = 14$ cm and height $h = 25$ cm

surface area	=	$\pi r^2 + 2\pi rh$	only one end face
	=	$\pi r(r + 2h)$	
	=	$\frac{22}{7} \times 7 \times (7 + (2 \times 25))$	since $r = \frac{14}{2} = 7$ cm
	=	22×57	
surface area	=	$1{,}254$ cm^2	

The surface area of the cylinder is $1{,}254$ cm^2.

16. Explain that in the case of a cylinder, it is advisable to make a quick sketch of the cylinder and net to help with calculating the surface area.

Practice *(15 minutes)*

1. Write questions b. and c. on the board.
2. Ask pupils to work independently to answer questions b. and c.
3. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.

Solutions:

b. i. Given: cylinder with radius $r = 4$ cm and height $h = 8$ cm

surface area	=	$2\pi r(r + h)$
	=	$2 \times 3.142 \times 4(4 + 8)$
		$2 \times 3.142 \times 4 \times 12$
		301.632
surface area	=	301.63 cm^2

ii Given: cylinder with radius $d = 4$ cm and height $h = 22$ cm

surface area	=	$2\pi r(r + h)$	
	=	$2 \times 3.142 \times 2(2 + 22)$	since $r = \frac{4}{2} = 2$ cm
		$2 \times 3.142 \times 2 \times 24$	
		301.632	
surface area	=	301.63 cm^2	

iii Given: cylinder with radius $r = 3$ cm and height $h = 13$ cm

surface area	=	$2\pi r(r + h)$
	=	$2 \times 3.142 \times 3(3 + 13)$
		$2 \times 3.142 \times 3 \times 16$
		301.632
surface area	=	301.63 cm^2

The three cylinders all have the same surface area $= 301.63$ cm^2.

c. Given: cylinder with radius $R = 40$ cm, $r = 25$ cm, height $h = 12$ cm

surface area of front and back	=	$2\pi(R^2 - r^2)$

$= \quad 2 \times 3.142 \times (40^2 - 25^2)$

$= \quad 6{,}126.9$ cm²

surface area of inner cylinder $= \quad 2\pi rh$

$= \quad 2 \times 3.142 \times 25 \times 12$

$= \quad 1{,}885.2$ cm

surface area of outer cylinder $= \quad 2\pi rh$

$= \quad 2 \times 3.142 \times 40 \times 12$

$= \quad 3{,}016.32$ cm

Total surface area of tyre $= \quad 6{,}126.9 + 1{,}885.2 + 3{,}016.32$

$11{,}028.42$ cm²

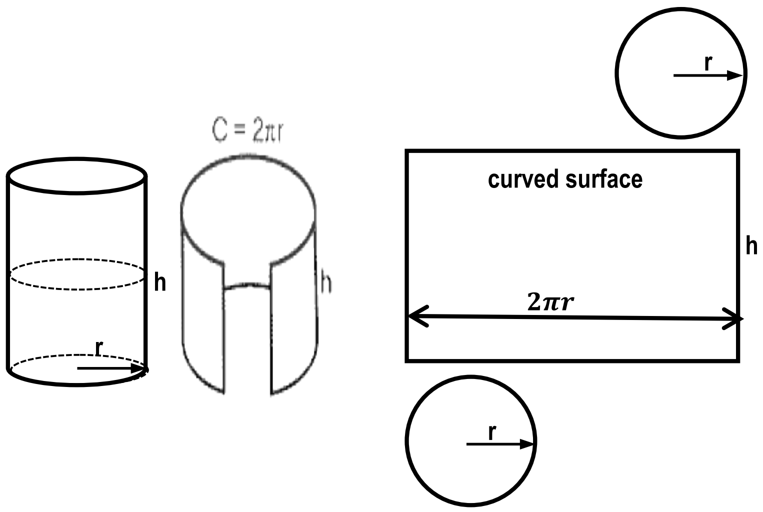
The total surface area of the tyre is $11{,}028.42$ cm².

Closing *(1 minute)*

1. For homework have pupils do the practice activity PHM3-L037 in the Pupil Handbook.

[DIAGRAM FOR TEACHING AND LEARNING]

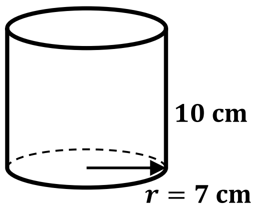
Draw on the board before the lesson.



[QUESTIONS]

a. Find the total surface area of the cylinders below. Take $\pi = \frac{22}{7}$

i.



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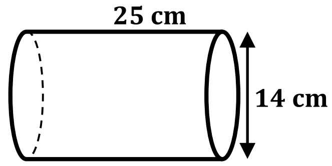
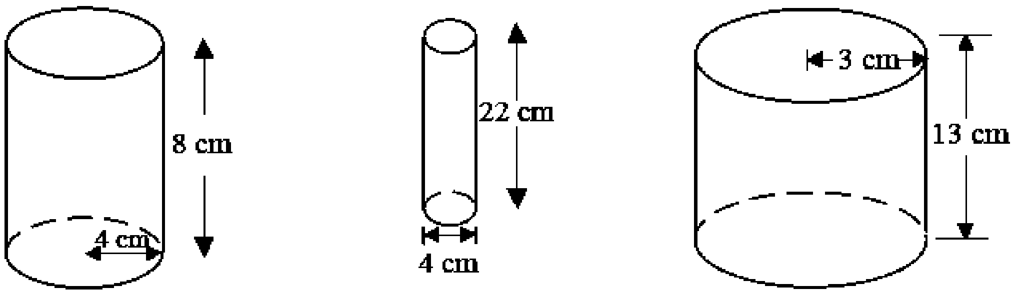
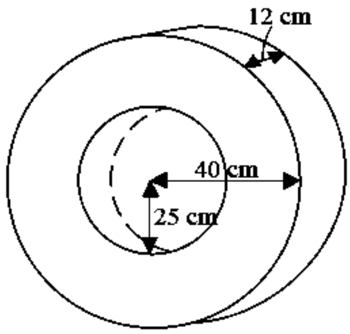


b. Show that the cylinders below have the same surface area.
Give your answer to 2 decimal places. Take $\pi = 3.142$



c. A car tyre is made up of a hollow cylinder with a hole cut out of the centre. Find the total surface area of the tyre.
Give your answer to the nearest cm^2.
Take $\pi = 3.142$



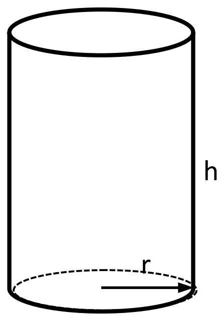
Lesson Title: Volume of a cylinder	Theme: Mensuration	
Lesson Number: M3-L038	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to calculate the volume of a cylinder using the appropriate formula.	**Preparation** Write questions a. i. and a. ii. on the board.	

Opening *(4 minutes)*

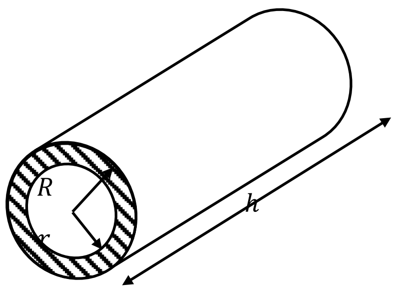
1. Ask pupils to find the surface area of a hollow water pipe with both ends open. Its radius is 21 cm and height 10 cm. Take $\pi = \frac{22}{7}$.
2. Invite a volunteer to say the formula they used to calculate the surface area. (Answer: surface area $= 2\pi rh$, since no circular end faces in the hollow open pipe)
3. Invite another volunteer to give the answer. (Answer: surface area $= 2 \times \frac{22}{7} \times 21 \times 10 = 2 \times 22 \times 3 \times 10 = 1{,}320$ cm^2)
4. Tell pupils that after today's lesson, they will be able to calculate the volume of a cylinder using the appropriate formula.

Teaching and Learning *(15 minutes)*

1. Explain:
   - We can use the formula for the volume of a cuboid or rectangular prism to find the volume of all prisms.
   - If we know the area A of the cross-section of the prism, then the volume $V = Al$ where l is the length of the prism.
2. Draw the cylinder shown at right on the board.
3. Invite a volunteer to give the area of the cross-section of the cylinder. (Answer: $A = \pi r^2$.)
4. Write on the board: The volume of the cylinder using the formula:
   Volume, $V = Al$
   $= \pi r^2 \times h$ since $l = h$ for a cylinder



5. Draw the hollow cylinder shown at right.
6. Explain:
   - There are 2 other types of volumes we can find for a cylinder.
   - This usually occurs when we have a pipe in the form of a hollow cylinder.
   - The cylindrical pipe shown has an outside or external radius R and an inside or internal radius r.
   - The cross-section of the pipe is shown shaded in the diagram.



7. The **volume of material** used to make the pipe is given by:
   volume, $V = Ah$ since $l = h$ for a pipe

But A = area of circle radius R − area of circle radius r

$= \pi R^2 - \pi r^2$

$= \pi(R^2 - r^2)$

$\therefore V = \pi(R^2 - r^2)h$

$= \pi h(R^2 - r^2)$

8. The **volume of any liquid** flowing through the pipe is given by:

   volume, $V = Ah$

   $= \pi r^2 h$ where r is the internal radius of the pipe
9. We will use the cylinders from the last lesson to practice how to find volumes of cylinders. They are reproduced here at the end of the lesson plan for convenience.
10. Invite a volunteer to draw the cylinder for question a. i. from last lesson.
11. Explain the solution to find the volume of a cylinder step by step.

    Solution:

    a. i. **Step 1.** Assess and extract the given information from the problem.

    Given: cylinder with radius $r = 7$ cm and height $h = 10$ cm

    Step 2. Find the area A of the cross-section using the appropriate formula

    area of cross-section $A = \pi r^2$ where r is the radius of the circle

    $= \frac{22}{7} \times 7^2$

    $22 \times 7 = 154$

    Step 3. Substitute into the appropriate formula.

    volume, $V = Ah$

    $= (\pi r^2)h$

    $= 154 \times 10$

    $V = 1{,}540$ cm^3

    Step 4. Write the answer.

    The volume of the cylinder $= 1{,}540$ cm^3.
12. Invite a volunteer to draw the cylinder for question a. ii. on the board.
13. Ask pupils to work with seatmates to find the area of the cross-section.
14. Invite a volunteer to say what the area is.

    (Answer: $A = \pi r^2 = \frac{22}{7} \times \left(\frac{14}{2}\right)^2 = \frac{22}{7} \times 7^2 = 154$ cm^2)
15. Ask pupils to work with seatmates to find the volume of the cylinder.
16. Invite another volunteer to say what the volume.

    (Answer: $V = 154 \times 25 = 3{,}850$ cm^3)
17. Ask pupils to write up the solution neatly in their exercise books. They can combine steps for efficiency.

    Solution:

    a. ii. Given: cylinder with diameter $d = 14$ cm and height $= 25$ cm

    volume, $V = Al$

    $\pi r^2 h$

    $= \frac{22}{7} \times \left(\frac{14}{2}\right)^2 \times 25$

$$= \frac{22}{7} \times 7^2 \times 25$$
$$= 22 \times 7 \times 25$$
$$V = 3{,}850 \text{ cm}^3$$

The volume of the cylinder is $3{,}850$ cm^3.

18. Explain that sometimes we are given the volume of the cylinder and we are asked to find the radius or the height given the other dimension.
19. Write on the board: Suppose the volume of the cylinder in question a. ii. is $11{,}000$ cm^3. If the height is 20 cm, what is the radius of the cylinder? Give answer to 2 decimal places. Take $\pi = 3.142$.
20. Ask pupils to work with seatmates to answer the question
21. Invite a volunteer to come to the board to show their solution. The rest of the class should check their solution and correct any mistakes.

Solution:

Given: cylinder with volume $V = 11{,}000$ cm^3 and height $h = 20$ cm

$$\text{volume } V = Al$$
$$= \pi r^2 h$$
$$r^2 = \frac{V}{\pi h}$$
$$= \frac{11000}{3.142 \times 20} = 175.0477$$
$$r = \sqrt{175.0477}$$
$$= 13.2306$$

The radius of the cylinder is 13.23 cm.

Practice *(20 minutes)*

1. Ask pupils to work independently to answer questions b and c.
2. Write the questions on the board.
3. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.

Solutions:

b. i. Given: cylinder with radius $r = 4$ cm and height $h = 8$ cm

$$\text{volume, } V = \pi r^2 h$$
$$= 3.142 \times 4^2 \times 8$$
$$\text{volume, } V = 402.176 \text{ cm}^3$$

The volume, V of the cylinder is 402 cm^3 to the nearest cm^3

ii. Given: cylinder with radius $d = 4$ cm and height $h = 22$ cm

$$\text{volume, } V = 3.142 \times 2^2 \times 22 \qquad \text{s since } r = \frac{4}{2} = 2 \text{ cm}$$
$$V = 276.496 \text{ cm}^3$$

The volume, V of the cylinder is 276 cm^3 to the nearest cm^3

iii. Given: cylinder with radius $r = 3$ cm and height $h = 13$ cm

$$\text{volume, } V = 3.142 \times 3^2 \times 13$$
$$V = 367.614 \text{ cm}^3$$

The volume, V of the cylinder is 368 cm^3 to the nearest cm^3.

Cylinder i. has the largest volume $= 402$ cm^3.

c. Given: cylinder with radius $R = 40$ cm, $r = 25$ cm, height $h = 12$ cm

volume, $V \quad = \quad \pi h(R^2 - r^2)$

$\quad = \quad 3.142 \times 12 \times (40^2 - 25^2)$

$V \quad = \quad 36{,}761.4$ cm^3

The volume of material used to make the tyre is $36{,}761.4$ cm^3.

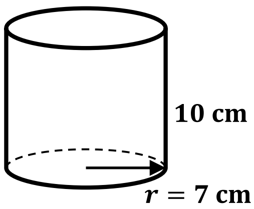
Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM3-L038 in the Pupil Handbook.

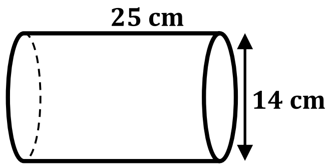
[QUESTIONS]

a. Find the volume of the cylinders below. Take $\pi = \frac{22}{7}$.

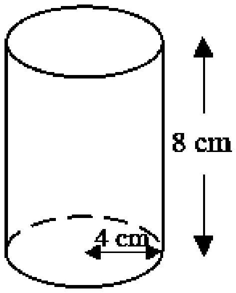
i.

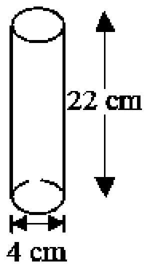


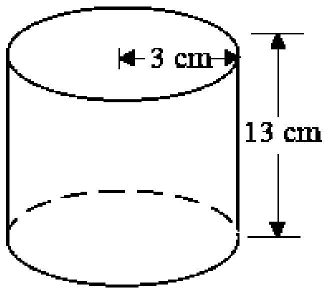
ii.



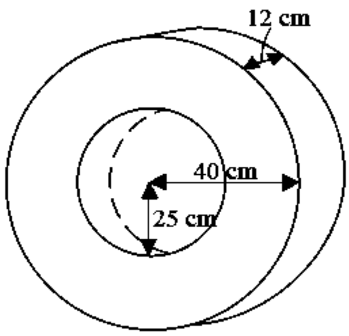
b. Each of the cylinders below has the same surface area. Which has the biggest volume? Give your answer to the nearest cm^3. Take $\pi = 3.142$.







c. A car tyre is made up of a hollow cylinder with a hole cut out of the centre. Find the volume of the material used to make the tyre.
Give your answer to 1 decimal place.
Take $\pi = 3.142$.



Lesson Title: Surface area of a cone	**Theme:** Mensuration	
Lesson Number: M3-L039	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to calculate the surface area of a cone using the appropriate formula.	**Preparation** Draw the diagrams for Teaching and Learning found at the end of this lesson plan on the board.	

Opening *(4 minutes)*

1. Invite a volunteer to tell the class what the formula to find the area of a sector is. (Answer: $\frac{\theta}{360} \times \pi r^2$)
2. Ask pupils to find the area of a sector of a circle with $r = 42$ cm, $\theta = 60°, \pi = \frac{22}{7}$.
3. Invite a volunteer to give the answer. (Answer: area of sector = $\frac{60}{360} \times \frac{22}{7} \times 42^2 = 924$ cm^2)
4. Tell pupils that after today's lesson, they will be able to calculate the surface area of a cone using the appropriate formula.

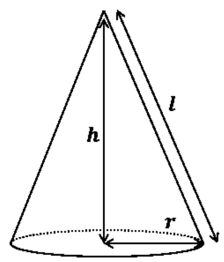
Teaching and Learning *(20 minutes)*

1. Refer to the diagram on the board. Point out on the diagram the different properties of the net of a cone, which is a sector of a circle and the cone it forms. Check for understanding before continuing.
2. Explain:
   - A sector of a circle can be bent to make the curved surface of an open cone such as the one on the board.
     [NOTE: This can be shown practically using a cut out piece of paper similar to the net.]
   - A **solid** cone will always have 2 pieces – one circular end face or base and the curved surface created by the sector.
   - A **hollow** cone with one end open will have just one piece – the curved surface created by the sector.
   - Assume a cone is solid unless otherwise stated.
3. Invite a volunteer to give the basic method of finding surface areas of solids. (Example answer: Find the area of the individual faces, add up the areas.)
4. Invite another volunteer to give this formula in words in the case of a cone. (Answer: surface area of cone = area of curved surface + area of circular base)
5. Explain:
   - Since the cone is made from the sector:
   – The area of the curved surface of the cone is equal to the area of the sector.
   – The length of the arc AXB is the same as the circumference of the circular base cone of the cone with radius r.
   – The radius of the circle l is the same as the slant length of the cone.

6. Invite a volunteer to give the area of the sector.
(Answer: $\frac{\theta}{360} \times \pi l^2$ where $l =$ radius of the sector OAXB, and θ is the angle subtended by the arc AXB at the centre).
7. Invite another volunteer to give the length of the arc of the sector. (Answer: $\frac{\theta}{360} \times 2\pi l$)
8. Write on the board:

surface area of cone	=	area of curved surface + area of circular base	
area of curved surface	=	$\frac{\theta}{360} \times \pi l^2$	area of sector, radius l and angle θ
Also, length of arc AXB	=	circumference of circular base of cone	
$\frac{\theta}{360} \times 2\pi l$	=	$2\pi r$	since the circular base has radius r
∴ $\frac{\theta}{360}$	=	$\frac{r}{l}$	
area of curved surface	=	$\frac{r}{l} \times \pi l^2$	
area of curved surface	=	πrl	radius r, slant length l of cone
area of circular base	=	πr^2	one circular base, radius r
∴ surface area of cone	=	$\pi rl + \pi r^2$	add the areas
	=	$\pi r(l + r)$	
surface area of hollow cone	=	πrl	since there is only the curved surface

9. Write question a. i. on the board:
Solution:
Step 1. Assess and extract the given information from the problem.
Given: cone with radius $r = 9$ cm and slant height $l = 15$ cm
Step 2. Find the surface area of the cone.



surface area $= \pi rl$
$= 3.142 \times 9 \times 15$
surface area $= 424.17$

Step 3. Write the final answer.

The surface area of the cone is 424 cm^2 to the nearest cm^2.
10. Write question a. ii. on the board.
11. Ask pupils to work with seatmates to find the surface area of the cone in question a. ii.
12. Invite a volunteer to come to the board to show their solution. The rest of the class should check their solution and correct any mistakes.
Solution:
Given: cone with radius $r = 7$ cm and height $h = 5$ cm
surface area $= \pi rl$
$l^2 = h^2 + r^2$ Find l using Pythagoras' Theorem
$= 5^2 + 7^2$
$= 25 + 49 = 74$

$l = \sqrt{74} = 8.60$ cm

surface area $= 3.142 \times 7 \times 8.60 = 189.15$

The surface area of the cone is 189 cm^2 to the nearest cm^2.

13. Ask pupils to work out the surface area of the cone if it had a base at one end.
14. Invite a volunteer to come to the board to show their solution.

Given: solid cone with radius $r = 7$ cm and height $h = 5$ cm

surface area $= \pi r(l + r)$

$= 3.142 \times 7 \times (8.60 + 7)$ $\quad$ $l = 8.60$ cm from previous

surface area $= 343.11$ cm^2

The surface area of the cone is 343 cm^2 to the nearest cm^2.

Practice *(15 minutes)*

1. Ask pupils to work independently to answer questions b. and c.
2. Write questions b. and c. on the board.
3. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.

Solutions:

b. i. Given: cone with radius $r = 5$ cm and slant height $l = 12$ cm

$\frac{\theta}{360} \times 2\pi l = 2\pi r$ $\quad$ since the length of the arc is the same as the circumference of the circular base

$\frac{\theta}{360} = \frac{r}{l}$

$\theta = \frac{360 \times r}{l}$

$= \frac{360 \times 5}{12}$

$\theta = 150°$

The angle of the sector used is $150°$ to the nearest whole number.

ii. surface area $= \pi r l$

$= 3.142 \times 5 \times 12$

$= 188.52$

surface area $= 189$ cm^2

The surface area of the cone is 189 cm^2 to the nearest whole number.

c. i. Given: circle with angle at centre $240°$ cm and radius $r = 25$ cm

$\frac{\theta}{360} = \frac{r}{l}$ $\quad$ From part b. above

$r = \frac{\theta \times l}{360}$

$= \frac{240 \times 25}{360}$

$= 16.666$

$r = 17$ cm

The radius of the sector is 17 cm to the nearest whole number.

ii. surface area $= \pi r l$

$$= \quad 3.142 \times 17 \times 25$$
$$\text{surface area} \quad = \quad 1{,}335.35$$

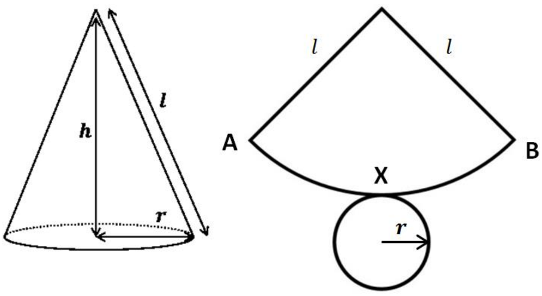
The surface area of the cone is $1{,}335.35$ cm^2

Closing *(1 minute)*

1. For homework have pupils do the practice activity PHM3-L039 in the Pupil Handbook.

[DIAGRAM FOR TEACHING AND LEARNING]

Draw on the board before the lesson.



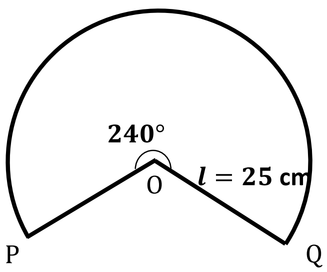
[QUESTIONS]

Give all answers to the nearest cm^2. Take $\pi = 3.142$ unless otherwise stated.

a. Find the curved surface area in the cone shown if:
   i. The base radius is 9 cm and slant height is 15 cm.
   ii. The base radius is 7 cm and height is 5 cm.

b. A cone of slant height 12 cm and radius 5 cm is made out of cardboard. Find to the nearest whole number:
   i. The angle of the sector used to make the cone.
   ii. The total surface area of cardboard used to make the cone. Exclude the base of the cone.

c. The diagram at right show the sector of a circle used to make a cone. If the angle at the centre $\theta = 240°$ and its radius 25 cm, find:
   i. The radius of the cone.
   ii. The surface area of the cone.



Lesson Title: Volume of a cone	**Theme:** Mensuration	
Lesson Number: M3-L040	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to calculate the volume of a cone using the appropriate formula.	**Preparation** Write questions a. i. and a. ii. on the board.	

Opening *(4 minutes)*

1. Ask pupils to find the surface area of an open cone with radius 21 cm and slant height 10 cm. Take $\pi = \frac{22}{7}$.
2. Invite a volunteer to say the formula they used to calculate the surface area. (Answer: surface area $= \pi r l$, since no base in the cone)
3. Invite another volunteer to give the answer. (Answer: surface area $= \frac{22}{7} \times 21 \times 10 = 22 \times 3 \times 10 = 660$ cm^2)
4. Tell pupils that after today's lesson, they will be able to calculate the volume of a cone using the appropriate formula.

Teaching and Learning *(20 minutes)*

1. Refer to the cone in the question on the board.
2. Explain: The formula for the volume of a cone is given by:

$$V = \frac{1}{3}\pi r^2 h$$

   where r is the radius and h the height of the cone.
3. Invite a volunteer to read question a. i. on the board.
4. Explain the solution to find the volume of a cone step by step.
   Solution:
   a. i. **Step 1.** Assess and extract the given information from the problem.
   given: cone with radius $r = 7$ cm and height $h = 5$ cm
   Step 2. Substitute into the appropriate formula.

$$\begin{aligned} \text{volume, } V &= \frac{1}{3}\pi r^2 h \\ &= \frac{1}{3} \times 3.142 \times 7^2 \times 5 \\ V &= 256.597 \text{ cm}^3 \end{aligned}$$

   Step 3. Write the answer.
   The volume of the cone $= 256.6$ cm^3 to 1 d.p.
5. Invite a volunteer to read question a. ii. on the board.
6. Invite another volunteer to assess the problem and extract the given information. (Example answer: given: cone with radius $r = 9$ cm and slant height $l = 15$ cm)
7. Invite another volunteer to say what we have been asked to find. (Answer: volume of the cone)
8. Ask pupils what formula is needed for volume of a cone. (Answer: $V = \frac{1}{3}\pi r^2 h$)

9. Ask pupils to look at what we are given and what is needed to find the volume. Is anything missing? Invite a volunteer to answer. (Example answer: We are given the slant height, l of the cone, we need to find the height, h)
10. Ask pupils to work with seatmates to find the height, h of the cone.
11. Invite a volunteer to show their solution on the board. The rest of the class should check their solution and correct any mistakes.

Solution:

a. ii.

$$\begin{aligned} l^2 &= h^2 + r^2 && \text{Find } h \text{ using Pythagoras' Theorem} \\ 15^2 &= h^2 + 9^2 \\ h^2 &= 15^2 - 9^2 \\ &= 225 - 81 = 144 \\ h &= \sqrt{144} = 12 \text{ cm} \end{aligned}$$

12. Ask pupils to now work out the volume of the cone.
13. Invite a volunteer to show their working out on the board. The rest of the class should check their solution and correct any mistakes.

Solution:

$$\begin{aligned} \text{volume, } V &= \frac{1}{3}\pi r^2 h \\ &= \frac{1}{3} \times 3.142 \times 9^2 \times 12 \\ V &= 1{,}018.008 \end{aligned}$$

The volume of the cone is $1{,}018.0$ cm^3 to 1 d.p.

14. Invite a volunteer to read question b. on the board.
15. Invite another volunteer to assess the problem and extract the given information. (Example answer: given: cone with 900 cm^3 volume and height 5 cm.)
16. Invite another volunteer to say what we have been asked to find. (Answer: radius of the cone)
17. Invite a volunteer to say the formula needed to find the radius of a cone when volume is known. (Answer: $V = \frac{1}{3}\pi r^2 h$)
18. Ask pupils to work with seatmates to find the radius of the cone.
19. Invite a volunteer to come to the board to show their solution. The rest of the class should check their solution and correct any mistakes

Solution:

b. Given: cone with 900 cm^3 volume and height 5 cm

$$\begin{aligned} \text{volume, } V &= \frac{1}{3}\pi r^2 h \\ \frac{3V}{\pi h} &= r^2 \\ r^2 &= \frac{3 \times 900}{3.142 \times 5} \\ &= 171.865 \\ r &= \sqrt{171.865} \\ r &= 13.109 \text{ cm} \end{aligned}$$

The radius of the cone is 13.1 cm to 1 d.p.

Practice *(15 minutes)*

1. Ask pupils to work independently to answer questions c.
2. Write question c. on the board.
3. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.

Solutions:

c. i. Given: area of sector $A = 631$ cm^2 and radius $l = 22$ cm

$$\text{area } A \text{ of sector } OPQ = \frac{\theta}{360} \times \pi l^2$$

where θ is the angle of the sector used to make the cone and l is the radius of the sector

$$\theta = \frac{360A}{\pi l^2}$$
$$= \frac{360 \times 631}{3.142 \times 22^2}$$
$$= 149.375$$
$$\theta = 149°$$

The angle of the sector is $149°$ to 3 s.f.

ii.

$$\text{arc length } PQ = \frac{\theta}{360} \times 2\pi l$$
$$= \frac{149}{360} \times 2 \times 3.142 \times 22$$
$$= 57.219$$
$$\text{arc length} = 57.2 \text{ cm}$$

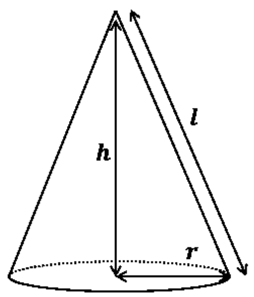
The length of the arc of the sector is 57.2 cm to 3 s.f.

iii. We are asked to find h. From the diagram, we know $l = 22$ cm.
We need to find the base radius r of the cone before we can find the height h.

$$\frac{\theta}{360} \times 2\pi l = 2\pi r$$

since the length of the arc is the same as the circumference of the circular base

$$\frac{\theta}{360} = \frac{r}{l}$$
$$r = \frac{\theta}{360} \times l$$
$$= \frac{149 \times 22}{360}$$
$$r = 9.106 \text{ cm}$$



$$l^2 = h^2 + r^2$$

Find h using Pythagoras' Theorem

$$22^2 = h^2 + (9.106)^2$$
$$h^2 = 22^2 - (9.106)^2$$
$$h = \sqrt{22^2 - (9.106)^2}$$
$$\sqrt{401.08}$$

$h = 20.027$
$h = 20.0$ cm

The height of the cone is 20.0 cm to 3 s.f.

iv. volume $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times 3.142 \times (9.106)^2 \times 20$ substitute $r = 9.106$ cm, $h = 20$ cm

$= 1{,}736.881$

volume $= 1{,}740$ cm^3

The volume of the cone is $1{,}740$ cm^3 to 3 s.f.

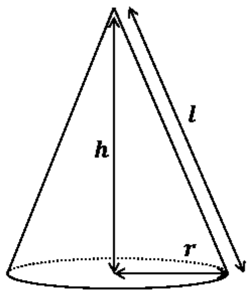
Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM3-L040 in the Pupil Handbook.

[QUESTIONS]

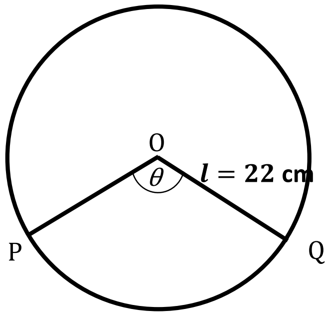
Give all answers to 1 decimal place unless otherwise stated.
Take $\pi = 3.142$ unless otherwise stated.

a. Find the volume of the cone shown if:
   i. Base radius is 7 cm and height is 5 cm.
   ii. Base radius is 9 cm and slant height is 15 cm.



b. A cone has volume of 900 cm^3. The height of the cone is 5 cm. Find its radius.

c. A sector of OPQ of area 631 cm^2 is cut out from a thin circular metal sheet of radius 22 cm. It is then folded with the straight edges meeting to make an open cone.
   Find:
   i. The angle of the sector.
   ii. The length of the arc of the sector.
   iii. The height of the cone.
   iv. The volume of the cone.
   Give your answers to 3 significant figures.



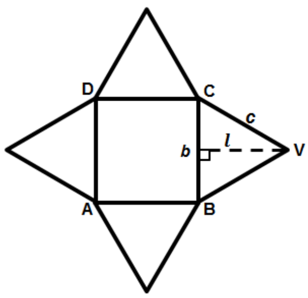
Lesson Title: Surface Area of a Rectangular Pyramid	**Theme:** Mensuration	
Lesson Number: M3-L041	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to calculate the surface area of a rectangular pyramid using the appropriate formula.	**Preparation** Draw the diagrams for Teaching and Learning found at the end of this lesson plan on the board.	

Opening *(4 minutes)*

1. Ask pupils to find the area of a triangle with base 10 cm and height 8 cm.
2. Invite a volunteer to give the answer.
   (Answer: area of triangle $= \frac{1}{2}bh = \frac{1}{2} \times 10 \times 8 = 40$ cm^2)
3. Tell pupils that after today's lesson, they will be able to calculate the surface area of a rectangular pyramid using the appropriate formula.

Teaching and Learning *(20 minutes)*

1. Refer to the diagrams on the board.
2. Explain:
   - The diagrams show a rectangular pyramid and its net.
   - A pyramid is any solid object that has a polygon for its base, and triangular sides which meet at a point or apex.
   - The type of polygon gives the pyramid its name.
   - A pyramid with a base shaped as a triangle is called a triangular pyramid.
   - If the base is a rectangle, it is called a rectangular pyramids.
3. Invite a volunteer to look at the net and say what type of pyramid the diagram shows. (Answer: rectangular pyramid)
4. Invite a volunteer to say how many triangular faces the rectangular pyramid has got. (Answer: 4 – one on each side of the base.)
   surface area of rectangular pyramid $=$ area of the net
   $=$ area of rectangular base + area of triangular faces
   $=$ area of ABCD $+$ area of VDC $+$ area of VAD $+$ area of VBC $+$ area of VAB
   since Δ VDC $=$ Δ VAB and Δ VBC $=$ Δ VAD
   surface area of rectangular pyramid $=$ area of ABCD $+ 2 \times$ (area of VDC) $+ 2 \times$ (area of VBC)
5. Explain:
   - For simplicity, most problems are based on a special rectangle with 4 equal sides – the square.
   - In a square, the slant height l of the pyramid is the same as the perpendicular height of the triangular faces. See diagram.
   - Write on the board:



surface area of square pyramid = area of ABCD + $4 \times$ (area of VBC)

$= b^2 + 4(\frac{1}{2} \times b \times l)$

surface area of square pyramid $= b^2 + 2bl$

6. Write question a. i. on the board.
   a. i. **Step 1.** Assess and extract the given information from the problem.
7. Invite a volunteer to say what information we are given. (Answer: given: square pyramid of side length = 8 cm and slant height = 16 cm)

   Step 2. Use the net of the square pyramid to calculate the surface area.

   total surface area = area of square base + area of triangular faces

   area of square base $= b^2$

   $= 8^2 = 64$ cm^2

   area of triangular faces $= 4\left(\frac{1}{2}bl\right)$ where l the slant height of the pyramid = perpendicular height of the triangular faces

   $= 2bl$

   $= 2 \times 8 \times 16 = 256$ cm^2

   Step 3. Find the surface area of the pyramid.

   total surface area $b^2 + 2bl$

   $= 64 + 256$

   $= 320$ cm^2

   Step 4. Write the final answer.

   The total surface area of the square pyramid is 320 cm^2.
8. Write question a. ii. on the board.
9. Invite a volunteer to assess the problem and tell the class what information we are given. (Answer: given: square pyramid with base sides 9 cm, triangular face with sides length 11 cm)
10. Invite another volunteer to say what we are required to find. (Answer: The surface area of the square pyramid.)
11. Invite a volunteer to say what measurement is missing from the given information. (Answer: perpendicular height of triangular faces / slant height of pyramid)
12. Invite a pupil to say what method can be used to find the perpendicular height. (Answer: Pythagoras' Theorem)
13. Ask pupils to work with seatmates to find the perpendicular height of the triangular faces and hence find the surface area of the square pyramid.
14. Invite a volunteer to come to the board to show their solution. The rest of the class should check their solution and correct any mistakes.

    Solution:

    a. ii. Given: square pyramid with base sides $b = 9$ cm, triangular face with sides of length $= 11$ cm)

    Find the perpendicular height l of the triangular faces.

    From the diagram $l = OE$

    .

    $OC^2 = OE^2 + EC^2$ Pythagoras' Theorem

    $11^2 = OE^2 + 4.5^2$ $EC = \frac{9}{2} = 4.5$ cm

    $OE^2 = 11^2 - 4.5^2 = 100.75$

$$OE = \sqrt{100.75} = 10.037$$
$$OE = 10 \text{ cm} = l$$

The height of the triangle OBC is 10 cm to the nearest cm

total surface area = area of square base + area of triangular faces

area of square base $= b^2$

$= 9^2 = 81$ cm^2

area of triangular faces $= 4\left(\frac{1}{2}bl\right) = 2bl$

$= 2 \times 9 \times 10 = 180$ cm^2

total surface area $= 81 + 180$

$= 261$ cm^2

The total surface area of the pyramid is 261 cm^2.

Practice *(15 minutes)*

1. Write the questions b. and c. on the board.
2. Ask pupils to work independently to answer questions b. and c.
3. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.

Solutions:

b. i.. Given: square pyramid of side length = 16 m and slant height = 20 m

total surface area = area of square base + area of triangular faces

area of square base $= b^2$

$= 16^2 = 256$ m^2

area of triangular faces $= 4\left(\frac{1}{2}bl\right) = 2bl$

$= 2 \times 16 \times 20 = 640$ m^2

total surface area $= 256 + 640$

$= 896$ m^2

The total surface area of the pyramid is 896 m^2.

ii. Given: square pyramid of side length = 22 mm and slant height = 27 mm

.

$OC^2 = OE^2 + EC^2$ Pythagoras' Theorem

$27^2 = OE^2 + 11^2$ $EC = \frac{22}{2} = 11$ mm

$OE^2 = 27^2 - 11^2 = 608$

$OE = \sqrt{608} = 24.658$

$OE = 25$ mm

The height of the triangle OBC is 25 mm.

total surface area = area of square base + area of triangular faces

area of square base $= b^2$

$= 22^2 = 484$ mm^2

area of triangular faces $= 4\left(\frac{1}{2}bl\right) = 2bl$

$= 2 \times 22 \times 25 = 1{,}100$ mm^2

total surface area $= 484 + 1100$

$= 1{,}584$ mm^2

The surface area of the pyramid is 1,584 mm^2.

c. Given: surface area of square pyramid = 105 mm^2 and base area = 25 mm^2

$$\begin{aligned}
\text{total surface area} &= \text{area of square base} + \text{area of triangular faces}\\
105 &= 25 + \text{area of triangular faces}\\
\text{area of triangular faces} &= 105 - 25 = 80 \text{ mm}^2\\
\text{But,}\quad \text{area of triangular faces} &= 4\left(\tfrac{1}{2}bl\right) = 2bl\\
80 &= 2bl \qquad\qquad (1)\\
\text{Find}\quad \text{area of square base} &= b^2\\
25 &= b^2\\
b &= \sqrt{25} = 5 \text{ mm}\\
\text{From (1)}\quad l &= \frac{80}{2b} = \frac{80}{2\times 5}\\
l &= 8 \text{ mm}
\end{aligned}$$

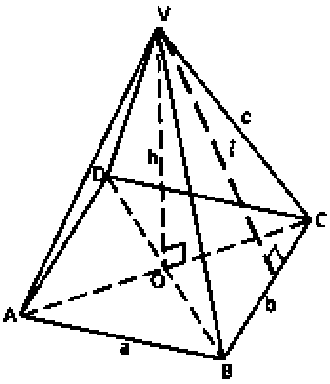
The height of the triangular faces = 8 mm.

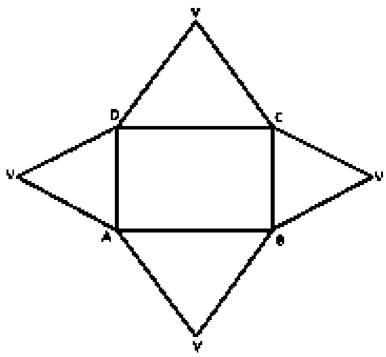
Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM3-L041 in the Pupil Handbook.

[DIAGRAMS FOR TEACHING AND LEARNING]

Draw on the board before the lesson.

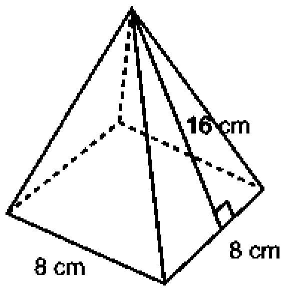




[QUESTIONS]

a. Find the surface area of the square pyramids below.

i.



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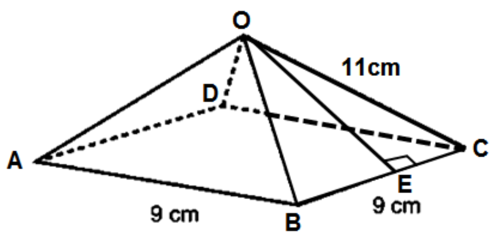
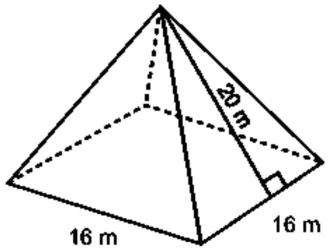


b. Find the surface area of the square pyramids below.

i.



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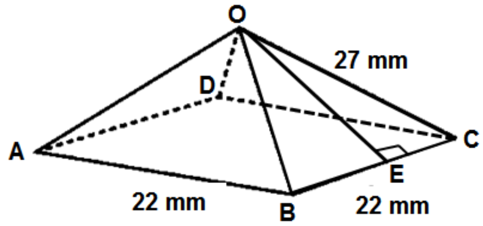


c. The surface area of a square pyramid is 105 mm^2. If the base area is 25 mm^2, find the height of the triangular faces.

Lesson Title: Volume of a rectangular pyramid	Theme: Mensuration	
Lesson Number: M3-L042	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to calculate the volume of a rectangular pyramid using the appropriate formula.	**Preparation** Write questions a. i. and a. ii. on the board.	

Opening *(4 minutes)*

1. Ask pupils to find the base area of a rectangular pyramid with a length of 5 cm and width of 7 cm.
2. Invite a volunteer to say the formula they used to calculate the surface area. (Answer: base area $= l \times w;$ length $\times$ width)
3. Invite another volunteer to give the answer. (Answer: base area $= 5 \times 7 = 35$ cm^2)
4. Tell pupils that after today's lesson, they will be able to calculate the volume of a rectangular using the appropriate formula.

Teaching and Learning *(20 minutes)*

1. Refer to the rectangular pyramid in question a. i. on the board.
2. Explain: The formula for the volume of a rectangular pyramid is given by:

$$V = \frac{1}{3} \times \text{area of base} \times \text{height}$$

$$= \frac{1}{3} lwh$$

where l is the length, w the width and h the height of the rectangular pyramid

3. Follow the standard procedure to solve question a. i.
   Solution:
   a. i. **Step 1.** Assess and extract the given information from the problem.
4. Invite a volunteer to assess the problem and tell the class what information we are given. (Answer: given: rectangular pyramid, length $l = 6$ cm, width $w = 4$ cm, height $h = 5$ cm.)
5. Invite another volunteer to say what we have been asked to find. (Answer: volume of the rectangular pyramid)

Step 2. Substitute into the appropriate formula.

$$\text{volume, } V = \frac{1}{3} lwh$$

$$= \frac{1}{3} \times 6 \times 4 \times 5$$

$$V = 40 \text{ cm}^3$$

Step 3. Write the answer.

The volume of the rectangular pyramid $= 40$ cm^3.

6. Invite a volunteer to assess question a. ii. on the board and extract the given information. (Example answer: given: square pyramid with sides $l = 8$ cm and height $h = 8$ cm)
7. Highlight to pupils that the pyramid has a square base.
8. Invite a volunteer to say what we have been asked to find. (Answer: volume of the square pyramid)
9. Ask pupils what formula is needed for volume of the pyramid.
   (Answer: $V = \frac{1}{3} \times$ area of base $\times h$).
10. Ask pupils to look at what we are given and what is needed to find the volume. Is anything missing?
11. Invite a volunteer to answer. (Example answer: We are given the slant height, l of the pyramid, we need to find the (perpendicular) height, h)
12. Carefully work through the procedure below to explain how to find the height of the square pyramid.

Solution:

Given: square pyramid with sides $l = 8$ cm and slant height $h = 8$ cm

$|OC|^2 = h^2 + |VC|^2$ Find h using Pythagoras Theorem

$h^2 = |OC|^2 - |VC|^2$ [ADD THE DETAILS BELOW TO THE PYRAMID ON THE BOARD]

But, $|VC| = \frac{1}{2}|AC|$

So, $|AC|^2 = |AB|^2 + |BC|^2$

$= 8^2 + 8^2$

$= 64 + 64 = 128$

$|AC| = \sqrt{128} = 11.31$ cm

$|VC| = \frac{1}{2} \times 11.31$ cm

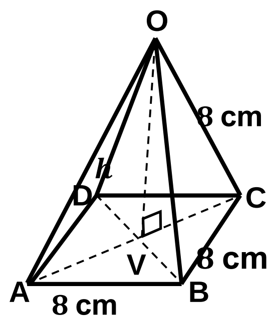
$= 5.66$ cm

$\therefore h^2 = 8^2 - 5.66^2$

$= 64 - 32.036 = 31.964$

$h = \sqrt{31.964} = 5.65$

$h = 5.65$ cm



13. Ask pupils to work with seatmates to find the volume of the square pyramid.
14. Invite a volunteer to show their working out on the board. The rest of the class should check their solution and correct any mistakes.

volume, $V = \frac{1}{3}lwh$

$= \frac{1}{3} \times 8 \times 8 \times 5.65$ substitute $l = w = 8$ cm, $h = 5.65$ cm

$V = 120.53$ cm^2

The volume of the square pyramid is 120.5 cm^3 to 1 d.p.

Practice *(15 minutes)*

1. Write questions b. and c. on the board.
2. Ask pupils to work independently to answer questions b. and c.
3. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.

Solutions:

b. Given: rectangular pyramid with 624 cm^3 volume and height 13 cm

$$\text{volume, } V = \frac{1}{3} \times \text{area of base} \times h$$
$$\text{area of base} = \frac{3V}{h} = \frac{3 \times 624}{13} = 144 \text{ cm}^2$$

The area of the base is 144 cm^2.

c. Given: square pyramid with sides $l = 9$ cm and height $h = 12$ cm

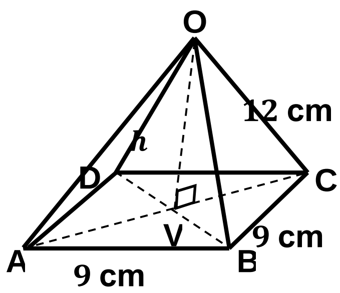
i. Find h using Pythagoras' Theorem [ADD THE DETAILS BELOW TO THE PYRAMID ON THE BOARD]

$$|OC|^2 = h^2 + |VC|^2$$
$$h^2 = |OC|^2 - |VC|^2$$

But, $|VC| = \frac{1}{2}|AC|$

So, $|AC|^2 = |AB|^2 + |BC|^2 = 9^2 + 9^2 = 81 + 81 = 162$

$$|AC| = \sqrt{162} = 12.728 \text{ cm}$$
$$|VC| = \tfrac{1}{2} \times 12.728 \text{ cm} = 6.364 \text{ cm}$$
$$\therefore h^2 = 12^2 - 6.364^2 = 144 - 40.50 = 103.5$$
$$h = \sqrt{103.5} = 10.173$$
$$h = 10.2 \text{ cm}$$



The height of the pyramid is 10.2 cm to 1 d.p.

ii. $$\text{volume, } V = \frac{1}{3}lwh$$
$$= \frac{1}{3} \times 9 \times 9 \times 10.2$$ substitute $l = w = 9$ cm, $h = 10.2$ cm
$$V = 275.4 \text{ cm}^2$$

The volume of the triangular pyramid is 275.4 cm^3 to 1 d.p.

Closing *(1 minute)*

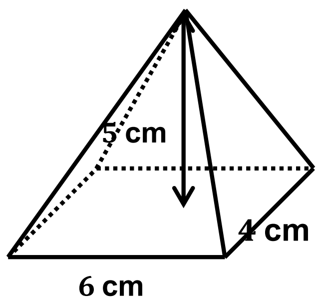
1. For homework, have pupils do the practice activity PHM3-L042 in the Pupil Handbook.

[QUESTIONS]

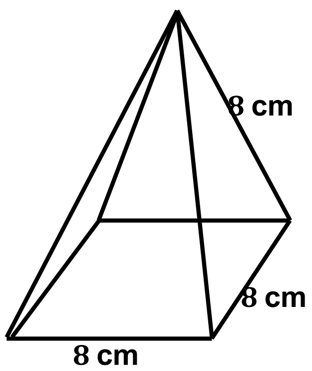
Give all answers to 1 decimal place unless otherwise stated.

a. Find the volume of the rectangular pyramids shown below:

i.

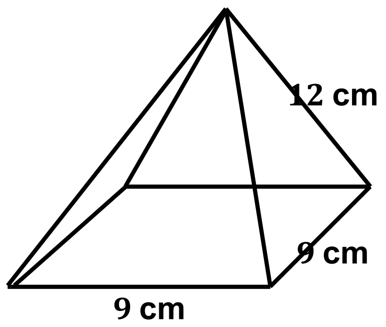


ii.



b. The volume of a rectangular pyramid is 624 cm^3. If the height of the pyramid is 13 cm, find the area of the base.

c. A pyramid with vertex *O* stands on a square base *ABCD*. Find:
   i. The height of the pyramid.
   ii. The volume of the square pyramid.



Lesson Title: Surface area of a triangular pyramid	**Theme:** Mensuration	
Lesson Number: M3-L043	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to calculate the surface area of a triangular pyramid using the appropriate formula.	**Preparation** Draw the diagrams for Teaching and Learning found at the end of this lesson plan on the board.	

Opening *(4 minutes)*

1. Ask pupils to find the area of a triangle with base 12 cm and height 15 cm.
2. Invite a volunteer to give the answer.
   (Answer: area of triangle $= \frac{1}{2}bh = \frac{1}{2} \times 12 \times 15 = 90$ cm^2)
3. Tell pupils that after today's lesson, they will be able to calculate the surface area of a triangular pyramid using the appropriate formula.

Teaching and Learning *(20 minutes)*

1. Invite a volunteer to recall what a pyramid is. (Example answer: Any solid object that has a polygon for its base, and triangular sides which meet at a point or apex.)
2. Refer to the diagrams on the board.
3. Invite a volunteer to look at the net and say what type of pyramid the diagram shows and why. (Answer: It shows a triangular pyramid, so called because the base is a triangle).
4. Explain that a triangular pyramid is also called a tetrahedron.
5. Invite a volunteer to say how many triangular faces the triangular pyramid has got. (Answer: 4 – the base of the pyramid plus one on each side of the base.)
6. Write on the board:
   surface area of triangular pyramid = area of the net
   = area of triangular base + area of triangular faces
   = area of ABC + area of VAB + area of VBC+ area of VAC
   since Δ VAB = Δ VBC = Δ VAC
   surface area of triangular pyramid = area of ABC + 3 × (area of VAB)
7. Explain:
   - For simplicity, most problems are based on an equilateral triangle as the base and either isosceles or equilateral triangles as the other faces of the pyramid.
   - If the triangular faces and the base are congruent equilateral triangles, the pyramid is called a regular tetrahedron.
   - For regular tetrahedrons, the slant height l of the pyramid is the same as the perpendicular height of the triangular faces.
   - Write on the board:
     surface area of triangular pyramid $= \frac{1}{2} \times b \times h + (3 \times \frac{1}{2} \times b \times l)$

surface area of triangular pyramid $= \frac{1}{2}bh + \frac{3}{2}bl$

$= \frac{1}{2}b(h + 3l)$

8. Write question a. i. on the board.

a. i. **Step 1.** Assess and extract the given information from the problem.

9. Invite volunteers to say what information we are given. (Answer: given: triangular pyramid with base $b = 9$ cm, base height $h = 7.8$ cm and height of the triangular faces $l = 10$ cm)

Step 2. Use the net of the triangular pyramid to calculate the surface area.

total surface area = area of triangular base + area of 3 triangular faces

$= \frac{1}{2}b(h + 3l)$ — base height $= h$, triangular faces height $= l$

Step 3. Find the surface area of the pyramid.

total surface area $= \frac{1}{2} \times 9 \times (7.8 + (3 \times 10))$ — substitute $b = 9$ cm, $h = 8$ cm, $l = 10$ cm

$= \frac{1}{2} \times 9 \times 37.8$

$= 170.1$ cm^2

Step 4. Write the final answer.

The total surface area of the triangular pyramid is 170 cm^2 to the nearest whole number.

10. Write question a. ii. on the board.
11. Invite volunteers to assess the problem and tell the class what information we are given. (Answer: given: triangular pyramid with base $b = 15$ cm, base height $h = 13$ cm and height of the triangular faces $l = 10$ cm)
12. Invite a volunteer to say what we are required to find. (Answer: The surface area of the triangular pyramid.)
13. Ask pupils to work with seatmates to find the surface area of the triangular pyramid.
14. Invite a volunteer to come to the board to show their solution. The rest of the class should check their solution and correct any mistakes.

Solution:

a. ii. Given: triangular pyramid with base $b = 15$ cm, base height $h = 13$ cm and height of the triangular faces $l = 10$ cm)

total surface area = area of triangular base + area of 3 triangular faces

$= \frac{1}{2}b(h + 3l)$

$= \frac{1}{2} \times 15 \times (13 + (3 \times 10))$

$= \frac{1}{2} \times 9 \times 38$

total surface area $= 322$ cm^2

The total surface area of the pyramid is 322 cm^2.

Practice *(15 minutes)*

1. Write questions b., c., and d. on the board.

2. Ask pupils to work independently to answer questions b., c. and d.
3. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.

Solutions:

b. Given: triangular pyramid with base $b = 10$ m, base height $h = 8.7$ m and height of the triangular faces $l = 14$ m

total surface area = area of triangular base + area of 3 triangular faces

$= \frac{1}{2}b(h + 3l)$

$= \frac{1}{2} \times 10 \times (8.7 + (3 \times 14))$

$= \frac{1}{2} \times 10 \times 50.7$

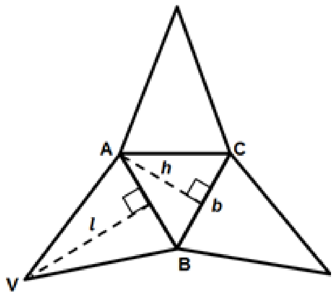
total surface area $= 253.5$ m^2

The total surface area of the pyramid is 322 m^2.

Given: triangular pyramid with equilateral base of length $b = 8$ cm, other faces isosceles triangles of height $l = 6$ cm

total surface area = area of triangular base + area of 3 triangular faces

$= \frac{1}{2}b(h + 3l)$



Find h $\quad |AC|^2 = h^2 + \left|\frac{1}{2}BC\right|^2$

$8^2 = h^2 + 4^2$ $\quad |AC| = |BC| = 8$ cm

$h^2 = 8^2 - 4^2$

$= 64 - 16 = 48$

$h = \sqrt{48} = 6.93$ cm

total surface area $= \frac{1}{2} \times 8 \times (6.93 + (3 \times 6))$

$= \frac{1}{2} \times 8 \times 24.93$

$= 99.72$ cm^2

The total surface area of the pyramid is 100 cm^2 to the nearest whole number.

d. Given: regular tetrahedron of total surface area $= 390$ cm^2, height $h = 13$ cm

total surface area = area of triangular base + area of 3 triangular faces

$= \frac{1}{2}b(h + 3l)$

$= \frac{1}{2}b(h + 3h)$ $\quad$ all 4 triangles in a tetrahedron are congruent and have the same perpendicular height h

$= \frac{1}{2}b \times 4h$

$390 = 2bh$

$b = \frac{390}{2h} = \frac{390}{2 \times 13} = 15$ cm

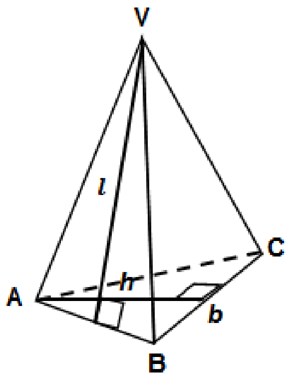
The length of the sides of the regular tetrahedron is 15 cm.

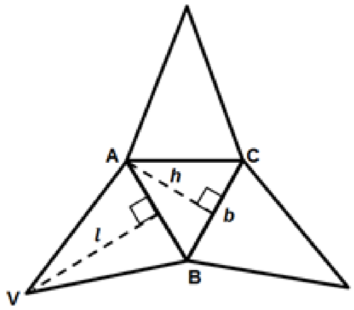
Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM3-L043 in the Pupil Handbook.

[DIAGRAMS FOR TEACHING AND LEARNING]

Draw on the board before the lesson.

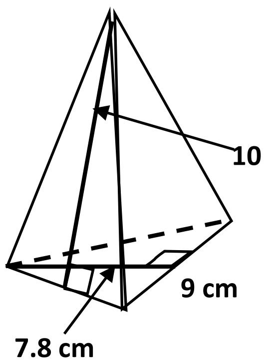




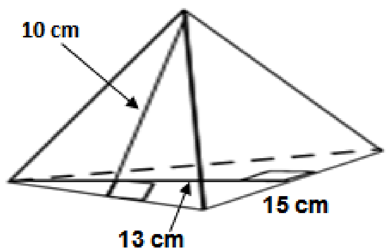
[QUESTIONS]

a. Find the total surface area of the triangular pyramids below.
   Give answers to the nearest whole number.

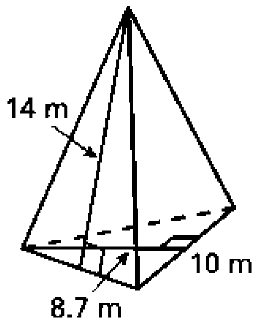
   i.



   ii.



b. Find the total surface area of the given triangular pyramid.



c. The base of a pyramid is an equilateral triangle with sides 8 cm long. Its other faces are isosceles triangles with perpendicular height of 6 cm. Find the total surface area of the pyramid.

d. A regular tetrahedron has a total surface area of 390 cm^2. If the perpendicular height of the base is 13 cm, find the length of its sides.

Lesson Title: Volume of a triangular pyramid	**Theme:** Mensuration	
Lesson Number: M3-L044	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to calculate the volume of a triangular pyramid using the appropriate formula.	**Preparation** Write questions a. i. and a. ii. on the board.	

Opening *(4 minutes)*

1. Ask pupils to find the base area of a triangular pyramid with a base length of 10 cm and perpendicular height of 7 cm.
2. Invite a volunteer to say the formula they used to calculate the base area. (Answer: base area $= \frac{1}{2}bh$)
3. Invite another volunteer to give the answer. (Answer: base area $= \frac{1}{2} \times 10 \times 7 = 35$ cm^2)
4. Tell pupils that after today's lesson, they will be able to calculate the volume of a triangular using the appropriate formula.

Teaching and Learning *(20 minutes)*

1. Refer to the triangular pyramid in question a. i. on the board.
2. Explain: The formula for the volume of a triangular pyramid is given by:

$$\begin{aligned} V &= \frac{1}{3} \times \text{area of base } \times \text{height} \qquad (1) \\ &= \frac{1}{3} \times \frac{1}{2} bh \times H \\ &= \frac{1}{6} bhH \end{aligned}$$

where b is the side length of the base, h the base height and H the height of the triangular pyramid [NOTE: It is advisable to use (1) to find the volume of the pyramid]

3. Follow the standard procedure to solve question a. i.
   Solution:
   a. i. **Step 1.** Assess and extract the given information from the problem.
4. Invite a volunteer to assess the problem and tell the class what information we are given. (Answer: given: triangular pyramid, base length $b = 12$ cm, base height $h = 10$ cm, height of pyramid $H = 14$ cm.)
5. Invite another volunteer to say what we have been asked to find. (Answer: volume of the triangular pyramid)

Step 2. Substitute into the appropriate formula.

$$\begin{aligned} \text{volume, } V &= \frac{1}{3} \times \text{area of base} \times \text{height} \\ \text{area of base} &= \frac{1}{2}bh \\ &= \frac{1}{2} \times 12 \times 10 \\ \text{area of base} &= 60 \text{ cm}^2 \\ \text{volume of pyramid} &= \frac{1}{3} \times 60 \times 14 \end{aligned}$$

$= \quad 280$ cm^3

Step 3. Write the answer.

The volume of the triangular pyramid $= 280$ cm^3.

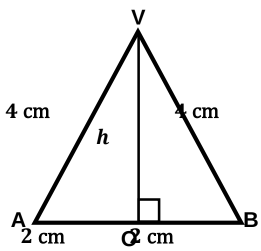
6. Invite a volunteer to assess question a. ii. on the board and extract the given information. (Example answer: given: equilateral triangular pyramid with sides $b = 4$ cm, height of pyramid $H = 6$ cm)
7. Invite a volunteer to say what we have been asked to find. (Answer: volume of the triangular pyramid)
8. Ask pupils what formula is needed for volume of the pyramid. (Answer: $V = \frac{1}{3} \times$ area of base $\times H$).
9. Ask pupils to look at what we are given and what is needed to find the volume. Is anything missing?
10. Invite a volunteer to answer. (Example answer: We are given the side lengths of the equilateral base of the pyramid. We need to find its perpendicular height, h.
11. Carefully work through the procedure below to explain how to find the height of the base.

Solution:

a. ii. Given: triangular pyramid with sides $b = 4$ cm and height $H = 6$ cm

To find h consider the base triangle VAB shown below

$$
\begin{aligned}
|VB|^2 &= h^2 + |OB|^2 \quad \text{Find } h \text{ using Pythagoras' Theorem} \\
h^2 &= |VB|^2 - |OB|^2 \\
&= 4^2 - 2^2 \\
&= 16 - 4 \\
&= 12 \\
h &= \sqrt{12} \\
h &= 3.464 \text{ cm}
\end{aligned}
$$



12. Ask pupils to work with seatmates to find the volume of the triangular pyramid.
13. Invite a volunteer to show their working out on the board. The rest of the class should check their solution and correct any mistakes.

$$
\begin{aligned}
\text{volume, } V &= \frac{1}{3} \times \text{area of base} \times H \\
\text{area of base} &= \frac{1}{2}bh \\
&= \frac{1}{2} \times 4 \times 3.464 \\
&= 6.928 \text{ cm}^2 \\
V &= \frac{1}{3} \times 6.928 \times 6 \\
V &= 13.856 \text{ cm}^3
\end{aligned}
$$

The volume of the triangular pyramid is 13.9 cm^3 to 1 d.p.

14. Write question b. on the board.
15. Ask pupils to continue to work with seatmates to find the area of the base of the triangular pyramid.
16. Invite a volunteer to show their working out on the board. The rest of the class should check their solution and correct any mistakes.

b. Given: triangular pyramid with volume of 948 cm^3 and height 11 cm

$$\text{volume, } V = \frac{1}{3} \times \text{area of base} \times H$$
$$\text{area of base} = \frac{3V}{H}$$
$$= \frac{3 \times 948}{11}$$
$$= 258.545 \text{ cm}^2$$

The area of the base is 258.5 cm^2 to 1 d.p.

Practice *(15 minutes)*

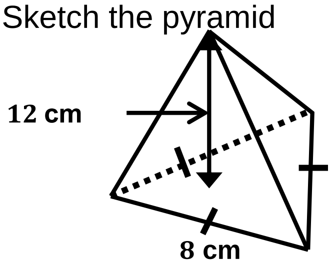
1. Write questions c. and d. on the board.
2. Ask pupils to work independently to answer questions c. and d.
3. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.

Solutions:

c. Given: triangular pyramid with sides $b = 8$ cm and height $H = 12$ cm
Draw a sketch of the pyramid.
To find h consider the triangle VAB of the base

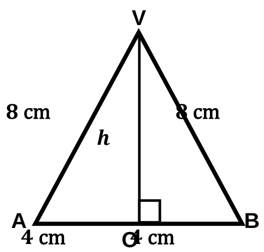
$$|VB|^2 = h^2 + |OB|^2 \quad \text{Find } h \text{ using Pythagoras Theorem}$$

Sketch the pyramid



$$h^2 = |VB|^2 - |OB|^2$$
$$= 8^2 - 4^2$$
$$= 64 - 16$$
$$= 48$$
$$h = \sqrt{48}$$
$$h = 6.928 \text{ cm}$$

The height of the pyramid is 6.9 cm to 1 d.p.



$$\text{volume, } V = \tfrac{1}{3} \times \text{area of base} \times H$$
$$\text{area of base} = \tfrac{1}{2} \times bh$$
$$= \tfrac{1}{2} \times 8 \times 6.928$$
$$\text{area of base} = 27.712 \text{ cm}^2$$
$$V = \tfrac{1}{3} \times 27.712 \times 12$$
$$= 110.848$$
$$V = 110.8 \text{ cm}^3$$

The volume of the triangular pyramid is 110.8 cm^3 to 1 d.p.

d. Given: total surface area of regular tetrahedron $= 600$ cm^2,
height of pyramid $h = 16$ cm

i. $$\text{total surface area} = \text{area of triangular base} + \text{area of 3 triangular faces}$$
$$= \tfrac{1}{2}b(h + 3l)$$

$$= \tfrac{1}{2}b(h + 3h) \quad \text{4 congruent triangles with the same perpendicular height } h$$

$$= \quad \frac{1}{2}b \times 4h = 2bh$$

$600 \quad = \quad 2 \times b \times 16$ substitute surface area $= 600$, $h = 16$

$b \quad = \quad \frac{600}{2\times16} = 18.75$ cm

The length of the sides of the regular tetrahedron b is 18.8 cm to 1 d.p.

ii. volume, $V \quad = \quad \frac{1}{3} \times$ area of base $\times H$

area of base $= \frac{1}{2} \times b \times h$

area of base $= \frac{1}{2} \times 18.8 \times 16 = 150.4$ cm^2

$V = \frac{1}{3} \times 150.4 \times 15$ substitute $H = 15$ cm

$= 752$

$V = 752$ cm^3

The volume of the regular tetrahedron is 752 cm^3.

Closing *(1 minute)*

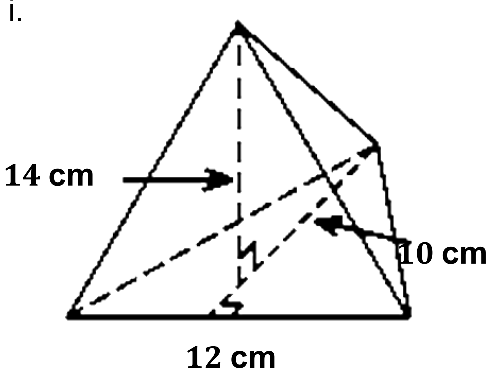
1. For homework, have pupils do the practice activity PHM3-L044 in the Pupil Handbook.

[QUESTIONS]

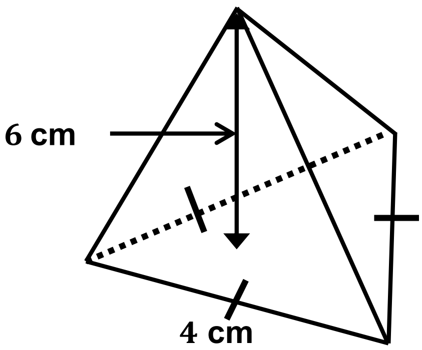
Give all answers to 1 decimal place unless otherwise stated.

a. Find the total surface area of the triangular pyramids below.
   Give answers to the nearest whole number.

i.



ii.



b. The volume of a triangular pyramid is 948 cm^3. If the height of the pyramid is 11 cm, find the area of the base.

c. The base of a pyramid is an equilateral triangle with sides 8 cm long. If the height of the pyramid is 12 cm, find the volume of the pyramid.

d. A regular tetrahedron has a total surface area of 600 cm^2. If the perpendicular height of the base is 16 cm, find i. the length of its sides; ii. the volume of the pyramid if the height of the pyramid is 15 cm.

Lesson Title: Surface area of a sphere	**Theme:** Mensuration	
Lesson Number: M3-L045	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to calculate the surface area of a sphere using the appropriate formula.	**Preparation** 1. Bring an orange, a ball or any other spherical object to class. 2. Draw the diagrams for Teaching and Learning found at the end of this lesson plan on the board. 3. Write questions a. i. and a. ii.	

Opening *(4 minutes)*

1. Show the spherical object to pupils.
2. Invite a volunteer to say what the shape is. (Example answers: the name of the object, ball, sphere)
3. Ask pupils to write down at least 2 objects they can think of that has the same shape
4. Invite a volunteer to give some examples. (Example answers: Various, but may include globe, calabash; any round fruit or vegetable)
5. Tell pupils that after today's lesson, they will be able to calculate the surface area of a sphere using the appropriate formula.

Teaching and Learning *(20 minutes)*

1. Refer to the diagrams on the board.
2. Explain:
   - The formula for the surface area of a sphere is given by:
     $$A = 4\pi r^2$$
     where r is the radius of the sphere.
   - Many problems require us to calculate the surface area of the hemisphere which is half of a sphere.
   - The formula for the surface area of a hemisphere is
     $$A = 2\pi r^2$$
   - When the hemisphere has a base, its surface area is given by:
     $$A = 2\pi r^2 + \pi r^2 = 3\pi r^2$$
3. Invite a volunteer to read question a. i. on the board.
4. Invite another volunteer to say what information we are given. (Answer: given sphere of radius = 7.5 cm)

Solution:

a. i. **Step 1.** Assess and extract the given information from the problem.

given sphere of radius = 7.5 cm

Step 2. Substitute into the appropriate formula.

surface area = $4\pi r^2$

$$= \quad 4 \times 3.142 \times 7.5^2$$
$$= \quad 706.95$$

Step 3. Write the answer.

The surface area of the sphere is 707 cm^2 to 3 s.f.

5. Ask pupils to work with seatmates to answer question a. ii.
6. Invite a volunteer to come to the board to show their solution. The rest of the class should check their solution and correct any mistakes.

Solution:

a. ii. Given sphere of diameter = 18.4 cm

$$\text{surface area} = 4\pi r^2$$
$$= 4 \times 3.142 \times 9.2^2 \qquad \text{since } r = \frac{d}{2} = \frac{18.4}{2} = 9.2$$
$$= 1{,}063.755$$
$$\text{surface area} = 1{,}060 \text{ cm}^2 \text{ to 3 s.f.}$$

7. Invite a volunteer to read question b. on the board.
8. Invite another volunteer to assess the problem and tell the class what information we are given. (Answer: given: hemisphere with surface area = 872 cm^2)
9. Invite another volunteer to say what we are required to find. (Answer: The radius of the hemisphere.)
10. Invite a volunteer to say what formula we will use to find the radius of the hemisphere with what we are given. (Answer: surface area = $2\pi r^2$)
11. Ask pupils to work with seatmates to find the radius of the hemisphere.
12. Invite a volunteer to come to the board to show their solution. The rest of the class should check their solution and correct any mistakes.

Solution:

b. Given: hemisphere with surface area = 872 cm^2

$$\text{surface area} = 2\pi r^2$$
$$872 = 2\pi r^2$$
$$r^2 = \frac{872}{2 \times 3.142}$$
$$r = \sqrt{\frac{872}{2 \times 3.142}}$$
$$r = 11.77986 \text{ cm}$$

The radius of the hemisphere is 12 cm to the nearest cm.

Practice *(15 minutes)*

1. Write the questions c., d., and e. on the board.
2. Ask pupils to work independently to answer questions c., d. and e.
3. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.

Solutions:

c. Given: surface area of sphere = surface area of cylinder, cylinder height $h = 28$ cm

surface area of sphere = surface area of cylinder

$$4\pi r^2 = 2\pi rh$$
$$2r = h$$
$$r = \frac{h}{2}$$
$$= \frac{28}{2} \quad \text{substitute } h = 28 \text{ cm}$$
$$r = 14 \text{ cm}$$

The common radius is 14 cm.

d. i. Given: diameter of hemispherical bowl $= 15$ cm

surface area of bowl + surface area of lid $= 2\pi r^2 + \pi r^2$

$= 3\pi r^2$

$= 3 \times 3.142 \times 7.5^2$ since $r = \frac{d}{2} = \frac{15}{2} = 7.5$ cm

$= 530.2125$

$= 530$ cm

The amount of material used to make the bowl and lid $= 530$ cm^2.

e. Given: 5 spherical balls radius $= 11$ cm each

surface area of one ball $= 4\pi r^2$

surface area of 5 balls $= 5 \times 4\pi r^2 = 20\pi r^2$

$= 20 \times 3.142 \times 11^2$

surface area of 5 balls $= 7{,}603.64$

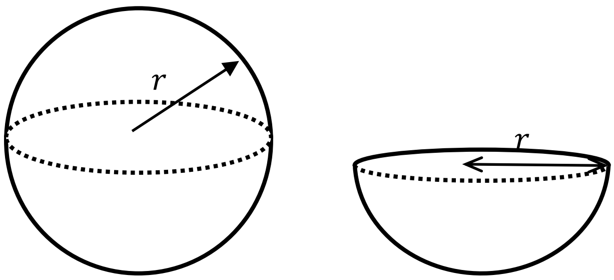
The surface area of the cube is $7{,}604$ cm^2 to the nearest cm^2.

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM3-L045 in the Pupil Handbook.

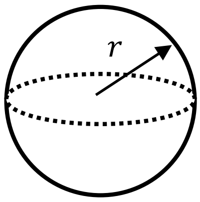
[DIAGRAM FOR TEACHING AND LEARNING]

Draw on the board before the lesson.



[QUESTIONS]

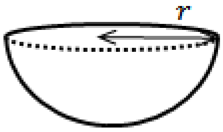
a. Find the surface area of the given sphere with:



i. $r = 7.5$ cm ii. $d = 18.4$ cm

Give your answers to 3 significant figures

b. A hemisphere has surface area of 872 cm^2. What is the radius of the hemisphere? Give your answer to the nearest cm.



c. A sphere and a cylinder both have the same diameter and surface area. If the cylinder has a height of 28 cm, find their common radius.

d. A bowl in the shape of a hemisphere together with its lid are both made from a type of plastic material. If the diameter of the bowl is 15 cm, what is the amount of material used to make the bowl and lid to the nearest cm^2?

e. A sports team wants to make 5 special balls for their anniversary game. If each ball is to be a sphere of radius 11 cm, how much material do they need to make the balls? Give your answer to the nearest cm^2.

Lesson Title: Volume of a sphere	**Theme:** Mensuration	
Lesson Number: M3-L046	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to calculate the volume of a sphere using the appropriate formula.	**Preparation** Write questions a. i. and a. ii. on the board.	

Opening *(4 minutes)*

1. Ask pupils to find the surface area of a sphere with a radius of 7 cm. Take $\pi = \frac{22}{7}$.
2. Invite a volunteer to say the formula they used to calculate the surface area. (Answer: surface area $= 4\pi r^2$)
3. Invite another volunteer to give the answer. (Answer: surface area $= 4 \times \frac{22}{7} \times 7^2 = 616$ cm^2)
4. Tell pupils that after today's lesson, they will be able to calculate the volume of a sphere using the appropriate formula.

Teaching and Learning *(15 minutes)*

1. Explain:
   - The formula for the volume of a sphere is given by:
     $$V = \frac{4}{3}\pi r^3$$
     where r is the radius of the sphere.
   - The volume of a hemisphere is half that of the sphere
   - The formula for the surface area of a hemisphere is:
     $$V = \frac{1}{2} \times \frac{4}{3}\pi r^3 = \frac{2}{3}\pi r^3$$
2. Invite a volunteer to read question a. i. on the board.
3. Explain the solution to find the volume of a sphere step by step.

   Solution:

   a. i. **Step 1.** Assess and extract the given information from the problem.

   given: sphere with radius $r = 3.5$ cm

   Step 2. Substitute into the appropriate formula.

   $$\begin{aligned} \text{volume, } V &= \frac{4}{3}\pi r^3 \\ &= \frac{4}{3} \times 3.142 \times 3.5^3 \\ V &= 179.59 \text{ cm}^3 \end{aligned}$$

   Step 3. Write the answer.

   The volume of the sphere $= 179.6$ cm^3 to 1 d.p.
4. Ask pupils to work with seatmates to answer question a. ii.

5. Invite a volunteer to come to the board to show their solution. The rest of the class should check their solution and correct any mistakes.
   Solution:
   a. ii. Given sphere of diameter $= 8.4$ cm

$$\text{volume, } V = \frac{4}{3}\pi r^3$$
$$= \frac{4}{3} \times 3.142 \times 4.2^3 \qquad \text{since } r = \frac{d}{2} = \frac{8.4}{2} = 4.2$$
$$= 310.379$$
$$\text{surface area} = 310.4 \text{ cm}^2 \text{ to 1 d.p.}$$

6. Invite a volunteer to read question b. on the board.
7. Invite another volunteer to assess the problem and extract the given information. (Example answer: given: hemisphere with 957 cm^3 volume)
8. Invite another volunteer to say what we have been asked to find. (Answer: radius of the hemisphere)
9. Ask pupils to write down the formula needed to find the radius when volume is known.
10. Invite a volunteer to say the formula. (Answer: $V = \frac{2}{3}\pi r^3$)
11. Ask pupils to work with seatmates to find the radius of the hemisphere.
12. Invite a volunteer to come to the board to show their solution. The rest of the class should check their solution and correct any mistakes.
    Solution:
    b. Given: hemisphere with 957 cm^3 volume

$$\text{volume, } V = \frac{2}{3}\pi r^3$$
$$\frac{3V}{2\pi} = r^3$$
$$r^3 = \frac{3 \times 957}{2 \times 3.142}$$
$$= 456.874$$
$$r = \sqrt[3]{456.874}$$
$$r = 7.702\text{cm}$$

The radius of the cone is 7.7 cm to 1 d.p.

Practice *(20 minutes)*

1. Write the questions c., d., and e. on the board.
2. Ask pupils to work independently to answer questions c., d. and e.
3. Ask volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.
   Solutions:
   c. Given: volume of sphere $=$ volume of cone,
   cone height $h = 36$ cm
   volume of sphere $=$ surface area of cylinder

$$\frac{4}{3}\pi r^3 = \frac{1}{3}\pi r^2 h$$
$$4r = h$$
$$r = \frac{h}{4}$$
$$= \frac{36}{4} \quad \text{substitute } h = 36 \text{ cm}$$
$$r = 9 \text{ cm}$$

The common radius is 9 cm.

d. Given: hemisphere with diameter $d = 15$ cm

volume, $V = \frac{2}{3}\pi r^3$

$\frac{2}{3} \times 3.142 \times 7.5^3$ since $r = \frac{15}{2} = 7.5$ cm

volume, $V = 883.6875$

The volume, V of the hemisphere is 883.7 cm^3.

e. Given: sphere with radius $r = 4$ cm

volume, $V = \frac{4}{3}\pi r^3$

$\frac{4}{3} \times 3.142 \times 4^3$

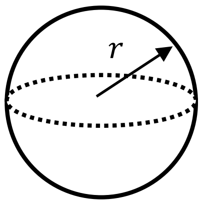
volume, $V = 268.117$

The volume, V of the hemisphere is 268.1 cm^3.

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM3-L046 in the Pupil Handbook.

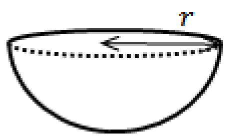
[QUESTIONS]



Give all answers to 1 decimal place unless otherwise stated.
Take $\pi = 3.142$ unless otherwise stated.

a. Find the volume of the given sphere with:
   i. $r = 3.5$ cm ii. $d = 8.4$ cm
   Give your answers to 3 significant figures.

b. A hemisphere has a volume of 957 cm^3. What is the radius of the hemisphere? Give your answer to the nearest cm.



c. A sphere and a cone both have the same diameter and volume. If the cone has a height of 36 cm, find their common radius.

d. A bowl is in the shape of a hemisphere. If the diameter of the bowl is 15 cm, what is the volume of the bowl?

e. Air is pumped into a spherical ball. If the radius of the ball is 4 cm, how much air is pumped into the ball?

Lesson Title: Surface area of composite solids	**Theme:** Mensuration	
Lesson Number: M3-L047	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to calculate the surface area of composite solids using the appropriate formulae.	**Preparation** Draw the diagrams and write the information found at the end of this lesson plan under QUESTIONS on the board.	

Opening *(1 minute)*

1. Tell pupils that after today's lesson, they will be able to calculate the surface area of composite solids using the appropriate formulae.

Teaching and Learning *(20 minutes)*

1. Refer to the solid in question a.
2. Invite a volunteer to tell the class the different solids that make up the one on the board. (Example answer: 2 different sized cuboids).
3. Ask a volunteer to tell the class how they think they can find the surface area of the composite solid. Example answers:
   - To find the surface area of a composite solid, first identify the solids it is made up of.
   - Find the individual surface area using the appropriate formula.
   - Finally, add the surface areas together.
4. Find the surface area of the composite solid on the board. Round answers to the nearest cm^2.

   Solution:

   a. **Step 1.** Assess and extract the given information from the problem.
   Given: composite solid made from 2 cuboids with given lengths, widths and heights

   Step 2. Identify and divide the shape into its individual parts
   Given: composite solid made up of a cylinder and a cone
   height of cone $h = 9$ cm; height of cylinder $H = 8$ cm; $r = 3$ cm

   Step 3. Find the individual surface area for each solid.

$$\begin{aligned} \text{surface area of solid} &= \text{surface area of cone} + \text{surface area of cylinder} \\ \text{surface area of cone} &= \pi r^2 + \pi r l \\ &= \pi(r^2 + rl) \\ \text{Find } l \qquad l^2 &= h^2 + r^2 \\ &= 9^2 + 3^2 \\ &= 81 + 9 \\ &= 90 \\ &= \sqrt{90} \\ l &= 9.487 \text{ cm} \end{aligned}$$

surface area of cone $= 3.142(3^2 + (3 \times 9.487))$
$= 117.702$ cm²

surface area of cylinder $= 2\pi r(r + h)$
$= 2 \times 3.142 \times 3 \times (3 + 8)$
$= 207.372$ cm²

Step 4. Find the sum of the surface areas of the cone and cylinder

surface area of solid $= 117.702 + 207.372$
$= 325.074$ cm²

Step 5. Write the final answer.

The surface area of the solid is 325 cm² to the nearest cm².

5. Ask pupils to work with seatmates to find the surface area of the solid in question b.
6. Invite a volunteer to come to the board to show their answer. The rest of the class should check their solution and correct any mistakes.

Solution:

b. Given: composite shape made up of a cuboid and a pyramid
cuboid size: $l = 4$ m, $w = 3$ m, $h = 2.5$ m; height of pyramid $H = 3$ m

surface area of solid = surface area of cuboid + surface area of pyramid

surface area of cuboid $= 2(lh + hw + lw)$
$= 2((4 \times 2.5) + (2.5 \times 3) + (3 \times 4))$
$= 2(10 + 7.5 + 12) = 2 \times 29.5$

surface area of cuboid $= 59$ m²

surface area of pyramid = area of base polygon + area of triangular faces

area of base polygon $= 4 \times 3$ since the base is the cuboid
$= 12$ m²

area of triangular faces $= 2\left(\frac{1}{2} \times lH\right) + 2\left(\frac{1}{2} \times wH\right)$ 2 sets of equal triangular faces with bases from the cuboid
$= 2\left(\frac{1}{2} \times 4 \times 3\right) + 2\left(\frac{1}{2} \times 3 \times 3\right)$
$= 12 + 9$

area of triangular faces $= 21$ m²

surface area of pyramid $= 12 + 21$
33 m²

∴surface area of solid $= 59 + 33$

surface area of solid $= 92$ m²

The surface area of the solid is 92 m².

Practice *(15 minutes)*

1. Ask pupils to work independently to answer questions c. and d.
2. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.

Solutions:

c. Given: hollow cylinder with one closed end, and size as given

Let A be the inner cylinder with radius $r = 30$ mm $= 3$ cm, $h = 20$ cm

surface area of A $= \pi r^2 + 2\pi rh = \pi r(r + 2h)$

$= \pi \times 3 \times (3 + (2 \times 20))$

$= \pi \times 3 \times 43$

surface area of A $= 405.318$ cm^2

The surface area of the inner surface is 405 cm^2 to the nearest cm^2.

d. Given: composite solid made up of a cylinder and a hemisphere

height of cylinder $h = 40$ cm

radius of cylinder = radius of hemisphere $r = 30$ cm

surface area of solid = curved surface area of cylinder + surface area of hemisphere

curved surface area of cylinder $= 2\pi rh$

$= 2 \times 3.142 \times 30 \times 40$

$= 7{,}540.8$ cm^2

surface area of hemisphere $= 2\pi r^2$

$= 2 \times 3.142 \times 30^2$

$= 5{,}655.6$ cm^2

surface area of solid $= 7{,}540.8 + 5{,}655.6$

$= 13{,}196.4$ cm^2

The surface area of the solid is $13{,}196$ cm^2 to the nearest cm^2.

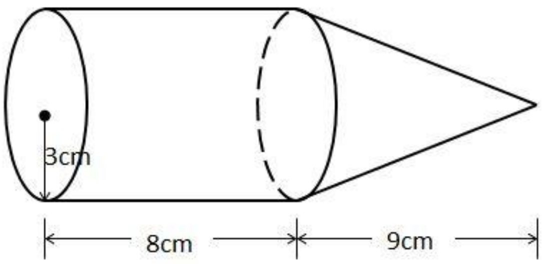
Closing *(4 minutes)*

1. Ask 3 to 5 volunteers to tell the class one new thing they learned during the lesson. (Answers: various)
2. For homework, have pupils do the practice activity PHM3-L047 in the Pupil Handbook.

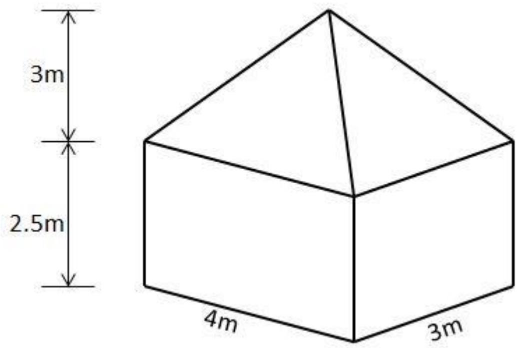
[QUESTIONS]

Find the surface area of the composite shapes shown. Assume all closed shapes except otherwise stated. Give answers to the nearest cm^2. Take $\pi = 3.142$ unless otherwise stated.

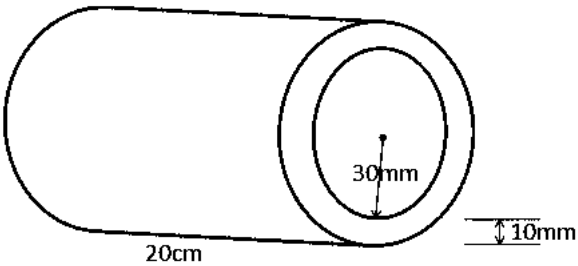
a.



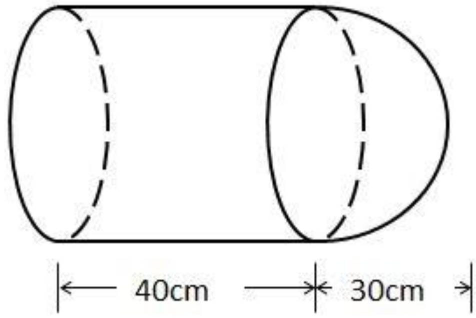
b.



c. Find the surface area of the inner surface of the hollow cylinder with one end closed shown below.



d.



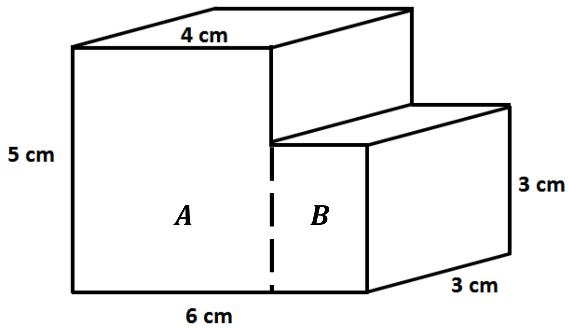
Lesson Title: Volume of composite solids	**Theme:** Mensuration	
Lesson Number: M3-L048	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to calculate the volume of composite solids using the appropriate formulae.	**Preparation** Draw the composite solids found at the end of this lesson plan on the board.	

Opening *(1 minute)*

1. Tell pupils that after today's lesson, they will be able to calculate the volume of composite solids using the appropriate formulae.

Teaching and Learning *(20 minutes)*

1. Explain to pupils that we are going to be finding the volumes of the solids we looked at during the previous lesson.
2. Ask a pupil to read question a. i. on the board.
3. Invite a volunteer to tell the class the different solids that make up the one on the board. (Example answer: 2 different sized cuboids).
4. Invite a volunteer to say how we can find the volume of the composite solid:
   - To find the volume of a composite solid, first identify the solids it is made up of.
   - Find the individual volume using the appropriate formula.
   - Finally, add the volumes together.
5. Find the volume of the composite solid in question a. on the board. Round answers to the nearest cm^3.



   Solution:

   a. **Step 1.** Assess and extract the given information from the problem.
   Given: composite solid made from 2 cuboids with given lengths, widths and heights

   Step 2. Identify and divide the shape into its individual parts
   Label the shape A and B – see diagram

   Step 3. Find the individual volume for each cuboid

$$\begin{aligned} \text{volume of solid} &= \text{volume of A} + \text{volume of B} \\ \text{volume of A} &= (l \times w \times h)_A \qquad l = 4\text{ cm}, w = 3\text{ cm}, h = 5\text{ cm} \\ &= 4 \times 3 \times 5 \\ &= 60\text{ cm}^3 \\ \text{volume of B} &= (l \times w \times h)_B \qquad l = 2\text{ cm}, w = 3\text{ cm}, h = 3\text{ cm} \\ &= 2 \times 3 \times 3 \\ &= 18\text{ cm}^3 \end{aligned}$$

   Step 4. Find the sum of the volume of the cuboids

volume of solid $= 60 + 18$
volume of solid $= 78$ cm^3

Step 5. Write the final answer.

The volume of the solid is 78 cm^3.

6. Ask pupils to work with seatmates to find the volume of the solid in question b.
7. Invite a volunteer to come to the board to show their answer. The rest of the class should check their solution and correct any mistakes.

Solution:

b. Given: hollow cylinder with dimensions as below.
Let A be the inner cylinder with radius $r = 30$ mm $= 3$ cm
B the outer cylinder with radius $R = 30 + 10$ mm $= 4$ cm, $h = 20$ cm

volume of material used = volume of B − volume of A
$= \pi R^2 h - \pi r^2 h$

volume of A $= \pi r^2 h$ $r = 3$ cm
$= 3.142 \times 3^2 \times 20$
$= 565.56$ cm^3

volume of B $= \pi R^2 h$ $R = 4$ cm
$= 3.142 \times 4^2 \times 20$
$= 1{,}005.44$ cm^3

volume of material used $= 1{,}005.44 - 565.56$
$= 439.88$

volume of material used $= 439.9$ cm^3

The volume of the material used is 440 cm^3 to the nearest cm^3.

8. Ask pupils to continue to work with seatmates to find the volume of the solid in question c.
9. Invite a volunteer to come to the board to show their answer. The rest of the class should check their solution and correct any mistakes.

Solution:

c. Given: composite solid made up of a cylinder and a cone
height of cylinder $h = 8$ cm; height of cone $H = 9$ cm; $r = 3$ cm
volume of solid = volume of cylinder + volume of cone

volume of cylinder $= \pi r^2 h$
$= 3.142 \times 3^2 \times 8$
$= 226.224 \text{ cm3}$

volume of cone $= \frac{1}{3}\pi r^2 H$
$= \frac{1}{3} \times 3.142 \times 3^2 \times 9$
$= 84.834 \text{ cm3}$

volume of solid $= 226.224 + 84.834$
volume of solid $= 311.058$ cm^3

The volume of the solid is 311 cm^3 to the nearest cm^3.

Practice *(15 minutes)*

1. Ask pupils to work independently to answer the questions.
2. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.

   Solutions:

   d. Given: composite shape made up of a cuboid and a pyramid
   cuboid size: $l = 4$ m, $w = 3$ m, $h = 2.5$ m; height of pyramid $H = 3$ m

volume of solid	=	volume of cuboid + volume of pyramid	
volume of cuboid	=	$l \times w \times h$	substitute:
	=	$4 \times 3 \times 2.5$	$l = 4$ m, $w = 3$ m, $h = 2.5$ m
	=	30 m^3	
volume of pyramid	=	$\frac{1}{3} \times$ base area $\times H$	
	=	$\frac{1}{3} \times (4 \times 3) \times 3$	since the base is the cuboid
	=	12 m^3	
volume of solid	=	$30 + 12$	
volume of solid	=	42 m^3	

   The volume of the solid is 42 m^3.

   e. Given: composite solid made up of a cylinder and a hemisphere
   height of cylinder $h = 40$ cm;
   radius of cylinder = radius of hemisphere $r = 30$ cm

volume of solid	=	volume of cylinder + volume of hemisphere
volume of cylinder	=	$\pi r^2 h$
	=	$3.142 \times 30^2 \times 40$
	=	$113{,}112$ cm^3
volume of hemisphere	=	$\frac{2}{3}\pi r^3$
	=	$\frac{2}{3} \times 3.142 \times 30^3$
volume of hemisphere	=	$56{,}556$ cm
volume of solid	=	$56{,}556 + 113{,}112$
volume of solid	=	$169{,}668$ cm^3

   The volume of the solid is $169{,}668$ cm^3.

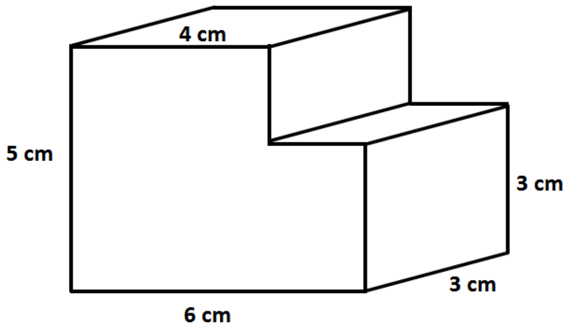
Closing *(4 minutes)*

1. Invite 3 to 5 volunteers to tell the class one new thing they learned during the lesson. (Answers: various)
2. For homework, have pupils do the practice activity PHM3-L048 in the Pupil Handbook.

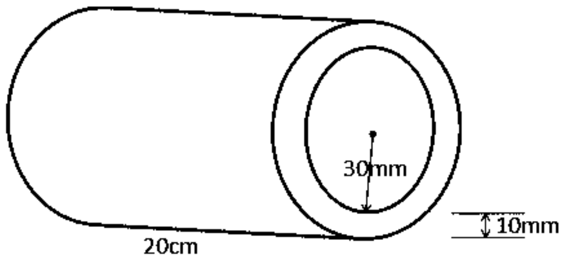
[QUESTIONS]

Find the volume of the composite shapes shown. Assume all closed shapes except otherwise stated. Give answers to the nearest cm^3. Take $\pi = 3.142$ unless otherwise stated.

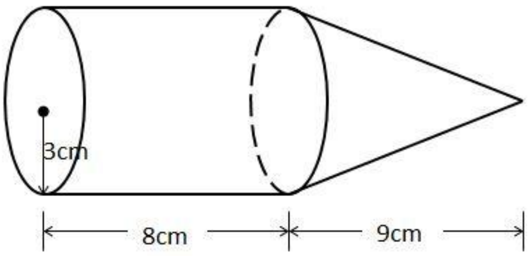
a.



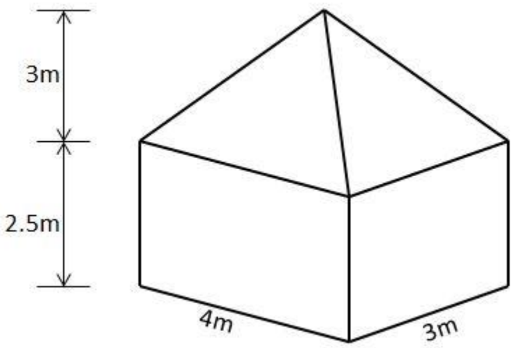
b. Hollow cylinder with one end closed.



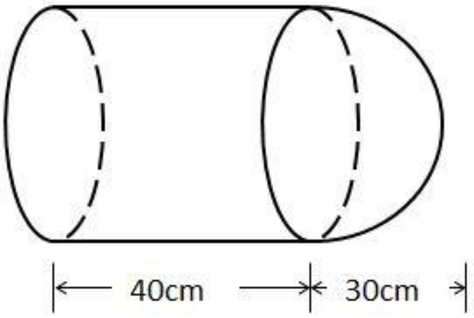
c.



d.



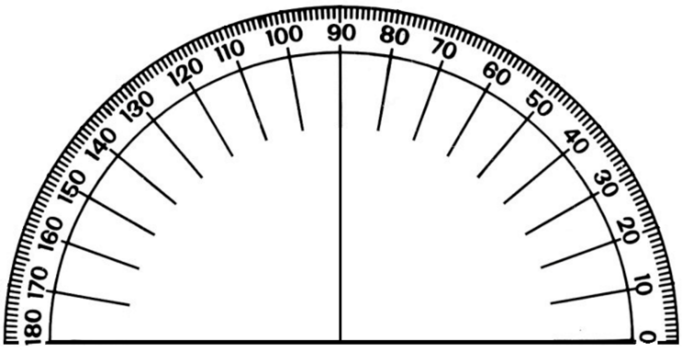
e.



f.

Appendix I: Protractor

You can use a protractor to measure angles. If you do not have a protractor, you can make one with paper. Trace this protractor with a pen onto another piece of paper. Then, cut out the semi-circle using scissors.



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