Free Quality School Education

Ministry of Basic and Senior Secondary Education

Lesson Plans for

Senior Secondary *Mathematics*

SSS III

Term II

STRICTLY NOT FOR SALE

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Please see final page for further information.

Foreword

These Lesson Plans and the accompanying Pupils' Handbooks are essential educational resources for the promotion of quality education in senior secondary schools in Sierra Leone. As Minister of Basic and Senior Secondary Education, I am pleased with the professional competencies demonstrated by the writers of these educational materials in English Language and Mathematics.

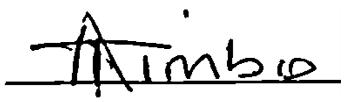
The Lesson Plans give teachers the support they need to cover each element of the national curriculum, as well as prepare pupils for the West African Examinations Council's (WAEC) examinations. The practice activities in the Pupils' Handbooks are designed to support self-study by pupils, and to give them additional opportunities to learn independently. In total, we have produced 516 lesson plans and 516 practice activities – one for each lesson, in each term, in each year, for each class. The production of these materials in a matter of months is a remarkable achievement.

These plans have been written by experienced Sierra Leoneans together with international educators. They have been reviewed by officials of my Ministry to ensure that they meet the specific needs of the Sierra Leonean population. They provide step-by-step guidance for each learning outcome, using a range of recognized techniques to deliver the best teaching.

I call on all teachers and heads of schools across the country to make the best use of these materials. We are supporting our teachers through a detailed training programme designed specifically for these new lesson plans. It is really important that the Lesson Plans and Pupils' Handbooks are used, together with any other materials they may have.

This is just the start of educational transformation in Sierra Leone as pronounced by His Excellency, the President of the Republic of Sierra Leone, Brigadier Rtd Julius Maada Bio. I am committed to continue to strive for the changes that will make our country stronger and better.

I do thank our partners for their continued support. Finally, I also thank the teachers of our country for their hard work in securing our future.



Mr. Alpha Osman Timbo

Minister of Basic and Senior Secondary Education

The policy of the Ministry of Basic and Senior Secondary Education, Sierra Leone, on textbooks stipulates that every printed book should have a lifespan of three years.

To achieve thus, DO NOT WRITE IN THE BOOKS.

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Introduction

to the Lesson Plans

These lesson plans are based on the National Curriculum and the West Africa Examination Council syllabus guidelines, and meet the requirements established by the Ministry of Basic and Senior Secondary Education.

1. The lesson plans will not take the whole term, so use spare time to review material or prepare for examinations.
2. Teachers can use other textbooks alongside or instead of these lesson plans.
3. Read the lesson plan before you start the lesson. Look ahead to the next lesson, and see if you need to tell pupils to bring materials for next time.
4. Make sure you understand the learning outcomes, and have teaching aids and other preparation ready – each lesson plan shows these using the symbols on the right.
5. If there is time, quickly review what you taught last time before starting each lesson.
6. Follow the suggested time allocations for each part of the lesson. If time permits, extend practice with additional work.
7. Lesson plans have a mix of activities for the whole class and for individuals or in pairs.
8. Use the board and other visual aids as you teach.
9. Interact with all pupils in the class – including the quiet ones.
10. Congratulate pupils when they get questions right! Offer solutions when they don't, and thank them for trying.

Learning outcomes

Preparation

KEY TAKEAWAYS FROM SIERRA LEONE'S PERFORMANCE IN WEST AFRICAN SENIOR SCHOOL CERTIFICATE EXAMINATION – GENERAL MATHEMATICS[1]

This section, seeks to outline key takeaways from assessing Sierra Leonean pupils' responses on the West African Senior School Certificate Examination. The common errors pupils make are highlighted below with the intention of giving teachers an insight into areas to focus on, to improve pupil performance on the examination. Suggestions are provided for addressing these issues.

Common errors

1. Errors in applying principles of BODMAS
2. Mistakes in simplifying fractions
3. Errors in application of Maths learned in class to real-life situations, and vis-a-versa.
4. Errors in solving geometric constructions.
5. Mistakes in solving problems on circle theorems.
6. Proofs are often left out from solutions, derivations are often missing from quadratic equations.

Suggested solutions

1. Practice answering questions to the detail requested
2. Practice re-reading questions to make sure all the components are answered.
3. If possible, procure as many geometry sets to practice geometry construction.
4. Check that depth and level of the lesson taught is appropriate for the grade level.

This information is derived from an evaluation of WAEC Examiners' Reports, as well as input from their examiners and Sierra Leonean teachers.

FACILITATION STRATEGIES

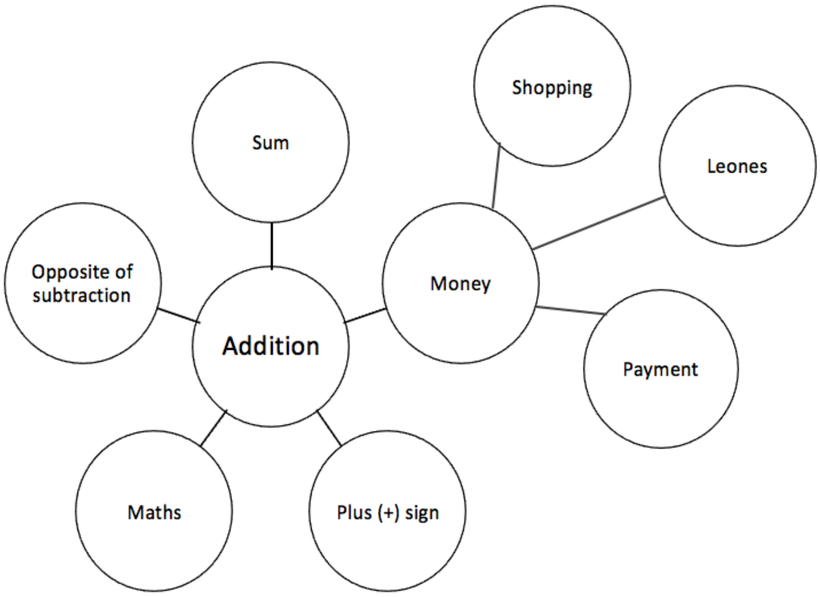
This section includes a list of suggested strategies for facilitating specific classroom and evaluation activities. These strategies were developed with input from national experts and international consultants during the materials development process for the Lesson Plans and Pupils' Handbooks for Senior Secondary Schools in Sierra Leone.

Strategies for introducing a new concept

- **Unpack prior knowledge:** Find out what pupils know about the topic before introducing new concepts, through questions and discussion. This will activate the relevant information in pupils' minds and give the teacher a good starting point for teaching, based on pupils' knowledge of the topic.
- **Relate to real-life experiences:** Ask questions or discuss real-life situations where the topic of the lesson can be applied. This will make the lesson relevant for pupils.
- **K-W-L:** Briefly tell pupils about the topic of the lesson, and ask them to discuss 'What I know' and 'What I want to know' about the topic. At the end of the lesson have pupils share 'What I learned' about the topic. This strategy activates prior knowledge, gives the teacher a sense of what pupils already know and gets pupils to think about how the lesson is relevant to what they want to learn.
- **Use teaching aids from the environment:** Use everyday objects available in the classroom or home as examples or tools to explain a concept. Being able to relate concepts to tangible examples will aid pupils' understanding and retention.
- **Brainstorming:** Freestyle brainstorming, where the teacher writes the topic on the board and pupils call out words or phrases related that topic, can be used to activate prior knowledge and engage pupils in the content which is going to be taught in the lesson.

Strategies for reviewing a concept in 3-5 minutes

- **Mind-mapping:** Write the name of the topic on the board. Ask pupils to identify words or phrases related to the topic. Draw lines from the topic to other related words. This will create a 'mind-map', showing pupils how the topic of the lesson can be mapped out to relate to other themes. Example below:



- **Ask questions:** Ask short questions to review key concepts. Questions that ask pupils to summarise the main idea or recall what was taught is an effective way to review a concept quickly. Remember to pick volunteers from all parts of the classroom to answer the questions.
- **Brainstorming:** Freestyle brainstorming, where the teacher writes the topic on the board and pupils call out words or phrases related that topic, is an effective way to review concepts as a whole group.
- **Matching:** Write the main concepts in one column and a word or a phrase related to each concept in the second column, in a jumbled order. Ask pupils to match the concept in the first column with the words or phrases that relate to in the second column.

Strategies for assessing learning without writing

- **Raise your hand:** Ask a question with multiple-choice answers. Give pupils time to think about the answer and then go through the multiple-choice options one by one, asking pupils to raise their hand if they agree with the option being presented. Then give the correct answer and explain why the other answers are incorrect.
- **Ask questions:** Ask short questions about the core concepts. Questions which require pupils to recall concepts and key information from the lesson are an effective way to assess understanding. Remember to pick volunteers from all parts of the classroom to answer the questions.
- **Think-pair-share:** Give pupils a question or topic and ask them to turn to seatmates to discuss it. Then, have pupils volunteer to share their ideas with the rest of the class.
- **Oral evaluation:** Invite volunteers to share their answers with the class to assess their work.

Strategies for assessing learning with writing

- **Exit ticket:** At the end of the lesson, assign a short 2-3 minute task to assess how much pupils have understood from the lesson. Pupils must hand in their answers on a sheet of paper before the end of the lesson.
- **Answer on the board:** Ask pupils to volunteer to come up to the board and answer a question. In order to keep all pupils engaged, the rest of the class can also answer the question in their exercise books. Check the answers together. If needed, correct the answer on the board and ask pupils to correct their own work.
- **Continuous assessment of written work:** Collect a set number of exercise books per day/per week to review pupils' written work in order to get a sense of their level of understanding. This is a useful way to review all the exercise books in a class which may have a large number of pupils.
- **Write and share:** Have pupils answer a question in their exercise books and then invite volunteers to read their answers aloud. Answer the question on the board at the end for the benefit of all pupils.
- **Paired check:** After pupils have completed a given activity, ask them to exchange their exercise books with someone sitting near them. Provide the answers, and ask pupils to check their partner's work.
- **Move around:** If there is enough space, move around the classroom and check pupils' work as they are working on a given task or after they have completed a given task and are working on a different activity.

Strategies for engaging different kinds of learners

- For pupils who progress faster than others:
  - Plan extension activities in the lesson.
  - Plan a small writing project which they can work on independently.
  - Plan more challenging tasks than the ones assigned to the rest of the class.
  - Pair them with pupils who need more support.

- For pupils who need more time or support:
  - Pair them with pupils who are progressing faster, and have the latter support the former.
  - Set aside time to revise previously taught concepts while other pupils are working independently.
  - Organise extra lessons or private meetings to learn more about their progress and provide support.
  - Plan revision activities to be completed in the class or for homework.
  - Pay special attention to them in class, to observe their participation and engagement.

Lesson Title: Expression of Ratios	**Theme:** Numbers and Numeration	
Lesson Number: M3-L049	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcomes By the end of the lesson, pupils will be able to: 1. Express ratios in their simplest terms. 2. Increase and decrease quantities in a given ratio.	**Preparation** Write the questions found at the end of this lesson plan on the board.	

Opening *(2 minutes)*

1. Ask pupils to discuss with seatmates everything they know about ratios.
2. After 1 minute, invite volunteers to tell the class what they know. (Example answers: A ratio is used to compare 2 or more quantities. The quantities must be measured in the same unit. We can write a ratio using a colon, **:**, or as a fraction).
3. Tell pupils that after today's lesson, they will be able to express ratios in their simplest terms. They will also increase and decrease quantities in a given ratio.

Teaching and Learning *(20 minutes)*

1. Explain:
   - We use ratios to compare quantities of the same type, e.g. length, weight, people, money and much more.
   - We describe ratios in 2 different ways.
   - An example will help in explaining the 2 ways:
     - Suppose in an SSS 3 class there are 24 girls and 36 boys.
2. Write on the board: 24 girls and 36 boys.
3. Invite volunteers to say how to write this as a ratio. (Answer: 24 : 36)
4. Explain: We read this ratio as "24 to 36".
5. Ask pupils to write the second way this ratio can be written.
6. Invite a volunteer to give their answer. (Answer: as a fraction, $\frac{24}{36}$)
7. Explain:
   - It does not matter which of the 2 ways we write a ratio, we should always simplify it to its lowest terms.
   - This is done by dividing by common factors.
8. Ask pupils to simplify the ratio on the board to its lowest terms.
9. Invite a volunteer to give the answer and explain what it means. (Answer: 2 : 3, $\frac{2}{3}$. This means that for every 2 girls there are 3 boys.)
10. Explain:
    - The order in which ratios are written is very important.
    - We must maintain the order as given in the problem
    - A ratio written as 2 : 3 means $\frac{2}{3}$, while a ratio written as 3 : 2 means $\frac{3}{2}$.

- The fractions are different and give different answers.

11. Ask pupils to work with seatmates to simplify the following ratios on the board to their lowest terms.
    a. 48 : 12   b. 9 : 27   c. 35 : 49
12. Invite volunteers to give their answers. (Answers: a. 4 : 1; b. 1 : 3; c. 5 : 7)
13. Explain:
    - We can only simplify ratios when the quantities are in the same units.
    - If the quantities are not in the same unit, we must convert one to the other before we simplify.
14. Write on the board: Simplify 5 cm to 5 m
15. Invite volunteers to tell the class what to do to solve the problem. (Example answer: convert the m to cm)
16. Ask pupils to work with seatmates to answer the question.
17. Invite a volunteer to give the answer. (Answer: 1 : 100, the simplified form of 5 : 500.)
18. Write on the board: Simplify to the lowest terms.
    d. 900 g to 4.5 kg   e. 9 litres to 1800 ml   f. 1 hour to 60 s
19. Ask pupils to continue to work with seatmates to simplify the ratios.
20. Invite volunteers to give their answers. (Answer : d. 1 : 5; e. 5 : 1; f. 60 : 1)
21. Explain: We may sometimes be asked to both simplify and also to solve for missing parts of equivalent ratios.
22. Write on the board; Express $24 : 14$ as a fraction in its lowest terms. If your result is equal to $x : 35$, find x.

    Solution:

    g. **Step 1.** Assess and extract the given information from the problem.
    given: $24 : 14; x : 35$

    Step 2. Change the ratio to its fraction form.
    $$\frac{24}{14} = \frac{12}{7}$$ Original ratio in lowest terms

    Step 3. Make the ratios equal
    $$\frac{12}{7} = \frac{x}{35}$$

    Step 4. Make x the subject of the equality:
    $$\frac{35 \times 12}{7} = x$$

    Step 5. Solve for x.
    $$x = 60$$
23. Ask pupils to work with seatmates to answer the following: h. Solve for x in the following ratio: 132 : x = 20 : 5
24. Ask volunteers to come to the board to show their solutions.
    The rest of the class should check their solutions and correct any mistakes.

    Solution:
    $$\frac{132}{x} = \frac{20}{5}$$
    $$\frac{132 \times 5}{20} = x$$

$x = 33$

25. Explain:
    - Ratio problems have to be interpreted in different ways depending on what we are required to find.
    - For example, we are sometimes required to increase or decrease quantities by a given ratio to find the new amounts.
    - The calculation to increase or decrease a quantity Q by a ratio $m : n$ is given by: $\frac{m}{n} \times Q$
26. Write on the board: i. Increase Le 60,000.00 in the ratio 8 : 5.
27. Explain:
    - This means that every Le 5.00 is increased to Le8.00.
    - We know it is an increase because the first part of the ratio is larger than the second part of the ratio.
28. Show on the board the calculation to do the increase.

    Solution:

    i. Increase Le 60,000.00 in the ratio 8 : 5

New amount	$= \frac{8}{5} \times 60{,}000$	Change the ratios to their fraction forms.
	$= 96000$	This is a reasonable result as we know the amount increased from before.

    The new amount is Le 96,000.00.
29. Ask pupils to work with seatmates to answer the following: j. Decrease 350 g in the ratio 2 : 7.
30. Ask volunteers to come to the board to show their solutions.
    The rest of the class should check their solutions and correct any mistakes.

    Solutions:

    j. Decrease 350 g in the ratio 2 : 7.

New amount	$= \frac{2}{7} \times 350$	Every 7 g is reduced to 2 g
The new amount is 100 g		This is reasonable because it is less than the original amount of 350 g

Practice *(17 minutes)*

1. Ask pupils to work independently to answer the questions from the QUESTIONS section on the board.
2. Walk around, if possible, to check the answers and clear any misconceptions.
3. Invite volunteers to come to the board to show their solutions.
   The rest of the class should check their solutions and correct any mistakes.

   Solutions:

   a. i. given: $12 : 36$

   $\frac{12}{36} = \frac{1}{3}$ divide by 12

   The simplified ratio is $1 : 3$

   ii. given: $52 : 13$

   $\frac{52}{13} = \frac{4}{1}$ divide by 13

   The simplified ratio is $4 : 1$

   b. i. given: 40 kg : 500 g

   40 kg $=$ $40{,}000$ g

   ii. given: 100 cm : 3 m

   3 m $=$ 300 cm

$\frac{40,000}{500} = \frac{80}{1}$ divide by 500

The simplified ratio is $80 : 1$

$\frac{100}{300} = \frac{1}{3}$ divide by 100

The simplified ratio is $1 : 3$

iii. given: 30 seconds : 5 hours

5 hours = 18,000 seconds

$\frac{30}{18,000} = \frac{1}{600}$ divide by 30

The simplified ratio is $1 : 600$

c. i. given: $x : 3 = 25 : 15$

$\frac{x}{3.} = \frac{25}{15}$

$x = \frac{25\times3}{15} = 5$

The value of $x = 5$

ii. given: $3 : 7 = 9 : x$

$\frac{3}{7} = \frac{9}{x}$

$x = \frac{9\times7}{3} = 21$

The value of $x = 21$

d. i. given: increase 150 g in the ratio $7 : 5$

$\frac{7}{5}\times 150 = 210$ g

The new amount is 210 g

ii. given: increase 132 cm in the ratio $9 : 4$

$\frac{9}{4}\times 132 = 297$ cm

The new amount is 297 cm

e. i. given: decrease 6 weeks in the ratio $2 : 3$

$\frac{2}{3}\times 6 = 4$ weeks

The new time is 4 weeks

ii. given: decrease 154 km in the ratio $3 : 7$

$\frac{3}{7}\times 154 = 66$ km

The new distance is 66 km

f. given: 28 stalls of vegetable and 7 stalls of fish

$\frac{28}{7} = \frac{4}{1}$

The ratio is $4 : 1$

g. given: increase Le 35,000.00 in the ratio $9 : 7$

$\frac{9}{7}\times 35000 =$ Le 45,000.00

The new price is Le 45,000.00

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM3-L049 in the Pupil Handbook.

[QUESTIONS]

a. Express the following ratios in their simplest terms:
   i. $12 : 36$ ii. $52 : 13$
b. Express the following ratios in their simplest terms:
   i. 40 kg to 500 g ii. 100 cm to 3 m iii. 30 seconds to 5 hours
c. Solve for x in the following ratios
   i. $x : 3 = 25 : 15$ ii. $3 : 7 = 9 : x$
d. Increase the following quantities in the ratios given:
   i. 150 g in the ratio $7 : 5$ ii. 132 cm in the ratio $9 : 4$
e. Decrease the following quantities in the ratios given:
   i. 6 weeks in the ratio $2 : 3$ ii. 154 km in the ratio $3 : 7$
f. A market contains 28 stalls selling vegetables and 7 stalls selling fish. Express the ratio of vegetable stalls to fish stalls in the lowest terms.
g. A fish monger increases the price of her fish in the ratio 9 : 7. What is the new price of fish she used to sell at Le 35,000.00?

Lesson Title: Comparison of Ratios	**Theme:** Numbers and Numeration	
Lesson Number: M3-L050	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to compare and simplify ratios.	**Preparation** 1. Write on the board: Solve for x in the ratio $2 : 3 = x : 12$ 2. Write the questions found at the end of this lesson plan on the board.	

Opening *(2 minutes)*

1. Ask pupils to solve for x in the ratio on the board.
2. Invite a volunteer to show their solution on the board.
3. The rest of the class should check their solutions and correct any mistakes. (Answer: $\frac{2}{3} = \frac{x}{12}; 2 \times 12 = 3x; 3x = 24; x = 8$)
4. Tell pupils that after today's lesson, they will be able to compare and simplify ratios.

Teaching and Learning *(20 minutes)*

1. Explain:
   - We are often asked to compare 2 or more ratios to find out which is biggest or smallest relative to the others.
   - One way we can compare ratios is by writing them as unit ratios.
   - If we have a ratio in the form $m : n$, we can write it either as $m : 1$ or $1 : n$.
   - To write as $m : 1$, we divide both ratios by n.
   - To write as $1 : n$, we divide both ratios by m.
2. Ask pupils to work with seatmates to write the ratio $2 : 10$ as $m : 1$ and $1 : n$.
3. Invite volunteers to give the answers. The rest should check and correct their answers. (Answer: $2 : 10$ as $m : 1$ is $\frac{1}{5} : 1$ since we divide both numerator and denominator by 10; $2 : 10$ as $1 : n$ is $1 : 5$ since we divide both numerator and denominator by 2.)
4. Explain:
   - Once we have converted the given ratios to unit fractions, we then determine which ratio is greatest or smallest in relation to the others.
   - A second way to compare ratios is to use LCM to convert each ratio into an equivalent fraction. Both fractions will then have the same denominator.
   - We then inspect the numerators and determine which ratio is greatest or smallest in relation to the others.
5. We will look at both methods using an example.
6. Write on the board: Express the ratios $3 : 8$ and $4 : 15$ in the form $m : 1$. Which ratio is greater? Use LCM to confirm your result.

   Solution:

   Step 1. Assess and extract the given information from the problem.

7. Invite a volunteer to assess the problem and tell the class what information we are given. (Answer: given: 2 ratios $3 : 8$ and $4 : 15$)
8. Invite another volunteer to say what we have been asked to find. (Answer: To find out which ratio is greater.)
9. Explain:

Step 2. Change the ratios to their fraction form.

Method 1

Simplify each ratio independently to a unit fraction.

$\frac{3}{8} = \frac{0.375}{1}$ divide the numerator and denominator by 8

$\frac{4}{15} = \frac{0.267}{1}$ divide the numerator and denominator by 15

Now, compare the 2 ratios.

$3 : 8 = 0.375 : 1$

$4 : 15 = 0.267 : 1$

$3 : 8 > 4 : 15$ since $0.375 > 0.267$

Method 2

Find the LCM of the 2 fractions.

$\frac{3}{8} = \frac{45}{120}$ since the LCM of 8 and 15 is 120

$\frac{4}{15} = \frac{32}{120}$

Now, compare the 2 ratios.

$3 : 8 = 45 : 120$

$4 : 15 = 32 : 120$

$3 : 8 > 4 : 15$ since $45 > 32$

Step 3. Write the answer.

$\therefore$ $3 : 8$ is the greater ratio

10. Ask pupils to work with seatmates to answer the next question.
11. Write on the board: Express the 2 ratios $9 : 12$ and $8 : 10$ in the form 1 : n. Which is greater?
12. Invite a volunteer to show the solution on the board.
The rest of the class should check their work and correct any mistakes.

Solution:

Given 2 ratios $9 : 12$ and $8 : 10$, find the greater ratio

$\frac{9}{12} = \frac{1}{1.333}$ divide the numerator and denominator by 9

$\frac{8}{10} = \frac{1}{1.375}$ divide the numerator and denominator by 8

Compare the 2 ratios.

$9 : 12 = 1 : 1.333$

$8 : 10 = 1 : 1.25$

$8 : 10 > 9 : 12$ since $1.333 > 1.25$, it will give a smaller result when divided into 1

$\therefore$ $8 : 10$ is the greater ratio

13. Explain:
    - We compare 2 ratios using 1 : n or m : 1 depending on the context of the question and what we are required to find.

- In other cases, it may be best to use the LCM method.

14. Ask pupils to work with seatmates to verify the results above by comparing the 2 ratios using LCM.
15. Invite a volunteer to show their solution on the board. The rest of the class should check and correct any mistakes.

Solution:

$\frac{8}{10} = \frac{48}{60}$ since the LCM of 10 and 12 is 60

$\frac{9}{12} = \frac{45}{60}$

Compare the 2 ratios.

$8:10 = 48:60$

$9:12 = 45:60$

$8:10 > 9:12$ since $48 > 45$

$\therefore$ $8:10$ is the greater ratio

Practice *(17 minutes)*

1. Ask pupils to work independently to answer the questions from the QUESTIONS section on the board.
2. Walk around, if possible, to check the answers and clear up any misconceptions.
3. Ask volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.

Solutions:

a. i. Given: $12:36$

$12:36 = \frac{12}{36}:\frac{36}{36}$

$12:36 = \frac{1}{3}:1$

$12:36$ in the form m $1 = \frac{1}{3}:1$

ii. Given: $52:13$

$52:13 = \frac{52}{13}:\frac{13}{13}$

$52:13 = \frac{4}{1}:1$

$52:13$ in the form $m:1 = 4:1$

$40:8$ in the form $m:1 = 5:1$

b. i. Given: $5:45$

$5:45 = \frac{5}{5}:\frac{45}{5}$

$5:45 = 1:\frac{9}{1}$

$5:45$ in the form $1:n = 1:9$

ii. Given: $20:35$

$20:35 = \frac{20}{20}:\frac{35}{20}$

$20:35 = 1:\frac{7}{4}$

$20:35$ in the form $1:n = 1:\frac{7}{4}$

c. i. Given: $2:5$ and $5:12$

$\frac{2}{5} = \frac{0.4}{1}$

$\frac{5}{12} = \frac{0.417}{1}$

$5:12 > 2:5$

$5:12$ is the greater ratio

ii. Given: $6:2$ and $15:10$

$\frac{6}{2} = \frac{3}{1}$

$\frac{15}{10} = \frac{1.5}{1}$

$6:2 > 15:10$

$6:2$ is the greater ratio

d. i. Given: $10:3$ and $40:8$

$\frac{10}{3} = \frac{1}{0.3}$

$\frac{40}{8} = \frac{1}{0.2}$

ii. Given: $8:32$ and $4:14$

$\frac{8}{32} = \frac{1}{4}$

$\frac{4}{14} = \frac{1}{3.5}$

$10 : 3 < 40 : 8$ $8 : 32 < 4 : 14$

$10 : 3$ is the smaller ratio $8 : 32$ is the smaller ratio

e. i. Given: $2.5 : 100$ and $6 : 150$

$$\frac{2.5}{100} = \frac{7.5}{300}$$

LCM of 100 and 150 is 300

$$\frac{6}{150} = \frac{12}{300}$$

$$12 : 300 > 7.5 : 300$$

$$\therefore 6 : 150 > 2.5 : 100$$

$6 : 150$ is the greater ratio

ii Given: $13 : 6$ and $12 : 8$

$$\frac{13}{6} = \frac{52}{24}$$

LCM of 6 and 8 is 24

$$\frac{12}{8} = \frac{36}{24}$$

$$52 : 24 > 36 : 24$$

$$\therefore 13 : 6 > 12 : 8$$

$13 : 6$ is the greater ratio

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM3-L050 in the Pupil Handbook.

[QUESTIONS]

a. Express the following ratios in the form $m : 1$.
   - iii. $12 : 36$ iv. $52 : 13$

b. Express the following ratios in the form $1 : n$.
   - iv. $5 : 45$ v. $20 : 35$

c. Use the form $m : 1$ to compare the given ratios. Which is greater?
   - i. $2 : 5$ and $5 : 12$ ii. $6 : 2$ and $15 : 10$

d. Use the form $1 : n$ to compare the given ratios. Which is smaller?
   - i. $10 : 3$ and $40 : 8$ ii. $8 : 32$ and $4 : 14$

e. Use LCM to compare the given ratios. Which is greater?
   - i. $2.5 : 100$ and $6 : 150$ ii. $13 : 6$ and $12 : 8$

Lesson Title: Rate	**Theme:** Numbers and Numeration	
Lesson Number: M3-L051	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to use rates to connect quantities of different kinds.	**Preparation** 1. Write the ratio 50 g to 1 kg in the form $1 : n$. 2. Write the questions found at the end of this lesson plan in the QUESTIONS section on the board.	

Opening *(4 minutes)*

1. Ask pupils to answer the question on the board.
2. Invite a volunteer to give their answer. (Answer: change quantities to g; 50 : 1,000; 1 : 20)
3. Tell pupils that after today's lesson, they will be able to use rates to connect quantities of different kinds.

Teaching and Learning *(20 minutes)*

1. Invite a volunteer to give the first step they did before expressing the quantities as a ratio. (Example answer: converted the quantity in kg to g; changed quantities to the same unit)
2. Explain:
   - We use ratios to compare two or more "like" quantities.
     Like quantities means they are of the same kind, e.g. height, temperature, mass, or weight as in our example.
   - The quantities must be expressed in the same unit for them to be compared. In the example, both quantities were written as grams.
   - We use rates when we want to compare quantities of different kinds e.g. how far a motorbike travels in kilometres for a particular length of time in hours or how much money someone is paid per month at their job.
3. Ask pupils to work with seatmates to decide which of the following is a ratio and which is a rate.
4. Ask them to give examples of the units of measurement of the quantities.
5. Invite a volunteer to answer after each question.
   Write each answer on the board.
   - The length of a rectangle compared to its width. (Answer: Ratio, both are measured in units of length, e.g. m, cm)
   - The number of kilometres a bike travels in one hour. (Answer: Rate (speed); units are kilometres and hour (kilometres per hour))
   - The area of a square compared to the area of a triangle. (Answer: Ratio, both are measured in units of an area, e.g. cm^2, m^2)

- The amount of money someone is paid every month. (Answer: rate called a "pay rate" or "rate of pay", units are Leones and month (Leones per month))
- The percentage of interest a lender pays in one year. (Answer: rate called "interest rate", units are percentage and year (percentage per annum))

6. Ask pupils to look at the units of measurements given in each example.
7. Invite a volunteer to say what they notice about the units. (Example answer: The quantities in the ratio are measured with one unit, the quantities in the rate are measured with 2 units.)
8. Explain:
   - When we write the ratio as a fraction, the units in the ratio cancel each other out because they are the same, e.g. $\frac{\text{area of square}}{\text{area of triangle}} = \frac{\text{cm}^2}{\text{cm}^2}$
   - The units in a rate take on the unit from the numerator and the unit from the denominator, e.g. $\frac{\text{distance}}{\text{time}} = \frac{\text{km}}{\text{hr}}$
9. Demonstrate a typical problem on rates on the board using question a.

   Solution:

   a. i. **Step 1.** Assess and extract the given information from the problem.
   Given: A car travels 240 km for every 3 hours.

   Step 2. Convert to a ratio and simplify.

240 km	**:**	3 hrs	Write as a ratio
rate	$=$	$\frac{240\text{ km}}{3\text{ hrs}}$	Write as a fraction
	$=$	$\frac{80\text{ km}}{1\text{ hr}}$	Write in the form $m : 1$ by dividing numerator and denominator by 3
	$=$	80 km/hr	Write as a rate in km/hr

   Step 3. Write the answer.
   The average speed is 80 km/hr.

   ii.

speed	$=$	80 km/hr	
	$=$	$\frac{80\text{ km}}{1\text{ hr}}$	80 km in 1 hr
Distance travelled in 5 hours	$=$	$\frac{80\text{ km}}{1\text{ hr}} \times 5$ hrs	Multiply by 5 for distance travelled in 5 hours
	$=$	400 km	The hours cancel each other

   The distance in 5 hours is 400 km.

10. Explain:
    - The above method is called the unitary method.
    - We first find the unit rate at which the car is travelling. This is the distance travelled for every 1 hour.
    - In our example, this is "80" kilometres for every "1" hour, i.e. 80 km/hr.
    - We then use the unit rate to find all other distances given the time taken.
    - A unit rate is a ratio that tells us how many units of one quantity there are for every one unit of the second quantity.
    - Rates use words and symbols such as "per" (/), "each" (ea) and "at" (@).

11. Invite a volunteer to assess question b. on the board and extract the given information. (Example answer: A water tank empties 500,000 litres of water In 2 days)
12. Invite a volunteer to say what we have been asked to find in part i. (Answer: the rate at which the tank empties)
13. Ask pupils to work with seatmates to answer question b.
14. Invite a volunteer to come to the board to show their solution. The rest of the class should check their solutions and correct any mistakes.

    Solution:

    b. i. Given: A 500,000 litre water tank empties in 2 days.

500,000 litres	:	2 days	Write as a ratio
500,000 litres	:	48 hrs.	Convert days to hours
rate	=	$\frac{500{,}000 \text{ litres}}{48 \text{ hrs}}$	Write as a fraction
	=	$\frac{10{,}416.667 \text{ litres}}{1 \text{ hr}}$	Write in the form $m:1$ by dividing numerator and denominator by 48
10,417 litres/hr	=	Write as a rate in litres/hr	

The tank empties at a rate of 10,417 litres/hr to the nearest litre.

ii. rate $= \frac{10{,}416.667 \text{ litres}}{1 \text{ hr}}$

Since both tanks empty at the same rate:

$\frac{10{,}416.667 \text{ litres}}{1 \text{ hr}}$	=	$\frac{750{,}000 \text{ litres}}{x}$	Where x is the number of hours it takes the second tank to empty
x	=	$\frac{750{,}000 \text{ litres}}{10{,}416.667 \text{ litres}} \times 1 \text{ hr}$	
	=	71.999 hrs	The litres cancel each other
	=	72 hrs or 3 days	

The 750,000 litre tank empties in 3 days.

Practice *(15 minutes)*

1. Write question c., d. and e. on the board.
2. Ask pupils to work independently to answer questions c., d. and e.
3. Walk around, if possible, to check the answers and clear up any misconceptions.
   Ask volunteers to come to the board to show their solutions.
   The rest of the class should check their solutions and correct any mistakes.

   Solutions:

   c. Given: Le720,000.00 for 3 workers

Le 720,000.00	:	3 workers
rate	=	$\frac{720{,}000}{3}$
	=	$\frac{240{,}000}{1}$

The men worked at a rate of Le 240,000.00/worker.

d. Given: 6 litres for 150 km

150 km : 6 litres

$$\text{rate} = \frac{150}{6}$$
$$= \frac{25}{1}$$

The car travels at the rate of 25 km/litre.

e. i. Given: 142 beats in 2 minutes

142 beats : 2 minutes

$$\text{rate} = \frac{142}{2}$$
$$= \frac{71}{1}$$

The man's heart beats at a rate of 71 beats/minutes.

ii. Given: 852 beats

$$\text{rate} = \frac{71}{1}$$
$$\frac{71}{1} = \frac{852}{x}$$
$$x = \frac{852}{71}$$
$$x = 12 \text{ minutes}$$

The man's heart beats 852 times in 12 minutes.

Closing *(1 minute)*

1. Invite volunteers to say one new thing they learned this lesson. (Example answer: How to calculate rates from information given.)
2. For homework, have pupils do the practice activity PHM3-L051 in the Pupil Handbook.

[QUESTIONS]

Give your answer to the nearest whole number unless otherwise stated.

a. A car travels a distance of 240 km in 3 hours.
   i. What is the average speed in kilometres per hour (km/hr.)?
   ii. How far will it travel in 5 hours?

b. A water tank contains 500,000 litres of water. It takes 2 days to empty the tank. If the tank empties at a constant rate
   i. Calculate the rate the tank empties in litres per hour.
   ii. How long will it take to empty another tank with 750,000 litres of water if it empties at the same rate?

c. A team of 3 workers charged a house owner Le 720,000.00 to paint her house. How much is it costing her per worker for the job?

d. A car uses 6 litres of fuel for a journey of 150 km. What is the average rate of fuel use in km per litre?

e. A man's heart rate was tested at 142 beats in 2 minutes.
   i. What is the rate of heart beats per minute?
   ii. How many minutes will it take his heart to beat 852 times?

<table>
<tr><td>Lesson Title: Proportional Division</td><td colspan="2">Theme: Numbers and Numeration</td></tr>
<tr><td>Lesson Number: M3-L052</td><td>Class: SSS 3</td><td>Time: 40 minutes</td></tr>
<tr><td>Learning Outcome
By the end of the lesson, pupils will be able to divide quantities into given proportions.</td><td colspan="2">Preparation
Write the questions found at the end of this lesson plan on the board.</td></tr>
</table>

Opening *(2 minutes)*

1. Ask pupils to answer question a. i. on the board.
2. Invite a volunteer to give the answer. (Answer: $\frac{750,000}{2}$ = Le $375,000.00$ each)
3. Tell pupils that after today's lesson, they will be able to divide quantities into given proportions.

Teaching and Learning *(20 minutes)*

1. Explain:
   - We are used to doing calculations where we share quantities equally as we did just now in question a. i.
   - Suppose we are asked instead to share the same amount of money according to the ages of the children as in question a. ii.
   - We perform a proportional division according to the given ratio.
2. Show the procedure for proportional division. Explain each step carefully.
   Solution:

   a. ii. **Step 1.** Assess and extract the given information from the problem.
3. Invite a volunteer to assess the problem and tell the class what information we are given. (Answer: Given: Le 750,000.00 to be shared in the ratio 8 : 7.)
4. Invite another volunteer to say what we have been asked to find. (Answer: How much each child will receive.)

Step 2. Find the total number of parts to the ratio.

Total number of parts $= 8+7 = 15$

This ratio means that for every Le 15.00 of the amount to be shared, Le 8.00 will go to Child 1 and Le 7.00 will go to Child 2.

Step 3: Find what proportion (fraction) of the total is given to each part

Child 1 receives: $\frac{8}{15}\times 750,000 = 400,000$

Child 2 receives: $\frac{7}{15}\times 750,000 = 350,000$

Step 4: Write the answer.
Child 1 receives Le 400,000.00
Child 2 receives Le 350,000.00

Step 5: Check whether the answer is reasonable.
This answer is reasonable as it adds up to Le 750,000.00.

5. Explain:

- A quantity shared equally will result in the same amount per share as in question a. i.
- A quantity shared in different proportions will result in different amounts per share as in question a. ii.
- Sharing according to a given ratio is called **proportional division.**

6. Ask pupils to work with seatmates to answer question b.
7. Invite volunteers to show their solution on the board.
   The rest of the class should check their solution and correct any mistakes.
   Solution:
   b. i. Given: Divide 500 g in the ratio 2 : 3

   $$\text{total number of parts} = 2 + 3 = 5$$

   2 parts give: $\frac{2}{5} \times 500 = 200$ g

   3 parts give: $\frac{3}{5} \times 500 = 300$ g

   This answer is reasonable as it adds up to 500 g.

   ii. Given: Divide Le 520,000.00 in the ratio 10 : 9 : 7

   $$\text{total number of parts} = 10 + 9 + 7 = 26$$

   10 parts give: $\frac{10}{26} \times 520{,}000 =$ Le $200{,}000.00$

   9 parts give: $\frac{9}{26} \times 520{,}000 =$ Le $180{,}000.00$

   7 parts give: $\frac{7}{26} \times 520{,}000 =$ Le $140{,}000.00$

   This answer is reasonable as it adds up to Le 520,000.00

Practice *(17 minutes)*

1. Write questions c., d. and e. on the board.
2. Ask pupils to work independently to answer questions c., d. and e.
3. Walk around, if possible, to check the answers and clear any misconceptions.
4. Invite volunteers to come to the board to show their solutions.
   The rest of the class should check their solutions and correct any mistakes.
   Solutions:
   c. Given: Le 900,000.00 contributed in the ratio 7:6:5

   $$\text{total number of parts} = 7 + 6 + 5 = 18$$

   Femi's share $= \frac{7}{18} \times 900{,}000$ = Le $350{,}000.00$

   Kemi's share $= \frac{6}{18} \times 900{,}000$ = Le $300{,}000.00$

   Yemi's share $= \frac{5}{18} \times 900{,}000$

$= \text{Le } 250{,}000.00$

Femi's share $= \text{Le } 350{,}000.00$

Kemi's share $= \text{Le } 300{,}000.00$

Yemi's share $= \text{Le } 250{,}000.00$

The answer is reasonable as it adds up to Le 900,000.00.

d. Given: An acid and water mixture in the ratio 1 : 5

total number of parts $= 1 + 5$

$= 6$

acid $= \frac{1}{6} \times 216$

$= 36 \text{ ml}$

water $= \frac{5}{6} \times 216$

$= 180 \text{ ml}$

amount of acid $= 36 \text{ ml}$

amount of water $= 180 \text{ ml}$

The answer is reasonable as it adds up to 216 ml.

e. Given: Le 200,000.00 shared between 3 partners for the first year, Le 500,000.00 in the second year in the same ratio.

1st year:

Ramatu $= \text{Le } 40{,}000.00$

Mohamed $= \text{Le } 120{,}000.00$

Isatu $= 200{,}000 - (40{,}000 + 120{,}000)$

$= \text{Le } 40{,}000.00$

Ramatu's share $= \frac{40{,}000}{200{,}000}$

$= \frac{1}{5}$

Mohamed's share $= \frac{120{,}000}{200{,}000}$

$= \frac{3}{5}$

Isatu's share $= \frac{40{,}000}{200{,}000}$

$= \frac{1}{5}$ same as Ramatu's share

2nd year:

Ramatu's share $= \frac{1}{5} \times 500{,}000$

$= \text{Le } 100{,}000.00$ = Isatu's share

Mohamed's share $\frac{3}{5} \times 500{,}000$

$\text{Le } 300{,}000.00$

Ramatu's share $= \text{Le } 100{,}000.00$

Isatu's share $= \text{Le } 100{,}000.00$

Mohamed's share = Le $300{,}000.00$

The answer is reasonable as it adds up to Le 500,000.00.

Closing *(1 minute)*

1. Invite volunteers to say one new thing they learned during the lesson. (Example answer: How to share a quantity in a given ratio.)
2. For homework, have pupils do the practice activity PHM3-L052 in the Pupil Handbook.

[QUESTIONS]

a. Compare the results of the 2 calculations below
   i. Share Le 750,000.00 equally between 2 children. How much will each child receive?
   ii. Share Le 750,000.00 between 2 children in the ratio 8 : 7. How much will each child receive?

b. Divide the quantities below in the given ratio.
   i. 500 g in the ratio 2 : 3
   ii. Le 520,000.00 in the ratio 10 : 9 : 7

c. To start a small business, Femi, Kemi and Yemi contributed money in the ratio 7 : 6 : 5 respectively. If the total amount contributed is Le 900,000.00, how much did each person contribute? Check that your answer is reasonable.

d. In a chemistry laboratory, acid and water were mixed together in a ratio 1:5 to give 216 ml of a mixture. How much acid and how much water was mixed together? Check that your answer is reasonable.

e. After the first year in business, Le 200,000.00 profit was shared between 3 partners. Ramatu received Le 40,000.00, Mohammed received Le 120,000.00 and Isata received the rest. After the second year they shared out a profit of Le 500,000.00 in the same ratio. How much did each person receive in the second year? Check that your answer is reasonable.

Lesson Title: Scales – Part 1	**Theme:** Number and Numeration	
Lesson Number: M3-L053	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to interpret scales used in drawing plans and maps.	**Preparation** Write the questions found at the end of this lesson plan on the board.	

Opening *(1 minute)*

1. Tell pupils that after today's lesson, they will be able to interpret scales used in drawing plans and maps.

Teaching and Learning *(20 minutes)*

1. Explain:
   - Plans and maps are diagrams of real-life objects and places.
   - Scales are used to reduce the size of the objects in the plans and maps in order to make them fit on to a piece of paper.
   - Scales allow the diagrams to be drawn in proportion to their original size. They are examples of a ratio and are usually written in the form $1 : n$.
   - For example, a scale of 1 : 50 on a plan, means that 1 cm on the plan is 50 cm in real-life.
   - On a map, scales are usually of the order 1 : 50,000. This means 1 cm on the map represents an actual distance of 50,000 cm (500 m or 0.5 km) on the ground.
   - We can read a plan or map and be able to deduce the actual sizes or distances of the objects or places they show.
2. Invite a volunteer to read question a. What are we required to do? (Answer: copy and complete table; draw accurate plan of room)

   Solution:

   a. **Step 1.** Assess and extract the given information from the problem.
      Given: table to copy and complete, sketch the plan of a bedroom

      Step 2. Complete the table following the given procedure.
      - Using the unitary method, the measurement is given by:

        $\frac{1}{50} = \frac{\text{size in plan}}{\text{actual size}}$ Take care to match the ratio of size in plan : actual size

        size in plan $= \frac{\text{actual size}}{50}$ Use the same units for both sizes

      - Invite volunteers to give the answer to each measurement.

Actual size m	cm	Size in plan cm		
3	300	$300 \div 50$	=	6
2.5	250	$250 \div 50$	=	5
3.5	350	$350 \div 50$	=	7
2.8	280	$280 \div 50$	=	5.6
0.7	70	$70 \div 50$	=	1.4

Step 3. Draw the plan.

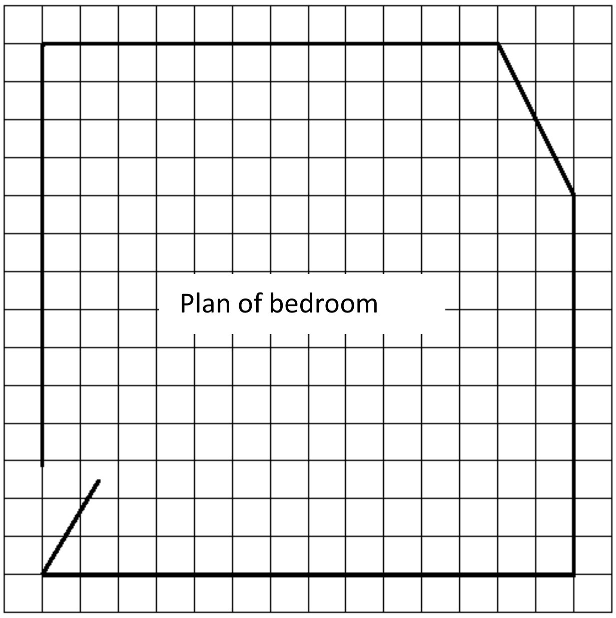
See the plan (drawn to scale).

Step 4. Measure the unmarked length and calculate its actual size.

Depending on the accuracy of the drawing the unmarked length measures between 2.2 and 2.4 cm.

$$\begin{aligned} \text{size in plan} &= 2.2 \text{ cm} && \text{for example} \\ \frac{1}{50} &= \frac{\text{size in plan}}{\text{actual size}} \\ \text{actual size} &= \text{size in plan} \times 50 \\ &= 2.2 \times 50 = 110 \text{ cm} \\ &= 1.1 \text{ m} \end{aligned}$$

The actual size of the unmarked length is 1.1 m (accept lengths between 1.1 and 1.2 m)



3. Invite a volunteer to assess question b. on the board and extract the given information. (Example answer: Given: line on the map 18 cm long, actual measurement is 90 km)
4. Invite a volunteer to say what we have been asked to find. (Answer: the scale of the map)
5. Ask pupils to work with seatmates to answer the question.
   The rest of the class should check their solution and correct any mistakes.
   Solution:

b. i. Given: line on map 18 cm long, actual distance 90 km

$$\begin{aligned} 90 \text{ km} &= 90{,}000 \text{ m} && \text{convert 90 km to cm} \\ &= 9{,}000{,}000 \text{ cm} \\ \text{scale} &= 18 : 9{,}000{,}000 && \text{length on map : actual distance} \\ &= 1 : 500{,}000 && \text{write in the form } 1 : n \end{aligned}$$

The scale of the map is $1 : 500{,}000$.

ii.

$$\begin{aligned} \text{length on map} &= 11 \text{ cm} \\ \frac{1}{500{,}000} &= \frac{\text{length on map}}{\text{actual distance}} \\ &= \frac{11}{x} && \text{where } x \text{ is the actual distance} \\ 1 \times x &= 11 \times 500{,}000 \\ x &= 5{,}500{,}000 \text{ cm} \\ &= 55{,}000 \text{ m} \\ x &= 55 \text{ km} \end{aligned}$$

The actual distance between 2 towns represented by 11 cm is 55 km.

Practice *(17 minutes)*

1. Ask pupils to work independently to answer questions c., d. and e.
2. Walk around, if possible, to check the answers and clear any misconceptions.
3. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.

Solutions:

c. Given: hall measuring 10 m wide by 15 m long

width: $10 \text{ m} = 1{,}000 \text{ cm}$

length: $15 \text{ m} = 1{,}500 \text{ cm}$

$$\text{scale} = 1 : n$$

$$\frac{1}{n} = \frac{\text{size in plan}}{\text{actual size}}$$

$$\text{size in plan} = \frac{\text{actual size}}{n}$$

i.

$$\text{scale} = 1 : 100$$

$$\text{size in plan} = \frac{\text{actual size}}{100}$$

$$\text{width: } 1{,}000 \text{ cm} = \frac{1{,}000}{100} = 10 \text{ cm}$$

$$\text{length: } 1{,}500 \text{ cm} = \frac{1{,}500}{100} = 15 \text{ cm}$$

On 1 : 100 scale:
width = 10 cm, length = 15 cm

ii.

$$\text{scale} = 1 : 200$$

$$\text{size in plan} = \frac{\text{actual size}}{200}$$

$$\text{width: } 1{,}000 \text{ cm} = \frac{1{,}000}{200} = 5 \text{ cm}$$

$$\text{length: } 1{,}500 \text{ cm} = \frac{1{,}500}{200} = 7.5 \text{ cm}$$

On 1 : 200 scale:
width = 5 cm, length = 7.5 cm

iii.

$$\text{scale} = 1 : 50$$

$$\text{size in plan} = \frac{\text{actual size}}{50}$$

$$\text{width: } 1{,}000 \text{ cm} = \frac{1{,}000}{50} = 20 \text{ cm}$$

$$\text{length: } 1{,}500 \text{ cm} = \frac{1{,}500}{50} = 30 \text{ cm}$$

On 1 : 50 scale:
width = 20 cm, length = 30 cm

iv.

$$\text{scale} = 1 : 20$$

$$\text{size in plan} = \frac{\text{actual size}}{20}$$

$$\text{width: } 1{,}000 \text{ cm} = \frac{1{,}000}{20} = 50 \text{ cm}$$

$$\text{length: } 1{,}500 \text{ cm} = \frac{1{,}500}{20} = 75 \text{ cm}$$

On 1 : 20 scale:
width = 50 cm, length = 75 cm

d. Given: 5 m is represented by 25 cm

$$5 \text{ m} = 500 \text{ cm}$$

$$25 : 500 \quad \text{Divide throughout by 25}$$

$$1 : 20$$

The scale of the plan is $1 : 20$

e. Given: Bo to Freetown is 174 km

$$174 \text{ km} = 17{,}400{,}000 \text{ cm}$$

$$\text{scale} = 1 : n$$

$$\frac{1}{n} = \frac{\text{distance on the map}}{\text{actual distance}}$$

$$\text{distance on the map} = \frac{\text{actual distance}}{n}$$

i.

$$\text{scale} = 1 : 500{,}000$$

$$\text{distance on the map} = \frac{17{,}400{,}000}{500{,}000}$$

ii.

$$\text{scale} = 1 : 1{,}000{,}000$$

$$\text{distance on the map} = \frac{17{,}400{,}000}{1{,}000{,}000}$$

$= \quad 34.8$ cm

iii. scale $= \quad 1 : 300,000$

distance on the map $= \frac{17,400,000}{300,000}$

$= \quad 58$ cm

$= \quad 17.4$ cm

iv. scale $= \quad 1 : 87,000$

distance on the map $= \frac{17,400,000}{87,000}$

$= \quad 200$ cm

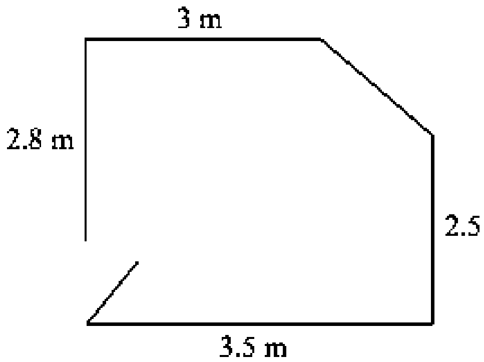
Closing *(2 minutes)*

1. Ask pupils to write down 2 jobs they think would find using scales on plans and maps useful.
2. Invite volunteers to share their views. (Example answers: architects (draw plans for building houses and other construction), builders, surveyors (measures land and represents it on a site plan), civil engineers (build roads, dams))
3. For homework, have pupils do the practice activity PHM3-L053 in the Pupil Handbook.

[QUESTIONS]

a. The diagram shows the sketch of a bedroom (not to scale).
   i. Copy the table below. Use a scale of $1 : 50$ to complete the table.
   The doorway is 0.7 m wide.

Actual size m	cm	Size in plan cm		
3	300	$300 \div 50$	=	6
2.5			=	
3.5			=	
2.8			=	
0.7			=	



   ii. Draw an accurate plan of the bedroom using the measurements from your table.
   iii. What is the actual measurement in m of the unmarked side?

b. i. A line on a map joining two towns is 18 cm long. If the towns are actually 90 km apart, what is the scale of the map?
   ii. What is the actual distance between 2 towns 11 cm apart on the map?

c. A hall measures 10 m wide by 15 m long. Give the dimensions of the hall on plans with the scales below.
   i. 1 : 100
   ii. 1 : 200
   iii. 1 : 50
   iv. 1 : 20

d. On a plan the actual distance of 5 m is represented by 25 cm. What is the scale of the plan?

e. The distance from Bo to Freetown is 174 km. What would be the distance between these 2 cities on a map with a scale of:
   i. 1 : 500,000
   ii. 1 : 1,000,000
   iii. 1 : 300,000
   iv. 1 : 87,000

Lesson Title: Scales – Part 2	**Theme:** Numbers and Numeration	
Lesson Number: M3-L054	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to use scales to calculate distance between two points.	**Preparation** Write the questions found at the end of this lesson plan on the board.	

Opening *(3 minutes)*

1. Ask pupils to answer question a. on the board.
2. After 1 minute, invite a volunteer to show the calculation on the board. (Answer: $\frac{1}{100} = \frac{\text{length on plan}}{\text{actual length}} = \frac{x}{900}; x = \frac{900\times1}{100} = 9$ cm)
3. Invite a volunteer to say what must happen if quantities are to be compared in a ratio. (Answer: The quantities must have (or be in) the same units.)
4. Tell pupils that after today's lesson, they will be able to use scales to calculate the distance between two points.

Teaching and Learning *(20 minutes)*

1. Invite a volunteer to read question b. on the board.
2. Work through the solution. Invite volunteers to give answers at every step.

   Solution:

   b. **Step 1.** Assess and extract the given information from the problem.
   Given: distance between 2 towns = 15 km, scale 1 : 300,000

   Step 2. Use the unitary method to find the missing distance

   $$\frac{1}{300{,}000} = \frac{\text{distance on map}}{\text{actual distance}}$$

   $$\text{distance on map} = \frac{\text{actual distance}}{300{,}000}$$

   $$= \frac{1{,}500{,}000}{300{,}000} = \frac{15}{3}$$

   $$= 5 \text{ cm}$$

   Step 3. Write the answer.

   The distance on the map between the 2 towns is 5 cm.
3. Invite a volunteer to assess question c. on the board and extract the given information. (Example answer: 32 cm represents 40 km, 14 cm represents unknown distance)
4. Invite a volunteer to say what we have been asked to find. (Answer: actual distance represented by 14 cm.)
5. Work through the solution. Invite volunteers to give answers at every step.

   Solution:

   c. Given: 32 cm represents 40 km, 14 cm represents unknown distance (x)

   40 km $= 40{,}000$ m   Change all measurements to cm

   $= 4{,}000{,}000$ cm

6. Explain:
   - Since we are reading the same map, the scale is the same for both distances.
   - We can therefore put the 2 ratios equal to each other.

$$\frac{32}{4{,}000{,}000} = \frac{\text{distance on map}}{\text{actual distance}} \quad \text{Ratio and scale for map}$$
$$\frac{32}{4{,}000{,}000} = \frac{14}{x} \quad \text{Make the ratios equal}$$
$$x = \frac{14 \times 4{,}000{,}000}{32}$$
$$= 1{,}750{,}000 \text{ cm}$$
$$= 17{,}500 \text{ m}$$
$$x = 17.5 \text{ km}$$

The actual distance on the map represented by 14 cm is 17.5 km.

7. Explain:
   - The solution shows we do not need to find the scale first.
   - We can make the ratios equal to each other, then solve to find the unknown quantity.
8. Invite a volunteer to assess question d. on the board and extract the given information. (Example answer: distance between 2 towns is 0.6 cm on a map with a scale of 1 : 3,000,000)
9. Invite a volunteer to say what we have been asked to find in part i. (Answer: distance between the 2 towns in km)
10. Ask pupils to work with seatmates to answer the question.
11. Invite a volunteer to answer each part of the question.
    The rest of the class should check their solution and correct any mistakes.
    Solution:
    d. i. Given: distance between 2 towns = 0.6 cm, scale 1 : 3,000,000

$$\frac{1}{3{,}000{,}000} = \frac{\text{distance on map}}{\text{actual distance}}$$
$$\text{actual distance} = \text{distance on map} \times 3{,}000{,}000$$
$$= 0.6 \times 3{,}000{,}000$$
$$= 1{,}800{,}000 \text{ cm}$$
$$= 18 \text{ km}$$

ii. given: distance between 2 towns = 18 km, scale 1 : 60,000

$$\frac{1}{60{,}000} = \frac{\text{distance on map}}{\text{actual distance}}$$
$$\text{distance on map} = \frac{\text{actual distance}}{60{,}000}$$
$$= \frac{1{,}800{,}000}{60{,}000}$$
$$= 30 \text{ cm}$$

Practice *(15 minutes)*

1. Ask pupils to work independently to answer questions e. through g.
2. Walk around, if possible, to check the answers and clear any misconceptions.

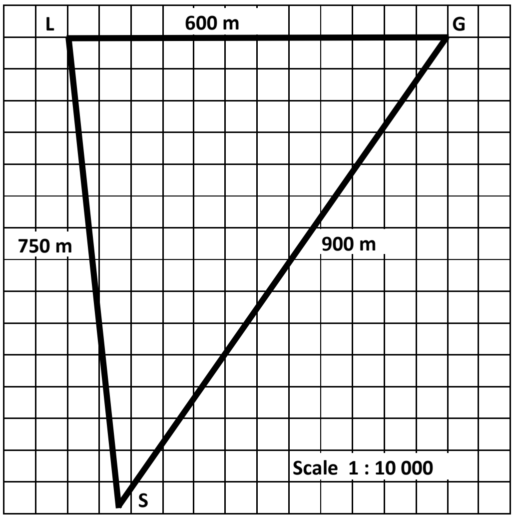
3. Ask volunteers to come to the board to show their solutions.
   The rest of the class should check their solutions and correct any mistakes.
   Solutions:

   e. Given: 3.5 cm : 875 m, 4.2 cm : x

$$875 \text{ m} = 87{,}500 \text{ cm}$$
$$\frac{3.5}{87{,}500} = \frac{4.2}{x} \qquad \text{same map means ratios are equal}$$
$$x = \frac{4.2 \times 87{,}500}{3.5}$$
$$x = 105{,}000 \text{ cm}$$
$$= 1{,}050 \text{ m}$$

   f. i. Given: library to gates $= 600$ m, gates to staffroom $= 900$ m, library to staffroom $= 750$ m , scale: 1:10,000

$$\frac{1}{10{,}000} = \frac{\text{distance on map}}{\text{actual distance}}$$
$$\text{distance on map} = \frac{\text{actual distance}}{10{,}000}$$
$$\text{library to gates} = 600 \text{ m}$$
$$= 60{,}000 \text{ cm}$$
$$= \frac{60{,}000}{10{,}000}$$
$$\text{distance on map} = 6 \text{ cm}$$
$$\text{gates to staffroom} = 900 \text{ m}$$
$$= 90{,}000 \text{ cm}$$
$$= \frac{90{,}000}{10{,}000}$$
$$\text{distance on map} = 9 \text{ cm}$$
$$\text{library to staffroom} = 750 \text{ m}$$
$$= 75{,}000 \text{ cm}$$
$$= \frac{75{,}000}{10{,}000}$$
$$\text{distance on map} = 7.5 \text{ cm}$$



   ii. measured: distance on map $= 5.1$ cm, scale: 1 : 10,000

$$\frac{1}{10{,}000} = \frac{\text{distance on map}}{\text{actual distance}}$$
$$\text{actual distance} = \text{distance on map} \times 10{,}000$$
$$= 5.1 \times 10{,}000$$
$$= 510 \text{ m}$$

   iii. The angle is measured on map at $55°$

   g. Given: 1.5 cm, 1.2 cm, scale: 1 : 25,000

$$\frac{1}{25{,}000} = \frac{\text{distance on map}}{\text{actual distance}}$$
$$\text{actual distance} = \text{distance on map} \times 25{,}000$$
$$\text{length} = 1.5 \times 25{,}000$$
$$= 37{,}500$$
$$= 375 \text{ m}$$
$$\text{width} = 1.2 \times 25{,}000$$
$$= 30{,}000$$

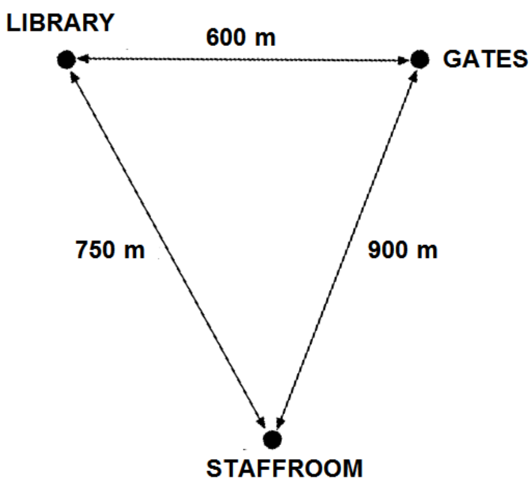
$$
\begin{aligned}
& = 300 \text{ m} \\
\text{area} & = lw \\
& = 375 \times 300 \\
& = 112{,}500 \text{ m}^2
\end{aligned}
$$

Closing *(2 minutes)*

1. Ask pupils to write down 2 new things they learned today.
2. Invite volunteers to answer. (Example answers: How to calculate distances between towns or places; how to draw a scaled map.)
3. For homework, have pupils do the practice activity PHM3-L054 in the Pupil Handbook.

[QUESTIONS]

a. A classroom wall measures 9 m. How much will it measure in a plan with a scale of 1 : 100 (cm)?
b. Two towns are 15 km apart. What would be the distance between the 2 towns on a map with a scale of 1 : 300,000?
c. On a map a distance of 40 km is represented by 32 cm. What actual distance would be represented by 14 cm on the map?
d. On a map with a scale of 1 : 3,000,000, the distance between 2 towns is 0.6 cm.
   i. Find the distance between the 2 towns in km.
   ii. How far apart will the towns be on a map with a scale of 1 : 60,000?
e. A sports field is shown on a map as a rectangle. One side of length 875 m is represented on the map by 3.5 cm. If the other side is represented by 4.2 cm, what is its actual length?
f. A pupil measures the distance between various points in her school compound. The various points are shown in the diagram which is not drawn to scale.



   i. Draw a map to show this information, using a scale of 1 : 10,000.
   ii. A pupil is exactly halfway between the gates and the staffroom. How far are they from the library?
   iii. Another pupil stands at the gates looking towards the library. They turn counter-clockwise so that they are looking at the staffroom. What angle does the pupil turn through?
g. On a map with a scale of 1 : 25,000 a plot of land is represented by a rectangle 1.5 cm by 1.2 cm. Find the area of the plot of land.

Lesson Title: Speed – Part 1	**Theme:** Numbers and Numeration	
Lesson Number: M3-L055	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to solve problems involving speed.	**Preparation** Write the questions found at the end of this lesson plan on the board.	

Opening *(4 minutes)*

1. Ask pupils to write down 2 things they know about speed.
2. Invite volunteers to give one answer each. (Example answers: Speed compares distance with time; Speed is the ratio of distance to time. Speed is measured in units of $\frac{\text{distance}}{\text{time}}$ e.g. km/hr or m/s.)
3. Tell pupils that after today's lesson, they will be able to solve problems involving speed.

Teaching and Learning *(20 minutes)*

1. Explain: This lesson reviews work done on speed from previous years.
2. Invite a volunteer to tell the class a formula connecting speed, distance and time. (Example answer: $d = st$ where d is the distance travelled, s is the speed and t is the time taken to cover the distance.)
   [NOTE: It does not matter which form of the formula is given as the other 2 will be asked for next]
3. Ask pupils to write the formulas for finding the other 2 variables in their exercise books.
4. Invite volunteers to give the formulas. (Answers: $s = \frac{d}{t}$; $t = \frac{d}{s}$)
5. Write all 3 formulas on the board.
6. Explain:
   - Use these formulas whenever a problem asks "how fast", "how far", or "how long".
   - The speed s can be defined either as a constant speed over a particular distance or the average speed for a journey.
   - If it is average speed it is given by: average speed $= \frac{\text{total distance travelled}}{\text{total time taken}}$
7. Invite a volunteer to read question a. on the board.
   Solution:
   a. **Step 1.** Assess and extract the given information from the problem.
8. Invite a volunteer to assess the problem and tell the class what information we are given. (Answer: Given: total time taken $= 3\frac{1}{2}$ hours, total distance travelled $=$ 126 km)
9. Invite another volunteer to say what we have been asked to find. (Answer: average speed)

Step 2. Substitute into the appropriate formula.

$$\text{average speed} = \frac{\text{total distance travelled}}{\text{total time taken}}$$

$$= \frac{126}{3.5} \text{ km/hr} \qquad \text{change } 3\tfrac{1}{2} \text{ to } 3.5$$

$$= 36 \text{ km/hr}$$

Step 3. Write the answer.

The average speed of the lorry is 36 km/hr.

10. Invite a volunteer to assess question b. on the board and extract the given information. (Answer: Given: average speed = 40 km/hr, total distance travelled = 300 km)
11. Invite a volunteer to say what we have been asked to find. (Answer: time taken for journey)
12. Ask pupils to work with seatmates to find the time taken for the journey.
13. Invite a volunteer to show the answer on the board. The rest of the class should check their solution and correct any mistakes.

Solution:

b. Given: average speed $= 40$ km/hr, total distance travelled $= 300$ km

$$\text{time} = \frac{\text{total distance travelled}}{\text{average speed}}$$

$$= \frac{300}{40}$$

$$\text{time} = 7.5 \text{ hrs}$$

The time taken to travel 300 km is 7.5 hrs.

14. Ask pupils to continue to work with seatmates to answer question c.
15. Invite a volunteer to show the answer on the board. The rest of the class should check their solution and correct any mistakes.

Solution:

c. i. Given: 2 part journey, 1st part: 3 hours at 48 km/hr

2nd part: 2 hours at 53 km/hr

$$\text{distance} = \text{speed} \times \text{time}$$

1st part: $\text{distance} = 48 \times 3 = 144$ km

2nd part: $\text{distance} = 53 \times 2 = 106$ km

$$\text{total distance travelled} = 144 + 106$$

$$= 250 \text{ km}$$

The total distance travelled $= 250$ km

ii. $$\text{average speed} = \frac{\text{total distance travelled}}{\text{total time taken}}$$

$$= \frac{250}{5} \text{ km/hr} \qquad \text{since total time} = 3 + 2 \text{ hrs}$$

$$= 50 \text{ km/hr}$$

The average speed for the journey is 50 km/hr.

Practice *(15 minutes)*

1. Ask pupils to work independently to answer questions d. and e.
2. Walk around, if possible, to check the answers and clear any misconceptions.

3. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.

Solutions:

d. i. Given: 2 part journey, 1st part: 2 km at speed of 6 km/hr
2nd part: 2 km at speed of 4 km/hr
total distance travelled = 4 km

$$\text{time} = \frac{\text{distance}}{\text{speed}}$$

1st part: $\text{time} = \frac{2}{6} = \frac{1}{3}$ hr

2nd part: $\text{time} = \frac{2}{4} = \frac{1}{2}$ hr

total time taken $= \frac{1}{3} + \frac{1}{2}$

$= \frac{5}{6}$ hr

The total time taken = $\frac{5}{6}$ hr or 50 minutes.

ii. average speed $= \frac{\text{total distance travelled}}{\text{total time taken}}$

$= \frac{4}{\frac{5}{6}}$ km/hr   since total distance = 4 km and total time = $\frac{5}{6}$ hr

$= 4.8$ km/hr

The average speed for the journey is 4.8 km/hr.

e. i. Given: 2 part journey: 1st part: 2 hr 30 min at speed x km/hr
2nd part: 2 hr 20 min at speed $x+2$ km/hr
total distance travelled = $2y$ km

where x is the initial speed and y is half the distance between the towns

distance = speed × time

1st part: $y = x \times 2\frac{1}{2}$ since 2 hr 30 min $= 2\frac{1}{2}$ hr

$= \frac{5}{2}x$

2nd part: $y = (x+2) \times 2\frac{1}{3}$ since 2 hr 20 min $= 2\frac{1}{3}$ hr

$= \frac{7}{3}(x+2)$

since the distances are equal:

$$\frac{5}{2}x = \frac{7}{3}(x+2)$$
$$= \frac{7}{3}x + \frac{14}{3}$$
$$\left(\frac{5}{2} - \frac{7}{3}\right)x = \frac{14}{3}$$
$$\frac{1}{6}x = \frac{14}{3}$$
$$x = \frac{14 \times 6}{3} = 28 \text{ km/hr}$$

The initial speed of the driver = 28 km/hr.

ii. distance $y = \frac{5}{2}x$

$= \frac{5}{2} \times 28$

$= 70$ km

The distance between the 2 towns is $70 -$ km.

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM3-L055 in the Pupil Handbook.

[QUESTIONS]

a. A lorry takes $3\frac{1}{2}$ hours to travel a distance of 126 km. What is its average speed?
b. How long does the lorry take to travel 300 km at an average speed of 40 km/hr?
c. A van travels for 3 hours at 48 km/hr. It then travels for 2 hours at 53 km/hr.
   i. What is the total distance travelled by the van?
   ii. What is the average speed for the whole journey?
d. Adama lives 2 km away from her grandmother. Her speed on the way to visit her is 6 km/hr and her speed on the way back is 4 km/hr. Find:
   i. The total time she took to get to her grandmother's house and back.
   ii. The average speed for the whole journey.
e. An Okada driver covered half the distance between two towns in 2 hr 30 mins. After that he increased his speed by 2 km/hr. He covered the second half of the distance in 2 hr 20 mins. Find:
   i. The initial speed of the driver.
   ii. The distance between the two towns.

Lesson Title: Speed – Part 2	**Theme:** Numbers and Numeration	
Lesson Number: M3-L056	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to solve more complex problems involving speed.	**Preparation** Write the questions found at the end of this lesson plan on the board.	

Opening *(4 minutes)*

1. Ask pupils to answer question a. on the board.
2. After 2 minutes, invite a volunteer to show the answer on the board. The rest of the class should check their solution and correct any mistakes. (Answer: Aruna's average speed $= \frac{30}{1.5} = 20$ km/hr; Sia's average speed $= \frac{42}{2} = 21$ km/hr. Sia had the higher average speed.)
3. Tell pupils that after today's lesson, they will be able to solve more complex problems involving speed.

Teaching and Learning *(20 minutes)*

1. Explain:
   - There are times when the connection between distance, speed and time leads to more complex equations.
   - We will look at 2 instances of these types of situations.
   - One leads to solving simultaneous linear equations. The other leads to solving a quadratic equation.
2. Invite a volunteer to read question b. on the board.
   Solution:
   b. i. **Step 1.** Assess and extract the given information from the problem.
3. Invite a volunteer to assess the problem and tell the class the information we are given. (Answer: Given: average speed of poda-poda is 30 km/hr slower than twice the speed of the bus; after 2 hours, the poda-poda is 20 miles ahead of the bus.)
4. Invite another volunteer to say what we have been asked to find in b. i. (Answer: speed of the bus)

Step 2. Assign variables to the unknown quantities.

distance of bus $= d$ distance of poda-poda $= d + 20$

speed of bus $= s$ speed of poda-poda $= 2s - 30$

Step 3. Set up the equations.

distance = speed × time

$d = 2s$ (1) $t = 2$ hours

$d + 20 = 2(2s - 30)$ (2)   same time

We now have 2 linear equations in d and s

Solve simultaneously by substitution

Step 4. Substitute equation (1) into equation (2) and simplify.

$$\begin{aligned} 2s + 20 &= 2(2s - 30) \\ &= 4s - 60 \end{aligned}$$

$$80 = 2s$$
$$s = 40 \text{ km/hr}$$

Step 5. Write the speed of the bus.

The speed of the bus is 40 km/hr.

ii. **Step 6.** Find the speed of the poda-poda.

$$\text{speed of poda-poda} = 2s - 30$$
$$= (2 \times 40) - 30$$
$$= 80 - 30 = 50 \text{ km/hr}$$

The speed of the poda-poda is 50 km/hr.

5. Invite a volunteer to assess question c. on the board and say the total distance and total time travelled. (Answer: total distance travelled = 200 km, total time taken = 4 hours)
6. Invite a volunteer to say what we have been asked to find. (Answer: Sam's 2 speeds.)
7. Explain:
   - If average speed was required, we would have been able to use the usual formula, average speed $= \frac{\text{total distance travelled}}{\text{total time taken}} = \frac{200}{4} = 50$ km/hr.
   - However, we are asked to find Sam's 2 speeds so we need to set up equations for the situation described in the question.
8. Explain the solution, step by step, ensuring that pupils understand the procedure.

   Solution:

   c. Given: 2 part journey, 1st part: 120 km at speed of x km/hr.
   2nd part: 80 km at speed of $x + 15$ km/hr
   total distance travelled = 200 km, total time taken $= 4$ hrs

	distance = speed × time	
1st part	$120 = x \times t_1$	where t_1 and t_2 are the times for the 1st and 2nd parts of the journey respectively
2nd part	$80 = (x + 15) \times t_2$	

$t_1 + t_2 = 4$	given
$\frac{120}{x} + \frac{80}{x+15} = 4$	since $t_1 = \frac{120}{x}$ and $t_2 = \frac{80}{x+15}$
$\frac{120(x+15)+80x}{x(x+15)} = 4$	convert using LCM
$120(x + 15) + 80x = 4x(x + 15)$	multiply throughout by LCM
$30x + 450 + 20x = x^2 + 15x$	expand brackets
$30x + 20x - 15x + 450 = x^2$	collect like terms
$35x + 450 = x^2$	simplify
$x^2 - 35x - 450 = 0$	

   We now have a quadratic equation in x, the speed in the first part

$(x - 45)(x + 10) = 0$	factorise
$x - 45 = 0$ or $x = 45$	solve for both factors
$x + 10 = 0$ or $x = -10$	

   However, as we cannot have a negative speed we disregard $x = -10$

   ∴ The speed in the first part of the journey is $x = 45$ km/hr.

   The speed in the second part of the journey is $x + 15 = 60$ km/hr.
9. Explain:

- It is not clear when reading the question what type of equations we will get.
- Use the information given to set up the equations and examine the result.
- This will guide you towards the correct method for finding the requested information.

Practice *(15 minutes)*

1. Ask pupils to work independently to answer questions d. and e. on the board.
2. Walk around, if possible, to check the answers and clear any misconceptions.
3. Invite volunteers to come to the board to show their solutions.
   The rest of the class should check their solutions and correct any mistakes.
   Solutions:

d. Given: 2 part journey, 1st part: x hours at 5 km/hr.
2nd part: y hours at 10 km/hr
average speed = 7 km/h, total distance 35 km

distance = speed × time
1st part distance = $5 \times x = 5x$
2nd part distance = $10 \times y = 10y$
$5x + 10y$ = total distance travelled
$5x + 10y = 35$ (1)
average speed = $\frac{\text{total distance travelled}}{\text{total time taken}}$
$7 = \frac{35}{x+y}$
$7(x + y) = 35$ divide throughout by 7
$x + y = 5$ (2)

We now have 2 linear equations in x and y
$5x + 10y = 35$
$x + y = 5$

Solve the two equations simultaneously by elimination
$5x + 10y = 35$ (1)
$5x + 5y = 25$ (3) Multiply equation (2) by 5
$5y = 10$ subtract (3) from (1)
$y = 2$
$x + y = 5$ (2)
$x + 2 = 5$ substitute $y = 2$ in equation (2)
$x = 5 - 2$
$x = 3$ solve for x

Amadu walks for $x = 3$ hours, and runs for $y = 2$ hours.

e. Given: 2 part journey, 1st part: 100 km, speed x km/hr, time t_1
2nd part: 200 km, speed $x + 30$ km/hr, time $t - 1$
total distance travelled = 300 km

distance = speed × time
1st part $100 = x \times t$ (1)
2nd part $200 = (x + 30) \times (t - 1)$ (2)

$$\begin{aligned}
200 &= (x+30)\times\left(\frac{100}{x}-1\right) && \text{from (1) } t=\frac{100}{x}\\
&= 100-x+\frac{3{,}000}{x}-30 && \text{expand brackets}\\
&= 70-x+\frac{3{,}000}{x} && \text{collect like terms}\\
200-70 &= -x+\frac{3{,}000}{x}\\
130 &= -x+\frac{3{,}000}{x}\\
130x &= -x^2+3{,}000 && \text{multiply throughout by } x\\
x^2+130x-3{,}000 &= 0 && \text{simplify}
\end{aligned}$$

We now have a quadratic equation in x

$$\begin{aligned}
(x-20)(x+150) &= 0 && \text{factorise}\\
x-20=0 \text{ or } x&=20 && \text{solve for both factors}\\
x+150=0 \text{ or } x&=-150
\end{aligned}$$

However, as we cannot have a negative speed we disregard $x=-150$

∴ The speed in the first part of the journey is $x=20$ km/hr.

$$\begin{aligned}
t &= \frac{100}{x}=\frac{100}{20}=5 \text{ km/hr} && \text{from (1)}\\
\text{total time taken} &= t+t-1=2t-1\\
&= 2\times10-1=9 \text{ hrs}\\
\text{average speed} &= \frac{\text{total distance travelled}}{\text{total time taken}}\\
&= \frac{300}{9}=33\tfrac{1}{3} \text{ km/hr}
\end{aligned}$$

The average speed is $33\frac{1}{3}$ km/hr or 33.33 km/hr to 2 d.p.

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM3-L056 in the Pupil Handbook.

[QUESTIONS]

a. Aruna travelled 30 km in 1.5 hours. Sia travelled 42 km in 2 hours. Who had the higher average speed?
b. A bus and a poda-poda both left the bus terminal at the same time heading in the same direction. The average speed of the poda-poda is 30 km/hr slower than twice the speed of the bus. In two hours, the poda-poda is 20 miles ahead of the bus. Find: i. the speed of the bus ii. the speed of the poda-poda.
c. On a journey of 200 km, Sam travels at a constant speed for the first 120km. He then increases his speed by 15km/h for the remainder. If the whole journey takes 4 hours, find his two speeds.
d. Amadu walks for x hours at 5 km/hr and runs for y hours at 10 km/hr. He travels a total of 35 km and his average speed is 7 km/hr. Find the values of x and y.
e. On a journey of 300 km, Mariama drives the first 100 km at a constant speed. She then increases her speed by 30 km/hr for the remainder of her journey. If the second part took 1 hour less than the first part, find the average speed of her journey.

Lesson Title: Travel Graphs	**Theme:** Numbers and Numeration	
Lesson Number: M3-L057	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to draw and interpret travel graphs.	**Preparation** Write the questions found at the end of this lesson plan on the board.	

Opening *(1 minute)*

1. Invite a volunteer to say what formula we have been using to calculate average speed. (Answer: average speed $= \frac{\text{total distance travelled}}{\text{total time taken}}$)
2. Tell pupils that after today's lesson, they will be able to draw and interpret travel graphs.

Teaching and Learning *(20 minutes)*

1. Refer to the graph in question a. on the board.
2. Explain:
   - A travel graph shows the relationship between distance and time of a moving object for a journey.
   - Travel graphs are also referred to as distance-time graphs.
   - The vertical scale shows the distance from the starting or reference point.
   - The horizontal graph shows the time taken.
   - Travel graphs can have 1, 2, 3 or more parts, which represent different parts of the journey.
   - The example on the board has 3 parts.
3. Invite a volunteer to say what the first part of the journey shows. (Example answer: A straight line sloping upwards shows travelling away from the starting point at a constant speed.) Note that the pupil may not use the words "at a constant speed". You will explain this later in the lesson.
4. Invite a volunteer to say how the graph represents when the object is not moving. (Answer: by a horizontal line.)
5. Explain:
   - The horizontal line shows that for a period of time, there was no increase or decrease of distance from the starting point.
   - The straight line sloping downwards shows the object coming back towards its starting point at a constant speed.
   - The steeper the slope of the line, the faster the object is travelling for a given time.
6. Invite a volunteer to say what measures distance travelled in a given time. (Answer: speed)
7. Write on the board: speed $= \frac{\text{distance}}{\text{time}}$
8. Explain:

- Because each part of the graph is a straight line, it shows that the object is travelling either at a constant or zero speed.
- The slope or gradient of the travel graph gives the speed of the object.
- A positive gradient means the object is moving away from the starting point.
- A negative gradient means the object is moving towards the starting point.
- The average speed can also be calculated for the whole journey.

9. Work through the solution on the board inviting volunteers to answer at each step.

   Solution:

   a. **Step 1.** Assess and extract the given information from the problem.
   Given: travel graph showing Fatu's journey
   Step 2. Describe each part of her journey.

   i. 1st part: Fatu moves away from the house at a constant speed.
   2nd part: Fatu remains at the same place for 180 seconds.
   3rd part: Fatu returns home at a constant speed.

   Step 3. Substitute into the appropriate formula

   ii. $\text{speed} = \frac{\text{distance}}{\text{time}}$

   1st part: Fatu travels 450 m in 150 seconds.

   $$\text{speed} = \frac{450\text{ m}}{150\text{ s}} = 3\text{ m/s}$$

   Fatu's speed in the first part of her journey is 3 m/s.
   2nd part: Fatu's speed is 0 (horizontal line has 0 gradient).
   3rd part: Fatu travels 450 m in 120 seconds.

   $$\text{speed} = \frac{450\text{ m}}{120\text{ s}} = 3.75\text{ m/s}$$

   Fatu's speed in the second part of her journey is 3.75 m/s.

   iii. $$\text{average speed} = \frac{\text{total distance travelled}}{\text{total time taken}} = \frac{900\text{ m}}{450\text{ s}} = 2\text{ m/s}$$

   Fatu's average speed is 2 m/s.

10. Invite a volunteer to assess question b. on the board and extract the given information. (Example answer: distance walked by Malay to shop = 420 m, time to walk to shop = 7 minutes, time at shop = 5 minutes time to walk home = 6 minutes)
11. Invite a volunteer to say what we have been asked to find. (Answer: the distance-time graph for Malay's shopping trip and the speed at which he walks on each part of the journey.)
12. Ask pupils to work with seatmates to find the solution. The rest of the class should check their solutions and correct any mistakes.

    Solution:

    b. Given: distance walked by Malay to shop = 420 m, time to walk to shop = 7 minutes, time at shop = 5 minutes time to walk home = 6 minutes

    i. distance-time graph: see next page

    ii. $\text{speed} = \frac{\text{distance}}{\text{time}}$

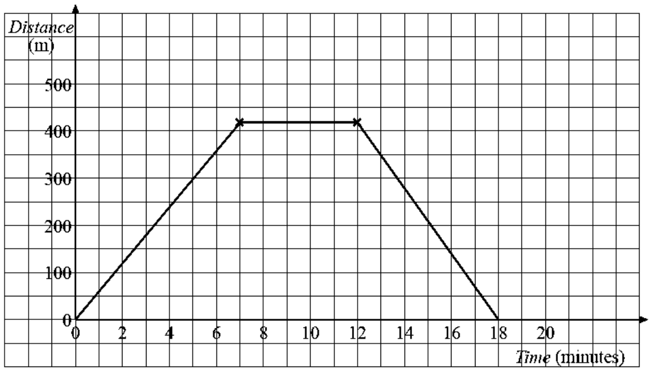
    1st part: Malay travels 420 m in 7 minutes speed $= \frac{420 \text{ m}}{7 \text{ min}}$

speed $= 60$ m/minute

$= \frac{60 \text{ m}}{60 \text{ s}}$

$= 1$ m/s

Malay's speed in the first part of his journey is 60 m/minute or 1 m/s.



Malay's speed in the 2nd part of his journey is 0 m/s.

3rd part: Malay travels 420 m in 6 minutes

speed $= \frac{420 \text{ m}}{6 \text{ min}}$

$= 70$ m/minute

$\frac{70 \text{ m}}{60 \text{ s}}$

$= 1.1666$ m/s

Malay's speed in the 3rd part of his journey is 1.17 m/s to 2 d.p.

Practice *(17 minutes)*

1. Ask pupils to work independently to answer question c.
2. Walk around, if possible, to check the answers and clear any misconceptions.
3. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.

Solutions:

c. i. Given: 10,000 m race, 2,000 m in 5 m/s, 7,400 m in 4 m/s, 600 m in 6 m/s
To draw the graph, we need to find the time taken for each part of the race.

speed $= \frac{\text{distance}}{\text{time}}$

time $= \frac{\text{distance}}{\text{speed}}$

1st part: time $= \frac{2{,}000}{5}$

$= 400$ s $= 6.7$ minutes

The time for the first part of the race was 6.7 minutes.

2nd part: time $= \frac{7{,}400}{4}$

$= 1{,}850$ s $= 30.8$ minutes

The time for the second part of the race was 30.8 minutes.

3rd part: time $= \frac{600}{6}$

$= 100$ s $= 1.7$ minutes

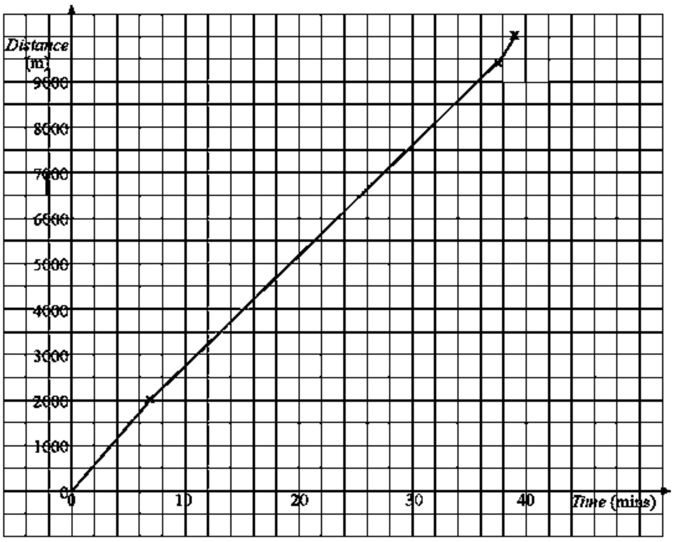
The time for the third part of the race was 1.7 minutes.

ii. total time taken $= 6.7 + 30.8 + 1.7$

$= 39.2$ minutes

$= 39$ mins

The total time taken for the race was 39 minutes to the nearest minute.

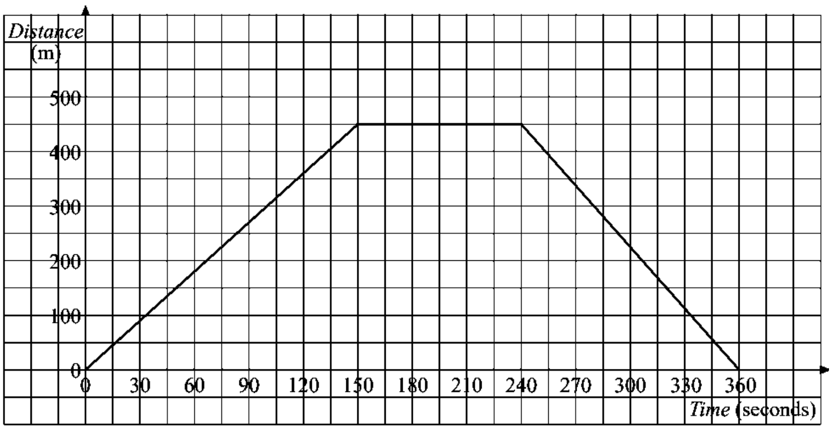


Closing *(2 minutes)*

1. Invite a volunteer to say how to find the speed of an object from a travel graph. (Answer: The slope or gradient of the graph gives the speed.)
2. For homework, have pupils do the practice activity PHM3-L057 in the Pupil Handbook.

[QUESTIONS]

a. The graph shows Fatu's journey from home.
   i. Describe how Fatu moves on each part of her journey.
   ii. Calculate her speed on each part.
   iii. What is her average speed for the whole journey?



b. Malay walks 420 m from his house to a shop in 7 minutes. He spends 5 minutes at the shop and then walks home in 6 minutes.
   i. Draw a distance-time graph for Malay's shopping trip.
   ii. Calculate the speed at which Malay walks on each part of the journey.

c. Khadija completes a 10,000 m race. She runs the first 2,000 m at 5 m/s, the next 7,400 m at 4m/s and the last 600 m at 6m/s.
   i. Draw a travel graph for Khadija's race.
   ii. How long does she take to complete the race?

Lesson Title: Density	Theme: Numbers and Numeration	
Lesson Number: M3-L058	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to calculate the density of a population or an object using ratio and proportion.	**Preparation** Write the questions found at the end of this lesson plan on the board.	

Opening *(2 minutes)*

1. Ask pupils to work with seatmates to write down their understanding of the word "density".
2. After 1 minute, invite a volunteer to give their answer. (Example answers: density is the ratio of the mass of an object to its volume; density $= \frac{\text{mass}}{\text{volume}}$)
3. Tell pupils that after today's lesson, they will be able to able to calculate the density of a population or an object using ratio and proportion.

Teaching and Learning *(20 minutes)*

1. Write on the board:

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

$$\text{population density} = \frac{\text{number of people in area}}{\text{area}}$$

2. Explain:
   1. The more familiar formula for the density of an object is a measure of the mass of the object per cm^3 of volume.
   2. Mass is how much matter an object has, and volume is the amount of space that the object takes up.
   3. Population density is a measure of how close people or organisms live to each other.
   4. It is the ratio of the number of people or organisms per unit area of available space.
   5. We are going to look at both the density of an object and population density problems.
3. Invite a volunteer to read question a. on the board. What information are we given? (Answer: Given: piece of silver with mass of 84 g and a volume of 8 cm^3)
4. Invite another volunteer to say what are we asked to find? (Answer: density).
5. Go through the solution below, making sure that pupils understand the procedure to find the density of an object.

   Solution:

   a. **Step 1.** Assess and extract the given information from the problem.

      Given: piece of silver with mass of 84 g and a volume of 8 cm^3

      Step 2. Substitute into the appropriate formula.

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

$$\text{density} = \frac{84}{8}$$

$$= 10.5 \text{ g/cm}^3$$

Step 3. Write the answer. The piece of silver has a density of 10.5 g/cm^3

6. Invite a volunteer to assess question b. on the board and extract the given information. (Example answer: village with an area of 70 km^2 and population of 6,200 people)
7. Invite a volunteer to say what we have been asked to find. (Answer: population density of the village in people/km^2)

   Solution:

   b. Given: village with an area of 70 km^2 and population of 6,200 people

   $$\text{population density} = \frac{\text{number of people in area}}{\text{area}}$$
   $$= \frac{6{,}200}{70}$$
   $$= 88.571$$
   $$= 88.6$$

   The population density of the village is 88.6 people/km^2.

8. Ask pupils to work with seatmates to answer question c.
9. Invite a volunteer to show their working out on the board. The rest of the class should check their solution and correct any mistakes.

   Solution:

   c. given: mass $= 3$ kg

   $$\text{density} = \frac{\text{mass}}{\text{volume}}$$
   $$\text{volume} = l \times w \times h$$
   $$= 50 \times 2 \times 2$$
   $$= 200 \text{ cm}^3$$

   i. $$\text{density} = \frac{3}{200}$$
   $$= 0.015 \text{ kg/cm}^3$$

   ii. $$3 \text{ kg} = 3{,}000 \text{ g}$$
   $$\text{density} = \frac{3{,}000}{200}$$
   $$= 15 \text{ g/cm}^3$$

Practice *(15 minutes)*

1. Ask pupils to work independently to answer questions d., e. and f.
2. Walk around, if possible, to check the answers and clear any misconceptions.
3. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.

   Solutions:

   d. Given: density $= 7700$ kg/m^3

   $$\text{density} = \frac{\text{mass}}{\text{volume}}$$

   i. volume $= 2.5$ m^3 ii. mass $= 1{,}540$ kg mass $= \text{density} \times \text{volume}$ volume $= \frac{\text{mass}}{\text{density}}$

$= 7{,}700 \times 2.5$ volume $= \frac{1{,}540}{7{,}700}$

$= 19{,}250$ kg $= 0.2\ m^3$

e. i. population density $= \frac{\text{number of people in area}}{\text{area}}$

population density for Ghana $= \frac{27{,}499{,}924}{227{,}533}$

$= 120.9$ people/km^2

population density for Guinea $= \frac{12{,}413{,}867}{245{,}717}$

$= 50.5$ people/km^2

population density for Liberia $= \frac{4{,}689{,}021}{96{,}320}$

$= 48.7$ people/km^2

population density for Nigeria $= \frac{190{,}632{,}261}{910{,}768}$

$= 209.3$ people/km^2

population density for Sierra Leone $= \frac{6{,}163{,}195}{71{,}620}$

$= 86.1$ people/km^2

Use the information to complete the table

Country	**Population (people)**	**Area (km^2)**	**Population density (people/km^2)**
Ghana	27,499,924	227,533	120.9
Guinea	12,413,867	245,717	50.5
Liberia	4,689,021	96,320	48.7
Nigeria	190,632,261	910,768	209.3
Sierra Leone	6,163,195	71,620	86.1

ii. Nigeria has the highest population density (209.3 people/km^2).

iii. Liberia has the lowest population density (48.7 people/km^2).

iv. p.d. Nigeria : p.d. Liberia $= \frac{209.3}{48.7}$

$= 4.3$

There are 4.3 as many people per km^2 in Nigeria as in Liberia.
The ratio is $4.3 : 1$.

f. population density of Western Sahara = 2.3 people/km^2,
population density of Mauritius = 668.2 people/km^2

i. Western Sahara : Mauritius

2.3 people/km^2 : 668.2 people/km^2

In Western Sahara, there are 2.3 people for every 1 km^2, while in Mauritius, there are 668.2 people for every 1 km^2.

ii. 2.3 : 668.2 divide throughout by 2.3

1 : 290.5

The amount of land available to 1 person in Western Sahara is the same as for 290.5 people in Mauritius.

Closing *(3 minutes)*

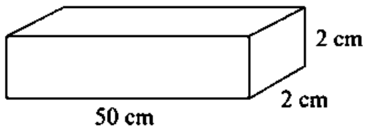
1. Ask pupils to write down 2 new things they learned today.
2. Invite volunteers to share their views. (Example answers: Various but could include that population density measures ratio of the number of people to area of available space; population density is measured in people per unit of area; density measures the amount of mass contained per a unit of volume))
3. For homework, have pupils do the practice activity PHM3-L058 in the Pupil Handbook.

[QUESTIONS]

a. A piece of silver has a mass of 84 g and a volume of 8 cm^3. Work out the density of the silver.

b. A village has an area of 70 km^2. It has a population of 6,200 people. Calculate the population density in people/ km^2. Give your answer to 3 significant figures.

c. The mass of the metal block shown is 3 kg.
   What is the density of the block in:
   i. kg/cm^3
   ii. $grams/cm^3$



d. The density of steel is 7,700 kg/m^3.
   i. A steel bar has a volume of 2.5 m^3. What is the mass of the bar?
   ii. A block of steel has a mass of 1,540 kg. What is the volume of the block?

e. The table below gives the 2017 population data and area for 5 West African countries.

Country	**Population (people)**	**Area (km^2)**	**Population density (people/km^2)**
Ghana	27,499,924	227,533	
Guinea	12,413,867	245,717	
Liberia	4,689,021	96,320	
Nigeria	190,632,261	910,768	
Sierra Leone	6,163,195	71,620	

Source: United States Census Bureau

   i. Calculate the population density for each country to 1 decimal place.
   ii. Which country has the highest population density?
   iii. Which country has the lowest population density?
   iv. How many more people per km^2 live in the highest compared to the lowest population density?

f. The country with the lowest population density in Africa is Western Sahara with 2.3 people/km^2. The country with the highest population density is Mauritius with 668.2 people/km^2.
   i. What is the ratio of the population density of Western Sahara to that of Mauritius? Interpret your result in terms of how many people live in 1 km^2 of land in both countries.
   ii. Give your ratio in the form $1 : n$. Interpret your answer.

Lesson Title: Rates of Pay	**Theme:** Numbers and Numeration	
Lesson Number: M3-L059	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to calculate rates of pay using percentages, ratio and proportion and data given.	**Preparation** Write the questions found at the end of this lesson plan on the board.	

Opening *(4 minutes)*

1. Ask pupils to answer question a. on the board.
2. Invite a volunteer to answer. (Answer: 5% of Le 120,000.00 $= \frac{5}{100} \times 120{,}000 = \frac{5 \times 120{,}000}{100} = \text{Le } 6{,}000.00$)
3. Tell pupils that after today's lesson, they will be able to calculate rates of pay using percentages, ratio and proportion, and data given.

Teaching and Learning *(20 minutes)*

1. Explain:
   - To calculate pay, we are usually given the pay rate for a period of time and asked to calculate how much was earned by a worker.
   - We are also sometimes asked to find the new pay after a percentage increase or decrease in salary.
   - Calculations on pay rates are best done by working through examples which show the different types of problems and methods.
2. Invite a volunteer to assess question b. and tell the class what information we are given. (Answer: given: Sia's hours of work and pay rate)
3. Invite another volunteer to say what we have been asked to find. (Answer: Sia's total weekly wage.)

   Solution:

   b. **Step 1.** Assess and extract the given information from the problem.
   Given: Sia works from 8:30 am to 1:00 pm for 5 days and from 2:00 pm to 5:30 pm for 3 days; rate of pay is Le 5,000.00 per hour.

   Step 2. Use Sia's daily wage to calculate how much she earns

   hours worked per day from 8:30 am to 1:00 pm $= 4.5$ hrs
   hours worked for 5 days $= 5 \times 4.5 = 22.5$ hrs
   hours worked per day from 2:00 pm to 5:30 pm $= 3.5$ hrs
   hours worked for 3 days $= 3 \times 3.5 = 10.5$ hrs
   total hours worked $= 22.5 + 10.5 = 33$ hrs
   weekly wage $= 33 \times 5{,}000 =$ Le $165{,}000.00$

   Step 3. Write the answer.
   Sia's total weekly wage is Le $165{,}000.00$.

4. Invite a volunteer to assess question c. and tell the class what information we are given. (Answer: Fatu's salary = Le 720,000.00; Mohammed's salary = Le 960,000.00)
5. Invite another volunteer to say what we have been asked to find. (Answer: new salaries after 4.2% pay rise)
6. Show 2 different methods of finding the new salaries.

   Solution:

   c. Given: Fatu's salary = Le 720,000.00; Mohammed's salary = Le 960,000.00

   Method 1: (Fatu's new salary)

   Step 1. Find increase in pay.

   increase in pay $= \frac{4.2}{100} \times 720{,}000 =$ Le $30{,}240.00$

   Step 2. Add increase in pay to original amount.

   Fatu's new salary $= 720{,}000 + 30{,}240 =$ Le $750{,}240.00$

   Method 2: (Mohammed's new salary)

   Step 1. Find the multiplier for the increase in pay

   multiplier $= 100\% + 4.2\% = 104.2\%$

   [NOTE]: 100% refers to the original salary before the increase.

   Step 2. Use multiplier to calculate new salary

   Mohammed's new salary $= \frac{104.2}{100} \times 960{,}000 =$ Le $1{,}000{,}032.00$

7. Explain: We can use either method as appropriate, to increase or decrease a quantity by a given percentage.
8. Ask pupils to work with seatmates to answer question d.
9. Invite a volunteer to show their answer on the board. The rest of the class should check their solution and correct any mistakes.

   Solution:

   d. Given: Ibrahim's pay cut $= 3\%$, new salary$= 727{,}500$

   $$\begin{aligned}
   \text{Let Ibrahim's initial salary} &= x \\
   \text{multiplier} &= 100\% - 3\% = 97\% \\
   \tfrac{97}{100} \text{ of } x &= 727{,}500 \\
   \tfrac{97x}{100} &= 727{,}500 \\
   0.97x &= 727{,}500 \\
   x &= \tfrac{727{,}500}{0.97} = 750{,}000
   \end{aligned}$$

   Ibrahim's initial salary = Le 750,000.00

Practice *(15 minutes)*

1. Ask pupils to work independently to answer questions e., f., g. and h.
2. Walk around, if possible, to check the answers and clear up any misconceptions.
3. Invite a volunteer to come to the board to show their solution. The rest of the class should check their solutions and correct any mistakes.

   Solutions:

e. Given: Momoh worked for 32 hours Monday to Thursday @ Le 4,000.00 per hour, also 8 hours on Sunday for 25% extra

hours worked from Monday to Thursday = 32 hrs

hours worked on Sunday. = 8 hrs

For Monday to Thursday, wage = $32 \times 4{,}000$ = Le $128{,}000.00$

For Sunday, 25% extra, wage = $1.25 \times 8 \times 6{,}000$ = Le $60{,}000.00$

this week's total wage = $128{,}000 + 60{,}000$ = Le$188{,}000.00$

Momoh's total wage this week is Le $188{,}000$.

f. Given: A: 4% increase this year followed by 5% increase next year.

B: 4½% increase this year followed by 4½% increase next year.

current salary is Le 6,000,000 per year

A: salary this year = $1.04 \times 6{,}000{,}000$ = Le $6{,}240{,}000.00$

salary next year = $1.05 \times 6{,}240{,}000$ = Le $6{,}552{,}000.00$

total % increase after 2 years = 1.04×1.05 = 1.092

B: salary this year = $1.045 \times 6{,}000{,}000$ = Le $6{,}270{,}000.00$

salary next year = $1.045 \times 6{,}270{,}000$ = Le $6{,}552{,}150.00$

total % increase after 2 years = 1.045×1.045 = 1.092025

She should accept B. It gives a higher salary each year as well as a higher total percentage increase after 2 years.

g. Given: Ratio of Agnes and Aruna salaries = $2:3$. Aruna's salary = Le 675,000.00

Agnes : Aruna = $2:3$

Total number of parts = $2 + 3 = 5$

Let x = combined salaries

Aruna's salary: $\frac{3}{5} \times x = 675{,}000$

$3x = 5 \times 675{,}000$

$x = \frac{5 \times 675{,}000}{3}$

= Le $1{,}125{,}000.00$

Agnes' salary = $1{,}125{,}000 - 675{,}000$ (or $\frac{2}{5} \times 1{,}125{,}000$)

= Le $450{,}000.00$

h. Given: Manager's salary = Le 3,000,000, Finance Director's salary = 80% of Manager's salary, Assistant Manager's salary = 65% of Manager's salary

Method 1.

Finance Director's salary = $\frac{80}{100} \times 3{,}000{,}000$ = Le $2{,}400{,}000.00$

Assistant Manager's salary = $\frac{65}{100} \times 3{,}000{,}000$ = Le $1{,}950{,}000.00$

difference in salary = $2{,}400{,}000 - 1{,}950{,}000$ = Le $450{,}000.00$

Method 2.

difference in salary = $\frac{80-65}{100} \times 3{,}000{,}000$

= $\frac{15}{100} \times 3{,}000{,}000$ = Le $450{,}000.00$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM3-L059 in the Pupil Handbook.

[QUESTIONS]

a. Find 5% of Le 120,000.00.

b. Each week Sia works from 8:30 am to 1:00 pm for 5 days and from 2:00 pm to 5:30 pm for 3 days. Her rate of pay is Le 5,000.00 per hour. What is her total weekly wage?

c. The table shows the salaries of two workers. If each worker receives a 4.2% salary increase, what is the new salary of each worker?

Fatu	Le 720,000.00
Mohammed	Le 960,000.00

d. Ibrahim received a 3% cut to his pay. If his new salary is Le 727,500.00, what was his old salary?

e. Momoh earns Le 4,000.00 per hour for working Mondays to Fridays. He earns 25% extra if he works on Saturdays and Sundays.
This week, Momoh worked for 32 hours from Monday to Thursday and a further 8 hours on Sunday. How much did he earn in total?

f. Saphie is offered the following pay deals:
A: 4% increase this year followed by 5% increase next year.
B: 4½% increase this year followed by 4½% increase next year.
If Saphie's current salary is Le 6,000,000.00 per year, which pay deal should she accept?

g. The salaries of Agnes and Aruna are in the ratio 2:3. If Aruna's salary is Le 675,000.00, what is Agnes' salary?

h. Company ABC has a sliding scale they use to pay their senior staff as follows:

Manager	Le 3,000,000.00
Finance Director	80% of the Manager's pay
Assistant Manager	65% of the Manager's pay

What is the difference in pay between the Finance Director and the Assistant Manage?

<table>
<tr><td>Lesson Title: Commission</td><td colspan="2">Theme: Numbers and Numeration</td></tr>
<tr><td>Lesson Number: M3-L060</td><td>Class: SSS 3</td><td>Time: 40 minutes</td></tr>
<tr><td>Learning Outcome
By the end of the lesson, pupils will be able to calculate commission on a transaction by applying percentages.</td><td colspan="2">Preparation
Write the questions found at the end of this lesson plan on the board.</td></tr>
</table>

Opening *(1 minute)*

1. Tell pupils that after today's lesson, they will be able to calculate commission on a transaction by applying percentage.

Teaching and Learning *(20 minutes)*

1. Explain:

- Some employees, particularly sales people, are given commission on top of (or instead of) their wages or salaries.
- The value of the commission is usually worked out as a percentage of the amount they sold during the month or year.
- The value of the amount sold is taken as 100%.
- To calculate x commission on a particular sales amount, use the formula:

$$\text{commission} = \frac{x}{100} \times \text{sales amount}$$

2. Invite a volunteer to assess question a. and tell the class what information we are given. (Answer: given: commission received by sales vendor = 10% of sales)
3. Invite another volunteer to say what we have been asked to find. (Answer: find commission for various sale amounts)
4. Write the calculation for the commission on the board and invite volunteers to work out the answer for each.

Solution:

a. **Step 1.** Assess and extract the given information from the problem.
Given: commission received by sales vendor = 10% of sales

Step 2. Calculate commission for each sales amount.

$$\text{commission} = \frac{12}{100} \times \text{sales amount} = 0.12 \times \text{sales amount}$$

i.	Le 2,000.00 sales:	commission	$= 0.12 \times 2{,}000$	= Le 240.00
ii.	Le 6,000.00 sales:	commission	$= 0.12 \times 5{,}000$	= Le 600.00
iii.	Le 340,000.00 sales:	commission	$= 0.12 \times 340{,}000$	= Le $40{,}800.00$
iv.	Le 18,000.00 sales:	commission	$= 0.12 \times 18{,}000$	= Le $2{,}160.00$

5. Invite a volunteer to assess question b. and tell the class what information we are given. (Answer: 10% commission on bread sold; Jenneh's commission was Le $45{,}000.00$)

6. Invite another volunteer to say what we have been asked to find. (Answer: how much bread Jenneh sold)

Solution:

b. Given: 10% commission on bread sold; Jenneh's commission was Le $45,000.00$

$$\begin{aligned} \text{Let amount of sales} &= x \\ \text{commission} &= \frac{10}{100} \times x = 0.1x \\ 45,000 &= 0.1x \\ x &= \frac{45,000}{0.1} \\ x &= \text{Le } 450,000.00 \end{aligned}$$

Jenneh sold Le $450,000.00$ worth of bread.

7. Ask pupils to work with seatmates to answer question c.
8. Invite a volunteer to show their answer on the board. The rest of the class should check their solution and correct any mistakes.

Solution:

c. Given: total sales = Le $500,000.00$, commission = Le $25,000.00$

$$\begin{aligned} \text{Let percentage commission} &= x \\ \text{commission} &= \frac{x}{100} \times 500,000 = 5,000x \\ 25,000 &= 5,000x \\ x &= \frac{25,000}{500,000} = \frac{5}{100} \\ &= 5\% \end{aligned}$$

The percentage commission received by the agent = 5%.

Practice *(15 minutes)*

1. Ask pupils to work independently to answer questions d., e., f., g. and h.
2. Walk around, if possible, to check the answers and clear up any misconceptions.
3. Invite a volunteer to come to the board to show their solution. The rest of the class should check their solutions and correct any mistakes.

Solution:

d. Given: Le $60,000.00$ commission on Le $800,000.00$ worth of goods received

$$\begin{aligned} \text{Let percentage commission} &= x \\ \text{commission} &= \frac{x}{100} \times 800,000 \\ 60,000 &= 8,000x \\ x &= \frac{60,000}{8,000} = 7.5\% \end{aligned}$$

Abu pays 7.5% commission on goods received.

e. Given: total sales Le600 million, commission of 5.2%

i.
$$\begin{aligned} \text{commission} &= \frac{5.2}{100} \times 600,000,000 \\ &= 31,200,000 \end{aligned}$$

The sales manager received commission of Le $31,200,000.00$ for the year.

ii average monthly income $= \frac{31,200,000}{12}$

$= \text{Le } 2,600,000.00$

The sales manager's average monthly income was Le $2,600,000.00$.

f. Given: Dahlina'salary = Le $700,000.00$ with 2% commission on all sales, monthly income is Le $1,000,000.00$

commission $= 1,000,000 - 700,000 = \text{Le } 300,000.00$

Let amount of sales $= x$

commission $= \frac{2}{100} \times x = 0.02x$

$300,000 = 0.02x$

$x = \frac{300,000}{0.02} = \text{Le } 15,000,000.00$

Dahlina made sales of Le$15,000,000.00$ for the month.

g. Given: sale price of house Le$500,000,000.00$ 2% commission on the first Le$200,000,000.00$ and 3% on the remainder

commission on the first Le $200,000,000.00$ $= \frac{2}{100} \times 200,000,000 = \text{Le } 4,000,000.00$

remainder $= 500,000,000 - 200,000,000$

$= \text{Le } 300,000,000$

commission on remainder $= \frac{3}{100} \times 300,000,000 = \text{Le } 9,000000$

total commission $= 4,000,000 + 9,000,000 = \text{Le } 13,000,000$

The estate agency made Le $13,000,000$ commission on the house sale.

h. Given: 3% commission on the first Le 2 million sales, 4% commission on the next Le 3 million and 5% on any sales above Le 5 million; sales in December = Le 16 million

commission on the first Le $200,000,000.00$ $= \frac{3}{100} \times 2,000,000 = \text{Le}60,000.00$

commission on the next Le $300,000,000.00$ $= \frac{4}{100} \times 3,000,000 = \text{Le}120,000.00$

remainder $= 16,000,000 - 5,000,000$

$= \text{Le } 11,000,000.00$

commission on remainder $= \frac{5}{100} \times 11,000,000 = \text{Le } 550,000.00$

total commission $= 60,000 + 120,000 + 550,000 = \text{Le } 730,000.00$

Total commission of sales agent = Le $730,000.00$

Closing *(4 minutes)*

1. Ask pupils to discuss with seatmates one new thing they learned during the lesson.
2. Invite volunteers to share their discussion with the class. (Answer: Various)

3. For homework, have pupils do the practice activity PHM3-L060 in the Pupil Handbook.

[QUESTIONS]

a. A newspaper vendor makes a commission of 12% on his sales. Calculate his commission on the following sales:
   i. Le $2,000.00$
   ii. Le $6,000.00$
   iii. Le $340,000.00$
   iv. Le $18,000.00$
b. Jenneh gets a commission of 10% on bread sold. In one week, Jenneh's commission was Le $45,000.00$. How much bread did she sell during that week?
c. An insurance agent received Le$25,000.00$ as his commission on a total sales of Le $500,000.00$. Calculate the percentage of his commission.
d. Abu pays Le $60,000.00$ commission on items received in his second-hand shop. If he received Le $800,000.00$ worth of goods, at what percentage commission does he pay?
e. A sales manager sold goods worth Le 600 million in her shop in one year. If she was paid a commission of 5.2% on her sales,
   i. How much money was she paid that year?
   ii. What was her average monthly income?
f. Dahlina receives a salary of Le $700,000.00$ and a 2% commission on all sales for the month. If her total income in a particular month was Le $1,000,000.00$, what was the amount of her sales for the month?
g. An estate agency sold a house for Le 500 million. The agreed commission was 2% of the first $200,000,000$ of the sales price and 3% on the remainder. How much commission did the agency make on the sale?
h. Every month, a sales agent selling electrical goods makes commission of 3% on the first Le 2 million of sales, 4% on the next Le 3 million of sales and 5% on any sales over Le 5 million. How much commission does he make on sales of Le 16 million in December?

Lesson Title: Income taxes	**Theme:** Numbers and Numeration	
Lesson Number: M3-L061	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to calculate the amount of income tax to be paid using percentages.	**Preparation** 1. Draw the PAYE table shown in Teaching and Learning on the board. 2. Write the questions found at the end of this lesson plan on the board.	

Opening *(1 minute)*

1. Tell pupils that after today's lesson, they will be able to calculate the amount of income tax to be paid using percentages.

Teaching and Learning *(20 minutes)*

1. Explain:
   - Tax is deducted every month by the government from the money people earn.
   - This tax is called **Income Tax** and is used to provide services to the country such as education, health, police, military and social welfare.
   - Employee taxes are deducted from their salaries by their employers using a method called PAYE.
   - PAYE stands for Pay As You Earn and the 2017 rates are shown on the table on the board.
   - Every employee has a tax-free income. This is a certain amount of income on which you do not have to pay any income tax.
   - The net income an employee earns is the income after tax has been deducted.
2. Ask pupils to examine the table and write down what the tax-free income is.
3. Invite a volunteer to give the answer. (Answer: Le 500,000.00)
4. Explain: We will now look at examples of how to use the modified PAYE Income tax table.

Sierra Leone PAYE Tax Rate	
Not over Le 500,000.00 per month	Nil
Next Le 500,000.00 per month	15%
Next Le 500,000.00 per month	20%
Next Le 500,000.00 per month	30%
Above Le 2 million per month	35%

5. Invite a volunteer to assess question a. and say what we have been asked to find. (Answer: Income tax for salaries of Le 850,000.00 and Le 1,700,000.00)
   Solution:
   a. **Step 1.** Assess and extract the given information from the problem.
      Given: Income tax for salaries of Le 850,000.00 and Le 1,700,000.00.
      Le 500,000.00 is tax- free.
   i. **Step 2.** Calculate the income tax paid per month on Le 850,000.00 salary

$$\text{taxable income} = 850{,}000 - 500{,}000 = \text{Le } 350{,}000.00$$
$$\text{income tax} = \frac{15}{100} \times 350{,}000 = \text{Le } 52{,}500.00$$

Step 3. Write the answer.

The income tax is Le $52,500.00$ per month

ii. **Step 4.** Calculate the income tax paid per month on Le 1,700,000.00 salary

taxable income $= 1,700,000 - 500,000 =$ Le $1,200,000.00$

Use a table to aid the calculation. Explain each line.

The remaining income is the income left after the taxation at each stage.

Remaining income (Le)	Amount to be taxed (Le)	Rate of tax (%)	Income Tax (Rate × Amount)
$1,200,000$	$500,000$	15	$\frac{15}{100} \times 500,000 = 75,000$
$1,200,000 - 500,000 = 700,000$	$500,000$	20	$\frac{20}{100} \times 500,000 = 100,000$
$700,000 - 500,000 = 200,000$	$200,000$	30	$\frac{30}{100} \times 200,000 = 60,000$

income tax $= 75,000 + 100,000 + 60,000 =$ Le $235,000.00$

Step 5. Write the answer.

The income tax is Le $235,000.00$ per month.

6. Invite a volunteer to assess question b. and tell the class what information we are given. (Answer: Sama's income tax of Le 187,000.00 each month)
7. Invite another volunteer to say what we have been asked to find. (Answer: find Sama's taxable income)

Solution:

b. Given: income tax of Le 187,000.00 each month

We do a reverse calculation to question a.

Remaining tax (Le)	Amount to be taxed (Le)	Rate of tax (%)	Income Tax
$187,000$	$500,000$	15	$\frac{15}{100} \times 500,000 = 75,000$
$187,000 - 75,000 = 112,000$	$500,000$	20	$\frac{20}{100} \times 500,000 = 100,000$
$112,000 - 100,000 = 12,000$	x	30	$\frac{30}{100} \times x = 0.3x$

From the amount of tax remaining in the last line, we know that the amount to be paid is less than Le 500,000.00. Let the amount $= x$, such that:

$0.3x = 12,000$ income tax = remaining tax

$x = \frac{12,000}{0.3} =$ Le$40,000.00$

taxable income $= 500,000 + 500,000 + x$ from the table

$= 500,000 + 500,000 + 40,000 = 1,040,000$

income $=$ tax-free income $+$ taxable income

$= 500,000 + 1,040,000 = 1,540,000$

Sama's earns Le $1,540,000.00$.

8. Ask pupils to work with seatmates to answer question c.
9. Invite volunteers to show their solution on the board. The rest of the class should check their solution and correct any mistakes.

Solution:

c. Given: Adama earns Le 1,200,000.00 per month, has 3 children for whom she can claim Le 50,000.00 per child.

i. total tax-free income $= 500{,}000 + (3 \times 50{,}000) =$ Le $650{,}000.00$

i. taxable income $= 1{,}200{,}000 - 650{,}000 =$ Le $550{,}000.00$

ii.

Remaining income (Le)	Amount to be taxed (Le)	Rate of tax (%)	Income Tax (Rate × Amount)
$550{,}000$	$500{,}000$	15	$\frac{15}{100} \times 500{,}000 = 75{,}000$
$550{,}000 - 500{,}000 = 50{,}000$	$50{,}000$	20	$\frac{20}{100} \times 50{,}000 = 10{,}000$

total income tax $= 75{,}000 + 10{,}000 = 85{,}000$

iv. net income $= 1{,}200{,}000 - 85{,}000 = 1{,}115{,}000$

Practice *(15 minutes)*

1. Ask pupils to work independently to answer questions d., e. and f.
2. Walk around, if possible, to check the answers and clear up any misconceptions.
3. Invite a volunteer to come to the board to show their solution. The rest of the class should check their solutions and correct any mistakes.

Solution:

d. Given: Mariama earns Le 6,600,000.00 per year

monthly pay $= \frac{6{,}600{,}000}{12} =$ Le $550{,}000.00$

taxable income $= 550{,}000 - 500{,}000 =$ Le $50{,}000.00$

income tax $= \frac{15}{100} \times 50{,}000 =$ Le $7{,}500.00$

Mariama's monthly tax is Le7,500.00.

e. Given: Musa earned Le9,000,000.00 last year

i. total tax-free income $= 12 \times 500{,}000 =$ Le $6{,}000{,}000.00$

ii. monthly pay $= \frac{9{,}000{,}000}{12} =$ Le $750{,}000.00$

taxable income $= 750{,}000 - 500{,}000 =$ Le $250{,}000.00$

income tax $= \frac{15}{100} \times 250{,}000 =$ Le $37{,}500.00$

iii. pay rise $= \frac{5}{100} \times 750{,}000 =$ Le $37{,}500.00$

new salary $= 37{,}500 + 750{,}000 =$ Le $787{,}500.00$

taxable income $= 787{,}500 - 500{,}000 =$ Le $287{,}500.00$

income tax $= \frac{15}{100} \times 287{,}500 =$ Le $43{,}125.00$

increase in income tax $= 43{,}125 - 37{,}500 =$ Le $6{,}125.00$

f. Given: Yeneva paid Le 4,740,000.00 in tax last year

i. monthly tax $= \frac{4{,}740{,}000}{12} =$ Le $395{,}000.00$

Remaining tax (Le)	Amount to be taxed (Le)	Rate of tax (%)	Income Tax
$395,000$	$500,000$	15	$\frac{15}{100} \times 500,000 = 75,000$
$395,000 - 75,000 = 320,000$	$500,000$	20	$\frac{20}{100} \times 500,000 = 100,000$
$320,000 - 100,000 = 220,000$	$500,000$	30	$\frac{30}{100} \times 500,000 = 150,000$
$220,000 - 150,000 = 70,000$	x	35	$\frac{35}{100} \times x = 0.35x$

$0.35x = 70,000$ income tax = remaining tax

$x = \frac{70,000}{0.35}$

= Le $200,000.00$

monthly taxable income = $500,000 + 500,000 + 500,000 + 200,000$

= Le $1,700,000.00$

ii. monthly income = tax-free income + taxable income

= $500,000 + 1,700,000$ = Le $2,200,000.00$

net monthly income = $2,200,000 - 395,000$ = Le $1,805,000.00$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM3-L061 in the Pupil Handbook.

[QUESTIONS]

a. Use the table to calculate how much tax is paid on the salaries below each month:
   i. Le 850,000.00 ii. Le 1,700,000.00
b. Sama pays income tax of Le 187,000.00 each month. How much does he earn per month?
c. Adama earns Le 1,200,000.00 per month. In addition to her tax-free income she can claim Le 50,000.00 for every dependent child. She has 3 children. Calculate:
   i. Her total tax-free income.
   ii. Her taxable income.
   iii. Her total tax per month.
   iv. Her net income per month.
d. Mariama earns Le 6,600,000.00 per year. How much tax does she pay each month?
e. Musa earned Le 9,000,000.00 last year.
   i. What was his total tax-free income?
   ii. How much tax did he pay each month?
   iii. This year, he received a pay rise of 5%. By how much does his income tax increase each month?
f. Yeneva paid Le 4,500,000,00 in tax last year.
   i. How much was her monthly taxable income?
   ii. What was her net income per month?

Lesson Title: Simple interest	**Theme:** Numbers and Numeration	
Lesson Number: M3-L062	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to calculate simple interest rates and time.	**Preparation** Write the questions found at the end of this lesson plan on the board.	

Opening *(1 minute)*

1. Tell pupils that after today's lesson, they will be able to calculate simple interest rates and time.

Teaching and Learning *(20 minutes)*

1. Invite a volunteer to say what they know about simple interest. (Example answer: The amount of money you have to pay the bank when they lend you money.)
2. Explain:
   - When someone deposits money in a bank, the bank pays them interest on the money deposited.
   - When a bank lends money to its customers, it charges them interest on the money borrowed.
   - There are two types of interest earned or charged on money – **simple interest** and **compound interest.**
   - We will look at simple interest in this lesson, and compound interest in the next.
   - Simple interest, I, is the amount earned or charged on the initial amount or principal, P, at a given rate, R, and for a given period of time, T (in years).
   $$I = \frac{PRT}{100}$$
   - It is in effect the percentage of the principal that is earned or charged for the use of the money.
   - The amount, A, at the end of the period is given by Principal + Interest.
   That is:
   $$A = P + I$$
3. Invite a volunteer to assess question a. and tell the class what information we are given. (Answer: given: Le 500,000.00 deposited by Alusine, 4% interest rate per annum for 2 years)
4. Invite another volunteer to say what we have been asked to find. (Answer: the simple interest paid.)

   Solution:

   a. **Step 1.** Assess and extract the given information from the problem.
   Given: Le 500,000.00 deposited by Alusine 4% interest rate per annum for 2 years

   Step 2. Calculate the interest.
   $$I = \frac{PRT}{100}$$

$$I = \frac{500{,}000 \times 4 \times 2}{100} = \text{Le } 40{,}000.00$$

Step 3. Write the answer.

The interest received by Alusine is Le $40{,}000.00$.

5. Invite a volunteer to assess question b. and tell the class what information we are given. (Answer: simple interest of Le 87,000.00 on Le 725,000.00 for 4 years)
6. Invite another volunteer to say what we have been asked to find. (Answer: interest rate per annum)

   Solution:

   b. Given: simple interest of Le 87,000.00 on Le 725,000.00 for 4 years

   $$I = \frac{PRT}{100}$$

   $$R = \frac{I \times 100}{PT}$$ make R the subject of the formula

   $$R = \frac{87{,}000 \times 100}{725{,}000 \times 4} = 3\%$$

   The interest rate per annum is 3%.
7. Invite a volunteer to assess question a. and tell the class what information we are given. (Answer: Given: interest of Le 90,000.00, after 3 years at 5% per annum)
8. Invite another volunteer to say what we have been asked to find. (Answer: the amount invested (principal))

   Solution:

   c. Given: interest of Le 90,000.00, after 3 years at 5% per annum

   $$P = \frac{I \times 100}{RT}$$ make P the subject of the formula

   $$P = \frac{90{,}000 \times 100}{5 \times 3}$$

   $$= \text{Le } 600{,}000.00$$

   amount after 3 years $= 600{,}000 + 90{,}000 =$ Le $690{,}000.00$

   The amount after 3 years $=$ Le $690{,}000.00$.
9. Ask pupils to work with seatmates to answer question d.
10. Invite a volunteer to show their solution on the board. The rest of the class should check their solution and correct any mistakes.

    Solution:

    d. Given: interest on Le 300,000.00 at 3% interest rate = Le 45,000.00.

    $$T = \frac{I \times 100}{PR}$$ make T the subject of the formula

    $$T = \frac{45{,}000 \times 100}{300{,}000 \times 3} = 5 \text{ years}$$

    The time period $= 5$ years

Practice *(15 minutes)*

1. Ask pupils to work independently to answer questions e., f., g. and h.
2. Walk around, if possible, to check the answers and clear up any misconceptions.
3. Invite a volunteer to come to the board to show their solution. The rest of the class should check their solutions and correct any mistakes.

Solution:

e. Given: Loan to Isata of Le 1,500,000.00 at 3.5% interest rate for 5 years

$$I = \frac{PRT}{100}$$

$$I = \frac{1{,}500{,}000\times 3.5\times 5}{100} \quad = \quad \text{Le } 262{,}500.00$$

$$A = P + I$$

$$= 1{,}500{,}000 + 262{,}500 \quad = \quad \text{Le } 1{,}762{,}500.00$$

Isata will pay back Le $1{,}762{,}500.00$ at the end of the loan period.

f. Given: Memuna has Le 371,000.00 after 1 year at 6% interest rate per annum

$$A = P + I \qquad (1)$$

$$I = \frac{PRT}{100}$$

$$I = \frac{P\times 6\times 1}{100} \qquad \text{substitute given values}$$

$$I = \frac{6P}{100} \quad = \quad 0.06P$$

$$371{,}000 = P + 0.06P \qquad \text{substitute in (1)}$$

$$= P(1+0.06) \quad = \quad 1.06P$$

$$371{,}000 = 1.06P$$

$$P = \frac{371{,}000}{1.06}$$

$$= \text{Le } 350{,}000.00$$

Memuna invested Le $350{,}000.00$ initially.

g. Given: Kasho invested Le 5,000,000.00 at 4% interest for 5 years, then 2 years at 5.5% interest.

$$I = \frac{PRT}{100}$$

After 5 years: $I = \frac{5{,}000{,}000\times 4\times 5}{100} = \text{Le } 1{,}000{,}000.00$

$A = 5{,}000{,}000 + 1{,}000{,}000 = \text{Le } 6{,}000{,}000.00$

After 2 years: $I = \frac{6{,}000{,}000\times 5.5\times 2}{100} = \text{Le } 660{,}000.00$

$A = 6{,}000{,}000 + 660{,}000 = \text{Le } 6{,}660{,}000.00$

The total amount after 7 years = Le $6{,}660{,}000.00$.

h. Given: Le 12,000,000.00 at the rate of 6% per annum for 2 years.

i. $I = \frac{PRT}{100}$

$I = \frac{12{,}000{,}000\times 6\times 2}{100} = \text{Le } 1{,}440{,}000.00$

ii. amount after 2 years $= 12{,}000{,}000 + 1{,}440{,}000$

$= \text{Le } 13{,}440{,}000.00$

iii. monthly instalment $= \frac{13{,}440{,}000}{24} = \text{Le } 560{,}000.00$ per month

Closing *(4 minutes)*

1. Ask pupils to write in their exercise books what formula to use to calculate the amount of money to repay the bank at the end of a loan period.
2. Invite a volunteer to give the answer. (Answer: $A = P + I$**)**

3. For homework, have pupils do the practice activity PHM3-L062 in the Pupil Handbook.

[QUESTIONS]

a. Alusine deposits Le 500,000.00 in the bank at a rate of 4% per annum for 2 years. How much interest does he receive?
b. The simple interest on Le 725,000.00 for 4 years is Le 87,000.00. How much per annum is the interest rate?
c. How much money should be invested if interest of Le 90,000.00 is to be paid after 3 years at 5% per annum? What is the amount after 3 years?
d. Find the time period in which the interest on Le 300,000.00 at 3% interest rate is Le 45,000.00.
e. A bank loaned Isata Le 1,500,000.00 at 3.5% interest rate for 5 years. How much will she pay back at the end of the loan period?
f. Memuna invests some money in a savings account. Interest is paid at a rate of 6% per annum. After 1 year, there is Le 371,000.00 in the account. How much did she invest initially?
g. Kasho invested Le 5,000,000.00 at 4% interest for 5 years. He then invested the amount at the end of the 5 years for a further 2 years at 5.5% interest. What was the total amount after the 7 years?
h. A businesswoman took a loan of Le 12,000,000.00 from the bank at a rate of 6% per annum for 2 years. She agreed to pay back the sum in monthly instalments over the 2-year period.
   Calculate:
   i. The amount of interest on the loan.
   ii. The amount to be paid at the end of the 2 years.
   iii. The monthly instalment.

Lesson Title: Compound interest – Part 1	**Theme:** Numbers and Numeration	
Lesson Number: M3-L063	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to calculate compound interest using successive addition.	**Preparation** Write the questions found at the end of this lesson plan on the board.	

Opening *(2 minutes)*

1. Ask pupils to answer question a. on the board.
2. Invite a volunteer to give the answer. (Answer: $I = \frac{PRT}{100} = \frac{500{,}000 \times 5 \times 4}{100} =$ Le $100{,}000.00$**)**
3. Tell pupils that after today's lesson, they will be able to calculate compound interest using successive addition.

Teaching and Learning *(20 minutes)*

1. Invite a volunteer to say what name is given to the interest we have just calculated. (Answer: simple interest)
2. Explain:
   - When we calculate simple interest, we are finding the percentage of the principal that is added to the investment or loan over the whole period at a given interest rate.
   - The principal remains unchanged for the entire period of the loan.
   - However, investments and loans are not usually calculated using simple interest.
3. Invite a volunteer to say the name of the other type of interest. (This was mentioned in the previous lesson). (Answer: compound interest).
4. Explain:
   - **Compound interest** is the interest calculated at given intervals over the loan period and added to the principal.
   - This new amount becomes the principal and changes every time the interest is calculated.
   - Each time we do the calculation, we compound the principal by adding the interest calculated for a given period to the previous principal.
   - We are in effect earning or paying interest on the interest.
   - Each period is called a **compounding period** and can be at intervals of 1 year, 6 months ($\frac{1}{2}$ year), 3 months ($\frac{1}{4}$ year) or any other agreed time period.
   - The compound interest, CI, is given by:

     $CI \quad = \quad A - P$ **where** $A =$ **Amount at the end of the period**
     $P =$ **Principal**
   - We will now do an example to show 2 different methods of calculating compound interest.
   - We will concentrate on calculating the compound interest annually.

5. Invite a volunteer to assess question b. and tell the class what we have been asked to find. (Answer: find the interest on a loan of Le 500,000.00 for 4 years at a compound interest rate of 5% per annum)

Solution:

b. **Step 1.** Assess and extract the given information from the problem.

Given: Loan of Le 500,000.00 for 4 years at a compound interest rate of 5% per annum.

Step 2. Calculate the amount at the end of the period.

Method 1. Using successive addition

Year	Principal at start of year (Le)	Interest (Le)	Amount at end of year (Le)
1	$500{,}000$	$\frac{5}{100}\times 500000 = 25{,}000$	$500{,}000+25{,}000 = 525{,}000$
2	$525{,}000$	$\frac{5}{100}\times 525000 = 26{,}250$	$525{,}000+26{,}250 = 551{,}250$
3	$551{,}250$	$\frac{5}{100}\times 551250 = 27{,}563$	$551{,}250+27{,}563 = 578{,}813$
4	$578{,}813$	$\frac{5}{100}\times 578{,}813 = 28{,}941$	$578{,}813+28{,}941 = 607{,}754$

Method 2. Using a multiplier

Explain:

- A multiplier is used whenever we wish to increase or decrease an amount by a given percentage.
- The original amount is 100% or 1. $\left(\frac{100}{100}\right)$
- We add to increase the amount by the given percentage – successive addition is embedded in the calculation.

Multiplier $= 1+\frac{5}{100} = 1.05$

Year	Principal at start of year (Le)	Amount at end of year (Le)
1	$500{,}000$	$500{,}000\times 1.05 = 525{,}000$
2	$525{,}000$	$525{,}000\times 1.05 = 551{,}250$
3	$551{,}250$	$551{,}250\times 1.05 = 578{,}813$
4	$578{,}813$	$578{,}813\times 1.05 = 607{,}754$

Step 3: Calculate the compound interest

$$CI = A - P$$
$$= 607{,}754 - 500{,}000 = \text{Le } 107{,}754.00$$

Step 4: Write the answer.

The compound interest at the end of 4 years = Le $107{,}754.00$.

Comparing the answer to question a., the additional interest earned is $107{,}754 - 100{,}000 =$ Le $7{,}754.00$.

6. Ask pupils to work with seatmates to answer question c. They should use Method 1 to solve the problem.
7. Invite a volunteer to show their solution on the board using Method 1. The rest of the class should check their solution and correct any mistakes.

Solution:

c. Given: Deposit of Le 1,500,000.00 in a bank at 4% for 3 years

Year	Principal at start of year (Le)	Interest (Le)	Amount at end of year (Le)
1	$1{,}500{,}000$	$\frac{4}{100}\times 1{,}500{,}000 = 60{,}000$	$60{,}000 + 1{,}500{,}000 = 1{,}560{,}000$
2	$1{,}545{,}000$	$\frac{4}{100}\times 1{,}560{,}000 = 62{,}400$	$62{,}400 + 1{,}560{,}000 = 1{,}622{,}400$
3	$1{,}591{,}350$	$\frac{4}{100}\times 1{,}622{,}400 = 64{,}896$	$64{,}896 + 1{,}622{,}400 = 1{,}687{,}296$

compound interest = $1{,}687{,}296 - 1{,}500{,}000 = 187{,}296$

The compound interest at the end of 3 years = Le $187{,}296.00$.

8. Ask pupils to continue work with seatmates to answer question d. They should now use Method 2 to solve the problem.
9. Invite a volunteer to show their solution on the board using Method 2. The rest of the class should check their solution and correct any mistakes.

Solution:

d. Given: Le 750,000.00 for 3 years at a rate of 6% per annum.

Multiplier $= 1 + \frac{6}{100} = 1.06$

Year	Principal at start of year (Le)	Amount at end of year (Le)
1	$750{,}000$	$750{,}000 \times 1.06 = 795{,}000$
2	$802{,}500$	$802{,}500 \times 1.06 = 842{,}700$
3	$858{,}675$	$858{,}675 \times 1.06 = 893{,}262$

compound interest = $893{,}262 - 750{,}000 = 143{,}262$

The compound interest at the end of 3 years = Le $143{,}262.00$.

Practice *(15 minutes)*

1. Ask pupils to work independently to answer question e. They can use whichever method they prefer. The solution is given using Method 2.
2. Walk around, if possible, to check the answers and clear up any misconceptions.
3. Ask a volunteer to come to the board to show their solution. The rest of the class should check their solutions and correct any mistakes.

Solution:

e. Given: Businesswoman deposits Le 3,000,000.00 at 7% rate per annum for 5 years. She withdraws Le 1,000,000.00 after 3 years.

Multiplier $= 1 + \frac{7}{100} = 1.07$

Year	Principal at start of year (Le)	Amount at end of year (Le)
1	$3{,}000{,}000$	$3{,}000{,}000 \times 1.07 = 3{,}210{,}000$
2	$3{,}210{,}000$	$3{,}210{,}000 \times 1.07 = 3{,}434{,}700$
3	$3{,}434{,}700$	$3{,}434{,}700 \times 1.07 = 3{,}675{,}129$
	withdrawal of Le 1,000,000.00: new principal	$= 3{,}675{,}129 - 1{,}000{,}000$
		= Le $2{,}675{,}129.00$

4	$2,675,129$	$2,675,129 \times 1.07 \quad = \quad 2,862,388.03$
5	$2,862,388.03$	$2,862,388.03 \times 1.07 \quad = \quad 3,062,755.19$
After 5 years, the businesswoman has Le $3,062,755.19$ to the nearest cent in her account.		

Closing *(3 minutes)*

1. Ask pupils to discuss with seatmates which method they prefer and why.
2. Invite volunteers to share their discussion with the class giving the reasons from both seatmates. (Answer: Various)
3. For homework, have pupils do the practice activity PHM3-L063 in the Pupil Handbook.

[QUESTIONS]

a. Find the interest on a loan of Le 500,000.00 for 4 years at 5% per annum.
b. Find the interest on a loan of Le 500,000.00 for 4 years at a compound interest rate of 5% per annum. What additional interest is earned using compound as compared to a simple interest rate?
c. Abdul deposited Le 1,500,000.00 in a bank at 4% compound interest rate per annum for 3 years. Find the interest at the end of the period.
d. Find the interest on a loan of Le 750,000.00 for 3 years at a compound interest rate of 6% per annum.
e. A businesswoman deposited Le 3,000,000.00 in her bank account at a 7% compound interest rate per annum for 5 years. At the end of the third year, she withdrew Le 1,000,000.00. Calculate the amount she has in her account after 5 years. Give your answer to the nearest cent.

<table>
<tr><td>Lesson Title: Compound interest – Part 2</td><td colspan="2">Theme: Numbers and Numeration</td></tr>
<tr><td>Lesson Number: M3-L064</td><td>Class: SSS 3</td><td>Time: 40 minutes</td></tr>
<tr><td>Learning Outcome
By the end of the lesson, pupils will be able to calculate compound interest using the formula.</td><td colspan="2">Preparation
1. Draw the table below on the board for question a.
<table><tr><td>Year (n)</td><td>Principal at start of year (Le)</td><td>Amount at end of year (Le)</td></tr><tr><td></td><td></td><td></td></tr><tr><td></td><td></td><td></td></tr><tr><td></td><td></td><td></td></tr></table>2. Write the questions found at the end of this lesson plan on the board.</td></tr>
</table>

Opening *(4 minutes)*

1. Ask pupils to answer question a. on the board.
2. Invite a volunteer to give the answer. (Answer: Compound interest = Le $62{,}432.00$)
3. Tell pupils that after today's lesson, they will be able to calculate compound interest using the formula.

Teaching and Learning *(20 minutes)*

1. Write the solution shown for question a. below using the table on the board.
   a. Given: Le 500,000.00 for 3 years at a rate of 4% per annum.

$$\text{Multiplier} = 1 + \frac{4}{100} = 1.04$$

Year (n)	Principal (Le)	Amount at end of year (Le)		
1	$500{,}000$	$500{,}000 \times 1.04 = 500{,}000 \times 1.04^1$	=	$520{,}000$
2	$520{,}000$	$520{,}200 \times 1.04 = 500{,}000 \times 1.04^2$	=	$540{,}800$
3	$540{,}800$	$540{,}800 \times 1.04 = 500{,}000 \times 1.04^3$	=	$562{,}432$

The compound interest = $562{,}432 - 500{,}000 = 62{,}432$

The compound interest at the end of the period = Le $62{,}432.00$.

2. Ask pupils to look at the expanded version of the calculation for the amount at the end of each year. What connects the year to the index of the multiplier?
3. Invite a volunteer to answer. (Answer: the year gives the index for the multiplier)
4. Ask pupils to discuss with seatmates what the calculation for the amount would be if the loan was extended to 4 years.
5. Invite a volunteer to answer. (Answer: amount at end of 4 years = $500{,}000 \times 1.04^4$)
6. Explain:
   - From the table, the amount for a particular year is calculated using the formula:

$$\text{amount at end of } n \text{ years} = 500{,}000 \times (1.04)^n$$
$$= 500{,}000 \times \left(1 + \frac{4}{100}\right)^n$$

   - From this, we can write a general formula to find the amount at the end of any period as:

$$A = P\left(1 + \frac{R}{100}\right)^n$$

where $A =$ Amount at the end of the period $P =$ Principal
$R =$ Rate $n =$ Period

- The compound interest, CI, is given as before by:

$CI \quad = \quad A - P$ where $A =$ Amount at the end of the period
$P =$ Principal

7. Invite a volunteer to assess question b. i. and tell the class what information we have been given. (Answer: Le 400,000.00 borrowed by Alice for 3 years at 10%)
8. Invite another volunteer to say what we have been asked to find. (Answer: find the amount on the loan at the end of the period)

Solution:

b. **Step 1.** Assess and extract the given information from the problem.
Given: Le 400,000.00 borrowed by Alice for 3 years at 10%

i. **Step 2.** Calculate the amount at the end of the loan period.

$$A = P\left(1 + \frac{R}{100}\right)^n$$
$$= 400{,}000\left(1 + \frac{10}{100}\right)^3 = 400{,}000 \times (1.1)^3$$
$$= \text{Le } 532{,}400.00$$

Step 3. Write the answer for i.
The amount = Le $532{,}400.00$

ii. **Step 4.** Calculate the compound interest

$$CI = A - P$$
$$= 532{,}400 - 400{,}000 = \text{Le } 132{,}400.00$$

Step 5. Write the answer for ii.
The compound interest = Le $132{,}400.00$**.**

9. Explain: We will now consider how to calculate the compound interest when the compounding period is not per year.
10. Invite a volunteer to assess question c. i. and tell the class what information we have been given. (Answer: Le 250,000.00 deposited by a market trader for 2 years at 4% per annum compounded half-yearly)
11. Invite another volunteer to say what we have been asked to find. (Answer: find the amount in his account at the end of 2 years)

Solution:

c. Given: Le 250,000.00 deposited by a market trader for 3 years at 4% per annum compounded half-yearly.

i. $$A = P\left(1 + \frac{R}{100}\right)^n$$

Since the loan is compounded half-yearly,

- the rate R is equivalent to $\frac{4}{2}$% or 2% per half-year
- there are 6 half-yearly periods in 3 years, so $n = 6$

$$\therefore \quad A = 250{,}000\left(1 + \frac{2}{100}\right)^6 = 250{,}000 \times (1.02)^6$$
$$= \text{Le } 281{,}540.60$$

The amount at the end of 2 years = Le $281{,}540.60$ to the nearest cent.

ii. $$CI = A - P$$

$$= \quad 281{,}540.60 - 250{,}000 \quad = \quad \text{Le } 31{,}540.60$$

The compound interest = Le $31{,}540.60$ to the nearest cent.

12. Ask pupils to work with seatmates to answer question d.
13. Invite a volunteer to show their solution on the board. The rest of the class should check their solution and correct any mistakes.

Solution:

d. Given: a final amount of Le 800,000.00, compounded quarterly at 8% for 3 years.

$$A = P\left(1 + \frac{R}{100}\right)^n$$

$$800{,}000 = P\left(1 + \frac{2}{100}\right)^{12}$$

$R = \frac{8}{4} = 2\%$ per quarter

$n = 12$ (4 quarters x 3 years)

$$800{,}000 = P \times (1.02)^{12}$$

$$P = \frac{800{,}000}{1.02^{12}}$$

$$= 630{,}794.54$$

The amount at the start = Le $630{,}794.54$ to the nearest cent.

Practice *(15 minutes)*

1. Ask pupils to work independently to answer questions e. and f.
   Walk around, if possible, to check the answers and clear up any misconceptions.
2. Ask a volunteer to come to the board to show their solution. The rest of the class should check their solutions and correct any mistakes.

   Solution:

e. Given: Le 3,000,000.00 for 2 years at rate of 8% per annum.

i

$$A = P\left(1 + \frac{R}{100}\right)^n$$

$$A = 3{,}000{,}000\left(1 + \frac{8}{100}\right)^2 \quad = \quad \text{Le } 3{,}499{,}200.00$$

Mr. Karimu has not saved enough to buy the bike

ii. $\text{difference} = 4{,}000{,}000 - 3{,}499{,}200 \quad = \quad \text{Le } 500{,}800.00$

Mr. Karimu still needs to save Le $500{,}800.00$ in order to buy the bike.

f. Given: A: Le 3,000,000.00 for 5 years at 2% compound interest rate per annum
B: Le 5,000,000.00 for 1 year at 3% compound interest rate per annum
C: Le 2,000,000.00 for 3 years at 8% compound interest rate per annum

$$A = P\left(1 + \frac{R}{100}\right)^n \qquad CI = A - P$$

A:

$$A = 3{,}000{,}000\left(1 + \frac{2}{100}\right)^5 = \text{Le } 3{,}312{,}242.41$$

$$CI = 3{,}312{,}242.41 - 3{,}000{,}000 = \text{Le } 312{,}242.41$$

B:

$$A = 5{,}000{,}000\left(1 + \frac{3}{100}\right)^1 = \text{Le } 5{,}150{,}000.00$$

$$CI = 5{,}150{,}000 - 5{,}000{,}000 = \text{Le } 150{,}000.00$$

C:

$$A = 2{,}000{,}000\left(1 + \frac{8}{100}\right)^3 = \text{Le } 2{,}519{,}424.00$$

$$CI = 2{,}519{,}424 - 2{,}000{,}000 = \text{Le } 519{,}424.00$$

C will earn more interest over the other two investments.

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM3-L064 in the Pupil Handbook.

[QUESTIONS]

a. A sum of Le 500,000.00 is to be invested for 3 years. Use the multiplier method to find the final value of the investment if the annual compound interest rate is 4%.
b. Alice borrowed Le 400,000.00 for 3 years at 10% compound interest rate.
   i. What was the amount at the end of the 3 years?
   ii. How much was the compound interest?
c. A market trader deposited Le 250,000.00 into his account in a bank at a compound interest rate of 4% per annum. If interest is compounded half-yearly:
   i. How much does he have in his account after 3 years?
   ii. How much compound interest did he earn?

   Give your answers to the nearest cent.
d. At the end of 3 years, there was Le 800,000.00 in a bank account. If the interest rate was 8% compounded quarterly (4 times a year) over the entire period, how much was there in the bank account at the start? Give your answers to the nearest cent.
e. Mr. Karimu is saving to buy a motor bike. He deposits Le 3,000,000.00 in his account which pays a compound interest rate of 8% per annum.
   i. If the bike costs Le 4,000,000.00, has Mr. Karimu saved enough after 2 years to buy the motor bike?
   ii. If not, how much more does he need to save?
f. Which of the following investments would earn the most interest?

   A: Le 3,000,000.00 for 5 years at 2% compound interest rate per annum

   B: Le 5,000,000.00 for 1 year at 3% compound interest rate per annum

   C: Le 2,000,000.00 for 3 years at 8% compound interest rate per annum

Lesson Title: Profit and loss – Part 1	**Theme:** Numbers and Numeration	
Lesson Number: M3-L065	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to calculate profit and loss on transactions by applying percentage.	**Preparation** Write the questions found at the end of this lesson plan on the board.	

Opening *(1 minute)*

1. Tell pupils that after today's lesson, they will be able to calculate profit and loss on transactions by applying percentage.

Teaching and Learning *(23 minutes)*

1. Explain:
   - An item is sold at a profit when the selling price is greater than the cost price of the item.
   - If, however the cost price of the item is greater than the selling price, the item is sold at a loss.
   - The profit or loss is calculated by taking the difference between the cost price (*CP)* and selling price (*SP).*
   - Note that as difference is always positive:

     profit $= SP - CP$

     loss $= CP - SP$
   - Percentage profit or loss based on the cost price is given by:

     Percentage profit = $\frac{SP-CP}{CP} \times 100$
   - Percentage loss based on the cost price is given by:

     Percentage loss = $\frac{CP-SP}{CP} \times 100$
2. Invite a volunteer to assess question a. and tell the class what information we are given. (Answer: Given: John buys a set of bicycle pumps for Le 40,000.00 and sells them for Le 50,000.00)
3. Invite another volunteer to say what we have been asked to find. (Answer: John's percentage profit)

   Solution:

   a. **Step 1.** Assess and extract the given information from the problem.

   Given: John buys a set of bicycle pumps for Le 40,000.00 and sells them for Le 50,000.00.

   Step 2. Calculate the percentage profit.

   $$\textbf{\% profit} = \frac{SP-CP}{CP} \times 100$$
   $$= \frac{50{,}000-40{,}000}{40{,}000} \times 100$$
   $$= \frac{10{,}000}{40{,}000} \times 100$$
   $$= 25\%$$

   Step 3. Write the answer.

John made 25% profit.

4. Invite a volunteer to assess question b. and tell the class what information we are given. (Answer: cost price of Le 15,000,000.00, selling price of Le 12,000,000.00)
5. Invite another volunteer to say what we have been asked to find. (Answer: percentage loss)

   Solution:

   b. Given: cost price of Le 15,000,000.00, selling price of Le 12,000,000.00

$$\text{percentage loss} = \frac{SP-CP}{CP}\times 100$$

$$\text{percentage loss} = \frac{15{,}000{,}000-12{,}000{,}000}{15{,}000{,}000}\times 100$$

$$= \frac{3{,}000{,}000}{15{,}000{,}000}\times 100$$

$$= 20\%$$

   The percentage loss was 20%.

6. Invite a volunteer to assess question c. and tell the class what information we are given. (Answer: television set bought by Akin, sold for Le 2,500,000.00, percentage profit 25%)
7. Invite another volunteer to say what we have been asked to find. (Answer: cost price of TV)

   Solution:

   c. Given: television set bought by Akin, sold for Le 2,500,000.00, percentage profit 25%

   Method 1. Use the formula for percentage profit.

$$\text{percentage profit} = \frac{SP-CP}{CP}\times 100$$

$$25 = \frac{2{,}500{,}000-CP}{CP}\times 100$$

$$25 = \frac{(2{,}500{,}000-CP)\times 100}{CP}$$

   Multiply throughout by the cost price, CP

$$25CP = (2{,}500{,}000 - CP)\times 100$$

$$\frac{25}{100}CP = 2{,}500{,}000 - CP$$

$$0.25CP + CP = 2{,}500{,}000$$

$$1.25CP = 2{,}500.000$$

$$CP = \frac{2{,}500{,}000}{1.25}$$

$$= \text{Le } 2{,}000{,}000.00$$

   Method 2. Use a multiplier

$$SP = CP + \frac{25}{100}CP$$

   since Akin made a profit, we add the percentage profit to 100% of the cost price

$$= CP\left(1+\frac{25}{100}\right)$$

$$\text{multiplier} = 1+\frac{25}{100} = 1.25$$

$$SP = 1.25\times CP$$

$$2{,}500{,}000 = 1.25CP$$

$$CP = \frac{2{,}500{,}000}{1.25}$$

$= \quad$ Le $2,000,000.00$

Akin bought the television set for Le $2,000,000.00$.

8. Ask pupils to work with seatmates to answer question d.
9. Invite a volunteer to show their solution on the board. Method 2 is shown here. The rest of the class should check their solution and correct any mistakes.

Solution:

d. Given: motor bike sold for Le 4,250,000.00 at a loss of 15%.

$$\text{multiplier} = 1 - \frac{15}{100} = 0.85$$

subtract the percentage loss from 100% of the cost price

$$SP = 0.85 \times CP$$
$$5,000,000 = 0.85CP$$
$$CP = \frac{4,250,000}{0.85}$$
$$= \text{Le } 5,000,000.00$$

The cost price of the motor bike was Le $5,000,000.00$.

Practice *(15 minutes)*

1. Ask pupils to work independently to answer questions e., f., g. and h.
2. Walk around, if possible, to check the answers and clear any misconceptions.
3. Invite a volunteer to come to the board to show their solution. The rest of the class should check their solutions and correct any mistakes.

Solution:

e. Given: cost price of Le 1,250,000.00, percentage profit of 20%

$$\text{multiplier} = 1 + \frac{20}{100} = 1.2$$
$$SP = 1.2 \times CP$$
$$SP = 1.2 \times 1,250,000$$
$$= \text{Le } 1,500,000.00$$

To make a profit of 20%, the factory should sell a tank for Le $1,500,000.00$.

f. Given: cost price of Le 15,000.00 per dozen, selling price of 3 for Le 10,000.00

total selling price per dozen $= 4 \times 10,000 =$ Le $40,000.00$

$$\text{percentage profit} = \frac{SP-CP}{CP} \times 100$$
$$= \frac{40,000-15,000}{15000} \times 100$$
$$= \frac{25,000}{15000} \times 100 \qquad = 166\tfrac{2}{3}\%$$

The trader made a percentage profit of $166\frac{2}{3}\%$ (or 166.67%).

g. Given: cost price of Le 180,000.00, percentage loss of 2.5%

$$\text{multiplier} = 1 - \frac{2.5}{100} = 0.975$$
$$SP = 0.975 \times CP$$
$$SP = 0.975 \times 180,000$$
$$= \text{Le } 175,500.00$$

The fishmonger sold the fish for Le $175,500.00$.

h. Given: profit of small shop increased this year by 10% to Le 22,000,000.00.

$$\begin{aligned} \text{Let } x &= \text{Last year's profit} \\ \text{profit for this year} &= 1.1x \\ 22{,}000{,}000 &= 1.1x \\ x &= \frac{22{,}000{,}000}{1.1} = \text{Le } 20{,}000{,}000.00 \end{aligned}$$

The shop made a profit last year of Le $20{,}000{,}000.00$.

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM3-L065 in the Pupil Handbook.

[QUESTIONS]

a. John buys a set of bicycle pumps for Le 40,000.00 and sells them for Le 50,000.00. Find his percentage profit.
b. A man bought a car for Le 15,000,000.00. He later sold it for Le 12,000,000.00. What was his percentage loss on the sale of the car?
c. Akin bought a television set at the second-hand shop. He sold it for Le 2,500,000.00. If he made a profit of 25%, how much did he buy the television for?
d. A motor bike was sold for Le 4,250,000.00 at a loss of 15%. Find the cost price.
e. A factory produces water tanks. The cost of making a tank is Le 1,250,000.00. How much should the factory sell a tank if they want to make a profit of 20%?
f. A trader bought oranges at Le 15,000.00 per dozen. He sold them at 3 for Le 10,000.00. Calculate his percentage profit.
g. A fishmonger bought fish at Le 180,000.00. If she made a loss of 2.5%, for how much did she sell them?
h. The end of year profit of a small shop increased this year by 10% to Le 22,000,000.00. How much profit was made last year?

Lesson Title: Profit and loss – Part 2	**Theme:** Numbers and Numeration	
Lesson Number: M3-L066	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to calculate profit and loss on transactions by applying percentage.	**Preparation** Write the questions found at the end of this lesson plan on the board.	

Opening *(1 minute)*

1. Tell pupils that after today's lesson they will be able to calculate profit and loss on transactions by applying percentage.

Teaching and Learning *(23 minutes)*

1. Explain: Today we will focus on more complex profit and loss questions.
2. Invite a volunteer to assess question a. and tell the class what information we are given. (Answer: frozen chickens imported for Le 7,500,000.00, sold for Le 11,000,000.00, import duty = 10%, sales tax = 15%)
3. Invite another volunteer to say what we have been asked to find. (Answer: % profit)

Solution:

a. **Step 1.** Assess and extract the given information from the problem.
Given: frozen chickens imported for Le 7,500,000.00, sold for Le 11,000,000.00, import duty $= 10\%$, sales tax $= 15\%$

Step 2. Calculate the cost price.

cost price including import duty $= 7{,}500{,}000\left(1+\frac{10}{100}\right)$

$= 7{,}500{,}000 \times 1.1 \quad = \quad$ Le $8{,}250{,}000.00$

cost price including sales tax $= 8{,}250{,}000\left(1+\frac{15}{100}\right)$

$= 8{,}250{,}000 \times 1.15 \quad = \quad$ Le $9{,}487{,}500.00$

Step 3. Calculate the percentage profit

percentage profit $= \frac{SP-CP}{CP} \times 100$

percentage profit $= \frac{11{,}000{,}000-9{,}487{,}500}{9{,}487{,}500} \times 100$

$= \frac{1{,}512{,}500}{9{,}487{,}500} \times 100$

$= 16\%$

Step 4. Write the answer.
The percentage profit is 16%.

4. Invite a volunteer to assess question b. i. and tell the class what information we are given. (Answer: fishmonger bought m fish for Le 480,000.00, number rotten $= 4$, selling price $=$ Le 10,000.00 more than the cost price)
5. Invite another volunteer to say what we have been asked to find. (Answer: cost price in terms of m)

Solution:

b. Given: fishmonger bought m fish for Le 480,000.00, number rotten $= 4$, selling price = Le 10,000.00 more than cost price

i. Let cost of one fish $= y$

$$y = \frac{480,000}{m}$$

ii. number of fish sold $= m - 4$ since 4 of the m fish that were rotten

iii. selling price of one fish $= y + 10,000$

$$= \frac{480,000}{m} + 10,000$$

iv. total sum from sales $= (m - 4) \times \left(\frac{480,000}{m} + 10,000\right)$ (1)

v. profit from sales $=$ Le $120,000.00$

total sum from sales $= 480,000 + 120,000 = 600,000$ (2)

Equation (1) = Equation (2)

$$(m - 4) \times \left(\frac{480,000}{m} + 10,000\right) = 600,000$$

$$480,000 + 10,000m - \frac{1,920,000}{m} - 40,000 = 600,000$$

$$10,000m - \frac{1,920,000}{m} = 600,000 - 440000$$

$$10000m - \frac{1920000}{m} = 160,000$$

Multiply throughout by m

$$10,000m^2 - 1,920,000 = 160,000m$$

Divide throughout by 10,000

$$m^2 - 192 = 16m$$

$$m^2 - 16m - 192 = 0$$

$$(m - 16)(m + 12) = 0$$

So, $(m - 16) = 0 \Rightarrow m = 16$

$(m + 12) = 0 \Rightarrow m = -12$

We ignore $m = -12$ as quantities cannot be negative

$\therefore$ number of fishes bought $= 16$

vi. cost of one fish $= \frac{480,000}{m}$

$= \frac{480,000}{16} =$ Le $30,000.00$

The cost of one fish is Le $30,000.00$.

6. Ask pupils to work with seatmates to answer question c.
7. Invite a volunteer to show their solution on the board. The rest of the class should check their solution and correct any mistakes.

Solution:

c. Given: 10 boxes of fruit at Le 20,000.00 each, sold 4 boxes for Le 25,000.00 each, 3 boxes for Le 30,000.00 and the remainder for Le 18,000.00 each

i. total cost $= 10 \times 20,000 =$ Le $200,000.00$

number of boxes sold for Le $18,000.00$ $= 10 - (4 + 3)) = 10 - 7 = 3$

total income $= 4(25,000) + 3(30,000) + 3(18,000) =$ Le $244,000.00$

profit $= 244,000 - 200,000 =$ Le$44,000.00$

The trader made Le $44{,}000.00$ profit.

ii. average selling price $= \frac{244{,}000}{10} =$ Le $24{,}400.00$

The average selling price per box = Le $24{,}400.00$.

Practice *(15 minutes)*

1. Ask pupils to work independently to answer questions d. and e.
2. Walk around, if possible, to check for understanding and clear misconceptions.
3. Invite a volunteer to come to the board to show their solution. The rest of the class should check their solutions and correct any mistakes.

Solution:

d. Given: cost price Le 30,000.00 for 12 bottles of soft drink, 2 broken bottles, sold 10 for a profit of 5%.

i. cost of one bottle $= \frac{30{,}000}{12} =$ Le $2{,}500.00$

ii. selling price = 5% more than cost price

$= \left(1 + \frac{5}{100}\right) \times$ cost price

$= 1.05 \times 30{,}000$

$=$ Le $31{,}500.00$

selling price of one bottle $= \frac{31{,}500}{10}$ since only 10 were sold

$=$ Le $3{,}150.00$

iii profit on one bottle $= 3{,}150 - 2{,}500 =$ Le 650.00

e. Given: profit made by dealer on generator =20%, loss made by man =10% on sale price of Le 810,000.00

selling price by man = 10% less than cost price

$810{,}000 = \left(1 - \frac{10}{100}\right) \times$ cost price

$= 0.9 \times$ cost price

cost price $= \frac{810{,}000}{0.9}$

$=$ Le $900{,}000.00$

= dealer's selling price

The dealer's selling price is 20% more than the cost of the generator.

Let cost of generator $= x$

$900{,}000 =$ 20% more than the cost of the generator

$= x\left(1 + \frac{20}{100}\right)$

$= 1.2x$

$\frac{900{,}000}{1.2} = x$

$x =$ Le $750{,}000.00$

The dealer bought the generator for Le $750{,}000.00$.

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM3-L066 in the Pupil Handbook.

[QUESTIONS]

a. A shop imported frozen chicken at a cost of Le 7,500,000.00. They paid an import duty of 10% of the cost. They also paid a sales tax of 15% of the total cost of the goods including the import duty. If they sold the chicken for Le 11,000,000.00, calculate the percentage profit to the nearest whole number.

b. A fishmonger bought m fish for Le 480,000.00. She found that 4 of them were rotten. She then sold all the remaining fish. The selling price of one fish was Le 10,000.00 more than the cost price. Find in terms of m:
   i. The cost price of one fish.
   ii. The total number of fish that she sold.
   iii. The selling price of one fish.
   iv. An expression for the total sum that she received from the sale.

If she made a profit of Le 120,000 from the sales, find:
   v. The total number of fish she originally bought.
   vi. The cost price of one fish.

c. A trader bought 10 boxes of fruit at Le 20,000.00 each. She sold 4 boxes for Le 25,000.00 each, 3 boxes for Le 30,000.00 and the remainder for Le 18,000.00 each.
   i. How much profit or loss did the trader make on the boxes of fruit?
   ii. What was the average selling price per box?

d. A petty trader paid Le 30,000.00 for 12 bottles of soft drink. Two of the bottles broke. He sold the remaining 10, making a profit of 5%. Calculate:
   i. The cost price of one bottle of soft drink.
   ii. The selling price of each of the 10 remaining bottles.
   iii. The profit made on each bottle sold.

e. A dealer sold a generator to a man and made a profit of 20%. The man then sold it to his uncle for Le 810,000.00 at a loss of 10%. What was the purchase price paid by the dealer for the generator?

<table>
<tr><td>Lesson Title: Hire purchase</td><td colspan="2">Theme: Numbers and Numeration</td></tr>
<tr><td>Lesson Number: M3-L067</td><td>Class: SSS 3</td><td>Time: 40 minutes</td></tr>
<tr><td>Learning Outcome
By the end of the lesson, pupils will be able to calculate hire purchase based on percentages.</td><td colspan="2">Preparation
Write the questions found at the end of this lesson plan on the board.</td></tr>
</table>

Opening *(1 minute)*

1. Tell pupils that after today's lesson, they will be able to calculate hire purchase based on percentages.

Teaching and Learning *(23 minutes)*

1. Explain:
   - There are instances when an item is bought and the full amount is paid for in regular instalments over several months or years.
   - Since the item is being paid for over time, it usually costs more than the cash price when bought outright.
   - This is because interest is usually added to the price of the item being sold.
   - In many cases, a deposit is paid for the item so that the buyer can make use of it right away. However, the item does not belong to the buyer until it has been paid in full.
   - The interest charged can be calculated using the simple interest rate based on the length of the loan. However, more complicated formulas are used to calculate the interest on hire purchase loans.
   - We will use the average time for the loan in the simple interest formula as it gives a good approximation of the interest.
2. Invite a volunteer to assess question a. and tell the class what information we are given. (Answer: Given: Le 25,000,000.00 at a SI rate of 15% per annum for 2 years)
3. Invite another volunteer to say what we have been asked to find. (Answer: the interest paid.)

   Solution:

   a. **Step 1.** Assess and extract the given information from the problem.
      Given: Le 25,000,000.00, simple interest rate 15% per annum for 2 years.

      Step 2. Calculate the interest.

      remainder to be paid $= 25{,}000{,}000 - 5{,}000{,}000 = 20{,}000{,}000$

      $$I = \frac{PRT}{100}$$

      $$I = \frac{20{,}000{,}000 \times 15 \times 2}{100}$$ use $T =$ length of loan

      $=$ Le $6{,}000{,}000.00$

      Step 3. Write the answer.

The interest received by Mr. Kargbo is Le $6,000,000.00$.

4. Invite a volunteer to assess question b. and tell the class what information we are given. (Answer: deposit 30% of the cash price, 4 monthly instalments charged at a simple interest rate of 20% on the remainder, cash price is Le 2,250,000.00)
5. Invite another volunteer to say what we have been asked to find. (Answer: the remainder on which interest is charged and the monthly instalments)

   Solution:

   b. Given: deposit 30% of the cash price, 4 monthly instalments charged at a simple interest rate of 20% on the remainder, cash price is Le 2,250,000.00

   i. remainder $= \left(1 - \frac{30}{100}\right) \times 2{,}250{,}000 =$ Le $1{,}575{,}000.00$

   ii. average time, $T = \frac{1+4}{2}$ average time is used as explained before

   $= \frac{5}{2} =$ 2.5 months $= \frac{2.5}{12}$ years

   $I = \frac{PRT}{100} = \frac{1{,}575{,}000 \times 20 \times 2.5}{100 \times 12}$

   $=$ Le $65{,}625.00$

   total cost over 4 months $= 1{,}575{,}000 + 65{,}625$

   monthly instalment $= \frac{1{,}640{,}625}{4}$

   $=$ Le $410{,}156.25.00$

   Interest is charged on Le $1{,}575{,}000.00$ with 4 monthly instalments of Le $410{,}156.25.00$.

6. Ask pupils to work with seatmates to answer question c.
7. Invite a volunteer to show their solution on the board. The rest of the class should check their solution and correct any mistakes.

   Solution:

   c. Given: cost of refrigerator is Le 2,000,000.00, amount deposited Le 500,000.00, balance paid in 12 monthly instalments of Le 150,000.00.

   i. total for instalment $= 150{,}000 \times 12 =$ Le $1{,}800{,}000.00$

   total cost $= 500{,}000 + 1{,}800{,}000 =$ Le $2{,}300{,}000.00$

   ii. interest paid $= 2{,}300{,}000 - 2{,}000{,}000 =$ Le $300{,}000.00$

   iii. **average time,** $T = \frac{1+12}{2}$

   $= \frac{13}{2} =$ 6.5 months $= \frac{6.5}{12}$ years

   $R = \frac{I \times 100}{PT}$

   $R = \frac{300{,}000 \times 100 \times 12}{2{,}000{,}000 \times 6.5}$

   $= 27.69\%$

   The total cost paid by Miss Koroma is Le $2{,}300{,}000.00$; interest paid is Le $300{,}000.00$ and the approximate interest rate is 27.7% to 1 d.p.

Practice *(15 minutes)*

1. Ask pupils to work independently to answer questions d. and e.
2. Walk around, if possible, to check the answers and clear any misconceptions.

3. Invite a volunteer to come to the board to show their solution. The rest of the class should check their solutions and correct any mistakes.

Solution:

d. Given: cost of oven = Le 1,687,500.00 = 12.5% more than cash price, initial deposit = 20% of cash price, balance in 6 monthly instalments

i. $\left(1+\frac{12.5}{100}\right)\times$ cash price of oven $= 1{,}687{,}500$

$1.125 \times$ cash price of oven $= 1{,}687{,}500$

cash price of oven $= \frac{1{,}687{,}500}{1.125}$ = Le $1{,}500{,}000.00$

initial deposit $= \frac{20}{100}\times 1{,}500{,}000$ = Le $300{,}000.00$

ii. **remainder to be paid** $= 1{,}500{,}000 - 300{,}000$ = Le $1{,}200{,}000.00$

amount per instalment $= \frac{1{,}200{,}000}{6}$ = Le $200{,}000.00$

iii. **average time,** $T = \frac{1+6}{2}$

$= \frac{7}{2}$ = 3.5 months $= \frac{3.5}{12}$ years

interest, $I = 1{,}687{,}500 - 1{,}500{,}000$ = Le $187{,}500.00$

$R = \frac{I\times 100}{PT}$

$R = \frac{187{,}500\times 100\times 12}{1{,}200{,}000\times 3.5}$

$= 53.57\%$

Mrs. Mansaray paid an initial deposit of Le $300{,}000.00$; the monthly instalment is Le $200{,}000.00$ and approximate interest rate is 53.6% to 1 d.p.

e. Given: deal from shop – deposit 50% of the cash price, pay remainder in 12 monthly instalments

i. For cash price of Le 3,000,000.00 and monthly instalments of Le 150,000.00

deposit = 50% of cash price $= \frac{50}{100}\times 3{,}000{,}000$

= Le $1{,}500{,}000.00$

total for instalments $= 150{,}000 \times 12$ = Le $1{,}800{,}000.00$

total cost $= 1{,}500{,}000 + 1{,}800{,}000$

= Le $3{,}300{,}000.00$

ii. interest paid $= 3{,}300{,}000 - 3{,}000{,}000$ = Le $300{,}000.00$

iii. average time, $T = \frac{1+12}{2} = \frac{13}{2}$

= 6.5 months $= \frac{6.5}{12}$ years

$R = \frac{I\times 100}{PT}$

$R = \frac{300{,}000\times 100\times 12}{1{,}500{,}000\times 6.5}$ $\quad P = 1{,}500{,}000$ is the remaining amount

$= 36.92\%$

The deposit is Le $1{,}500{,}000.00$, with interest paid of Le $300{,}000.00$ at an approximate interest rate of 36.9% to 1 d.p.

Closing *(1 minute*)

1. For homework, have pupils do the practice activity PHM3-L067 in the Pupil Handbook.

[QUESTIONS]

a. Mr. Kargbo wants to buy a car on sale at Le 25,000,000.00 cash. He paid Le 5,000,000.00 deposit and 15% simple interest charged on the remainder for 2 years. How much interest did he pay?
b. A retailer offers the following hire purchase terms on generators:
   Deposit 30% of the cash price, then 4 monthly instalments charged at a simple interest rate of 20% on the remainder. If the cash price is Le 2,250,000.00, find:
   i. The remainder on which interest is charged.
   ii. The monthly instalments.
c. The cash price for a refrigerator is Le 2,000,000.00. Miss Koroma paid a deposit of Le 500,000.00 for the refrigerator. If the balance was paid in 12 monthly instalments of Le 150,000.00, find:
   i. The total amount paid for the refrigerator.
   ii. The interest charged.
   iii. The approximate rate of interest to 1 decimal place.
d. Mrs. Mansaray bought an oven on hire purchase for Le 1,687,500. She paid 12.5% more than if she had paid cash for the oven. If she made an initial deposit of 20% of the cash price and then paid the rest in 6 monthly instalments, find:
   iii. The initial deposit.
   iv. The amount of each instalment.
   v. The approximate rate of interest to 1 decimal place.
e. A shop selling television sets has the following deal on offer:
   Deposit 50% of the cash price and pay the rest in 12 monthly instalments.
   If the cash price is Le 3,000,000.00, and the monthly instalments is Le 150,000.00:
   i. How much will a buyer pay for the television set?
   ii. What is the interest charged on the television set?
   iii. What is the approximate rate of interest? Give your answer to 1 decimal place.

Lesson Title: Discount	Theme: Numbers and Numeration	
Lesson Number: M3-L068	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to calculate discount on a transaction by applying percentage.	**Preparation** Write the questions found at the end of this lesson plan on the board.	

Opening *(1 minute)*

1. Tell pupils that after today's lesson, they will be able to calculate discount on a transaction by applying percentage.

Teaching and Learning *(23 minutes)*

1. Explain:
   - A discount is given in shops when customers buy in bulk or when there is a special offer.
   - The discount is usually given as a percentage of the original price.
   - The original price is 100% or 1 $\left(\frac{100}{100}\right)$.
   - We use a multiplier which is given by $1 - \frac{R}{100}$ where R is the percentage discount.
2. Invite a volunteer to assess question a. and tell the class what information we are given. (Answer: Given: cost for gas cooker is Le1,250,000.00 with a 20% discount offer)
3. Invite another volunteer to say what we have been asked to find. (Answer: how much the customer pays)

   Solution:

   a. **Step 1.** Assess and extract the given information from the problem.

   Given: cost for gas cooker is Le 1,250,000.00 with 20% discount offer.

   Step 2. Calculate how much the customer pays.

   $$\begin{aligned} \text{multiplier} &= 1 - \frac{20}{100} = 1 - 0.2 \\ &= 0.8 \\ \text{amount paid} &= 0.8 \times 1{,}250{,}000 = \text{Le } 1{,}000{,}000.00 \end{aligned}$$

   Step 3. Write the answer.

   The customer pays Le 1,000,000.00
4. Invite a volunteer to assess question b. and tell the class what information we are given. (Answer: cost price Le 250,000.00, discounted price Le 212,500.00)
5. Invite another volunteer to say what we have been asked to find. (Answer: percentage discount)

   Solution:

   b. Given: cost price Le 250,000.00, discounted price Le 212,500.00

   $$\text{discount} = 250{,}000 - 212{,}500 = \text{Le } 37{,}500.00$$

percentage discount $= \frac{37,500}{250,000} \times 100$

$= 15\%$

The percentage discount is 15%.

6. Invite a volunteer to assess question c. and tell the class what information we are given. (Answer: Given: cost price Le 4,500,000.00, percentage discount $33\frac{1}{3}\%$)
7. Invite another volunteer to say what we have been asked to find. (Answer: discount)

Solution:

c. Given: cost price Le 4,500,000.00, percentage discount $33\frac{1}{3}\%$

i. percentage discount $= 33\frac{1}{3}\%$

$= \frac{1}{3}$ since $33\frac{1}{3}\% = \frac{100}{3}\% = \frac{\frac{100}{3}}{100} = \frac{100}{3\times100} = \frac{1}{3}$

discount $= \frac{1}{3} \times 4,500,000$

$=$ Le $1,500,000.00$

ii. amount paid $= 4,500,000 - 1,500,000$

$=$ Le $3,000,000.00$

The buyer pays Le $3,000,000.00$.

8. Ask pupils to work with seatmates to answer question d.
9. Invite a volunteer to show their solution on the board. The rest of the class should check their solution and correct any mistakes.

Solution:

d. Given: cost per book is Le 1,500.00, 15% discount for more than 500 books, 20% discount for more than 1,000 books.

i. 750 books: discount per book $= \frac{15}{100} \times 1,500 =$ Le 225.00

amount saved $= 225 \times 750$

$=$ Le $168,750.00$

ii. 1,250 books discount per book $= \frac{20}{100} \times 1,500 =$ Le 300.00

amount saved $= 300 \times 1,250$

$=$ Le $375,000.00$

The school saved Le $168,750.00$ when they bought 750 books and Le $375,000.00$ when they bought 1,250 books.

Practice *(15 minutes)*

1. Ask pupils to work independently to answer questions e., f., g. and h.
2. Walk around, if possible, to check the answers and clear misconceptions.
3. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.

Solutions:

e. Given: 5% discount on all items in the shop

multiplier $= 1 - \frac{5}{100} = 1 - 0.05$

$= 0.95$

i. bag of flour at Le 55,000.00:

$$\text{new price} = 0.95 \times 55{,}000 = \text{Le } 52{,}250.00$$

ii. bag of onions at Le 75,000.00

$$\text{new price} = 0.95 \times 75{,}000 = \text{Le } 71{,}250.00$$

iii. box of tomato puree at Le 60,000.00

$$\text{new price} = 0.95 \times 60{,}000 = \text{Le } 57{,}000.00$$

The new prices are Le $52{,}250.00$ for a bag of flour, Le $71{,}250.00$ for a bag of onions and Le $57{,}000.00$ for a box of tomato puree.

f. Given: discounted price Le 2,800,000.00, discount 20%

$$\text{Let original price} = x$$
$$\text{multiplier} = 1 - \frac{20}{100} = 1 - 0.2$$
$$= 0.8$$
$$0.8 \times x = 2{,}800{,}000$$
$$x = \frac{2{,}800{,}000}{0.8}$$
$$x = \text{Le } 3{,}500{,}000.00$$

The original price of the television set was Le $3{,}500{,}000.00$.

g. Given: original discount is 15%, further additional 10% discount, cost of item is Le 48,000.00

i.
$$\text{multiplier} = 1 - \frac{15}{100} = 1 - 0.15$$
$$= 0.85$$
$$\text{discounted price} = 0.85 \times 48{,}000 = \text{Le } 40{,}800.00$$
$$\text{multiplier} = 1 - \frac{10}{100} = 1 - 0.1$$
$$= 0.9$$
$$\text{new price} = 0.9 \times 40{,}800$$
$$= \text{Le } 36{,}720.00$$

ii.
$$\%\text{ profit lost} = \frac{48{,}000 - 36{,}720}{48{,}000} \times 100$$
$$= \frac{11{,}280}{48{,}000} \times 100$$
$$= 23.5\%$$

The retailer sells the item for Le $36{,}720.00$ at a percentage loss of 23.5%.

h. Given: Le 52,000.00 per bag for the first 12 bags and Le 50,000.00 per bag for any additional bags bought.

$$\text{first 12 bags} = 12 \times 52{,}000 = \text{Le } 624{,}000.00$$
$$\text{remainder} = 8 \times 50{,}000 = \text{Le } 400{,}000.00$$
$$\text{total cost} = 624{,}000 + 400{,}000$$
$$= \text{Le } 1{,}024{,}000.00$$

20 bags of cement cost Le $1{,}024{,}000.00$.

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM3-L068 in the Pupil Handbook.

[QUESTIONS]

a. A gas cooker costs Le 1,250,000.00. If the shop offers a customer 20% discount, how much will the customer pay for the cooker?
b. What percentage discount was given on an item reduced from Le 250,000.00 to Le 212,500.00?
c. A motor bike costs Le 4,500,000.00. The shop gives a discount of $33\frac{1}{3}$% for cash.
    iv. How much will a buyer save by paying cash for the motor bike?
    v. How much will the buyer pay for the motor bike?
d. A school buys exercise books from a supplier. He gives the school 15% discount if they buy more than 500 and 20% discount for buying over 1,000 books. If each book costs Le 1,500.00, how much will they save if they buy:
    i. 750 books ii. 1,250 books?
e. A shop decided to reduce all its price by 5% for a month. What is the new price of each of the items below?
    iv. Bag of flour, Le 55,000.00.
    v. Bag of onions, Le 75,000.00.
    vi. Box of tomato puree, Le 60,000.00.
f. Mrs. Davies bought a television set reduced by 20% What was the original price if the discounted price was Le 2,800,000.00?
g. A retailer discounted her prices by 15% for a month. She then gave a further 10% off the discounted price.
    i. How much will an item originally costing Le 48,000.00 now cost?
    ii. How much percentage profit will she lose by selling at this price?
h. A building materials store sells cement for Le 52,000.00 per bag for the first 12 bags and Le 50,000.00 per bag for any additional bags bought. How much will you pay for 20 bags of cement?

Lesson Title: Depreciation	**Theme:** Numbers and Numeration	
Lesson Number: M3-L069	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to calculate depreciation using percentages.	**Preparation** Write the questions found at the end of this lesson plan on the board.	

Opening *(1 minute)*

1. Tell pupils that after today's lesson, they will be able to calculate depreciation by applying percentage.

Teaching and Learning *(23 minutes)*

1. Explain:
   - Many goods lose their value over time as they get older and are no longer in prime condition. Examples are cars, computers, mobile phones and most electrical appliances.
   - This decrease in value is called **depreciation**.
   - In a previous lesson, we looked at calculating compound interest using a formula.
2. Invite a volunteer to give the formula. (Answer: $A = P\left(1 + \frac{R}{100}\right)^n$)
3. **Explain:**
   - We can use a similar formula to calculate depreciation.
   - With compound interest the value appreciates or increases over time, but with depreciation it depreciates, or decreases, over time.
4. Ask pupils to discuss how they think the formula will change.
5. Invite a volunteer to give the answer. (Example answer: for depreciation, the percentage rate will be subtracted)
6. Write on the board:
   - The value at the end of a particular time period is given by:

     $V = P\left(1 - \frac{R}{100}\right)^n$ where V = Value at the end of the period
     R = rate of depreciation
     P = Original price
     n = Period
   - The rate of depreciation can be found using the formula:

     $R = \frac{P-V}{P} \times 100$
7. Invite a volunteer to assess question a. and tell the class what information we are given. (Answer: original cost of car = Le 25,000,000.00, depreciates at 20% per annum)
8. Invite another volunteer to say what we have been asked to find. (Answer: value at the end of 1 and 3 years)

Solution:

a. **Step 1.** Assess and extract the given information from the problem.
Given: original cost of car = Le 25,000,000.00, depreciates at 20% per annum.

Step 2. Calculate the value at the end of each period.

$$V = P\left(1-\frac{R}{100}\right)^n$$

i. 1 year: $V = 25{,}000{,}000\left(1-\frac{20}{100}\right)^1$
$= \text{Le } 20{,}000{,}000.00$

ii. 3 years: $V = 25{,}000{,}000\left(1-\frac{20}{100}\right)^3$
$= \text{Le } 12{,}800{,}000.00$

Step 3. Write the answers.
The value after 1 year is Le 20,000,000.00.
The value after 3 years is Le 12,800,000.00.

9. Invite a volunteer to assess question b. and tell the class what information we are given. (Answer: motor bike costs Le 4,500,000.00, depreciates 18% the first year and 15% the second and subsequent years)
10. Invite another volunteer to say what we have been asked to find. (Answer: value and average rate of depreciation after 5 years)

Solution:

b. Given: motor bike costs Le 4,500,000.00, depreciates 18% the first year and 15% the second and subsequent years.

$$V = P\left(1-\frac{R}{100}\right)^n$$

i. After 1 year, $V = 4{,}500{,}000\left(1-\frac{18}{100}\right)^1 = \text{Le } 3{,}690{,}000.00$

After 4 more years, $V = 3{,}690{,}000\left(1-\frac{15}{100}\right)^4 = \text{Le } 1{,}926{,}203.06$

ii. average rate $= \frac{18+15}{2} = 16.5\%$

After 5 years, the motor bike is worth Le 1,926,203.06.
The average rate of depreciation is 16.5%.

11. Ask pupils to work with seatmates to answer question c.
12. Invite a volunteer to show their solution on the board.
13. The rest of the class should check their solution and correct any mistakes.

Solution:

c. Given: gas cooker depreciates 15% per annum, value after 2 years is Le 614,125.00.

$$\begin{aligned} V &= P\left(1-\frac{R}{100}\right)^n \\ 614{,}125 &= P\left(1-\frac{15}{100}\right)^2 \\ &= P \times 0.85^2 \\ 614{,}125 &= 0.7225P \\ P &= \frac{614{,}125}{0.7225} \\ &= \text{Le } 850{,}000.00 \end{aligned}$$

The original price of the gas cooker was Le $850{,}000.00$.

Practice *(15 minutes)*

1. Ask pupils to work independently to answer questions d., e. and f.
2. Walk around, if possible, to check for understanding and clear misconceptions.
3. Invite a volunteer to come to the board to show their solution. The rest of the class should check their solutions and correct any mistakes.

   Solution:

   d. Given: refrigerator costs Le 2,250,000.00 new, Le 1,845,000.00 after 1 year.

$$\begin{aligned} R &= \frac{P-V}{P} \times 100 \\ R &= \frac{2{,}250{,}000-1{,}845{,}000}{2{,}250{,}000} \times 100 \\ &= \frac{405{,}000}{2{,}250{,}000} \times 100 \\ R &= 18\% \end{aligned}$$

   The rate of depreciation is 18%.

   e. Given: rate of depreciation is 15%, initial cost of car = Le 30,000,000.00, kept for 4 years

   i.

$$\begin{aligned} V &= P\left(1-\frac{R}{100}\right)^n \\ V &= 30{,}000{,}000\left(1-\frac{15}{100}\right)^4 \\ &= \text{Le } 15{,}660{,}187.50 \end{aligned}$$

   ii.

$$\begin{aligned} \text{selling price as percentage} &= \frac{15{,}660{,}187.50}{30{,}000{,}000} \times 100 \\ &= 52.2\% \end{aligned}$$

   The car cost Le $15{,}660{,}187.50$ after 4 years, 52.2% of the original price.

   f. Given: computer costs Le 2,500,000.00, depreciates by 20% in the first year, 15% the second year and 12% the third year.

$$V = P\left(1-\frac{R}{100}\right)^n$$

   i.

$$\begin{aligned} \text{After 1 year, } V &= 2{,}500{,}000\left(1-\frac{20}{100}\right)^1 = \text{Le } 2{,}000{,}000.00 \\ \text{After 2 years, } V &= 2{,}000{,}000\left(1-\frac{15}{100}\right)^1 = \text{Le } 1{,}700{,}000.00 \\ \text{After 3 years, } V &= 1{,}700{,}000\left(1-\frac{12}{100}\right)^1 = \text{Le } 1{,}496{,}000.00 \end{aligned}$$

   The computer's value after 3 years is Le $1{,}496{,}000.00$

   ii.

$$\begin{aligned} \text{percentage loss} &= \frac{CP-SP}{CP} \times 100 \\ \text{percentage loss} &= \frac{2{,}500{,}000-1{,}496{,}000}{2{,}500{,}000} \times 100 \\ &= \frac{1{,}004{,}000}{2{,}500{,}000} \times 100 \\ &= 40.16\% \end{aligned}$$

   The computer sold at a loss of 40%.

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM3-L069 in the Pupil Handbook.

[QUESTIONS]

a. A car costs Le 25,000,000.00. It depreciates by 20% per annum. Find its value after:
   i. 1 year
   ii. 3 years
b. A motor bike costs Le 4,500,000.00. Its value depreciates by 18% the first year and 15% in the second and subsequent years.
   i. What is its value at the end of 5 years? Give your answer to 2 decimal places.
   ii. What was the average rate of depreciation over the 5 years?
c. A gas cooker depreciates at a rate of 15% per annum. If its value after 2 years is Le 614,125.00, what was its original price?
d. Find the rate of depreciation of a refrigerator which costs Le 2,250,000.00 to buy new and is valued at Le 1,845,000.00 after 1 year.
e. The value of a car depreciates at 15% per annum. A man keeps a car for 4 years and then sells it. If the car initially costs Le 30,000,000.00 find:
   i. Its value after 4 years.
   ii. The selling price as a percentage of the original value.
f. A computer costs Le 2,500,000.00. Its value depreciates by 20% the first year, 15% the second year and 12% the third year.
   i. What is its value at the end of the third year?
   ii. If the owner decides to sell it at that price, what is the percentage loss on the original price to the nearest whole number?

Lesson Title: Financial partnerships	**Theme:** Numbers and Numeration	
Lesson Number: M3-L070	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to calculate financial partnerships using percentage.	**Preparation** Write the questions found at the end of this lesson plan on the board.	

Opening *(1 minute)*

1. Tell pupils that after today's lesson, they will be able to calculate financial partnership using percentage.

Teaching and Learning *(20 minutes)*

1. Explain:
   - When 2 or more people come together and invest money for the purpose of providing goods or services at a profit, it is called a financial or business partnership.
   - Partnerships are usually formed by professionals such as lawyers, doctors, architects and engineers who wish to pool their resources together.
   - In many instances, the partners pay out profit in proportion to the money or capital invested.
2. Invite a volunteer to assess question a. i. and tell the class what information we are given. (Answer: Given: Mustapha and Ahmed invested Le 15,000,000.00 in a business in the ratio 3 : 2 respectively)
3. Invite another volunteer to say what we have been asked to find. (Answer: each partner's investment)

   Solution:

   a. **Step 1.** Assess and extract the given information from the problem.
   Given: Mustapha and Ahmed invested Le 15,000,000.00 in a business in the ratio 3 : 2 respectively.

   Step 2. Calculate each partner's investment.

   i. total number of parts $= 3 + 2 = 5$
   Mustapha's investment $= \frac{3}{5} \times 15{,}000{,}000 =$ Le $9{,}000{,}000.00$
   Ahmed's investment $= \frac{2}{5} \times 15{,}000{,}000 =$ Le $6{,}000{,}000.00$

   Step 3. Calculate each partner's profit.

   ii. Mustapha's profit $= \frac{3}{5} \times 6{,}000{,}000 =$ Le $3{,}600{,}000.00$
   Ahmed's profit $= \frac{2}{5} \times 6{,}000{,}000 =$ Le $2{,}400{,}000.00$

   Step 4. Write the answers.
   Mustapha invested Le $9{,}000{,}000.00$ and made a profit of Le $3{,}600{,}000.00$.
   Ahmed invested Le $6{,}000{,}000.00$ and made a profit of Le $2{,}400{,}000.00$.

4. Invite a volunteer to assess question b. and tell the class how much each sister contributed. (Answer: Kemi contributed Le 5,600,000.00, Yemi contributed Le 2,400,000.00)
5. Invite another volunteer to say what we have been asked to find in b. i. (Answer: the amount reserved for re-investment)

Solution:

b. Given: Kemi contributed Le 5,600,000.00, Yemi contributed Le 2,400,000.00,

i. total contribution $= 5{,}600{,}000 + 2{,}400{,}000 =$ Le $8{,}000{,}000.00$

profit $= \frac{70}{100} \times 8{,}000{,}000 =$ Le $5{,}600{,}000.00$

amount reserved for re-investment $= \frac{20}{100} \times 5{,}600{,}000 =$ Le $1{,}120{,}000.00$

The amount reserved for re-investment is Le $1{,}120{,}000.00$.

ii. remaining profit $= 5{,}600{,}000 - 1{,}120{,}000 =$ Le $4{,}480{,}000.00$

amount paid into a trust fund $= \frac{2.5}{100} \times 4{,}480{,}000 =$ Le $112{,}000.00$

The amount paid into a trust fund is Le $112{,}000.00$.

iii. remaining profit $= 4{,}480{,}000 - 112{,}000 =$ Le $4{,}368{,}000.00$

ratio of contribution $= \frac{5{,}600{,}000}{2{,}400{,}000} = \frac{7}{3}$

total number of parts $= 7 + 3 = 10$

Kemi's share $= \frac{7}{10} \times 4{,}368{,}000 =$ Le $3{,}057{,}600.00$

Yemi's share $= \frac{3}{10} \times 4{,}368{,}000 =$ Le $1{,}310{,}400.00$

iv. percentage share for Kemi $= \frac{3{,}057{,}600}{5{,}600{,}000} \times 100 = 54.6\%$

percentage share for Femi $= \frac{1{,}310{,}400}{2{,}400{,}000} \times 100 = 54.6\%$

Kemi's share of the profit is Le $3{,}057{,}600.00$ which is 54.6% of her contribution.

Yemi's share of the profit is Le $1{,}310{,}400.00$ which is 54.6% of her contribution.

6. Ask pupils to work with seatmates to answer question c.
7. Invite a volunteer to show their solution on the board. The rest of the class should check their solution and correct any mistakes.

Solution:

c. Given: Peter, John and Mary invested Le 3,000,000.00, Le 2,400,000.00 and Le 3,600,000.00 respectively

i. ratio of contribution

Peter: John: Mary $= 3{,}000{,}000 : 2{,}400{,}000 : 3{,}600{,}000$

divide throughout by 600,000

Peter : John : Mary $= 5 : 4 : 6$

total ratio $= 5 + 4 + 6 = 15$

Peter's share of the profit $= \frac{5}{15} \times 2{,}700{,}000 =$ Le $900{,}000.00$

John's share of profit $= \frac{4}{15} \times 2{,}700{,}000 =$ Le $720{,}000.00$

Mary's share of profit $= \frac{6}{15} \times 2{,}700{,}000 =$ Le $1{,}080{,}000.00$

ii. percentage share for Peter $= \frac{900{,}000}{3{,}000{,}000} \times 100 = 30\%$

Practice *(18 minutes)*

1. Ask pupils to work independently to answer question d.
2. Walk around, if possible, to check for understanding and clear misconceptions.
3. Invite a volunteer to come to the board to show their solution. The rest of the class should check their solutions and correct any mistakes.

Solution:

d. Given: total capital invested of Le 45,000,000.00 by Mr. Koroma and Mr. Kamara in the ratio 2 : 1 respectively.

i. total number of parts $= 2+1 = 3$

Mr. Koroma's contribution $= \frac{2}{3} \times 45{,}000{,}000 =$ Le $30{,}000{,}000.00$

Mr. Kamara's contribution $= \frac{1}{3} \times 45{,}000{,}000 =$ Le $15{,}000{,}000.00$

Let total profit $= x$

Mr. Koroma's share of the profit

6% as manager $= \frac{6}{100} \times x = 0.06x$

4% of investment $= \frac{4}{100} \times 30{,}000{,}000 =$ Le $1{,}200{,}000.00$

Mr. Kamara's share of profit

4% of investment $= \frac{4}{100} \times 15{,}000{,}000 =$ Le $600{,}000.00$

profit shared so far $= 0.06x + 1{,}200{,}000 + 600{,}000$
$= 0.06x + 1{,}800{,}000$

remaining profit $= x - (0.06x + 1{,}800{,}000)$
$= 0.94x - 1{,}800{,}000$

The remaining profit is shared in the ratio of the partners' investments.

Mr. Koroma's share $= \frac{2}{3}(0.94x - 1{,}800{,}000) = 0.626x - 1{,}200{,}000$

Mr. Kamara's share $= \frac{1}{3}(0.94x - 1{,}800{,}000) = 0.313x - 600{,}000$

Mr. Koroma's share of the total profit $=$ Le $4{,}000{,}000.00$

$4{,}000{,}000 = 0.06x + 1{,}200{,}000 + 0.626x - 1{,}200{,}000$

$4{,}000{,}000 = 0.686x$

$x = \frac{4{,}000{,}000}{0.686} =$ Le $5{,}830{,}903.79$

The total profit to the nearest thousand Leones is Le $5{,}831{,}000.00$

ii. Mr. Kamara's share of the total profit $= 5{,}831{,}000 - 4{,}000{,}000$
$=$ Le $1{,}831{,}000.00$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM3-L070 in the Pupil Handbook.

[QUESTIONS]

a. Mustapha and Ahmed invested Le 15,000,000.00 in a business in the ratio 3 : 2 respectively.

i. How much did each partner invest?
ii. If they shared Le 6,000,000.00 profit in the same ratio as their investments, how much did each receive?

b. Two sisters, Kemi and Yemi, entered into a business partnership. Kemi contributed Le 5,600,000.00 and Yemi contributed Le 2,400,000.00. At the end of the year, they made a profit of 70% of their total contribution. Twenty percent of the profit was reserved for re-investment and 2.5% of the remaining profit was paid into a trust fund for their children. If they shared the remaining profit in the ratio of their contributions, find:
   i. The amount reserved for re-investment.
   ii. The amount paid into the trust fund.
   iii. The amount received by each partner as her share of the profit.
   iv. Each sister's share as a percentage of her contribution.
c. Peter, John and Mary invested Le 3,000,000.00, Le 2,400,000.00 and Le 3,600,000.00 respectively in a small shop.
   i. How should they share a profit of Le 2,700,000.00 if they agree to share it in proportion to their investments?
   ii. Express Peter's profit as a percentage of his investment.
d. Mr. Koroma and Mr. Kamara entered into a financial partnership with a total capital of Le 45,000,000.00. They agreed to contribute capital in the ratio 2 : 1 respectively. The profit was shared as follows: Mr. Koroma was paid 6% of the total profit for his services as a manager. Each partner was paid 4% of the capital he invested. The remainder of the profit was then shared in the ratio of the capital invested. If Mr. Koroma's share of the total profits was Le 4,000,000.00, find:
   i. The total profit for the year to the nearest thousand Leones.
   ii. Mr. Kamara's share of the total profits.

Lesson Title: Foreign exchange	Theme: Numbers and Numeration	
Lesson Number: M3-L071	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to convert one type of currency to another based on given rates using ratio and proportion.	**Preparation** 1. Write the tables below on the board. 2. Write the questions found at the end of this lesson plan on the board.	

Opening *(1 minute)*

1. Tell pupils that after today's lesson, they will be able to convert one type of currency to another based on given rates using ratio and proportion.

Teaching and Learning *(20 minutes)*

1. Explain:
   - Every country has its own currency which it uses for its money.
   - The table on the right gives some countries and currencies.
   - The exchange rate is the rate at which one unit of a particular currency is converted to another currency.
   - There are usually two rates – the **buying** and the **selling** rate.
   - The table shows the buying and selling rates in a bank for various currencies on a particular day.
   - The bank buys from customers at the buying rate and sells at the selling rate.
   - The selling rate is higher than the buying rate. This allows the bank to make a profit in trading in the currency.

Country	Currency	Symbol
Ghana	Cedi	GH¢
Gambia	Dalasi	D
Germany	Euro	€
Great Britain	Pounds	£
Nigeria	Naira	₦
Sierra Leone	Leones	Le
United Sates	Dollars	$

Currency	Buying	Selling
€ 1.00	Le 8,600	Le 8,900
GH¢ 1.00	Le 1,500	Le 1,560
GMD 1.00	Le 150	Le 156
₦ 1.00	Le 20	Le20.80
£ 1.00	Le 9,500	Le 9,800
$ 1.00	Le 7,600	Le 7,900

2. Invite a volunteer to assess question a. i. and tell the class what the given information is and what we have to do. (Answer: given: Le 5,000,000.00 to buy US $, GB £ and Gambia D)

   Solution:

   a. **Step 1.** Assess and extract the given information from the problem.
   Given: Le 5,000,000.00 to buy US $, GB £ and Gambia D

   Step 2. Use the unitary method and conversion rate to calculate the amount in the required currency.

   i.
$$\text{Le } 7{,}900.00 = \$1.00$$
$$\text{Le } 1.00 = \$\tfrac{1.00}{7{,}900}$$
$$\text{Le } 5{,}000{,}000.00 = \$\tfrac{1.00}{7900} \times 5{,}000{,}000$$
$$= \$632.91$$

   ii.
$$\text{Le } 9{,}800.00 = £1.00$$

Le $5,000,000.00$ = £$\frac{1.00}{9,800} \times 5,000,000$
= £510.20

iii. Le 156.00 = Gambian D1.00
Le $5,000,000.00$ = $\frac{1.00}{156} \times 5,000,000$
= Gambian D$32,051.28$

Step 3: Write the answer.

Le $5,000,000.00$ buys \$$632.91$, £$510.20$ and Gambian D$32,051.28$.

3. Invite a volunteer to assess question b. and tell the class what information we are given. (Answer: bank buys then sells \$500.00)
4. Invite another volunteer to say what we have been asked to find in b. i. (Answer: bank's profit.)

Solution:

b. Given: bank buys then sells \$500.00

\$$1.00$ = Le $7,600.00$ buying rate
\$$500.00$ = $7,600 \times 500$ = Le $3,800,000.00$
\$$1.00$ = Le $7,900.00$ selling rate
\$$500.00$ = $7,900 \times 500$ = Le $3,950,000.00$
profit = $3,950,000 - 3,800,000$
= Le $150,000.00$

The bank makes Le $150,000.00$ profit.

5. Invite a volunteer to assess question c. and tell the class what information we are given. (Answer: Olu received £20.00 for his birthday)
6. Invite another volunteer to say what we have been asked to find in b. i. (Answer: amount in Leones)

Solution:

c. Given: Olu received £20.00 for his birthday

£1.00 = Le $9,500.00$ bank buys from Olu at Le 9,500.00
£20.00 = $20 \times 9,500$ = Le $190,000.00$

Olu receives Le $190,000.00$ for his £20.00.

7. Invite a volunteer to assess question d. and tell the class what information we are given. (Answer: Mr. Samuels wants to order goods costing GH¢45,000)
8. Invite another volunteer to say what we have been asked to find in b. i. (Answer: the cost in Leones.)

Solution:

d. Given: Mr. Samuels wants to order goods costing GH¢45,000.

GH¢1.00 = Le $1,560.00$ bank sells to Mr. Samuels
GH¢$45,000$ = $45,000 \times 1,560$ = Le $70,200,000.00$

It costs Mr. Samuels Le $70,200,000$ for GH¢45,000.

9. Ask pupils to work with seatmates to answer question e.
10. Invite a volunteer to show their solution on the board.
The rest of the class should check their solution and correct any mistakes.

Solution:

e. Given: Memuna wants to change Le 15,600,000.00 for her visit to Nigeria.

$₦1.00 = \text{Le } 20.80$ bank sells to Memuna

$\text{Le } 15,600,000.00 = \frac{15,600,000}{20.80} = ₦750,000.00$

Memuna changes Le 15,600,000.00 for ₦750,000.00**.**

Practice *(15 minutes)*

1. Ask pupils to work independently to answer question f., g., and h.
2. Walk around, if possible, to check the answers and clear misconceptions.
3. Invite a volunteer to come to the board to show their solution.
   The rest of the class should check their solutions and correct any mistakes.
   Solution:
   f. Given: sell various amounts of foreign currency (bank buys from customer).
   i. $GH¢1.00 = \text{Le } 1,500.00$
   GH¢5000: $GH¢5000 = 5,000 \times 1,500 = \text{Le } 7,500,000.00$
   ii. $€1.00 = \text{Le } 8,600.00$
   €200.00: $€200.00 = 200 \times 8,600 = \text{Le } 1,720,000.00$
   $₦1.00 = \text{Le } 20$
   iii. ₦1,500.00: $₦1,500.00 = 1,500 \times 20 = \text{Le } 30,000.00$
   The customer gets Le 7,500,000.00 for GH¢5000, Le 1,720,000.00 for €200.00 and Le30,000.00 for ₦1,500.00.
   g. Given: sell then buy $200.00 – bank buys from customer and sells to customer.
   $\$1.00 = \text{Le } 7,600.00$ buying rate
   $\$200.00 = 7,600 \times 200 = \text{Le } 1,520,000.00$
   $\$1.00 = \text{Le} 7,900$ selling rate
   $\$200.00 = 7,900 \times 200 = \text{Le } 1,580,000.00$
   $\text{amount lost} = 1,580,000 - 1,520,000 = \text{Le } 60,000.00$
   You will lose Le 60,000.00 from selling then buying $200.00**.**
   h. Given: Mrs. Sesay spends Le 10,000,000.00 to buy foreign currency.
   $₦1.00 = \text{Le } 20.80$
   $\text{Le } 10,000,000.00 = \frac{10,000,000}{20.80} = ₦480,769.23$
   $£1.00 = \text{Le } 9,800.00$
   $\text{Le } 10,000,000.00 = \frac{10,000,000}{9,800} = £1,020.41$
   $€1.00 = \text{Le } 8,900.00$
   $\text{Le } 10,000,000.00 = \frac{10,000,000}{8,900} = €1,123.60$
   She needs ₦480,769.23 for Nigeria, £1,020.41 for GB and €1,123.60 for Germany.

Closing *(4 minutes)*

1. Ask pupils to write down which rate they should use if they want to buy $100.00 from a bank. How much will they spend?
2. Invite a volunteer to answer. (Answer: selling rate, Le 790,000.00)
3. For homework, have pupils do the practice activity PHM3-L071 in the Pupil Handbook.

[QUESTIONS]

For each question, decide whether the bank is buying foreign currency from you or selling foreign currency to you. Then use the appropriate rate.

Where appropriate give your answer to 2 decimal places.

a. How much will Le 5,000,000.00 give you in the following currencies? Use the selling rate.
   i. US $ ii. GB £ iii. Gambia D
b. How much profit will a bank make if they buy then sell $500.00?
c. Olu received £20.00 from his uncle in London for his birthday. How much in Leones can he buy with his birthday money?
d. Mr. Samuels wants to order goods for his shop from Ghana. The goods cost GH¢45,000. How much does this cost him in Leones?
e. Memuna is going on an exchange visit to Nigeria. She wants to change Le 15,000,000.00 for the visit. How much will this be in Naira?
f. How much in Leones will you get for the following amounts? Use the buying rate.
   i. GH¢5,000 ii. €200.00 iii. ₦1,500.00
g. How much will you lose if you sell then buy $200.00?
h. Mrs. Sesay buys goods from all over the world for her shop. She wants to order Le 10,000,000.00 worth of goods each from Nigeria, Great Britain and Germany. How much of each country's currency will she need?

Lesson Title: Additional practice with applications of percentage	**Theme:** Numbers and Numeration	
Lesson Number: M3-L072	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcomes By the end of the lesson, pupils will be able to: 1. Calculate value added tax using percentages. 2. Calculate the amount to be paid for employer health insurance based on percentages.	**Preparation** Write the questions found at the end of this lesson plan on the board.	

Opening *(1 minute)*

1. Tell pupils that after today's lesson, they will be able to calculate value added tax and the amount to be paid for employer health insurance using percentages.

Teaching and Learning *(23 minutes)*

1. Explain:
   - Value Added Tax (VAT) is the tax charged on goods and services in some West African countries such as Ghana.
     It is similar to the Goods and Services (GST) tax charged in Sierra Leone.
   - If the VAT / GST is given as x%, then

     $$\text{VAT / GST} = \text{basic cost} \times \frac{x}{100}$$

     $$\text{Cost of goods/services} = \text{basic cost} \times \left(1 + \frac{x}{100}\right)$$

   - Basic cost is **exclusive** of VAT / GST. This is the cost before the tax is added.
   - In the questions below, take VAT to also mean GST
2. Invite a volunteer to assess question a. and tell the class what information we are given. (Answer: electric oven sold at Le 1,250,000.00 +VAT, VAT charged at 15%)
3. Invite another volunteer to say what we have been asked to find. (Answer: cost of the oven)

   Solution:

   a. **Step 1.** Assess and extract the given information from the problem.
      Given: electric oven sold at Le 1,250,000.00 + VAT, VAT is charged at 15%

      Step 2. Calculate the cost of the oven.

      $$\text{cost of oven} = \text{basic cost} \times \left(1 + \frac{x}{100}\right)$$

      $$\text{cost of oven} = 1{,}250{,}000 \times \left(1 + \frac{15}{100}\right) = 1{,}250{,}000 \times 1.15$$

      $$= \text{Le } 1{,}437{,}500.00$$

      Step 3. Write the answer.
      The electric oven cost Le $1{,}437{,}500.00$.
4. Invite a volunteer to fully assess question b. (Answer: woman's restaurant bill of Le 69,000.00, VAT charged at 15%, find the VAT charged)

Solution:

b. Given: woman's restaurant bill of Le 69,000.00, VAT charged at 15%)

$$\text{cost of service} = \text{basic cost} \times \left(1 + \frac{x}{100}\right)$$

$$\text{Let basic cost} = y$$

$$69{,}000 = y \times \left(1 + \frac{15}{100}\right)$$

$$69{,}000 = 1.15y$$

$$y = \frac{69{,}000}{1.15} = \text{Le } 60{,}000.00$$

$$\text{VAT} = 69{,}000 - 60{,}000 \quad (\text{or } 0.15 \times 60{,}000)$$

$$= \text{Le } 9{,}000.00$$

The woman paid VAT of Le 9,000.00.

5. Explain:
   - Some countries in West Africa add a levy called the National Health Insurance Levy (NHIL) to goods and services to cover National Health Insurance.
   - It is added to the VAT and charged on the basic cost of goods and services.
6. Invite a volunteer to fully assess question c. (Answer: VAT and NHIL inclusive price of computer is Le 5,170,000.00. VAT charged at 15%, NHIL charged at 2.5%, find the VAT and NHIL charged)

   Solution:

   c. Given: VAT and NHIL inclusive price of computer is Le 5,170,000.00. VAT charged at 15%, NHIL charged at 2.5%.

$$\text{total rate} = 15 + 2.5 = 17.5$$

i. $$\text{cost of computer} = \text{basic cost} \times \left(1 + \frac{17.5}{100}\right)$$

$$5{,}170{,}000 = \text{basic cost} \times (1.175)$$

$$\text{basic cost} = \frac{5{,}170{,}000}{1.175} = \text{Le } 4{,}400{,}000.00$$

ii. $$\text{NHIL} = \text{basic cost} \times \frac{\text{NHIL rate}}{100}$$

$$= 4{,}400{,}000 \times \frac{2.5}{100} = \text{Le } 110{,}000.00$$

iii. $$\text{VAT} = \text{basic cost} \times \frac{\text{VAT rate}}{100}$$

$$= 4{,}400{,}000 \times \frac{15}{100} = \text{Le } 660{,}000.00$$

The basic cost of the computer is Le 4,400,000.00, the NHIL is Le 110,000.00 and the VAT is Le 660,000.00.

7. Ask pupils to work with seatmates to answer question d.
8. Invite a volunteer to show their solution on the board. The rest of the class should check their solution and correct any mistakes.

   Solution:

   d. Given: VAT rate is $12\frac{1}{2}$%, NHIL rate is 2%, basic cost of item was Le 675,000.00

$$\text{total rate charged} = 12\tfrac{1}{2} + 2 = 14\tfrac{1}{2}\%$$

$$\text{cost of item} = \text{basic cost} \times \left(1 + \frac{14.5}{100}\right)$$

$$= 675{,}000 \times 1.145 = \text{Le } 772{,}875.00$$

The cost of the item was Le 772,875.00 inclusive of VAT and NHIL.

Practice *(15 minutes)*

1. Ask pupils to work independently to answer questions e., f. and g.
2. Walk around, if possible, to check for understanding and clear any misconceptions.
3. Invite a volunteer to come to the board to show their solution. The rest of the class should check their solutions and correct any mistakes.

Solution:

e. Given: basic cost Le 600,000.00. The VAT inclusive cost Le 705,000.00.

$$\text{VAT} = 705{,}000 - 600{,}000 = \text{Le } 105{,}000.00$$
$$\text{VAT} = \text{basic cost} \times \frac{x}{100}$$
$$105{,}000 = 600{,}000 \times \frac{x}{100}$$
$$105{,}000 = 6{,}000x$$
$$x = \frac{105{,}000}{6{,}000} = 17.5\%$$

The VAT rate is 17.5%.

f. Given: Mr. Cole bought 5 shirts at Le 25,000.00 each, 2 pairs of trousers at Le 40,000.00 each, a pair of shoes for Le 75,000.00, and a leather belt for Le 10,000. The VAT rate is 12.5%.

i.
$$\text{cost of shirts} = 25{,}000 \times 5 = \text{Le } 125{,}000.00$$
$$\text{cost of trousers} = 40{,}000 \times 2 = \text{Le } 80{,}000.00$$
$$\text{total amount} = 125{,}000 + 80{,}000 + 75{,}000 + 10{,}000 = \text{Le } 290{,}000.00$$

ii.
$$\text{total VAT charged} = \text{basic cost} \times \frac{x}{100}$$
$$= 290{,}000 \times \frac{12.5}{100} = \text{Le } 36{,}250.00$$

iii.
$$\text{total amount paid} = 290{,}000 + 36{,}250 = \text{Le } 326{,}250.00$$

Mr. Cole paid a VAT exclusive amount of Le $290{,}000.00$, VAT of Le $36{,}250.00$ and a total inclusive amount paid of Le $326{,}250.00$.

g. Given: cost of motor bike Le 12,925,000.00, VAT 15%, NHIL 2.5%.

i.
$$\text{total rate} = 2.5 + 15 = 17.5$$
$$\text{Let basic cost} = y$$
$$\text{cost of motor bike} = \text{basic cost} \times \left(1 + \frac{x}{100}\right)$$
$$12{,}925{,}000 = y \times \left(1 + \frac{17.5}{100}\right) = 1.175y$$
$$y = \frac{12{,}925{,}000}{1.175}$$
$$= \text{Le } 11{,}000{,}000.00$$

ii.
$$\text{NHIL} = \text{basic cost} \times \frac{x}{100} = 11{,}000{,}000 \times \frac{2.5}{100}$$
$$= \text{Le } 275{,}000.00$$

iii.
$$\text{VAT} = \text{basic cost} \times \frac{x}{100} = 11{,}000{,}000 \times \frac{15}{100}$$
$$= \text{Le } 1{,}650{,}000.00$$

The basic cost of the motor bike is Le $11{,}000{,}000.00$, NHIL is Le $275{,}000.00$ and the VAT is Le $1{,}650{,}000.00$.

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM3-L072 in the Pupil Handbook.

[QUESTIONS]

a. An electric oven is sold at Le 1,250,000.00 + VAT. How much will the oven cost inclusive of VAT charged at 15%?
b. After eating in a restaurant, a woman's bill came to Le 69,000.00 inclusive of VAT. If VAT is charged at 15%, how much did she pay for VAT?
c. The VAT and NHIL marked inclusive price of a computer is Le 5,170,000.00. The VAT is charged at 15% and the NHIL is charged at 2.5%. Find:
    i. The cost of the computer (VAT and NHIL exclusive);
    ii. The NHIL charged;
    iii. The VAT charged.
d. The VAT rate of a country is $12\frac{1}{2}$% and the NHIL rate is 2%. The basic cost of an item was Le 675,000.00. Find the full cost of the item.
e. Goods sold exclusive of VAT cost Le 600,000.00. When VAT is added they cost Le 705,000.00. How much is the VAT rate?
f. Mr. Cole bought the following items exclusive of VAT: 5 shirts at Le 25,000.00 each, 2 pairs of trousers at Le 40,000.00 each, a pair of shoes at Le 75,000.00 and a leather belt at Le 10,000.00. If the VAT rate is 12.5% Calculate:
    i. The total amount exclusive of VAT;
    ii. The amount charged for VAT;
    iii. The total amount Mr. Cole paid for the items.
g. The VAT and NHIL marked inclusive price of a motor bike is Le 12,925,000.00. The VAT is charged at 15% and the NHIL is charged at 2.5%. Find:
    i. The cost of the motor bike (VAT and NHIL exclusive);
    ii. The NHIL charged;
    iii. The VAT charged.

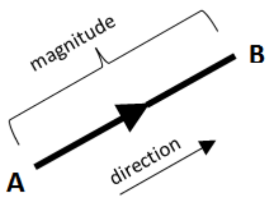
<table>
<tr><td>Lesson Title: Introduction to vectors and scalars</td><td colspan="2">Theme: Vectors and Transformations</td></tr>
<tr><td>Lesson Number: M3-L073</td><td>Class: SSS 3</td><td>Time: 40 minutes</td></tr>
<tr><td>Learning Outcomes
By the end of the lesson, pupils will be able to:
1. Define and describe vectors and scalars and their uses.
2. Use correct notation and representation for vectors.</td><td colspan="2">Preparation
1. Draw grid measuring 16 squares across and 16 squares down on the board.
2. Write the INFORMATION FOR TEACHING AND LEARNING found at the end of this lesson plan on the board.
3. Write questions a. and b. on the board.</td></tr>
</table>

Opening *(4 minutes)*

1. Ask pupils to write down 2 things they notice about the lines on the board.
2. Invite volunteers to give their answers. (Example answers: The lines are all the same length (5 cm); the lines go in different directions.)
3. Tell pupils that after today's lesson, they will be able to define and describe vectors and scalars and their uses.

Teaching and Learning *(20 minutes)*

1. Explain:
   - The lines on the board are all the same length or size but go in different directions.
   - If we are asked to draw a line 5 cm long, we could draw any one of the lines on the board.
   - To draw a specific line we would need both the length and the direction in which to draw them.
   - A line which has both length and direction is called a vector.
   - The length of the vector is called its **magnitude** or size.
2. Ask pupils to write the definitions below in their exercise books.
   - A **vector** is any quantity which has both magnitude and direction**.** Examples of vectors are displacement (translation), velocity, and force.
   - A **scalar** is any quantity which has only magnitude but no direction. Examples of scalars are distance, speed, and time.
3. Ask pupils to look at the list on the board and to write down which words describe a vector and which a scalar quantity.
4. After 1 minute, invite volunteers to answer. (Answers: vectors – weight, momentum, acceleration, impulse; scalars – mass, temperature, length, area, volume)
5. Draw a vector similar to the one on the right.



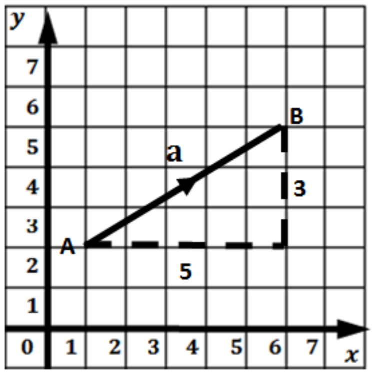
6. Explain:
   - Vectors are represented in various ways.

- The simplest representation is as a line segment with the length equal to the magnitude of the vector and an arrow indicating its direction.
- This vector shows a displacement of a point from position A to position B.
- It can be written in many ways:

  $\overrightarrow{AB}, \overline{AB}, \mathbf{AB}, \vec{a}, \bar{a}, \underline{a}, \mathbf{a}$

  [NOTE: Hand-written vectors can be represented using arrows, over- or under-bars. Use whatever is familiar to you.]
- Vectors written in lowercase letters are called position vectors.
  We will learn more about position vectors in a later lesson.

7. Explain:
   - Vectors can be represented on a Cartesian plane as shown at right.
     Consider the vector $\overrightarrow{AB}$:
     it can be written as a column matrix or column vector:



$$\overrightarrow{AB} = \mathbf{a} = \binom{5}{3}$$

   - The vector is drawn by starting at point A, moving 5 units to the right and 3 units up.
   - In general, any vector $\overrightarrow{AB} = \binom{a}{b}$ has 2 components:
     the horizontal component a measured along the x-axis, and the vertical component, b measured along the y-axis from point A to point B.
   - Any move to the left or downwards is movement in the negative direction.
8. Invite a volunteer to assess problem a. and tell the class what information we are given. (Answer: given: line segments $\overrightarrow{AB}$, $\overrightarrow{HI}$, $\overrightarrow{IJ}$ and $\overrightarrow{JK}$)
9. Invite another volunteer to say what we have been asked to find. (Answer: write as column vectors in the form $\binom{x}{y}$)

   Solution:

   a. **Step 1.** Assess and extract the given information from the problem.
      Given: line segments $\overrightarrow{AB}$, $\overrightarrow{HI}$, $\overrightarrow{IJ}$ and $\overrightarrow{JK}$

      Step 2. Write each line segment as a column vector.
      Show clearly how the positive and negative components of the vector are measured on the grid.

$$\overrightarrow{AB} = \binom{5}{6}$$
$$\overrightarrow{HI} = \binom{-2}{-5}$$
$$\overrightarrow{IJ} = \binom{-8}{0}$$
$$\overrightarrow{JK} = \binom{5}{6}$$

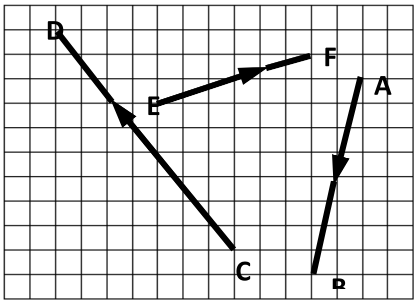
10. Invite a volunteer to assess question b. and tell the class what information we are given. (Answer: Given: column vectors: $\overrightarrow{AB} = \binom{-2}{-8}, \overrightarrow{CD} = \binom{-7}{9}, \overrightarrow{EF} = \binom{6}{2}$)

11. Invite another volunteer to say what we have been asked to find. (Answer: draw given column vectors as line segments)

Solution:

b. Given: column vectors: $\overrightarrow{AB} = \begin{pmatrix} -2 \\ -8 \end{pmatrix}, \overrightarrow{CD} = \begin{pmatrix} -7 \\ 9 \end{pmatrix}, \overrightarrow{EF} = \begin{pmatrix} 6 \\ 2 \end{pmatrix}$

- Draw the column vectors as line segments on the grid on the board. (shown below)



12. Explain:
- There is one important exception to vectors having magnitude and direction.
- The **zero vector**, denoted by $\mathbf{0} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$, is the vector of zero length or magnitude.
- It has no length, and does not point in any particular direction.
- We call it **the zero vector** since there is only one vector of zero length.

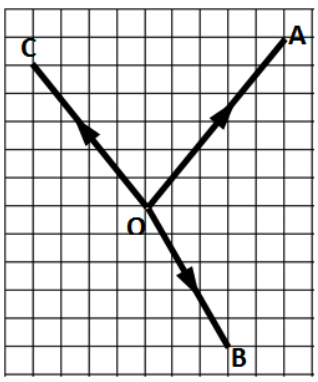
Practice *(15 minutes)*

1. Write questions c. and d. on the board.
2. Ask pupils to work independently to answer questions c. and d.
3. Walk around, if possible, to check the answers and clear any misconceptions.
4. Invite volunteers to come to the board to show their solutions.
The rest of the class should check their solutions and correct any mistakes.

Solutions:

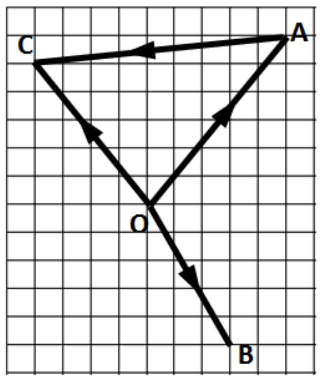
c. Given: column vectors: $\overrightarrow{OA} = \begin{pmatrix} 5 \\ 6 \end{pmatrix}$, $\overrightarrow{OB} = \begin{pmatrix} 3 \\ -5 \end{pmatrix}$ and $\overrightarrow{OC} = \begin{pmatrix} -4 \\ 5 \end{pmatrix}$

i.



ii. $\overrightarrow{AC} = \begin{pmatrix} -9 \\ -1 \end{pmatrix}$

iii. $\overrightarrow{CA} = \begin{pmatrix} 9 \\ 1 \end{pmatrix}$

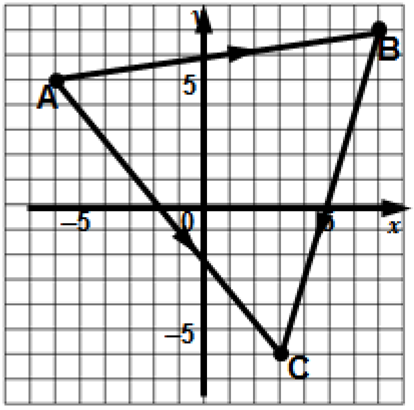


d. See diagram on the next page.

i. $\overrightarrow{AB} = \begin{pmatrix} 13 \\ 2 \end{pmatrix}$

ii. $\overrightarrow{BC} = \begin{pmatrix} -4 \\ -13 \end{pmatrix}$

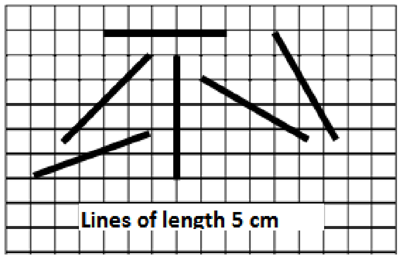
iii. $\overrightarrow{AC} = \begin{pmatrix} 9 \\ -11 \end{pmatrix}$



Closing *(1 minute)*

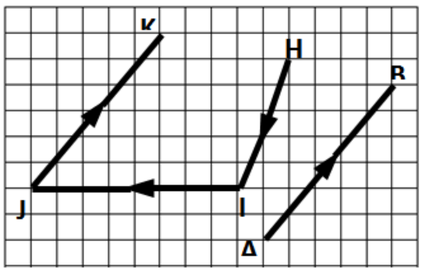
1. For homework, have pupils do the practice activity PHM3-L073 in the Pupil Handbook.

[INFORMATION FOR TEACHING AND LEARNING]



- Draw 5 cm lines on the grid on the board.
- Write the words below on the board:
  temperature, volume, weight, momentum, length, mass, impulse, acceleration, area

[QUESTIONS]



a. The line segments shown at right represent column vectors $\overrightarrow{AB}$, $\overrightarrow{HI}$, $\overrightarrow{IJ}$ and $\overrightarrow{JK}$. Write these as vectors in the form $\begin{pmatrix} x \\ y \end{pmatrix}$.

b. Draw line segments on a graph paper to represent the following column vectors:
$$\overrightarrow{AB} = \begin{pmatrix} -2 \\ -8 \end{pmatrix}, \overrightarrow{CD} = \begin{pmatrix} -7 \\ 9 \end{pmatrix}, \overrightarrow{EF} = \begin{pmatrix} 6 \\ 2 \end{pmatrix}$$

c. Draw line segments with respect to *O* to represent the following column vectors.

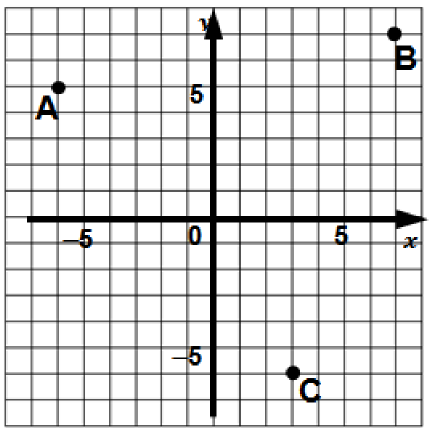
  i. $\overrightarrow{OA} = \begin{pmatrix} 5 \\ 6 \end{pmatrix}$, $\overrightarrow{OB} = \begin{pmatrix} 3 \\ -5 \end{pmatrix}$ and $\overrightarrow{OC} = \begin{pmatrix} -4 \\ 5 \end{pmatrix}$

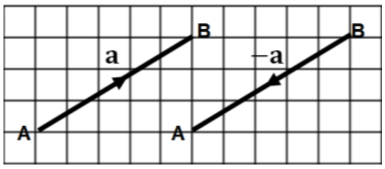
  ii. Hence give the vector $\overrightarrow{AC}$ in the form $\begin{pmatrix} a \\ b \end{pmatrix}$.

  iii. What is $\overrightarrow{CA}$?

d. The points $A\,(-6,5)$, $B(7,7)$ and $C(3,-6)$ are shown on a grid.
Write each of the following vectors in the form $\begin{pmatrix} a \\ b \end{pmatrix}$. (Hint: Draw the lines joining the points.)

  i. $\overrightarrow{AB}$ ii. $\overrightarrow{BC}$ iii. $\overrightarrow{AC}$



Lesson Title: Basic vector properties	**Theme:** Vectors and Transformations	
Lesson Number: M3-L074	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able identify and use basic properties of vectors.	**Preparation** 1. Draw the diagram below on the board.  2. Write the questions found at the end of this lesson plan on the board.	

Opening *(4 minutes)*

1. Invite a volunteer to give the difference between a vector and a scalar quantity. (Answer: A vector is any quantity which has both magnitude and direction whilst a scalar has only magnitude but no direction.)
2. Tell pupils that after today's lesson, they will be able to use the basic properties of vectors.

Teaching and Learning *(20 minutes)*

1. Refer to the diagram on the board.
2. Explain:
   - Consider the vector $\overrightarrow{AB}$ shown in the diagram
     $$\overrightarrow{AB} = \mathbf{a} = \binom{5}{3}$$
   - Then the vector:
     $$\overrightarrow{BA} = -\overrightarrow{AB} = -\mathbf{a} = -\binom{5}{3} = \binom{-5}{-3}$$
     is the inverse vector of $\mathbf{a}$.
   - $-\mathbf{a}$ is equal in magnitude (or length) to $\mathbf{a}$, but opposite in direction.
   - The direction changes from $A \to B$ to $B \to A$.
3. Refer to question a. i. on the board.
4. Invite a volunteer to assess question a. i. and tell the class what information we are given. (Answer: Given: vector $\overrightarrow{PQ} = \binom{3}{4}$)
5. Invite another volunteer to say what we have been asked to find. (Answer: find $\overrightarrow{QP}$)
   Solution:
   a. i. **Step 1.** Assess and extract the given information from the problem.
   given: vector $\overrightarrow{PQ} = \binom{3}{4}$
   Step 2. Find the inverse vector.

$$\overrightarrow{QP} = -\overrightarrow{PQ}$$
$$= -\binom{3}{4}$$
$$= \binom{-3}{-4}$$

Step 3. Write the answer.

The inverse vector of $\overrightarrow{PQ} = \binom{3}{4}$ is $\overrightarrow{QP} = \binom{-3}{-4}$.

6. Refer to question a. ii on the board.
7. Invite a volunteer to assess the problem and tell the class what information we are given. (Answer: given: vector $\overrightarrow{RS} = \binom{6}{-2}$)
8. Invite another volunteer to say what we have been asked to find. (Answer: find $\overrightarrow{SR}$)

Solution:

a. ii. Given: vector $\overrightarrow{RS} = \binom{6}{-2}$**,** find $\overrightarrow{SR}$.

$$\overrightarrow{SR} = -\overrightarrow{RS}$$
$$= -\binom{6}{-2}$$
$$= \binom{-6}{2}$$

The inverse vector of $\overrightarrow{RS} = \binom{6}{-2}$ is $\overrightarrow{SR} = \binom{-6}{2}$

9. Ask pupils to work with seatmates to answer questions a. iii and a. iv.
10. Invite a volunteer to give the answer. The rest of the class should check their solutions and correct any mistakes.

Solutions:

a. iii. Given: vector $\overrightarrow{XY} = \binom{-3}{1}$**,** find $\overrightarrow{YX}$.

$$\overrightarrow{YX} = -\overrightarrow{XY}$$
$$= -\binom{-3}{1}$$
$$= \binom{3}{-1}$$

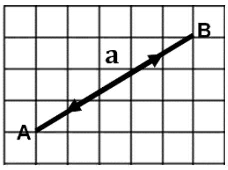
The inverse vector of $\overrightarrow{XY} = \binom{-3}{1}$is $\overrightarrow{YX} = \binom{3}{-1}$.

iv. Given: vector $\overrightarrow{AB} = \binom{-8}{-5}$**,** find $\overrightarrow{BA}$.

$$\overrightarrow{BA} = -\overrightarrow{AB}$$
$$= -\binom{-8}{-5}$$
$$= \binom{8}{5}$$

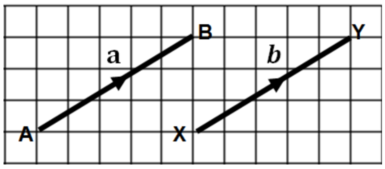
The inverse vector of $\overrightarrow{AB} = \binom{-8}{-5}$ is $\overrightarrow{BA} = \binom{8}{5}$.

11. Draw the diagram at right on the board.
12. Explain:
    - Every vector has an inverse vector which is equal in magnitude and opposite in direction to it.
    - When a point moves along a vector $\overrightarrow{AB}$ and then along its inverse $\overrightarrow{BA}$, the effect is that of zero movement.
    - The end result is a vector of zero magnitude and no direction.
    - This is an example of the zero vector.



13. Draw the diagrams at right on the board.

14. Explain:



- We can also have vectors which are equal in both magnitude and direction.
- We say that if $\mathbf{a} = \mathbf{b}$, then
  - $|\mathbf{a}| = |\mathbf{b}|$ (absolute value of $\mathbf{a}$ = absolute value of $\mathbf{b}$)
  - $\mathbf{a} \parallel \mathbf{b}$ (**a** is parallel to **b**)
- Also, if $\mathbf{a} = \binom{x_1}{y_1}$ and $\mathbf{b} = \binom{x_2}{y_2}$, then
  - $x_1 = x_2$, and $y_1 = y_2$

15. Invite a volunteer to assess question b. and tell the class what information we are given. (Answer: Given: $\mathbf{p} = \binom{a+6}{2-b}$, $\mathbf{q} = \binom{8}{4}$, $\mathbf{p} = \mathbf{q}$)
16. Invite another volunteer to say what we have been asked to find. (Answer: find a and b)

Solution:

b. Given: $\mathbf{p} = \binom{a+6}{2-b}$, $\mathbf{q} = \binom{8}{4}$, $\mathbf{p} = \mathbf{q}$

$$\mathbf{p} = \mathbf{q} \quad \Longrightarrow \quad \binom{a+6}{2-b} = \binom{8}{4}$$

equate corresponding components

$$\begin{aligned} a+6 &= 8 \\ \Longrightarrow a &= 8-6 = 2 \\ 2-b &= 4 \\ \Longrightarrow b &= 2-4 = -2 \\ a &= 2, b = -2 \end{aligned}$$

17. Ask pupils to work with seatmates to answer question c.
18. Invite a volunteer to show their solution on the board. The rest of the class should check their solution and correct any mistakes.

Solution:

c. Given: $\mathbf{a} = \binom{2x}{x+y}$, $\mathbf{b} = \binom{6}{7}$, $\mathbf{a} = \mathbf{b}$

$$\mathbf{a} = \mathbf{b} \quad \Longrightarrow \quad \binom{2x}{x+y}, = \binom{6}{7}$$

equate corresponding components

$$\begin{aligned} 2x &= 6 \\ \Longrightarrow x &= \tfrac{6}{2} = 3 \\ x+y &= 7 \\ \Longrightarrow y &= 7-x \\ &= 7-3 = 4 \\ x &= 3, y = 4 \end{aligned}$$

Practice *(15 minutes)*

1. Ask pupils to work independently to answer questions d. through f.
2. Walk around, if possible, to check the answers and clear any misconceptions.
3. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.

Solutions:

d. i Given: $\overrightarrow{AB} = \begin{pmatrix}4\\3\end{pmatrix}$

$$\begin{aligned}\overrightarrow{BA} &= -\overrightarrow{AB}\\ &= -\begin{pmatrix}4\\3\end{pmatrix}\\ &= \begin{pmatrix}-4\\-3\end{pmatrix}\end{aligned}$$

ii. given: $\overrightarrow{XY} = \begin{pmatrix}-7\\5\end{pmatrix}$

$$\begin{aligned}\overrightarrow{YX} &= -\overrightarrow{XY}\\ &= -\begin{pmatrix}-7\\5\end{pmatrix}\\ &= \begin{pmatrix}7\\-5\end{pmatrix}\end{aligned}$$

iii. Given: $\overrightarrow{RS} = \begin{pmatrix}1\\-2\end{pmatrix}$

$$\begin{aligned}\overrightarrow{SR} &= \overrightarrow{-RS}\\ &= -\begin{pmatrix}1\\-2\end{pmatrix}\\ &= \begin{pmatrix}-1\\2\end{pmatrix}\end{aligned}$$

e. Given: $\mathbf{p} = \begin{pmatrix}2a+5\\-2\end{pmatrix}$**,** $\mathbf{q} = \begin{pmatrix}7\\b+6\end{pmatrix}$

$$\begin{aligned}\mathbf{p} &= \mathbf{q}\\ \begin{pmatrix}2a+5\\-2\end{pmatrix} &= \begin{pmatrix}7\\b+6\end{pmatrix}\\ 2\text{a}+5 &= 7 && \text{Equate } x \text{ components}\\ 2\text{a} &= 7-5\\ 2a &= 2 && \text{Divide throughout by 2}\\ a &= 1\\ -2 &= b+6 && \text{Equate } y \text{ components}\\ -2-6 &= b\\ b &= -8\end{aligned}$$

$a = 1, b = -8$. $\mathbf{p} = \mathbf{q} = \begin{pmatrix}7\\-2\end{pmatrix}$

f. Given: $\overrightarrow{OA} = \begin{pmatrix}6\\6\end{pmatrix}, \overrightarrow{OB} = \overrightarrow{AO} = \begin{pmatrix}x+1\\y-3\end{pmatrix}$,

$$\begin{aligned}\overrightarrow{OB} &= \overrightarrow{AO} = -\overrightarrow{OA}\\ &= -\begin{pmatrix}6\\6\end{pmatrix} = \begin{pmatrix}-6\\-6\end{pmatrix}\\ \begin{pmatrix}x+1\\y-3\end{pmatrix} &= \begin{pmatrix}-6\\-6\end{pmatrix}\\ x &= -6-1 = -7\\ y &= -6+3 = -3\end{aligned}$$

$x = -7, y = -3$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM3-L074 in the Pupil Handbook.

[QUESTIONS]

a. i. If $\overrightarrow{PQ} = \begin{pmatrix}3\\4\end{pmatrix}$, find $\overrightarrow{QP}$. ii. If $\overrightarrow{RS} = \begin{pmatrix}6\\-2\end{pmatrix}$, find $\overrightarrow{SR}$.

iii. If $\overrightarrow{XY} = \begin{pmatrix}-3\\1\end{pmatrix}$**,** find $\overrightarrow{YX}$. iv. If $\overrightarrow{AB} = \begin{pmatrix}-8\\-5\end{pmatrix}$**,** find $\overrightarrow{BA}$.

b. Find a and b given that $\mathbf{p} = \begin{pmatrix}a+6\\2-b\end{pmatrix}$**,** $\mathbf{q} = \begin{pmatrix}8\\4\end{pmatrix}$ and $\mathbf{p} = \mathbf{q}$.

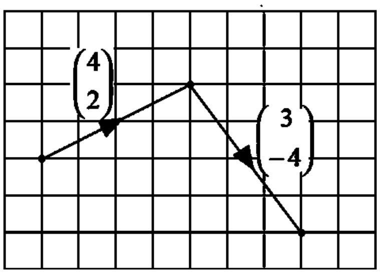
c. Find x and y given that $\mathbf{a} = \begin{pmatrix} 2x \\ x+y \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 6 \\ 7 \end{pmatrix}$ and $\mathbf{a} = \mathbf{b}$.

d. i. If $\overrightarrow{AB} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$, find $\overrightarrow{BA}$. ii. If $\overrightarrow{XY} = \begin{pmatrix} -7 \\ 5 \end{pmatrix}$, find $\overrightarrow{YX}$.

iii. If $\overrightarrow{RS} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$, find $\overrightarrow{SR}$.

e. Find a and b given that $\mathbf{p} = \begin{pmatrix} 2a+5 \\ -2 \end{pmatrix}$, $\mathbf{q} = \begin{pmatrix} 7 \\ b+6 \end{pmatrix}$ and $\mathbf{p} = \mathbf{q}$.

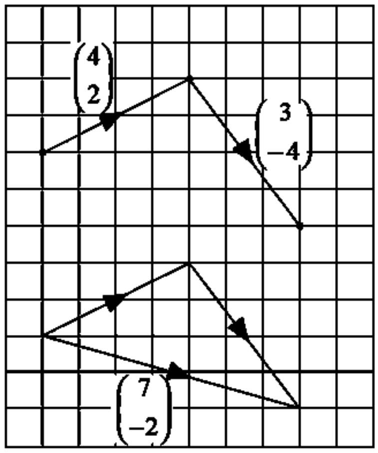
f. Find x and y given that $\overrightarrow{OA} = \begin{pmatrix} 6 \\ 6 \end{pmatrix}$ and $\overrightarrow{OB} = \overrightarrow{AO} = \begin{pmatrix} x+1 \\ y-3 \end{pmatrix}$

Lesson Title: Addition and subtraction of vectors	**Theme:** Vectors and Transformations	
Lesson Number: M3-L075	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to add or subtract vectors based on information given.	**Preparation** 1. Draw the diagram below on the board.  2. Write questions a. and b. found at the end of this lesson plan on the board.	

Opening *(4 minutes)*

1. Ask pupils to write down in their exercise books the inverse vector to $\binom{5}{0}$.
2. Invite a volunteer to give the answer. (Answer: $-\binom{5}{0} = \binom{-5}{0}$)
3. Tell pupils that after today's lesson, they will be able to add or subtract vectors based on information given.

Teaching and Learning *(20 minutes)*



1. Refer to the diagram on the board.
2. Explain:
   - Consider the vectors $\mathbf{a} = \binom{4}{2}$ and $\mathbf{b} = \binom{3}{-4}$ shown in the diagram on the board.
   - When we join the 2 end points by a line, we end up with a new diagram. (Join the 2 end points. Do not write the vector at this time.
3. Write on the board:

$$\mathbf{a} + \mathbf{b} = \binom{4}{2} + \binom{3}{-4}$$
$$= \binom{7}{-2} \qquad \text{From the diagram}$$

4. Explain: The column vector for the new line is $\binom{7}{-2}$.
5. This is the same as adding the corresponding x and y components together

$$\mathbf{a} + \mathbf{b} = \binom{4+3}{2+(-4)}$$
$$= \binom{7}{-2} \qquad \text{By calculation}$$

6. Explain:
   - In general, if $\mathbf{a} = \binom{x_1}{y_1}$ and $\mathbf{b} = \binom{x_2}{y_2}$, then

     $$\mathbf{a} + \mathbf{b} \quad = \quad \binom{x_1+x_2}{y_1+y_2}$$

   - Similarly,

     $$\mathbf{a} - \mathbf{b} \quad = \quad \binom{x_1-x_2}{y_1-y_2}$$

7. Invite a volunteer to assess problem a. from the board, and tell the class what information we are given. (Answer: Given: $\mathbf{a} = \binom{4}{7}$, $\mathbf{b}= \binom{3}{-5}$, and $\mathbf{c} = \binom{0}{4}$)
8. Invite another volunteer to say what we have been asked to find in question a. i., a. ii, and a. iii.) (Answers: (vector addition) **a + b, and b + c**, and (vector subtraction) **a - c**)

   Solutions:

   a. **Step 1.** Assess and extract the given information from the problem.

      given**:** $\mathbf{a} = \binom{4}{7}$, $\mathbf{b}= \binom{3}{-5}$, and $\mathbf{c} = \binom{0}{4}$

      Step 2. Complete the vector addition/subtraction.

      Step 3. Write the answers as shown below.

   i.
   $$\begin{aligned}\mathbf{a} + \mathbf{b} &= \binom{4}{7} + \binom{3}{-5} \\ &= \binom{4+3}{7+(-5)} \\ &= \binom{7}{2}\end{aligned}$$

   ii.
   $$\begin{aligned}\mathbf{b} + \mathbf{c} &= \binom{3}{-5} + \binom{0}{4} \\ &= \binom{3+0}{-5+4} \\ &= \binom{3}{-1}\end{aligned}$$

   iii.
   $$\begin{aligned}\mathbf{a} - \mathbf{c} &= \binom{4}{7} - \binom{0}{4} \\ &= \binom{4-0}{7-4} \\ &= \binom{4}{3}\end{aligned}$$

   - Ask pupils to work with seatmates to answer iv. and v.
   - Invite volunteers to show their answers on the board.

   iv.
   $$\begin{aligned}\mathbf{a} - \mathbf{b} &= \binom{4}{7} - \binom{3}{-5} \\ &= \binom{4-3}{7-(-5)} \\ &= \binom{1}{12}\end{aligned}$$

   v.
   $$\begin{aligned}\mathbf{a} + \mathbf{b} - \mathbf{c} &= \binom{4}{7} + \binom{3}{-5} - \binom{0}{4} \\ &= \binom{4+3-0}{7-5-4} \\ &= \binom{7}{-2}\end{aligned}$$

9. Ask pupils to continue to work with seatmates to answer question b.
10. Invite volunteers to show their answers on the board. The rest of the class should check their solution and correct any mistakes.

Solutions:

b. Given: **a** = $\binom{2}{1}$, **b**= $\binom{4}{-1}$, and **c** = $\binom{-2}{-4}$,

i. $$\begin{aligned} \mathbf{a}+\mathbf{b} &= \binom{2}{1} + \binom{4}{-1} \\ &= \binom{2+4}{1+(-1)} \\ &= \binom{6}{0} \end{aligned}$$

ii. $$\begin{aligned} \mathbf{a}+\mathbf{c} &= \binom{2}{1} + \binom{-2}{-4} \\ &= \binom{2+(-2)}{1+(-4)} \\ &= \binom{0}{-3} \end{aligned}$$

iii. $$\begin{aligned} \mathbf{a}+\mathbf{c}+\mathbf{b} &= \binom{2}{1} + \binom{-2}{-4} + \binom{4}{-1} \\ &= \binom{2+(-2)+4}{1+(-4)+(-1)} \\ &= \binom{4}{-4} \end{aligned}$$

iv. $$\begin{aligned} \mathbf{a}+\mathbf{b}-\mathbf{c} &= \binom{2}{1} + \binom{4}{-1} - \binom{-2}{-4} \\ &= \binom{2+4-(-2)}{1+(-1)-(-4)} \\ &= \binom{8}{4} \end{aligned}$$

Practice *(15 minutes)*

1. Ask pupils to work independently to answer questions c. and d.
2. Walk around, if possible, to check the answers and clear any misconceptions.
3. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.

Solutions:

c. Given: **p** = $\binom{2}{4}$, **q**= $\binom{5}{1}$, and **r** = $\binom{7}{5}$,

i $$\begin{aligned} \mathbf{p}+\mathbf{r} &= \binom{2}{4} + \binom{7}{5} \\ &= \binom{2+7}{4+5} \\ &= \binom{9}{9} \end{aligned}$$

ii. $$\begin{aligned} \mathbf{p}-\mathbf{r} &= \binom{2}{4} - \binom{7}{5} \\ &= \binom{2-7}{4-5} \\ &= \binom{-5}{-1} \end{aligned}$$

iii. $$\begin{aligned} \mathbf{r}-\mathbf{p}+\mathbf{q} &= \binom{7}{5} - \binom{2}{4} + \binom{5}{1} \\ &= \binom{7-2+5}{5-4+1} \\ &= \binom{10}{2} \end{aligned}$$

iv. $$\begin{aligned} \mathbf{p}-\mathbf{r}-\mathbf{q} &= \binom{2}{4} - \binom{7}{5} - \binom{5}{1} \\ &= \binom{2-7-5}{4-5-1} \\ &= \binom{-10}{-2} \end{aligned}$$

d. Given: **a** = $\binom{4}{2}$, **b**= $\binom{3}{-1}$., and **c** = $\binom{-2}{1}$

i. $$\begin{aligned} \mathbf{a}+\mathbf{x} &= \mathbf{b} \\ \binom{4}{2}+\mathbf{x} &= \binom{3}{-1} \\ \mathbf{x} &= \binom{3}{-1} - \binom{4}{2} \\ &= \binom{3-4}{-1-2} \\ &= \binom{-1}{-3} \end{aligned}$$

ii. $$\begin{aligned} \mathbf{x}-\mathbf{c} &= \mathbf{a} \\ \mathbf{x}-\binom{-2}{1} &= \binom{4}{2} \\ \mathbf{x} &= \binom{4}{2} + \binom{-2}{1} \\ &= \binom{4+(-2)}{2+1} \\ &= \binom{2}{3} \end{aligned}$$

iii. $$\begin{aligned} \mathbf{x}+\mathbf{b} &= \mathbf{c} \\ \mathbf{x}+\binom{3}{-1} &= \binom{-2}{1} \\ \mathbf{x} &= \binom{-2}{1} - \binom{3}{-1} \end{aligned}$$

iv. $$\begin{aligned} \mathbf{b}+\mathbf{x} &= \mathbf{a} \\ \binom{3}{-1}+\mathbf{x} &= \binom{4}{2} \\ \mathbf{x} &= \binom{4}{2} - \binom{3}{-1} \end{aligned}$$

$$= \begin{pmatrix} -2-3 \\ 1-(-1) \end{pmatrix} \qquad = \begin{pmatrix} 4-3 \\ 2-(-1) \end{pmatrix}$$
$$= \begin{pmatrix} -5 \\ 2 \end{pmatrix} \qquad = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM3-L075 in the Pupil Handbook.

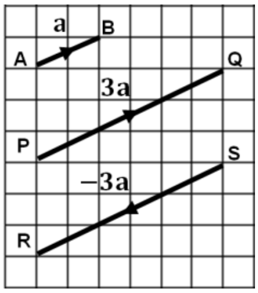
[QUESTIONS]

a. If $\mathbf{a} = \binom{4}{7}$, $\mathbf{b} = \binom{3}{-5}$, and $\mathbf{c} = \binom{0}{4}$, find:
   i. $\mathbf{a} + \mathbf{b}$ ii. $\mathbf{b} + \mathbf{c}$ iii. $\mathbf{a} - \mathbf{c}$
   iv. $\mathbf{a} - \mathbf{b}$ v. $\mathbf{a} + \mathbf{b} - \mathbf{c}$

b. if $\mathbf{a} = \binom{2}{1}$, $\mathbf{b} = \binom{4}{-1}$, and $\mathbf{c} = \binom{-2}{-4}$, find:
   i. $\mathbf{a} + \mathbf{b}$ ii. $\mathbf{a} + \mathbf{c}$
   iii. $\mathbf{a} + \mathbf{c} + \mathbf{b}$ iv. $\mathbf{a} + \mathbf{b} - \mathbf{c}$

c. If $\mathbf{p} = \binom{2}{4}$, $\mathbf{q} = \binom{5}{1}$, and $\mathbf{r} = \binom{7}{5}$, find:
   i. $\mathbf{p} + \mathbf{r}$ ii. $\mathbf{r} - \mathbf{p}$
   iii. $\mathbf{r} - \mathbf{p} + \mathbf{q}$ iv. $\mathbf{p} - \mathbf{r} - \mathbf{q}$

d. If $\mathbf{a} = \binom{4}{2}$, $\mathbf{b} = \binom{3}{-1}$, and $\mathbf{c} = \binom{-2}{1}$, solve the equations below to find the column vector **x**.
   i. $\mathbf{a} + \mathbf{x} = \mathbf{b}$ ii. $\mathbf{x} - \mathbf{c} = \mathbf{a}$
   iii. $\mathbf{x} + \mathbf{b} = \mathbf{c}$ iv. $\mathbf{b} + \mathbf{x} = \mathbf{a}$

<table>
<tr><td>Lesson Title: Multiplication of vectors by scalars</td><td colspan="2">Theme: Vectors and Transformations</td></tr>
<tr><td>Lesson Number: M3-L076</td><td>Class: SSS 3</td><td>Time: 40 minutes</td></tr>
<tr><td>Learning Outcome
By the end of the lesson, pupils will be able to multiply a vector by a scalar to find the scalar multiple.</td><td colspan="2">Preparation
1. Draw the diagram below on the board.


2. Draw a grid 16 squares across by 16 squares down on the board.
3. Write the questions found at the end of this lesson plan on the board.</td></tr>
</table>

Opening *(4 minutes)*

1. Ask pupils to write down the vectors $\overrightarrow{AB}$, $\overrightarrow{PQ}$ and $\overrightarrow{RS}$ as shown on the board.
2. Invite volunteers to give their answers. (Answer: $\overrightarrow{AB} = \binom{2}{1}, \overrightarrow{PQ} = \binom{6}{3}, \overrightarrow{RS} = \binom{-6}{-3}$)
3. Tell pupils that after today's lesson, they will be able to multiply a vector by a scalar to find the scalar multiple.

Teaching and Learning *(20 minutes)*

1. Refer to the diagram on the board.
2. Invite volunteers to describe what they notice about the components of the vectors. (Example answers: The absolute value of each component in $\overrightarrow{PQ}$ and $\overrightarrow{RS}$ is three times that of corresponding components in $\overrightarrow{AB}$. $\overrightarrow{AB}$ and $\overrightarrow{PQ}$ are in the same direction but opposite to $\overrightarrow{RS}$.)
3. Explain:
   - Consider the vector $\overrightarrow{AB} = \mathbf{a} = \binom{2}{1}$ shown in the diagram,
   - It can be seen from the diagram that:

$$\begin{aligned}\overrightarrow{PQ} &= \binom{6}{3}\\ &= 3\binom{2}{1}\\ &= 3\mathbf{a} \quad \text{3 times vector } \mathbf{a} \text{ in the same direction}\\ \overrightarrow{RS} &= \binom{-6}{-3}\\ &= -3\binom{2}{1}\\ &= -3\mathbf{a} \quad \text{3 times vector } \mathbf{a} \text{ in the opposite direction}\end{aligned}$$

4. Explain:

- In general, if $\mathbf{a} = \binom{x}{y}$ then

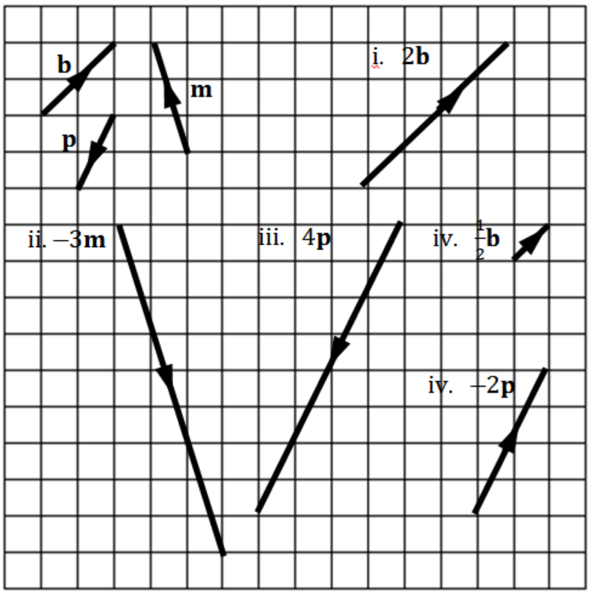
$$k\mathbf{a} = \binom{kx}{ky}$$ where k is a scalar or number which can be a positive or negative whole number or fraction

- Multiplication of a vector by a scalar is called scalar multiplication.
- Each component of the vector is multiplied by the scalar amount.
- It has the effect of “scaling” the vector up or down by the factor of the scalar quantity.
- If the scalar is positive, the resulting vector is in the same direction as the original vector.
- If the scalar is negative, the resulting vector is in the opposite direction as the original vector.

5. Invite a volunteer to assess problem a. and tell the class what information we are given. (Answer: Given: vectors **b**, **p** and **m**)
6. Invite another volunteer to say what we have been asked to find. (Answer: Draw and label the vectors on a grid showing its direction with an arrow.)
7. Do questions a. i. and ii. on the board. The solution is shown below.
8. Once you have completed a. i and a. ii, ask pupils to do questions a. iii., iv. and v.
9. Invite volunteers to show their solutions on the board.

Solution:

a. **Step 1.** Assess and extract the given information from the problem.
Given: vectors **b**, **p** and **m** (shown below)
Step 2. Draw and label each vector on the grid showing its direction with an arrow.



10. Invite a volunteer to assess question b. on the board and extract the given information. (Answer: Given: $\mathbf{a} = \binom{2}{1}$, $\mathbf{b} = \binom{4}{-1}$, and $\mathbf{c} = \binom{-2}{-4}$)
11. Invite a volunteer to say what we have been asked to find. (Answer: find the sum of vectors as given)
12. Do questions b. i. and ii. on the board. The solution is shown below.

13. Once you have completed b. i and b. ii, ask pupils to work with seatmates to answer question b. iii. and iv.
14. Invite volunteers to show their answer on the board. The rest of the class should check their solution and correct any mistakes.

Solution:

b. Given: $\mathbf{a} = \begin{pmatrix}2\\1\end{pmatrix}$, $\mathbf{b} = \begin{pmatrix}4\\-1\end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix}-2\\-4\end{pmatrix}$

i.
$$\begin{aligned}3\mathbf{a}+2\mathbf{b} &= 3\begin{pmatrix}2\\1\end{pmatrix}+2\begin{pmatrix}4\\-1\end{pmatrix}\\ &= \begin{pmatrix}6\\3\end{pmatrix}+\begin{pmatrix}8\\-2\end{pmatrix}\\ &= \begin{pmatrix}6+8\\3+(-2)\end{pmatrix}\\ &= \begin{pmatrix}14\\1\end{pmatrix}\end{aligned}$$

ii.
$$\begin{aligned}4\mathbf{a}+3\mathbf{c} &= 4\begin{pmatrix}2\\1\end{pmatrix}+3\begin{pmatrix}-2\\-4\end{pmatrix}\\ &= \begin{pmatrix}8\\4\end{pmatrix}+\begin{pmatrix}-6\\-12\end{pmatrix}\\ &= \begin{pmatrix}8+(-6)\\4+(-12)\end{pmatrix}\\ &= \begin{pmatrix}2\\-8\end{pmatrix}\end{aligned}$$

iii.
$$\begin{aligned}6\mathbf{a}-3\mathbf{b} &= 6\begin{pmatrix}2\\1\end{pmatrix}-3\begin{pmatrix}4\\-1\end{pmatrix}\\ &= \begin{pmatrix}12\\6\end{pmatrix}-\begin{pmatrix}12\\-3\end{pmatrix}\\ &= \begin{pmatrix}12-12\\6-(-3)\end{pmatrix}\\ &= \begin{pmatrix}0\\9\end{pmatrix}\end{aligned}$$

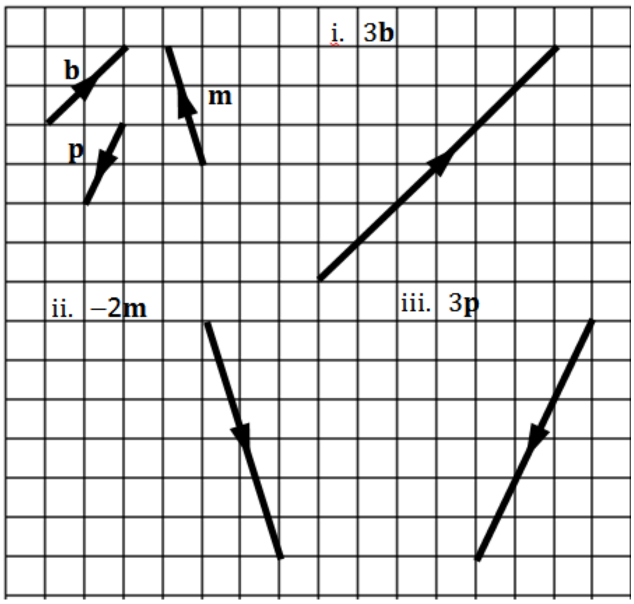
iv.
$$\begin{aligned}5\mathbf{a}+2\mathbf{b}-4\mathbf{c} &= 5\begin{pmatrix}2\\1\end{pmatrix}+2\begin{pmatrix}4\\-1\end{pmatrix}-4\begin{pmatrix}-2\\-4\end{pmatrix}\\ &= \begin{pmatrix}10\\5\end{pmatrix}+\begin{pmatrix}8\\-2\end{pmatrix}-\begin{pmatrix}-8\\-16\end{pmatrix}\\ &= \begin{pmatrix}10+8+8\\5+(-2)+16\end{pmatrix}\\ &= \begin{pmatrix}26\\19\end{pmatrix}\end{aligned}$$

Practice *(15 minutes)*

1. Ask pupils to work independently to answer questions c. and d.
2. Walk around, if possible, to check the answers and clear any misconceptions.
3. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.

Solutions:

c. Given: $\mathbf{b} = \begin{pmatrix}2\\2\end{pmatrix}$, $\mathbf{m} = \begin{pmatrix}-1\\3\end{pmatrix}$, $\mathbf{p} = \begin{pmatrix}-1\\-2\end{pmatrix}$,



iv.
$$\begin{aligned}2\mathbf{b} &= 2\begin{pmatrix}2\\2\end{pmatrix}\\ &= \begin{pmatrix}4\\4\end{pmatrix}\end{aligned}$$

v.
$$\begin{aligned}-\mathbf{m} &= -\begin{pmatrix}-1\\3\end{pmatrix}\\ &= \begin{pmatrix}1\\-3\end{pmatrix}\end{aligned}$$

vi.
$$\tfrac{1}{3}\mathbf{m} = \tfrac{1}{3}\begin{pmatrix}-1\\3\end{pmatrix}$$

vii.
$$-2\mathbf{p} = -2\begin{pmatrix}-1\\-2\end{pmatrix}$$

$= \begin{pmatrix} -1 \\ 9 \end{pmatrix}$ $\qquad\qquad = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$

viii
$$\begin{aligned} 2\mathbf{m} + \mathbf{p} &= 2\begin{pmatrix} -1 \\ 3 \end{pmatrix} + \begin{pmatrix} -1 \\ -2 \end{pmatrix} \\ &= \begin{pmatrix} -2 \\ 6 \end{pmatrix} + \begin{pmatrix} -1 \\ -2 \end{pmatrix} \\ &= \begin{pmatrix} -2+(-1) \\ 6+(-2) \end{pmatrix} \\ &= \begin{pmatrix} -3 \\ 4 \end{pmatrix} \end{aligned}$$

d. Given: $\mathbf{a} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$, $\mathbf{c} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$

i
$$\begin{aligned} 3\mathbf{a} + 2\mathbf{x} &= 4\mathbf{b} \\ 3\begin{pmatrix} 4 \\ 2 \end{pmatrix} + 2\mathbf{x} &= 4\begin{pmatrix} 3 \\ -1 \end{pmatrix} \\ \begin{pmatrix} 12 \\ 6 \end{pmatrix} + 2\mathbf{x} &= \begin{pmatrix} 12 \\ -4 \end{pmatrix} \\ 2\mathbf{x} &= \begin{pmatrix} 12 \\ -4 \end{pmatrix} - \begin{pmatrix} 12 \\ 6 \end{pmatrix} \\ &= \begin{pmatrix} 12-12 \\ -4-6 \end{pmatrix} \\ &= \begin{pmatrix} 0 \\ -10 \end{pmatrix} \\ \mathbf{x} &= \begin{pmatrix} 0 \\ -10 \end{pmatrix} \div 2 \\ \mathbf{x} &= \begin{pmatrix} 0 \\ -5 \end{pmatrix} \end{aligned}$$

ii.
$$\begin{aligned} \mathbf{a} - 2\mathbf{x} &= 4\mathbf{c} \\ \begin{pmatrix} 4 \\ 2 \end{pmatrix} - 2\mathbf{x} &= 4\begin{pmatrix} -2 \\ 1 \end{pmatrix} \\ \begin{pmatrix} 4 \\ 2 \end{pmatrix} - 2\mathbf{x} &= \begin{pmatrix} -8 \\ 4 \end{pmatrix} \\ \begin{pmatrix} 4 \\ 2 \end{pmatrix} - \begin{pmatrix} -8 \\ 4 \end{pmatrix} &= 2\mathbf{x} \\ \begin{pmatrix} 4-(-8) \\ 2-4 \end{pmatrix} &= 2\mathbf{x} \\ 2\mathbf{x} &= \begin{pmatrix} 12 \\ -2 \end{pmatrix} \\ \mathbf{x} &= \begin{pmatrix} 12 \\ -2 \end{pmatrix} \div 2 \\ &= \begin{pmatrix} 6 \\ -1 \end{pmatrix} \end{aligned}$$

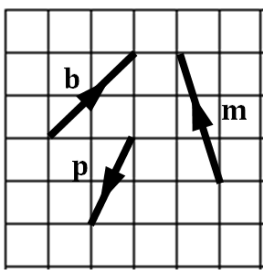
iii.
$$\begin{aligned} 2\mathbf{x} + 3\mathbf{b} &= \mathbf{c} \\ 2\mathbf{x} + 3\begin{pmatrix} 3 \\ -1 \end{pmatrix} &= \begin{pmatrix} -2 \\ 1 \end{pmatrix} \\ 2\mathbf{x} + \begin{pmatrix} 9 \\ -3 \end{pmatrix} &= \begin{pmatrix} -2 \\ 1 \end{pmatrix} \\ 2\mathbf{x} &= \begin{pmatrix} -2 \\ 1 \end{pmatrix} - \begin{pmatrix} 9 \\ -3 \end{pmatrix} \\ &= \begin{pmatrix} -2-9 \\ 1-(-3) \end{pmatrix} \\ &= \begin{pmatrix} -11 \\ 4 \end{pmatrix} \\ \mathbf{x} &= \begin{pmatrix} -11 \\ 4 \end{pmatrix} \div 2 \\ \mathbf{x} &= \begin{pmatrix} -5.5 \\ 2 \end{pmatrix} \end{aligned}$$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM3-L076 in the Pupil Handbook.

[QUESTIONS]

a. Using the vectors **b**, **p** and **m** from the grid shown, draw the following vectors on a grid. Label each vector and show its direction with an arrow.



i. 2**b**   ii. –3**m**   iii. 4**p**
iv. $\frac{1}{2}$**b**   v. –2**p**

b. If $\mathbf{a} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix} -2 \\ -4 \end{pmatrix}$ find:

i. **a** + 2**b**   ii. 4**a** + 3**c**   iii. 6**a** – 3**b**   iv. 5**a** + 2**b** –4**c**

c. Using the vectors **b**, **p** and **m** from the grid in question a.:
Draw the vectors below.

i. $3\mathbf{b}$ ii. $-2\mathbf{m}$ iii. $3\mathbf{p}$

Write the vectors below in the form $\begin{pmatrix} x \\ y \end{pmatrix}$

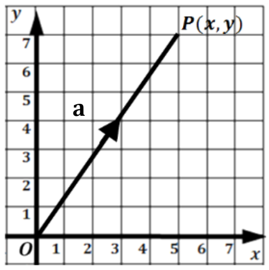
iv. $2\mathbf{b}$ v. $-\mathbf{m}$ vi. $\frac{1}{3}\mathbf{m}$ vii. $-2\mathbf{p}$ viii. $2\mathbf{m} + \mathbf{p}$

d. If $\mathbf{a} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$, $\mathbf{c} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$ solve for $\mathbf{x}$ in the equations below.

i. $3\mathbf{a} + 2\mathbf{x} = 4\mathbf{b}$

ii. $4\mathbf{a} - \mathbf{x} = \mathbf{c}$

iii. $2\mathbf{x} + 3\mathbf{b} = \mathbf{c}$

Lesson Title: Position vectors	**Theme:** Vectors and Transformations	
Lesson Number: M3-L077	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcomes By the end of the lesson, pupils will be able to: 1. Define the position vector of a point. 2. Express two given points as a vector.	**Preparation** 1. Draw the diagram below on the board.  2. Write the questions found at the end of this lesson plan on the board.	

Opening *(4 minutes)*

1. Ask pupils to write the co-ordinates of the point P shown in the diagram on the board.
2. Invite a volunteer to give their answer. (Answer: $P(5,7)$)
3. Tell pupils that after today's lesson, they will be able to define the position vector of a point. They will also be able to express two given points as a vector.

Teaching and Learning *(20 minutes)*

1. Refer again to the diagram on the board.
2. Explain:
   - $P(x, y)$ is a point on the Cartesian plane with origin O.
   - Vector **a** is the displacement of P from O.
3. Invite a volunteer to say what displacement is. (Example answer: Displacement is the shortest distance travelled in a given direction).
4. Explain:
   - Since displacement gives the position of P relative to the origin O, **a** is called the position vector of P.
   - From the diagram $\overrightarrow{OP} = \mathbf{a} = \binom{5}{7}$
   - We already know the co-ordinates of $P = (5,7)$.
   - Therefore, if a point has co-ordinates (x, y), its position vector is $\binom{x}{y}$.
5. Invite a volunteer to assess question a. and tell the class what information we are given. (Answer: Given: graph showing points A, B, C and D)
6. Invite another volunteer to say what we have been asked to find. (Answer: the position vectors relative to the origin, O)
7. Work through the solution for point A.

   Solution:

   a. i. **Step 1.** Assess and extract the given information from the problem.

Given: graph showing point A

Step 2. Write the co-ordinates and position vector of the given point.

co-ordinates of point A $=$ $(3,7)$

position vector of point A $= \overrightarrow{OA}$

$= \binom{3}{7}$

Step 3. Write the answer.

The position vector relative to the origin of point $A(3,7)$: $\overrightarrow{OA} = \binom{3}{7}$.

8. Ask pupils to work with seatmates to answer the rest of question a.
9. Invite volunteers to give their answers. The rest of the class should check their solution and correct any mistakes.

Solution:

ii. The position vector relative to the origin of $B(5,-3)$: $\overrightarrow{OB} = \binom{5}{-3}$

iii. The position vector relative to the origin of $C(-7,-4)$: $\overrightarrow{OC} = \binom{-7}{-4}$

iv. The position vector relative to the origin of $D(-7,3)$: $\overrightarrow{OD} = \binom{-7}{3}$

10. Explain:
    - Position vectors can be used to express 2 given points as a vector.
    - Let $A(x_1, y_1)$ and $B(x_2, y_2)$ be any 2 points on a Cartesian plane as shown in Figure A.
    - From the diagram, we can see that:

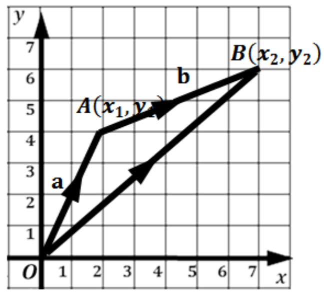


Figure A

$$\begin{aligned} \overrightarrow{OA} + \overrightarrow{AB} &= \overrightarrow{OB} \\ \overrightarrow{AB} &= \overrightarrow{OB} - \overrightarrow{OA} && \text{position vector of } B \text{ –position vector of } A \\ &= \mathbf{b} - \mathbf{a} && \text{equivalent representation of position vectors} \\ &= \binom{x_2}{y_2} - \binom{x_1}{y_1} \\ &= \binom{x_2 - x_1}{y_2 - y_1} \end{aligned}$$

The vector joining A and B is given by: $\overrightarrow{AB} = \binom{x_2 - x_1}{y_2 - y_1}$.

11. Invite a volunteer to assess question b. on the board and extract the given information. (Answer: Given: $A(2,4)$ and $B(7,6)$)
12. Invite a volunteer to say what we have been asked to find in b. i. (Answer: the position vector of points A and B relative to the origin O)

Solution:

b. Given: $A(2,4)$ and $B(7,6)$

i. position vector of point A $= \overrightarrow{OA}$ relative to the origin O

$= \binom{2}{4} = \binom{x_1}{y_1}$

position vector of point B $= \overrightarrow{OB}$ relative to the origin O

$= \binom{7}{6} = \binom{x_2}{y_2}$

ii. $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$= \begin{pmatrix} x_2 - x_1 \\ y_2 - y_1 \end{pmatrix} = \begin{pmatrix} 7-2 \\ 6-4 \end{pmatrix}$

$= \begin{pmatrix} 5 \\ 2 \end{pmatrix}$

This is clearly the case as can be verified from Figure A.

iii. $\overrightarrow{BA} = -\overrightarrow{AB}$

$\overrightarrow{BA} = -\begin{pmatrix} 5 \\ 2 \end{pmatrix}$

$= \begin{pmatrix} -5 \\ -2 \end{pmatrix}$

13. Ask pupils to work with seatmates to answer question c.
14. Invite a volunteer to show their working out on the board. The rest of the class should check their solution and correct any mistakes.

Solution:

c. Given: $P(3{,}5)$ and $Q(6, -2)$

i. position vector of point P $= \overrightarrow{OP}$ relative to the origin O

$= \begin{pmatrix} 3 \\ 5 \end{pmatrix} = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$

position vector of point Q $= \overrightarrow{OQ}$ relative to the origin O

$= \begin{pmatrix} 6 \\ -2 \end{pmatrix} = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$

$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$

$= \begin{pmatrix} x_2 - x_1 \\ y_2 - y_1 \end{pmatrix} = \begin{pmatrix} 6-3 \\ -2-5 \end{pmatrix}$

$= \begin{pmatrix} 3 \\ -7 \end{pmatrix}$

$\overrightarrow{QP} = -\overrightarrow{PQ}$

$= -\begin{pmatrix} 3 \\ -7 \end{pmatrix}$

$= \begin{pmatrix} -3 \\ 7 \end{pmatrix}$

ii. Given: $P(4, -5)$ and $Q(-2, -8)$

position vector of point P $= \overrightarrow{OP}$ relative to the origin O

$= \begin{pmatrix} 4 \\ -5 \end{pmatrix}$

position vector of point Q $= \overrightarrow{OQ}$ relative to the origin O

$= \begin{pmatrix} -2 \\ -8 \end{pmatrix}$

. $\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$

$= \begin{pmatrix} -2-4 \\ -8-(-5) \end{pmatrix} = \begin{pmatrix} -6 \\ -3 \end{pmatrix}$

$\overrightarrow{QP} = -\overrightarrow{PQ}$

$= -\begin{pmatrix} -6 \\ -3 \end{pmatrix}$

$= \begin{pmatrix} 6 \\ 3 \end{pmatrix}$

Practice *(15 minutes)*

1. Ask pupils to work independently to answer questions d. and e.
2. Walk around, if possible, to check the answers and clear any misconceptions.
3. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.

Solutions:

d. Given: $R(1,6)$, $S(x,y)$, $\overrightarrow{RS} = \binom{3}{-5}$

position vector of point R $= \overrightarrow{OR}$ relative to the origin O

$= \binom{1}{6}$

position vector of point S $= \overrightarrow{OS}$ relative to the origin O

$= \binom{x}{y}$

.

$$\overrightarrow{RS} = \overrightarrow{OS} - \overrightarrow{OR}$$

$$\overrightarrow{RS} = \binom{x}{y} - \binom{1}{6}$$

$$\binom{3}{-5} = \binom{x-1}{y-6}$$

Equating the components of the vectors gives:

$$3 = x - 1$$

$$\Rightarrow x = 4$$

$$-5 = y - 6$$

$$\Rightarrow y = 1$$

The co-ordinates of S are $(4,1)$.

e. Given: $P(3,4)$, $\overrightarrow{PQ} = \binom{6}{3}$.

position vector of point P $= \overrightarrow{OP}$ relative to the origin O

$= \binom{3}{4}$

position vector of point Q $= \overrightarrow{OQ}$ relative to the origin O

$= \binom{x}{y}$

$$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$$

$$\binom{6}{3} = \binom{x}{y} - \binom{3}{4} = \binom{x-3}{y-4}$$

this gives $6 = x - 3 \Rightarrow x = 9$

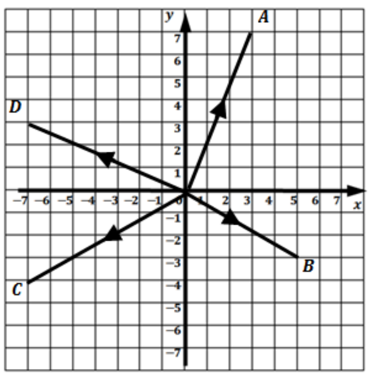
$3 = y - 4 \Rightarrow y = 7$

The position vector of point $Q = \binom{9}{7}$.

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM3-L077 in the Pupil Handbook.

[QUESTIONS]



a. State the position vectors relative to the origin of the points:
   i. A ii. B iii. C iv. D

b. $A(2,4)$ and $B(7,6)$) are points on a Cartesian plane (see Figure A). Find:
   i. the position vector of points A and B relative to the origin O
   ii. the vector $\overrightarrow{AB}$
   iii. the vector $\overrightarrow{BA}$

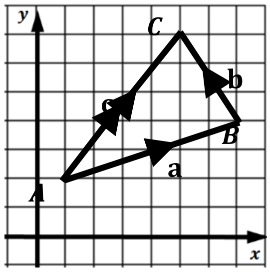
c. Find $\overrightarrow{PQ}$ and $\overrightarrow{QP}$ given the following:

i. $P(3,5)$ and $Q(6,-2)$

ii. $P(4,-5)$ and $Q(-2-8)$

d. $R(1,6)$ and $S(x,y)$ are points on a Cartesian plane such that $\overrightarrow{RS} = \binom{3}{-5}$.
Find the co-ordinates of S.

e. P is the point $(3,4)$ such that $\overrightarrow{PQ} = \binom{6}{3}$. Find the position vector of Q.

<table>
<tr><td>Lesson Title: Triangle law of vector addition</td><td colspan="2">Theme: Vectors and Transformations</td></tr>
<tr><td>Lesson Number: M3-L078</td><td>Class: SSS 3</td><td>Time: 40 minutes</td></tr>
<tr><td>Learning Outcome
By the end of the lesson, pupils will be able to add two vectors using the triangle law of vector addition.</td><td colspan="2">Preparation
1. Draw the diagram below on the board.


2. Write the questions found at the end of this lesson plan on the board.</td></tr>
</table>

Opening *(3 minutes)*

1. Ask pupils to add the 2 vectors in question a.
2. After 1 minute, invite a volunteer to give their answer.
   (Answer: $\mathbf{a}+\mathbf{b}=\binom{6+(-2)}{2+3}=\binom{4}{5}$)
3. Tell pupils that after today's lesson, they will be able to add two vectors using the triangle law of vector addition.

Teaching and Learning *(20 minutes)*

1. Explain: To find the sum of the given vectors, we used what we learned from a previous lesson, namely that:

   If $\mathbf{a}=\binom{x_1}{y_1}$ and $\mathbf{b}=\binom{x_2}{y_2}$, then

   $$\mathbf{a}+\mathbf{b} = \binom{x_1+x_2}{y_1+y_2} \quad (1)$$
2. Refer to the diagram on the board.
   - Vector addition can be shown on a diagram.
   - A point moving from A to B, then from B to C performs the same journey as a point moving from A to C.
   - We can write this using vector notation as:

   $\overrightarrow{AB}+\overrightarrow{BC} = \overrightarrow{AC}$ — $\overrightarrow{AC}$ is called the **resultant** of the 2 vectors. Note the end point (B) of the first vector must be the start point (B) of the second vector.

   $$\mathbf{a}+\mathbf{b} = \mathbf{c} \quad (2)$$

   $$\Rightarrow \quad \mathbf{a}+\mathbf{b}-\mathbf{c} = \mathbf{0} \quad (3)$$ This is the zero vector, $\mathbf{0}=\binom{0}{0}$,
3. Explain:
   - Equation (3) is the triangle law of vector addition which states that:

     If three vectors are represented by the sides of a triangle **taken in order**, then their vector sum must be equal to the zero vector.

- Equation (2) is the usual form of the triangle law to use in solving problems.

4. Invite a volunteer to read the resultant of the 2 vectors from the diagram. (Answer: $\overrightarrow{AC} = \binom{4}{5}$).
5. Invite another volunteer to say what they notice. (Example answer: the vector found by drawing is the same magnitude and direction as the vector found by calculating.)
6. Explain:
   - Using the triangle law of vector addition gives the same result as adding 2 vectors together.

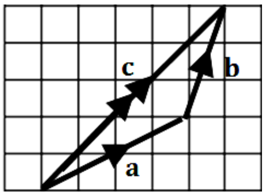
     $$\mathbf{c} \quad = \quad \binom{x_1+x_2}{y_1+y_2} \quad (4)$$
   - We will now do examples to show we get the same answer by drawing using the triangle law of vector addition as by calculating.
7. Invite a volunteer to assess question b. and tell the class what information we are given. (Answer: given: 2 vectors $\mathbf{a}$ and $\mathbf{b}$)
8. Invite another volunteer to say what we have been asked to find. (Answer: Find $\mathbf{a} +$ $\mathbf{b}$**,** by using the triangle law of vector addition and by calculation.)

   Solution:

   b. i. **Step 1.** Assess and extract the given information from the problem.

   given: $\mathbf{a} = \binom{4}{2}$, $\mathbf{b} = \binom{1}{3}$

   Step 2. Add by using the triangle law of vector addition $\mathbf{a} + \mathbf{b} = \mathbf{c}$.



   From the diagram: $\mathbf{a} + \mathbf{b} = \binom{5}{5}$

   Step 3. Add by calculation using $\mathbf{a} + \mathbf{b} = \binom{x_1+x_2}{y_1+y_2}$

   $$\mathbf{a} + \mathbf{b} = \binom{4}{2} + \binom{1}{3} = \binom{4+1}{2+3}$$
   $$= \binom{5}{5}$$

   Step 4. Write the answer.

   The resultant vector $\mathbf{c} = \binom{5}{5}$
9. Invite a volunteer to assess question b. ii. on the board and extract the given information. (Answer: Given: $\mathbf{a} = \binom{6}{3}$, $\mathbf{b} = \binom{-2}{2}$)
10. Invite a volunteer to say what we have been asked to find. (Answer: Find $\mathbf{a} + \mathbf{b}$**,** by using the triangle law of vector addition and by calculation.))

    The rest of the class should check their solution and correct any mistakes.

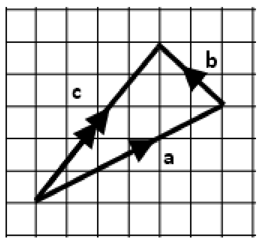
    Solution:

    b. ii. Given: $\mathbf{a} = \binom{6}{3}$, $\mathbf{b} = \binom{-2}{2}$

    $\mathbf{a} + \mathbf{b} = \binom{4}{5}$ from the diagram

    $\mathbf{a} + \mathbf{b} = \binom{6}{3} + \binom{-2}{2} = \binom{6+(-2)}{3+2}$

    $= \binom{4}{5}$



The sum $\mathbf{c} = \binom{4}{5}$

11. Ask pupils to work with seatmates answer question b. iii on the board.
12. Invite a volunteer to show their solution on the board. The rest of the class should check their solution and correct any mistakes.

Solution:

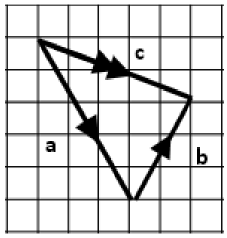
b. iii. Given: $\mathbf{a} = \binom{3}{-5}$, $\mathbf{b} = \binom{2}{3}$

$\mathbf{a} + \mathbf{b} = \binom{5}{-2}$ from the diagram

$\mathbf{a} + \mathbf{b} = \binom{3}{-5} + \binom{2}{3} = \binom{3+2}{-5+3}$

$= \binom{5}{-2}$

The sum $\mathbf{c} = \binom{5}{-2}$



Practice *(15 minutes)*

1. Ask pupils to work independently to answer questions c. and d.
2. Walk around, if possible, to check the answers and clear any misconceptions.
3. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.

Solutions:

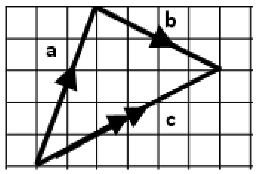
c. i. Given: $\mathbf{a} = \binom{2}{5}$, $\mathbf{b} = \binom{4}{-2}$

$\mathbf{a} + \mathbf{b} = \binom{6}{3}$ from the diagram

$\mathbf{a} + \mathbf{b} = \binom{2}{5} + \binom{4}{-2} = \binom{2+4}{5+(-2)}$

$= \binom{6}{3}$

The sum $\mathbf{c} = \binom{6}{3}$



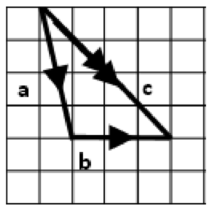
ii. Given: $\mathbf{a} = \binom{1}{-4}$, $\mathbf{b} = \binom{3}{0}$

$\mathbf{a} + \mathbf{b} = \binom{4}{-4}$ from the diagram

$\mathbf{a} + \mathbf{b} = \binom{1}{-4} + \binom{3}{0} = \binom{1+3}{-4+0}$

$= \binom{4}{-4}$

The sum $\mathbf{c} = \binom{4}{-4}$



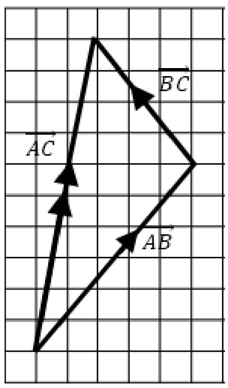
d. i. Given: $\overrightarrow{AB} = \binom{5}{6}$, $\overrightarrow{BC} = \binom{-3}{4}$

$\overrightarrow{AB} + \overrightarrow{BC} = \binom{2}{10}$ from the diagram

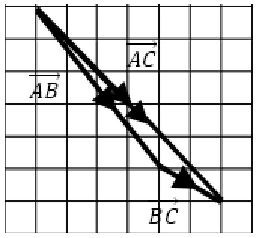
$\overrightarrow{AB} + \overrightarrow{BC} = \binom{5}{6} + \binom{-3}{4} = \binom{5+(-3)}{6+4}$

$= \binom{2}{10}$

The sum $\overrightarrow{AC} = \binom{2}{10}$



ii. Given: $\overrightarrow{AB} = \binom{4}{-5}$, $\overrightarrow{CB} = \binom{-2}{1}$



$$\begin{aligned}
\overrightarrow{AB} + \overrightarrow{BC} &= \binom{6}{-6} \quad \text{from the diagram} \\
\overrightarrow{BC} &= -\overrightarrow{CB} \\
&= -\binom{-2}{1} \quad = \binom{2}{-1} \\
\overrightarrow{AB} + \overrightarrow{BC} &= \binom{4}{-5} + \binom{2}{-1} \quad = \binom{4+2}{-5+(-1)} \\
&= \binom{6}{-6}
\end{aligned}$$

The sum $\overrightarrow{AC} = \binom{6}{-6}$

Closing *(2 minutes)*

1. Ask pupils to write down 1 new thing they learned in this lesson.
2. Invite volunteers to give their answer. (Example answer: How to add vectors using the triangle law of vector addition.)
3. For homework, have pupils do the practice activity PHM3-L078 in the Pupil Handbook.

[QUESTIONS]

a. $\mathbf{a} = \binom{6}{2}$ and $\mathbf{b} = \binom{-2}{3}$, find $\mathbf{a} + \mathbf{b}$.

b. Find $\mathbf{a} + \mathbf{b}$:
- By using the triangle law of vector addition $\mathbf{a} + \mathbf{b} = \mathbf{c}$. (Draw a diagram for each)
- By calculation using $\mathbf{a} + \mathbf{b} = \binom{x_1+x_2}{y_1+y_2}$.

i. $\mathbf{a} = \binom{4}{2}$, $\mathbf{b} = \binom{1}{3}$ ii. $\mathbf{a} = \binom{6}{3}$, $\mathbf{b} = \binom{-2}{2}$ iii. $\mathbf{a} = \binom{3}{-5}$, $\mathbf{b} = \binom{2}{3}$

c. Find the sum of the vectors $\mathbf{a}$ and $\mathbf{b}$.
- By using the triangle law of vector addition $\mathbf{a} + \mathbf{b} = \mathbf{c}$. (Draw a diagram)
- By calculation using $\mathbf{a} + \mathbf{b} = \binom{x_1+x_2}{y_1+y_2}$.

i. $\mathbf{a} = \binom{2}{5}$, $\mathbf{b} = \binom{4}{-2}$ ii. $\mathbf{a} = \binom{1}{-4}$, $\mathbf{b} = \binom{3}{0}$

d. Find the sum of the given vectors.
- By using the triangle law of vector addition. (Draw a diagram)
- By calculation.

i. If $\overrightarrow{AB} = \binom{5}{6}$ and $\overrightarrow{BC} = \binom{-3}{4}$, find $\overrightarrow{AC}$ ii. If $\overrightarrow{AB} = \binom{4}{-5}$ and $\overrightarrow{CB} = \binom{-2}{1}$, find $\overrightarrow{AC}$

Additional questions

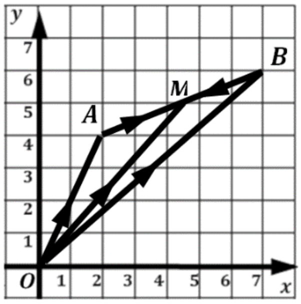
Find the sum of the given vectors**.**

- by using the triangle law of vector addition. (draw a diagram)
- by calculation.

a. $\mathbf{a} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 8 \\ -4 \end{pmatrix}$

b. If $\overrightarrow{XY} = \begin{pmatrix} 6 \\ -7 \end{pmatrix}$ and $\overrightarrow{YZ} = \begin{pmatrix} -4 \\ 0 \end{pmatrix}$, find $\overrightarrow{ZX}$

c. If $\overrightarrow{OA} = \begin{pmatrix} 4 \\ -5 \end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$, find $\overrightarrow{AB}$

Lesson Title: Mid-point of a line segment	**Theme:** Vectors and Transformations	
Lesson Number: M3-L079	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to calculate the mid-point of a line segment.	**Preparation** 1. Draw the diagram below on the board.  2. Write the questions found at the end of this lesson plan on the board.	

Opening *(4 minutes)*

1. Ask pupils to answer question a. on the board.
2. Invite a volunteer to give their answer.
   (Answer: $(2,4), \overrightarrow{OA} = \binom{2}{4}$; $B(7,6)$, $\overrightarrow{OB} = \binom{7}{6}$)
3. Tell pupils that after today's lesson, they will be able to calculate the mid-point of a line segment.

Teaching and Learning *(20 minutes)*

1. Refer to the diagram on the board.
2. Explain:
   - The points $A(x_1, y_1)$ and $B(x_2, y_2)$ lie in a Cartesian plane.
   - M is the mid-point of the line segment AB.
   - From the diagram, we can see that there are two routes from O to M.
3. Invite volunteers to say what the 2 routes are. (Answer: from O via A to M; from O via B to M).
4. Explain: We can write the position vector $\overrightarrow{OM}$ in terms of the position vectors of A and B and the vector $\overrightarrow{AB}$.
5. Write on the board:

$$\overrightarrow{AM} = \overrightarrow{MB} = \frac{1}{2}\overrightarrow{AB} \quad \text{since } M \text{ is the mid-point of } AB$$

$$\overrightarrow{OM} = \overrightarrow{OA} + \overrightarrow{AM}$$

$$= \overrightarrow{OA} + \frac{1}{2}\overrightarrow{AB} \quad (1)$$

$$\overrightarrow{OM} = \overrightarrow{OB} + \overrightarrow{BM}$$

$$= \overrightarrow{OB} - \overrightarrow{MB}$$

$$= \overrightarrow{OB} - \frac{1}{2}\overrightarrow{AB} \qquad (2)$$

$$2\overrightarrow{OM} = \overrightarrow{OA} + \frac{1}{2}\overrightarrow{AB} + \overrightarrow{OB} - \frac{1}{2}\overrightarrow{AB} \qquad \text{add equations (1) and (2)}$$

$$\overrightarrow{OA} + \overrightarrow{OB}$$

$$\overrightarrow{OM} = \frac{\overrightarrow{OA}+\overrightarrow{OB}}{2}$$

$$= \frac{1}{2}\left(\overrightarrow{OA} + \overrightarrow{OB}\right)$$

6. Explain:
   - The position vector of the mid-point of the line segment is an average of the position vectors of the 2 end points.
   - The co-ordinates of the mid-point can similarly be found by finding the average of the x coordinates and y coordinates respectively.

     The mid-point M will have co-ordinates $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$
   - This is called the Mid-point Theorem.
7. Invite a volunteer to assess question b. on the board and extract the given information. (Answer: Given: from part a. position vectors $\overrightarrow{OA} = \binom{2}{4}$ and $\overrightarrow{OB} = \binom{7}{6}$)
8. Invite a volunteer to say what we have been asked to find. (Answer: mid-point of line segment AB)

   Solution:

   b. **Step 1.** Assess and extract the given information from the problem.

   Given: from part a. position vectors $\overrightarrow{OA} = \binom{2}{4}$ and $\overrightarrow{OB} = \binom{7}{6}$

   Step 2. Find the position vector and co-ordinates of the mid-point.

   i.
   $$\overrightarrow{OM} = \frac{1}{2}\left(\overrightarrow{OA} + \overrightarrow{OB}\right)$$
   $$= \frac{1}{2}\left(\binom{2}{4} + \binom{7}{6}\right)$$
   $$= \frac{1}{2}\binom{2+7}{4+6} = \frac{1}{2}\binom{9}{10}$$
   $$\overrightarrow{OM} = \binom{4.5}{5}$$

   ii.
   $$\text{co-ordinates of } M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$
   $$= (4.5, 5) \qquad \text{write directly using } \overrightarrow{OM}$$

   Step 3. Write the answer.

   The position vector $\overrightarrow{OM} = \binom{4.5}{5}$.

   The co-ordinates of $M = (4.5, 5)$.
9. Invite a volunteer to assess question c. on the board and extract the given information. (Answer: Given: $P(p, 2)$ and $Q(-1, q)$)
10. Invite a volunteer to say what we have been asked to find. (Answer: Find the values of p and q so that $\binom{-2}{3}$ gives the position vector of the mid-point of PQ)

    Solution:

    c. Given: $P(p, 2)$ and $Q(-1, q)$ $\overrightarrow{OM} = \binom{-2}{3}$

    Let the position vectors $\overrightarrow{OP} = \binom{p}{2}$ and $\overrightarrow{OQ} = \binom{-1}{q}$

    $$\overrightarrow{OM} = \frac{1}{2}\left(\overrightarrow{OP} + \overrightarrow{OQ}\right)$$

$$\begin{pmatrix}-2\\3\end{pmatrix} = \frac{1}{2}\left(\begin{pmatrix}p\\2\end{pmatrix} + \begin{pmatrix}-1\\q\end{pmatrix}\right)$$

Multiply both sides by two:

$$\begin{aligned}
\begin{pmatrix}-4\\6\end{pmatrix} &= \begin{pmatrix}p\\2\end{pmatrix} + \begin{pmatrix}-1\\q\end{pmatrix}\\
&= \begin{pmatrix}p-1\\2+q\end{pmatrix}\\
-4 &= p-1\\
-4+1 &= p\\
p &= -3\\
6 &= 2+q\\
6-2 &= q\\
q &= 4
\end{aligned}$$

$p=-3, \quad q=4$

11. Ask pupils to work with seatmates to find the mid-point of the given vectors for problem d.
12. Invite a volunteer to show their working out on the board. The rest of the class should check their solution and correct any mistakes.

Solution:

d. Given: $P(1,3)$ and $Q(2,7)$

Let the position vectors $\overrightarrow{OP} = \begin{pmatrix}1\\3\end{pmatrix}$ and $\overrightarrow{OQ} = \begin{pmatrix}2\\7\end{pmatrix}$

$$\begin{aligned}
\overrightarrow{OM} &= \frac{1}{2}(\overrightarrow{OP} + \overrightarrow{OQ})\\
&= \frac{1}{2}\left(\begin{pmatrix}1\\3\end{pmatrix} + \begin{pmatrix}2\\7\end{pmatrix}\right)\\
&= \frac{1}{2}\begin{pmatrix}1+2\\3+7\end{pmatrix} = \frac{1}{2}\begin{pmatrix}3\\10\end{pmatrix}\\
\overrightarrow{OM} &= \begin{pmatrix}1.5\\5\end{pmatrix}
\end{aligned}$$

The position vector of M is $\begin{pmatrix}1.5\\5\end{pmatrix}$

Practice *(15 minutes)*

1. Ask pupils to work independently to answer questions e. and f.
2. Walk around, if possible, to check the answers and clear any misconceptions.
3. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.

Solutions:

e. Given: $\overrightarrow{OA} = \begin{pmatrix}0\\8\end{pmatrix}$ and $\overrightarrow{OM} = \begin{pmatrix}-4\\8\end{pmatrix}$

$$\begin{aligned}
\overrightarrow{OM} &= \frac{1}{2}(\overrightarrow{OA} + \overrightarrow{OB})\\
\begin{pmatrix}-4\\8\end{pmatrix} &= \frac{1}{2}\left(\begin{pmatrix}0\\8\end{pmatrix} + \overrightarrow{OB}\right)\\
2\times\begin{pmatrix}-4\\8\end{pmatrix} &= \begin{pmatrix}0\\8\end{pmatrix} + \overrightarrow{OB}\\
\begin{pmatrix}-8\\16\end{pmatrix} &= \begin{pmatrix}0\\8\end{pmatrix} + \overrightarrow{OB}\\
\begin{pmatrix}-8\\16\end{pmatrix} - \begin{pmatrix}0\\8\end{pmatrix} &= \overrightarrow{OB}\\
\overrightarrow{OB} &= \begin{pmatrix}-8-0\\16-8\end{pmatrix}\\
&= \begin{pmatrix}-8\\8\end{pmatrix}
\end{aligned}$$

Point B has coordinate $(-8, 8)$

f. Given: $A(4,-7)$, $B(-2,3)$ and $Y(5,1)$

Let the position vectors $\overrightarrow{OA} = \binom{4}{-7}$, $\overrightarrow{OB} = \binom{-2}{3}$ and $\overrightarrow{OY} = \binom{5}{1}$

$$\begin{aligned}
\overrightarrow{OX} &= \tfrac{1}{2}\left(\overrightarrow{OA} + \overrightarrow{OB}\right) \\
&= \tfrac{1}{2}\left(\binom{4}{-7} + \binom{-2}{3}\right) \\
&= \tfrac{1}{2}\binom{4+(-2)}{-7+3} = \tfrac{1}{2}\binom{4-2}{-7+3} \\
&= \tfrac{1}{2}\binom{2}{-4} \\
\overrightarrow{OX} &= \binom{1}{-2} \\
\overrightarrow{XY} &= \overrightarrow{OY} - \overrightarrow{OX} \\
\overrightarrow{XY} &= \binom{5}{1} - \binom{1}{-2} \\
&= \binom{5-1}{1-(-2)} \\
\overrightarrow{XY} &= \binom{4}{3}
\end{aligned}$$

The vector $\overrightarrow{XY} = \binom{4}{3}$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM3-L079 in the Pupil Handbook.

[QUESTIONS]

a. State the position vectors relative to the origin of $A(2, 4)$ and $B(7,6)$.

b. Find:
   i. The position vector of point M from question a.
   ii. The mid-point of the line segment AB.

c. $P(p, 2)$ and $Q(-1, q)$ are points in a Cartesian plane. Find the values of p and q so that $\binom{-2}{3}$ gives the position vector of the mid-point of PQ.

d. $P(1,3)$ and $Q(2,7)$ are points in a Cartesian plane. If M is the mid-point of PQ, find the position vector of M.

e. The position vector of the mid-point of the line segment AB is given by $\binom{-4}{8}$. If point A has co-ordinates $(0,8)$, find the co-ordinates of point B.

f. $A(4,-7)$, $B(-2,3)$ and $Y(5,1)$ are three points in a Cartesian plane. If X is the mid-point of AB, find $\overrightarrow{XY}$.

Additional question

g. $R(9,1)$ and $S(-\frac{u}{2},3)$ are points in a Cartesian plane. Find the values of u and v such that $\binom{2u}{v}$ gives the position vector of the mid-point of RS.

Solution:

g. given: $R(9,1)$ and $S = \left(-\frac{u}{2},3\right)$, $\overrightarrow{OM} = \binom{2u}{v}$

Let the position vectors $\overrightarrow{OR} = \binom{9}{1}$ and $\overrightarrow{OS} = \begin{pmatrix}-\frac{u}{2}\\3\end{pmatrix}$

$$\overrightarrow{OM} = \frac{1}{2}\left(\overrightarrow{OR} + \overrightarrow{OS}\right)$$

$$\binom{2u}{v} = \frac{1}{2}\left(\binom{9}{1} + \begin{pmatrix}-\frac{u}{2}\\3\end{pmatrix}\right)$$

$$\binom{4u}{2v} = \left(\binom{9}{1} + \begin{pmatrix}-\frac{u}{2}\\3\end{pmatrix}\right)$$ ` Multiply both sides by 2

$$= \left(\begin{pmatrix}9+(-\frac{u}{2})\\1+3\end{pmatrix}\right) = \left(\begin{pmatrix}9-\frac{u}{2}\\1+3\end{pmatrix}\right)$$

$$4u = 9 - \frac{u}{2}$$

$$8u = 18 - u$$ Multiply throughout by 2

$$8u + u = 18$$

$$9u = 18$$

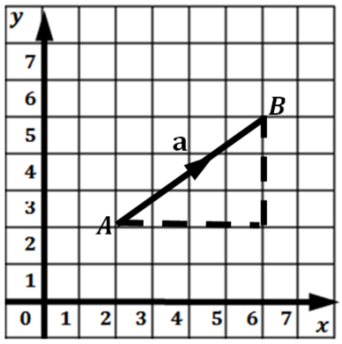
$$u = 2$$

$$2v = 1 + 3$$

$$2v = 4$$

$$v = 2$$

$u = 2, v = 2$ giving $S = (-1, 3)$, $\overrightarrow{OM} = \binom{4}{2}$

Lesson Title: Magnitude of a vector	**Theme:** Vectors and Transformations	
Lesson Number: M3-L080	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to calculate the magnitude of a vector.	**Preparation** 1. Draw the diagram below on the board.  2. Write the questions found at the end of this lesson plan on the board.	

Opening *(4 minutes)*

1. Ask pupils to answer question a. on the board.
2. Invite a volunteer to give their answer. (Answer: $A(2,2), B(6,5)$; $\overrightarrow{AB} = \binom{6-2}{5-2} = \binom{4}{3}$)
3. Tell pupils that after today's lesson, they will be able to calculate the magnitude and direction of a vector.

Teaching and Learning *(20 minutes)*

1. Refer to the diagram on the board.
2. Explain:
   - Consider the 2 points $A(x_1, y_1)$ and $B(x_2, y_2)$.
   - We already know that vector $\overrightarrow{AB} = \binom{x_2-x_1}{y_2-y_1} = \binom{x}{y}$.
   - From the diagram, we can use Pythagoras' Theorem to find the magnitude or length of the vector $\overrightarrow{AB}$.
   - The magnitude of vector $\overrightarrow{AB}$ can be written with the modulus or absolute value notation.

     Examples: $|\overrightarrow{AB}|$, $\overline{AB}$ $|\mathbf{a}|$, $|\underline{a}|$.
   - So, since $\overrightarrow{AB} = \mathbf{a} = \binom{x}{y}$ then

     $$|\overrightarrow{AB}|^2 \quad = \quad \sqrt{x^2 + y^2} \qquad (1)$$
   - Alternatively, we can find the magnitude of $\overrightarrow{AB}$ by substituting directly in equation (1) using the co-ordinates of the given points:

     $$|\overrightarrow{AB}|^2 \quad = \quad \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad (2)$$
3. Invite a volunteer to assess question b. and tell the class what information we are given. (Answer: Given: vector $\overrightarrow{AB} = \binom{4}{3}$ from part a.)

4. Invite another volunteer to say what we have been asked to find in part b. i. (Answer: the magnitude of $\overrightarrow{AB}$)

   Solution:

   b. i. **Step 1.** Assess and extract the given information from the problem.

   Given: vector $\overrightarrow{AB} = \binom{4}{3}$ from part a.

   Step 2. Substitute into the appropriate formula.

$$
\begin{aligned}
|\overrightarrow{AB}|^2 &= \sqrt{x^2 + y^2} \\
&= \sqrt{4^2 + 3^2} \qquad \text{Pythagoras' Theorem} \\
&= \sqrt{16 + 9} \\
|\overrightarrow{AB}| &= \sqrt{25} \\
&= 5
\end{aligned}
$$

   Step 3. Write the answer.

   The magnitude of $\overrightarrow{AB}$: $|\overrightarrow{AB}| = 5$ units.

5. Invite a volunteer to assess question c. on the board and extract the given information. (Answer: Given: $\overrightarrow{PQ} = \binom{-8}{4}$ and $\overrightarrow{RS} = \binom{1}{3}$)
6. Invite a volunteer to say what we have been asked to find. (Answer: find the magnitude of the given vectors to the nearest whole number)

   Solution:

   c. i. Given: $\overrightarrow{PQ} = \binom{-8}{4}$

$$
\begin{aligned}
|\overrightarrow{PQ}| &= \sqrt{x^2 + y^2} \\
&= \sqrt{-8^2 + 4^2} \\
&= \sqrt{64 + 16} \\
&= \sqrt{80} \\
&= 8.94 \\
|\overrightarrow{PQ}| &= 9
\end{aligned}
$$

   The magnitude of $\overrightarrow{PQ} = 9$ units.

7. Ask pupils to work with seatmates to find the magnitude of $\overrightarrow{RS}$ in problem c. ii.
8. Invite a volunteer to show their working out on the board. The rest of the class should check their solution and correct any mistakes.

   Solution:

   c. ii. Given: $\overrightarrow{RS} = \binom{1}{3}$

$$
\begin{aligned}
|\overrightarrow{RS}| &= \sqrt{x^2 + y^2} \\
&= \sqrt{1^2 + 3^2} \\
&= \sqrt{1 + 9} \\
&= \sqrt{10} \\
&= 3.16 \\
|\overrightarrow{RS}| &= 3
\end{aligned}
$$

   The magnitude of $\overrightarrow{RS} = 3$ units.

Practice *(15 minutes)*

1. Ask pupils to work independently to answer questions d. through f.
2. Walk around, if possible, to check the answers and clear any misconceptions.
3. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.

   Solutions:

d. i. Given: $\mathbf{p} = \binom{4}{-6}$.

$$\begin{aligned} |\mathbf{p}| &= \sqrt{x^2 + y^2} \\ &= \sqrt{4^2 + (-6)^2} \\ &= \sqrt{16 + 36} \\ &= \sqrt{52} \end{aligned}$$

ii.

$$\begin{aligned} |\mathbf{p}| &= 2\sqrt{13} \\ &= 7.211 \\ |\mathbf{p}| &= 7.21 \text{ units} \end{aligned}$$

e. i. Given: $\overrightarrow{AB} = \binom{3}{4}$

$$\begin{aligned} |\overrightarrow{AB}| &= \sqrt{x^2 + y^2} \\ &= \sqrt{3^2 + 4^2} \\ &= \sqrt{9 + 16} \\ &= \sqrt{25} \\ |\overrightarrow{AB}| &= 5 \end{aligned}$$

The magnitude of $\overrightarrow{AB} = 5$ units.

ii. Given: $\overrightarrow{BC} = \binom{5}{0}$

$$\begin{aligned} |\overrightarrow{BC}| &= \sqrt{x^2 + y^2} \\ &= \sqrt{5^2 + 0^2} \\ &= \sqrt{25 + 0} \\ &= \sqrt{25} \\ |\overrightarrow{BC}| &= 5 \end{aligned}$$

The magnitude of $\overrightarrow{BC} = 5$ units.

iii. Given: $\overrightarrow{CD} = \binom{-3}{4}$

$$\begin{aligned} |\overrightarrow{CD}| &= \sqrt{x^2 + y^2} \\ &= \sqrt{-3^2 + 4^2} \\ &= \sqrt{9 + 16} \\ &= \sqrt{25} \\ |\overrightarrow{CD}| &= 5 \end{aligned}$$

The magnitude of $\overrightarrow{CD} = 5$ units.

vi. Given: $\overrightarrow{DA} = \binom{-5}{0}$

$$\begin{aligned} |\overrightarrow{CD}| &= \sqrt{x^2 + y^2} \\ &= \sqrt{-5^2 + 0^2} \end{aligned}$$

$$= \sqrt{25 + 0}$$
$$= \sqrt{25}$$
$$|\overrightarrow{CD}| = 5$$

The magnitude of $\overrightarrow{CD} = 5$ units.

v. The shape is a square.

f. Given: $\binom{x}{6}$

$$\sqrt{x^2 + y^2} = 10$$
$$\sqrt{x^2 + 6^2} = 10$$

Square both sides

$$x^2 + 36 = 100$$
$$x^2 = 100 - 36$$
$$x^2 = 64$$
$$x = \sqrt{64}$$
$$= 8$$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM3-L080 in the Pupil Handbook.

[QUESTIONS]

a. Write the vector joining the points A and B on the diagram on the board.

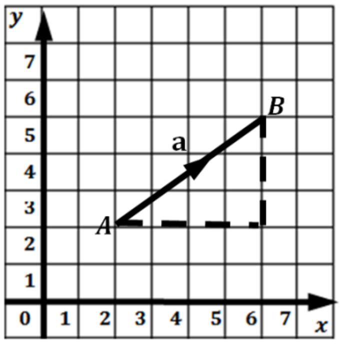
b. Find the magnitude of $\overrightarrow{AB}$ from part a.

c. Find the magnitude of the given vectors to the nearest whole number:
   i. $\overrightarrow{PQ} = \binom{-8}{4}$ ii. $\overrightarrow{RS} = \binom{1}{3}$

d. Find the magnitude of the vector $\mathbf{p} = \binom{4}{-6}$.
   Give your answer: i. as a surd ($p\sqrt{q}$) ii. to 3 significant figures

e. Find the magnitude of the vectors below.
   i. $\overrightarrow{AB} = \binom{3}{4}$ ii. $\overrightarrow{BC} = \binom{5}{0}$ iii. $\overrightarrow{CD} = \binom{-3}{4}$ iv. $\overrightarrow{DA} = \binom{-5}{0}$
   v. What shape is $ABCD$?

f. A column vector $\binom{x}{6}$ has a magnitude of 10. Find x.

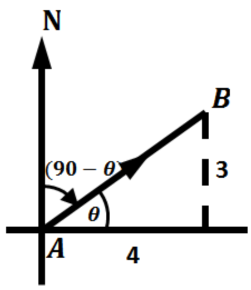
<table>
<tr><td>Lesson Title: Direction of a vector</td><td colspan="2">Theme: Vectors and Transformations</td></tr>
<tr><td>Lesson Number: M3-L081</td><td>Class: SSS 3</td><td>Time: 40 minutes</td></tr>
<tr><td>Learning Outcome
By the end of the lesson, pupils will be able to calculate the direction of a vector.</td><td colspan="2">Preparation
1. Draw the diagram below on the board.

2. Write the questions found at the end of this lesson plan on the board.</td></tr>
</table>

Opening *(4 minutes)*

1. Ask pupils to answer question a. on the board.
2. Invite a volunteer to give their answer. (Answer: $A(2,2), B(6,5)$; $\overrightarrow{AB} = \binom{6-2}{5-2} = \binom{4}{3}$)
3. Tell pupils that after today's lesson, they will be able to calculate the magnitude and direction of a vector.

Teaching and Learning *(20 minutes)*

1. Refer to the diagram on the board.
2. Explain:
   - Before the direction of a vector can be found, we need a diagram of the problem.
   - If not given, draw a sketch of the problem, as shown, to assist in finding the direction of the vector.
   - The direction of the vector is given by the angle it makes when measured from the north in a clockwise direction.
   - We find this angle by first finding the acute angle θ, the vector makes with the x-axis.
   - This angle is given by $\tan\theta = \frac{y}{x}$, where x, y are the components of the resultant vector.
   - From our sketch, we can then deduce the angle the vector makes when measured from the north in a clockwise direction.
   - In our example, this angle is given by $(90 - \theta)$.
   - This is the same as finding the bearing of B from A.



3. Invite a volunteer to assess question b. and tell the class what information we are given. (Answer: Given: vector $\overrightarrow{AB} = \binom{4}{3}$ from part a.)

4. Invite another volunteer to say what we have been asked to find. (Answer: the direction or bearing of $\overrightarrow{AB}$)

Solution:

b. **Step 1.** Assess and extract the given information from the problem.

Given: vector $\overrightarrow{AB} = \binom{4}{3}$ from part a

Step 2. Draw a sketch of the vector (shown on the previous page).

Step 3. Find the direction (bearing) of the vector.

From the diagram the direction of $\overrightarrow{AB}$ is at an angle $(90 - \theta)$ when measured clockwise from the north

$$\tan\theta = \frac{3}{4} = 0.75 \qquad \text{from diagram, use tan ratio}$$

$$\theta = \tan^{-1}(0.75)$$

$$= 36.87°$$

The direction of $\overrightarrow{AB}$ measured from the north:

$$= 90 - 36.87$$

$$= 53.13°$$

Step 4. Write the answer.

The direction of $\overrightarrow{AB}$ measured from the north $= 53°$ to the nearest degree.

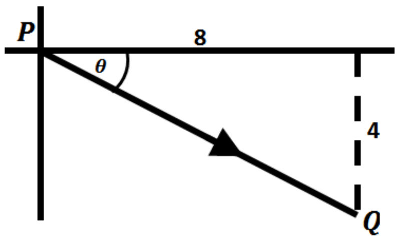
5. Invite a volunteer to assess question c. on the board and extract the given information. (Answer: Given: $\overrightarrow{PQ} = \binom{-8}{4}$)
6. Invite a volunteer to say what we have been asked to find. (Answer: find direction of $\overrightarrow{PQ}$)

Solution:

c. i. given: $\overrightarrow{PQ} = \binom{8}{-4}$

Explain:

- We do not need a scaled diagram. Just a representation of the vector to guide us.
- Find the acute angle θ the vector makes with the x-axis.



$$\tan\theta = \frac{4}{8} = 0.5$$

$$\theta = \tan^{-1}(0.5)$$

$$= 26.57°$$

The direction of $\overrightarrow{PQ}$ measured from the north:

$$= 90 + 26.57$$

$$= 116.57°$$

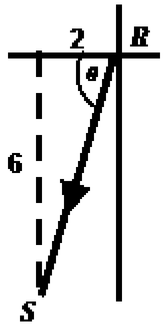
The direction of $\overrightarrow{PQ}$ measured from the north $= 117°$ to the nearest degree.

7. Ask pupils to work with seatmates to find the magnitude and direction question c. ii.
8. Invite a volunteer to show their working out on the board. The rest of the class should check their solution and correct any mistakes.

Solution:

ii. Given: $\overrightarrow{RS} = \binom{-2}{-6}$

Find the acute angle θ, the vector makes with the x-axis.



$$\begin{aligned} \tan\theta &= \tfrac{6}{2} = 3 \\ \theta &= \tan^{-1}(3) \\ &= 71.57° \end{aligned}$$

The direction of $\overrightarrow{RS}$ measured from the north:

$$\begin{aligned} &= 270 - 71.57 \\ &= 198.43° \end{aligned}$$

The direction of $\overrightarrow{PQ}$ measured from the north $= 198°$ to the nearest degree.

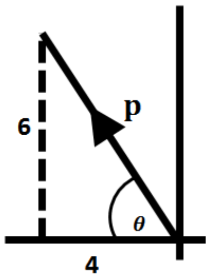
Practice *(15 minutes)*

1. Ask pupils to work independently to answer questions d. through f.
2. Walk around, if possible, to check the answers and clear any misconceptions.
3. Ask volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.

Solutions:

d. Given: $\mathbf{p} = \binom{-4}{6}$.

Find the acute angle θ, the vector makes with the x-axis.



$$\begin{aligned} \tan\theta &= \frac{6}{4} \\ \theta &= \tan^{-1}(1.5) \\ &= 56.3° \end{aligned}$$

The direction of $\mathbf{p}$ measured from the north:

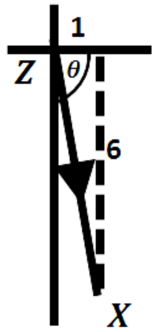
$$\begin{aligned} &= 270 + 56.3 \\ &= 326.3° \end{aligned}$$

The direction of $\mathbf{p}$ measured from the north $= 326°$ to the nearest degree.

e. Given: $\overrightarrow{XY} = \binom{2}{1}$, $\overrightarrow{ZY} = \binom{3}{-5}$,

i.
$$\begin{aligned} \overrightarrow{YZ} &= -\binom{3}{-5} \\ &= \binom{-3}{5} \\ \overrightarrow{XZ} &= \overrightarrow{XY} + \overrightarrow{YZ} \\ &= \binom{2}{1} + \binom{-3}{5} \\ &= \binom{2+(-3)}{1+5} \\ \overrightarrow{XZ} &= \binom{-1}{6} \\ \overrightarrow{ZX} &= -\binom{-1}{6} \\ &= \binom{1}{-6} \end{aligned}$$

ii. Find the acute angle θ, the vector makes with the x-axis:



$$\tan\theta = \frac{6}{1}$$
$$\theta = \tan^{-1}(6)$$
$$= 80.54°$$

The direction of $\overrightarrow{ZX}$ measured from the north:

$$= 90 + 80.54$$
$$= 170.54°$$

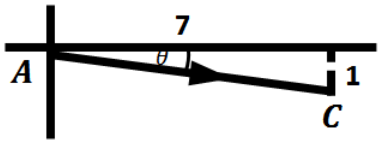
The direction of $\overrightarrow{XZ}$ measured from the north = 171° to the nearest degree.

f. Given: $A(0,1)$ and $C(7,-2)$

i. $\overrightarrow{AC} = \binom{0}{1} + \binom{7}{-2}$
$$= \binom{0+7}{1+(-2)}$$
$$\overrightarrow{AC} = \binom{7}{-1}$$

ii. Find the acute angle θ, the vector makes with the x-axis:

$$\tan\theta = \frac{1}{7}$$
$$\theta = \tan^{-1}(\tfrac{1}{7})$$
$$= 8.13°$$



The direction of $\overrightarrow{AC}$ measured from the north:

$$= 90 + 8.13$$
$$= 98.13°$$

The direction of $\overrightarrow{AC}$ measured from the north = 98° to the nearest degree.

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM3-L081 in the Pupil Handbook.

[QUESTIONS]

a. Write the vector joining the points A and B on the diagram on the board.

b. Find the direction of $\overrightarrow{AB}$ in part a.

c. Find the direction of the given vectors to the nearest whole number:

iii. $\overrightarrow{PQ} = \binom{8}{-4}$ iv. $\overrightarrow{RS} = \binom{-2}{-6}$

d. Find the direction of the vector $\mathbf{p} = \binom{-4}{6}$.

e. If $\overrightarrow{XY} = \binom{2}{1}$ and $\overrightarrow{ZY} = \binom{3}{-5}$, find:

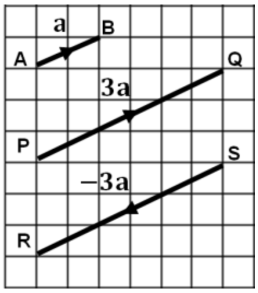
i. $\overrightarrow{ZX}$

ii. The bearing of Z from X correct to the nearest degree.

f. The points $A(0,1), B(4,1), C(7,-2)$ and $D(3,-2)$ are the vertices of a parallelogram. Find:

i. $\overrightarrow{AC}$

ii. The bearing of A from C correct to the nearest degree.

<table>
<tr><td>Lesson Title: Parallel and perpendicular vectors</td><td colspan="2">Theme: Vectors and Transformations</td></tr>
<tr><td>Lesson Number: M3-L082</td><td>Class: SSS 3</td><td>Time: 40 minutes</td></tr>
<tr><td>Learning Outcome
By the end of the lesson, pupils will be able to solve problems with parallel and perpendicular vectors.</td><td colspan="2">Preparation
1. Draw the diagram below on the board.


2. Write the questions found at the end of this lesson plan on the board.</td></tr>
</table>

Opening *(3 minutes)*

1. Ask pupils to answer question a. on the board.
2. After 2 minutes, invite a volunteer to answer. (Answer: $\mathbf{a} = \binom{2}{1}$, $3\binom{2}{1} = \binom{3\times2}{3\times1} = \binom{6}{3}$)
3. Tell pupils that after today's lesson, they will be able to solve problems with parallel and perpendicular vectors.

Teaching and Learning *(20 minutes)*

1. Explain:
   - We know from a previous lesson that:

     If $\mathbf{a} = \binom{x}{y}$ then

     $k\mathbf{a} = \binom{kx}{ky}$ where k is a scalar or number which can be a positive or negative whole number or fraction
   - From this, we know that the vectors $\mathbf{a}$, $\mathbf{3a}$ and $-\mathbf{3a}$ are all scalar multiples of each other.
   - From the diagram, we can see that they are also all parallel to each other.
   - They all make the same acute angle with the x-axis.

     $$\tan\theta = \frac{2}{1} = \frac{6}{3} = 2$$
     $$\theta = \tan^{-1}(2) = 63.42$$
     $$= 63° \text{ to the nearest degree}$$
   - By definition,

     If $\mathbf{b} = k\mathbf{a}$ then $\mathbf{a}$ and $\mathbf{b}$ are parallel
   - If $k > 0$, then $\mathbf{a}$ and $\mathbf{b}$ have the same direction.
   - If $k < 0$, then $\mathbf{a}$ and $\mathbf{b}$ have opposite directions.
   - If $k = 1$, then $\mathbf{a}$ and $\mathbf{b}$ are equal (that is they have the same magnitude and direction).
2. Explain:

- The corresponding components of 2 parallel vectors are in the same ratio with each other.

  If $\binom{a}{b} = k\binom{c}{d}$ that is, they are parallel vectors

  then $a : c = b : d$ corresponding ratios in the same order are equal

3. Invite a volunteer to assess question b. and tell the class what information we are given. (Answer: Given: vector $\binom{10}{15}$, 5 other vectors)
4. Invite another volunteer to say what we have been asked to find. (Answer: Which of the 5 other given vectors is parallel to $\binom{10}{15}$.)

   Solution:

   b. **Step 1.** Assess and extract the given information from the problem.

   given: vector $\binom{10}{15}$, 5 other vectors

   Step 2. Compare each vector with $\binom{10}{15}$.

   If the vectors are parallel, then:

   $a : 10 = b : 15$ where a and b are components of the vector to be compared

   $\frac{a}{10} = \frac{b}{15}$

   i. $\binom{6}{9}$: $\frac{6}{10} = \frac{9}{15} = \frac{3}{5}$ parallel

   ii. $\binom{2}{3}$: $\frac{2}{10} = \frac{3}{15} = \frac{1}{5}$ parallel

   iii. $\binom{8}{-12}$: $\frac{8}{10} \neq \frac{-12}{15}$ not parallel

   iv. $\binom{-4}{-6}$: $\frac{-4}{10} = \frac{-6}{15} = -\frac{2}{5}$ parallel

   v. $\binom{15}{10}$: $\frac{15}{10} \neq \frac{10}{15}$ not parallel

   Step 3. Write the answer.

   The vectors $\binom{6}{9}$, $\binom{2}{3}$ and $\binom{-4}{-6}$ are parallel to $\binom{10}{15}$.

5. Ask pupils to work with seatmates to answer question c.
6. Invite a volunteer to show their solution on the board.

   The rest of the class should check their solution and correct any mistakes.

   Solution:

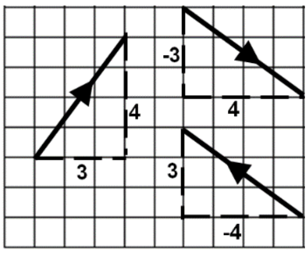
   c. Given: vector $\binom{12}{x}$ is parallel to vector $\binom{9}{3}$

   for parallel vectors: $\frac{12}{9} = \frac{x}{3}$

   $$x = \frac{12\times 3}{9}$$
   $$= 4$$

   The missing component $x = 4$.

7. Explain:
   - There are times when we are asked to find vectors which are perpendicular to each other.
   - Consider the diagram shown on the right.
   - We can see that the vectors $\binom{4}{-3}$ and $\binom{-4}{3}$ are perpendicular to vector $\binom{3}{4}$.
   - Their scalar multiples $\binom{4k}{-3k}$ and $\binom{-4k}{3k}$ where k is a **positive** number are also perpendicular to $\binom{3}{4}$.



- In general, the vectors perpendicular to $\binom{x}{y}$ are:

  $\binom{y}{-x}$ $\binom{-y}{x}$

  and scalar multiples: $\binom{ky}{-kx}$ $\binom{-ky}{kx}$

8. Invite a volunteer to assess question d. on the board and extract the given information. (Example answer: Given: vector $\binom{7}{5}$, 5 other vectors)
9. Invite a volunteer to say what we have been asked to find. (Answer: Which of the 5 other given vectors is perpendicular to $\binom{7}{5}$?)

   Solution:

   d. Given: vector $\binom{7}{5}$, 5 other vectors

   The vectors perpendicular to $\binom{7}{5}$ are $\binom{5}{-7}$, $\binom{-5}{7}$ and their scalar multiples.

   i. $\binom{-5}{-7}$: not perpendicular

   ii. $\binom{10}{-14}$: perpendicular – scalar multiple of $\binom{5}{-7}$

   iii. $\binom{-5}{7}$: perpendicular

   iv. $\binom{-14}{-10}$: not perpendicular

   v. $\binom{5}{-7}$: perpendicular

   The vectors$\binom{10}{-14}$, $\binom{-5}{-7}$ and $\binom{5}{-7}$ are perpendicular to $\binom{7}{5}$.

Practice *(15 minutes)*

1. Ask pupils to work independently to answer questions e. through h.
2. Walk around, if possible, to check the answers and clear any misconceptions.
3. Invite volunteers to come to the board to show their solutions.

   The rest of the class should check their solutions and correct any mistakes.

   Solutions:

   e. Given: vector $\binom{6}{-8}$, 5 other vectors

   $a : 6 \quad = \quad b : -8$ where a and b are components of the vector to be compared

   $\frac{a}{6} = \frac{b}{-8}$

   i. $\binom{-6}{8}$: $\frac{-6}{6} = \frac{8}{-8} = -1$ parallel and opposite

   ii. $\binom{3}{-4}$: $\frac{3}{6} = \frac{-4}{-8} = \frac{1}{2}$ parallel but not opposite

   iii. $\binom{-4}{-3}$: $\frac{-4}{6} \neq \frac{-3}{-8}$ not parallel

   iv. $\binom{8}{6}$: $\frac{8}{6} \neq \frac{6}{-8}$ not parallel

   v. $\binom{-3}{4}$: $\frac{-3}{6} = \frac{4}{-8} = -\frac{1}{2}$ parallel and opposite

   The vectors $\binom{-6}{8}$ and $\binom{-3}{4}$ are parallel and opposite to $\binom{6}{-8}$.

   f. given: vector $\binom{6}{-8}$, 5 other vectors

   The vectors perpendicular to $\binom{6}{-8}$ are $\binom{8}{6}$, $\binom{-8}{-6}$ and their scalar multiples.

   i. $\binom{-6}{8}$: not perpendicular

   ii. $\binom{3}{-4}$: not perpendicular

iii. $\begin{pmatrix}-4\\-3\end{pmatrix}$: perpendicular – $\frac{1}{2}\times\begin{pmatrix}-8\\-6\end{pmatrix}$ iv. $\begin{pmatrix}8\\6\end{pmatrix}$: perpendicular

v. $\begin{pmatrix}-3\\4\end{pmatrix}$: not perpendicular

The vectors $\begin{pmatrix}-4\\-3\end{pmatrix}$ and $\begin{pmatrix}8\\6\end{pmatrix}$ are perpendicular to $\begin{pmatrix}6\\-8\end{pmatrix}$.

g. Given: 5 vectors

i. $\overrightarrow{AB} = 2\mathbf{m} + 4\mathbf{n} = 2(\mathbf{m} + 2\mathbf{n})$

ii. $\overrightarrow{CD} = 6\mathbf{m} - 12\mathbf{n} = 6(\mathbf{m} - 2\mathbf{n})$

iii. $\overrightarrow{EF} = 4\mathbf{m} + 8\mathbf{n} = 4(\mathbf{m} + 2\mathbf{n})$

iv. $\overrightarrow{GH} = -\mathbf{m} - 2\mathbf{n} = -(\mathbf{m} + 2\mathbf{n})$

v. $\overrightarrow{IJ} = 6\mathbf{m} + 16\mathbf{n} = 2(\mathbf{3m} + 8\mathbf{n})$

From the above, it is clear that $\overrightarrow{AB}, \overrightarrow{EF}$ and $\overrightarrow{GH}$ are parallel, as they are scalar multiples of $(\mathbf{m} + 2\mathbf{n})$

h. Given: $\overrightarrow{DC} : \overrightarrow{AB} = 2 : 1$, $\overrightarrow{AB} = \mathbf{m}$

$$\overrightarrow{DC} : \overrightarrow{AB}$$
$$\overrightarrow{DC} : \mathbf{m}$$
$$2 : 1$$
$$\overrightarrow{DC} = 2\mathbf{m}$$

Closing *(2 minutes)*

1. Ask pupils to discuss with seatmates one task they found difficult to do during the lesson.
2. Invite volunteers to share their views with the class. (Example answers: various)
3. Tell pupils to improve their understanding they should do the practice activity PHM3-L082 in the Pupil Handbook.

[QUESTIONS]

a. If $\mathbf{a} = \begin{pmatrix}2\\1\end{pmatrix}$, find $3\mathbf{a}$.

b. Which of the following vectors are parallel to $\begin{pmatrix}10\\15\end{pmatrix}$?

i. $\begin{pmatrix}6\\9\end{pmatrix}$ ii. $\begin{pmatrix}2\\3\end{pmatrix}$ iii. $\begin{pmatrix}8\\-12\end{pmatrix}$ iv. $\begin{pmatrix}-4\\-6\end{pmatrix}$ v. $\begin{pmatrix}15\\10\end{pmatrix}$

c. The vector $\begin{pmatrix}12\\x\end{pmatrix}$ is parallel to the vector $\begin{pmatrix}9\\3\end{pmatrix}$, find x.

d. Which of the following vectors are perpendicular to $\begin{pmatrix}7\\5\end{pmatrix}$?

i. $\begin{pmatrix}-5\\-7\end{pmatrix}$ ii. $\begin{pmatrix}10\\-14\end{pmatrix}$ iii. $\begin{pmatrix}-5\\7\end{pmatrix}$ iv. $\begin{pmatrix}-14\\-10\end{pmatrix}$ v. $\begin{pmatrix}5\\-7\end{pmatrix}$

e. Which of the following vectors are parallel and opposite to $\begin{pmatrix}6\\-8\end{pmatrix}$?

i. $\begin{pmatrix}-6\\8\end{pmatrix}$ ii. $\begin{pmatrix}3\\-4\end{pmatrix}$ iii. $\begin{pmatrix}-4\\-3\end{pmatrix}$ iv. $\begin{pmatrix}8\\6\end{pmatrix}$ v. $\begin{pmatrix}-3\\4\end{pmatrix}$

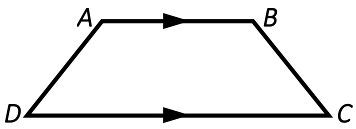
f. Which of the vectors in question e. are perpendicular to $\begin{pmatrix}6\\-8\end{pmatrix}$?

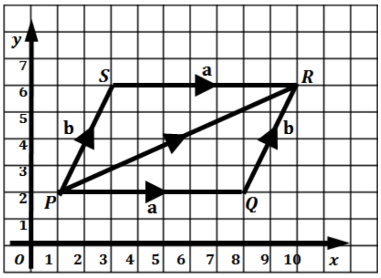
g. Three of the vectors below are parallel. Which are the parallel vectors? Give reasons for your answer.

i. $\overrightarrow{AB} = 2\mathbf{m} + 4\mathbf{n}$ ii. $\overrightarrow{CD} = 6\mathbf{m} - 12\mathbf{n}$ iii. $\overrightarrow{EF} = 4\mathbf{m} + 8\mathbf{n}$

iv. $\overrightarrow{GH} = -\mathbf{m} - 2\mathbf{n}$ v. $\overrightarrow{IJ} = 6\mathbf{m} + 16\mathbf{n}$

h. A regular trapezium $ABCD$ is shown in the diagram.
$\overrightarrow{DC}$ is parallel to $\overrightarrow{AB}$ and $\overrightarrow{DC} : \overrightarrow{AB} = 2 : 1$.
If $\overrightarrow{AB} = \mathbf{m},$ express $\overrightarrow{CD}$ in terms of $\mathbf{m}.$



<table>
<tr><td>Lesson Title: Parallelogram law of vector addition</td><td colspan="2">Theme: Vectors and Transformations</td></tr>
<tr><td>Lesson Number: M3-L083</td><td>Class: SSS 3</td><td>Time: 40 minutes</td></tr>
<tr><td>Learning Outcome
By the end of the lesson, pupils will be able to add two vectors using the parallelogram law of vector addition.</td><td colspan="2">Preparation
1. Draw the diagram below on the board.

2. Write the questions found at the end of this lesson plan on the board.</td></tr>
</table>

Opening (*3 minutes)*

1. Ask pupils to answer question a. on the board.
2. After 1 minute, invite volunteers to give their answers. (Answer: $\overrightarrow{PQ} = \overrightarrow{SR} = \mathbf{a}$, $\overrightarrow{PS} = \overrightarrow{QR} = \mathbf{b}$)
3. Tell pupils that after today's lesson they will be able to add two vectors using the parallelogram law of vector addition.

Teaching and Learning *(20 minutes)*

1. Refer to the diagram on the board.
2. Explain:
   - We are often required to find the resultant vector of 2 vectors starting from the same origin.
   - Consider the parallelogram $PQRS$:
   - Opposite sides are equal in length and are parallel

   Since $\overrightarrow{PQ}$ and $\overrightarrow{SR}$ are in the same direction,

   $$\begin{aligned} \overrightarrow{PQ} &= \overrightarrow{SR} \qquad (1) \\ &= \mathbf{a} \end{aligned}$$

   Similarly,

   $$\begin{aligned} \overrightarrow{PS} &= \overrightarrow{QR} \qquad (2) \\ &= \mathbf{b} \end{aligned}$$

   From the triangle law of vector addition,

   $$\begin{aligned} \overrightarrow{PQ} + \overrightarrow{QR} &= \overrightarrow{PR} \\ &= \mathbf{a} + \mathbf{b} \\ \Rightarrow \quad \overrightarrow{PR} &= \overrightarrow{PQ} + \overrightarrow{PS} \quad \text{from equations (1) and (2)} \end{aligned}$$

   Let $\overrightarrow{PR} = \mathbf{c}$

   then $\mathbf{a} + \mathbf{b} = \mathbf{c}$ (3) where $\mathbf{c}$ is the **resultant vector** of the two vectors $\mathbf{a}$ and $\mathbf{b}$
3. Write on the board:

- The parallelogram law of vector addition states that when two vectors are represented by two adjacent sides of a parallelogram by magnitude and direction, then the resultant of these vectors is represented in magnitude and direction by the diagonal of the parallelogram starting from the same point.

  If $\quad \mathbf{a} = \binom{x_1}{y_1}$ and $\mathbf{b} = \binom{x_2}{y_2}$

  then $\quad \mathbf{a} + \mathbf{b} = \mathbf{c} = \binom{x_1+x_2}{y_1+y_2}$ as before

4. Invite a volunteer to assess question b. and tell the class what information we are given. (Answer: Given: from question a $\overrightarrow{PQ} = \overrightarrow{SR} = \mathbf{a}$, $\overrightarrow{PS} = \overrightarrow{QR} = \mathbf{b}$.)
5. Invite another volunteer to say what we have been asked to find. (Answer: Find $\mathbf{a} + \mathbf{b}$, by using the parallelogram law of vector addition.)

Solution:

b. **Step 1.** Assess and extract the given information from the problem.

Given: from question a. $\overrightarrow{PQ} = \overrightarrow{SR} = \mathbf{a}$, $\overrightarrow{PS} = \overrightarrow{QR} = \mathbf{b}$

Step 2. Write down the column vectors for a and b,

From the diagram: $\mathbf{a} = \binom{7}{0}$

$\mathbf{b} = \binom{2}{4}$

Step 3. Add by using the parallelogram law of vector addition $\mathbf{a} + \mathbf{b} = \mathbf{c}$, where $\mathbf{c} = \binom{x_1+x_2}{y_1+y_2}$.

$$\begin{aligned} \mathbf{a} + \mathbf{b} &= \mathbf{c} \\ &= \binom{7}{0} + \binom{2}{4} = \binom{7+2}{0+4} \\ &= \binom{9}{4} \end{aligned}$$

Step 4. Write the answer.

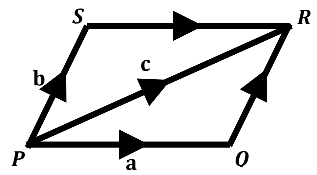
The resultant vector $\mathbf{c} = \binom{9}{4}$.

6. Invite a volunteer to assess question c. on the board and extract the given information. (Answer: Given: the co-ordinates of vertices $P(-6,-3), Q(1,-3)$, $R(4,3)$ and $S(-3,3)$)
7. Invite a volunteer to say what we have been asked to find. (Answer: $\mathbf{a}, \mathbf{b}$ and their resultant vector $\mathbf{c}$ as column vectors.)
8. Work through the solution on the board. Clear any misconceptions.

Solution:

c. Given: the co-ordinates of vertices $P(-6,-3), Q(1,-3)$, $R(4,3)$ and $S(-3,3)$

Write down the position vectors of each point (remember position vectors are relative to the origin O).



$$\overrightarrow{OP} = \binom{-6}{-3} \qquad \overrightarrow{OQ} = \binom{1}{-3}$$

$$\overrightarrow{OR} = \binom{4}{13} \qquad \overrightarrow{OS} = \binom{-3}{3}$$

$$\begin{aligned} \overrightarrow{PQ} &= \overrightarrow{OQ} - \overrightarrow{OP} \\ &= \binom{1}{-3} - \binom{-6}{-3} \\ &= \binom{7}{0} = \mathbf{a} \end{aligned}$$

$$\begin{aligned} \overrightarrow{PS} &= \overrightarrow{OS} - \overrightarrow{OP} \\ &= \binom{-3}{3} - \binom{-6}{-3} \end{aligned}$$

$$= \binom{3}{6} \qquad = \mathbf{b}$$

$$\mathbf{c} = \mathbf{a} + \mathbf{b} \qquad \text{parallelogram law of vector addition}$$
$$= \binom{7}{0} + \binom{3}{6}$$
$$= \binom{10}{6}$$

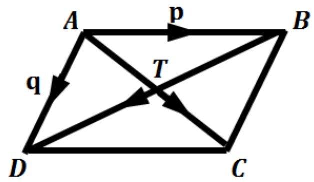
The vectors are: $\mathbf{a} = \binom{7}{0}$, $\mathbf{b} = \binom{3}{6}$, resultant $\mathbf{c} = \binom{10}{6}$.

9. Ask pupils to work with seatmates to answer question d. on the board.
10. Invite a volunteer to show their working out on the board. The rest of the class should check their solution and correct any mistakes.

Solution:

d. Given: $\overrightarrow{AB} = \mathbf{p}$, $\overrightarrow{AD} = \mathbf{q}$

i.
$$\overrightarrow{DC} = \mathbf{p} \qquad \text{parallel}$$
$$\overrightarrow{AC} = \overrightarrow{AD} + \overrightarrow{DC}$$
$$\overrightarrow{AC} = \mathbf{p} + \mathbf{q}$$



ii.
$$\overrightarrow{BC} = \mathbf{q} \qquad \text{parallel}$$
$$\overrightarrow{BD} = \overrightarrow{BC} + \overrightarrow{CD}$$
$$\overrightarrow{BD} = \mathbf{q} + (-\mathbf{p})$$
$$= \mathbf{q} - \mathbf{p}$$

iii
$$\overrightarrow{AT} = \overrightarrow{AC} \qquad T \text{ is the mid-point of } \overrightarrow{AC}$$
$$= \frac{1}{2}\overrightarrow{AC}$$
$$= \frac{1}{2}(\mathbf{p} + \mathbf{q})$$

Practice *(15 minutes)*

1. Ask pupils to work independently to answer questions e. and f.
2. Walk around, if possible, to check the answers and clear any misconceptions.
3. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.

Solutions:

e. Given: $\overrightarrow{PQ} = \mathbf{a} = s\binom{3}{2}$, $\overrightarrow{PS} = \mathbf{b} = t\binom{2}{1}$ and $\mathbf{c} = \binom{7}{1}$

$$\mathbf{a} + \mathbf{b} = \mathbf{c} \qquad \text{parallelogram law of vector addition}$$
$$s\binom{3}{2} + t\binom{2}{1} = \binom{7}{1}$$
$$\binom{3s}{2s} + \binom{2t}{t} = \binom{7}{1}$$
$$3s + 2t = 7 \qquad (1)$$
$$2s + t = 1 \qquad (2)$$
$$\Rightarrow \quad t = 1 - 2s \qquad (3)$$
$$3s + 2(1 - 2s) = 7 \qquad \text{substitute equation (3) into equation (1)}$$
$$3s + 2 - 4s = 7$$
$$-s = 7 - 2$$
$$-s = 5$$
$$s = -5$$

$$t = 1 - 2 \times (-5) \quad \text{substitute s into equation (3)}$$
$$= 1 + 10$$
$$= 11$$

f. given: $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$

i.
$$\overrightarrow{AB} = \overrightarrow{OA} + \overrightarrow{OB}$$
$$\overrightarrow{AB} = \mathbf{a} + \mathbf{b}$$
$$\mathbf{p} = \frac{1}{2}(\mathbf{a} + \mathbf{b})$$

ii.
$$\overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{BC}$$
$$\overrightarrow{OC} = \mathbf{b} + \mathbf{a}$$
$$\boldsymbol{q} = \frac{1}{2}(\mathbf{b} + \mathbf{a})$$

iii. They are the same point.

iv. The diagonals of a parallelogram bisect each other.

Closing *(2 minutes)*

1. Ask pupils to write down 1 new thing they learned in this lesson.
2. Invite volunteers to give their answer. (Example answer: How to add vectors using the parallelogram law of vector addition.)
3. For homework, have pupils do the practice activity PHM3-L083 in the Pupil Handbook.

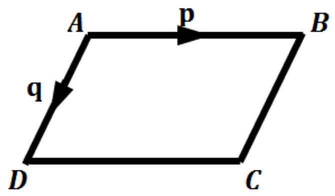
[QUESTIONS]

a. $PQRS$ is a parallelogram.
   i. What vectors are equal to $\mathbf{a}$?
   ii. What vectors are equal to $\mathbf{b}$**?**

b. Use the result from question a. to write the column vectors for $\mathbf{a}$ and $\mathbf{b}$. Find the resultant vector $\mathbf{c}$ using the parallelogram law of vector addition.

c. The parallelogram $PQRS$ has $P(-6,-3), Q(1,-3)$, $R(4,3)$ and $S(-3,3)$. If $\overrightarrow{PQ} = \mathbf{a}$ and $\overrightarrow{PS} = \mathbf{b}$, write $\mathbf{a}, \mathbf{b}$ and their resultant vector $\mathbf{c}$ as column vectors.

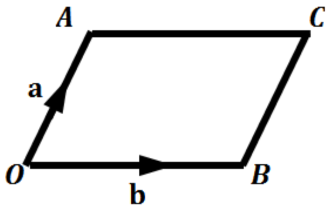
d. $ABCD$ is a parallelogram.
   $\overrightarrow{AB} = \mathbf{p}$ and is parallel to $\overrightarrow{DC}$. $\overrightarrow{AD} = \mathbf{q}$ and is parallel to $\overrightarrow{BC}$.
   Express in terms of $\mathbf{p}$ and $\mathbf{q}$:



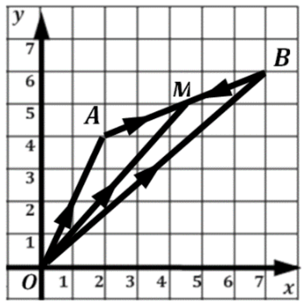
   i. $\overrightarrow{AC}$ ii. $\overrightarrow{BD}$
   iii. AC and BD intersect at T. Express $\overrightarrow{AT}$ in terms of $\mathbf{p}$ and $\mathbf{q}$.

e. In a parallelogram $\overrightarrow{PQ} = \mathbf{a} = s\binom{3}{2}$ and $\overrightarrow{PS} = \mathbf{b} = t\binom{2}{1}$. Find s and t such that the resultant vector $\mathbf{c} = \binom{7}{1}$

f. $OACB$ is a parallelogram with $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.
   P is the mid-point of AB. Q is the mid-point of OC.
   Express in terms of $\mathbf{a}$ and $\mathbf{b}$:



   i. $\overrightarrow{OP}$ ii. $\overrightarrow{QO}$
   iii. What do your answers show about the points P and Q?
   iv. What property of a parallelogram has been proved by this question?

<table>
<tr><td>Lesson Title: Application of vectors – Part 1</td><td colspan="2">Theme: Vectors and Transformations</td></tr>
<tr><td>Lesson Number: M3-L084</td><td>Class: SSS 3</td><td>Time: 40 minutes</td></tr>
<tr><td>Learning Outcome
By the end of the lesson, pupils will be able to apply vectors to solve simple geometric problems.</td><td colspan="2">Preparation
1. Draw the diagram below on the board.

2. Write the questions found at the end of this lesson plan on the board.</td></tr>
</table>

Opening *(1 minute)*

1. Tell pupils that after today's lesson, they will be able to apply vectors to solve simple geometric problems.

Teaching and Learning *(20 minutes)*

1. Explain:
   - Vectors can be used to solve simple problems in geometry.
   - We can describe any point in the Cartesian plane relative to the origin using position vectors.
     We know that given any point $A(x, y)$, the position vector relative to O is $\overrightarrow{OA} = \begin{pmatrix} x \\ y \end{pmatrix}$.
   - We also know that line segments can be described using position vectors.
     For example, $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$, where AB is a line segment and $\overrightarrow{OA}$ and $\overrightarrow{OB}$ are position vectors of the end-points of AB.
   - We can also find the position vector $\overrightarrow{OM}$ of the mid-point of any line segment using the formula $\overrightarrow{OM} = \frac{1}{2}\left(\overrightarrow{OA} + \overrightarrow{OB}\right)$.
2. Invite a volunteer to assess question a. and tell the class what information we are given. (Answer: given: co-ordinates of vertices $A(-6, -3), B(1, -3)$, and $C(3,1)$)
3. Invite another volunteer to say what we have been asked to find. (Answer: the co-ordinates of the vertex D)

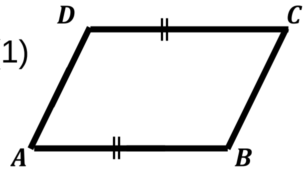
   Solution:

   a. **Step 1.** Assess and extract the given information from the problem.
      Given: co-ordinates of vertices $A(-6, -3), B(1, -3)$, and $C(3,1)$
      Step 2. Write down the position vectors of each point.
      (remember position vectors are relative to the origin)

Let $\overrightarrow{OA} = \mathbf{a} = \begin{pmatrix} -6 \\ -3 \end{pmatrix}$ $\quad \overrightarrow{OB} = \mathbf{b} = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$

$\overrightarrow{OC} = \mathbf{c} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ $\quad \overrightarrow{OD} = \mathbf{b} = \begin{pmatrix} x \\ y \end{pmatrix}$

Since $ABCD$ is a parallelogram.



$\overrightarrow{AB} = \overrightarrow{DC}$ (1)

$$\begin{aligned}
\overrightarrow{AB} &= \overrightarrow{OB} - \overrightarrow{OA} \\
&= \mathbf{b} - \mathbf{a} \\
&= \begin{pmatrix} 1 \\ -3 \end{pmatrix} - \begin{pmatrix} -6 \\ -3 \end{pmatrix} = \begin{pmatrix} 7 \\ 0 \end{pmatrix} \\
\overrightarrow{DC} &= \overrightarrow{OC} - \overrightarrow{OD} \\
&= \mathbf{c} - \mathbf{d} \\
&= \begin{pmatrix} 3 \\ 1 \end{pmatrix} - \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3-x \\ 1-y \end{pmatrix} \\
\begin{pmatrix} 7 \\ 0 \end{pmatrix} &= \begin{pmatrix} 3-x \\ 1-y \end{pmatrix} \quad \text{from equation (1)} \\
7 &= 3 - x \\
\Longrightarrow \quad x &= -4 \\
0 &= 1 - y \\
\Longrightarrow \quad y &= 1
\end{aligned}$$

Step 3. Write the answer.

The co-ordinates of $D = (-4,1)$

4. Invite a volunteer to assess question b. i. and b. ii. on the board and extract the given information. (Example answer: triangle ABC with $A(3,7)$, $\overrightarrow{BA} = \begin{pmatrix} -1 \\ 6 \end{pmatrix}$, $\overrightarrow{BC} = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$)
5. Invite a volunteer to say what we have been asked to find. (Answer: co-ordinates of $B \text{ and } C$)

Solution:

b. Given: triangle ABC with $A(3,7)$, $\overrightarrow{BA} = \begin{pmatrix} -1 \\ 6 \end{pmatrix}$, $\overrightarrow{BC} = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$

Let position vectors for A: $\overrightarrow{OA} = \begin{pmatrix} 3 \\ 7 \end{pmatrix}$, B: $\overrightarrow{OB}$ and C: $\overrightarrow{OC}$

i.
$$\begin{aligned}
\overrightarrow{BA} &= \overrightarrow{OA} - \overrightarrow{OB} \\
\begin{pmatrix} -1 \\ 6 \end{pmatrix} &= \begin{pmatrix} 3 \\ 7 \end{pmatrix} - \overrightarrow{OB} \\
\overrightarrow{OB} &= \begin{pmatrix} 3 \\ 7 \end{pmatrix} - \begin{pmatrix} -1 \\ 6 \end{pmatrix} \\
&= \begin{pmatrix} 3--1 \\ 7-6 \end{pmatrix} \\
&= \begin{pmatrix} 4 \\ 1 \end{pmatrix}
\end{aligned}$$

The co-ordinates of $B = (4,1)$

ii.
$$\begin{aligned}
\overrightarrow{BC} &= \overrightarrow{OC} - \overrightarrow{OB} \\
\begin{pmatrix} 4 \\ -2 \end{pmatrix} &= \overrightarrow{OC} - \begin{pmatrix} 4 \\ 1 \end{pmatrix} \\
\overrightarrow{OC} &= \begin{pmatrix} 4 \\ -2 \end{pmatrix} + \begin{pmatrix} 4 \\ 1 \end{pmatrix} \\
&= \begin{pmatrix} 4+4 \\ -2+1 \end{pmatrix} \\
&= \begin{pmatrix} 8 \\ -1 \end{pmatrix}
\end{aligned}$$

The co-ordinates of $C = (8,-1)$

6. Ask pupils to work with seatmates to answer question c.
7. Invite a volunteer to show their answer on the board. The rest of the class should check their solution and correct any mistakes.

Solution:

c. Given: pentagon $ABCDE$, origin O,

$\overrightarrow{AB} = \begin{pmatrix} 0 \\ -2 \end{pmatrix}$, $\overrightarrow{CB} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$, $\overrightarrow{CD} = \begin{pmatrix} -5 \\ 2 \end{pmatrix}$ and $\overrightarrow{DE} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$.

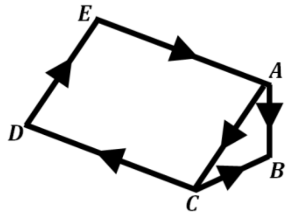
not to scale

Draw a sketch of the problem (shown at right).

If $ACDE$ is a parallelogram, then $\overrightarrow{AC}$ should be parallel to $\overrightarrow{DE}$ and $\overrightarrow{EA}$ should be parallel to $\overrightarrow{CD}$.

$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$ triangle law

$\overrightarrow{BC} = \begin{pmatrix}-2\\-1\end{pmatrix}$



$\overrightarrow{AC} = \begin{pmatrix}0\\-2\end{pmatrix} + \begin{pmatrix}-2\\-1\end{pmatrix}$

$= \begin{pmatrix}0+(-2)\\-2+(-1)\end{pmatrix} = \begin{pmatrix}-2\\-3\end{pmatrix}$

$\overrightarrow{DE} = \begin{pmatrix}2\\3\end{pmatrix}$ and $\overrightarrow{AC} = \begin{pmatrix}-2\\-3\end{pmatrix}$ therefore they are parallel

$\overrightarrow{EC} = \overrightarrow{EA} + \overrightarrow{AC}$ triangle law

$\overrightarrow{EA} = \overrightarrow{EC} - \overrightarrow{AC}$

$\overrightarrow{CE} = \overrightarrow{CD} + \overrightarrow{DE}$ find $\overrightarrow{EC} = -\overrightarrow{CE}$

$= \begin{pmatrix}-5\\2\end{pmatrix} + \begin{pmatrix}2\\3\end{pmatrix}$

$= \begin{pmatrix}-5+(2)\\2+3\end{pmatrix} = \begin{pmatrix}-3\\5\end{pmatrix}$

$\overrightarrow{EC} = \begin{pmatrix}3\\-5\end{pmatrix}$

$\overrightarrow{EA} = \begin{pmatrix}3\\-5\end{pmatrix} - \begin{pmatrix}-2\\-3\end{pmatrix}$

$= \begin{pmatrix}3-(-2)\\-5-(-3)\end{pmatrix} = \begin{pmatrix}5\\-2\end{pmatrix}$

$\overrightarrow{CD} = \begin{pmatrix}-5\\2\end{pmatrix}$ and $\overrightarrow{EA} = \begin{pmatrix}5\\-2\end{pmatrix}$ therefore they are parallel

Hence $ACDE$ is a parallelogram.

Practice *(15 minutes)*

1. Ask pupils to work independently to answer questions d.
2. Walk around, if possible, to check the answers and clear any misconceptions.
3. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.

Solution:

d. Given: triangle PQR with vertices $P(-1,1)$, $Q(2,5)$, and $R(-3,5)$, origin O

Let position vectors for P: $\overrightarrow{OP} = \begin{pmatrix}-1\\1\end{pmatrix}$, Q: $\overrightarrow{OQ} = \begin{pmatrix}2\\5\end{pmatrix}$ and R: $\overrightarrow{OR} = \begin{pmatrix}-3\\5\end{pmatrix}$

i. $\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$

$= \begin{pmatrix}2\\5\end{pmatrix} - \begin{pmatrix}-1\\1\end{pmatrix}$

$= \begin{pmatrix}2-(-1)\\5-1\end{pmatrix} = \begin{pmatrix}3\\4\end{pmatrix}$

Vector $\overrightarrow{PQ} = \begin{pmatrix}3\\4\end{pmatrix}$

$\overrightarrow{QR} = \overrightarrow{OR} - \overrightarrow{OQ}$

$= \begin{pmatrix}-3\\5\end{pmatrix} - \begin{pmatrix}2\\5\end{pmatrix}$

$= \begin{pmatrix}-3-2\\5-5\end{pmatrix} = \begin{pmatrix}-5\\0\end{pmatrix}$

Vector $\overrightarrow{QR} = \begin{pmatrix}-5\\0\end{pmatrix}$

$\overrightarrow{RP} = \overrightarrow{OP} - \overrightarrow{OR}$

$= \begin{pmatrix}-1\\1\end{pmatrix} - \begin{pmatrix}-3\\5\end{pmatrix}$

$= \begin{pmatrix}-1-(-3)\\1-5\end{pmatrix} = \begin{pmatrix}2\\-4\end{pmatrix}$

Vector $\overrightarrow{RP} = \begin{pmatrix}2\\-4\end{pmatrix}$

ii. PQR is isosceles if 2 of its sides are equal in length.

Use Pythagoras' Theorem $c^2 = \sqrt{a^2 + b^2}$ to find the length of the sides

$|\overrightarrow{PQ}| = \sqrt{3^2 + 4^2}$

$= \sqrt{9 + 16}$

$|\overrightarrow{QR}| = \sqrt{(-5)^2 + 0^2}$

$= \sqrt{25 + 0}$

$= \sqrt{25} = 5$ units $\qquad\qquad$ $= \sqrt{25} = 5$ units

$|\overrightarrow{RP}| = \sqrt{2^2 + (-4)^2}$

$= \sqrt{4 + 16}$

$= \sqrt{20} = 4.47$ units

Since $|\overrightarrow{PQ}| = |\overrightarrow{QR}|$, PQR is isosceles.

iii. $\overrightarrow{OM} = \frac{1}{2}(\overrightarrow{OP} + \overrightarrow{OR})$

$= \frac{1}{2}\left(\binom{-1}{1} + \binom{-3}{5}\right)$

$= \frac{1}{2}\binom{-1+(-3)}{1+5}$

$= \frac{1}{2}\binom{-4}{6}$

$= \binom{-2}{3}$

Vector $\overrightarrow{OM} = \binom{-2}{3}$

iv. $\overrightarrow{QM} = \overrightarrow{OM} - \overrightarrow{OQ}$

$= \binom{-2}{3} - \binom{2}{5}$

$= \binom{-2-2}{3-5} = \binom{-4}{-2}$

v. $\overrightarrow{QM} = \binom{-4}{-2}$, $\overrightarrow{RP} = \binom{2}{-4}$

Hence $\overrightarrow{QM}$ is perpendicular to $\overrightarrow{RP}$

Closing *(4 minutes)*

1. Ask pupils to tell seatmates one new thing they learned or understood better after this lesson.
2. Invite volunteers to tell the class what they learned/understood better. (Answers: various)
3. Tell pupils that they can get more practice from the practice activity PHM3-L084 in the Pupil Handbook.

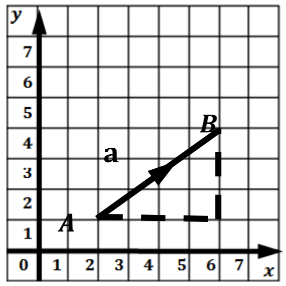
[QUESTIONS]

a. A parallelogram $ABCD$ has vertices $A(-6,-3), B(1,-3)$, and $C(3,1)$. Find the co-ordinates of the vertex D.

b. A triangle ABC has vertex $A(3,7)$. $\overrightarrow{BA} = \binom{-1}{6}$ and $\overrightarrow{BC} = \binom{4}{-2}$.
   Find the co-ordinates of: i. B $\qquad$ ii. C

c. The points A, B, C, D and E are the vertices of a pentagon. O is the origin. Show that $ACDE$ is a parallelogram.
   $\overrightarrow{AB} = \binom{0}{-2}, \overrightarrow{CB} = \binom{2}{1}, \overrightarrow{CD} = \binom{-5}{2}$ and $\overrightarrow{DE} = \binom{2}{3}$.

d. PQR is a triangle with vertices $P(-1,1)$, $Q(2,5)$, and $R(-3,5)$
   i. If O is the origin, express $\overrightarrow{PQ}$, $\overrightarrow{QR}$ and $\overrightarrow{RP}$ as column vectors.
   ii. Hence, show that triangle PQR is isosceles.
   iii. M is the mid-point of the side PR. Express $\overrightarrow{OM}$ as a vector.
   iv. Express $\overrightarrow{QM}$ as a column vector.
   v. State the relationship between $\overrightarrow{QM}$ and $\overrightarrow{RP}$.

<table>
<tr><td>Lesson Title: Application of vectors – Part 2</td><td colspan="2">Theme: Vectors and Transformations</td></tr>
<tr><td>Lesson Number: M3-L085</td><td>Class: SSS 3</td><td>Time: 40 minutes</td></tr>
<tr><td>Learning Outcome
By the end of the lesson, pupils will be able to apply vectors to solve simple real-world problems.</td><td colspan="2">Preparation
1. Draw the diagram below on the board.

2. Write the questions found at the end of this lesson plan on the board.</td></tr>
</table>

Opening *(1 minute)*

1. Tell pupils that after today's lesson, they will be able to apply vectors to solve simple real-world problems.

Teaching and Learning *(20 minutes)*

1. Explain:
   - Vectors can be used to solve simple problems in the real world.
   - Problems usually require us to find the magnitude and direction of the resultant vector of forces, velocities or displacement.
2. Invite a volunteer to give the formula to find the magnitude of a vector. (Answer: magnitude $= \sqrt{x^2 + y^2}$ (Pythagoras' Theorem))
3. Invite a volunteer to give the method to find the direction of a vector. (Example answer: Draw a sketch of the problem; find the acute angle θ, the vector makes with the x-axis given by $\tan\theta = \frac{y}{x}$; use sketch to find the direction of the vector)
4. Invite a volunteer to assess question a. and tell the class what information we are given. (Answer: given: speed of canoe = 2 m/s, speed of river current = 1.5 m/s)
5. Invite another volunteer to say the first thing we have been asked to find. (Answer: resultant velocity of canoe as a column vector)

   Solution:

   a. **Step 1.** Assess and extract the given information from the problem.
      Given: speed of canoe east = 2 m/s,
      speed of river current south = 1.5 m/s

      Step 2. Write the position vectors of canoe and current.

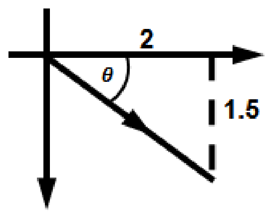
   i. Sketch the diagram (not to scale).
      Since movement east is positive on the x-axis
      and movement south is negative on the y-axis,

position vector of canoe $= \binom{2}{0}$

position vector of current $= \binom{0}{-1.5}$

$\therefore$ resultant velocity of canoe $= \binom{2}{0} + \binom{0}{-1.5}$

$= \binom{2}{-1.5}$



Step 3. Find the magnitude of the velocity.

magnitude of velocity $= \sqrt{(2)^2 + (-1.5)^2} = \sqrt{4 + 2.25}$

$= \sqrt{6.25}$

$= 2.5$ m/s

Step 4. Write the answer to i.

The velocity has a magnitude of 2.5 m/s.

ii. **Step 5.** Find the acute angle θ, the vector makes with the x-axis.

$\tan\theta = \frac{1.5}{2} = 0.75$ from diagram

$\theta = \tan^{-1}(0.75)$

$= 36.87°$

Step 6. Find the direction of the velocity.

direction of velocity $= 90 + 36.87$ from diagram

$= 126.87°$

The velocity has a direction of $127°$.

iii. **Step 7.** Find the time it takes the man to cross river.

width of river $= 20$ m given

time taken to cross $t = \frac{20}{2.5}$ from distance formula $d = st$

$= 8$ s

The time taken to cross the river is 8 s.

6. Invite a volunteer to assess question b. i. on the board and extract the given information. (Example answer: the co-ordinates of vertical poles placed at the vertices $A(4,4)$, $B(1,-1)$, $C(-5,-1)$ and $D(-2,4)$ relative to a point O)
7. Invite a volunteer to say the first thing we have been asked to find. (Answer: vector $\overrightarrow{AC}$)

Solution:

b. Given: the co-ordinates of vertical poles placed at the vertices $A(4,4)$, $B(1,-1)$, $C(-5,-1)$ and $D(-2,4)$ relative to a point O.

Let position vectors for A: $\overrightarrow{OA} = \binom{4}{4}$, B: $\overrightarrow{OB} = \binom{1}{-1}$,

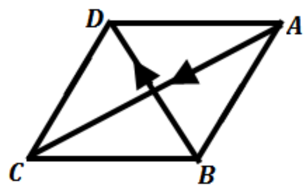
C: $\overrightarrow{OC} = \binom{-5}{-1}$ and D: $\overrightarrow{OD} = \binom{-2}{4}$

i. $\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA}$

$= \binom{-5}{-1} - \binom{4}{4} = \binom{-5-4}{-1-4}$

$\binom{-9}{-5}$

Vector $\overrightarrow{AC} = \binom{-9}{-5}$



ii. $|\overrightarrow{AC}| = \sqrt{(-9)^2 + (-5)^2}$

$= \sqrt{81 + 25} = \sqrt{106}$

$= 10.296$

$= 10.3$ units

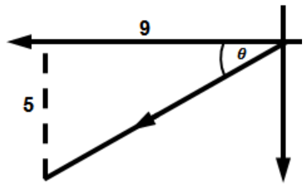
Magnitude of $\overrightarrow{AC}$ = 10.3 units to 1 d.p

iii. $\tan\theta = \frac{5}{9} = 0.5556$

$\theta = \tan^{-1}(0.5556) = 29.05°$

bearing $= 270 - 29.05$

$= 240.95$



Bearing of $\overrightarrow{AC} = 241°$ to the nearest degree.

8. Ask pupils to work with seatmates to answer question b. iv, v. and vi.
9. Invite a volunteer to show their answer on the board. The rest of the class should check their solution and correct any mistakes.

Solution:

Given: as above

iv. $\overrightarrow{BD} = \overrightarrow{OD} - \overrightarrow{OB}$

$= \binom{-2}{4} - \binom{1}{-1}$

$= \binom{-2-1}{4-(-1)}$

$= \binom{-3}{5}$

Vector $\overrightarrow{BD} = \binom{-3}{5}$

v. $|\overrightarrow{BD}| = \sqrt{(-3)^2 + (5)^2}$

$= \sqrt{9 + 25}$

$= \sqrt{34} = 5.831$

$= 5.8$ units

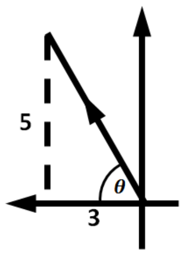
Magnitude of $\overrightarrow{BD}$ = 5.8 units to 1 d.p.

vi. $\tan\theta = \frac{5}{3} = 1.6667$

$\theta = \tan^{-1}(1.6667) = 59.036$

bearing $= 270 + 59.036$

$= 329.036$



Bearing of $\overrightarrow{BD} = 329°$ to the nearest degree.

Practice *(15 minutes)*

1. Ask pupils to work independently to answer questions c., d. and e.
2. Walk around, if possible, to check the answers and clear any misconceptions.
3. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.

Solutions:

c. Given: Muniratu walks 3 km west, then 2 km north to go to school.

i. vector of walk west $= \binom{-3}{0}$

vector of walk north $= \binom{0}{2}$

resultant displacement $= \binom{-3}{0} + \binom{0}{2}$

$= \binom{-3}{2}$

Displacement vector $= \binom{-3}{2}$

magnitude $= \sqrt{(-3)^2 + (2)^2}$

$= \sqrt{9 + 4}$

$= \sqrt{13} = 3.606$

$= 3.6$ km

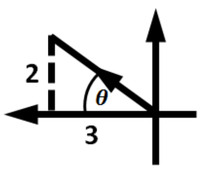
Magnitude = 3.6 km to 1 d.p.

ii. $\tan\theta = \frac{2}{3} = 0.6667$

$\theta = \tan^{-1}(0.6667) = 33.690$

bearing $= 270 + 33.69$

$= 303.69$



The bearing of $\overrightarrow{BD} = 304°$ to the nearest degree.

d. Given: $X(3,5)$, $Y(\text{a},\text{b})$ and $\overrightarrow{XY} = \binom{4}{-2}$

Let position vectors for X: $\overrightarrow{OX} = \binom{3}{5}$, Y: $\overrightarrow{OY} = \binom{a}{b}$

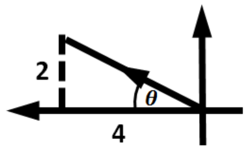
i.
$$\overrightarrow{XY} = \overrightarrow{OY} - \overrightarrow{OX}$$
$$\binom{4}{-2} = \binom{a}{b} - \binom{3}{5}$$
$$\binom{a}{b} = \binom{4}{-2} + \binom{3}{5}$$
$$= \binom{4+3}{-2+5}$$
$$= \binom{7}{3}$$

Position of Y on map $= (7,3)$

ii. distance between the towns, $\overrightarrow{XY}$
$$= \sqrt{(4)^2 + (-2)^2}$$
$$= \sqrt{16+4}$$
$$= \sqrt{20}$$
$$= 4.472$$
$$= 4.5 \text{ units}$$

The distance on the map between the towns $= 4.5$ units to 1 d.p.

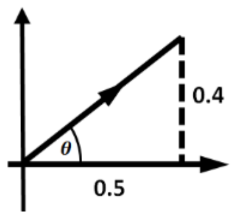
iii.
$$\tan\theta = \frac{2}{4} = 0.5$$
$$\theta = \tan^{-1}(0.5) = 26.565$$
$$\text{bearing} = 270 + 26.565$$
$$= 296.565$$



The bearing of X from $Y = 297°$ to the nearest degree.

e. Given: Abu's speed $= 0.4$ m/s, speed of the current $= 0.5$ m/s

$$\text{position vector of Abu} = \binom{0}{0.4}$$
$$\text{position vector of the current} = \binom{0.5}{0}$$
$$\text{resultant velocity} = \binom{0}{0.4} + \binom{0.5}{0}$$
$$= \binom{0.5}{0.4}$$
$$\text{Abu's velocity} = \sqrt{(0.5)^2 + (0.4)^2}$$
$$= \sqrt{0.25 + 0.16} = \sqrt{0.41}$$
$$= 0.64 \text{ m/s to 2 d.p.}$$
$$\tan\theta = \frac{0.4}{0.5} = 0.8$$
$$\theta = \tan^{-1}(0.8)$$
$$= 38.6598$$
$$= 39°$$
$$\text{bearing} = 90 - 39° = 51°$$



Abu swam across the river at 0.64 m/s on a bearing of $51°$.

Closing *(4 minutes)*

1. Ask pupils to write down one new thing they learned or understood better after this lesson.
2. Invite volunteers to share their answers with the class. (Answers: various)
3. Tell pupils that they can get more practice from the practice activity PHM3-L085 in the Pupil Handbook.

[QUESTIONS]

a. A man paddles his canoe east at 2 m/s across a river. If the river flows south with a current of 1.5 m/s.
   i. Express the resultant velocity of the canoe as a column vector.
   ii. Find the resultant velocity of the canoe giving the direction as a bearing.
   iii. If the river is 20 m across, how long does it take the man to cross the river?

b. Vertical poles are placed in a field in the shape of a parallelogram. Each pole is at a corner of the parallelogram with vertices $A(4,4)$, $B(1,-1)$, $C(-5,-1)$ and $D(-2,4)$ relative to a point O in the field. Wires are to be stretched along the diagonals AC and BD of the parallelogram. Find:
   i. $\overrightarrow{AC}$ ii. $|\overrightarrow{AC}|$ to 1 decimal place.
   iii. The bearing of A from C correct to the nearest degree.
   iv. $\overrightarrow{BD}$ v. $|\overrightarrow{BD}|$ to 1 decimal place.
   vi. The bearing of B from D correct to the nearest degree.

c. Every morning, Muniratu walks 3 km west, then 2 km north to go to school.
   i. Express her resultant displacement as a column vector.
   ii. Find her resultant displacement giving the direction as a bearing.

d. The location of two towns are shown on a map as $X(3,5)$ and $Y(\mathrm{a},\mathrm{b})$ relative to a point O on the map. The displacement between the towns is given as $\overrightarrow{XY} = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$. Find:
   i. The position of Y on the map.
   ii. The distance on the map between the towns.
   iii. The bearing of X from Y correct to the nearest degree.

e. Abu can swim 0.4 m/s in still water. He tries to swim across a stream with a current flowing 0.5 m/s east. If Abu is swimming north, find his actual velocity giving his direction as a bearing. Give the magnitude to 2 decimal places and bearing to the nearest degree.

Lesson Title: Application of Vectors – Part 3	**Theme:** Vectors and Transformations	
Lesson Number: M3-L086	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to apply vectors to solve more real-world problems.	**Preparation** Write the questions found at the end of this lesson plan on the board.	

Opening *(4 minutes)*

1. Ask pupils work with seatmates to answer question a. on the board.
2. After 2 minutes, invite volunteers to give the answers.

   Answers: i. $\begin{pmatrix}10\\0\end{pmatrix}$ ii. $\begin{pmatrix}0\\-9\end{pmatrix}$ iii. $\begin{pmatrix}-25\\0\end{pmatrix}$ iv. $\begin{pmatrix}0\\5\end{pmatrix}$
3. Tell pupils that after today's lesson, they will be able to apply vectors to solve more real-world problems.

Teaching and Learning *(20 minutes)*

1. Explain:
   - In the last lesson, we looked at some simple real-world problems that can be solved by applying vector methods.
   - For this lesson, we will look at more problems using vectors to solve real-world problems.
2. Invite a volunteer to assess question b. and tell the class what information we are given. (Answer: plane headed east with a speed of 150 m/s, jet stream wind blows from the west with a speed of 50 m/s)
3. Invite another volunteer to say the first thing we have been asked to find. (Answer: time taken for the plane to reach a destination 200 km east)

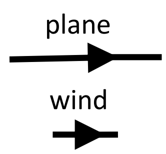
   Solution:

   b. **Step 1.** Assess and extract the given information from the problem.
   Given: plane headed east with a speed of 150 m/s,
   jet stream wind blows from the west with a speed of 50 m/s

   i. **Step 2.** Sketch the diagram – shown below (not to scale).

   Step 3. Write the velocities of the plane and wind as position vectors.

   $$\text{position vector of plane} = \begin{pmatrix}150\\0\end{pmatrix}$$
   $$\text{position vector of wind} = \begin{pmatrix}50\\0\end{pmatrix}$$
   $$\text{resultant velocity} = \begin{pmatrix}150\\0\end{pmatrix} + \begin{pmatrix}50\\0\end{pmatrix} = \begin{pmatrix}200\\0\end{pmatrix}$$



   ii. **Step 4.** Write the magnitude and direction of the resultant velocity
   magnitude of velocity $= 200$ m/s
   direction of velocity is east since resultant velocity is positive

   Step 5. Find the time for the plane to reach a destination 200 km east

   $$\text{distance} = 200 \text{ km} = 200{,}000 \text{ m}$$

time taken t $= \frac{200{,}000}{200}$ from distance formula $d = st$

$= 1{,}000$ s

$= 16.67$ minutes change to minutes

The time to reach the destination 200 km east is 16.67 minutes.

iii. **Step 6.** Find the time for the plane to reach a destination 200 km west.

resultant velocity $= \binom{-150}{0} + \binom{50}{0}$ plane now heading west

$= \binom{-100}{0}$

time taken t $= \frac{20{,}000}{100}$ from distance formula $d = st$

$= 2{,}000$ s

$= 33.33$ minutes change to minutes

The time to reach the destination 200 km west is 33.33 minutes.

4. Invite a volunteer to assess question c. on the board and extract the given information. (Example answer: given: boat travels a distance of 6 km in a direction 030° from A to B)
5. Invite a volunteer to say what we have been asked to find. (Answer: the resultant displacement $\overrightarrow{AB}$ as a column vector)

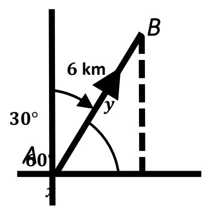
Solution:

c. Given: boat travels a distance of 6 km in a direction 030° from A to B

i. Sketch the diagram – shown at right (not to scale)

ii. Write vector $\overrightarrow{AB}$ in the form $\binom{x}{y}$

From the diagram, a 030° bearing makes an acute angle of 60° with the x-axis



$\therefore \cos 60° = \frac{x}{6}$ use cosine ratio

$x = 6\cos 60° = 6 \times 0.5000$

$= 3$ km

Similarly $\sin 60° = \frac{y}{6}$ use sine ratio

$y = 6\sin 60° = 6 \times 0.8660$

$= 5.2$ km to 2 significant figures

$\therefore \overrightarrow{AB} = \binom{3}{5.2}$

The resultant displacement $\overrightarrow{AB} = \binom{3}{5.2}$.

6. Explain:
   - If we are given the magnitude and direction (bearing) of a vector, we can use the method in question c. to find the column vector.
   - It is useful to always draw a diagram to help us with writing the vector.
7. Ask pupils to work with seatmates to answer question d.
8. You may want to stop the class to check that pupils have drawn the correct diagram in question d. i.
9. Invite a volunteer to show their answer on the board. The rest of the class should check their solution and correct any mistakes.

Solution:

d. Given: 2 stage journey

1st stage: from A to B in direction 060° for 10 km

2nd stage: from B to C north for 7 km

i. Sketch the diagram – shown at right (not to scale)

ii. Write vector $\overrightarrow{AB}$ in the form $\binom{x}{y}$

From the diagram, a $060°$ bearing makes an acute angle of $30°$ with the x-axis

$\therefore \quad \cos 30° = \frac{x}{10}$ use cosine ratio

$x = 10\cos 30° = 10 \times 0.8660$

$= 8.660$

$= 8.7$ km

Similarly $\sin 30° = \frac{y}{10}$

$y = 10\sin 30° = 10 \times 0.5000$

$= 5$ km

$\therefore \quad \overrightarrow{AB} = \binom{8.7}{5}$

$\overrightarrow{BC} = \binom{0}{7}$ since $\overrightarrow{BC}$ is 7 km North

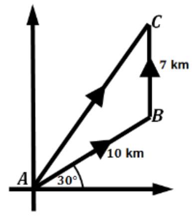
$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$

$= \binom{8.7}{5} + \binom{0}{7}$

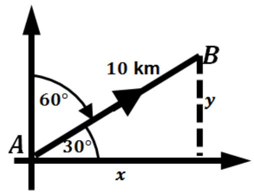
$= \binom{8.7+0}{5+7}$

$\overrightarrow{AC} = \binom{8.7}{12}$

i.



ii.



iii. Magnitude of $\overrightarrow{AC} = \sqrt{(8.7)^2 + (12)^2}$ use Pythagoras' Theorem

$= \sqrt{75.69 + 144}$

$= \sqrt{219.69}$

$= 14.862$

$= 14.9$ km

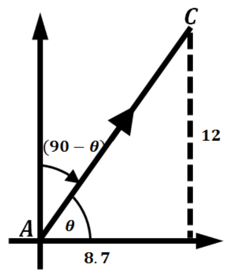
From the diagram

$\tan\theta = \frac{12}{8.7}$

$\theta = \tan^{-1}\left(\frac{12}{8.7}\right)$

$= 54.058$

Direction of $\overrightarrow{AC} = 90 - 54.058 = 35.942$



The cyclist is at a distance of 14.9 km from A at direction $036°$ to the nearest degree.

10. Explain: In general the components of a vector given as a magnitude and direction or bearing is given by:

$\binom{x}{y} = \binom{a\cos\theta}{a\sin\theta}$ where a is the magnitude of the vector

θ is the acute angle the bearing makes with the x-axis

Practice *(15 minutes)*

1. Ask pupils to work independently to answer question e.
2. Walk around, if possible, to check the answers and clear any misconceptions.

3. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.

Solution:

f. Given: 2 stage journey

1st stage: 300 km from P to Q at a bearing of $150°$ to Q; 2nd stage: 350 km from Q to R at a bearing of $60°$ to R.

i. sketch of journey shown (not to scale)

ii. From the sketch:

$\overrightarrow{PQ} = \begin{pmatrix} 300\cos 60° \\ -300\sin 60° \end{pmatrix}$

$= \begin{pmatrix} 150 \\ -260 \end{pmatrix}$

$\overrightarrow{QR} = \begin{pmatrix} 350\cos 30° \\ 350\sin 30° \end{pmatrix}$

$= \begin{pmatrix} 303 \\ 175 \end{pmatrix}$

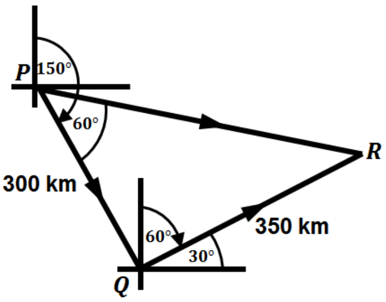
$\overrightarrow{PR} = \overrightarrow{PQ} + \overrightarrow{QR}$

$= \begin{pmatrix} 150 \\ -260 \end{pmatrix} + \begin{pmatrix} 303 \\ 175 \end{pmatrix}$

$= \begin{pmatrix} 150+303 \\ -260+175 \end{pmatrix}$

$\overrightarrow{PR} = \begin{pmatrix} 453 \\ -85 \end{pmatrix}$

i.



Magnitude of $\overrightarrow{PR}$

$|\overrightarrow{PR}| = \sqrt{(453)^2 + (-85)^2}$

$= \sqrt{205209 + 7225}$

$= \sqrt{212434}$

$= 460.906$

$= 461$ km

Direction of $\overrightarrow{PR}$

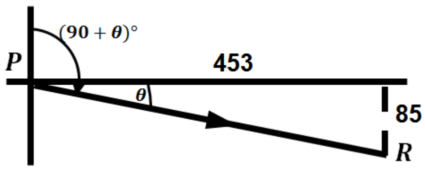
$\tan\theta = \frac{85}{453} = 0.1876$

$\theta = \tan^{-1}(0.1876)$

$= 10.625°$

bearing $= 90 + 10.625°$

$= 100.625°$



The plane is 461 km from P at a bearing of $101°$.

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM3-L086 in the Pupil Handbook.

[QUESTIONS]

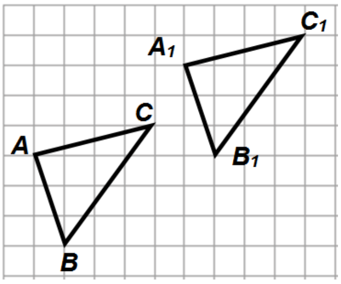
a. Write the following as position vectors.
   i. 10 km east   ii. 9 km south   iii. 25 m/s west   iv. 5 m/s north

b. A plane is headed east at a speed of 150 m/s while a jet stream wind blows from the west with a speed of 50 m/s.
   i. Write the resultant velocity as a column vector.
   ii. How much time will it take for this plane to reach a destination 200 km east?
   iii. How much time will it take for this plane to return the same 200 km west?

c. A boat starts from A and travels a distance of 6 km in the direction $030°$ to B.
   i. Draw a sketch of the journey from A to B.
   ii. Write the resultant displacement $\overrightarrow{AB}$ as a column vector.

d. A cyclist starts from A and travels a distance of 10 km in the direction $060°$ to B. She then travels 7 km north to C.
   i. Draw a sketch of the journey from A to C.
   ii. Write $\overrightarrow{AB}$, $\overrightarrow{BC}$ and $\overrightarrow{AC}$ as column vectors.
   iii. Find the cyclist's distance and bearing from A.

e. A plane flies from P to Q traveling a distance of 300 km at a bearing of $150°$ to Q. It then travels 350 km at a bearing of $60°$ to R.
   i. Draw a sketch of the journey from P to R.
   ii. Write $\overrightarrow{PQ}$, $\overrightarrow{QR}$ and $\overrightarrow{PR}$ as column vectors.
   iii. Find the plane's distance and bearing from P to the nearest whole number.

<table>
<tr><td>Lesson Title: Translation – Part 1</td><td colspan="2">Theme: Vectors and Transformations</td></tr>
<tr><td>Lesson Number: M3-L087</td><td>Class: SSS 3</td><td>Time: 40 minutes</td></tr>
<tr><td>Learning Outcomes
By the end of the lesson, pupils will be able to:
1. Identify that translation moves an object without changing its size or shape.
2. Use vectors to translate given points and images.</td><td colspan="2">Preparation
1. Draw a grid 16 squares across and 16 squares down on the board.
2. Draw the diagram below on the grid on the board.


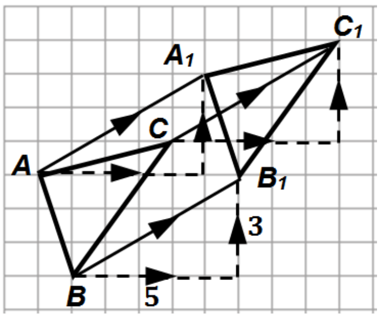
3. Write the questions found at the end of this lesson plan on the board.</td></tr>
</table>

Opening *(4 minutes)*

1. Ask pupils to discuss with seatmates and write down what they notice about triangle *ABC* and its movement on the grid to $A_1B_1C_1$.
2. After 2 minutes, invite volunteers to share their discussion with the class. (Example answers: the triangle moved from its original position – 5 left and 3 up; all the points moved by the same vector – $\binom{5}{3}$; the triangle moved without changing its size or shape, the triangle was translated)
3. Tell pupils that after today's lesson, they will be able to identify that translation moves an object without changing its size or shape. They will also use vectors to translate given points and images.

Teaching and Learning *(20 minutes)*

1. Draw the vectors showing the movement of triangle *ABC* as shown below.
2. Explain:
   - The object *ABC* was translated from its original position 5 units right and 3 units up.
   - A translation moves all the points of an object in the same direction and the same distance without changing its shape or size.



3. Ask pupils to write the column vectors which show the movements of *A, B* and *C* to their new positions A_1, B_1 and C_1.
4. Invite volunteers to give their answers. What do they notice? (Answers: $\overrightarrow{AA_1} = \binom{5}{3}$, $\overrightarrow{BB_1} = \binom{5}{3}$ and $\overrightarrow{CC_1} = \binom{5}{3}$; all 3 points move according to the same column vector)
5. Explain:

- We can conclude that all the points on triangle ABC moved by the same column vector $\binom{5}{3}$ to $A_1B_1C_1$.
- The vector $\binom{5}{3}$ is called a **translation** vector, $\mathbf{v}$.
- In general a translation vector $\binom{a}{b}$ moves a point $A(x, y)$ along the x- and y-axes by the amount of the components of the vector.
- The new point $A_1(x_1, y_1)$, called the **image point,** will have co-ordinates $x_1 = x + a, y_1 = y + b$.
- We can write a mapping for the translation as:

$$\binom{x}{y} \longrightarrow \binom{x}{y} + \binom{a}{b} = \binom{x+a}{y+b}$$

- Similarly, $(x, y) \longrightarrow (x + a, y + b)$

6. Invite a volunteer to assess question a. and tell the class what information we are given. (Answer: given: point $P(4,2)$, translation vector $\binom{-4}{3}$)
7. Invite another volunteer to say what we have been asked to find. (Answer: find coordinates of the image of the point P)

Solution:

a. i. **Step 1.** Assess and extract the given information from the problem.

Given: point $P(4,2)$, translation vector $\binom{-4}{3}$

Step 2. Write the mapping for the translation.

$$\binom{x}{y} \longrightarrow \binom{x}{y} + \binom{a}{b} = \binom{x+a}{y+b}$$

$$\binom{4}{2} \longrightarrow \binom{4}{2} + \binom{-4}{3} = \binom{4+(-4)}{2+3} = \binom{0}{5}$$

$$\binom{4}{2} \longrightarrow \binom{0}{5}$$

Step 3. Write the answer.

The co-ordinates of the image $= (0,5)$.

8. Invite a volunteer to say what information we have been given, and what we have been asked to find in question b. ii. (Answer: given: image A' $(-2, 4)$ of point A under translation vector $\binom{2}{-1}$; find co-ordinates of point A.)

Solution:

a. ii. Given: image A' $(-2, 4)$ of point A under translation vector $\binom{2}{-1}$

$$\binom{x}{y} \longrightarrow \binom{x}{y} + \binom{a}{b} = \binom{x+a}{y+b}$$

$$\binom{x}{y} \longrightarrow \binom{x}{y} + \binom{2}{-1} \qquad \text{using the translation vector}$$

$$\binom{x}{y} \longrightarrow \binom{-2}{4} \qquad \text{using the image point}$$

$$\Rightarrow \binom{x}{y} + \binom{2}{-1} = \binom{-2}{4}$$

$$\binom{x}{y} = \binom{-2}{4} - \binom{2}{-1} = \binom{-2-2}{4-(-1)}$$

$$= \binom{-4}{5}$$

$$\binom{-4}{5} \longrightarrow \binom{-2}{4} \qquad \text{mapping for the translation}$$

The co-ordinates of point A under translation vector $\binom{2}{-1}$ to image point $A'(-2, 4) = (-4,5)$.

9. Invite a volunteer to say what information we have been given and what we have been asked to find in question b. iii. (Answer: point $Q(5,2)$, image $Q'(1,-3)$ under a translation vector; find the vector.)

   Solution:

   a. iii. Given: point $Q(5,2)$, image $Q'(1,-3)$ under a translation vector

$$\begin{pmatrix}x\\y\end{pmatrix} \longrightarrow \begin{pmatrix}x\\y\end{pmatrix} + \begin{pmatrix}a\\b\end{pmatrix} = \begin{pmatrix}x+a\\y+b\end{pmatrix}$$

$$\begin{pmatrix}5\\2\end{pmatrix} \longrightarrow \begin{pmatrix}5\\2\end{pmatrix} + \begin{pmatrix}a\\b\end{pmatrix}$$ using the translation vector

$$\begin{pmatrix}5\\2\end{pmatrix} \longrightarrow \begin{pmatrix}1\\-3\end{pmatrix}$$ using the image point

$$\Longrightarrow \begin{pmatrix}5\\2\end{pmatrix} + \begin{pmatrix}a\\b\end{pmatrix} = \begin{pmatrix}1\\-3\end{pmatrix}$$

$$\begin{pmatrix}a\\b\end{pmatrix} = \begin{pmatrix}1\\-3\end{pmatrix} - \begin{pmatrix}5\\2\end{pmatrix} = \begin{pmatrix}1-5\\-3-2\end{pmatrix}$$

$$= \begin{pmatrix}-4\\-5\end{pmatrix}$$ translation vector

   The translation vector which maps $Q(5,2)$ to $Q'(1,-3) = \begin{pmatrix}-4\\-5\end{pmatrix}$.

10. Ask pupils to work with seatmates to answer question b.
11. Invite volunteers to show their answer on the board. The rest of the class should check their solution and correct any mistakes.

    Solution:

    b. Given: point $S(-1,-2)$, translation vector $\begin{pmatrix}6\\-2\end{pmatrix}$

$$\begin{pmatrix}x\\y\end{pmatrix} \longrightarrow \begin{pmatrix}x\\y\end{pmatrix} + \begin{pmatrix}a\\b\end{pmatrix} = \begin{pmatrix}x+a\\y+b\end{pmatrix}$$

$$\begin{pmatrix}-1\\-2\end{pmatrix} \longrightarrow \begin{pmatrix}-1\\-2\end{pmatrix} + \begin{pmatrix}6\\-2\end{pmatrix} = \begin{pmatrix}-1+6\\-2+(-2)\end{pmatrix}$$

$$= \begin{pmatrix}5\\-4\end{pmatrix}$$

$$\begin{pmatrix}-1\\-2\end{pmatrix} \longrightarrow \begin{pmatrix}5\\-4\end{pmatrix}$$ mapping for the translation

    The co-ordinates of the image of the point $S(-1,-2)$ under translation vector $\begin{pmatrix}6\\-2\end{pmatrix} = (5,-4)$

Practice *(15 minutes)*

1. Ask pupils to work independently to answer questions c., d. and e.
2. Walk around, if possible, to check the answers and clear any misconceptions.
3. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.

   Solutions:

   c. Given: image $T'(2,6)$ of point T under translation vector $\begin{pmatrix}-2\\-5\end{pmatrix}$

$$\begin{pmatrix}x\\y\end{pmatrix} \longrightarrow \begin{pmatrix}x\\y\end{pmatrix} + \begin{pmatrix}a\\b\end{pmatrix} = \begin{pmatrix}x+a\\y+b\end{pmatrix}$$

$$\begin{pmatrix}x\\y\end{pmatrix} \longrightarrow \begin{pmatrix}x\\y\end{pmatrix} + \begin{pmatrix}-2\\-5\end{pmatrix}$$ using the translation vector

$$\begin{pmatrix}x\\y\end{pmatrix} \longrightarrow \begin{pmatrix}2\\6\end{pmatrix}$$ using the image point

$$\Longrightarrow \begin{pmatrix}x\\y\end{pmatrix} + \begin{pmatrix}-2\\-5\end{pmatrix} = \begin{pmatrix}2\\6\end{pmatrix}$$

$$\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}2\\6\end{pmatrix} - \begin{pmatrix}-2\\-5\end{pmatrix} = \begin{pmatrix}2-(-2)\\6-(-5)\end{pmatrix}$$

$$\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}4\\11\end{pmatrix}$$

The co-ordinates of point T under translation by vector $\mathbf{v} = (4,11)$.

d. Given: A, B and C with coordinates $(4,7), (2,-6)$ and $(-3,-6)$ respectively

$$\begin{pmatrix} x \\ y \end{pmatrix} \longrightarrow \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} x+a \\ y+b \end{pmatrix}$$

i. vector from $A(4,7)$ to $B(2,-6)$

$$\begin{aligned}
\begin{pmatrix} 4 \\ 7 \end{pmatrix} &\longrightarrow \begin{pmatrix} 4 \\ 7 \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix} \\
\begin{pmatrix} 4 \\ 7 \end{pmatrix} &\longrightarrow \begin{pmatrix} 2 \\ -6 \end{pmatrix} \\
\begin{pmatrix} 4 \\ 7 \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix} &= \begin{pmatrix} 2 \\ -6 \end{pmatrix} \\
\begin{pmatrix} a \\ b \end{pmatrix} &= \begin{pmatrix} 2 \\ -6 \end{pmatrix} - \begin{pmatrix} 4 \\ 7 \end{pmatrix} \\
&= \begin{pmatrix} 2-4 \\ -6-7 \end{pmatrix} \\
&= \begin{pmatrix} -2 \\ -13 \end{pmatrix}
\end{aligned}$$

The translation vector from $A(4,7)$ to $B(2,-6) = \begin{pmatrix} -2 \\ -13 \end{pmatrix}$.

ii. vector from $B(2,-6)$ to $C(-3,-6)$

$$\begin{aligned}
\begin{pmatrix} 2 \\ -6 \end{pmatrix} &\longrightarrow \begin{pmatrix} -3 \\ -6 \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix} \\
\begin{pmatrix} 2 \\ -6 \end{pmatrix} &\longrightarrow \begin{pmatrix} -3 \\ -6 \end{pmatrix} \\
\begin{pmatrix} 2 \\ -6 \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix} &= \begin{pmatrix} -3 \\ -6 \end{pmatrix} \\
\begin{pmatrix} a \\ b \end{pmatrix} &= \begin{pmatrix} -3 \\ -6 \end{pmatrix} - \begin{pmatrix} 2 \\ -6 \end{pmatrix} \\
&= \begin{pmatrix} -3-2 \\ -6-(-6) \end{pmatrix} \\
&= \begin{pmatrix} -5 \\ 0 \end{pmatrix}
\end{aligned}$$

The translation vector from $B(2,-6)$ to $C(-3,-6) = \begin{pmatrix} -5 \\ 0 \end{pmatrix}$.

e. point $P(5,2)$, image $P'(2,-5)$, translation vector $\mathbf{v}$

$$\begin{pmatrix} x \\ y \end{pmatrix} \longrightarrow \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} x+a \\ y+b \end{pmatrix}$$

i. vector from $P(5,2)$ to $P'(2,-5)$

$$\begin{aligned}
\begin{pmatrix} 5 \\ 2 \end{pmatrix} &\longrightarrow \begin{pmatrix} 5 \\ 2 \end{pmatrix} + \mathbf{v} \qquad \mathbf{v} = \begin{pmatrix} a \\ b \end{pmatrix} \\
\begin{pmatrix} 5 \\ 2 \end{pmatrix} &\longrightarrow \begin{pmatrix} 2 \\ -5 \end{pmatrix} \\
\begin{pmatrix} 5 \\ 2 \end{pmatrix} + \mathbf{v} &= \begin{pmatrix} 2 \\ -5 \end{pmatrix} \\
\mathbf{v} &= \begin{pmatrix} 2 \\ -5 \end{pmatrix} - \begin{pmatrix} 5 \\ 2 \end{pmatrix} \\
&= \begin{pmatrix} 2-5 \\ -5-2 \end{pmatrix} \\
&= \begin{pmatrix} -3 \\ -7 \end{pmatrix}
\end{aligned}$$

The translation vector $= \begin{pmatrix} -3 \\ -7 \end{pmatrix}$.

ii. Q given $Q'(-5,-2)$, $\mathrm{v} = \begin{pmatrix} -3 \\ -7 \end{pmatrix}$

$$\begin{aligned}
\begin{pmatrix} x \\ y \end{pmatrix} &\longrightarrow \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} -3 \\ -7 \end{pmatrix} \\
\begin{pmatrix} x \\ y \end{pmatrix} &\longrightarrow \begin{pmatrix} -5 \\ -2 \end{pmatrix} \\
\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} -3 \\ -7 \end{pmatrix} &= \begin{pmatrix} -5 \\ -2 \end{pmatrix} \\
\begin{pmatrix} x \\ y \end{pmatrix} &= \begin{pmatrix} -5 \\ -2 \end{pmatrix} - \begin{pmatrix} -3 \\ -7 \end{pmatrix} \\
&= \begin{pmatrix} -5-(-3) \\ -2-(-7) \end{pmatrix} \\
&= \begin{pmatrix} -2 \\ 5 \end{pmatrix}
\end{aligned}$$

Co-ordinates of $Q = (-2,5)$.

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM3-L087 in the Pupil Handbook.

[QUESTIONS]

a. i. Find the coordinates of the image of the point $P(4,2)$ when it is translated by the vector $\begin{pmatrix} -4 \\ 3 \end{pmatrix}$.

   ii. $A'(-2,4)$ is the image of a point A under the translation by the vector $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$. Find the co-ordinates of point A.

   iii. $Q'(1,-3)$ is the image of the point $Q(5,2)$ under translation by a vector. Find the translation vector**.**

b. Find the coordinates of the image of the point $S(-1,-2)$ when they are translated by the vector $\begin{pmatrix} 6 \\ -2 \end{pmatrix}$

c. $T'(2,6)$ is the image of a point T under the translation by the vector $\begin{pmatrix} -2 \\ -5 \end{pmatrix}$. Find the co-ordinates of point T.

d. The points A, B and C have coordinates $(4, 7), (2, -6)$ and $(-3, -6)$ respectively. Find the vector which would be used to translate:

   i. A to B ii. B to C

e. $P'(5\,, 2)$ is the image of the point $P(2\,, -5)$ by the translation vector $\mathbf{v}$. Find

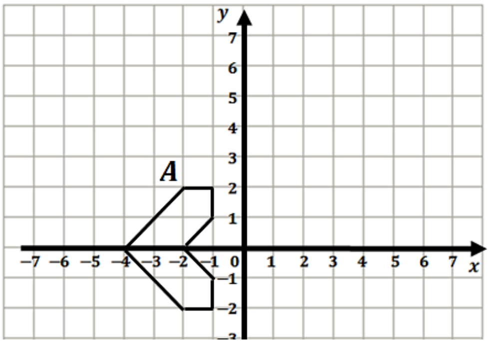
   i. The vector $\mathbf{v}$

   ii. The coordinates of point Q which maps onto point $Q'(-5, -2)$ under $\mathbf{v}$.

Lesson Title: Translation – Part 2	**Theme:** Vectors and Transformations	
Lesson Number: M3-L088	**Class:** SSS 3	**Time:** 40 minutes

Learning Outcome
By the end of the lesson, pupils will be able to translate plane figures.

Preparation
1. Draw the diagram below on the board.



2. Write the questions found at the end of this lesson plan on the board.

Opening *(4 minutes)*

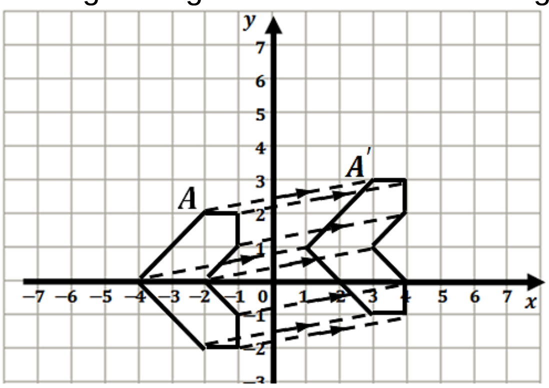
1. Ask pupils answer question a. on the board.
2. Invite a volunteer to answer. (Answer: co-ordinates of $A = (-2,2)$; $\binom{-2}{2} \rightarrow \binom{-2}{2} + \binom{5}{1} = \binom{-2+5}{2+1} = \binom{3}{3}$; co-ordinates of $A' = (3,3)$).
3. Tell pupils that after today's lesson, they will be able to translate plane figures.

Teaching and Learning *(20 minutes)*

1. Invite a volunteer to read question b. i. What are we asked to find? (Answer: Draw the image of the shape under the translation vector $\binom{5}{1}$)
2. Show on the board how the image point A' is used as the reference point to translate the object 5 units right and 1 unit up.

Solution:

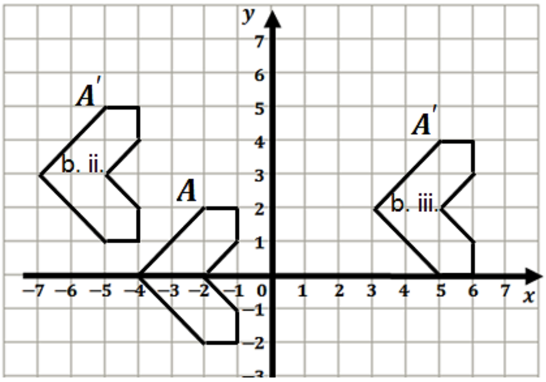
b. i. **Step 1.** Assess and extract the given information from the problem.
Given: a shape with point A marked, translation vector $\binom{5}{1}$
Step 2. Use the answer to question a. to mark point A' on the grid.
Step 3. Draw the image using A' as the reference image point.



3. Invite a volunteer to say what we have been asked to do in question b. ii. (Answer: Use translation vector $\binom{-3}{3}$ to translate given object.)
4. Ask pupils to find the image point A' using the translation vector $\binom{-3}{3}$.
3. Invite a volunteer to give the answer. (Answer: co-ordinates of $A = (-2,2)$; $\binom{-2}{2} \longrightarrow \binom{-2}{2} + \binom{-3}{3} = \binom{-2+(-3)}{2+3} = \binom{-5}{5}$; co-ordinates of $A' = (-5,5)$)
5. Mark point A' and use as a reference image point for the translation.
Solution:
Shown under solution b. iii.
Draw the answer to b. ii. on board using same axes as before.
6. Ask pupils to work with seatmates to answer question b. iii.
7. Invite a volunteer to show their answer on the board.
The rest of the class should check their solution and correct any mistakes.
Solution:

b. iii. Given: Use translation vector $\binom{7}{2}$ to translate the given object.

co-ordinates of $A = (-2,2)$;
$\binom{-2}{2} \longrightarrow \binom{-2}{2} + \binom{7}{2} = \binom{-2+7}{2+2} = \binom{5}{4}$
co-ordinates of $A' = (5,4)$



Practice *(15 minutes)*

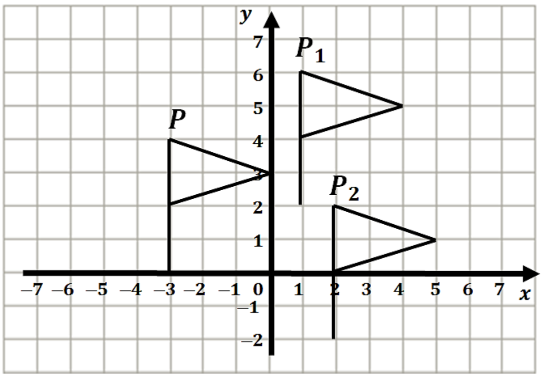
1. Ask pupils to work independently to answer questions c. and d.
2. Walk around, if possible, to check the answers and clear any misconceptions.
3. Ask volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.
Solutions:
c. Given: shape to be translated
i. shape shown below
ii. use translation vector $\binom{4}{2}$ to translate the given object
co-ordinates of $P = (-3,4)$;
$\binom{-3}{4} \longrightarrow \binom{-3}{4} + \binom{4}{2} = \binom{-3+4}{4+2} = \binom{1}{6}$
co-ordinates of $P_1 = (1,6)$

iii. use translation vector $\binom{1}{-4}$ to translate the given object
co-ordinates of $P_1 = (1,6)$
$\binom{1}{6} \longrightarrow \binom{1}{6} + \binom{1}{-4} = \binom{1+1}{6+(-4)} = \binom{2}{2}$
co-ordinates of $P_2 = (2,2)$
image shown below

iv. since the object was translated using the vector sum $\binom{4}{2} + \binom{1}{-4}$, the inverse vector will take the final image back to the position of the original shape

$$\begin{aligned}\text{translation vector } \mathbf{v} &= -\left(\binom{4}{2} + \binom{1}{-4}\right) \\ &= -\binom{4+1}{2+(-4)} = -\binom{5}{-2} \\ &= \binom{-5}{2}\end{aligned}$$

Translation vector $\mathbf{v} = \binom{-5}{2}$ takes the final image back to the position of the original shape



d. Given: shape to be translated

i. shape shown below

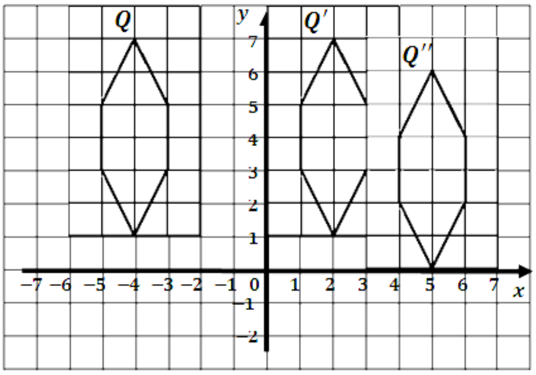
ii. use translation vector $\binom{6}{0}$ to translate the given object
co-ordinates $Q = (-4,7)$
$\binom{-4}{7} \longrightarrow \binom{-4}{7} + \binom{6}{0} = \binom{-4+6}{7+0} = \binom{2}{7}$
co-ordinates of $Q' = (2,7)$
image shown below

iii. use translation vector $\binom{3}{-1}$ to translate the given object
co-ordinates of $Q' = (2,7)$
$\binom{2}{7} \longrightarrow \binom{2}{7} + \binom{3}{-1} = \binom{2+3}{7+(-1)} = \binom{5}{6}$
co-ordinates of $Q'' = (5,6)$
image shown below

iv. since the object was translated using the vector sum $\binom{6}{0} + \binom{3}{-1}$, this vector sum will take the original shape to the final image

$$\begin{aligned}\text{translation vector } \mathbf{v} &= \binom{6}{0} + \binom{3}{-1}, \\ &= \binom{6+3}{0+(-1)} \\ &= \binom{9}{-1}\end{aligned}$$

Translation vector $\mathbf{v} = \binom{9}{-1}$ takes the original shape to the final image.



Closing *(1 minute)*

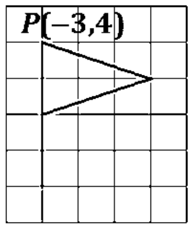
1. For homework, have pupils do the practice activity PHM3-L088 in the Pupil Handbook.

[QUESTIONS]

a. Find the image A' if A is translated by the vector $\begin{pmatrix}5\\1\end{pmatrix}$,

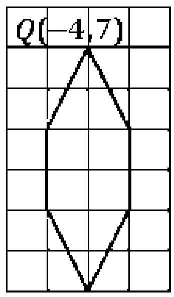
b. On the same axes, draw the image of the shape under the translation by the vectors below such that A maps onto A'.
For each translation, find the co-ordinates of A' and use it as the reference point to translate the object.
i. $\begin{pmatrix}5\\1\end{pmatrix}$ ii. $\begin{pmatrix}-3\\3\end{pmatrix}$, iii. $\begin{pmatrix}7\\2\end{pmatrix}$,

c. Using axes as before:



i. Draw the shape given.
ii. Translate the shape using the vector $\begin{pmatrix}4\\2\end{pmatrix}$ and $P(-3,4)$ as the reference point to give P_1.
iii. Translate the image using the vector $\begin{pmatrix}1\\-4\end{pmatrix}$ and P_1 the reference image point to give P_2.
iv. Which vector would be needed to translate the final image back to the position of the original shape?

d. Using axes as before:



i. Draw the shape given.
ii. Translate the shape using the vector $\begin{pmatrix}6\\0\end{pmatrix}$ and $Q(-4,7)$ as the reference point to give Q'.
iii. Translate the image using the vector $\begin{pmatrix}3\\-1\end{pmatrix}$ and Q' as the reference image point to give Q''.
iv. Which vector would be needed to translate the original shape to the final image?

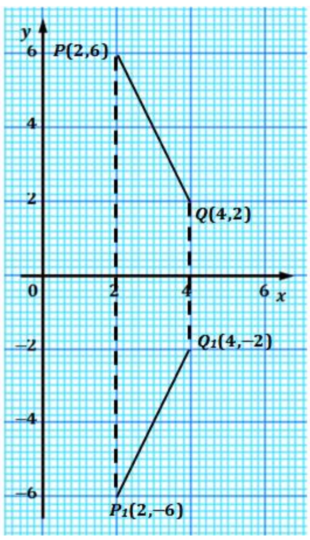
Lesson Title: Reflection – Part 1	Theme: Geometry	
Lesson Number: M3-L089	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcomes By the end of the lesson, pupils will be able to: 1. Identify and explain the reflection of an object in the line $y = k$ 2. Identify and explain the reflection of an object in the line $x = k$	**Preparation** 1. Using a scale of 2 cm to 2 units on both axes, draw two perpendicular axes Ox and Oy for $-6 \leq x \leq 6$ and $-6 \leq y \leq 6$. 2. Write the questions found at the end of this lesson plan on the board.	

Opening *(4 minutes)*

1. Ask pupils to write down what they understand by the term reflection.
2. Invite 1-2 volunteers to answer. (Example answer: Reflection is what you see when you look in a mirror.)
3. Tell pupils that after today's lesson, they will be able to identify and explain the reflection of an object in the lines $y = k$ and $x = k$.

Teaching and Learning *(20 minutes)*

1. Explain:
   - Reflection is the image we see when we look at objects in a mirror.
   - Every point on the reflected image is the same distance away from the **line of reflection** or **mirror line** as the object.
   - Distances are always measured at right angles to the mirror line.
   - The image has the same angles, lengths and area as the object, but its figure is reversed.
2. Invite a volunteer to assess question a. and tell the class what information we are given. (Answer: given: points $P(2,6), Q(4,2)$)
3. Invite another volunteer to say what we have been asked to find. (Answer: find coordinates of the image of the line joining the 2 points under reflection in the x-axis)
   Solution:



a. **Step 1.** Assess and extract the given information from the problem. Given: points $P(2{,}6),\ Q(4{,}2))$

Step 2. Draw the x- and y- axes (if not already drawn). Locate the points P and Q on the graph. Draw the line joining the points.

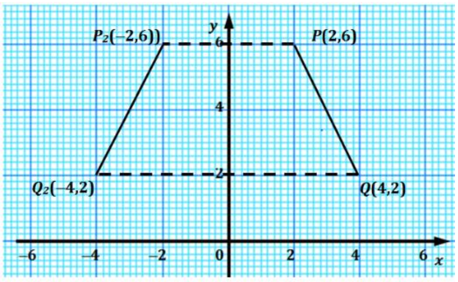
Step 3. Draw a line at right angles from P to the mirror line (the x-axis). Measure this distance.

Step 4. Measure the same distance on the opposite side of the mirror line (the x-axis) to locate the point P_1 on the graph.

Step 5. Write the co-ordinates of $P_1 = (2, -6)$.

Step 6. Follow the same procedure for the point Q to give $Q_1(4, -2)$; see graph.

4. Ask pupils to discuss with seatmates anything they notice when they compare the co-ordinates of P with P_1 and Q with Q_1.
5. Invite a volunteer to answer. (Answer: the x-co-ordinates are the same, the y-co-ordinates have opposite signs)
6. Explain:
   - We can write a mapping of the **reflection in the x-axis (i.e. $y = 0$)** as:
     $$\binom{x}{y} \longrightarrow \binom{x}{-y} \quad \text{giving} \quad (x, y) \longrightarrow (x, -y)$$
   - We can use this procedure to reflect any point in any given line of reflection.
7. Ask pupils to work with seatmates to answer question b.
8. Invite a volunteer to show their answer on the board. The rest of the class should check their solution and correct any mistakes.
   Solution:



b. From the graph, the co-ordinates are $P_2(-2, 6)$ and $Q_2(-4{,}2)$

mapping for P $\binom{2}{6} \longrightarrow \binom{-2}{6}$ giving $P_2(-2{,}6)$

mapping for Q $\binom{4}{2} \longrightarrow \binom{-4}{2}$ giving $Q_2(-4{,}2)$

The x-co-ordinates have opposite signs, the y-co-ordinates are the same.

9. Explain:
   - We can write a mapping of the **reflection in the y-axis (i.e. $x = 0$)** as:
     $$\binom{x}{y} \longrightarrow \binom{-x}{y} \quad \text{giving} \quad (x, y) \longrightarrow (-x, y)$$
   - If we continue to follow the procedure we will find the mappings given below.
   - A mapping of the **reflection in the line $y = k$ or $y - k = 0$** is given by:
     $$\binom{x}{y} \longrightarrow \binom{x}{2k-y} \quad \text{giving} \quad (x, y) \longrightarrow (x, 2k - y)$$

- A mapping of the **reflection in the line $x = k$ or $x - k = 0$** is given by:

$$\begin{pmatrix} x \\ y \end{pmatrix} \longrightarrow \begin{pmatrix} 2k-x \\ y \end{pmatrix} \quad \text{giving} \quad (x, y) \longrightarrow (2k - x, y)$$

10. Ask pupils to work with seatmates to answer question c.
11. Ask pupils to give their answers. The rest of the class should check their answers.

Solution:

c. Given: $A(1, -6)$

i. mapping for reflection in $x = 0$ (y-axis) is given by:

$$\begin{pmatrix} x \\ y \end{pmatrix} \longrightarrow \begin{pmatrix} -x \\ y \end{pmatrix}$$
$$\begin{pmatrix} 1 \\ -6 \end{pmatrix} \longrightarrow \begin{pmatrix} -1 \\ -6 \end{pmatrix}$$

giving image point $(-1, -6)$

ii. mapping for reflection in $y = 0$ (x-axis) is given by:

$$\begin{pmatrix} x \\ y \end{pmatrix} \longrightarrow \begin{pmatrix} x \\ -y \end{pmatrix}$$
$$\begin{pmatrix} 1 \\ -6 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 \\ 6 \end{pmatrix}$$

giving image point $(1, 6)$

iii. mapping for reflection in $x = 2$ is given by:

$$\begin{pmatrix} x \\ y \end{pmatrix} \longrightarrow \begin{pmatrix} 2k-x \\ y \end{pmatrix}$$
$$\begin{pmatrix} 1 \\ -6 \end{pmatrix} \longrightarrow \begin{pmatrix} 2(2)-1 \\ -6 \end{pmatrix} = \begin{pmatrix} 4-1 \\ -6 \end{pmatrix}$$
$$\begin{pmatrix} 1 \\ -6 \end{pmatrix} \longrightarrow \begin{pmatrix} 3 \\ -6 \end{pmatrix}$$

giving image point $(3, -6)$

iv. mapping for reflection in $y + 1 = 0$ i.e. $y = -1$ is given by:

$$\begin{pmatrix} x \\ y \end{pmatrix} \longrightarrow \begin{pmatrix} x \\ 2k-y \end{pmatrix}$$
$$\begin{pmatrix} 1 \\ -6 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 \\ 2(-1)-(-6) \end{pmatrix} = \begin{pmatrix} 1 \\ -2+6 \end{pmatrix}$$
$$\begin{pmatrix} 1 \\ -6 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 \\ 4 \end{pmatrix}$$

giving image point $(1, 4)$

d. Given: $A(1, 3), B(4, 4), C(4, 2), D(2, 2)$

i. All diagrams for this question can be found at the end of Question e.

ii. reflect in $y = 0$ (x-axis): $\begin{pmatrix} x \\ y \end{pmatrix} \longrightarrow \begin{pmatrix} x \\ -y \end{pmatrix}$

$$\begin{pmatrix} 1 \\ 3 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 \\ -3 \end{pmatrix} \qquad \begin{pmatrix} 4 \\ 2 \end{pmatrix} \longrightarrow \begin{pmatrix} 4 \\ -2 \end{pmatrix}$$
$$\begin{pmatrix} 4 \\ 4 \end{pmatrix} \longrightarrow \begin{pmatrix} 4 \\ -4 \end{pmatrix} \qquad \begin{pmatrix} 2 \\ 2 \end{pmatrix} \longrightarrow \begin{pmatrix} 2 \\ -2 \end{pmatrix}$$

Co-ordinates:: $A_1(1, -3)$, $B_1(4, -4)$, $C_1(4, -2), D_1(2, -2)$

Practice *(15 minutes)*

1. Ask pupils to work independently to answer question e.
2. Walk around, if possible, to check the answers and clear any misconceptions.
3. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.

Solution:

e. Given: : $A_1(1, -3)$, $B_1(4, -4)$, $C_1(4, -2), D_1(2, -2)$

i. reflect in $x = 0$ (y-axis): $\begin{pmatrix} x \\ y \end{pmatrix} \longrightarrow \begin{pmatrix} -x \\ y \end{pmatrix}$

$$\begin{pmatrix} 1 \\ -3 \end{pmatrix} \longrightarrow \begin{pmatrix} -1 \\ -3 \end{pmatrix} \qquad \begin{pmatrix} 4 \\ -2 \end{pmatrix} \longrightarrow \begin{pmatrix} -4 \\ -2 \end{pmatrix}$$
$$\begin{pmatrix} 4 \\ -4 \end{pmatrix} \longrightarrow \begin{pmatrix} -4 \\ -4 \end{pmatrix} \qquad \begin{pmatrix} 2 \\ -2 \end{pmatrix} \longrightarrow \begin{pmatrix} -2 \\ -2 \end{pmatrix}$$

Co-ordinates: $A_2(-1, -3), B_2(-4, -4), C_2(-4, -2), D_2(-2, -2)$

ii. reflect $A_2(-1, -3), B_2(-4, -4), C_2(-4, -2), D_2(-2, -2)$ in $x = -5 \Longrightarrow k = -5$

$$\begin{pmatrix} x \\ y \end{pmatrix} \longrightarrow \begin{pmatrix} 2k-x \\ y \end{pmatrix} = \begin{pmatrix} 2(-5)-x \\ y \end{pmatrix} = \begin{pmatrix} -10-x \\ y \end{pmatrix}$$
$$\begin{pmatrix} -1 \\ -3 \end{pmatrix} \longrightarrow \begin{pmatrix} -10-(-1) \\ -3 \end{pmatrix} = \begin{pmatrix} -9 \\ -3 \end{pmatrix} \qquad \begin{pmatrix} -4 \\ -2 \end{pmatrix} \longrightarrow \begin{pmatrix} -10-(-4) \\ -2 \end{pmatrix} = \begin{pmatrix} -6 \\ -2 \end{pmatrix}$$

$\begin{pmatrix} -4 \\ -4 \end{pmatrix} \longrightarrow \begin{pmatrix} -10-(-4) \\ -4 \end{pmatrix} = \begin{pmatrix} -6 \\ -4 \end{pmatrix}$ $\qquad$ $\begin{pmatrix} -2 \\ -2 \end{pmatrix} \longrightarrow \begin{pmatrix} -10-(-2) \\ -2 \end{pmatrix} = \begin{pmatrix} -8 \\ -2 \end{pmatrix}$

Co-ordinates: $A_3(-9,-3), B_3(-6,-4), C_3(-6,-2), D_3(-8,-2)$

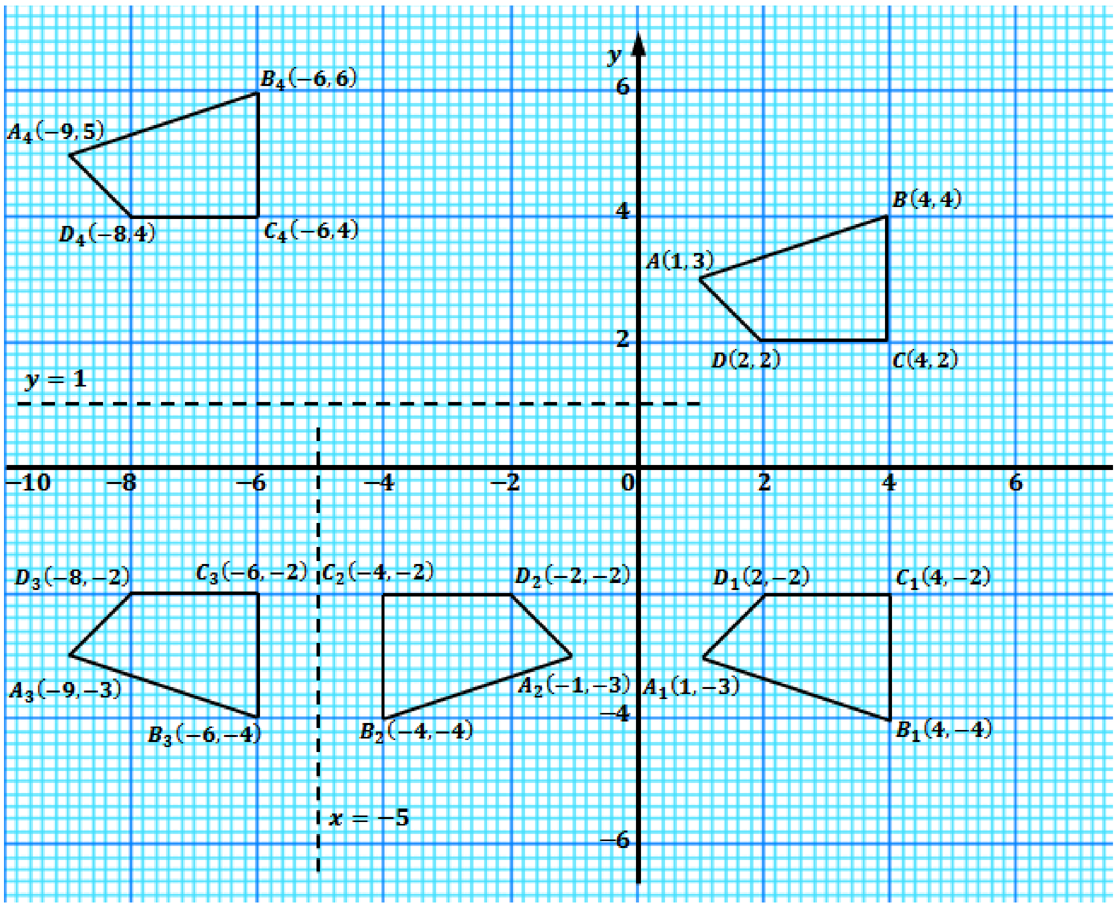
iii. reflect $A_3(-9,-3), B_3(-6,-4), C_3(-6,-2), D_3(-8,-2)$ in $y-1=0 \Rightarrow k=1$

$\begin{pmatrix} x \\ y \end{pmatrix} \longrightarrow \begin{pmatrix} x \\ 2k-y \end{pmatrix} = \begin{pmatrix} x \\ 2(1)-y \end{pmatrix} = \begin{pmatrix} x \\ 2-y \end{pmatrix}$

$\begin{pmatrix} -9 \\ -3 \end{pmatrix} \longrightarrow \begin{pmatrix} -9 \\ 2-(-3) \end{pmatrix} = \begin{pmatrix} -9 \\ 5 \end{pmatrix}$ $\qquad$ $\begin{pmatrix} -6 \\ -2 \end{pmatrix} \longrightarrow \begin{pmatrix} -6 \\ 2-(-2) \end{pmatrix} = \begin{pmatrix} -6 \\ 4 \end{pmatrix}$

$\begin{pmatrix} -6 \\ -4 \end{pmatrix} \longrightarrow \begin{pmatrix} -6 \\ 2-(-4) \end{pmatrix} = \begin{pmatrix} -6 \\ 6 \end{pmatrix}$ $\qquad$ $\begin{pmatrix} -8 \\ -2 \end{pmatrix} \longrightarrow \begin{pmatrix} -8 \\ 2-(-2) \end{pmatrix} = \begin{pmatrix} -8 \\ 4 \end{pmatrix}$

Co-ordinates: $A_4(-9,5), B_4(-6,6), C_4(-6,4), D_4(-8,4)$



Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM3-L089 in the Pupil Handbook.

[QUESTIONS]

a. Points $P(2,6)$, $Q(4,2)$ are two points on the given axes. Find the co-ordinates, P_1 and Q_1, of the image of the line joining the points under reflection in the x-axis

b. For the given points in question a., and using the same axes, find the co-ordinates, P_2 and Q_2, of the image under reflection in the y-axis ($x = 0$).
Write the mapping for each point. What do you notice?

c. Use the appropriate formula to find the image of the point $A(1,-6)$ when reflected in the lines:

i. $y = 0$ ii. $x = 0$ iii. $x = 2$ iv. $y + 1 = 0$

d. Using a scale of 2 cm to 2 units on both axis. Draw on a graph sheet two perpendicular axes Ox and Oy for the intervals $-10 \leq x \leq 10$ and $-6 \leq y \leq 6$.
   i. Plot the points with coordinates: $A(1, 3),\ B(4, 4),\ C(4, 2),\ D(2, 2)$. Join the points in that order to form shape $ABCD$.
   ii. Reflect the object in the x- axis. Write down the coordinates of the corners of the image $A_1B_1C_1D_1$.

e. On the same axes as in Question d.:
   i. Reflect the image obtained in Question d. ii, $A_1B_1C_1D_1$, in the y- axis. Write down the coordinates of the image $A_2B_2C_2D_2$.
   ii. Reflect $A_2B_2C_2D_2$ in the line $x = -5$. Write down the coordinates of $A_3B_3C_3D_3$.
   iii. Reflect $A_3B_3C_3D_3$ in the line $y - 1 = 0$. Write down the coordinates of $A_4B_4C_4D_4$.

Lesson Title: Reflection – Part 2	**Theme:** Geometry	
Lesson Number: M3-L090	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcomes By the end of the lesson, pupils will be able to: 1. Identify and explain the reflection of an object in the line $y = x$ 2. Identify and explain the reflection of an object in the line $y = -x$.	**Preparation** 1. Using a scale of 2 cm to 2 units on both axes, draw two perpendicular axes Ox and Oy for $-6 \leq x \leq 6$ and $-6 \leq y \leq 6$. 2. Write the questions found at the end of this lesson plan on the board.	

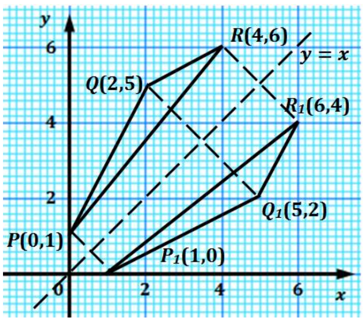
Opening *(4 minutes)*

1. Ask pupils to answer question a. on the board.
2. Invite a volunteer to answer. (Answer: for reflection in line $x - 4 = 0, k = 4$; $\begin{pmatrix} x \\ y \end{pmatrix} \longrightarrow \begin{pmatrix} 2k-x \\ y \end{pmatrix}$, gives $\begin{pmatrix} 3 \\ -5 \end{pmatrix} \longrightarrow \begin{pmatrix} 2(4)-3 \\ -5 \end{pmatrix} = \begin{pmatrix} 5 \\ -5 \end{pmatrix}$)
3. Tell pupils that after today's lesson, they will be able to identify and explain the reflection of an object in the lines $y = x$ and $y = -x$

Teaching and Learning *(20 minutes)*

1. Explain:
   - Reflection of points or shapes can be done on other lines apart from horizontal and vertical lines.
   - We follow the procedure in the previous lesson to reflect in the given line.
2. Invite a volunteer to assess question b. and tell the class what information we are given. (Answer: given: points $P(0,1), Q(2,5)$ and $R(4,6)$, line $y = x$)
3. Invite another volunteer to say what we have been asked to find. (Answer: find co-ordinates of the image of the triangle formed under reflection in line $y = x$.)

   Solution:



   b. **Step 1.** Assess and extract the given information from the problem. given: points $P(0,1), Q(2,5)$ and $R(4,6)$, line $y = x$.
   Step 2. Draw the x- and y- axes (if not already drawn). Locate the points P, Q and R on the graph. Draw the lines joining the points.
   Step 3. Draw the line $y = x$.
   Step 4. Draw a line at right angles from P to the mirror line ($y = x$). Measure this distance.
   Step 5. Measure the same distance on the opposite side of the mirror line ($y = x$) to locate the point P_1 on the graph.
   Step 6. Write the co-ordinates of $P_1 = (1,0)$.
   Step 7. Follow the same procedure for points Q and R giving $Q_1(5,2)$ and $R_1(6,4)$ (see graph).

4. Ask pupils to discuss with seatmates anything they notice when they compare the co-ordinates of P with P_1, Q with Q_1 and R with R_1.
5. Invite a volunteer to answer. (Answer: the x- and y-co-ordinates have interchanged)
6. Explain: We can write a mapping of the **reflection in the line** $y = x$ as:

$$\binom{x}{y} \longrightarrow \binom{y}{x} \qquad \text{giving} \qquad (x, y) \longrightarrow (y, x)$$

7. Ask pupils to work with seatmates to answer question c.
8. Invite a volunteer to show their answer on the board. The rest of the class should check their solution and correct any mistakes.

Solution:

c. Given: points $P(0,1), Q(2,5)$ and $R(4,6)$, line $y = -x$

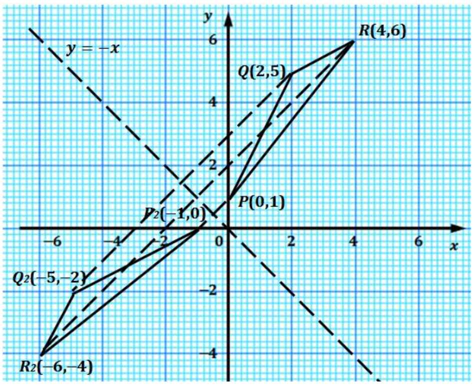
mapping for P $\binom{0}{1} \longrightarrow \binom{-1}{0}$

mapping for Q $\binom{2}{5} \longrightarrow \binom{-5}{-2}$

mapping for R $\binom{4}{6} \longrightarrow \binom{-6}{-4}$

The co-ordinates are $P_2(-1,0), Q_2(-5,-2)$ and $R_2(-6,-4)$

The x and y-co-ordinates have interchanged and have opposite signs.



9. Explain:
   - We can write a mapping of the **reflection in in the line** $y = -x$ as:

$$\binom{x}{y} \longrightarrow \binom{-y}{-x} \qquad \text{giving} \qquad (x, y) \longrightarrow (-y, -x)$$

   - The general mapping for $y = kx$ is beyond the scope of this lesson (TEACHERS PLEASE NOTE IT IS **NOT** AS STATED IN SOME FREQUENTLY USED TEXTBOOKS):

$$\binom{x}{y} \longrightarrow \binom{\frac{1}{k}y}{kx} \quad ✘$$

10. Invite a volunteer to assess question d. i. and tell the class what information we are given. (Answer: given: points $A(3,4)$, $B(-2,-1)$, line $y = x$)
11. Invite another volunteer to say what we have been asked to find. (Answer: find the image of the points under reflection in the given line)

Solution:

d. given: points $A(3,4)$, $B(-2,-1)$, line $y = x$

i. mapping is given by $\binom{x}{y}$ — $\binom{y}{x}$

$\binom{3}{4} \longrightarrow \binom{4}{3}$; image point $(4, 3)$ $\qquad \binom{-2}{-1} \longrightarrow \binom{-1}{-2}$; image point $(-1, -2)$

12. Ask pupils to work with their seatmates to answer question c. ii.
13. Invite volunteers to give their answers. The rest of the class should check their solution and correct any mistakes.

Solution:

d. Given: points $A(3,4)$, $B(-2,-1)$, line $y = -x$

ii. mapping is given by $\binom{x}{y} \longrightarrow \binom{-y}{-x}$

$\binom{3}{4} \longrightarrow \binom{-4}{-3}$; image point$(-4,-3)$ $\binom{-2}{-1} \longrightarrow \binom{-(-1)}{-(-2)} = \binom{1}{2}$; image point $(1,2)$

14. Invite a volunteer to assess questions e. i. and ii. and tell the class what information we are given. (Answer: given: quadrilateral with vertices $A(2,8), B(4,8), C(4,6)$, $D(3,5)$)
15. Invite another volunteer to say what we have been asked to do. (Answer: draw the quadrilateral $ABCD$; find image points under reflection in the line $y = x$ and draw the image $A_1B_1C_1D_1$)
16. Write on the board the formula to find the mapping for $A \longrightarrow A_1$ and hence find the image point A_1 $(8,2)$.
17. Ask pupils to continue to work with seatmates to complete question e.
18. Invite a volunteer to show the solution on the board. The rest of the class should check their solution and correct any mistakes.

Solution:

e. Given: quadrilateral with vertices $A(2,8), B(4,8), C(4,6)$ and $D(3,5)$; line $y = x$ mapping is given by $\binom{x}{y} \longrightarrow \binom{y}{x}$

i. All diagrams for this question can be found at the end of question f.

ii. $\binom{2}{8} \longrightarrow \binom{8}{2}$ giving $A_1(8,2)$ $\binom{4}{8} \longrightarrow \binom{8}{4}$ giving $B_1(8,4)$

$\binom{4}{6} \longrightarrow \binom{6}{4}$ giving $C_1(6,4)$ $\binom{3}{5} \longrightarrow \binom{5}{3}$ giving $D_1(5,3)$

Practice *(15 minutes)*

1. Ask pupils to work independently to answer question f.
2. Walk around, if possible, to check the answers and clear any misconceptions.
3. Invite volunteers to come to the board to show their solutions.
   The rest of the class should check their solutions and correct any mistakes.

Solution:

f. Given: quadrilateral with vertices $A(2,8), B(4,8), C(4,6)$ and $D(3,5)$; line $y = -x$ mapping is given by $\binom{x}{y} \longrightarrow \binom{-y}{-x}$

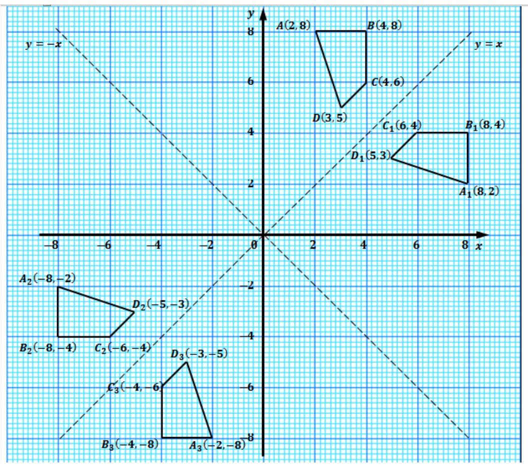
i. $\binom{2}{8} \longrightarrow \binom{-8}{-2}$ giving $A_2(-8,-2)$ $\binom{4}{8} \longrightarrow \binom{-8}{-4}$ giving $B_2(-8,-4)$

$\binom{4}{6} \longrightarrow \binom{-6}{-4}$ giving $C_2(-6,-4)$ $\binom{3}{5} \longrightarrow \binom{-5}{-3}$ giving $D_2(-5,-3)$

ii. Given: quadrilateral with vertices $A_1(8,2), B_1(8,4), C_1(6,4)$ and $D_1(5,3)$

$\binom{8}{2} \longrightarrow \binom{-2}{-8}$ giving $A_3(-2,-8)$ $\binom{8}{4} \longrightarrow \binom{-4}{-8}$giving $B_3(-4,-8)$

. $\binom{6}{4} \longrightarrow \binom{-4}{-6}$ giving $C_3(-4,-6)$ $\binom{5}{3} \longrightarrow \binom{-3}{-5}$giving $D_3(-3,-5)$



Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM3-L090 in the Pupil Handbook.

[QUESTIONS]

a. Use the appropriate formula to find the image of the point $(3, -5)$ when reflected in the line $x - 4 = 0$.

b. Points $P(0,1), Q(2,5)$ and $R(4,6)$ are points on the given axes. Find the co-ordinates, P_1, Q_1 and R_1 of the image of the triangle formed under reflection in the line $y = x$.

c. For the given points in question b., and using the same axes, find the co-ordinates, P_2, Q_2, and R_2 of the image under reflection in the line $y = -x$.
Write the mapping for each point. What do you notice?

d. Use the appropriate formula to find the image of the points $A(3,4)$ and $B(-2,-1)$ when reflected in the line: i. $y = x$ ii. $y = -x$

e. Using a scale of 2 cm to 2 units on both axes, draw and label on a sheet of graph paper two perpendicular axes Ox and Oy for $-8 \leq x \leq 8$ and $-8 \leq y \leq 8$.
Draw on the same axes, showing clearly the co-ordinates of all vertices:
   i. The quadrilateral with vertices $A(2,8), B(4,8), C(4,6)$ and $D(3,5)$.
   ii. The image $A_1B_1C_1D_1$ of $ABCD$ under a reflection in the line $y = x$ where $A \longrightarrow A_1$, $B \longrightarrow B_1$, $C \longrightarrow C_1$ and $D \longrightarrow D_1$.

f. On the same axes as for question e.:
   i. For the quadrilateral $ABCD$ from question e., draw the image $A_2B_2C_2D_2$ under a reflection in the line $y = -x$ where $A \longrightarrow A_2$, $B \longrightarrow B_2$, $C \longrightarrow C_2$ and $D \longrightarrow D_2$.
   ii. For the quadrilateral $A_1B_1C_1D_1$ from question e., draw the image $A_3B_3C_3D_3$ under a reflection in the line $y = -x$ where $A_1 \longrightarrow A_3$, $B_1 \longrightarrow B_3$, $C_1 \longrightarrow C_3$ and $D_1 \longrightarrow D_3$.

Lesson Title: Rotation – Part 1	**Theme:** Geometry	
Lesson Number: M3-L091	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcomes By the end of the lesson, pupils will be able to: 1. Identify that rotation is a movement around a fixed point. 2. Find the image of an object under rotation about the origin.	**Preparation** 1. Using a scale of 2 cm to 2 units on both axes, draw two perpendicular axes Ox and Oy for $-6 \leq x \leq 6$ and $0 \leq y \leq 6$. 2. Write the questions found at the end of this lesson plan on the board.	

Opening *(1 minute)*

1. Tell pupils that after today's lesson, they will be able to identify that rotation is a movement around a fixed point. They will also be able to find the image of an object under rotation about the origin.

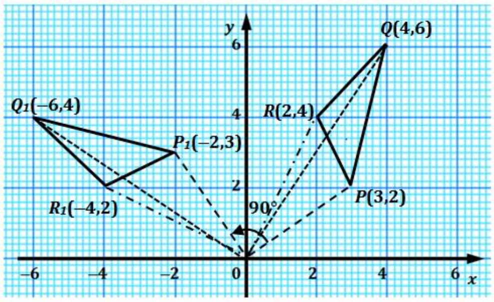
Teaching and Learning *(20 minutes)*

1. Explain:
   - Rotation is a movement around a fixed point, called the **centre of rotation**.
   - Rotation is always done in a specified angle and direction about a specified point.
   - Anti-clockwise rotation is the standard rotation used, and can be assumed if no direction is given.
   - For example, an object can be rotated $90°$ anti-clockwise about the origin.
2. Invite a volunteer to say what clockwise rotation is, the same as $90°$ anti-clockwise. (Answer: $270°$ clockwise)
3. Explain: We can rotate a given point or shape by following a procedure similar to what we have done previously for translation and reflection.
4. Invite a volunteer to assess question a. and tell the class what information we are given. (Answer: given: points $P(3,2), Q(4,6)$ and $R(2,4)$, origin, $O(0,0)$)
5. Invite another volunteer to say what we have been asked to find. (Answer: find co-ordinates of the image of the triangle formed under an anti-clockwise rotation of $90°$ about the origin.)

Solution:

a. **Step 1.** Assess and extract the given information from the problem.
given: points $P(3,2), Q(4,6)$ and $R(2,4)$, origin $O(0,0)$

Step 2. Draw the x- and y- axes (if not already drawn).
Locate the points P, Q and R on the graph.
Draw the lines joining the points.



Step 3. Mark the centre of rotation $O(0,0)$.
Draw a straight line from O to P.
Measure the distance OP.

Step 4. Measure an angle of $90°$ in an anti-clockwise direction.

Step 5. Measure the distance OP along this line to locate the point P_1.

Step 6. Write the co-ordinates of $P_1 = (-2,3)$.

Step 7. Follow the same procedure for points Q and R giving $Q_1(-6,4)$ and $R_1(-4,2)$. (see graph)

6. Ask pupils to discuss with seatmates anything they notice when they compare the co-ordinates of P with P_1, Q with Q_1 and R with R_1.
7. Invite a volunteer to answer. (Answer: the x- and y-co-ordinates have interchanged, and the y-co-ordinate has also changed signs)
8. Explain:
   - We can write a mapping of the **rotation through $90°$ anti-clockwise or $270°$ clockwise about the origin O** as:

     $$\binom{x}{y} \longrightarrow \binom{-y}{x} \qquad \text{giving} \qquad (x, y) \longrightarrow (-y, x)$$

   - We can follow the same procedure to draw the **rotation through $270°$ anti-clockwise or $90°$ clockwise about the origin O.** The mapping is given as:

     $$\binom{x}{y} \longrightarrow \binom{y}{-x} \qquad \text{giving} \qquad (x, y) \longrightarrow (y, -x)$$

   - And the mapping for the **rotation through $180°$ (half turn) anti-clockwise about the origin O** is given as:

     $$\binom{x}{y} \longrightarrow \binom{-x}{-y} \qquad \text{giving} \qquad (x, y) \longrightarrow (-x, -y)$$

     Since $180°$ is a half-turn, both anti-clockwise and clockwise rotation result in the same image.
9. Invite a volunteer to assess question b. and tell the class what information we are given. (Answer: given: $A(-4, -7)$, $A_1(-7,4)$ and $B(3, -5)$)
10. Invite another volunteer to say what we have been asked to find. (Answer: find co-ordinates of B_1)

Solution:

b. Given: $A(-4, -7)$, $A_1(-7,4)$ and $B(3, -5)$

$$\binom{-4}{-7} \longrightarrow \binom{-7}{4}$$

this is the same as $$\binom{x}{y} \longrightarrow \binom{y}{-x}$$

$$\therefore \binom{3}{-5} \longrightarrow \binom{-5}{-3}$$

The point B was rotated $270°$ anti-clockwise about the origin *O.*

11. Invite a volunteer to assess questions c. i. and ii. and tell the class what information we are given. (Answer: given: the triangle PQR with $P(5,3), Q(8,4)$ and $R(2,6)$)
12. Invite another volunteer to say what we have been asked to do. (Answer: draw the triangle PQR; find the image triangle $P_1Q_1R_1$ of triangle PQR under an anti-clockwise rotation of $90°$ about the origin)
13. Write on the board the formula to find the mapping for $P \longrightarrow P_1$ and hence find the image point P_1. (=$\binom{-3}{5}$).
14. Ask pupils to write down the image points for $Q(8,4)$ and $R(2,6)$.
15. Invite a volunteer to give the image points. (Answer: $Q_1(-4,8)$ and $R_1(-6,2)$
16. Ask pupils to work with seatmates to complete questions c. ii.
17. Invite a volunteer to show the solution on the board. The rest of the class should check their solution and correct any mistakes.

    Solutions:

    c. Given: triangle PQR with $P(5,3), Q(8,4)$ and $R(2,6)$

    mapping is given by $\binom{x}{y} \longrightarrow \binom{-y}{x}$

    i. All diagrams for this question can be found at the end of question d.

    ii. $\binom{5}{3} \longrightarrow \binom{-3}{5}$ giving $P_1(-3, 5)$ $\binom{8}{4} \longrightarrow \binom{-4}{8}$ giving $Q_1(-4, 8)$

    $\binom{2}{6} \longrightarrow \binom{-6}{2}$ giving $R_1(-6, 2)$

Practice *(15 minutes)*

1. Ask pupils to work independently to answer question d.
2. Walk around, if possible, to check the answers and clear misconceptions.
3. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.

   Solutions:

   d. Given: triangle PQR with $P(5,3), Q(8,4)$ and $R(2,6)$

   mapping is given by $\binom{x}{y} \longrightarrow \binom{y}{-x}$

   i. $\binom{5}{3} \longrightarrow \binom{3}{-5}$ giving $P_2(3, -5)$ $\binom{8}{4} \longrightarrow \binom{4}{-8}$ giving $Q_2(4, -8)$

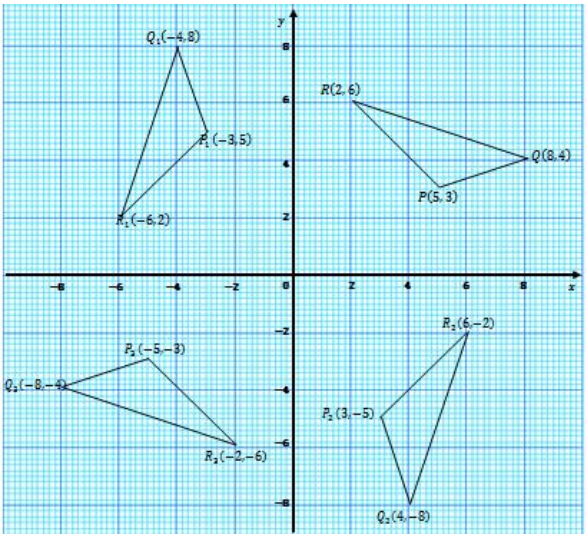
   $\binom{2}{6} \longrightarrow \binom{6}{-2}$ giving $R_2(6, -2)$

   ii. Given: triangle PQR with $P(5,3), Q(8,4)$ and $R(2,6)$

   mapping is given by $\binom{x}{y} \longrightarrow \binom{-x}{-y}$

   $\binom{5}{3} \longrightarrow \binom{-5}{-3}$ giving $P_2(-5, -3)$ $\binom{8}{4} \longrightarrow \binom{-8}{-4}$ giving $Q_2(-8, -4)$

   $\binom{2}{6} \longrightarrow \binom{-2}{-6}$ giving $R_2(-2, -6)$



Closing *(4 minutes)*

1. Ask pupils to tell seatmates one new thing they learned or understood better after this lesson.
2. Invite volunteers to share their answers with the class. (Answer: various)
3. For homework, have pupils do the practice activity PHM3-L091 in the Pupil Handbook.

[QUESTIONS]

a. Points $P(3,2), Q(4,6)$ and $R(2,4)$ are points on the given axes. Find the co-ordinates, P_1, Q_1 and R_1 of the image of the triangle formed under an anti-clockwise rotation of $90°$ about the origin, O.
b. A rotation about the origin maps $A(-4,-7)$ to $A_1(-7,4)$ and $B(3,-5)$ to B_1. Find the co-ordinates of B_1. Describe the rotation for the mapping.
c. Using a scale of 2 cm to 2 units on both axes, draw two perpendicular axes Ox and Oy for $-8 \leq x \leq 8$ and $-8 \leq y \leq 8$ on a sheet of graph paper.
   Draw on the same axes clearly showing the co-ordinates of all vertices:
   i. The triangle PQR with $P(5,3), Q(8,4)$ and $R(2,6)$.
   ii. The image $\Delta P_1Q_1R_1$ of ΔPQR under an anti-clockwise rotation of $90°$ about the origin where $P \longrightarrow P_1,\ Q \longrightarrow Q_1\ R \longrightarrow R_1$.
d. On the same axes as for question c.;
   i. For ΔPQR from question c. draw the image $\Delta P_2Q_2R_2$ under an anti-clockwise rotation of $270°$ about the origin where $P \longrightarrow P_2,\ Q \longrightarrow Q_2\ R \longrightarrow R_2$.
   ii. For ΔPQR from question c. draw the image $\Delta P_3Q_3R_3$ under a rotation of $180°$ about the origin where $P \longrightarrow P_3,\ Q \longrightarrow Q_3\ R \longrightarrow R_3$..

Lesson Title: Rotation – Part 2	**Theme:** Geometry	
Lesson Number: M3-L092	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to find the image of an object under rotation about any point (a, b).	**Preparation** Write the questions found at the end of this lesson plan on the board.	

Opening *(4 minutes)*

1. Ask pupils to answer question a. on the board.
2. Invite a volunteer to give the answer. (Answer: for rotation under $180°$, $(x, y) \longrightarrow (-x, -y)$, therefore $X(4, -3) \longrightarrow (-4, 3)$)
3. Tell pupils that after today's lesson, they will be able to find the image of an object under rotation about any point (a, b).

Teaching and Learning *(20 minutes)*

1. Explain:
   - We can rotate a given point or shape about any point (a, b) by following the procedure from the last lesson.
   - We can also use a formula to calculate the new co-ordinates obtained after a specified rotation.
   - We follow the steps given below to find the formula for **rotation through $90°$ anti-clockwise or $270°$ clockwise about the point (a, b)**.
2. Write on the board:

Step 1.	Subtract the co-ordinates of the centre of rotation (a, b) from (x, y)	$\begin{pmatrix} x-a \\ y-b \end{pmatrix}$
Step 2.	Apply the appropriate rotation formula	$\begin{pmatrix} x-a \\ y-b \end{pmatrix} \longrightarrow \begin{pmatrix} -(y-b) \\ x-a \end{pmatrix}$
Step 3.	Add the result in Step 2 to the centre of the rotation to get the image point	$\begin{pmatrix} -(y-b)+a \\ (x-a)+b \end{pmatrix}$
Step 4.	Write the co-ordinates of the image point	$(-(y-b)+a,\ (x-a)+b)$

3. Ask pupils to work with seatmates to answer question b.
4. Invite volunteers to show how they worked out the formulae on the board. The rest of the class should check their solution and correct any mistakes.

   Solution:

   b. i. **Rotation through $270°$ anti-clockwise or $90°$ clockwise about (a, b)**

$\begin{pmatrix} x-a \\ y-b \end{pmatrix}$	subtract components of the centre of rotation from the given point
$\begin{pmatrix} x-a \\ y-b \end{pmatrix} \longrightarrow \begin{pmatrix} y-b \\ -(x-a) \end{pmatrix}$	apply the appropriate formula
$\begin{pmatrix} (y-b)+a \\ -(x-a)+b \end{pmatrix}$	add result to (a, b)
$((y-b)+a, -(x-a)+b)$	co-ordinates of the image point

ii. **Rotation through 180° (half turn) anti-clockwise about (a, b)**

$\begin{pmatrix} x-a \\ y-b \end{pmatrix}$ subtract components of the centre of rotation from the given point

$\begin{pmatrix} x-a \\ y-b \end{pmatrix} \longrightarrow \begin{pmatrix} -(x-a) \\ -(y-b) \end{pmatrix}$ apply the appropriate formula

$\begin{pmatrix} -(x-a)+a \\ -(y-b)+b \end{pmatrix} = \begin{pmatrix} -(x-2a) \\ -(y-2b) \end{pmatrix}$ add result to (a, b)

$(-(x-2a), -(y-2b))$ co-ordinates of the image point

5. Ask pupils to continue to work with seatmates to answer question c.
6. Invite a volunteer to show how they worked out the formulae on the board. The rest of the class should check their solution and correct any mistakes.

Solution:

c. Given: point $Y(-2, -5)$, rotated $90°$ clockwise about the point $(1, -4)$

$\begin{pmatrix} x-a \\ y-b \end{pmatrix} = \begin{pmatrix} -2-1 \\ -5-(-4) \end{pmatrix} = \begin{pmatrix} -2-1 \\ -5+4 \end{pmatrix}$ subtract components of the centre of the rotation from the given point

$= \begin{pmatrix} -3 \\ -1 \end{pmatrix}$

$\begin{pmatrix} x-a \\ y-b \end{pmatrix} \longrightarrow \begin{pmatrix} y-b \\ -(x-a) \end{pmatrix}$ apply the appropriate formula

$\begin{pmatrix} -3 \\ -1 \end{pmatrix} \longrightarrow \begin{pmatrix} -1 \\ -(-3) \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$

$\begin{pmatrix} -1 \\ 3 \end{pmatrix} \longrightarrow \begin{pmatrix} -1+1 \\ 3+(-4) \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$ add result to $(1, -4)$

$Y(-2, -5)$ rotated $90°$ clockwise about the point $(1, -4)$ gives $(0, -1)$

7. Invite a volunteer to assess questions d. i. and ii. and tell the class what information we are given. (Answer: given: the triangle PQR with $P(2,4), Q(4,8)$ and $R(8,7)$)
8. Invite another volunteer to say what we have been asked to do. (Answer: draw the triangle PQR; find the image triangle $P_1Q_1R_1$ of triangle PQR under an anti-clockwise rotation of $90°$ about the point $(1,2)$)
9. Write on the board the formula to find the mapping for $P \longrightarrow P_1$ and hence find the image point P_1. (see solution below)
10. Ask pupils to work with seatmates to complete questions d. ii.
11. Invite a volunteer to show the solution on the board. The rest of the class should check their solution and correct any mistakes.

Solution:

d. Given: triangle PQR with $P(2,4), Q(4,8)$ and $R(8,7)$

i. All diagrams for this question can be found at the end of question d.

ii. mapping under an anti-clockwise rotation of $90°$ about the point (a, b) is given by: $\begin{pmatrix} x \\ y \end{pmatrix} \longrightarrow \begin{pmatrix} -(y-b)+a \\ (x-a)+b \end{pmatrix}$

for rotation about $C(1,2)$ $\begin{pmatrix} x \\ y \end{pmatrix} \longrightarrow \begin{pmatrix} -(y-2)+1 \\ (x-1)+2 \end{pmatrix} = \begin{pmatrix} -y+3 \\ x+1 \end{pmatrix}$

$\begin{pmatrix} 2 \\ 4 \end{pmatrix} \longrightarrow \begin{pmatrix} -4+3 \\ 2+1 \end{pmatrix}$

$\longrightarrow \begin{pmatrix} -1 \\ 3 \end{pmatrix}$ giving $P_1(-1, 3)$

$\begin{pmatrix} 4 \\ 8 \end{pmatrix} \longrightarrow \begin{pmatrix} -8+3 \\ 4+1 \end{pmatrix}$

$\longrightarrow \begin{pmatrix} -5 \\ 5 \end{pmatrix}$ giving $Q_1(-5, 5)$

$\binom{8}{7} \longrightarrow \binom{-7+3}{8+1}$

$\longrightarrow \binom{-4}{9}$ giving $R_1(-4, 9)$

Practice *(15 minutes)*

1. Ask pupils to work independently to answer question e.
2. Walk around, if possible, to check the answers and clear up any misconceptions.
3. Ask volunteers to come to the board to show their solutions.
   The rest of the class should check their solutions and correct any mistakes.
   Solutions:
   d. Given: triangle PQR with $P(2,4), Q(4,8)$ and $R(8,7)$

i. mapping is given by $\binom{x}{y} \longrightarrow \binom{(y-b)+a}{-(x-a)+b} = \binom{(y-2)+1}{-(x-1)+2} = \binom{y-1}{-x+3}$

$\binom{2}{4} \longrightarrow \binom{4-1}{-2+3}$

$\longrightarrow \binom{3}{1}$ giving $P_2(3, 1)$

$\binom{4}{8} \longrightarrow \binom{8-1}{-4+3}$

$\longrightarrow \binom{7}{-1}$ giving $Q_2(7, -1)$

$\binom{8}{7} \longrightarrow \left(\binom{7-1}{-8+3}\right)$

$\longrightarrow \binom{6}{-5}$ giving $R_2(6, -5)$

Ii. mapping is given by $\binom{x}{y} \longrightarrow \binom{-(x-2a)}{-(y-2b)} = \binom{-(x-2(1))}{-(y-2(2))} = \binom{-x+2}{-y+4}$

$\binom{2}{4} \longrightarrow \binom{-2+2}{-4+4}$

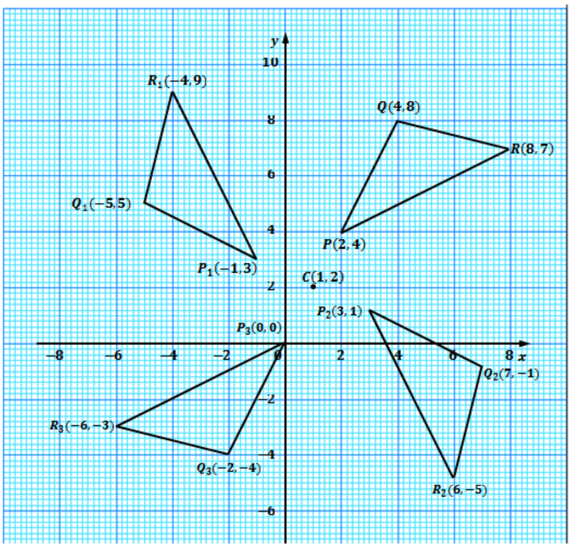
$\longrightarrow \binom{0}{0}$ giving $P_3(0, 0)$

$\binom{4}{8} \longrightarrow \binom{-4+2}{-8+4}$

$\longrightarrow \binom{-2}{-4}$ giving $Q_3(-2, -4)$

$\binom{8}{7} \longrightarrow \binom{-8+2}{-7+4}$

$\longrightarrow \binom{-6}{-3}$ giving $R_3(-6, -3)$



Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM3-L092 in the Pupil Handbook.

[QUESTIONS]

a. Use the appropriate formula to find the co-ordinates of the image point when point $X(4,-3)$ is rotated $180°$ about the origin.

b. Find the formula for image point for:
   i. Rotation through $270°$ anti-clockwise or $90°$ clockwise about (a,b)
   ii. Rotation through $180°$ (half turn) anti-clockwise about (a,b)

c. Use the appropriate formula to find the co-ordinates of the image point when point $Y(-2,-5)$ is rotated $90°$ clockwise about the point $(1,-4)$.

d. Using a scale of 2 cm to 2 units on both axes, draw on a sheet of graph paper, two perpendicular axes Ox and Oy for $-8 \leq x \leq 8$ and $-6 \leq y \leq 10$.
   Draw on the same axes showing clearly the co-ordinates of all vertices:
   iii. The triangle PQR with $P(2,4), Q(4,8)$ and $R(8,7)$.
   iv. The image $\Delta P_1Q_1R_1$ of triangle PQR under an anti-clockwise rotation of $90°$ about the point $(1,2)$ where $P \longrightarrow P_1,\ Q \longrightarrow Q_1\ R \longrightarrow R_1$.

e. Draw on the same axes as for question d. showing clearly the co-ordinates of all vertices:
   i. For ΔPQR from question d. draw the image $\Delta P_2Q_2R_2$ under an anti-clockwise rotation of $270°$ about the point $(1,2)$ where $P \longrightarrow P_2,\ Q \longrightarrow Q_2\ R \longrightarrow R_2$.
   ii. For ΔPQR from question d. draw the image $\Delta P_3Q_3R_3$ under a rotation of $180°$ about the point $(1,2)$ where $P \longrightarrow P_3,\ Q \longrightarrow Q_3\ R \longrightarrow R_3$.

Lesson Title: Enlargement – Part 1	Theme: Geometry	
Lesson Number: M3-L093	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to use scalar multiplication to enlarge given shapes.	**Preparation** 1. Using a scale of 2 cm to 2 units on both axes, draw on a sheet of graph paper, two perpendicular axes Ox and Oy for $-8 \le x \le 8$ and $-10 \le y \le 10$. 2. Write the questions found at the end of this lesson plan on the board.	

Opening *(4 minutes)*

1. Ask pupils to answer question a. on the board.
2. Invite a volunteer to give the answer. (Answer: $\mathbf{a} = \binom{3}{-2}, 3\mathbf{a} = \mathbf{3}\binom{3}{-2} = \binom{9}{-6}$)
3. Tell pupils that after today's lesson, they will be able to use scalar multiplication to enlarge given shapes.

Teaching and Learning *(20 minutes)*

1. Explain:
   - An enlargement is a transformation which enlarges or reduces the size of an image.
   - It is described by a centre of enlargement and a scale factor, k.
   - Two different formulas are given for enlargement:
   - The formula **for enlargement from the origin $\boldsymbol{O}$ by a scale factor $\boldsymbol{k}$** is given by:

$$\binom{x}{y} \longrightarrow k\binom{x}{y} = \binom{kx}{ky}$$
$$(x, y) \longrightarrow (kx, ky)$$

where k is positive or negative whole number or fraction

   - The formula for **enlargement from any point $(\boldsymbol{a}, \boldsymbol{b})$ other than the origin $\boldsymbol{O}$ by a scale factor $\boldsymbol{k}$** can be found by following the steps given below.
2. Write on the board:

Step 1. Subtract the co-ordinates of the centre of rotation (a, b) from (x, y) $\qquad \binom{x-a}{y-b}$

Step 2. Enlarge using the given scale factor $\qquad \binom{x-a}{y-b} \longrightarrow \binom{k(x-a)}{k(y-b)}$

Step 3. Add the result in Step 2 to the centre of rotation to get the image point $\qquad \binom{k(x-a)+a}{k(y-b)+b}$

Step 4. Write the co-ordinates of the image point $\qquad (k(x-a)+a,\ k(y-b)+b)$

3. Ask pupils to work with seatmates to answer question b.
4. Invite volunteers to show how they worked out the formulae on the board. The rest of the class should check their solution and correct any mistakes.
   Solution:

b. Given: $(-1,-6)$, enlarge with scale factor 4

i. $\begin{pmatrix}-1\\-6\end{pmatrix} \longrightarrow 4\begin{pmatrix}-1\\-6\end{pmatrix} = \begin{pmatrix}4\times(-1)\\4\times(-6)\end{pmatrix} = \begin{pmatrix}-4\\-24\end{pmatrix}$

ii. Given: $(-1,-6)$ enlarge about the point $(2,4)$ with scale factor 4

$\begin{pmatrix}x-a\\y-b\end{pmatrix} \longrightarrow \begin{pmatrix}-1-2\\-6-4\end{pmatrix} = \begin{pmatrix}-3\\-10\end{pmatrix}$ subtract components of the centre of rotation from given point

$\begin{pmatrix}-3\\-10\end{pmatrix} \longrightarrow 4\begin{pmatrix}-3\\-10\end{pmatrix} = \begin{pmatrix}-12\\-40\end{pmatrix}$ enlarge using given scale factor

$\begin{pmatrix}-12\\-40\end{pmatrix} \longrightarrow \begin{pmatrix}-12+2\\-40+4\end{pmatrix} = \begin{pmatrix}-10\\-36\end{pmatrix}$ add back components of the centre of rotation

5. Invite a volunteer to assess questions c. i. and ii. and tell the class what information we are given. (Answer: given: triangle PQR with points $P(1,1)$, $Q(3,1)$ and $R(1,4)$)
6. Invite another volunteer to say what we have been asked to do. (Answer: draw the triangle PQR; find the image triangle $P_1Q_1R_1$ of triangle PQR under an enlargement from the origin with scale factor 2)
7. Show on the board how to enlarge using the given information.

Solution:

c. Given: triangle PQR with points $P(1,1)$, $Q(3,1)$ and $R(1,4)$

i. All diagrams for this question can be found at the end of the question.

ii. Mapping under an enlargement from the origin with scale factor 2 $\begin{pmatrix}x\\y\end{pmatrix} \longrightarrow \begin{pmatrix}kx\\kx\end{pmatrix} = \begin{pmatrix}2x\\2x\end{pmatrix}$

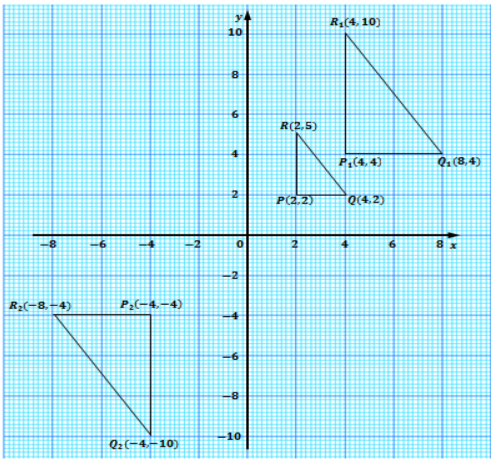
$$\begin{pmatrix}1\\1\end{pmatrix} \longrightarrow 2\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}2\\2\end{pmatrix}$$
$$\begin{pmatrix}3\\1\end{pmatrix} \longrightarrow 2\begin{pmatrix}3\\1\end{pmatrix} = \begin{pmatrix}6\\2\end{pmatrix}$$
$$\begin{pmatrix}1\\4\end{pmatrix} \longrightarrow 2\begin{pmatrix}1\\4\end{pmatrix} = \begin{pmatrix}2\\8\end{pmatrix}$$

$P_1(2,2), Q_1(6,2)$ and $R_1(2,8)$

8. Ask pupils to work with seatmates to complete questions c. iii.
9. Invite a volunteer to show the solution on the board. The rest of the class should check their solution and correct any mistakes.

iii.
$$\begin{pmatrix}1\\1\end{pmatrix} \longrightarrow -2\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}-2\\-2\end{pmatrix}$$
$$\begin{pmatrix}3\\1\end{pmatrix} \longrightarrow -2\begin{pmatrix}3\\1\end{pmatrix} = \begin{pmatrix}-6\\-2\end{pmatrix}$$
$$\begin{pmatrix}1\\4\end{pmatrix} \longrightarrow -2\begin{pmatrix}1\\4\end{pmatrix} = \begin{pmatrix}-2\\-8\end{pmatrix}$$

$P_2(-2,-2), Q_2(-6,-2)$ and $R_2(-2,-8)$



iv. The image $P_2Q_2R_2$ is upside down.

10. Explain: Negative scale factors give images at the opposite side of the centre of enlargement. The image is turned upside down (or inverted).
11. Ask pupils to work with seatmates to complete question d.
12. Invite a volunteer to show the solution on the board. The rest of the class should check their solution and correct any mistakes.

Solution:

d. Given: $A(8,4)$, $B(8,-8)$, $C(-4,-8)$ and $D(-4,4)$

i.
$$\begin{pmatrix}8\\4\end{pmatrix} \longrightarrow \frac{1}{2}\begin{pmatrix}8\\4\end{pmatrix} = \begin{pmatrix}4\\2\end{pmatrix}$$
$$\begin{pmatrix}8\\-8\end{pmatrix} \longrightarrow \frac{1}{2}\begin{pmatrix}8\\-8\end{pmatrix} = \begin{pmatrix}4\\-4\end{pmatrix}$$
$$\begin{pmatrix}-4\\-8\end{pmatrix} \longrightarrow \frac{1}{2}\begin{pmatrix}-4\\-8\end{pmatrix} = \begin{pmatrix}-2\\-4\end{pmatrix}$$
$$\begin{pmatrix}-4\\4\end{pmatrix} \longrightarrow \frac{1}{2}\begin{pmatrix}-4\\4\end{pmatrix} = \begin{pmatrix}-2\\2\end{pmatrix}$$

$A_1(4,2), B_1(4,-4), C_1(-2,-4),$ and $D_1(-2,2)$

ii. The co-ordinates of $A_1B_1C_1D_1$ are half that of the original $ABCD$.

13. Explain:
    - An object (shape) under enlargement with a scale factor which is a fraction, results in a smaller image than the object. It is a **reduction**.
    - The diagram in the next question will show this clearly.

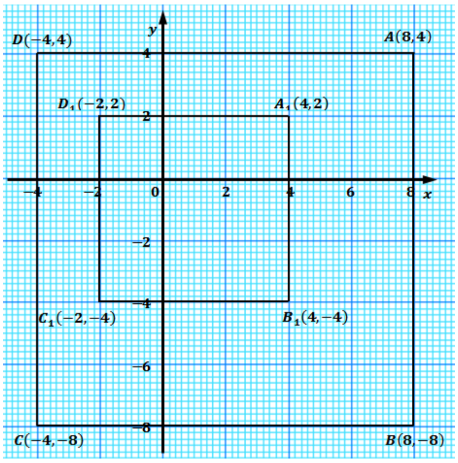
Practice *(15 minutes)*

1. Ask pupils to work independently to answer question e.
2. Walk around, if possible, to check the answers and clear any misconceptions.
3. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.

   Solution:

   e. Given: $A(8,4)$, $B(8,-8)$, $C(-4,-8)$ and $D(-4,4)$ from question d.

i.
ii.



Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM3-L093 in the Pupil Handbook.

[QUESTIONS]

a. Find $3\mathbf{a}$ when $\mathbf{a} = \binom{3}{-2}$.

b. Find the image of $(-1, -6)$ under the enlargement with scale factor of 4 from:
   i. The origin   ii. The point $(2,4)$

c. Draw on the given axes showing clearly the co-ordinates of all vertices:
   v. The triangle PQR with $P(1,1), Q(3,1)$ and $R(1,4)$.
   vi. The image triangle $P_1Q_1R_1$ of triangle PQR under an enlargement from the origin with scale factor 2 where $P \longrightarrow P_1,\ Q \longrightarrow Q_1\ R \longrightarrow R_1$.
   vii. The image triangle $P_2Q_2R_2$ of triangle PQR under an enlargement from the origin with scale factor -2 where $P \longrightarrow P_2,\ Q \longrightarrow Q_2\ R \longrightarrow R_2$.
   viii. What do you notice about the enlargement $P_2Q_2R_2$?

d. A square has vertices $A(8,4),\ B(8,-8),\ C(-4,-8)$ and $D(-4,4)$.
   i. Find the co-ordinates of the vertices of the image square $A_1B_1C_1D_1$ of $ABCD$ under an enlargement from the origin with scale factor $\frac{1}{2}$ where $A \longrightarrow A_1,\ B \longrightarrow B_1,\ C \longrightarrow C_1$ and $D \longrightarrow D_1$.
   ii. What do you notice about the co-ordinates of $A_1B_1C_1D_1$?

e. Using a scale of 2 cm to 2 units on both axes, draw on a sheet of graph paper, two perpendicular axes Ox and Oy for $-4 \leq x \leq 8$ and $-8 \leq y \leq 4$.
   Draw on the same axes, showing clearly the co-ordinates of all vertices:
   iii. The square $ABCD$ from question d.
   iv. The image $A_1B_1C_1D_1$ of $ABCD$ from question d.

Lesson Title: Enlargement – Part 2	**Theme:** Geometry	
Lesson Number: M3-L094	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to find the relationship between scale factor, length, area and volume of enlarged shapes.	**Preparation** 1. Using a scale of 2 cm to 2 units on both axes, draw on a sheet of graph paper, two perpendicular axes Ox and Oy for $0 \leq x \leq 16$ and $0 \leq y \leq 16$. 2. Write the questions found at the end of this lesson plan on the board.	

Opening *(4 minutes)*

1. Ask pupils to answer question a. on the board.
2. Invite a volunteer to answer. (Example answers: Fractional scale factor of $\frac{1}{2}$ means the image is half the size of the object. Negative scale factor means the image is on the opposite side of the centre of enlargement as the object is inverted.)
3. Tell pupils that after today's lesson, they will be able to find the relationship between scale factor, length, area and volume of enlarged shapes.

Teaching and Learning *(20 minutes)*

1. Explain: We will look at question b. to help us understand the relationship between the scale factor used to enlarge a figure and the resulting lengths of the sides of the enlarged figure.
2. Invite a volunteer to assess question b. and tell the class what information we are given. (Answer: given: triangle with vertices $A(2,1), B(5,1)$ and $C(5,5)$; enlarged from the origin to give image having vertices $A_1(6,3)$, $B_1(15,3)$ and $C_1(15,15)$)
3. Invite another volunteer to say what we have been asked to find. (Answer: find scale factor of enlargement.
   Solution:

b. **Step 1.** Assess and extract the given information from the problem.
given: triangle with vertices $A(2,1), B(5,1)$ and $C(5,5)$; enlarged from the origin to give image having vertices $A_1(6,3)$, $B_1(15,3)$ and $C_1(15,15)$

i. **Step 2.** Find the formula for the mapping

mapping for A $\begin{pmatrix}2\\1\end{pmatrix} \longrightarrow \begin{pmatrix}6\\3\end{pmatrix}$ for A_1

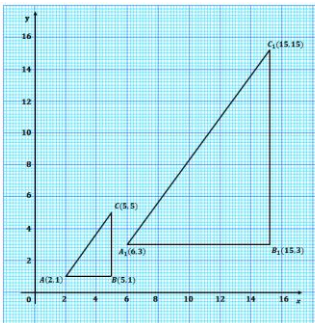
mapping for B $\begin{pmatrix}5\\1\end{pmatrix} \longrightarrow \begin{pmatrix}15\\3\end{pmatrix}$ for B_1

mapping for C $\begin{pmatrix}5\\5\end{pmatrix} \longrightarrow \begin{pmatrix}15\\15\end{pmatrix}$ for C_1

from the images it is clear the mapping is $\begin{pmatrix}x\\y\end{pmatrix} \longrightarrow 3\begin{pmatrix}x\\y\end{pmatrix}$, $\therefore$ the scale factor is 3.

ii. **Step 3.** Draw the object and image triangles

iii. **Step 4.** Measure corresponding lengths and complete the table



Length of triangle		Length of image	
AB	3	A_1B_1	9
BC	4	B_1C_1	12
AC	5	A_1C_1	15

iv.
- Ask pupils to discuss with seatmates what they notice when they compare the lengths of the sides of the triangle with that of the image.
- Invite a volunteer to answer. (Answer: the lengths of the sides of the image are 3 times the lengths of the sides of the object.

Step 4. Write the relationship connecting scale factor to length.

length of image $= 3 \times$ length of corresponding side of object
where $3 =$ scale factor, k, of the enlargement

4. Explain: We can write the ratio of a length of the image to the corresponding length of the object as the scale factor of the enlargement.

$$k = \frac{\text{length of image}}{\text{length of corresponding side of object}} \qquad (1)$$

5. Invite a volunteer to say what type of triangles have the ratios of the lengths of their sides the same? (Answer: similar triangles)
6. Explain:
   - Enlargement always produces images which are similar figures to the objects in proportion to the scale factor of the enlargement.
   - For triangles, if either one of the following is true, they are **similar triangles**:
     – The ratio of corresponding sides are equal.
     – The angles of one triangle are equal to corresponding angles in the other triangle.
7. Invite a volunteer to assess question c. i. and tell the class what information we are given. (Answer: given: ΔABE with $AB = 6$ cm, $AE = x$, $EB = 6$ cm and ΔADE with $BC = y$, $ED = 4$ cm, $DC = 9$ cm)
8. Invite another volunteer to say what we have been asked to find. (Answer: explain why triangles ABE and ADE are similar.)

Solution:

c. Given: ΔABE with $AB = 6$ cm, $AE = x, EB = 6$ cm and ΔADE with $BC = y$, $ED = 4$ cm, $DC = 9$ cm

i.
$\angle ABE = \angle ACD$
$\angle AEB = \angle ADC$ — since the lines BE and CD are parallel
$\angle EAB = \angle DAC$ — common vertex A

Since the 3 angles are the same in both triangles, they are similar triangles.

ii. $k = \frac{\text{length of image}}{\text{length of corresponding side of object}}$

For $EB:DC$ $\quad k = \frac{6}{9} = 1.5$

For $AC:AB$ $\quad 1.5 = \frac{AC}{6}$ $\qquad$ For $AD:AE$ $\quad 1.5 = \frac{4+x}{x}$

$$AC = 6 \times 1.5 = 9 \text{ cm} \qquad \therefore BC = y = 3 \text{ cm}$$

$$1.5x = 4 + x \qquad 0.5x = 4 \qquad \therefore AE = x = 8 \text{ cm}$$

9. Explain:
   - Ratios can also be found linking the scale factor to the area and volume of the enlarged and original figures.
   - The ratio of the area of the enlarged figure to the area of the original figure is the square of the scale factor of the enlargement

     $k^2 = \frac{\text{area of enlarged image}}{\text{area of original object}}$ (2)
   - We can also show that the ratio of the volume of the enlarged image to the volume of the original object is the cube of the scale factor of the enlargement.

     $k^3 = \frac{\text{volume of enlarged image}}{\text{volume of original object}}$ (3)
10. Ask pupils to work with seatmates to answer question d.
11. Invite a volunteer to show their answer on the board. The rest of the class should check their solution and correct any mistakes.

    Solution:

    d. Given: triangle from question b. Use the lengths in previously completed table.

    i area $= \frac{1}{2} \times$ base $\times$ height

    For ΔABC

    area $= \frac{1}{2} \times 3 \times 4$

    $= 6$ cm^2

    For $\Delta A_1B_1C_1$

    area $= \frac{1}{2} \times 9 \times 12$

    $= 54$ cm^2

    ii. ratio $= \frac{\text{area of } \Delta A_1B_1C_1}{\text{area of } \Delta ABC}$

    $= \frac{54}{6} = 9 = 3^2$ where $k = 3$ is the scale factor of the enlargement.

    The ratio of the areas is the square of the scale factor (as given in equation 2)

Practice *(15 minutes)*

1. Ask pupils to work independently to answer question e.
2. Walk around, if possible, to check the answers and clear up any misconceptions.
3. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.

   Solution:

   e. Given: $\frac{\text{length of one side of pentagon A}}{\text{length of corresponding side of pentagon B}}$ = scale factor $= 4$

   i. $\frac{\text{area of pentagon A}}{\text{area of pentagon B}} = 4^2 = 16$

   ii. $\frac{\text{volume of pentagon A}}{\text{volume of pentagon B}} = 4^3$

   $\frac{\text{volume of pentagon A}}{4} = 64$

   $\therefore$ Volume of pentagon A $= 4 \times 64 = 256$ cm^3

Closing *(1 minute)*

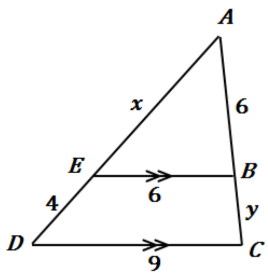
1. For homework, have pupils do the practice activity PHM3-L094 in the Pupil Handbook.

[QUESTIONS]

a. Describe the image obtained when an object is enlarged with a scale factor of $-\frac{1}{2}$.
b. A triangle has vertices with co-ordinates $A(2,1), B(5,1)$ and $C(5,5)$. It is enlarged from the origin to give an image having vertices with co-ordinates $A_1(6,3)$, $B_1(15,3)$ and $C_1(15,15)$.
   i. What is the scale factor of the enlargement?
   ii. Draw on the given axes showing clearly the co-ordinates of all vertices triangles ABC and $A_1B_1C_1$.
   iii. Measure the corresponding lengths of the triangle and image on the graph. Copy and complete the table below.

Length of triangle		Length of image	
AB		A_1B_1	
BC		B_1C_1	
AC		A_1C_1	

   iv. What do you notice?
c. For the figure shown on the right:
   i. Explain why triangles ABE and ADE are similar.
   ii. Find the lengths of x and y. All measurements are in cm.



d. Prove that the ratio of the area of triangles $A_1B_1C_1$ to ABC in question b. is equal to the square of the ratio of the scale factor.
e. The length of a side of a regular pentagon A is 4 times the length of a side of another regular pentagon B.
   i. Find the ratio of the area of A to B.
   ii. If the volume of B is 4 cm^3, what is the volume of A?

Lesson Title: Combination of Transformations	**Theme:** Vectors and Transformations	
Lesson Number: M3-L095	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to perform a combination of transformations on a plane shape.	**Preparation** 1. Using a scale of 2 cm to 2 units on both axes, draw two perpendicular axes Ox and Oy for $-8 \leq x \leq 8$ and $-8 \leq y \leq 8$. 2. Write the questions found at the end of this lesson plan on the board.	

Opening *(1 minute)*

1. Tell pupils that after today's lesson, they will be able to perform a combination of transformations on a plane shape.

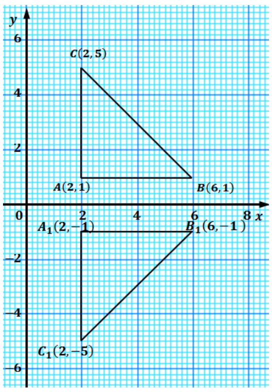
Teaching and Learning *(20 minutes)*

1. Explain:
   - An object can undergo more than one transformation.
   - This **combination of transformations** can be described by a mapping which gives the single transformation that produces the same end result.
2. Invite a volunteer to assess questions a. i. and ii. and tell the class what information we are given. (Answer: given: triangle ABC with vertices $A\ (2,1), B(6,1)$ and $C(2,5)$.)
3. Invite another volunteer to say what we have been asked to do. (Answer: draw the image $\Delta A_1B_1C_1$ under a reflection in the x-axis $(y = 0)$)
4. Show the mapping and transformation on the board.

   Solution:

   a. **Step 1.** Assess and extract the given information from the problem.
   Given: $A\ (2,1), B(6,1)$ and $C(2,5)$

   i **Step 2.** Draw the triangle with the given vertices.
   ΔABC shown right.



   ii. **Step 3.** Apply the appropriate mapping formula.

   $\begin{pmatrix} x \\ y \end{pmatrix} \longrightarrow \begin{pmatrix} x \\ -y \end{pmatrix}$

   $\begin{pmatrix} 2 \\ 1 \end{pmatrix} \longrightarrow \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ $\quad$ $\begin{pmatrix} 2 \\ 5 \end{pmatrix} \longrightarrow \begin{pmatrix} 2 \\ -5 \end{pmatrix}$

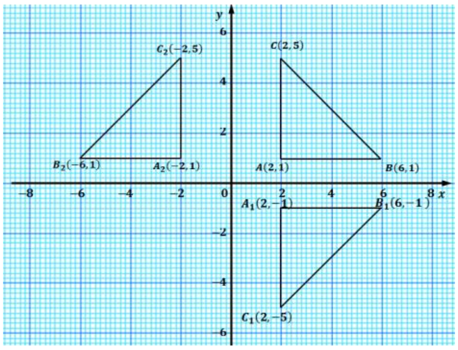
   $\begin{pmatrix} 6 \\ 1 \end{pmatrix} \longrightarrow \begin{pmatrix} 6 \\ -1 \end{pmatrix}$

   Step 4. Write the answer and draw the image triangle
   $A_1(2,-1), B_1(6,-1)$ and $C_1(2,-5)$

5. Ask pupils to work with seatmates to calculate the mapping for question a. ii.
6. Invite volunteers to give their mapping. The rest of the class should check their solution and correct any mistakes.
7. Show the mapping and transformation on the board.

Solution:

iii. Given: $A_1(2,-1), B_1(6,-1)$ and $C_1(2,-5)$



$\begin{pmatrix} x \\ y \end{pmatrix} \longrightarrow \begin{pmatrix} -x \\ -y \end{pmatrix}$

$\begin{pmatrix} 2 \\ -1 \end{pmatrix} \longrightarrow \begin{pmatrix} -2 \\ 1 \end{pmatrix}$

$\begin{pmatrix} 6 \\ -1 \end{pmatrix} \longrightarrow \begin{pmatrix} -6 \\ 1 \end{pmatrix}$

$\begin{pmatrix} 2 \\ -5 \end{pmatrix} \longrightarrow \begin{pmatrix} -2 \\ 5 \end{pmatrix}$

$A_2(-2,1), B_2(-6,1)$ and $C_2(-2,5)$

iv. From the graph, the reflection in the y-axis $(x = 0)$ maps ΔABC onto $\Delta A_2B_2C_2$

mapping can be written: $\begin{pmatrix} x \\ y \end{pmatrix} \longrightarrow \begin{pmatrix} x \\ -y \end{pmatrix} \longrightarrow \begin{pmatrix} -x \\ -(-y) \end{pmatrix}$

i.e. $\begin{pmatrix} x \\ y \end{pmatrix} \longrightarrow \begin{pmatrix} -x \\ y \end{pmatrix}$ reflection in the y-axis $(x = 0)$.

8. Ask pupils to work with seatmates to answer questions b.
9. Invite a volunteer to show the solution on the board. The rest of the class should check their solution and correct any mistakes.

Solution:

b. Given: $P(2,4), Q\ (3,5)$ and $R\ (2,6)$

i. All diagrams for this question can be found below.

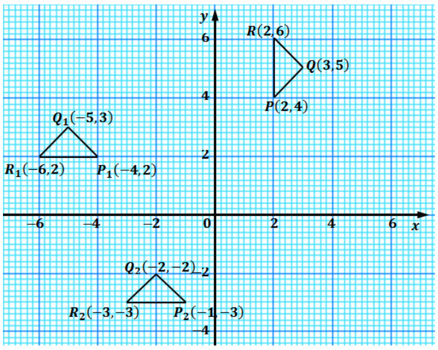
ii. $\begin{pmatrix} x \\ y \end{pmatrix} \longrightarrow \begin{pmatrix} -y \\ x \end{pmatrix}$ $\quad$ $\begin{pmatrix} 2 \\ 4 \end{pmatrix} \longrightarrow \begin{pmatrix} -4 \\ 2 \end{pmatrix}$

$\begin{pmatrix} 3 \\ 5 \end{pmatrix} \longrightarrow \begin{pmatrix} -5 \\ 3 \end{pmatrix}$ $\quad$ $\begin{pmatrix} 2 \\ 6 \end{pmatrix} \longrightarrow \begin{pmatrix} -6 \\ 2 \end{pmatrix}$

$P_1(-4,2), Q_1(-5,3)$ and $R_1(-6,2)$

iii. Given: $P_1(-4,2), Q_1(-5,3)$ and $R_1(-6,2)$, translation vector $\begin{pmatrix} 3 \\ -5 \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} \longrightarrow \begin{pmatrix} x+a \\ y+b \end{pmatrix} = \begin{pmatrix} x+3 \\ y+(-5) \end{pmatrix} = \begin{pmatrix} x+3 \\ y-5 \end{pmatrix}$



$\begin{pmatrix} -4 \\ 2 \end{pmatrix} \longrightarrow \begin{pmatrix} -4+3 \\ 2-5 \end{pmatrix} = \begin{pmatrix} -1 \\ -3 \end{pmatrix}$

$\begin{pmatrix} -5 \\ 3 \end{pmatrix} \longrightarrow \begin{pmatrix} -5+3 \\ 3-5 \end{pmatrix} = \begin{pmatrix} -2 \\ -2 \end{pmatrix}$

$\begin{pmatrix} -6 \\ 2 \end{pmatrix} \longrightarrow \begin{pmatrix} -6+3 \\ 2-5 \end{pmatrix} = \begin{pmatrix} -3 \\ -3 \end{pmatrix}$

$P_2(-1,-3),\ Q_2(-2,-2)$ and $R_1(-3,-3)$

iv. From the graph, the single transformation is a clockwise rotation of $90°$ about a point.

Combined mapping can be written: $\begin{pmatrix} x \\ y \end{pmatrix} \longrightarrow \begin{pmatrix} -y \\ x \end{pmatrix}$ from ii.

$\longrightarrow \begin{pmatrix} -y+3 \\ x-5 \end{pmatrix}$ from iii. since $x = -y$ and $y = x$

Practice *(15 minutes)*

1. Ask pupils to work independently to answer question c.
2. Walk around, if possible, to check the answers and clear up any misconceptions.
3. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.

Solutions:

c. Given: $A(1,5)$, $B(3,5)$, $C(3,8)$ and $D\ (1,7)$

i. All diagrams for this question can be found below.

ii $\begin{pmatrix} x \\ y \end{pmatrix} \longrightarrow \begin{pmatrix} y \\ -x \end{pmatrix}$

$\begin{pmatrix} 1 \\ 5 \end{pmatrix} \longrightarrow \begin{pmatrix} 5 \\ -1 \end{pmatrix}$ $\qquad$ $\begin{pmatrix} 3 \\ 8 \end{pmatrix} \longrightarrow \begin{pmatrix} 8 \\ -3 \end{pmatrix}$

$\begin{pmatrix} 3 \\ 5 \end{pmatrix} \longrightarrow \begin{pmatrix} 5 \\ -3 \end{pmatrix}$ $\qquad$ $\begin{pmatrix} 1 \\ 7 \end{pmatrix} \longrightarrow \begin{pmatrix} 7 \\ -1 \end{pmatrix}$

$A_1(5,-1)$, $B_1(5,-3)$, $C_1(8,-3)$ and $D_1(7,-1)$

iii. Given: $A_1(5,-1)$, $B_1(5,-3)$, $C_1(8,-3)$, $D_1(7,-1)$, translation vector $\begin{pmatrix} -4 \\ -2 \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} \longrightarrow \begin{pmatrix} x+a \\ y+b \end{pmatrix} = \begin{pmatrix} x+(-4) \\ y+(-2) \end{pmatrix} = \begin{pmatrix} x-4 \\ y-2 \end{pmatrix}$

$\begin{pmatrix} 5 \\ -1 \end{pmatrix} \longrightarrow \begin{pmatrix} 5-4 \\ -1-2 \end{pmatrix} = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$

$\begin{pmatrix} 5 \\ -3 \end{pmatrix} \longrightarrow \begin{pmatrix} 5-4 \\ -3-2 \end{pmatrix} = \begin{pmatrix} 1 \\ -5 \end{pmatrix}$

$\begin{pmatrix} 8 \\ -3 \end{pmatrix} \longrightarrow \begin{pmatrix} 8-4 \\ -3-2 \end{pmatrix} = \begin{pmatrix} 4 \\ -5 \end{pmatrix}$

$\begin{pmatrix} 7 \\ -1 \end{pmatrix} \longrightarrow \begin{pmatrix} 7-4 \\ -1-2 \end{pmatrix} = \begin{pmatrix} 3 \\ -3 \end{pmatrix}$

$A_2(1,-3)$, $B_2(1,-5)$, $C_2(4,-5)$ and $D_2(3,-3)$

iv. Given: $A_1(5,-1)$, $B_1(5,-3)$, $C_1(8,-3)$ and $D_1(7,-1)$ and line $x = 1$

$\begin{pmatrix} x \\ y \end{pmatrix} \longrightarrow \begin{pmatrix} 2k-x \\ y \end{pmatrix} = \begin{pmatrix} 2(1)-x \\ y \end{pmatrix} = \begin{pmatrix} 2-x \\ y \end{pmatrix}$

$\begin{pmatrix} 5 \\ -1 \end{pmatrix} \longrightarrow \begin{pmatrix} 2-5 \\ -1 \end{pmatrix} = \begin{pmatrix} -3 \\ -1 \end{pmatrix}$

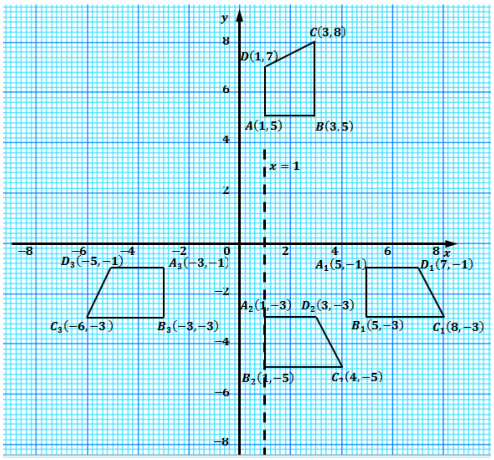
$\begin{pmatrix} 5 \\ -3 \end{pmatrix} \longrightarrow \begin{pmatrix} 2-5 \\ -3 \end{pmatrix} = \begin{pmatrix} -3 \\ -3 \end{pmatrix}$

$\begin{pmatrix} 8 \\ -3 \end{pmatrix} \longrightarrow \begin{pmatrix} 2-8 \\ -3 \end{pmatrix} = \begin{pmatrix} -6 \\ -3 \end{pmatrix}$

$\begin{pmatrix} 7 \\ -1 \end{pmatrix} \longrightarrow \begin{pmatrix} 2-7 \\ -1 \end{pmatrix} = \begin{pmatrix} -5 \\ -1 \end{pmatrix}$

$A_3(-3,-1)$, $B_3(-3,-3)$,

$C_3(-6,-3)$ and $D_3(-5,-1)$



v. The single transformation is given by the combined mapping:

$\begin{pmatrix} x \\ y \end{pmatrix} \longrightarrow \begin{pmatrix} y \\ -x \end{pmatrix}$ $\qquad$ from ii.

$\longrightarrow \begin{pmatrix} 2-y \\ -x \end{pmatrix}$ $\qquad$ from iv. since $x = y$ and $y = -x$

Closing *(4 minutes)*

1. Ask pupils to discuss with seatmates one new thing they learned in this lesson.
2. Invite a volunteer to give their answer. (Example answer: A single mapping can be written for combined transformations.)
3. For homework, have pupils do the practice activity PHM3-L095 in the Pupil Handbook.

[QUESTIONS]

Unless otherwise stated, use a scale of 2 cm to 2 units on both axes, draw two perpendicular axes Ox and Oy for $-8 \leq x \leq 8$ and $-8 \leq y \leq 8$.

a. Draw on the given axes, showing clearly the co-ordinates of all vertices:
   i. Triangle ABC with vertices $A\ (2,1), B(6,1)$ and $C(2,5)$.
   ii. The image $\Delta A_1B_1C_1$ of ΔABC under a reflection in the x-axis $(y = 0)$ where $A \longrightarrow A_1,\ B \longrightarrow B_1$ and $C \longrightarrow C_1$.
   iii. The image $\Delta A_2B_2C_2$ of $\Delta A_1B_1C_1$ under a rotation of $180°$ about the origin where $A_1 \longrightarrow A_2,\ B_1 \longrightarrow B_2$ and $C_1 \longrightarrow C_2$.
   iv. Describe fully the single transformation that will map ΔABC onto $\Delta A_2B_2C_2$.

b. Draw on the given axes, showing clearly the co-ordinates of all vertices:
   i. The triangle PQR with $P(2,4), Q\ (3,5)$ and $R(2,6)$.
   ii. The image $\Delta P_1Q_1R_1$ of ΔPQR under an anti-clockwise rotation of $90°$ about the origin where $P \longrightarrow P_1,\ Q \longrightarrow Q_1,\ R \longrightarrow R_1$.
   iii. The image $\Delta P_2Q_2R_2$ of $\Delta P_1Q_1R_1$ under a translation by the vector $\binom{3}{-5}$ where $P_1, \longrightarrow P_2,\ Q_1 \longrightarrow Q_2,\ R_2 \longrightarrow R_2$
   iv. What type of transformation will map $\Delta P_2Q_2R_2$ back on to ΔPQR?

c. Draw on the given axes, showing clearly the co-ordinates of all vertices:
   i. The quadrilateral $ABCD$ with $A(1,5),\ B(3,5),\ C(3,8)$ and $D\ (1,7)$.
   ii. The image $A_1B_1C_1D_1$ of $ABCD$ under a clockwise rotation of $90°$ about the origin where $A \longrightarrow A_1,\ B \longrightarrow B_1,\ C \longrightarrow C_1$ and $D \longrightarrow D_1$
   iii. The image $A_2B_2C_2D_2$ of $A_1B_1C_1D_1$ under a translation by the vector $\binom{-4}{-2}$ where. $A_1 \longrightarrow A_2,\ B_1 \longrightarrow B_2,\ C_1 \longrightarrow C_2$ and $D_1 \longrightarrow D_2$.
   iv. The image $A_3B_3C_3D_3$ of $A_1B_1C_1D_1$ under a reflection in the line $x = 1$ where. $A_1 \longrightarrow A_3,\ B_1 \longrightarrow B_3,\ C_1 \longrightarrow C_3$ and $D_1 \longrightarrow D_3$.
   v. Describe the single transformation that will map $ABCD$ onto $A_3B_3C_3D_3$.

Lesson Title: Application of Transformations	**Theme:** Vectors and Transformations	
Lesson Number: M3-L096	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to solve problems using transformations.	**Preparation** 1. Using a scale of 2 cm to 2 units on both axes, draw two perpendicular axes Ox and Oy for $-8 \leq x \leq 8$ and $0 \leq y \leq 8$. 2. Write the questions found at the end of this lesson plan on the board.	

Opening *(1 minute)*

1. Tell pupils that after today's lesson, they will be able to solve problems using transformations.

Teaching and Learning *(20 minutes)*

1. Invite a volunteer to assess questions a. i. and ii. and tell the class what information we are given. (Answer: given: triangle PQR with $P(3,2)$, $\overrightarrow{PQ} = \binom{4}{1}$ and $\overrightarrow{QR} = \binom{-1}{2}$)
2. Invite another volunteer to say what we have been asked to do. (Answer: Draw the image triangle $P_1Q_1R_1$ of triangle PQR under an anti-clockwise rotation of $90°$ about the origin.)

   Solution:

   a. **Step 1.** Assess and extract the given information from the problem.

   Given: $P(3,2)$, $\overrightarrow{PQ} = \binom{4}{1}$ and $\overrightarrow{QR} = \binom{-1}{2}$

   i **Step 2.** Find the co-ordinates of Q and R.

$$\begin{aligned} \overrightarrow{PQ} &= \overrightarrow{OQ} - \overrightarrow{OP} \\ \binom{4}{1} &= \overrightarrow{OQ} - \binom{3}{2} \\ \overrightarrow{OQ} &= \binom{4}{1} + \binom{3}{2} \\ &= \binom{7}{3} \end{aligned} \qquad \begin{aligned} \overrightarrow{QR} &= \overrightarrow{OR} - \overrightarrow{OQ} \\ \binom{-1}{2} &= \overrightarrow{OR} - \binom{7}{3} \\ \overrightarrow{OR} &= \binom{-1}{2} + \binom{7}{3} \\ &= \binom{6}{5} \end{aligned}$$

   $P(3,2), Q(7,3)$ and $R\ (6,5)$.

   Step 3. Draw the triangle with vertices above.

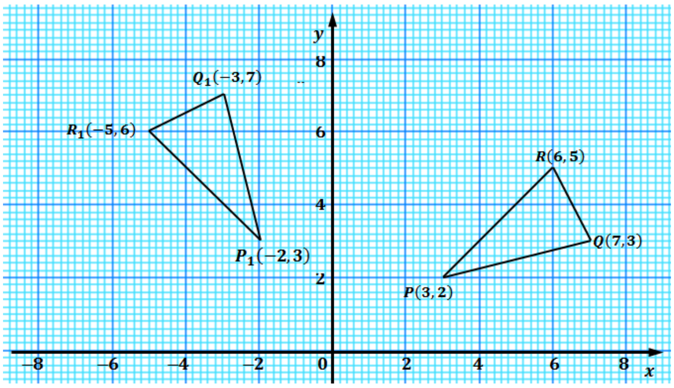
   ΔPQR shown below.

   ii. **Step 4.** Apply the appropriate mapping formula.

$$\binom{x}{y} \longrightarrow \binom{-y}{x}$$
$$\binom{3}{2} \longrightarrow \binom{-2}{3}$$
$$\binom{7}{3} \longrightarrow \binom{-3}{7}$$
$$\binom{6}{5} \longrightarrow \binom{-5}{6}$$

   Step 5. Write the answer and draw the image triangle (shown below).

   $P_1(-2,3), Q_1(-3,7)$ and $R_1(-5,6)$



iii. **Step 6.** Use the graph to find $\overrightarrow{P_1R_1}$ and hence $|\overrightarrow{P_1R_1}|$.

$$\overrightarrow{P_1R_1} = \binom{-3}{3}$$

$$|\overrightarrow{P_1R_1}| = \sqrt{(-3)^2 + 3^2}$$

$$= \sqrt{9+9} = \sqrt{18}$$

$$= 3\sqrt{2} \text{ units}$$

3. Ask pupils to work with seatmates to answer question b.
4. Invite volunteers to show their solution on the board. The rest of the class should check their solution and correct any mistakes.

Solution:

b. $A(2,1), B(6,1)$ and $C(2,5)$

The translation of A by a vector $\binom{4}{0}$ maps it to B

$$\binom{x}{y} \longrightarrow \binom{x+a}{y+b}$$

$$\binom{2}{1} \longrightarrow \binom{2+4}{1+0}$$

$$\longrightarrow \binom{6}{1}$$

The reflection in the line $x = 4$ maps $A\ to\ B$, $k = 4$

$$\binom{x}{y} \longrightarrow \binom{2k-x}{y} = \binom{2(4)-x}{y} = \binom{8-x}{y}$$

$$\binom{2}{1} \longrightarrow \binom{8-2}{1}$$

$$\longrightarrow \binom{6}{1}$$

Practice *(15 minutes)*

1. Ask pupils to work independently to answer questions c. and d.
2. Walk around, if possible, to check the answers and clear up any misconceptions.
3. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.

Solutions:

c. Given: $A(1,0),\ B(1,3),\ C\ (4,3)$, scale factor $k = -2$, centre of enlargement $(0,0)$

ii. $$\binom{x}{y} \longrightarrow k\binom{x}{y} = -2\binom{x}{y}$$

$$\begin{pmatrix}1\\0\end{pmatrix} \longrightarrow -2\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}-2\\0\end{pmatrix}$$

$$\begin{pmatrix}1\\3\end{pmatrix} \longrightarrow -2\begin{pmatrix}1\\3\end{pmatrix} = \begin{pmatrix}-2\\-6\end{pmatrix}$$

$$\begin{pmatrix}4\\3\end{pmatrix} \longrightarrow -2\begin{pmatrix}4\\3\end{pmatrix} = \begin{pmatrix}-8\\-6\end{pmatrix}$$

$A_1(-2,0)$, $B_1(-2,-6)$ and $C_1(-8,-6)$

iii. Given: $B(1,3), B_1(-2,-6)$,

gradient $= \frac{y_2-y_1}{x_2-x_1}$ Let $x_1 = 1,\ y_1 = 3, x_2 = -2,\ y_2 = -6$

$= \frac{-6-3}{-2-1}$ substitute the assigned variables

$= \frac{-9}{-3}$

$= 3$

Using point $B(1,3)$

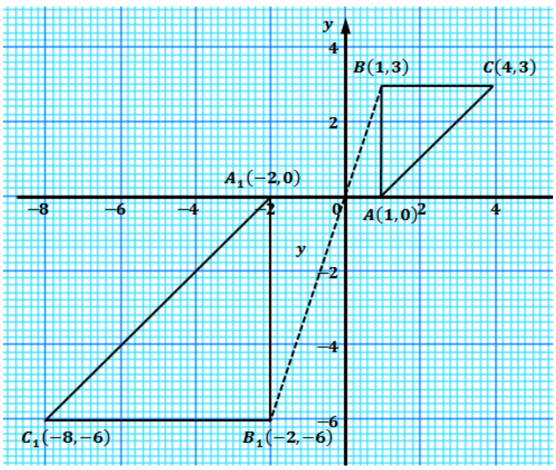
$\frac{y-3}{x-1} = 3$ standard formula for equation of a straight line

$y - 3 = 3(x-1)$

$y - 3 = 3x - 3$

$y = 3x$

The equation of line BB_1 is $y = 3x$.



d. Given: scale factor $k = -0.5$, $AB = 6.4$ cm

$$k = \frac{\text{length of image}}{\text{length of object}}$$

$$-0.5 = \frac{\text{length of image}}{6.4}$$

$$\text{length of image} = 6.4 \times (-0.5) = -3.2$$

Since lengths cannot be negative, length of image = 3.2 cm.

Closing *(4 minutes)*

1. Ask pupils to discuss with seatmates the main differences between enlargements and the other transformations.
2. Invite volunteers to give their answer. (Example answer: Enlargements change the size of the object while the other transformations maintain the size of the object)
3. For homework, have pupils do the practice activity PHM3-L096 in the Pupil Handbook.

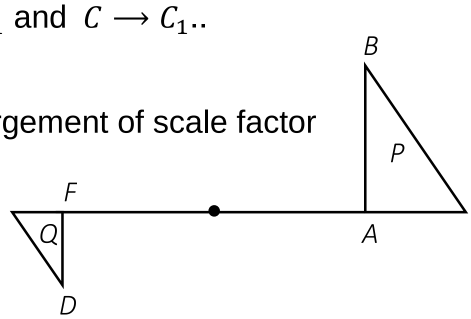
[QUESTIONS]

a. Draw on the given axes, showing clearly the co-ordinates of all vertices:
   i. The triangle PQR with $P(3,2)$, $\overrightarrow{PQ} = \binom{4}{1}$ and $\overrightarrow{QR} = \binom{-1}{2}$
   ii. The image triangle $P_1Q_1R_1$ of triangle PQR under an anti-clockwise rotation of $90°$ about the origin where $P \longrightarrow P_1$, $Q \longrightarrow Q_1$ $R \longrightarrow R_1$.
   iii. Use the graph to find $\overrightarrow{P_1R_1}$ and hence $|\overrightarrow{P_1R_1}|$ leaving the answer in surd form.

b. A triangle ABC has vertices $A(2,1), B(6,1)$ and $C(2,5)$.
   Describe **two** different transformations that fully map A to B.

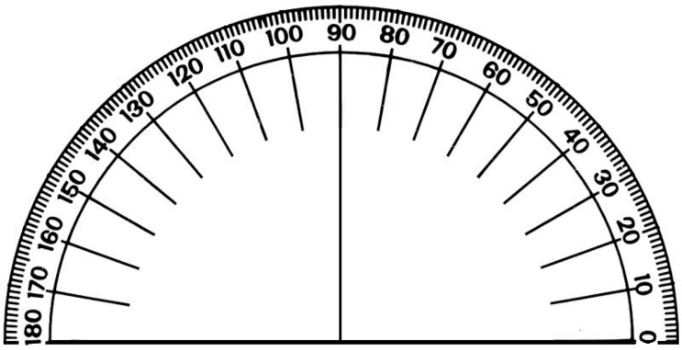
c. Using a scale of 2 cm to 2 units on both axes, draw two perpendicular axes Ox and Oy for $-8 \leq x \leq 4$ and $-8 \leq y \leq 4$.
   a. Draw on the same axes, showing clearly the co-ordinates of all vertices:
      vi. The triangle $A(1,0)$, $B(1,3)$ and C $(4,3)$
      vii. The image $\Delta A_1B_1C_1$ of ΔABC under an enlargement about the origin with scale factor –2 where $A \longrightarrow A_1$, $B \longrightarrow B_1$ and $C \longrightarrow C_1$..
   b. Find the equation of the line BB_1.

d. Triangle P is mapped to triangle Q by an enlargement of scale factor -0.5. If AB is 6.4 cm long, how long is FD?



Appendix I: Protractor

You can use a protractor to measure angles. If you do not have a protractor, you can make one with paper. Trace this protractor with a pen onto another piece of paper. Then, cut out the semi-circle using scissors.



Appendix II: Sines of Angles

Sines of Angles (*x* in degrees)

$x \rightarrow \sin x$

x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	ADD Differences 1	2	3	4	5	6	7	8	9
0	0·0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	2	3	5	7	9	10	12	14	16
1	0·0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	2	3	5	7	9	10	12	14	16
2	0·0349	0366	0384	0401	0419	0436	0454	0471	0488	0506	2	3	5	7	9	10	12	14	16
3	0·0523	0541	0558	0576	0593	0610	0628	0645	0663	0680	2	3	5	7	9	10	12	14	16
4	0·0698	0715	0732	0750	0767	0785	0802	0819	0837	0854	2	3	5	7	9	10	12	14	16
5	0·0872	0889	0906	0924	0941	0958	0976	0993	1011	1028	2	3	5	7	9	10	12	14	16
6	0·1045	1063	1080	1097	1115	1132	1149	1167	1184	1201	2	3	5	7	9	10	12	14	16
7	0·1219	1236	1253	1271	1288	1305	1323	1340	1357	1374	2	3	5	7	9	10	12	14	16
8	0·1392	1409	1426	1444	1461	1478	1495	1513	1530	1547	2	3	5	7	9	10	12	14	16
9	0·1564	1582	1599	1616	1633	1650	1668	1685	1702	1719	2	3	5	7	9	10	12	14	15
10	0·1736	1754	1771	1788	1805	1822	1840	1857	1874	1891	2	3	5	7	9	10	12	14	15
11	0·1908	1925	1942	1959	1977	1994	2011	2028	2045	2062	2	3	5	7	9	10	12	14	15
12	0·2079	2096	2113	2130	2147	2164	2181	2198	2215	2233	2	3	5	7	9	10	12	14	15
13	0·2250	2267	2284	2300	2317	2334	2351	2368	2385	2402	2	3	5	7	8	10	12	14	15
14	0·2419	2436	2453	2470	2487	2504	2521	2538	2554	2571	2	3	5	7	8	10	12	14	15
15	0·2588	2605	2622	2639	2656	2672	2689	2706	2723	2740	2	3	5	7	8	10	12	13	15
16	0·2756	2773	2790	2807	2823	2840	2857	2874	2890	2907	2	3	5	7	8	10	12	13	15
17	0·2924	2940	2957	2974	2990	3007	3024	3040	3057	3074	2	3	5	7	8	10	12	13	15
18	0·3090	3107	3123	3140	3156	3173	3190	3206	3223	3239	2	3	5	7	8	10	12	13	15
19	0·3256	3272	3289	3305	3322	3338	3355	3371	3387	3404	2	3	5	7	8	10	12	13	15
20	0·3420	3437	3453	3469	3486	3502	3518	3535	3551	3567	2	3	5	7	8	10	11	13	15
21	0·3584	3600	3616	3633	3649	3665	3681	3697	3714	3730	2	3	5	6	8	10	11	13	15
22	0·3746	3762	3778	3795	3811	3827	3843	3859	3875	3891	2	3	5	6	8	10	11	13	15
23	0·3907	3923	3939	3955	3971	3987	4003	4019	4035	4051	2	3	5	6	8	10	11	13	14
24	0·4067	4083	4099	4115	4131	4147	4163	4179	4195	4210	2	3	5	6	8	10	11	13	14
25	0·4226	4242	4258	4274	4289	4305	4321	4337	4352	4368	2	3	5	6	8	9	11	13	14
26	0·4384	4399	4415	4431	4446	4462	4478	4493	4509	4524	2	3	5	5	8	9	11	12	14
27	0·4540	4555	4571	4586	4602	4617	4633	4648	4664	4679	2	3	5	6	8	9	11	12	14
28	0·4695	4710	4726	4741	4756	4772	4787	4802	4818	4833	2	3	5	6	8	9	11	12	14
29	0·4848	4863	4879	4894	4909	4924	4939	4955	4970	4985	2	3	5	6	8	9	11	12	14
30	0·5000	5015	5030	5045	5060	5075	5090	5105	5120	5135	2	3	5	6	8	9	11	12	14
31	0·5150	5165	5180	5195	5210	5225	5240	5255	5270	5284	1	3	4	6	7	9	10	12	13
32	0·5299	5314	5329	5344	5358	5373	5388	5402	5417	5432	1	3	4	6	7	9	10	12	13
33	0·5446	5461	5476	5490	5505	5519	5534	5548	5563	5577	1	3	4	6	7	9	10	12	13
34	0·5592	5606	5621	5635	5650	5664	5678	5693	5707	5721	1	3	4	6	7	9	10	12	13
35	0·5736	5750	5764	5779	5793	5807	5821	5835	5850	5864	1	3	4	6	7	9	10	11	13
36	0·5878	5892	5906	5920	5934	5948	5962	5976	5990	6004	1	3	4	6	7	8	10	11	13
37	0·6018	6032	6046	6060	6074	6088	6101	6115	6129	6143	1	3	4	6	7	8	10	11	12
38	0·6157	6170	6184	6198	6211	6225	6239	6252	6266	6280	1	3	4	5	7	8	10	11	12
39	0·6293	6307	6320	6334	6347	6361	6374	6388	6401	6414	1	3	4	5	7	8	9	11	12
40	0·6428	6441	6455	6468	6481	6494	6508	6521	6534	6547	1	3	4	5	7	8	9	11	12
41	0·6561	6574	6587	6600	6613	6626	6639	6652	6665	6678	1	3	4	5	7	8	9	10	12
42	0·6691	6704	6717	6730	6743	6756	6769	6782	6794	6807	1	3	4	5	6	8	9	10	12
43	0·6820	6833	6845	6858	6871	6884	6896	6909	6921	6934	1	3	4	5	6	8	9	10	11
44	0·6947	6959	6972	6984	6997	7009	7022	7034	7046	7059	1	2	4	5	6	7	9	10	11
x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9

x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	ADD Differences 1	2	3	4	5	6	7	8	9
45	0·7071	7083	7096	7108	7120	7133	7145	7157	7169	7181	1	2	4	5	6	7	9	10	11
46	0·7193	7206	7218	7230	7242	7254	7266	7278	7290	7302	1	2	4	5	6	7	8	10	11
47	0·7314	7325	7337	7349	7361	7373	7385	7396	7408	7420	1	2	4	5	6	7	8	9	11
48	0·7431	7443	7455	7466	7478	7490	7501	7513	7524	7536	1	2	3	5	6	7	8	9	10
49	0·7547	7559	7570	7581	7593	7604	7615	7627	7638	7649	1	2	3	5	6	7	8	9	10
50	0·7660	7672	7683	7694	7705	7716	7727	7738	7749	7760	1	2	3	4	6	7	8	9	10
51	0·7771	7782	7793	7804	7815	7826	7837	7848	7859	7869	1	2	3	4	5	7	8	9	10
52	0·7880	7891	7902	7912	7923	7934	7944	7955	7965	7976	1	2	3	4	5	6	7	8	10
53	0·7986	7997	8007	8018	8028	8039	8049	8059	8070	8080	1	2	3	4	5	6	7	8	9
54	0·8090	8100	8111	8121	8131	8141	8151	8161	8171	8181	1	2	3	4	5	6	7	8	9
55	0·8192	8202	8211	8221	8231	8241	8251	8261	8271	8281	1	2	3	4	5	6	7	8	9
56	0·8290	8300	8310	8320	8329	8339	8348	8358	8368	8377	1	2	3	4	5	6	7	8	9
57	0·8387	8396	8406	8415	8425	8434	8443	8453	8462	8471	1	2	3	4	5	6	7	8	8
58	0·8480	8490	8499	8508	8517	8526	8536	8545	8554	8563	1	2	3	4	5	5	6	7	8
59	0·8572	8581	8590	8599	8607	8616	8625	8634	8643	8652	1	2	3	4	4	5	6	7	8
60	0·8660	8669	8678	8686	8695	8704	8712	8721	8729	8738	1	2	3	3	4	5	6	7	8
61	0·8746	8755	8763	8771	8780	8788	8796	8805	8813	8821	1	2	2	3	4	5	6	7	7
62	0·8829	8838	8846	8854	8862	8870	8878	8886	8894	8902	1	2	2	3	4	5	6	6	7
63	0·8910	8918	8926	8934	8942	8949	8957	8965	8973	8980	1	2	2	3	4	5	5	6	7
64	0·8988	8996	9003	9011	9018	9026	9033	9041	9048	9056	1	2	2	3	4	5	5	6	7
65	0·9063	9070	9078	9085	9092	9100	9107	9114	9121	9128	1	1	2	3	4	4	5	6	7
66	0·9135	9143	9150	9157	9164	9171	9178	9184	9191	9198	1	1	2	3	3	4	5	6	6
67	0·9205	9212	9219	9225	9232	9239	9245	9252	9259	9265	1	1	2	3	3	4	5	5	6
68	0·9272	9278	9285	9291	9298	9304	9311	9317	9323	9330	1	1	2	3	3	4	4	5	6
69	0·9336	9342	9348	9354	9361	9367	9373	9379	9385	9391	1	1	2	2	3	4	4	5	6
70	0·9397	9403	9409	9415	9421	9426	9432	9438	9444	9449	1	1	2	2	3	3	4	5	5
71	0·9455	9461	9466	9472	9478	9483	9489	9494	9500	9505	1	1	2	2	3	3	4	4	5
72	0·9511	9516	9521	9527	9532	9537	9542	9548	9553	9558	1	1	2	2	3	3	4	4	5
73	0·9563	9568	9573	9578	9583	9588	9593	9598	9603	9608	0	1	1	2	2	3	3	4	4
74	0·9613	9617	9622	9627	9632	9636	9641	9646	9650	9655	0	1	1	2	2	3	3	4	4
75	0·9659	9664	9668	9673	9677	9681	9686	9690	9694	9699	0	1	1	2	2	3	3	3	4
76	0·9703	9707	9711	9715	9720	9724	9728	9732	9736	9740	0	1	1	2	2	2	3	3	4
77	0·9744	9748	9751	9755	9759	9763	9767	9770	9774	9778	0	1	1	2	2	2	3	3	3
78	0·9781	9785	9789	9792	9796	9799	9803	9806	9810	9813	0	1	1	1	2	2	2	3	3
79	0·9816	9820	9823	9826	9829	9833	9836	9839	9842	9845	0	1	1	1	2	2	2	3	3
80	0·9848	9851	9854	9857	9860	9863	9866	9869	9871	9874	0	1	1	1	1	2	2	2	3
81	0·9877	9880	9882	9885	9888	9890	9893	9895	9898	9900	0	1	1	1	1	2	2	2	2
82	0·9903	9905	9907	9910	9912	9914	9917	9919	9921	9923	0	0	1	1	1	1	2	2	2
83	0·9925	9928	9930	9932	9934	9936	9938	9940	9942	9943	0	0	1	1	1	1	1	2	2
84	0·9945	9947	9949	9951	9952	9954	9956	9957	9959	9960	0	0	1	1	1	1	1	1	2
85	0·9962	9963	9965	9966	9968	9969	9971	9972	9973	9974	0	0	0	1	1	1	1	1	1
86	0·9976	9977	9978	9979	9980	9981	9982	9983	9984	9985	0	0	0	0	1	1	1	1	1
87	0·9986	9987	9988	9989	9990	9990	9991	9992	9993	9993	0	0	0	0	0	0	1	1	1
88	0·9994	9995	9995	9996	9996	9997	9997	9997	9998	9998									
89	0·9998	9999	9999	9999	9999	1·0000	1·0000	1·0000	1·0000	1·0000									
x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9

Appendix III: Cosines of Angles

x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9
											SUBTRACT Differences								
0	1·0000	1·000	1·000	1·000	1·000	1·000	·9999	·9999	·9999	·9999									
1	0·9998	·9998	·9998	·9997	·9997	·9997	·9996	·9996	·9995	·9995									
2	0·9994	·9993	·9993	·9992	·9991	·9990	·9990	·9989	·9988	·9987	0	0	0	0	0	0	1	1	1
3	0·9986	·9985	·9984	·9983	·9982	·9981	·9980	·9979	·9978	·9977	0	0	0	0	1	1	1	1	1
4	0·9976	·9974	·9973	·9972	·9971	·9969	·9968	·9966	·9965	·9963	0	0	0	1	1	1	1	1	1
5	0·9962	·9960	·9959	·9957	·9956	·9954	·9952	·9951	·9949	·9947	0	0	1	1	1	1	1	1	2
6	0·9945	·9943	·9942	·9940	·9938	·9936	·9934	·9932	·9930	·9928	0	0	1	1	1	1	1	2	2
7	0·9925	·9923	·9921	·9919	·9917	·9914	·9912	·9910	·9907	·9905	0	0	1	1	1	1	2	2	2
8	0·9903	·9900	·9898	·9895	·9893	·9890	·9888	·9885	·9882	·9880	0	1	1	1	1	2	2	2	2
9	0·9877	·9874	·9871	·9869	·9866	·9863	·9860	·9857	·9854	·9851	0	1	1	1	1	2	2	2	3
10	0·9848	·9845	·9842	·9839	·9836	·9833	·9829	·9826	·9823	·9820	0	1	1	1	2	2	2	3	3
11	0·9816	·9813	·9810	·9806	·9803	·9799	·9796	·9792	·9789	·9785	0	1	1	1	2	2	2	3	3
12	0·9781	·9778	·9774	·9770	·9767	·9763	·9759	·9755	·9751	·9748	0	1	1	2	2	2	3	3	3
13	0·9744	·9740	·9736	·9732	·9728	·9724	·9720	·9715	·9711	·9707	0	1	1	2	2	2	3	3	4
14	0·9703	·9699	·9694	·9690	·9686	·9681	·9677	·9673	·9668	·9664	0	1	1	2	2	3	3	3	4
15	0·9659	·9655	·9650	·9646	·9641	·9636	·9632	·9627	·9622	·9617	0	1	1	2	2	3	3	4	4
16	0·9613	·9608	·9603	·9598	·9593	·9588	·9583	·9578	·9573	·9568	0	1	1	2	2	3	3	4	4
17	0·9563	·9558	·9553	·9548	·9542	·9537	·9532	·9527	·9521	·9516	1	1	2	2	3	3	4	4	5
18	0·9511	·9505	·9500	·9494	·9489	·9483	·9478	·9472	·9466	·9461	1	1	2	2	3	3	4	4	5
19	0·9455	·9449	·9444	·9438	·9432	·9426	·9421	·9415	·9409	·9403	1	1	2	2	3	3	4	5	5
20	0·9397	·9391	·9385	·9379	·9373	·9367	·9361	·9354	·9348	·9342	1	1	2	2	3	4	4	5	6
21	0·9336	·9330	·9323	·9317	·9311	·9304	·9298	·9291	·9285	·9278	1	1	2	3	3	4	4	5	6
22	0·9272	·9265	·9259	·9252	·9245	·9239	·9232	·9225	·9219	·9212	1	1	2	3	3	4	5	5	6
23	0·9205	·9198	·9191	·9184	·9178	·9171	·9164	·9157	·9150	·9143	1	1	2	3	3	4	5	6	6
24	0·9135	·9128	·9121	·9114	·9107	·9100	·9092	·9085	·9078	·9070	1	1	2	3	4	4	5	6	7
25	0·9063	·9056	·9048	·9041	·9033	·9026	·9018	·9011	·9003	·8996	1	2	2	3	4	5	5	6	7
26	0·8988	·8980	·8973	·8965	·8957	·8949	·8942	·8934	·8926	·8918	1	2	2	3	4	5	5	6	7
27	0·8910	·8902	·8894	·8886	·8878	·8870	·8862	·8854	·8846	·8838	1	2	2	3	4	5	6	6	7
28	0·8829	·8821	·8813	·8805	·8796	·8788	·8780	·8771	·8763	·8755	1	2	2	3	4	5	6	7	7
29	0·8746	·8738	·8729	·8721	·8712	·8704	·8695	·8686	·8678	·8669	1	2	3	3	4	5	6	7	8
30	0·8660	·8652	·8643	·8634	·8625	·8616	·8607	·8599	·8590	·8581	1	2	3	4	4	5	6	7	8
31	0·8572	·8563	·8554	·8545	·8536	·8526	·8517	·8508	·8499	·8490	1	2	3	4	5	5	6	7	8
32	0·8480	·8471	·8462	·8453	·8443	·8434	·8425	·8415	·8406	·8396	1	2	3	4	5	6	7	8	8
33	0·8387	·8377	·8368	·8358	·8348	·8339	·8329	·8320	·8310	·8300	1	2	3	4	5	6	7	8	9
34	0·8290	·8281	·8271	·8261	·8251	·8241	·8231	·8221	·8211	·8202	1	2	3	4	5	6	7	8	9
35	0·8192	·8181	·8171	·8161	·8151	·8141	·8131	·8121	·8111	·8100	1	2	3	4	5	6	7	8	9
36	0·8090	·8080	·8070	·8059	·8049	·8039	·8028	·8018	·8007	·7997	1	2	3	4	5	6	7	8	9
37	0·7986	·7976	·7965	·7955	·7944	·7934	·7923	·7912	·7902	·7891	1	2	3	4	5	6	7	8	10
38	0·7880	·7869	·7859	·7848	·7837	·7826	·7815	·7804	·7793	·7782	1	2	3	4	5	6	7	9	10
39	0·7771	·7760	·7749	·7738	·7727	·7716	·7705	·7694	·7683	·7672	1	2	3	4	6	7	8	9	10
40	0·7660	·7649	·7638	·7627	·7615	·7604	·7593	·7581	·7570	·7559	1	2	3	5	6	7	8	9	10
41	0·7547	·7536	·7524	·7513	·7501	·7490	·7478	·7466	·7455	·7443	1	2	3	5	6	7	8	9	10
42	0·7431	·7420	·7408	·7396	·7385	·7373	·7361	·7349	·7337	·7325	1	2	4	5	6	7	8	9	11
43	0·7314	·7302	·7290	·7278	·7266	·7254	·7242	·7230	·7218	·7206	1	2	4	5	6	7	8	10	11
44	0·7193	·7181	·7169	·7157	·7145	·7133	·7120	·7108	·7096	·7083	1	2	4	5	6	7	9	10	11
x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9

x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9
											SUBTRACT Differences								
45	0·7071	·7059	·7046	·7034	·7022	·7009	·6997	·6984	·6972	·6959	1	2	4	5	6	7	9	10	11
46	0·6947	·6934	·6921	·6909	·6896	·6884	·6871	·6858	·6845	·6833	1	3	4	5	6	8	9	10	11
47	0·6820	·6807	·6794	·6782	·6769	·6756	·6743	·6730	·6717	·6704	1	3	4	5	6	8	9	10	12
48	0·6691	·6678	·6665	·6652	·6639	·6626	·6613	·6600	·6587	·6574	1	3	4	5	7	8	9	10	12
49	0·6561	·6547	·6534	·6521	·6508	·6494	·6481	·6468	·6455	·6441	1	3	4	5	7	8	9	11	12
50	0·6428	·6414	·6401	·6388	·6374	·6361	·6347	·6334	·6320	·6307	1	3	4	5	7	8	9	11	12
51	0·6293	·6280	·6266	·6252	·6239	·6225	·6211	·6198	·6184	·6170	1	3	4	5	7	8	10	11	12
52	0·6157	·6143	·6129	·6115	·6101	·6088	·6074	·6060	·6046	·6032	1	3	4	5	7	8	10	11	12
53	0·6018	·6004	·5990	·5976	·5962	·5948	·5934	·5920	·5906	·5892	1	3	4	6	7	8	10	11	13
54	0·5878	·5864	·5850	·5835	·5821	·5807	·5793	·5779	·5764	·5750	1	3	4	6	7	9	10	11	13
55	0·5736	·5721	·5707	·5693	·5678	·5664	·5650	·5635	·5621	·5606	1	3	4	6	7	9	10	12	13
56	0·5592	·5577	·5563	·5548	·5534	·5519	·5505	·5490	·5476	·5461	1	3	4	6	7	9	10	12	13
57	0·5446	·5432	·5417	·5402	·5388	·5373	·5358	·5344	·5329	·5314	1	3	4	6	7	9	10	12	13
58	0·5299	·5284	·5270	·5255	·5240	·5225	·5210	·5195	·5180	·5165	1	3	4	6	7	9	10	12	13
59	0·5150	·5135	·5120	·5105	·5090	·5075	·5060	·5045	·5030	·5015	2	3	5	6	8	9	11	12	14
60	0·5000	·4985	·4970	·4955	·4939	·4924	·4909	·4894	·4879	·4863	2	3	5	6	8	9	11	12	14
61	0·4848	·4833	·4818	·4802	·4787	·4772	·4756	·4741	·4726	·4710	2	3	5	6	8	9	11	12	14
62	0·4695	·4679	·4664	·4648	·4633	·4617	·4602	·4586	·4571	·4555	2	3	5	6	8	9	11	12	14
63	0·4540	·4524	·4509	·4493	·4478	·4462	·4446	·4431	·4415	·4399	2	3	5	6	8	9	11	12	14
64	0·4384	·4368	·4352	·4337	·4321	·4305	·4289	·4274	·4258	·4242	2	3	5	6	8	9	11	13	14
65	0·4226	·4210	·4195	·4179	·4163	·4147	·4131	·4115	·4099	·4083	2	3	5	6	8	10	11	13	14
66	0·4067	·4051	·4035	·4019	·4003	·3987	·3971	·3955	·3939	·3923	2	3	5	6	8	10	11	13	14
67	0·3907	·3891	·3875	·3859	·3843	·3827	·3811	·3795	·3778	·3762	2	3	5	6	8	10	11	13	15
68	0·3746	·3730	·3714	·3697	·3681	·3665	·3649	·3633	·3616	·3600	2	3	5	6	8	10	11	13	15
69	0·3584	·3567	·3551	·3535	·3518	·3502	·3486	·3469	·3453	·3437	2	3	5	6	8	10	11	13	15
70	0·3420	·3404	·3387	·3371	·3355	·3338	·3322	·3305	·3289	·3272	2	3	5	7	8	10	12	13	15
71	0·3256	·3239	·3223	·3206	·3190	·3173	·3156	·3140	·3123	·3107	2	3	5	7	8	10	12	13	15
72	0·3090	·3074	·3057	·3040	·3024	·3007	·2990	·2974	·2957	·2940	2	3	5	7	8	10	12	13	15
73	0·2924	·2907	·2890	·2874	·2857	·2840	·2823	·2807	·2790	·2773	2	3	5	7	8	10	12	13	15
74	0·2756	·2740	·2723	·2706	·2689	·2672	·2656	·2639	·2622	·2605	2	3	5	7	8	10	12	13	15
75	0·2588	·2571	·2554	·2538	·2521	·2504	·2487	·2470	·2453	·2436	2	3	5	7	8	10	12	14	15
76	0·2419	·2402	·2385	·2368	·2351	·2334	·2317	·2300	·2284	·2267	2	3	5	7	8	10	12	14	15
77	0·2250	·2233	·2215	·2198	·2181	·2164	·2147	·2130	·2113	·2096	2	3	5	7	9	10	12	13	15
78	0·2079	·2062	·2045	·2028	·2011	·1994	·1977	·1959	·1942	·1925	2	3	5	7	9	10	12	14	15
79	0·1908	·1891	·1874	·1857	·1840	·1822	·1805	·1788	·1771	·1754	2	3	5	7	9	10	12	14	15
80	0·1736	·1719	·1702	·1685	·1668	·1650	·1633	·1616	·1599	·1582	2	3	5	7	9	10	12	14	15
81	0·1564	·1547	·1530	·1513	·1495	·1478	·1461	·1444	·1426	·1409	2	3	5	7	9	10	12	14	16
82	0·1392	·1374	·1357	·1340	·1323	·1305	·1288	·1271	·1253	·1236	2	3	5	7	9	10	12	14	16
83	0·1219	·1201	·1184	·1167	·1149	·1132	·1115	·1097	·1080	·1063	2	3	5	7	9	10	12	14	16
84	0·1045	·1028	·1011	·0993	·0976	·0958	·0941	·0924	·0906	·0889	2	3	5	7	9	10	12	14	16
85	0·0872	·0854	·0837	·0819	·0802	·0785	·0767	·0750	·0732	·0715	2	3	5	7	9	10	12	14	16
86	0·0698	·0680	·0663	·0645	·0628	·0610	·0593	·0576	·0558	·0541	2	3	5	7	9	10	12	14	16
87	0·0523	·0506	·0488	·0471	·0454	·0436	·0419	·0401	·0384	·0366	2	3	5	7	9	10	12	14	16
88	0·0349	·0332	·0314	·0297	·0279	·0262	·0244	·0227	·0209	·0192	2	3	5	7	9	10	12	14	16
89	0·0175	·0157	·0140	·0122	·0105	·0087	·0070	·0052	·0035	·0017	2	3	5	7	9	10	12	14	16
x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9

Appendix IV: Tangents of Angles

Tangents of Angles (*x* in degrees)

$x \rightarrow \tan x$

x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	ADD Differences 1	2	3	4	5	6	7	8	9
0	0·0000	·0017	·0035	·0052	·0070	·0087	·0105	·0122	·0140	·0157	2	3	5	7	9	10	12	14	16
1	0·0175	·0192	·0209	·0227	·0244	·0262	·0279	·0297	·0314	·0332	2	3	5	7	9	10	12	14	16
2	0·0349	·0367	·0384	·0402	·0419	·0437	·0454	·0472	·0489	·0507	2	3	5	7	9	10	12	14	16
3	0·0524	·0542	·0559	·0577	·0594	·0612	·0629	·0647	·0664	·0682	2	4	5	7	9	11	12	14	16
4	0·0699	·0717	·0734	·0752	·0769	·0787	·0805	·0822	·0840	·0857	2	4	5	7	9	11	12	14	16
5	0·0875	·0892	·0910	·0928	·0945	·0963	·0981	·0998	·1016	·1033	2	4	5	7	9	11	12	14	16
6	0·1051	·1069	·1086	·1104	·1122	·1139	·1157	·1175	·1192	·1210	2	4	5	7	9	11	12	14	16
7	0·1228	·1246	·1263	·1281	·1299	·1317	·1334	·1352	·1370	·1388	2	4	5	7	9	11	12	14	16
8	0·1405	·1423	·1441	·1459	·1477	·1495	·1512	·1530	·1548	·1566	2	4	5	7	9	11	12	14	16
9	0·1584	·1602	·1620	·1638	·1655	·1673	·1691	·1709	·1727	·1745	2	4	5	7	9	11	13	14	16
10	0·1763	·1781	·1799	·1817	·1835	·1853	·1871	·1890	·1908	·1926	2	4	5	7	9	11	13	14	16
11	0·1944	·1962	·1980	·1998	·2016	·2035	·2053	·2071	·2089	·2107	2	4	5	7	9	11	13	15	16
12	0·2126	·2144	·2162	·2180	·2199	·2217	·2235	·2254	·2272	·2290	2	4	5	7	9	11	13	15	16
13	0·2309	·2327	·2345	·2364	·2382	·2401	·2419	·2438	·2456	·2475	2	4	6	7	9	11	13	15	17
14	0·2493	·2512	·2530	·2549	·2568	·2586	·2605	·2623	·2642	·2661	2	4	6	7	9	11	13	15	17
15	0·2679	·2698	·2717	·2736	·2754	·2773	·2792	·2811	·2830	·2849	2	4	6	8	9	11	13	15	17
16	0·2867	·2886	·2905	·2924	·2943	·2962	·2981	·3000	·3019	·3038	2	4	6	8	9	11	13	15	17
17	0·3057	·3076	·3096	·3115	·3134	·3153	·3172	·3191	·3211	·3230	2	4	6	8	10	12	13	15	17
18	0·3249	·3269	·3288	·3307	·3327	·3346	·3365	·3385	·3404	·3424	2	4	6	8	10	12	14	16	17
19	0·3443	·3463	·3482	·3502	·3522	·3541	·3561	·3581	·3600	·3620	2	4	6	8	10	12	14	16	18
20	0·3640	·3659	·3679	·3899	·3719	·3739	·3759	·3779	·3799	·3819	2	4	6	8	10	12	14	16	18
21	0·3839	·3859	·3879	·3899	·3919	·3939	·3959	·3979	·4000	·4020	2	4	6	8	10	12	14	16	18
22	0·4040	·4061	·4081	·4101	·4122	·4142	·4163	·4183	·4204	·4224	2	4	6	8	10	12	14	16	18
23	0·4245	·4265	·4286	·4307	·4327	·4348	·4369	·4390	·4411	·4431	2	4	6	8	10	12	15	17	19
24	0·4452	·4473	·4494	·4515	·4536	·4557	·4578	·4599	·4621	·4642	2	4	6	8	11	13	15	17	19
25	0·4663	·4684	·4706	·4727	·4748	·4770	·4791	·4813	·4834	·4856	2	4	6	9	11	13	15	17	19
26	0·4877	·4899	·4921	·4942	·4964	·4986	·5008	·5029	·5051	·5073	2	4	7	9	11	13	15	17	20
27	0·5095	·5117	·5139	·5161	·5184	·5206	·5228	·5250	·5272	·5295	2	4	7	9	11	13	16	18	20
28	0·5317	·5340	·5362	·5384	·5407	·5430	·5452	·5475	·5498	·5520	2	5	7	9	11	14	16	18	20
29	0·5543	·5566	·5589	·5612	·5635	·5658	·5681	·5704	·5727	·5750	2	5	7	9	12	14	16	18	21
30	0·5774	·5797	·5820	·5844	·5867	·5890	·5914	·5938	·5961	·5985	2	5	7	9	12	14	16	19	21
31	0·6009	·6032	·6056	·6080	·6104	·6128	·6152	·6176	·6200	·6224	2	5	7	10	12	14	17	19	22
32	0·6249	·6273	·6297	·6322	·6346	·6371	·6395	·6420	·6445	·6469	2	5	7	10	12	15	17	20	22
33	0·6494	·6519	·6544	·6569	·6594	·6619	·6644	·6669	·6694	·6720	3	5	8	10	13	15	18	20	23
34	0·6745	·6771	·6796	·6822	·6847	·6873	·6899	·6924	·6950	·6976	3	5	8	10	13	15	18	21	23
35	0·7002	·7028	·7054	·7080	·7107	·7133	·7159	·7186	·7212	·7239	3	5	8	11	13	16	18	21	24
36	0·7265	·7292	·7319	·7346	·7373	·7400	·7427	·7454	·7481	·7508	3	5	8	11	14	16	19	22	24
37	0·7536	·7563	·7590	·7618	·7646	·7673	·7701	·7729	·7757	·7785	3	6	8	11	14	17	19	22	25
38	0·7813	·7841	·7869	·7898	·7926	·7954	·7983	·8012	·8040	·8069	3	6	9	11	14	17	20	23	26
39	0·8098	·8127	·8156	·8185	·8214	·8243	·8273	·8302	·8332	·8361	3	6	9	12	15	18	21	23	26
40	0·8391	·8421	·8451	·8481	·8511	·8541	·8571	·8601	·8632	·8662	3	6	9	12	15	18	21	24	27
41	0·8693	·8724	·8754	·8785	·8816	·8847	·8878	·8910	·8941	·8972	3	6	9	12	16	19	22	25	28
42	0·9004	·9036	·9067	·9099	·9131	·9163	·9195	·9228	·9260	·9293	3	6	10	13	16	19	22	26	29
43	0·9325	·9358	·9391	·9424	·9457	·9490	·9523	·9556	·9590	·9623	3	7	10	13	17	20	23	27	30
44	0·9657	·9691	·9725	·9759	·9793	·9827	·9861	·9896	·9930	·9965	3	7	10	14	17	21	24	27	31
x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9

x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	ADD Differences 1	2	3	4	5	6	7	8	9
45	1·000	1·003	1·007	1·011	1·014	1·018	1·021	1·025	1·028	1·032	0	1	1	1	2	2	2	3	3
46	1·036	1·039	1·043	1·046	1·050	1·054	1·057	1·061	1·065	1·069	0	1	1	1	2	2	3	3	3
47	1·072	1·075	1·080	1·084	1·087	1·091	1·095	1·099	1·103	1·107	0	1	1	2	2	2	3	3	3
48	1·111	1·115	1·118	1·122	1·126	1·130	1·134	1·138	1·142	1·146	0	1	1	2	2	2	3	3	4
49	1·150	1·154	1·159	1·163	1·167	1·171	1·175	1·179	1·183	1·188	0	1	1	2	2	2	3	3	4
50	1·192	1·196	1·200	1·205	1·209	1·213	1·217	1·222	1·226	1·230	0	1	1	2	2	3	3	4	4
51	1·235	1·239	1·244	1·248	1·253	1·257	1·262	1·266	1·271	1·275	0	1	1	2	2	3	3	4	4
52	1·280	1·285	1·289	1·294	1·299	1·303	1·308	1·313	1·317	1·322	0	1	1	2	2	3	3	4	4
53	1·327	1·332	1·337	1·342	1·347	1·351	1·356	1·361	1·366	1·371	0	1	1	2	2	3	3	4	4
54	1·376	1·381	1·387	1·392	1·397	1·402	1·407	1·412	1·418	1·423	1	1	2	2	3	3	4	4	5
55	1·428	1·433	1·439	1·444	1·450	1·455	1·460	1·466	1·471	1·477	1	1	2	2	3	3	4	4	5
56	1·483	1·488	1·494	1·499	1·505	1·511	1·517	1·522	1·528	1·534	1	1	2	2	3	3	4	5	5
57	1·540	1·546	1·552	1·558	1·564	1·570	1·576	1·582	1·588	1·594	1	1	2	2	3	4	4	5	5
58	1·600	1·607	1·613	1·619	1·625	1·632	1·638	1·645	1·651	1·658	1	1	2	3	3	4	4	5	6
59	1·664	1·671	1·678	1·684	1·691	1·698	1·704	1·711	1·718	1·725	1	1	2	3	3	4	5	5	6
60	1·732	1·739	1·746	1·753	1·760	1·767	1·775	1·782	1·789	1·797	1	1	2	3	4	4	5	6	6
61	1·804	1·811	1·819	1·827	1·834	1·842	1·849	1·857	1·865	1·873	1	2	2	3	4	5	5	6	7
62	1·881	1·889	1·897	1·905	1·913	1·921	1·929	1·937	1·946	1·954	1	2	2	3	4	5	6	7	7
63	1·963	1·971	1·980	1·988	1·997	2·006	2·014	2·023	2·032	2·041	1	2	3	4	4	5	6	7	8
64	2·050	2·059	2·069	2·078	2·087	2·097	2·106	2·116	2·125	2·135	1	2	3	4	5	6	7	8	8
65	2·145	2·154	2·164	2·174	2·184	2·194	2·204	2·215	2·225	2·236	1	2	3	4	5	6	7	8	9
66	2·246	2·257	2·267	2·278	2·289	2·300	2·311	2·322	2·333	2·344	1	2	3	4	5	7	8	9	10
67	2·356	2·367	2·379	2·391	2·402	2·414	2·426	2·438	2·450	2·463	1	2	4	5	6	7	8	10	11
68	2·475	2·488	2·500	2·513	2·526	2·539	2·552	2·565	2·578	2·592	1	3	4	5	7	8	9	10	12
69	2·605	2·619	2·633	2·646	2·660	2·675	2·689	2·703	2·718	2·733	1	3	4	6	7	9	10	11	13
70	2·747	2·762	2·778	2·793	2·808	2·824	2·840	2·856	2·872	2·888	2	3	5	6	8	9	11	13	14
71	2·904	2·921	2·937	2·954	2·971	2·989	3·006	3·024	3·042	3·060	2	3	5	7	9	10	12	14	16
72	3·078	3·096	3·115	3·133	3·152	3·172	3·191	3·211	3·230	3·251	2	4	6	8	10	12	14	15	17
73	3·271	3·291	3·312	3·333	3·354	3·376	3·398	3·420	3·442	3·465	2	4	6	9	11	13	15	17	19
74	3·487	3·511	3·534	3·558	3·582	3·606	3·630	3·655	3·681	3·706	2	5	7	10	12	15	17	20	22
75	3·732	3·758	3·785	3·812	3·839	3·867	3·895	3·923	3·952	3·981	3	6	8	11	14	17	20	22	25
76	4·011	4·041	4·071	4·102	4·134	4·165	4·198	4·230	4·264	4·297	3	6	10	13	16	19	22	26	29
77	4·331	4·366	4·402	4·437	4·474	4·511	4·548	4·586	4·625	4·665									
78	4·705	4·745	4·787	4·829	4·872	4·915	4·959	5·005	5·050	5·097									
79	5·145	5·193	5·242	5·292	5·343	5·396	5·449	5·503	5·558	5·614									
80	5·671	5·730	5·789	5·850	5·912	5·976	6·041	6·107	6·174	6·243	differences unreliable:								
81	6·314	6·386	6·460	6·535	6·612	6·691	6·772	6·855	6·940	7·026									
82	7·115	7·207	7·300	7·396	7·495	7·596	7·700	7·806	7·916	8·028									
83	8·144	8·264	8·386	8·513	8·643	8·777	8·915	9·058	9·205	9·357									
84	9·514	9·677	9·845	10·02	10·20	10·39	10·58	10·78	10·99	11·20									
85	11·43	11·66	11·91	12·16	12·43	12·71	13·00	13·30	13·62	13·95									
86	14·30	14·67	15·06	15·46	15·89	16·35	16·83	17·34	17·89	18·46									
87	19·08	19·74	20·45	21·20	22·02	22·90	23·86	24·90	26·03	27·27									
88	28·64	30·14	31·82	33·69	35·80	38·19	40·92	44·07	47·74	52·08									
89	57·29	63·66	71·62	81·85	95·46	114·6	143·2	191·0	286·5	573·0									
x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9

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