Free Quality School Education

Ministry of Basic and Senior Secondary Education

Pupils' Handbook for Senior Secondary *Mathematics*

SSS I

Term I

STRICTLY NOT FOR SALE

DOI: 10.5281/zenodo.3745362
Please see final page for further information.

Foreword

These Lesson Plans and the accompanying Pupils' Handbooks are essential educational resources for the promotion of quality education in senior secondary schools in Sierra Leone. As Minister of Basic and Senior Secondary Education, I am pleased with the professional competencies demonstrated by the writers of these educational materials in English Language and Mathematics.

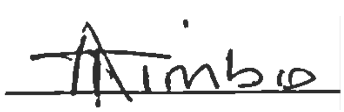
The Lesson Plans give teachers the support they need to cover each element of the national curriculum, as well as prepare pupils for the West African Examinations Council's (WAEC) examinations. The practice activities in the Pupils' Handbooks are designed to support self-study by pupils, and to give them additional opportunities to learn independently. In total, we have produced 516 lesson plans and 516 practice activities – one for each lesson, in each term, in each year, for each class. The production of these materials in a matter of months is a remarkable achievement.

These plans have been written by experienced Sierra Leoneans together with international educators. They have been reviewed by officials of my Ministry to ensure that they meet the specific needs of the Sierra Leonean population. They provide step-by-step guidance for each learning outcome, using a range of recognized techniques to deliver the best teaching.

I call on all teachers and heads of schools across the country to make the best use of these materials. We are supporting our teachers through a detailed training programme designed specifically for these new lesson plans. It is really important that the Lesson Plans and Pupils' Handbooks are used, together with any other materials they may have.

This is just the start of educational transformation in Sierra Leone as pronounced by His Excellency, the President of the Republic of Sierra Leone, Brigadier Rtd Julius Maada Bio. I am committed to continue to strive for the changes that will make our country stronger and better.

I do thank our partners for their continued support. Finally, I also thank the teachers of our country for their hard work in securing our future.



Mr. Alpha Osman Timbo

Minister of Basic and Senior Secondary Education

The policy of the Ministry of Basic and Senior Secondary Education, Sierra Leone, on textbooks stipulates that every printed book should have a lifespan of three years.

To achieve thus, DO NOT WRITE IN THE BOOKS.

Table of Contents

Introduction
to the Pupils' Handbook

These practice activities are aligned to the Lesson Plans, and are based on the National Curriculum and the West Africa Examination Council syllabus guidelines. They meet the requirements established by the Ministry of Basic and Senior Secondary Education.

1. The practice activities will not take the whole term, so use any extra time to revise material or re-do activities where you made mistakes.
2. Use other textbooks or resources to help you learn better and practise what you have learned in the lessons.
3. Read the questions carefully before answering them. After completing the practice activities, check your answers using the answer key at the end of the book.
4. Make sure you understand the learning outcomes for the practice activities and check to see that you have achieved them. Each lesson plan shows these using the symbol to the right.

   Learning Outcomes
5. Organise yourself so that you have enough time to complete all of the practice activities. If there is time, quickly revise what you learned in the lesson before starting the practice activities. If it is taking you too long to complete the activities, you may need more practice on that particular topic.
6. Seek help from your teacher or your peers if you are having trouble completing the practice activities independently.
7. Make sure you write the answers in your exercise book in a clear and systematic way so that your teacher can check your work and you can refer back to it when you prepare for examinations.
8. Congratulate yourself when you get questions right! Do not worry if you do not get the right answer – ask for help and continue practising!

KEY TAKEAWAYS FROM SIERRA LEONE'S PERFORMANCE IN WEST AFRICAN SENIOR SCHOOL CERTIFICATE EXAMINATION – GENERAL MATHEMATICS[1]

This section, seeks to outline key takeaways from assessing Sierra Leonean pupils' responses on the West African Senior School Certificate Examination. The common errors pupils make are highlighted below with the intention of giving teachers an insight into areas to focus on, to improve pupil performance on the examination. Suggestions are provided for addressing these issues.

Common errors

1. Errors in applying principles of BODMAS
2. Mistakes in simplifying fractions
3. Errors in application of Maths learned in class to real-life situations, and vis-a-versa.
4. Errors in solving geometric constructions.
5. Mistakes in solving problems on circle theorems.
6. Proofs are often left out from solutions, derivations are often missing from quadratic equations.

Suggested solutions

1. Practice answering questions to the detail requested
2. Practice re-reading questions to make sure all the components are answered.
3. If possible, procure as many geometry sets to practice geometry construction.
4. Check that depth and level of the lesson taught is appropriate for the grade level.

[1] This information is derived from an evaluation of WAEC Examiner Reports, as well as input from WAEC examiners and Sierra Leonean teachers.

Lesson Title: Review of Numbers and Numeration	**Theme:** Numbers and Numeration
Practice Activity: PHM1-L001	**Class:** SSS 1

Learning Outcomes
By the end of the lesson, you will be able to:
1. Identify prime numbers and prime factors.
2. Calculate LCM and HCF.

Overview

To find the LCM and HCF of given numbers, you must first understand factors, prime factorisation, and multiples.

Factors are numbers that can divide another number exactly. For example, 6 can divide 12 two times, therefore 6 is a factor of 12. Factors of 12 include 1, 2, 3, 6, and 12.

A **prime number** is a number that is greater than 1 and cannot be divided evenly by any other number except 1 and itself. Zero and 1 are not considered prime numbers. Examples of prime numbers between 1 and 30 are 2, 3, 5, 7, 11, 13, 17, 19, 23, and 29.

A **prime factor** is a prime number that divides exactly into another given number. For example, 2 and 3 are factors of 12 and they are also prime numbers. Therefore, 2 and 3 are prime factors of 12.

Prime factorisation is the process of expressing a number as the product of its prime factors. For example, the prime factorisations of 12 and 18 are: $12 = 2 \times 2 \times 3$, $18 = 2 \times 3 \times 3$.

Multiples are numbers that can be divided by another number without a remainder. 12 is a multiple of 4, because 4 divides 12 evenly. The first 5 multiples of 4 are: 4, 8, 12, 16, 20.

The **least common multiple (LCM)** is the smallest multiple that two numbers have in common. For example, 12 is the LCM of 3 and 4. 12 is a multiple of both 3 and 4, and it is the smallest multiple that these two numbers share.

To find the **LCM** of a set of numbers:

- Find the prime factorisation of each number.
- Find the prime factors that appear in **any one** of the prime factorisations.
- Find the product of these primes using each prime the greatest number of times that it appears in **any one** of the prime factorisations.

- That product is the LCM.

The **highest common factor (HCF)** of two numbers is the largest factor that the two numbers share. For example, the HCF of 8 and 12 is 4. This is the largest number that divides evenly into both 8 and 12.
To find the **HCF** of a set of numbers, use the following steps.

- Find the prime factorisation of each number.
- Choose common prime factors.
- Multiply the common prime factors.
- Multiply the common prime factors to get the HCF.

Solved Examples

1. What is the sum of the first 6 prime numbers in our counting numerals?

   Identify the first 6 Prime numbers: $2, 3, 5, 7, 11, 13$
   Add the 6 numbers together: $2 + 3 + 5 + 7 + 11 + 13 = 41$

2. Express these numbers as a product of their prime factors: a. 72 b. 90

   a. Identify the prime factors and multiply them to get 72: $72 = 2 \times 2 \times 2 \times 3 \times 3$
   b. Identify the prime factors and multiply them to get 90: $90 = 2 \times 3 \times 3 \times 5$

3. Find the LCM of 24 and 60.
   Solution:
   Step 1. Express each number as a product of it prime factors.
   $24 = 2 \times 2 \times 2 \times 3$
   $60 = 2 \times 2 \times 3 \times 5$
   Step 2. Find the prime factors that appear in **any one** of the prime factorisations.

   - 2 appears three times in 24.
   - 3 appears once in 24 and 60.
   - 5 appears once in 60.

   The factors that appear in any one of the prime factorisations are: $2, 2, 2, 3, 5$.
   Step 3. Find the product.
   LCM $= 2 \times 2 \times 2 \times 3 \times 5$
   $= 120$

4. Find the HCF of 24, 36 and 48.
   Solution:
   Step 1. Express each number as a product of its prime factors.
   $24 = 2 \times 2 \times 2 \times 3$
   $36 = 2 \times 2 \times 3 \times 3$
   $48 = 2 \times 2 \times 2 \times 2 \times 3$

Step 2. Choose common prime factors:

- 2 appears at least twice in each factorisation
- 3 appears at least once.

The common factors are: $2, 2, 3$

Step 3. Find the product:

HCF $= 2 \times 2 \times 3$

$= 12$

5. Find the HCF and the LCM of $36, 72,$ and 90.

Solution:

Step 1. Express each number as a product of its prime factors:

$36 = 2 \times 2 \times 3 \times 3$

$72 = 2 \times 2 \times 2 \times 3 \times 3$

$90 = 2 \times 3 \times 3 \times 5$

Step 2. Calculate HCF by multiplying common factors:

HCF= $2 \times 3 \times 3 = 18$

Step 3. Calculate LCM by multiplying the factors that appear in any one of the factorisations:

LCM= $2 \times 2 \times 2 \times 3 \times 3 \times 5 = 360$

Practice

1. Identify the non-prime numbers in the following numbers: 27, 37, 47, 51, 57, 67, 79, 81, 87, 91.
2. Express these numbers as product of their prime factors: a. 54 b. 64
3. Find the HCF of 42, 36 and 72.
4. Find the LCM of 12, 15 and 18.
5. Find the HCF and LCM of 24, 36 and 48.

Lesson Title: Addition and Subtraction of Fractions	**Theme:** Numbers and Numeration
Practice Activity: PHM1-L002	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to add and subtract fractions including word problems.

Overview

To add and subtract fractions, you must have a common denominator. Fractions that have common denominators are called 'like fractions' (For example: $\frac{1}{8}$, $\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{8}$). Fractions that have different denominators are called 'unlike fractions' (For example: $\frac{2}{3}$, $\frac{4}{5}$, $\frac{7}{8}$).

To add or subtract **like fractions**, simply add or subtract the numerators and keep the same (or like) denominator.

To add or subtract **unlike fractions**, follow these steps:

- Find the least common multiple (LCM) of the denominators.
- Change each fraction to have the LCM in the denominator.
- Add the 2 fractions, because they are like fractions now.

Here are some notes on solving addition and subtraction problems:

- Simplify answers to their lowest terms. (Example: $\frac{2}{4} \rightarrow \frac{1}{2}$)
- When performing operations on mixed fractions, always convert them to improper fractions first. (Example: $3\frac{1}{2} \rightarrow \frac{7}{2}$)
- If the answer to a problem is an improper fraction, convert it to a mixed fraction. (Example: $\frac{7}{2} \rightarrow 3\frac{1}{2}$)

Word problems are Maths problems that are expressed as stories. Here are key words to look for in word problems:

- Addition words: sum, total, more than
- Subtraction words: difference, less than, how much more than

Solved Examples

1. Simplify: $\frac{1}{5}+\frac{3}{5}$

   Solution:

   $\frac{1}{5}+\frac{3}{5} \quad = \frac{1}{5}+\frac{3}{5}$ The denominators are the same so keep it and add the numerators.

$= \frac{1+3}{5}$

$= \frac{4}{5}$

2. Simplify: $\frac{4}{7} + \frac{1}{3}$

   Solution:

   $\frac{4}{7} + \frac{1}{3} = \frac{4}{7} + \frac{1}{3}$

   $= \frac{4\times3}{21} + \frac{1\times7}{21}$ Change the denominator to the LCM, 21

   $= \frac{12}{21} + \frac{7}{21}$

   $= \frac{12+7}{21}$ With like denominators now just simply add the numerators.

   $= \frac{19}{21}$

3. Simplify: $2\frac{1}{5} - 1\frac{3}{4}$

   Solution:

   $2\frac{1}{5} - 1\frac{3}{4} = 2\frac{1}{5} - 1\frac{3}{4}$ Convert the mixed fraction to improper fraction.

   $= \frac{11}{5} - \frac{7}{4}$ The LCM of 4 and 5 is 20

   $= \frac{11\times4}{20} - \frac{7\times5}{20}$

   $= \frac{44}{20} - \frac{35}{20}$ Same denominators now so simply subtract the numerators

   $= \frac{44-35}{20}$

   $= \frac{9}{20}$

4. A school wants to make a new playground in an empty field. They give the job of planning the playground to a group of students. The pupils decide to use $\frac{1}{4}$ of the playground for a basketball court and $\frac{3}{8}$ for a football field. How much of the playground is left?

   Solution:

   Step 1. Add to find the total area used by the basketball court and football field.

   $\frac{1}{4} + \frac{3}{8} = \frac{2}{8} + \frac{3}{8}$ Change each denominator to the LCM, 8

$$= \frac{2+3}{8} = \frac{5}{8} \quad \text{Add the numerators}$$

Step 2. Subtract to find the area of the playground left.

$1 - \frac{5}{8}$	$=$	$\frac{1}{1} - \frac{5}{8}$	Subtract the area used from 1 whole
	$=$	$\frac{8}{8} - \frac{5}{8}$	Change each denominator to the LCM, 8
	$=$	$\frac{8-5}{8} = \frac{3}{8}$	Subtract the numerators

Answer: $\frac{3}{8}$ of the playground is left for other purposes.

5. A boy plays football for $1\frac{3}{4}$ hours, watches TV for $\frac{3}{4}$ hours and then spends $1\frac{1}{4}$ hours doing his homework. How much time does he spend altogether?

   Solution:

   To find the total amount of time he spends, add all 3 fractions together:

$1\frac{3}{4} + \frac{3}{4} + 1\frac{1}{4}$	$=$	$\frac{7}{4} + \frac{3}{4} + \frac{5}{4}$	Convert to improper fractions
	$=$	$\frac{7+3+5}{4}$	Add the numerators
	$=$	$\frac{15}{4}$	
	$=$	$3\frac{3}{4}$ hours	Convert to a mixed fraction

Practice

1. Simplify $\frac{3}{8} + \frac{1}{4}$
2. Simplify $3\frac{1}{2} + 2\frac{2}{3}$
3. Simplify $2\frac{1}{3} - 1\frac{1}{4} + 3\frac{1}{2}$
4. You give $\frac{1}{3}$ of your cake to Susan and $\frac{1}{6}$ of the cake to Patrick. How much of the cake did you give away?
5. You got out for a long walk. You walk $\frac{3}{4}$ mile and then sit down to take rest. Then you walk another $\frac{3}{8}$ mile. How far did you walk altogether?
6. Joseph walks $\frac{7}{8}$ mile to school. Paul walks $\frac{1}{2}$ mile to school. How much farther does Joseph walk than Paul?

Lesson Title: Multiplication and division of fractions	**Theme:** Number and numeration
Practice Activity: PHM1-L003	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to multiply and divide fractions including word problems.

Overview

When multiplying fractions, simply multiply the numerators (top numbers of the fractions) together and multiply the denominators (bottom numbers of the fractions) together. Cancelling is done when the numerator and denominator can be divided evenly by the same number. We cancel top-to-bottom and or diagonally but **never** across.

To divide by a fraction, simply multiply by the reciprocal of that fraction and then simplify. The reciprocal of a fraction is the result when you invert it, or turn it over.

Always convert mixed fractions to improper fractions before multiplying or dividing fractions.

Solved Examples

1. Simplify: $\frac{35}{40}\times\frac{100}{1000}$

$\frac{35}{40}\times\frac{100}{1000}=\frac{35}{40}\times\frac{100}{1000}$	Cancel (divide by) 100 on the right fraction
$=\frac{35}{40}\times\frac{1}{10}$	Cancel 5 from the left fraction
$=\frac{7}{8}\times\frac{1}{10}$	
$=\frac{7\times1}{8\times10}$	Multiply
$=\frac{7}{80}$	

2. Simplify: $4\frac{1}{3}\times1\frac{7}{8}$

$4\frac{1}{3}\times1\frac{7}{8}=\frac{13}{3}\times\frac{15}{8}$	Change to improper fractions
$=\frac{13}{1}\times\frac{5}{8}$	Cancel 3 diagonally
$=\frac{13\times5}{1\times8}$	Multiply
$=\frac{65}{8}$	

$= 8\frac{1}{8}$ Change to a mixed fraction

3. Simplify: $\frac{5}{8} \div \frac{15}{16}$

$\frac{5}{8} \div \frac{15}{16} = \frac{5}{8} \times \frac{16}{15}$ Change to multiplication

$= \frac{1}{1} \times \frac{2}{3}$ Cancel 5 and 8

$= \frac{2}{3}$ Multiply

4. Simplify $1\frac{3}{4} \div 2\frac{5}{8}$:

$1\frac{3}{4} \div 2\frac{5}{8} = \frac{7}{4} \div \frac{21}{8}$ Change to improper fractions

$= \frac{7}{4} \times \frac{8}{21}$ Change to multiplication

$= \frac{1}{1} \times \frac{2}{3}$ Cancel 7 and 4

$= \frac{2}{3}$ Multiply

5. Juliet studied for $3\frac{1}{3}$ hours during each of the 4 days before her last Mathematics test. How much time did she spend studying for the test?
   Solution:
   Juliet studied 4 **times**, once each day for 4 days. Although the question doesn't say "times", we know to multiply by thinking about the problem. To find the answer, multiply the amount of time spent each day ($3\frac{1}{3}$) by the number of days (4).

$3\frac{1}{3} \times 4 = \frac{10}{3} \times 4$ Convert to an improper fraction

$= \frac{40}{3}$ Multiply

$= 13\frac{1}{3}$ hours

6. Mrs. Nyalloma brought $5\frac{1}{2}$ yards of material. She used $\frac{2}{3}$ of the material to make a dress for herself, and the rest she kept for her daughter. How much material did she use?

Solution:

To find the amount used, find $\frac{2}{3}$ of $5\frac{1}{2}$. Recall that finding a fraction of a number involves multiplication. The problem to be solved is $\frac{2}{3} \times 5\frac{1}{2}$.

$\frac{2}{3} \times 5\frac{1}{2}$	$= \frac{2}{3} \times \frac{11}{2}$	Change to an improper fraction
	$= \frac{11}{3}$	Multiply
	$= 3\frac{2}{3}$ yards	Change to a mixed fraction

7. One lecture at an evening course lasts $2\frac{1}{4}$ hours. If the course last for 36 hours altogether, how many lectures are there?

   Solution:

   To find the number of lectures, divide the total amount of time for the course by the length of each lecture. The problem to be solved is $36 \div 2\frac{1}{4}$.

$36 \div 2\frac{1}{4}$	$= 36 \div \frac{9}{4}$	Convert to an improper fraction
	$= 36 \times \frac{4}{9}$	Change to multiplication
	$= 16$ lectures	Multiply

Practice

1. Simplify: $\frac{1}{4} \times \frac{6}{13}$
2. Simplify: $\frac{1}{4} \div \frac{1}{12}$
3. Simplify: $2\frac{1}{3} \times 1\frac{1}{2}$
4. Simplify: $3\frac{1}{7} \div 4\frac{5}{7}$
5. It takes $1\frac{3}{4}$ metres of a cloth to make a skirt. How many skirts can be made from $10\frac{1}{2}$ metres of cloth?
6. There are 20 books in a stack. The weight of each book is$1\frac{3}{4}$ kg. Find the total weight of the books.
7. In a school, $\frac{8}{9}$ of the students play sports. If $\frac{3}{4}$ of these play football, what fraction of the students play football?

Lesson Title: Addition and subtraction of decimals	Theme: Numbers and Numeration
Practice Activity: PHM1-L004	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to add and subtract decimals, including word problems.

Overview

The number 376.492 is an example of a decimal number. The number on the left of the decimal point are whole numbers and those on the right are the fractional part. The place value of each digit is shown below:

3	7	6	.	4	9	2
Hundreds	Tens	Ones		Tenths	Hundredths	Thousandths

When adding and subtracting decimals, always arrange the numbers vertically according to the place value. Ensure that you have one under ones, tens under tens, tenths under tenths, and so on. The decimal points must be lined up together vertically. Remember to fill up empty spaces with zeroes when you write the problem vertically.

Solved Examples

1. Add 0.47, 47 and 6.47
   Solution:
   Line up the decimal places and fill the empty spaces with zeroes. Then, add as you would whole numbers:

$$
\begin{array}{rcccc}
 & ^{1} & & & ^{1} \\
 & 0 & 0 & . & 4\ 7 \\
 & 4 & 7 & . & 0\ 0 \\
+ & 0 & 6 & . & 4\ 7 \\
\hline
 & 5 & 3 & . & 9\ 4
\end{array}
$$

2. Subtract 0.0364 from 0.969.
   Solution:

$$\begin{array}{rcccccc} & & & & & 8 & \\ & 0 & . & 9 & 6 & \not{9} & {}^{1}0 \\ - & 0 & . & 0 & 3 & 6 & 4 \\ \hline & 0 & . & 9 & 3 & 2 & 6 \end{array}$$

3. Subtract the sum of 0.23 and 0.023 from 2.3.
   Solution:
   Write the problem to be solved: $2.3 - (0.23 + 0.023)$
   First get the sum (add 0.23 and 0.023) and then subtract from 2.3.

$$\begin{array}{rccccc} & 0 & . & 2 & 3 & 0 \\ + & 0 & . & 0 & 2 & 3 \\ \hline & 0 & . & 2 & 5 & 3 \end{array} \qquad \begin{array}{rccccc} & & & 2 & 9 & \\ & 2 & . & \not{3} & \not{0} & {}^{1}0 \\ - & 0 & . & 2 & 5 & 3 \\ \hline & 2 & . & 0 & 4 & 7 \end{array}$$

   Answer: $2.3 - (0.23 + 0.023) = 2.047$

4. Simplify: $2.683 - 6.808 + 5.316$
   Solution:
   The operations can be applied in either order. Below, 2.683 and 5.316 are added together first. Then, 6.808 is subtracted from the sum. This way, we avoid working with negative numbers.

$$\begin{array}{rccccc} & 2 & . & 6 & 8 & 3 \\ + & 5 & . & 3 & 1 & 6 \\ \hline & 7 & . & 9 & 9 & 9 \end{array} \qquad \begin{array}{rccccc} & 7 & . & 9 & 9 & 9 \\ - & 6 & . & 8 & 0 & 8 \\ \hline & 1 & . & 1 & 9 & 1 \end{array}$$

   Answer: $2.683 - 6.808 + 5.316 = 1.191$

5. Five girls weight 26.4 kg, 21.6 kg, 9.94 kg, 23.7 kg and 25.8 kg. What is their total weight?
   Solution:
   Add all the weights together:

$$\begin{array}{rcccccl} & & 2 & 3 & & & \\ & & 2 & 6 & . & 4 & 0 \\ & & 2 & 1 & . & 6 & 0 \\ & & 0 & 9 & . & 9 & 4 \\ & & 2 & 3 & . & 7 & 0 \\ + & & 2 & 5 & . & 8 & 0 \\ \hline & 1 & 0 & 7 & . & 4 & 4 \text{ kg} \end{array}$$

   Answer: Their total weight is 107.44 kg.

6. By how much is the sum of 0.45 and 0.312 different from 0.96?
   Solution:
   Write the problem to be solved: $0.96 - (0.45 + 0.312)$
   Do the addition first, and then find the difference between the sum and 0.96.

$$\begin{array}{cccccc} & 0 & . & 4 & 5 & 0 \\ + & 0 & . & 3 & 1 & 2 \\ \hline & 0 & . & 7 & 6 & 2 \end{array} \qquad \begin{array}{cccccc} & & & 8 & 15 & \\ & 0 & . & \cancel{9} & \cancel{6} & {}^{1}0 \\ - & 0 & . & 7 & 6 & 2 \\ \hline & 0 & . & 1 & 9 & 8 \end{array}$$

   Answer: $0.96 - (0.45 + 0.312) = 0.198$

7. Abu packed his school bag for the first day of school. It weighed 9.327 kg. He decided to reduce the weight by removing a book that weighs 2.15 kg. How heavy is Abu's bag now?
   Solution:
   To find the weight of his bag now, we subtract 2.15 kg from 9.327 kg.

$$\begin{array}{cccccc} & & & 2 & & \\ & 9 & . & \cancel{3} & {}^{1}2 & 7 \\ - & 2 & . & 1 & 5 & 0 \\ \hline & 7 & . & 1 & 7 & 7 \end{array}$$

   Answer: The bag weighs 7.177 kg now.

Practice

1. Add 0.32, 32 and 5.32.
2. Subtract 0.092 from 0.87.
3. Subtract the sum of 0.27 and 0.027 from 2.1.
4. Simplify 3.214 – 7.31 + 6.15.
5. Five boys weigh 19.4 kg, 22.8 kg, 29.1 kg, 18.7 kg and 23.9 kg. Find their total weight.
6. Princess weighs her hand luggage. It weighs 5.24 kg. She decided to remove some items that weigh 1.29 kg. How heavy is her hand luggage now?

Lesson Title: Multiplication and division of decimals	**Theme:** Numbers and Numeration
Practice Activity: PHM1-L005	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to multiply and divide decimals including word problems.

Overview

To multiply a decimal number by a power of ten (such as 10, 100, 1000), shift the decimal point to the right according to the number of zeroes. For example: $4.386 \times 100 = 438.6$.

To divide a decimal number by a power of ten, shift the decimal point to the left according to the number of zeroes. For example: $36.4 \div 10 = 3.64$

To multiply a decimal number by another decimal number, omit the decimal point and treat the numbers as whole numbers. Multiply as you do for whole numbers. After multiplying, count the number of decimal places in the numbers in the question, and give the answer the same number of decimal places.

To divide decimals, make the divisor (the second number) a whole number by shifting the decimal point to the right. Shift the decimal point in the dividend by the same number of decimal places.

Solved Examples

1. Multiply: 0.456×100
   Solution:
   Move the decimal point 2 places to the right: 45.6

2. Divide: $3678.6 \div 10$
   Solution:
   Move the decimal point 1 place to the left: 367.86

3. 0.346×1.2
   Solution:
   Multiply as whole numbers:

$$\begin{array}{ccccc} & & & {}^{1} & \\ & & 3 & 4 & 6 \\ \times & & & 1 & 2 \\ \hline & & 6 & 9 & 2 \\ + & 3 & 4 & 6 & \\ \hline & 4 & 1 & 5 & 2 \end{array}$$

There are 4 decimal places in total in the problem. Therefore, the answer is 0.4152.

4. Michael has 12 exercise books in his bag. If each weigh 1.5 kg, find the total weight of books he carries.
Solution:
Total weight of books $= 12 \times 1.5$
Multiply as whole numbers:

$$\begin{array}{cccc}
 & & {}^{1} & \\
 & & 1 & 2 \\
 & \times & 1 & 5 \\
\hline
 & & 6 & 0 \\
+ & 1 & 2 & \\
\hline
 & 1 & 8 & 0
\end{array}$$

Give the answer one decimal place: 18.0. Michael carries 18 kg.

5. Simplify: 18.36×3.65
Solution:
Multiply as whole numbers:

$$\begin{array}{ccccccc}
 & & & {}^{4} & {}^{2}{}_{1} & {}^{3}{}_{3} & \\
 & & & 1 & 8 & 3 & 6 \\
 & \times & & & 3 & 6 & 5 \\
\hline
 & & & 9 & 1 & 8 & 0 \\
 & 1 & 1 & 0 & 1 & 6 & \\
+ & 5 & 5 & 0 & 8 & & \\
\hline
 & 6 & 7 & 0 & 1 & 4 & 0
\end{array}$$

Give the answer 4 decimal places: 67.0140

6. Simplify: $0.364 \div 1.6$
Solution:
Convert the divisor to a whole number by moving the decimal point 1 place to the right. Do the same to the dividend, to keep the same ratio:

$$\frac{0.364}{1.6} = \frac{3.64}{16}$$

Divide:

$$\begin{array}{r|ccccc}
 & 0. & 2 & 2 & 7 & 5 \\
\hline
16 & 3. & 6 & 4 & 0 & 0 \\
- & 3 & 2 & & & \\
\hline
 & & 4 & 4 & & \\
 & - & 3 & 2 & \downarrow & \\
\hline
 & & 1 & 2 & 0 & \\
 & - & 1 & 1 & 2 & \downarrow \\
\hline
 & & & & 8 & 0 \\
 & & & - & 8 & 0 \\
\hline
 & & & & & 0
\end{array}$$

The answer is left as 0.2275.

7. Margret shared 34.8 kg of milk powder among her 4 children. How many kilogrammes did each child get?
   Solution:
   Write the division problem to be solved: $34.8 \div 4$

		8.	7
4	3	4.	8
-	3	2	
		2	8
	-	2	8
			0

   Each child received 8.7 kg milk.

Practice

1. $94.87 \times 1{,}000$
2. $473.62 \div 100$
3. 0.248×1.23
4. Divide 10.8 by 1.2
5. 47.84 kg of rice was shared equally among 8 labourers. How many kg of rice did each receive?
6. Issa harvested 16.1 kg of cassava from his farm during one week. How much did he harvest each day, on average?
7. Fatu is writing a story. If she writes 1.5 pages each day, how much can she write in 20 days?

Lesson Title: Conversion of fractions, percentages and decimals	**Theme:** Numbers and numeration
Practice Activity: PHM1-L006	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to convert between fractions, percentages and decimals.

Overview

To convert **fractions to percentages**, multiply the fraction by 100% and then simplify.

To convert **percentages to fractions**, divide the percentage by 100% and then reduce the fraction to its lowest terms.

To convert **fractions to decimals**, divide the numerator by the denominator. The answer is the decimal number result.

To convert **decimals to fractions**, follow these steps:

- Write down the decimal divided by 1, like this: $\frac{decimal}{1}$
- Multiply both the top and bottom by 10 for every number after the decimal point. (For example, if there are two numbers after the decimal point, then use 100, if there are three then use 1000, and so on.)
- Simplify the fraction to its lowest terms.

To convert **percentages to decimals**, divide by 100%. Remember that the decimal place moves 2 spaces to the left.

To convert **decimals to percentages**, multiply by 100%. Remember that the decimal place moves 2 spaces to the right.

Solved Examples

1. Convert these fractions to percentages: a. $\frac{3}{4}$ b. $\frac{17}{20}$

   Solutions:

a.	$\frac{3}{4}$	$= \frac{3}{4} \times 100\%$	Multiply by 100%
		$= \frac{3}{1} \times 25\%$	Cancel 4 from 4 and 100
		$= 75\%$	
b.	$\frac{17}{20}$	$= \frac{17}{20} \times 100\%$	Multiply by 100%
		$= \frac{17}{1} \times 5$	Cancel 20 from 20 and 100
		$= 85\%$	

2. Convert these percentages to fractions: a. 45% b. 82%

Solutions:

a. $45\% = \frac{45\%}{100\%}$ Divide by 100%

$= \frac{9}{20}$ Cancel 5

b. $82\% = \frac{82\%}{100\%}$ Divide by 100%

$= \frac{41}{50}$ Cancel 2

3. Convert these fractions to decimals: a. $\frac{5}{8}$ b. $\frac{3}{4}$

Solutions:

a. Divide 5 by 8:

```
    0 .6  2  5
 8 |5 .0  0  0
 -  4  8
   ---------
       2  0
    -  1  6
   -----------
          4  0
       -  4  0
      --------
             0
```

Answer: $\frac{5}{8} = 0.625$

b. Divide 3 by 4:

```
    0 .7  5
 4 |3 .0  0
 -  2  8
   -------
       2  0
    -  2  0
   ---------
          0
```

Answer: $\frac{3}{4} = 0.75$

4. Convert these decimals to fractions: a. 0.025 b. 1.35

Solutions:

a. $0.025 = \frac{0.025}{1} = \frac{0.025\times1000}{1000} = \frac{25}{1000} = \frac{1}{40}$

b. $1.35 = \frac{1.35}{1} = \frac{1.35\times100}{100} = \frac{135}{100} = \frac{27}{20} = 1\frac{7}{20}$

5. Convert these percentages to decimals: a. 65% b. 72%

Solutions:

a. $65\% = \frac{65\%}{100\%} = 0.65$ (move the decimal 2 places to the left)

b. $72\% = \frac{72\%}{100\%} = 0.72$

6. Convert these decimals to percentages: a. 0.45 b. 0.278

Solutions:

a. $0.45 = 0.45 \times 100\% = 45\%$ (move the decimal 2 places to the right)

b. $0.278 = 0.278 \times 100\% = 27.8\%$

Practice

1.	Convert these fractions to percentages:	a. $\frac{1}{4}$	b. $\frac{9}{25}$
2.	Convert these percentages to fractions:	a. 64%	b. 25%
3.	Convert these fractions to decimals:	a. $\frac{3}{10}$	b. $\frac{7}{8}$
4.	Convert these decimals to fractions:	a. 0.65	b. 0.125
5.	Convert these percentages to decimals:	a. 95%	b. 7%
6.	Convert these decimals to percentages:	a. 0.18	b. 0.04

Lesson Title: Finding the percentage of a quantity	**Theme:** Numbers and numeration
Practice Activity: PHM1-L007	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to find the percentage of a quantity (including word problems).

Overview

Percent means out of a total of 100. For example, 30% means 30 out of 100. We can write percent as fraction by dividing the percent by 100. For example 30% $= \frac{30}{100}$, 25%$= \frac{25}{100}$.

To find the percentage of a given quantity, we express the percentage as a fraction and then multiply the fraction by the given quantity.

Solved Examples

1. Find 20% of 90 mangoes.
   Solution:
   Step 1. Express the percent as a fraction: $20\% = \frac{20}{100}$
   Step 2. Multiply the fraction by 90: $\frac{20}{100} \times 90 \ = \frac{1}{5} \times 90 = 18$ mangoes

2. Musu gave 30% of her oranges to her sister. How many oranges did she give away?
   Solution:
   Step 1. Express the percent as a fraction: $30\% = \frac{30}{100}$
   Step 2. Multiply the fraction by 50: $\frac{30}{100} \times 50 \ = \frac{3}{10} \times 50 = 3 \times 5 = 15$ oranges

3. Joe is given $Le\ 15{,}000.00$ as lunch and transport to and from school every day. If he spends 40% of this amount as transport to and from school, how much is left for lunch?
   Solution:
   We must first find how much he spends on transportation. Then, subtract that amount from 15,000 to find the amount left for lunch.
   Money spent on transportation $= \frac{40}{100} \times 15{,}000 = Le\ 6{,}000.00$
   Money left for lunch $= Le\ 15{,}000 - Le\ 6{,}000 = Le\ 9{,}000.00$

4. Mabel was given $Le\ 600{,}000.00$ to celebrate her birthday. She spent 15% to purchase drinks, 5% to do her hair, 30% to purchase a dress. The rest she kept for food. How much was left for food?

Solution:

We first find how much she spent on each of the items in the question. Then we subtract the sum from the total amount given to her.

Step 1. Money spent on drinks $= \frac{15}{100} \times 600{,}000 = \text{Le } 90{,}000$

Step 2. Money spent to do her hair $= \frac{5}{100} \times 600{,}000 = \text{Le } 30{,}000$

Step 3. Money spent to purchase dress $= \frac{30}{100} \times 600{,}000 = \text{Le } 180{,}000$

Step 4. Total amount spent $= \text{Le } 90{,}000 + \text{Le } 30{,}000 + \text{Le } 180{,}000$
$= \text{Le } 300{,}000$

Step 5. Money left for food $= \text{Le } 600{,}000 - \text{Le } 300{,}000$
$= \text{Le } 300{,}000$

Answer: She has $\text{Le } 300{,}000.00$ left for food.

5. In a school with a pupil population of 900, 55% are girls. How many boys are there in the school?

   Solution:

   We first find the number of girls. Then we subtract our answer from the total population.

   Step 1. Number of girls $= \frac{55}{100} \times 900 = 55 \times 9 = 495$ girls

   Step 2. Number of boys: $= 900 - 495 = 405$ boys

Practice

1. Find 35% of 120 mangoes.
2. Fatu bought a bag containing 150 oranges. If 10% of it got rotten before she got home, how many oranges were left with her?
3. A village has a population of 1,500 people. If there are 28% children and 32% men, how many women are there in the village?
4. A newspaper vendor has 500 newspapers to sell. He sold 25% of them in the morning and 18% in the afternoon. How many newspapers remain unsold?

Lesson Title: Express one quantity as a percentage of another	Theme: Number and numeration
Practice Activity: PHM1-L008	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to express one quantity as a percentage of another (including word problems).

Overview

To express one quantity as a percentage of another, make sure both are in the same unit. You may see problems with different units, for example kilometres and metres. Always convert the bigger units to the smaller one to avoid complications. If you have a problem with both kilometres and metres, convert the kilometres to metres before solving.

To express a quantity as a percentage of another (both in the same unit) write the given quantity as a fraction of the total. Multiply by 100% and simplify.

Solved Examples

1. In a bag containing 250 mangoes, 30 got rotten. What percentage of the mangoes got rotten?
   Solution:

$$\begin{aligned} \text{Percentage of rotten mangoes} &= \frac{\text{number of rotten mangoes}}{\text{total number of mangoes}} \times 100\% \\ &= \frac{30}{250} \times \frac{100}{1}\% \\ &= \frac{3}{25} \times \frac{100}{1}\% \\ &= \frac{3}{1} \times \frac{4}{1}\% \\ &= 12\% \end{aligned}$$

2. Express 30 g as a percentage of 1 kg.
   Solution:
   Step 1. Use the fact that 1 kg = 1000 g. We can convert 1 kg to 1,000 grammes without doing any calculation.
   Step 2. Write the given quantity (30 g.) as a fraction of the total (1,000 g.): $\frac{30}{1,000}$
   Step 3. Multiply the fraction by 100%: $\frac{30}{1,000} \times 100\% = 3\%$

3. What percentage of Le 72,000.00 is Le 1,800.00?
   Solution:
   Calculate : Le 1,800 as a percentage of Le 72,000:

$$\frac{1,800}{72,000} \times 100\% = \frac{18}{720} \times 100\% = \frac{180}{72} = \frac{5}{2} = 2.5\%$$

4. In a class of 50 pupils, 35 are girls. Find the percentage of:

   a. Girls in the class
   b. Boys in the class

Solution:

a.

$$\text{Percentage of girls} = \frac{number\ of\ girls\ in\ class}{number\ of\ pupils\ in\ class} \times 100$$
$$= \frac{35}{50} \times 100$$
$$= 70\%$$

b.

$$\text{Number of boys} = number\ of\ pupils\ - number\ of\ girls$$
$$= 50 - 35$$
$$= 15\ boys$$
$$\text{Percentage of boys} = \frac{number\ of\ boys\ in\ class}{number\ of\ pupils\ in\ class} \times 100$$
$$= \frac{15}{50} \times 100$$
$$= 30\%$$

Practice

1. Express $Le\ 100.00$ as a percentage of $Le\ 1{,}000.00$.
2. Express 400 g as a percentage of 2 kg.
3. During a Mathematics test lasting 1 hour a student took 9 minutes to answer one question. What percentage of the test time was used to answer the question?
4. Koroma had 300 mangoes and sold 240 of them.
   a. What percentage of the mangoes did he sell?
   b. What is the percentage of mangoes left?
5. In a farm there are 100 chickens, 800 goats, 200 sheep and 500 pigs. What percentage of the total number of animals on the farm are?
   a. Chicken
   b. Pigs
   c. Goats

Lesson Title: Percentage change	**Theme:** Number and numeration
Practice Activity: PHM1-L009	**Class:** SSS 1

Learning Outcome
By the end of the lesson, pupils will be able to calculate percentage increase and decrease (including word problems).

Overview

Percentage change is all about comparing old to new values. A change can be either an increase or a decrease. When the new value is greater than the old value, it is a percentage increase. When the new value is less than the old value, it is a decrease.

To find percentage change, we express the change in quantity as a fraction of the original quantity and then multiply by 100.

The formula for calculating percentage change is:

$$Percentage\ change = \frac{change\ in\ quantity}{(old)original\ quantity}(new) \times 100.$$

You can also find the new amount of a quantity after a given percentage change. To calculate the new amount, use the formula:

For an **increase:** $new\ quantity = \frac{100+percentage\ increase}{100} \times original\ quantity$

For a **decrease:** $new\ quantity = \frac{100-percentage\ decrease}{100} \times original\ quantity$

You can also find the new quantity if you are given the original quantity and the change in quantity. To find the new quantity after an **increase**, **add** the original quantity to change in quantity. To find the new quantity after a **decrease**, **subtract** the change in quantity from the original quantity.

Solved Examples

1. The cost of petrol increased from $Le4{,}500.00$ to $Le6{,}300.00$ per litre. Calculate the percentage increase.
   Solution:
   Step 1. Calculate the change in quantity: $6{,}300 - 4{,}500 = Le\ 1{,}800.00$
   Step 2. Calculate percentage increase using the formula:
   Percentage increase = $\frac{1{,}800}{4500} \times 100 = 40\%$

2. A new health centre was built in a particular town and the number of babies dying per month decreased from 20 to 8. Calculate the percentage decrease.
   Solution:
   Step 1. Calculate the change in quantity: $20 - 8 = 12$

Step 2. Calculate percentage increase using the formula:

Percentage decrease = $\frac{12}{20} \times 100 = 60\%$

3. Increase a length of 80 cm by 30%.

   Solution:

   $$\text{The new length} = \frac{100+30}{100} \times 80 \text{ cm}$$
   $$= \frac{130}{100} \times 80 \text{ cm}$$
   $$= 104 \text{ cm}$$

4. The cost of petrol was $Le\ 4{,}500.00$ per litre. The cost increases by 40% per litre. Find:
   a. The increase in cost per litre.
   b. The new price.

   Solutions:

   a.

   $$\text{Increase in cost} = 40\% \text{ of } Le\ 4{,}500$$
   $$= \frac{40}{100} \times Le\ 4{,}500$$
   $$= 40 \times Le\ 45$$
   $$= Le\ 1{,}800.00$$

   b.

   $$\text{The new price} = \frac{100+40}{100} \times\ Le\ 4{,}500$$
   $$= \frac{140}{100} \times Le\ 4{,}500$$
   $$= 140 \times Le\ 45$$
   $$= Le\ 6{,}300.00$$

   Alternative method: The increase can be added to the original price:

   $$\text{The new price} = \text{original price + increase in price}$$
   $$= Le\ 4{,}500 + Le\ 1{,}800$$
   $$= Le\ 6{,}300\ .00$$

5. A man brought a piece of land for $Le\ 500{,}000.00$. Ten years later, the value of the land had increased by 60%. Calculate the new value of the land.

   Solution:

   $$\text{The new value} = \frac{100+60}{100} \times\ Le\ 500{,}000$$
   $$= \frac{160}{100} \times Le\ 500{,}000$$

$= 160 \times Le\ 5{,}000$
$= Le\ 8{,}000{,}000.00$

6. A track which was 60 m long is decreased by 15%. Calculate the new length of the track.
   Solution:

$$
\begin{aligned}
\text{The new length} &= \frac{100-15}{100} \times 60 \\
&= \frac{85}{100} \times 60 \\
&= \frac{85}{10} \times 6 \\
&= 8.5 \times 6 \\
&= 51 \text{ m}
\end{aligned}
$$

7. A messenger received a salary of $Le\ 68{,}500.00$. He is promoted to a higher grade and his salary increases by 14%. Calculate his new salary.
   Solution:

$$
\begin{aligned}
\text{His new salary} &= \frac{100+14}{100} \times Le\ 68{,}500.00 \\
&= \frac{114}{100} \times Le\ 68{,}500.00 \\
&= 114 \times Le\ 685 \\
&= Le\ 78{,}090.00
\end{aligned}
$$

Practice

1. A seamstress gives a discount of 5% for customers who pay beforehand. Calculate the reduced price of a dress that originally cost $Le\ 70{,}000.00$.
2. A factory increases its annual production of shoes from 4,325 to 4,671. Calculate percentage increase in the number of shoes.
3. An athlete took 10 seconds to sprint 100 m during practice. If in the actual race he reduced his time by 8%, how long did it take him to run the actual race?
4. A 325 acre farm is reduced by 16%. Calculate:
   a. The area removed from the farm.
   b. The new area of the farm.
5. A man gets a 20% pay raise. If his current salary is $Le\ 60{,}000.00$ monthly, find:
   a. His monthly salary increase.
   b. His new monthly salary.

Lesson Title: Real world use of fractions	**Theme:** Number and numeration
Practice Activity: PHM1-L010	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to solve real-life problems using fractions.

Overview

Fractions are applicable in some instances in real life. For example:

- In construction, a carpenter that wants to build a table, might need to have it six feet two and a quarter inches long (6' $2\frac{1}{4}$")
- In cooking, you need to measure things when you follow a recipe. You have half cups, quarter of a teaspoon, and a whole bunch of other measurements.
- In the hospital, the doctor can prescribe a syrup for treatment. For example, you may take half of a tablespoon twice a day.

This lesson is on solving word problems that involve fractions. The following lists of words help to identify which operation to apply:

- Addition: sum, total, more than
- Subtraction: difference, less than, left, remaining
- Multiplication: times, of, total, per, each
- Division: average, share, per, each

These words are not used in every word problem. Read each problem and think carefully about the situation before deciding which operation to use.

Solved Examples

1. One recipe for cake requires $1\frac{2}{3}$ cups of sugar. If I wanted to make four times as much cake for my daughter's party, what could I do to find the amount of sugar I need?
   Solution:
   Here you multiply the fraction by 4. Note the use of the word "times" in the question.

$$\begin{aligned} 1\frac{2}{3} \times 4 &= \frac{5}{3} \times 4 \\ &= \frac{20}{3} \\ &= 6\frac{2}{3} \text{ cups of sugar} \end{aligned}$$

2. You are driving along and notice that your tank is only $\frac{3}{8}$ full. The tank holds 14 gallons of gas. Your car goes $22\frac{1}{2}$ miles for every gallon of gas. How far can you drive before you run out of gas?

Solution:

Step 1. Find how much gas you have:

$\frac{3}{8} \times 14 = \frac{42}{8}$ gallons

This can be left as an improper fraction because it is not the final answer, and we will use it in the next step.

Step 2. Find how far you can go with $\frac{42}{8}$ gallons.

$\frac{42}{8} \times 22\frac{1}{2}$	$=$	$\frac{42}{8} \times \frac{45}{2}$	Multiply (gallons) × (miles/gallon) to get miles
	$=$	$\frac{21}{8} \times \frac{45}{1}$	Cancel 2
	$=$	$\frac{21}{8} \times \frac{45}{1}$	
	$=$	$\frac{945}{8} = 118\frac{1}{8}$	Miles

3. A test was conducted for a class of 90 pupils, of which $\frac{2}{5}$ are boys and $\frac{3}{5}$ are girls. If $\frac{1}{3}$ of the girls failed and $\frac{2}{3}$ of the boys failed how many pupils passed the test?

Solution:

Step 1. Find the number of boys and number of girls in the class.

Number of boys $= \frac{2}{5} \times 90 = 36$

Number of girls $= \frac{3}{5} \times 90 = 54$

Step 2. Find the number of boys that failed and the number of girls that failed. Then, find the sum of these.

Number of boys that failed $= \frac{2}{3} \times 36 = 24$

Number of girls that failed $= \frac{1}{3} \times 54 = 18$

Total failure $= 24 + 18 = 42$ pupils

Step 3. Subtract the number who failed from the total number of pupils to find the number who passed:

Number who passed $= 90 - 42 = 48$ pupils

4. A gardener uses $\frac{8}{15}$ of his land for growing groundnut. He uses $\frac{3}{7}$ of the remainder for growing beans. What fraction of his land is used for growing beans?

Solution:

Step 1. Find the land that remains after growing groundnut. Take the whole land to be 1, and subtract.

Remaining land $= 1 - \frac{8}{15}$

$= \frac{15}{15} - \frac{8}{15} = \frac{7}{15}$

Step 2. Since $\frac{3}{7}$ of the remainder is used for beans, we multiply the two fractions.

Fraction use for growing beans $= \frac{3}{7} of \frac{7}{15}$

$= \frac{3}{7} \times \frac{7}{15}$

$= \frac{3}{15}$

$= \frac{1}{5}$

Answer: $\frac{1}{5}$ of his land is used for growing beans.

Practice

1. During the last 6 days of preparing for the BECE examination, Musu spend $3\frac{2}{3}$ hours each day to study. How many hours did she study in total?
2. A farmer used $\frac{9}{16}$ of his land for growing cassava. He uses $\frac{4}{7}$ of the remainder to grow potatoes. What fraction of his land is used for growing potatoes?
3. In a class of 60 pupils, $\frac{4}{5}$ took a test, of which $\frac{2}{3}$ passed. The rest failed. Find:
    a. The number of pupils that did not take the test.
    b. The number of pupils that failed the test.
4. Mabel brought $10\frac{1}{2}$ yards of material. She used $4\frac{1}{3}$ yards of it to make a dress for herself. She then shared the remainder equally between her two daughters. How much material was given to each girl?

Lesson Title: Real world use of decimals	Theme: Number and numeration
Practice Activity: PHM1-L011	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to solve real life problems using decimals.

Overview

We use decimals in several instances in everyday life. Examples:

- When measuring weights on scales, especially the digital ones. (example: 3.46 kg)
- Prices are sometimes given with Leones and cents. (example: $Le5.50$)
- Report cards, especially when showing an average grade. (example: 58.4%)
- Medical doctors use decimals often. (example: reading the patient's temperature or measuring their weight)

Look for the same key words as in the previous lesson to tell you which operation to use.

Solved Examples

1. If 4 mangoes weigh 1.2 kg, what is the weight of 10 mangoes?
   Solution:
   Step 1. Find the weight of 1 mango by dividing 1.2 kg by 4:

$$\begin{array}{r|l} & 0.3 \\ \hline 4 & 1.2 \\ - & 1.2 \\ \hline & 0 \end{array}$$

   Step 2. Find the weight of 10 mangoes. Multiply the weight of 1 mango by 10:

   $10 \times 0.3 = 3$ kg

2. Bentu's child has a temperature of 38.7°C. Normal body temperature is 37°C. How much should the child's temperature be reduced to be normal?
   Solution:
   Subtract normal body temperature from the child's temperature.
   $38.7 - 37 = 1.7$ →

$$\begin{array}{r} 38.7 \\ -\ 37.0 \\ \hline 1.7 \end{array}$$

3. Five boys weigh 28.9 kg, 20.5 kg, 9.7 kg, 27.6 kg and 23.4 kg. What is their total weight?

   Solution:

   Add all the weights.

   $$\begin{array}{r} 28.9\text{ kg} \\ 20.5\text{ kg} \\ 9.7\text{ kg} \\ 27.6\text{ kg} \\ \underline{+23.4\text{ kg}} \\ 110.1\text{ kg} \end{array}$$

4. Jane had 10 books on her study table, each weighing 2.5 kg. She decided to carry 8 of these books to school. The weight of her school bag is 1.5 kg before adding books. Find the weight she was carrying.

   Solution:

   Find the total weight of the 8 books by multiplying. Add the weight of the school bag to find the total weight.

Weight of books	$= 8 \times 2.5$ kg
	$= 20$ kg
Weight she was carrying	$= 20 + 1.5$ kg
	$= 21.5$ kg

Practice

1. A woman was carrying a basket containing 10 coconuts each weighing 2.1 kg. She added 3 mangoes each weighing 1.3 kg. Find the total weight she was carrying.
2. Margret had a fever and she went to the hospital. At first her temperature was 39.6°C. After receiving treatment her temperature dropped to 37.4°C. By how many degrees has her temperature dropped?
3. Mary checked her weight on a scale, and it was 38.4 kg. After eating breakfast, her weight increased to 40.1 kg. Find how much weight she gained after eating.
4. Princess scored the following grades in the last examination: Science 52.4, English 64.5, Mathematics 84.5 and Biology 61.4. Find her average grade.

Lesson Title: Approximation of whole numbers	**Theme:** Numbers and Numeration
Practice Activity: PHM1-L012	**Class:** SSS 1

Learning Outcome

By the end of the lesson, you will be able to round numbers to tens, hundreds, thousands, millions, billions and trillions.

Overview

Rounding makes numbers easier to work with. It is important to have a good understanding of place value, because the place of the digit is used in rounding. The place value of a digit gives its position in a number. For example, in the number 7,659: 7 is in the thousands place, 6 is in the hundreds place, 5 is in the tens place, and 9 is in the ones place.

To round a whole number, look at the digit to the right of the place you want to round to. For example, if you are rounding to the nearest ten you will look at the value of the digit in the ones place.

If the digit to the right is between 0 and 4, round down. Leave the digit in the place you are rounding to as it is, and change all digits to the right of it to zeros.

If the digit to the right is between 5 and 9, round up. Add 1 to the digit in the place value you are rounding to, and change all digits to the right of it to zeros.

The following key words tell you to round: approximate, correct to, round off.

The following key words tell you that a number has been rounded: roughly, approximately, close to.

Solved Examples

1. Approximate 7,582 to:
    a. The nearest thousand
    b. The nearest hundred
    c. The nearest ten

    Solutions:

    a. Round up because the next digit (5) is in the range 5 to 9.
    7,582 = 8,000 to the nearest thousand
    b. Round up because the next digit (8) is in the range 5 to 9.
    7,582 = 7,600 to the nearest hundred
    c. Round down because the next digit (2) is in the range 0-4.
    7,582 = 7,580 to the nearest ten

2. Round 7,852,785 to the nearest million.
Solution:
Seven is the digit in millions place value. Eight is the digit immediately after 7, and it is in the range 5 to 9. Therefore, add 1 to 7 in the millions position, and replace all other digits after it with zeros.

7,852,785 = 8,000,000 to the nearest million

3. Approximate 1,036,478,244 to the nearest billion
Solution:
One is the digit in the billions place value. Zero is the digit immediately after 1, and it's in the range 0 to 4. Therefore, leave 1 as it is in the billions position and replace all other digits after it with zeros.

1,036,478,244 = 1,000,000,000 to the nearest billion

4. Complete the table by rounding the numbers in the first column to each of the given place values.

Number	To the nearest ten	To the nearest hundred	To the nearest thousand	To the nearest million	To the nearest billion	To the nearest trillion
67		X	X	X	X	X
416			X	X	X	X
6,785				X	X	X
7,458,262					X	X
6,115,279,081						X
8,526,930,074,784						

Solution:

Number	To the nearest ten	To the nearest hundred	To the nearest thousand	To the nearest million	To the nearest billion	To the nearest trillion
67	70	X	X	X	X	X
416	420	400	X	X	X	X
6,785	6,790	6,800	7,000	X	X	X
7,458,262	7,458,260	7,458,300	7,458,000	7,000,000	X	X
6,115,279,081	6,115,279,080	6,115,279,100	6,115,279,000	6,115,000,000	6,000,000,000	X
8,526,930,074,784	8,526,930,074,780	8,526,930,074,800	8,526,930,075,000	8,526,930,000,000	8,527,000,000,000	9,000,000000,000

Practice

1. Approximate 8,752,587 to:
    a. The nearest ten
    b. The nearest hundred
    c. The nearest thousand
    d. The nearest ten thousand
    e. The nearest hundred thousand
    f. The nearest million
2. Approximate 2,576,310,442 to the nearest hundred million.
3. Approximate 6,467,345,279 to the nearest billion.
4. Complete the table by rounding the numbers in the first column to each of the given place values.

Number	To the nearest ten	To the nearest hundred	To the nearest thousand	To the nearest million	To the nearest billion	To the nearest trillion
98		X	X	X	X	X
568			X	X	X	X
1,115				X	X	X
3,756,235					X	X
9,567,815,395						X
2,886,711,231,121						

Lesson Title: Approximation in everyday life	**Theme:** Number and numeration
Practice Activity: PHM1-L013	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to round numbers in everyday life.

Overview

In certain areas in life, we may find it difficult to give the exact number value, therefore we usually give a rough estimate that brings us close to the actual value. For example:

- You may find it helpful to know roughly the answer to a sum.
- You may need to know roughly the population of a country.
- You may need to know roughly the distance from Freetown to Kono.

Rounding can be used in everyday situations to perform calculations more easily. Often, calculations on large numbers are easier if the numbers are rounded first. Remember that if you round numbers, the answer will not be precise. It will be an estimate.

Solved Examples

1. The population of a certain country is given as 7,649,300. Round this figure to the nearest 100 thousand.
   Solution:
   $7{,}649{,}300 = 7{,}600{,}000$ to the nearest 100 thousand.

2. You want to buy five watches that cost Le 203,500.00 each. When you go to buy them the cost is Le 1,217,500.00. Is that right?
   Solution:
   Step 1. Round 203,500 to a whole number that is easy to multiply. Round it to the nearest 100 thousand: 200,000.

   Step 2. Multiply this rounded figure by 5 to get the total approximate cost of the five watches $200{,}000 \times 5 = Le\ 1{,}000{,}000.00$

   We can see that the cost in the shop is higher than the actual cost of the watches. The amount charged is not right.

3. Mr. Kabba wants to cover the floor of his rectangular room. If the measurements are 4.1 meters by 2.75 meters, estimate the area of the room.
   Solution:
   Step 1. Round the measurements given to the nearest whole number.

4.1 metres → 4 metres
2.75 metres → 3 metres.

Step 2. Use the formula for the area of a rectangle: $Area = length \times width$

$$\begin{aligned}\text{Area of the room} &= length \times width \\ &= 4\text{ m} \times 3\text{ m} \\ &= 12\text{ m}^2\end{aligned}$$

4. Jattu is a petty trader. She keeps her profits on a daily basis. On Monday, she kept Le 55,500.00, Tuesday Le 30,150.00, and Wednesday Le 42,200.00. Estimate the sum of her profit to the nearest thousand.
   Solution:
   Step 1.Round the numbers to the nearest thousand.
   Le 55,500.00 → Le 56,000.00
   Le 30,150.00 → Le 30,000.00
   Le 42,200.00 → Le 42,000.00

   Step 2. Add the rounded numbers:
   Le 56,000.00
   Le 30,000.00
   + Le 42,000.00
   Le 128,000.00

   Jattu's profit can be estimated to Le 128,000.00 during the 3 days.

5. Mabinty wanted to buy 6 school bags for her children. Each bag costs Le 104,300.00. The shop owner asked her to pay Le 585,000. Was she asked to pay more or less than the actual cost?
   Solution:
   Step 1. Round the cost of each bag to the nearest hundred thousand.
   Le 104,300.00 → Le 100,000.00

   Step 2. Multiply to estimate the cost of 6 bags:

$$\begin{aligned}\text{Cost of the 6 bags} &= 6 \times \text{Le } 100{,}000.00 \\ &= \text{Le } 600{,}000.00.00\end{aligned}$$

   She was asked to pay less.

Practice

1. In an election, a candidate got 8,657,482 votes. Round his vote to the nearest hundred thousand.

2. Margret collected these amounts on the following days from her debtors. Monday Le 3,247,500.00, Tuesday Le 4,862,900.00, Wednesday Le 2,182,625.00 and on Thursday Le 1,872,125.00. Find the total money collected to the nearest hundred thousand.
3. Jane, Tom and Michael are partners in a business. At the end of the year, their total profit was Le 9,632,085.00. If they are to share this amount equally, find the share of each person to the nearest hundred thousand
4. Mr. Sandy has a rectangular garden at the back of his house. It measures 5.47 m by 2.681 m. He wants to fence round this garden. Estimate the distance round the garden.

Lesson Title: Conversion from any other base to base ten	Theme: Numbers and Numeration
Practice Activity: PHM1-L014	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to convert from any other base to base 10.

Overview

To understand conversion from any other base to base 10, it is helpful to recall how we write numbers in expanded form in base 10. For example:

$$2{,}134 = (2 \times 1000) + (1 \times 100) + (3 \times 10) + (4 \times 1)$$

This is the same as:

$$2{,}134 = (2 \times 10^3) + (1 \times 10^2) + (3 \times 10^1) + (4 \times 10^0)$$

From the expression, you can see that each digit in the number is multiplied by its place value (multiples of 10). This is because our counting numbers (such as 2,134) are in base 10.

Numbers can, however, be written in other bases apart from base 10. For example: base two, base three, and so on. To indicate bases other than 10, a subscript is added at the bottom right of the number. Example: 1101_{two}, 213_4, 7861_{nine}, 415_6.

The highest possible digit in any number is one less than the base. For example:

- The digits in base two numbers are: 0, 1 (no 2 or higher digit)
- The digits in base three numbers are: 0, 1, 2 (no 3 or higher digit)
- The digits in base nine numbers are: 0, 1, 2, 3, 4, 5, 6, 7, 8 (no nine)

A number in any base can be expanded according to the place value of each digit in it. To convert from any other base to base ten, first expand each digit. Each digit of the number must be converted using powers of the base you are converting from. The ones digit is multiplied by the base number to the power of 0. The powers on the base increase as you move to the left.

For example, the number 1011_2 can be written:

$$1011_2 = (1 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (1 \times 2^0)$$

This is its expanded form. The expanded form can be simplified to give the conversion of 1011_2 to base 10.

When converting a decimal number, digits to the right of the decimal point are assigned negative powers. For example:

$$10.11_2 = (1 \times 2^1) + (0 \times 2^0) + (1 \times 2^{-1}) + (1 \times 2^{-2})$$

Solved Examples

1. Convert 1101_{two} to base ten.
   Solution:
   Step 1. Label each digit of 1101_{two} with the power we will apply to the base:

Power	3	2	1	0
Digit	1	1	0	1

   Step 2. Expand and simplify:

$$\begin{aligned} 1101_{two} &= (1\times 2^3)+(1\times 2^2)+(0\times 2^1)+(1\times 2^0) \\ &= (1\times 8)+(1\times 4)+(0\times 2)+(1\times 1) \\ &= 8+4+0+1 \\ 1101_{two} &= 13 \end{aligned}$$

2. Convert 11.001_{two} to base ten.
   Solution:

Power	1	0	-1	-2	-3
Digit	1	1.	0	0	1

$$\begin{aligned} 11.001_{two} &= (1\times 2^1)+(1\times 2^0)+(0\times 2^{-1})+(0\times 2^{-2})+(1\times 2^{-3}) \\ &= (1\times 2)+(1\times 1)+\left(0\times\frac{1}{2}\right)+\left(0\times\frac{1}{2^2}\right)+\left(1\times\frac{1}{2^3}\right) \\ &= (1\times 2)+(1\times 1)+\left(0\times\frac{1}{2}\right)+\left(0\times\frac{1}{4}\right)+\left(1\times\frac{1}{8}\right) \\ &= 2+1+0+0+\frac{1}{8} \\ &= 3\frac{1}{8} \end{aligned}$$

3. Convert 3214_{six} to base ten.
   Solution:

Power	3	2	1	0
Digit	3	2	1	4

$$\begin{aligned} 3214_{six} &= (3\times 6^3)+(2\times 6^2)+(1\times 6^1)+(4\times 6^0) \\ &= (3\times 216)+(2\times 36)+(1\times 6)+(4\times 1) \\ &= 648+72+6+4 \\ &= 730 \end{aligned}$$

4. Convert 21.40_{five} to base ten.

Solution:

Power	1	0	-1	-2
Digit	2	1	4	0

$$
\begin{aligned}
21.40_{five} &= (2 \times 5^1) + (1 \times 5^0) + (4 \times 5^{-1}) + (0 \times 5^{-2}) \\
&= (2 \times 5) + (1 \times 1) + \left(4 \times \frac{1}{5}\right) + \left(0 \times \frac{1}{5^2}\right) \\
&= 10 + 1 + \frac{4}{5} + \left(0 \times \frac{1}{25}\right) \\
&= 10 + 1 + \frac{4}{5} + 0 \\
&= 11\frac{4}{5}
\end{aligned}
$$

5. Convert 1287_{nine} to base ten.

Solution:

Power	3	2	1	0
Digit	1	2	8	7

$$
\begin{aligned}
1287_{nine} &= (1 \times 9^3) + (2 \times 9^2) + (8 \times 9^1) + (7 \times 9^0) \\
&= (1 \times 729) + (2 \times 81) + (8 \times 9) + (7 \times 1) \\
&= 729 + 162 + 72 + 7 \\
&= 970
\end{aligned}
$$

Practice

1. Covert 111101_{two} to base ten.
2. Convert 111.101_{two} to base ten.
3. Convert 5412_{six} to base ten.
4. Convert 43.212_{five} to base ten.
5. Convert 7821_{nine} to base ten.

Lesson Title: Conversion from base ten to any other bases	**Theme:** Numbers and numeration
Practice Activity: PHM1-L015	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to convert numbers from base ten to any other base.

Overview

To get you started on this lesson, recall how to convert numbers in the other bases to base 10. For example, the conversion of 132_{four} to base 10 is:

$$134_{four} = (1 \times 4^2) + (3 \times 4^1) + (4 \times 4^0) = 16 + 12 + 4 = 32$$

This lesson covers the reverse, converting base 10 numbers to other base numbers.

To convert from base ten to any other base, repeatedly divide the base ten number by the base you are converting to. As you divide, follow these steps:

- Write down the remainder at each stage of the division.
- Continue dividing until nothing is left. That is, when it gets to zero.
- The answer is obtained by reading the remainders upwards.

Solved Examples

1. Convert 30_{ten} to base four.
   Solution:

4	30	Divide 30 by 4, write the answer (7 rem 2) on the next line
	7 *rem* 2	Divide 7 by 4, write the answer (1 rem 3) on the next line
	1 *rem* 3	Divide 1 by 4, write the answer (0 rem 1) on the next line
	0 *rem* 1	

$30_{\text{ten}} = 132_{\text{four}}$ Read the remainders upwards, and give it in base 4.

2. Convert 226_{ten} to base eight.
   Solution:

8	226
	28 *rem* 2
	3 *rem* 4
	0 *rem* 3

$$226_{\text{ten}} = 342_{\text{eight}}$$

3. Convert 105_{ten} to a binary number (base two).

Solution:

2	105
	52 *rem* 1
	26 *rem* 0
	13 *rem* 0
	6 *rem* 1
	3 *rem* 0
	1 *rem* 1
	0 *rem* 1

$$105_{\text{ten}} = 1101001_{\text{two}}$$

4. Convert:
   a. 372_{ten} to base eight
   b. 431_{ten} to base three

   Solutions:

a.

8	372
	46 *rem* 4
	5 *rem* 6
	0 *rem* 5

$$372_{\text{ten}} = 564_{\text{eight}}$$

b.

3	431
	143 *rem* 2
	47 *rem* 2
	15 *rem* 2
	5 *rem* 0
	1 *rem* 2
	0 *rem* 1

$$431_{\text{ten}} = 120222_{\text{two}}$$

5. Convert 123_{ten} to base 9.

   Solution:

9	123
	13 *rem* 6
	1 *rem* 4
	0 *rem* 1

$$123_{\text{ten}} = 146_{\text{nine}}$$

Practice

1. Convert 40_{ten} to base three.
2. Convert 622_{ten} to base eight.
3. Convert 501_{ten} to a binary number (base two).
4. Convert :
    a. 273_{ten} to base eight
    b. 134_{ten} to base three
5. Convert:
    a. 92_{ten} to base two
    b. 609_{ten} to base five
    c. 409_{ten} to base four

Lesson Title: Practice conversion between bases	**Theme:** Numbers and Numeration
Practice Activity: PHM1-L016	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to convert from one base to another.

Overview

To convert one base to another base we follow the following steps:

1. Convert the given base to base ten.
2. Convert your answer from base ten to the required base.

Solved Examples

1. Convert 324_{five} to base three.

   Solution:

   Step 1. Convert to base 10:

$$\begin{aligned} 324_{\text{five}} &= (3 \times 5^2) + (2 \times 5^1) + (4 \times 5^0) \\ &= (3 \times 25) + (2 \times 5) + (4 \times 1) \\ &= 75 + 10 + 4 \\ 324_{\text{five}} &= 89_{\text{ten}} \end{aligned}$$

   Step 2. Convert the result to base 3:

3	89
	$29 \; rem \; 2$
	$9 \; rem \; 2$
	$3 \; rem \; 0$
	$1 \; rem \; 0$
	$0 \; rem \; 1$

$$89_{\text{ten}} = 10{,}022_{\text{three}}$$

   Therefore, $324_{\text{five}} = 10{,}022_{\text{three}}$

2. Convert 402_{eight} to base six.

Solution:

Convert to base 10:

$$
\begin{aligned}
402_{\text{eight}} &= (4 \times 8^2) + (0 \times 8^1) + (2 \times 8^0) \\
&= (4 \times 64) + (0 \times 8) + (2 \times 1) \\
&= 256 + 0 + 2 \\
402_{\text{eight}} &= 258_{\text{ten}}
\end{aligned}
$$

Convert 258 to base 6:

6	258
	$43\ rem\ 0$
	$7\ rem\ 1$
	$1\ rem\ 1$
	$0\ rem\ 1$

$258_{\text{ten}} = 1{,}110_{\text{six}}$

Therefore, $402_{\text{eight}} = 1{,}110_{\text{six}}$

3. Convert 1221_{three} to base five.

Solution:

Convert to base 10:

$$
\begin{aligned}
1{,}221_{\text{three}} &= (1 \times 3^3) + (2 \times 3^2) + (2 \times 3^1) + (1 + \\
&= 3^0) \\
&= (1 \times 27) + (2 \times 9) + (2 \times 3) + (1 \times 1) \\
1{,}221_{\text{three}} &= 27 + 18 + 6 + 1 \\
& \quad 52_{\text{ten}}
\end{aligned}
$$

Convert 52 to base 5:

5	52
	$10\ rem\ 2$
	$2\ rem\ 0$
	$0\ rem\ 2$

$52_{\text{ten}} = 202_{\text{five}}$

Therefore, $1{,}221_{\text{three}} = 202_{\text{five}}$

4. Convert $22{,}222_{\text{three}}$ to base six.

Solution:

Convert to base 10:

$$
\begin{aligned}
22{,}222_{\text{three}} &= (2 \times 3^4) + (2 \times 3^3) + (2 \times 3^2) + (2 + 3^1) + (3 \times 3^0) \\
&= (2 \times 81) + (2 \times 27) + (2 \times 9) + (2 \times 3) + (2 \times 1) \\
&= 162 + 54 + 18 + 6 + 2 \\
22{,}222_{\text{three}} &= 242_{\text{ten}}
\end{aligned}
$$

Convert 242 to base 6:

6	242
	$40\ rem\ 2$
	$6\ rem\ 4$
	$1\ rem\ 0$
	$0\ rem\ 1$

$242_{\text{ten}} = 1042_{\text{six}}$

Therefore, $22{,}222_{\text{three}} = 1042_{\text{six}}$

5. Convert 501_{six} to base eight.

Solution:

Convert to base 10:

$$\begin{aligned} 501_{six} &= (5 \times 6^2) + (0 \times 6^1) + (1 \times 6^0) \\ &= (5 \times 36) + (0 \times 6) + (1 \times 1) \\ 501_{six} &= 180 + 0 + 1 \end{aligned}$$

181_{ten}

Convert 181 to base 8:

8	181
	$22\ rem\ 5$
	$2\ rem\ 6$
	$0\ rem\ 2$

$181_{ten} = 265_{eight}$

Therefore, $501_{six} = 265_{eight}$

Practice

1. Convert 423_{five} to base three.
2. Convert 703_{eight} to base six.
3. Convert $2,112_{three}$ to base five.
4. Convert $3,333_{four}$ to base six.
5. Convert $8,786_{nine}$ to base eight.

Lesson Title:	Theme:
Lesson Title: Addition and subtraction of number bases	**Theme:** Numbers and Numeration
Practice Activity: PHM1-L017	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to perform addition and subtraction operations on numbers involving number bases other than base 10 including binary numbers.

Overview

The method we use to add and subtract vertically in base 10 is the same which we use for other bases. However, care must be taken with carrying and borrowing.

For addition, when the sum of a column is equal to or greater than the base you are working in, you need to carry. Divide the sum of the column by the base, write the remainder underneath and carry the quotient. See Solved Examples 1 and 2.

For subtraction, when the bottom number is greater than the top number, you must borrow from the column to the left. When borrowing you must note that the borrowed number is equal to the base you are working in. For example, if you are working in base 4, you will borrow 1 digit from the column on the left, but you will add 4 to the column on the right. See Solved Examples 3 and 4.

Solved Examples

1. Add: $311_{\text{four}} + 213_{\text{four}}$
   Solution:
   Step 1. Write it as a vertical addition problem:

	3	1	1_{four}
+	2	1	3_{four}

   Step 2. Apply vertical addition, following these steps:
   - Add the right column: $1 + 3 = 4$. We have 1 four and no remainder. Write 0 underneath, carry 1 (representing the 4).
   - Add the middle column: $1 + 1 + 1 = 3$, write 3 underneath. Do not carry, because 3 is less than 4.
   - Add the left column: $3 + 2 = 5$, we have 1 four and remainder 1. Write 1 underneath and carry 1 (representing the 4)
   - Bring down the 1 you carried.

1		1	
	3	1	1_{four}
+	2	1	3_{four}
1	1	3	0_{four}

2. Add: $11{,}010_{\text{two}} + 1{,}011_{\text{two}}$

Solution:

Step 1. Write it as a vertical addition problem:

$$\begin{array}{cccccc} & 1 & 1 & 0 & 1 & 0_{\text{two}} \\ + & & 1 & 0 & 1 & 1_{\text{two}} \\ \hline \end{array}$$

Step 2. Apply vertical addition, following these steps:

- Add the right column: $0 + 1 = 1$, write 1 underneath.
- Add the next column: $1 + 1 = 2$, we have 1 two and no remainder, write 0 and carry 1.
- Add the next column: $1 + 0 + 0 = 1$, write 1
- Add the next column: $1 + 1 = 2$, we have 1 two and no remainder, write 0 and carry 1.
- Add the next column: $1 + 1 = 2$, we have 1 two and no remainder, write 0 and carry 1.
- Write 1 underneath.

$$\begin{array}{cccccc} {\scriptstyle 1} & {\scriptstyle 1} & & {\scriptstyle 1} & & \\ & 1 & 1 & 0 & 1 & 0_{\text{two}} \\ + & & 1 & 0 & 1 & 1_{\text{two}} \\ \hline 1 & 0 & 0 & 1 & 0 & 1_{\text{two}} \end{array}$$

3. Subtract: $314_{\text{five}} - 24_{\text{five}}$

Solution:

Step 1. Write it as a vertical subtraction problem:

$$\begin{array}{cccc} & 3 & 1 & 4_{\text{five}} \\ - & & 2 & 4_{\text{five}} \\ \hline \end{array}$$

Step 2. Apply vertical subtraction, following these steps:

- Subtract the right column: $4 - 4 = 0$, write 0 underneath.
- Subtract the middle column: $1 - 2$. We need to borrow 1 from the next column and convert it to base we are working in. We borrow one 5 and add it to the 1. In the middle column we now have $5 + 1 - 2 = 4$. Write 4 underneath.
- Carry the 2 down in the left column.

$$\begin{array}{cccc} & {}^{2}\!\not{3} & {}^{5+}1 & 4_{\text{five}} \\ - & & 2 & 4_{\text{five}} \\ \hline & 2 & 4 & 0_{\text{five}} \end{array}$$

4. Subtract: $11{,}101_{\text{two}} - 1{,}011_{\text{two}}$

Solution:

Step 1. Write as a vertical subtraction problem:

$$\begin{array}{rccccc} & 1 & 1 & 1 & 0 & 1_{\text{two}} \\ - & & 1 & 0 & 1 & 1_{\text{two}} \\ \hline & 1 & 0 & 0 & 1 & 0_{\text{two}} \end{array}$$

Step 2. Apply vertical subtraction, following these steps:

- Subtract the right column: $1 - 1 = 0$, write 0 underneath.
- Subtract the next column. We have $0 - 1$. We need to borrow 1 two from the next column. We now have $2 + 0 - 1 = 1$. Write 1 underneath.
- Subtract the next column. We borrowed 1 two from this column, so we are left with 0. We have $0 - 0 = 0$. Write 0.
- Subtract the next column: $1 - 1 = 0$, write 0.
- Carry the 1 down in the left column.

$$\begin{array}{rccccc} & 1 & 1 & {}^{0}\not{1} & {}^{2+}0 & 1_{\text{two}} \\ - & & 1 & 0 & 1 & 1_{\text{two}} \\ \hline & 1 & 0 & 0 & 1 & 0_{\text{two}} \end{array}$$

5. Evaluate: $103_{\text{four}} + 111_{\text{four}} + 321_{\text{four}}$

Solution:

Apply vertical addition to all 3 numbers at once, applying the same steps as described above:

$$\begin{array}{rccc} {}^{1} & {}^{1} & {}^{1} & \\ & 1 & 0 & 3_{\text{four}} \\ & 1 & 1 & 1_{\text{four}} \\ + & 3 & 2 & 1_{\text{four}} \\ \hline 1 & 2 & 0 & 1_{\text{four}} \end{array}$$

Practice

1. Evaluate:
   a. $1{,}130_{\text{five}} + 311_{\text{five}}$
   b. $110{,}101_{\text{two}} + 100{,}101_{\text{two}}$
2. Evaluate:
   a. $434_{\text{five}} – 42_{\text{five}}$
   b. $11{,}111_{\text{two}} – 1{,}010_{\text{two}}$
3. Evaluate: $556_{\text{seven}} + 415_{\text{seven}} + 225_{\text{seven}}$

Lesson Title: Multiplication of number bases	Theme: Numbers and Numeration
Practice Activity: PHM1-L018	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to perform multiplication of numbers involving number bases other than base 10 including binary numbers.

Overview

To multiply numbers in other bases, we follow a similar process as we do for multiplying numbers in base ten. Multiplying numbers in base two (binary numbers) is straight forward, because it does not involve carrying. Multiplying numbers in other bases requires carrying, so it is more complicated.

Solved Examples

1. $101_2 \times 11_2$

   Solution:

   Step 1. Multiply each digit in the top number (101) by the 1 on the right side of the bottom number.
   Step 2. Multiply each digit in the top number (101) by the 1 on the left side of the bottom number.
   Step 3. Add the result, following the rules for adding numbers in base 2.
   Answer: 1111_2

$$\begin{array}{rcccc} & & 1 & 0 & 1_2 \\ \times & & & 1 & 1_2 \\ \hline & & 1 & 0 & 1 \\ + & 1 & 0 & 1 & \\ \hline & 1 & 1 & 1 & 1_2 \end{array}$$

2. $23_{five} \times 12_{five}$

   Solution:

   Step 1. Multiply each digit in the top number (23) by the 2 on the right side of the bottom number:
   - $2 \times 3 = 6$, we have 1 five and remainder 1. Write 1 (the remainder) underneath and carry 1 five to the next column.
   - $2 \times 2 + 1 = 5$, we have 1 five and no remainder. Write 0 and carry 1.
   - Carry the 1 down.

$$\begin{array}{rccc} & {}^{1} & {}^{1} & \\ & & 2 & 3_{five} \\ & \times & 1 & 2_{five} \\ \hline & 1 & 0 & 1 \\ + & 2 & 3 & \\ \hline & 3 & 3 & 1_{five} \end{array}$$

   Step 2. Multiply each digit in the top number (23) by the 1 on the left side of the bottom number:
   - $3 \times 1 = 3$, write 3.
   - $2 \times 1 = 2$, write 2.

   Step 3. Add the numbers in base 5, as in the previous lesson.
   Answer: 331_{five}

3. $421_{six} \times 35_{six}$

Solution:

Step 1. Multiply each digit in the top (421) by the 5 on the right side of the bottom number

- $5 \times 1 = 5$, write 5.
- $5 \times 2 = 10$, we have 1 six and remainder 4, write 4 and carry 1 six to the next column.
- $5 \times 4 + 1 = 21$, we have 3 six and remainder 3, write 3 (remainder) and carry 3 six to the next column
- Write down the 3 that you carried.

		$^2 3$	1		
			4	2	1_{six}
		×		3	5_{six}
		3	3	4	5
+	2	1	0	3	
	2	4	4	1	5_{six}

Step 2. Multiply each digit in the top (421) by the 3 on the left of the bottom number.

- $3 \times 1 = 3$, write 3
- $3 \times 2 = 6$, we have 1 six and no remainder, write 0 and carry the 1 six to the next column.
- $3 \times 4 + 1 = 13$, we have 2 six and 1 remainder, write 1 and carry 2 six to the next column.
- Write down the 2.

Step 3. Add the numbers in base 6, as we did in the previous lesson.

Answer: 24415_{six}

4. If $231_{four} \times 10101_{two} = M_{six}$, find the value of M.

Solution:

When multiplying numbers in different bases, follow these steps:

Step 1. Convert the numbers to base ten.

$$\begin{aligned} 231_{four} &= (2 \times 4^2) + (3 \times 4^1) + (1 \times 4^0) \\ &= (2 \times 16) + (3 \times 4) + (1 \times 1) \\ &= 32 + 12 + 1 \\ &= 45_{ten} \\ 10101_{two} &= (1 \times 2^4) + (0 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0) \\ &= (1 \times 16) + (0 \times 8) + (1 \times 4) + (0 \times 2) + (1 \times 1) \\ &= 16 + 4 + 1 \\ &= 21_{ten} \end{aligned}$$

Step 2. Multiply your answer in base 10.

$$45 \times 21 = 945$$

Step 3. Convert your answer in base 10 to the required base, 6.

6	945
	$157\ rem\ 3$
	$26\ rem\ 1$
	$4\ rem\ 2$
	$0\ rem\ 4$

$$945_{\text{ten}} = 4213_{\text{six}}$$

Answer: $M = 4213$

5. Multiply 204_{six} by 122_{three} and give your answer in base nine.

Solution:

Step 1. Convert the numbers to base ten

$$\begin{aligned} 204_{\text{six}} &= (2 \times 6^2) + (0 \times 6^1) + (4 \times 6^0) \\ &= (2 \times 36) + (4 \times 1) \\ &= 72 + 4 \\ &= 76_{\text{ten}} \\ 112_{\text{three}} &= (1 \times 3^2) + (1 \times 3^1) + (2 \times 3^0) \\ &= (1 \times 9) + (1 \times 3) + (2 \times 1) \\ &= 9 + 3 + 2 \\ &= 14_{\text{ten}} \end{aligned}$$

Step 2. Multiply your answer in base ten.

$$76 \times 14 = 1{,}064$$

Step 3. Convert your answer in base 10 to the required base, base 9. →

Answer: 1412_{nine}

9	1,064
	$118\ rem\ 2$
	$13\ rem\ 1$
	$1\ rem\ 4$
	$0\ rem\ 1$

$$1{,}064_{\text{ten}} = 1{,}412_{\text{nine}}$$

Practice

1. $24_{\text{six}} \times 15_{\text{six}}$
2. $232_{\text{five}} \times 14_{\text{five}}$
3. $1101_{\text{two}} \times 111_{\text{two}}$
4. If $1022_{\text{four}} \times 15_{\text{six}} = P_{\text{five}}$, find the value of P.
5. Multiply 10110_{two} by 121_{three} and give your answer in base eight.
6. If $154_{\text{six}} \times 113_{\text{four}} = M_{\text{seven}}$, find the value of M.

Lesson Title: Division of number bases	**Theme:** Numbers and Numeration
Practice Activity: PHM1-L019	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to perform division of numbers involving number bases other than base 10, including binary numbers.

Overview

To divide number bases other than base ten, follow these steps:

Step 1. Convert the numbers to base ten.
Step 2. Do the division in base ten.
Step 3. Convert your final answer to the required base.

We can also divide numbers in different bases just like we did in multiplication. Simply follow the same steps stated above.

Solved Examples

1. $101{,}010_{\text{two}} \div 111_{\text{two}}$

Solution:

Step 1. Convert both numbers to base 10:

$$\begin{aligned} 101{,}010_{\text{two}} &= (1 + 2^5) + (0 \times 2^4) + (1 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (0 \times 2^0) \\ &= (1 \times 32) + (0 \times 16) + (1 \times 8) + (0 \times 4) + (1 \times 2) + (0 \times 1) \\ &= 32 + 8 + 2 \\ &= 42_{\text{ten}} = 42 \\ 111_{\text{two}} &= (1 \times 2^2) + (1 \times 2^1) + (1 \times 2^0) \\ &= 4 + 2 + 1 \\ &= 7_{\text{ten}} = 7 \end{aligned}$$

Step 2. Divide the numbers in base 10: $42 \div 7 = 6$

Step 3. Convert the result to base 2: $6_{\text{ten}} = 110_{\text{two}}$ →

2	6
	$3\ rem\ 0$
	$1\ rem\ 1$
	$0\ rem\ 1$

Step 4. Write the answer: $101{,}010_{\text{two}} \div 111_{\text{two}} = 110_{\text{two}}$

2. If $202_3 \div 101_2 = \text{Q}_2$, find the value of Q.

Solution:

Step 1. Convert the numbers to base ten.

$$\begin{aligned} 202_3 &= (2 \times 3^2) + (0 \times 3^1) + (2 \times 3^0) \\ &= (2 \times 9) + (0 \times 3) + (2 \times 1) \end{aligned}$$

$$= 18 + 2$$
$$= 20_{10} = 20$$
$$101_2 = (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0)$$
$$= 4 + 1$$
$$= 5_{10} = 5$$

Step 2. Divide the numbers in base 10:

$20 \div 5 = 4$

Step 3. Convert to base two: $4_{10} = 100_2$ →

Step 4. Write the answer: $Q = 100$

2	4
	$2\ rem\ 0$
	$1\ rem\ 0$
	$0\ rem\ 1$

3. Solve $2173_{\text{nine}} \div 101_{\text{two}}$ and give your answer in base seven.

Solution:

Step 1. Convert the numbers to base ten:

$$2173_{\text{nine}} = (2 \times 9^3) + (1 \times 9^2) + (7 \times 9^1) + (3 \times 9^0)$$
$$= (2 \times 729) + (1 \times 81) + (7 \times 9) + (3 \times 1)$$
$$= 1458 + 81 + 63 + 3$$
$$= 1605_{\text{ten}}$$
$$101_{\text{two}} = (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0)$$
$$= (1 \times 4) + (0 \times 2) + (1 \times 1)$$
$$= 4 + 1$$
$$= 5_{\text{ten}}$$

Step 2. Divide the numbers in base 10:

$1{,}605 \div 5 = 321$

Step 3. Convert your answer to base seven. →

$321 = 636_{\text{seven}}$

7	321
	$45\ rem\ 6$
	$6\ rem\ 3$
	$0\ rem\ 6$

Step 4. Write the answer:

$$2{,}173_{\text{nine}} \div 101_{\text{two}} = 636_{\text{seven}}$$

4. Divide 1310_{four} by 1002_{three} and give your answer in base two.

Solution:

Step 1. Convert to base ten.

$$1{,}310_{\text{four}} = (1 \times 4^3) + (3 \times 4^2) + (1 \times 4^1) + (0 \times 4^0)$$
$$= (1 \times 64) + (3 \times 16) + (1 \times 4) + (0 \times 1)$$
$$= 64 + 48 + 4$$
$$= 116_{\text{ten}}$$
$$1{,}002_{\text{three}} = (1 \times 3^3) + (0 \times 3^2) + (0 \times 3^1) + (2 \times 3^0)$$
$$= (1 \times 27) + (0 \times 9) + (0 \times 3) + (2 \times 1)$$
$$= 27 + 2$$
$$= 29_{\text{ten}}$$

Step 2. Divide the numbers in base 10:

116 ÷ 29 = 4

Step 3. Convert 4 to base two. →

2	4
	$2\ rem\ 0$
	$1\ rem\ 0$
	$0\ rem\ 1$

$4 = 100_2$

Step 4. Write the answer:

$1{,}310_{\text{four}} \div 1{,}002_{\text{three}} = 100_{\text{two}}$

Practice

1. $240_{\text{five}} \div 20_{\text{five}}$
2. $356_{\text{eight}} \div 7_{\text{eight}}$
3. $100011_{\text{two}} \div 101_{\text{two}}$
4. Evaluate $2{,}115_{\text{seven}} \div 12_{\text{seven}}$
5. Divide $2{,}321_{\text{four}}$ by $1{,}101_{\text{three}}$ and give your answer in base two.

Lesson Title: Basic equations involving number bases	**Theme:** Numbers and Numeration
Practice Activity: PHM1-L020	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to solve basic equations involving number bases.

Overview

This lesson focuses on solving equations involving number bases where a variable is in the base. For example, $25_x = 17_{10}$.

When you are given equations involving number bases to solve, change the base on both sides of the equation to base ten. Then you will have an equation to solve for x.

In the example problem above, the right-hand side is already in base 10. We can rewrite it as 17. Therefore, we only need to change the left-hand side to base 10.

Solved Examples

1. If $25_x = 17_{10}$, find the value of x.
   Solution:
   Convert 25_x from base x to base ten:
   $$\begin{aligned} 25_x &= (2\times x^1) + (5\times x^0) \\ &= 2x+5 \end{aligned}$$
   Now that both sides of the equation are in base 10, we can set them equal to each other and solve for x:
   $$\begin{aligned} 2x+5 &= 17 \\ 2x &= 17-5 && \text{Transpose 5} \\ 2x &= 12 \\ x &= \tfrac{12}{2} && \text{Divide throughout by 2} \\ x &= 6 \end{aligned}$$

2. If $34_x = 10011_2$, find the value of x.
   Solution:
   In this problem, neither side of the equation is in base 10. We must convert both sides to base 10, then set them equal.
   Convert the left hand side from base x to base 10:
   $$\begin{aligned} 34_x &= (3\times x^1) + (4\times x^0) \\ &= 3x+4 \end{aligned}$$
   Convert the right hand side from base 2 to base 10:
   $$10011_2 = (1\times 2^4) + (0\times 2^3) + (0\times 2^2) + (1\times 2^1) + (1\times 2^0)$$

$$= \quad 16 + 0 + 0 + 2 + 1$$
$$= \quad 19$$

Set the two sides equal and solve for x:

$$3x + 4 = 19$$
$$3x = 19 - 4 \qquad \text{Transpose 4}$$
$$3x = 15$$
$$x = \frac{15}{3} \qquad \text{Divide throughout by 3}$$
$$x = 5$$

3. Find the value of y in the number base $1011_{\text{two}} + 27_y = 28_{\text{ten}}$.
   Solution:
   Convert the left-hand side to base ten:

$$1011_2 = (1 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (1 \times 2^0)$$
$$= 8 + 0 + 2 + 1$$
$$= 11$$
$$27_y = (2 \times y^1) + (7 \times y^0)$$
$$= 2y + 7$$

   Equate the numbers in base 10 and solve for y:

$$11 + 2y + 7 = 28$$
$$2y + 18 = 28 \qquad \text{Collect like terms}$$
$$2y = 28 - 18 \qquad \text{Transpose 18}$$
$$2y = 10$$
$$y = \frac{10}{2} \qquad \text{Divide by 2}$$
$$y = 5$$

Practice

1. Find the value of n in $28_n = 102_4$
2. If $47_x = 39_{\text{ten}}$, find the value of x
3. If $12_y = 102$, find the value of y
4. If $50_6 + 21_x = 63_{10}$, find the value of x
5. If $102_{five} + 36_m = 54_{ten}$, find the value of m

Lesson Title: Introduction to modular arithmetic	**Theme:** Numbers and Numeration
Practice Activity: PHM1-L021	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to describe and interpret cyclical events.

Overview

Considering the digits that make up different number bases, you have;

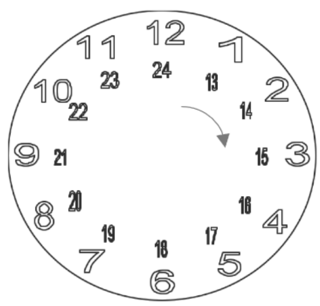
For a base 10 number, the digits are 0,1, 2, 3, 4, 5, 6, 7, 8, 9.
For a base 8 number, the digits are 0,1, 2, 3, 4, 5, 6, 7.
For a base 2 number, the digits are 0,1.

From the above, you can deduce that for every base, the digits do not have the base as a digit.

Recall the process for carrying numbers in addition of number bases. When you get a number equal to or greater than the base you divide the number by the base, write down the remainder and carry the quotient.

This lesson looks at how to describe and interpret cyclic events. Carrying in number bases is like dealing with numbers that go a full cycle or more.

Consider a clock with digits up to 24 drawn inside the digits 1-12, as shown.



The numbers go from 1-12, but when you get to 13, it becomes 1 o' clock again. So 13 is 1 o' clock, 14 is 2 o 'clock, 15 is 3 o' clock and so on. This can continue until you get to 25 which will actually be 1 o' clock again (also the same as 13 o' clock).

Modular arithmetic deals with cycles of a certain number.

When you go around the clock the second time you can divide each digit (13, 14, 15, ...) by 12, and you will get 1 remainder something. On the second round (25, 26, …) if you divide by 12, you get 2 remainder something.

Generally, conversion of each digit to the form $\frac{A}{B} = Q$ remainder R is interpreted as "A divided by B is Q remainder R". The idea of the remainder is the key in interpreting cyclic events and is going to be your main tool through this lesson.

Solved Examples

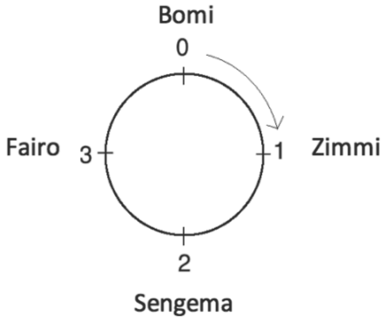
1. Fatu travels to 4 villages to sell her goods in a cycle one after another, as shown in the table below:

Day number	Village
0	Bomi
1	Zimmi
2	Sengema
3	Fairo

a. Represent Fatu's rounds on a cyclic diagram.
b. If Fatu's is at Bomi today, in how many days will she be in Sengema?
c. Where will Fatu be in: i. 3 days; ii. 5 days; iii. 11 days; iv. 16 days.

Solutions:

a.



b. Counting forward from day 0 in Bomi, she will be in Sengema in 2 days.
c. Count forward on the cycle, or divide and use the remainders to locate Fatu in a given number of days.
   i. In 3 days, she will be in Fairo. Count forward 3 days on the cycle.
   ii. In 5 days, she will be in Zimmi. Find this by counting forward 5 days on the cycle, or divide and count forward by the remainder. Divide 5 by the number of days in the cycle, which is 4: $5 \div 4 = 1$ remainder 1.
   The remainder is 1, which is Zimmi.
   iii. In 11 days, she will be in Fairo. Count forward 11 spaces, or divide and count forward 3 spaces (the remainder): $11 \div 4 = 2 \text{ remainder } 3$.
   iv. In 16 days, she will be in Bomi: $(16 \div 4) = 4$ remainder 0.

2. In a school, five teachers take turns to conduct devotion every morning for the 5 days of the school week as shown below.

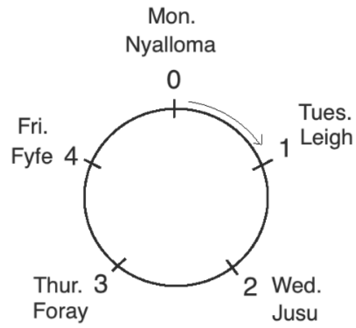
School day	Teacher in charge of devotion
Monday (Mon.)	Mrs. Nyalloma
Tuesday (Tue.)	Mr. Leigh
Wednesday (Wed.)	Mr. Jusu
Thursday (Thur.)	Mrs. Foray
Friday (Fri.)	Mr. Fyfe

a. Represent the roster on a cyclic diagram.
b. If today is Monday, find out:
   i. Who will conduct devotion after 15 days, excluding weekends.
   ii. Who will conduct devotion after 22 working days.
   iii. What day of the week it will be in 17 days, excluding weekends.

iv. What day of the week it will be in 29 days, excluding weekends.

Solutions:

a.



b.

i. $15 \div 5 = 3$ remainder 0. It will be Mrs. Nyalloma's turn.

ii. $22 \div 5 = 4$ remainder 2. It will be Mr. Jusu's turn.

iii. $17 \div 5 = 3$ remainder 2. It will be Wednesday.

iv. $29 \div 5 = 5$ remainder 4. It will be Friday.

Practice

1. In a hospital ward, 4 doctors are scheduled to take turns in making rounds every hour of an eight hour shift from 6:00 pm - 2:00 am as shown in the table below.

Hour number	Doctor
0	Dr. W
1	Dr. X
2	Dr. Y
3	Dr. Z

a. Represent the schedule of the doctors on a cyclic diagram.

b. If hours 0 is 6.00pm:

i. Who should be on duty at 10 pm

ii. Who should be on duty at 1:00 am

c. Using division show who will be making rounds in:

i. 3 hours from 6:00 pm

ii. 5 hours from 6:00 pm

iii. 7 hours from 6:00 pm

2. A ferry leaves the Freetown Ferry terminal every hour for the Tar green terminal. If the first ferry leaves at 8:00 am:

a. For 4 Ferries (F1, F2, F3, and F4) scheduled to run sequentially for 24 hours of the day, draw a departure schedule for them and represent it on a cyclic diagram.

b. Which Ferry should be leaving:

i. After 6 hours

ii. After 12 hours

iii. At 7:00 pm

iv. At midnight

Lesson Title: Simplest form of a given modulo	**Theme:** Numbers and Numeration
Practice Activity: PHM1-L022	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to reduce numbers to their simplest form with a given modulo.

Overview

Recall that for cyclic activity, the digits we use are from 0 to one less than the number of events. For example, if there are 7 events in a cycle, it can be represented by the numbers 0 through 6. That is, we code the events as 0, 1, 2, 3, 4, 5, 6.

You are going to learn in this lesson how to reduce numbers to their simplest form within a given modulo. The concept of cycles applies to the operation that we call modulo and is abbreviate as "mod".

The equation $A \bmod B = R$ can be read as "A modulo B equals R".

Any cyclical event that can be written as a division problem with a remainder can also be written in modulo form. As we saw with cycles, the digits in a modulo start with zero through one less the modulo. For example, the digits in modulo 12 are 0 through 11.

Consider the equation written as a division problem: $14 \div 12 = 1$ remainder 2.
It can be written in modular form: $14 \bmod 12 = 2$.

Here we see that the dividend is written on the left side of "mod" and the divisor is on the right. The remainder after dividing is the answer to the modulo.
If a number is less than the modulo, you leave the number as the answer. For example, $2 \bmod 4 = 2$. This is because 2 is the remainder: $\frac{2}{4} = 0$ remainder 2.

The modulo of a negative number is obtained by adding the modulo to the number continuously until you get a positive number. For example, $-2 \bmod 4 = -2 + 4 = 2$. In some cases, you will need to add the modulo multiple times until you get a positive number.

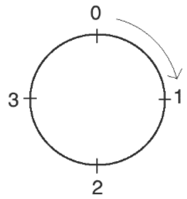
Solved Examples

1. Evaluate:
    a. Write the digits in modulo 4 and represent them on a cyclic diagram.
    b. What is 8 mod 4?

Solutions:

a. The digits in modulo 4 are: 0, 1, 2, 3.

Cyclically the digits can be represented as follows:



b. 8 mod 4 = 0; this can be found in 2 ways:

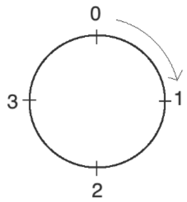
Using the cycle: Count up to 8 in a clockwise direction, and where you land on the 8th count is the answer. It is 0.

Using division: Divide $8 \div 4 = 2 \text{ remainder } 0$. The remainder is 0, which is the answer.

2. Simplify 25 mod 4 using a cycle and using division.

Solution:

Using the cycle:



Count up to 25 in a clockwise direction starting at 0. On the 25th count, you land on 1. This is your answer.

Using division: $\frac{25}{4} = 6$ remainder 1, therefore 25 mod 4 = 1.

3. Evaluate – 5 mod 2.

Solution:

Add 2 to -5 repeatedly until you get a positive number:

$-5 + 2 = -3 \rightarrow -3 + 2 = -1 \rightarrow -1 + 2 = 1$

Answer: – 5 mod 2 = 1

4. Evaluate:

a. 12 mod 5
b. 4 mod 9
c. 9 mod 6
d. 60 mod 9

Solutions:

a. 12 mod 5 → $\frac{12}{5} = 2$ remainder 2

Answer: 12 mod 5 = 2

b. 4 mod 9 = 4 (4 is less than 9, it is the answer.)
c. 9 mod 6 → $\frac{9}{6} = 1$ remainder 3

Answer: 9 mod 6 = 3

d. 60 mod 9 → $\frac{60}{9} = 6$ remainder 6

Answer: 60 mod 9 = 6

5. Evaluate:
   a. – 9 mod 6
   b. –15 mod 4

Solutions:

a. Add 6 until you reach a positive number:
$-9 + 6 = -3 \rightarrow -3 + 6 = 3$

Answer: – 9 mod 6 = 3

b. Add 4 until you reach a positive number:
$-15 + 4 = -11 \rightarrow -11 + 4 = -7 \rightarrow -7 + 4 = -3 \rightarrow -3 + 4 = 1$

Answer: – 15 mod 4 = 1

Practice

1. Write the digits in modulo 5 on a cyclic diagram.
2. Simplify 10 mod 5 using:
   a. A cyclic diagram
   b. Division
3. Evaluate:
   a. 13 mod 6
   b. – 11 mod 2
   c. 54 mod 7
4. Evaluate:
   a. 3 mod 10
   b. 7 mod 12
   c. – 10 mod 7

Lesson Title: Operations in various moduli	**Theme:** Numbers and Numeration
Practice Activity: PHM1-L023	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to add, subtract and multiply numbers in various moduli.

Overview

The following symbols are used in carrying out the following operations in modular arithmetic: $\oplus$ (addition), $\ominus$ (subtraction), $\otimes$ (multiplication).

Consider the example addition problem: $2 \oplus 3 (\text{mod } 4)$**.** This problem says “2 plus 3 in modulo 4”. We apply the operation to the numbers on either side of the symbol. Then, convert the result to modulo 4.

Subtraction and multiplication are carried out in a similar way. For example, $9 \ominus 2 (\text{mod } 5)$ says “9 minus 2 in modulo 5”. The problem $2 \otimes 4 (\text{mod } 4)$ says “2 multiplied by 4 in modulo 3”.

Note also that we can write addition and multiplication tables in given moduli, which can be done by only recording the remainders for every operation. See Solved Example 7 for examples.

Solved Examples

1. Simplify: $2 \oplus 3 \ (\text{mod } 4)$
   Solution:
   Step 1. Add the digits: $2 + 3 = 5$
   Step 2. Convert the result to modulus 4: $5 \div 4 = 1 \text{ remainder } 1$
   The remainder is the answer, in mod 4: $1 \ (\text{mod } 4)$
   Answer: $2 \oplus 3 \ (\text{mod } 4) = 1 \ (\text{mod } 4)$

2. Simplify: $9 \ominus 2 \ (\text{mod } 5)$
   Solution:
   Step 1. Subtract the digits: $9 - 2 = 7$
   Step 2. Convert the result to modulus 5: $7 \div 5 = 1 \text{ remainder } 2$
   The remainder is the answer, in mod 5: $2 \ (\text{mod } 5)$
   Answer: $9 \ominus 2 \ (\text{mod } 5) = 2 (\text{mod } 5)$

3. Simplify: $2 \otimes 4 \ (\text{mod } 3)$
   Solution:
   Step 1. Multiply the digits: $2 \times 4 = 8$

Step 2. Convert the result to modulus 3: $8 \div 3 = 2 \text{ remainder } 2$

The remainder is the answer, in mod 3: $2 \pmod 3$

Answer: $2 \otimes 4 \pmod 3 = 2 \pmod 3$

4. Simplify in the given moduli:
   a. $4 \oplus 5 \pmod 2$
   b. $80 \oplus 15 \pmod 4$

   Solutions:

   a. Add: $4 + 5 = 9$
      Convert to mod 2: $9 \div 2 = 4 \text{ remainder } 1$
      Answer: $4 \oplus 5 \pmod 2 = 1 \pmod 2$
   b. Add: $80 + 5 = 95$
      Convert to mod 4: $95 \div 4 = 23 \text{ remainder } 3$
      Answer: $80 \oplus 15 \pmod 4 = 3 \pmod 4$

5. Simplify the following in the given moduli:
   a. $6 \ominus 3 \pmod 2$
   b. $80 \ominus 15 \pmod 4$

   Solutions:

   a. Subtract: $6 - 3 = 3$
      Convert to mod 2: $3 \div 2 = 1 \text{ remainder } 1$
      Answer: $6 \ominus 3 \pmod 2 = 1 \pmod 2$
   b. Subtract: $80 - 5 = 65$
      Convert to mod 4: $65 \div 4 = 16 \text{ remainder } 1$
      Answer: $80 \ominus 15 \pmod 4 = 1 \pmod 4$

6. Find the value of the operations in the given moduli:
   a. $9 \otimes 13 \pmod 5$
   b. $5 \otimes 8 \pmod 6$

   Solutions:

   a. Multiply: $9 \times 13 = 117$
      Convert to mod 5: $117 \div 5 = 23 \text{ remainder } 2$
      Answer: $9 \otimes 13 \pmod 5 = 2 \pmod 5$
   b. Multiply: $5 \times 8 = 40$
      Convert to mod 6: $40 \div 6 = 6 \text{ remainder } 4$
      Answer: $5 \otimes 8 \pmod 6 = 4 \pmod 6$

7. Complete the following addition and multiplication table in the given modulo:

a.

Modulo 3			
$\oplus$	0	1	2
0			
1			
2			

b.

Modulo 3			
$\otimes$	0	1	2
0			
1			
2			

Solutions:

As with any addition or multiplication table, add or multiply the numbers in each row by the numbers in each column. Convert each result to modulo 3, and write the result (the remainder) in the table.

a.

Modulo 3			
$\oplus$	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

b.

Modulo 3			
$\otimes$	0	1	2
0	0	0	0
1	0	1	2
2	0	2	1

Practice

1. Simplify in the given moduli:
   a. $5 \oplus 6 \pmod 3$
   b. $10 \oplus 11 \pmod 6$
2. Simplify in the given moduli:
   a. $15 \ominus 6 \pmod 5$
   b. $30 \ominus 20 \pmod{12}$
3. Simplify in the given moduli:
   a. $10 \otimes 3 \pmod 9$
   b. $6 \otimes 6 \pmod{10}$
4. Complete the following addition and multiplication tables in the given modulo:

a.

Modulo 9				
$\oplus$	2	4	6	8
2				
4				
6				
8				

b.

Modulo 9				
$\otimes$	2	4	6	8
2				
4				
6				
8				

Lesson Title: Modular arithmetic in real-life situations	**Theme:** Numbers and Numeration
Practice Activity: PHM1-L024	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to apply modular arithmetic to real-life situations.

Overview

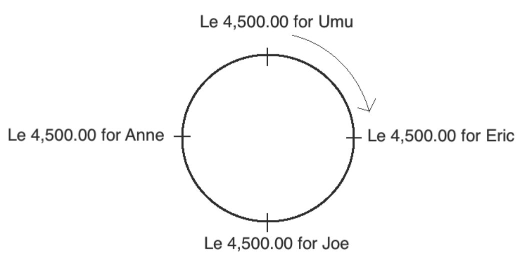
You are going to learn today how to solve real-life problems using modular arithmetic. You have already seen how cycles are applied to real-life situations, such as when people take turns, or a person travels between different places on a schedule. Modular arithmetic is used in similar situations, including those demonstrated in the examples below.

Solved Examples

1. The sum of Le 18,000.00 is to be shared evenly among 4 people, Umu, Eric, Joe and Anne. How much would each person get? How much change would be left?
   Solution:
   Using modulo arithmetic to solve the problem we have:
   $\frac{18,000}{4} = 4,500$ remainder 0, which is $18,000 \text{ mod } 4 = 0$

   From the computation you can now say Le 18,000.00 shared evenly among 4 people will result in each person getting Le 4,500.00 (an even sum). The remainder will be 0 Leones.

   Representing the situation on a cyclic diagram we have:



2. For a practical session, a class of 50 pupils is to be grouped into 6 equal groups such that they work in pairs in every group.
   a. How many pupils will there be per group?
   b. How many pupils will not be grouped?

Solutions:

a. Divide the number of pupils by number of groups desired:

$\frac{50}{6} = 8$ Remainder 2, which is $50 \bmod 6 = 2$

There will be 6 equal groups of 8 people per group.

b. The number of pupils left ungrouped is the result of $50 \bmod 6 = 2$. 2 pupils will be left without a group.

3. If it is Tuesday today, what day will it be 29 days later?

Solution:

Since there are 7 days in the week you are going to work in modulo 7. Tuesday is the third day of the week. We want to go forward 29 days from day 3. You will add 3 to 29 days. You will then divide by 7 to get the remainder, and the remainder will tell you the day of the week 29 days from now.

Add: $3 \oplus 29 \pmod 7 = 32 \bmod 7$

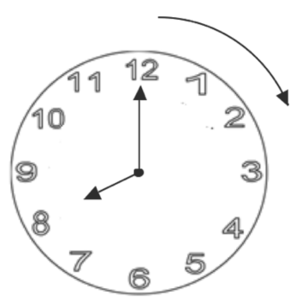
Divide: $\frac{32}{7} = 4$ Remainder 4

The remainder 4 indicates the day that comes 29 days after Tuesday. That day will be day 4 of the week, which is **Wednesday.**

4. On an analogue clock, the hour hand points to 8.
   a. What number did it point to 25 hours ago?
   b. What number will it point to in 20 hours from now?

Solutions:

Consider the clock:



a. To determine what the hour hand pointed at 25 hours ago, use modulo 12 since a cycle is completed every 12 hours. To find where the hand was 25 hours ago, subtract: $8 \ominus 25 \pmod{12} = -17 \pmod{12}$.
Simplify by adding to -17: $-17 + 12 = -5 \rightarrow -5 + 12 = 7$
25 hours ago, the hand pointed at 7.

b. To determine what the hour hand will point at in 20 hours from 8, add:
$20 \oplus 8 \pmod{12} = 28 \pmod{12}$

Simplify: $\frac{28}{12} = 2$ Remainder 4 → $28 \pmod{12} = 4$

20 hours from now, the hand will point at 4.

Practice

1. At a conference of 45 delegates, discussions are to be held in 6 equal groups.
   a. How many members will there be per seminar group?
   b. How many delegates will be left out of a seminar group?
2. Today is Monday.
   a. What day was it 19 days ago?
   b. What day will it be 6 days from today?
3. The hour hand of a clock points at 1.
   a. What will it be pointing at in 14 hours before now?
   b. What will it be pointing at in 16 hours from now?

Lesson Title: Rational and irrational numbers	**Theme:** Numbers and Numeration
Practice Activity: PHM1-L025	**Class:** SSS 1

Learning Outcomes
By the end of the lesson, you will be able to:
1. Define rational and irrational numbers.
2. Classify numbers as rational and irrational numbers.

Overview

A rational number is a number that can be expressed in the form $\frac{P}{Q}$ such that P and Q are integers and Q is not zero.

From the above definition you can see that the following are **rational numbers:**

- All **whole numbers** and **integers**. For example, 2 can be written as $\frac{2}{1}$ and -3 can be written as $-\frac{3}{1}$.
- All **fractions** (proper, improper and mixed). For example, the fraction $\frac{3}{4}, \frac{4}{3}, 1\frac{2}{3}$ are all rational numbers.
- **Decimal numbers** that are **terminating** or **recurring**. These are rational because they can be written as fractions. For example, the decimal numbers $0.75 = \frac{3}{4}$ and $1.\overline{3} = \frac{4}{3}$ are rational.
- The **square roots of perfect squares** are all rational because they can be written as integers. Example: $\sqrt{4}$ is rational since $\sqrt{4} = \frac{2}{1}$.

With the above description and examples of rational numbers, you can now define irrational numbers as numbers that are not rational. That is to say they cannot be written as fractions $\left(\frac{P}{Q}\right)$ such that P and Q are integers. When an irrational number is expressed in decimal form, it goes on forever without repeating or terminating.

The following are **irrational numbers:**

- All **decimal numbers** that **do not terminate.** For example, the numbers 0.1235610… and 1.22133450… are irrational.
- The **square root** of all numbers that are **not perfect squares.** For example, if you evaluate $\sqrt{2}$ the result will be a decimal number that is non-terminating (1.414213562...). It continues on forever.
- The constant **pi (π)** that is often used in geometry is a decimal that does not terminate ($\pi = 3.1459265\ldots$), and is therefore irrational. The values of $\frac{22}{7}$ or 3.14 for π are only approximations. These are not exact values.

Solved Examples

1. Identify the following numbers as rational or irrational:
   a. $\sqrt{2}$
   b. $x = 0.123456789101112$ ...
   c. $y = 3.454545$ ...
   d. -17π
   e. $-\frac{\sqrt{9}}{3}$

   Solutions:
   a. $\sqrt{2}$ is irrational because it is not a perfect square, so its square root does not terminate.
   b. x is irrational because the pattern in the number does not repeat.
   c. y is rational because the decimal number has a repeating sequence: $y = 3.\overline{45}$
   d. 17π is irrational. The product of an irrational number π and -17 will also be irrational.
   e. $-\frac{\sqrt{9}}{3}$ is rational because it can be simplified: $-\frac{\sqrt{9}}{3} = -\frac{3}{3} = -1$

2. Show whether the following numbers are rational or irrational:
   a. $\sqrt{5}$ b. $\sqrt{25}$ c. $\sqrt{24}$ d. $\frac{\sqrt{25}}{\sqrt{36}}$ e. $-\frac{\sqrt{9}}{\sqrt{2}}$

   Solutions:
   a. $\sqrt{5}$ is irrational. Five is not a perfect square so its square root does not terminate.
   b. $\sqrt{25}$ is rational, because it is a perfect square and $\sqrt{25} = 5$.
   c. $\sqrt{24}$ is irrational. Twenty-four is not a perfect square so its square root does not terminate.
   d. $\frac{\sqrt{25}}{\sqrt{36}}$ is rational because it can be simplified: $\frac{\sqrt{25}}{\sqrt{36}} = \frac{5}{6}$.
   e. $-\frac{\sqrt{9}}{\sqrt{2}}$ is irrational. Nine is a perfect square, but 2 is not. It can be simplified to $-\frac{3}{\sqrt{2}}$, which is irrational because it has an irrational part.

3. Determine whether the following numbers are rational or irrational:
   a. $\sqrt{12}$ b. 0.626262… c. $\sqrt{25}$ d. $\frac{\pi}{12}$ e. $-\sqrt{100}$

   Solutions:
   a. $\sqrt{12}$ is irrational. Twelve is not a perfect square so its square root does not terminate.
   b. 0.626262… is rational because it repeats.
   c. $\sqrt{144}$ is rational because it is a perfect square and $\sqrt{144} = 12$.
   d. $\frac{\pi}{12}$ is irrational because π divided by any number will also be irrational.

e. $-\sqrt{100}$ is rational because the surd is a perfect square and $-\sqrt{100} = -10$.

Practice

1. Determine whether the following numbers are rational or irrational:

   a. $\sqrt{7}$ b. $x = 0.10111213 \ldots$ c. $y = 1.21121121$

   d. 3π e. $\sqrt{\pi}$

2. Identify the following numbers as rational or irrational:

   a. $\sqrt{6}$ b. $\sqrt{36}$ c. $\sqrt{0.01}$ d. $3\sqrt{4}$ e. $\frac{\sqrt{3}}{2}$

3. Identify the following numbers as rational or irrational:

   a. $\sqrt{12}$ b. 0.323232….. c. $\sqrt{0.36}$ d. π^2 e. $1\frac{115}{201}$

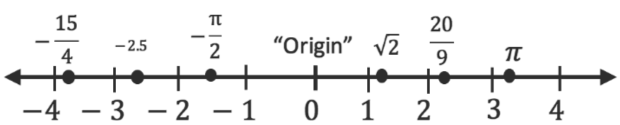
Lesson Title: Real numbers on the number line	Theme: Numbers and Numeration
Practice Activity: PHM1-L026	**Class:** SSS 1
Learning Outcome By the end of the lesson, you will be able to locate integers, fractions, and decimals on the number line.	

Overview

Today you are going to learn how to locate real numbers on a number line. In order to do so, you must understand the following:

- Real numbers include all rational and irrational numbers.
- Real numbers can be positive, negative or zero. Examples of real numbers include: 0, 1, 12, 0.38, -0.865, π, $\sqrt{2}$.
- There are numbers that are not real (they are neither rational nor irrational, neither positive, negative or zero). Such numbers include imaginary numbers represented by i and infinity written as ∞.

A number line is a line where all real numbers can be located. This implies that any point that can be identified on the number line is a real number. Each real number is represented with an x or a dot at the appropriate point on the number line. Example: the numbers; $-\frac{15}{4}$, $-2.5, -\frac{\pi}{2}$, $\sqrt{2}$, $\frac{20}{9}$, and π can be shown on the number line as follows:



To show some numbers on the number line, it is sometimes easier to graph its decimal. For example, π is approximately 3.14, which is a little more than 3. Therefore, we graph it slightly to the right of 3 on the number line.

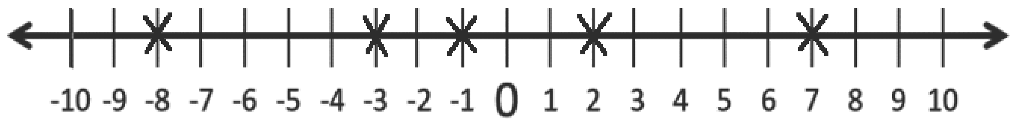
Solved Examples

1. Represent on a number line: $-3, -8, 2, -1, 7$

   Solution:

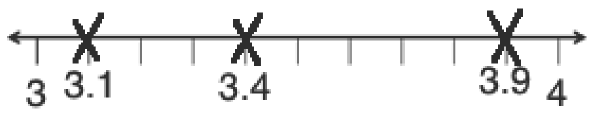
   First, choose an appropriate size and scale for the number line. The numbers given are positive and negative whole numbers between -10 and 10. Therefore, a number line showing whole numbers between -10 and 10 is best.

   Draw the number line, and draw an x at each number listed:



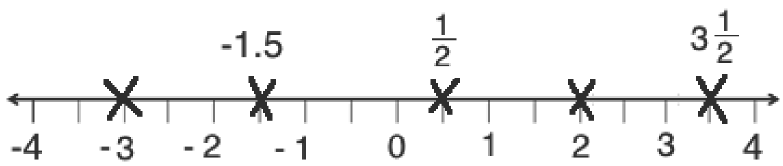
2. Represent the following on a number line: $3.1, 3.4, 3.9$.
   Solution:
   First, choose an appropriate size and scale for the number line. These numbers are between 3 and 4, and have decimal places with tenths. Therefore, a number line showing tenths from 3 to 4 is best.

   Draw the number line, and draw an x at each number listed:



3. Represent the following on a number line: $-1.5, \frac{1}{2}, 2, 3\frac{1}{2}, -3$
   Solution:
   First, choose an appropriate size and scale for the number line. These numbers are between -4 and 4, and have halves. Therefore, a number line showing halves from -4 to 4 is best.

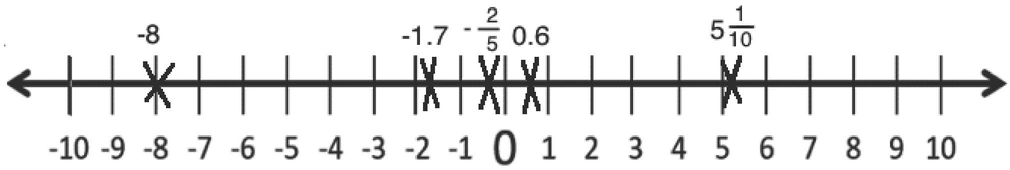
   Draw the number line, and draw an x at each number listed:



4. Represent the following on the number line:
   a. $-\frac{2}{5}$ b. $5\frac{1}{10}$ c. 0.6 d. –8 e. –1.7
   Solutions:
   We can show all of these on a number line from -10 to 10. For the decimals and fractions, we will show approximately where they are.

   First, convert fractions to decimals so they are easy to plot:
   a. $-\frac{2}{5} = -0.4$ b. $5\frac{1}{10} = 5.1$

   Draw the number line, and draw an x at each number listed:



5. Represent the following numbers on the number line:

   a. –6 b. –3.6 c. 2π d. $\sqrt{8}$ e. 9.82

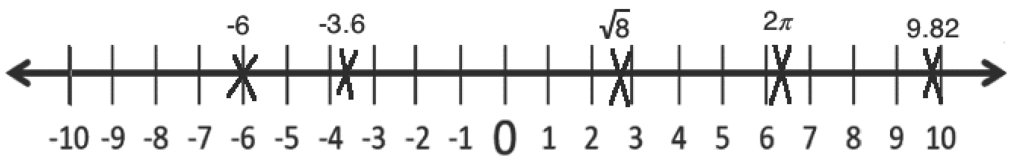
   Solutions:

   First, convert each number to a decimal:

   c. 2π is approximately $2 \times 3.14 = 6.28 = 6.3$ (to 1 decimal place)

   d. $\sqrt{8} = 2.8$ (to 1 decimal place)

   Draw the number line, and draw an x at each number listed:



Practice

1. Represent the following number on the number line:

   a. -4, b. 4, c. 0, d. 6 e. –8, f.–1

2. Represent the following numbers on the number line:

   a. $-\frac{3}{2}$, b. $\frac{5}{2}$, c. $-\frac{1}{2}$, d. $-2\frac{1}{2}$

3. Represent the following numbers on the number line:

   a. $\sqrt{2}$, b. $\sqrt{5}$, c. $-\sqrt{10}$, d. $-\sqrt{6}$, e. $\sqrt{7}$

4. Represent the following numbers on the number line:

   a. $-\pi$, b. 3π, c. $\frac{4}{5}$, d. $\sqrt{21}$, e. $\frac{5\pi}{2}$, f. $-\frac{3\pi}{2}$

Lesson Title: Comparing and ordering of rational numbers	**Theme:** Numbers and Numeration
Practice Activity: PHM1-L027	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to compare and order rational numbers.

Overview

You will recall that a rational number is any number which can be written as a fraction; that is, in the form $\frac{P}{Q}$ such that P and Q are integers. For example, $\frac{1}{3}$, $-\frac{7}{5}$, 1, $\sqrt{9}$, and $0.\overline{3}$ are all rational numbers.

In order to compare and order rational numbers that are in the form of fractions, note the following:

- Express the given rational numbers as common fractions with the same denominator. Where the denominators are different, change them so they are the same. This can be done by using the LCM or by multiplying all of the denominators.
- Once the rational numbers have the same denominator, compare the numerators. Use the following guidelines to order them:
  - For positive numbers, the rational number with a larger numerator is larger.
  - For negative numbers, the rational number with a smaller numerator is larger.
  - Positive numbers are always larger than negative numbers.

Numbers can be ordered either in ascending order (from least to greatest), or in descending order (from greatest to least).

Numbers that are not fractions can also be compared and ordered. For example, we can order $0.2, \frac{1}{4}, 30\%,$ and 10. The first step is to convert each number to a fraction.

Then proceed with the steps above for ordering fractions.

Solved Examples

1. Arrange the following numbers in ascending order: $\frac{1}{7}, \frac{3}{7}, \frac{5}{7}, \frac{2}{7}$

   Solution:
   Since all the numbers are positive and have the same denominator, consider the numerators as the basis of ordering.

   $\frac{1}{7} < \frac{2}{7} < \frac{3}{7} < \frac{5}{7}$

2. Arrange the following rational numbers in ascending order: $\frac{2}{3}, \frac{1}{2}, \frac{2}{5}, \frac{3}{4}$

Solution:

Since the denominators are different, multiply all the denominators, and use this as your new denominator:

$3 \times 2 \times 5 \times 4 = 120$

Change each fraction to have a denominator of 120:

$$\frac{2}{3} = \frac{2\times40}{3\times40} = \frac{80}{120}$$
$$\frac{1}{2} = \frac{1\times60}{2\times60} = \frac{60}{120}$$
$$\frac{2}{5} = \frac{2\times24}{5\times24} = \frac{48}{120}$$
$$\frac{3}{4} = \frac{3\times30}{4\times30} = \frac{90}{120}$$

Now with the common denominator, order the fractions on the basis of the numerators:

$$\frac{48}{120} < \frac{60}{120} < \frac{80}{120} < \frac{90}{120}$$

Therefore, $\frac{2}{5} < \frac{1}{2} < \frac{2}{3} < \frac{3}{4}$

3. Arrange the following in descending order: $\frac{1}{8}, -\frac{2}{8}, \frac{7}{8}, -\frac{8}{8}, -\frac{3}{8}, \frac{5}{8}$

Solution:

Recall that positive numbers are always greater than negative numbers. Therefore, the positive numbers come first when writing in descending order:

$$\frac{7}{8} > \frac{5}{8} > \frac{1}{8} > -\frac{2}{8} > -\frac{3}{8} > -\frac{8}{8}$$

4. Arrange the following rational numbers in ascending order:

–2.87, –2.8, –2.73

Solution:

Write the decimal numbers as common fractions. This can be done by multiplying them by $\frac{100}{100}$.

$$-2.87 = -\frac{287}{100}$$
$$-2.8 = -\frac{280}{100}$$
$$-2.73 = -\frac{273}{100}$$

With the same denominators, now compare using the numerators. Remember that for common fractions that are negative, the smaller numerator is larger.

$$-\frac{287}{100} < -\frac{280}{100} < -\frac{273}{100}$$

Therefore, $-2.87 < -2.8 < -2.73$

5. Arrange the following numbers in ascending order: $0.2, \frac{1}{4}, 30\%, 10$.

Solution:

First, express the numbers that are not already fractions as fractions:

$0.2 = \frac{2}{10} = \frac{1}{5}$

$30\% = \frac{30}{100} = \frac{3}{10}$

$10 = \frac{10}{1}$

Multiply all the denominators of the fractions: $5 \times 4 \times 10 \times 1 = 200$

Convert the fractions to like fractions:

$\frac{1}{5} = \frac{1\times40}{5\times40} = \frac{40}{200}$

$\frac{1}{4} = \frac{1\times50}{4\times50} = \frac{50}{200}$

$\frac{3}{10} = \frac{3\times20}{10\times20} = \frac{60}{200}$

$\frac{10}{1} = \frac{10\times200}{1\times200} = \frac{2,000}{200}$

Arrange the fractions in ascending order: $\frac{40}{200} < \frac{50}{200} < \frac{60}{200} < \frac{2,000}{200}$

Therefore: $0.2 < \frac{1}{4} < 30\% < 10$

Practice

1. Write the following numbers in ascending order.
   a. $\frac{1}{7}, \frac{8}{7}, \frac{3}{7}, \frac{9}{7}$
   b. $-\frac{3}{12}, -\frac{1}{12}, -\frac{6}{12}, -\frac{5}{12}$
   c. $\frac{1}{9}, -\frac{4}{9}, -\frac{9}{9}, \frac{8}{9}$
2. Arrange the following numbers in descending order.
   a. $\frac{1}{2}, \frac{1}{3}, \frac{1}{5}, \frac{1}{4}$
   b. $\frac{4}{5}, \frac{6}{7}, \frac{2}{3}, \frac{3}{4}$
3. Arrange the numbers in ascending order.
   a. 3.02, 3.2, 3.21, 3.12
   b. –2.45, –2.50, –2.51, –2.22
4. Arrange the following numbers in descending order.
   a. 0.3, 60%, $\frac{1}{5}$, 0.14
   b. $\frac{5}{6}$, 1, $-\frac{4}{5}$, 0.1
   c. 40%, 0.25, $\frac{1}{2}$, $\frac{3}{4}$

Lesson Title: Approximating of decimals	**Theme:** Numbers and Numeration
Practice Activity: PHM1-L028	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to round decimals to a given number of decimal places.

Overview

Decimal places (d.p.) in a number come after the decimal point. To number the places (1^{st}, 2^{nd}, 3^{rd}, ...), start at the decimal point and count to the right. For example, in the number 0.345, 3 is the 1^{st} decimal place, 4 is the 2^{nd} decimal place and 5 is the 3^{rd} decimal place.

Each digit after the decimal also has a place value. The 1^{st} decimal place is the tenths digit, the 2^{nd} is the hundredths, the 3^{rd} is the thousandths, and so on. For instance, 0.345 can be expanded as 3 tenths, 4 hundredths, and 5 thousandths.

0	.	3	4	5
		Tenths	Hundredths	Thousandths

To approximate (round) a decimal is to reduce the number of decimal places it has. When rounding decimal numbers, they are given a stated number of decimal places. This can be done as follows:

- If the digit after the decimal place you want is less than 5, **round down**. Just discard the decimal places you do not want.
- If the digit after the decimal place you want is 5 or more, **round up**. Add one to the last place you want and discard the remaining decimal places.

Solved Examples

1. Correct 9.23564 to:
   a. 1 d.p.
   b. 3 d.p.
   Solutions:
   a. The first d.p. is 2, and the digit next to it is 3, which is less than 5. Round down:
      $9.23564 = 9.2$ to 1d.p.

b. The 3rd d.p is 5, and next to it is 6, which is more than 5. Round up by adding 1 to 5 to make 6.

   9.23564 = 9.236 to 3d.p

2. Round 4.4325 to:
   a. The nearest hundredth
   b. The nearest thousandth

   Solutions:

   Remember that hundredth is the 2nd d.p. and thousandth is the 3rd d.p.

   a. 3 is in the hundredths place, and the next digit is 2, which is less than 5. Round down:

      4.4325 = 4.43 to the nearest hundredth

   b. 2 is in the thousandths place, and the next digit is 5. Round up by adding 1 to 2 to make 3.

      4.4325 = 4.433 to the nearest thousandth

3. Approximate 5.39642 to 2 d.p.

   Solution:

   The 2nd d.p. is 9, and next to it is 6. Round up by adding 1 to 9 to make 10. Because 10 is more than 1 digit, we write zero and carry 1 to the 3 in front of 9 to make 4.

   5.39642 = 5.40 to 2 d.p.

4. Round up 2.9547 to the nearest tenth.

   Solution:

   Nine is in the tenths place, and next to it is 5. Round up by adding 1 to 9 to make 10. Write 0 and carry the 1 to the 2 in front of 9.

   2.9547 = 3.0 to 1 d.p.

5. Round 1.25738 to:
   a. 3 d.p.
   b. 4 d.p.

   Solutions:

   a. The 3rd decimal place is 7 and next to it is 3, which is less than 5. Round down.

      1.25738 = 1.257 to 3 d.p.

   b. The 4th d.p. is 3 and next to is it is 8, which is greater than 5. Round up.

      1.25738 = 1.2574 to 4 d.p.

Practice

1. Correct 5.24608 to:
    a. 1 d.p.
    b. 2 d.p.
2. Round 6.9047 to the nearest tenth.
3. Approximate 8.1973 to the nearest hundredth.
4. Round 0.101501 to:
    a. 3 d.p.
    b. 4 d.p.
5. Approximate 15.495408 to the nearest:
    a. Tenth
    b. Hundredth
    c. Thousandth

Lesson Title: Recurring decimals as common fractions	**Theme:** Numbers and Numeration
Practice Activity: PHM1-L029	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to convert recurring decimals into common fraction.

Overview

When we try to convert a fraction to a decimal and the division comes to an end, we refer to that decimal as a terminating decimal.

For example, consider $\frac{3}{4}$:

	0.	7	5
4	3.	0	
-	2	8	
		2	0
	-	2	0
			0

Note that in this example, the division works out exactly and the decimal comes to an end (terminates).

In some cases, the division does not stop, and the decimal does not terminate. The same digit or group of digits keeps repeating indefinitely. We call such decimals **recurring decimals**. For example, consider $\frac{2}{9}$:

	0.	2	2	2
9	2.	0	0	0
-	1	8		
		2	0	
	-	1	8	
			2	0
		-	1	8
				2

In this example, you will notice that the 2 will never stop repeating. Repeating numbers are shown with a dot or line over the repeating number, for example: $0.\dot{2}$ or $0.\overline{2}$.

Now to convert recurring decimals to a fraction follow these steps:

Step 1. Multiply the recurring decimal by the smallest power of ten which will produce another number with the same repeating pattern of digits in the decimal place.

Step 2. Subtract the smaller number from the larger number and then eliminate the digits in the decimal place.

Solved Examples

1. Convert 3.222 to the form $\frac{a}{b}$

   Solution:

   Step 1. Let $\mathrm{r} = 3.222 \ldots$ (1)

   Step 2. Multiply both sides by 10, because this will give another number with repeating 2 in the decimal place.

   $$10\mathrm{r} = 32.222 \ldots \quad (2)$$

   Step 3. Subtract equation (1) from (2).

   $$10r - \mathrm{r} = (32.22222\ldots.) - (3.2222222 \ldots.)$$
   $$9r = 29$$

   Step 4. Divide both sides by 9.

   $$\frac{\not{9}\mathrm{r}}{\not{9}} = \frac{29}{9}$$
   $$r = \frac{29}{9} = 3\frac{2}{9}$$

2. Convert 0.454545…. to a fraction.

   Solution:

r	$=$	$0.454545 \ldots$ (1)	
$100r$	$=$	$45.4545 \ldots$ (2)	Multiply by 100 because 2 digits repeat
$100r - r$	$=$	$45.4545 \ldots - 0.4545 \ldots$	Subtract (2) – (1)
$99r$	$=$	45	Simplify
$\frac{99r}{99}$	$=$	$\frac{45}{99}$	Divide throughout by 99
r	$=$	$\frac{5}{11}$	Simplify

3. Convert 0.1666… to a fraction.

   Solution:

   The first decimal place of this number is not repeating. We need to multiply the number by 10 to get a repeating decimal. Then, we can follow the steps above.

r	$=$	$0.1\overline{6}$	
$10r$	$=$	$1.\overline{6}$ (1)	Multiply by 10 to get a repeating decimal
$100r$	$=$	$16.\overline{6}$ (2)	Multiply by another 10 because 1 digit repeats
$100r - 10r$	$=$	$16.\overline{6} - 1.\overline{6}$	Subtract (2) – (1)
$90r$	$=$	15	Simplify
$\frac{90r}{90}$	$=$	$\frac{15}{90}$	Divide throughout by 90

$$r = \frac{1}{6} \qquad \text{Simplify}$$

4. Convert $0.\overline{431}$ to a fraction.

   Solution:

$$\begin{aligned} r &= 0.\overline{431} \quad (1) \\ 1{,}000r &= 431.\overline{431} \quad (2) && \text{Multiply by 1,000 because 3 digits repeat} \\ 1{,}000r - r &= 431.\overline{431} - 0.\overline{431} && \text{Subtract (2) – (1)} \\ 999r &= 431 && \text{Simplify} \\ \frac{999r}{999} &= \frac{431}{999} && \text{Divide throughout by 999} \\ r &= \frac{431}{999} && \text{Simplify} \end{aligned}$$

5. Write 1.507507 ... in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

   Solution:

$$\begin{aligned} r &= 1.\overline{507} \quad (1) \\ 1{,}000r &= 1{,}507.\overline{507} \quad (2) && \text{Multiply by 1,000 because 3 digits repeat} \\ 1{,}000r - r &= 1{,}507.\overline{507} - 1.\overline{507} && \text{Subtract (2) – (1)} \\ 999r &= 1{,}506 && \text{Simplify} \\ \frac{999r}{999} &= \frac{1{,}506}{999} && \text{Divide throughout by 999} \\ r &= \frac{502}{333} && \text{Simplify} \end{aligned}$$

Practice

1. Convert 0.333333… to a fraction.
2. Convert $0.1\overline{2}$ to a fraction.
3. Write $0.\overline{12}$ in the form $\frac{m}{n}$ where m and n are integers and $n \neq 0$.
4. Change $0.\overline{72}$ to a fraction.
5. Express these recurring decimals as common fractions:
   a. 2.054054....
   b. 0.252525…
6. Write 0.090090… in the form $\frac{u}{v}$ where u and v are integers, and $v \neq 0$.

Lesson Title: Operations on real numbers	**Theme:** Numbers and Numeration
Practice Activity: PHM1-L030	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to perform operations on real numbers.

Overview

The four simple operations in Mathematics are addition, subtraction, multiplication and division. However, we have other operations, which involve some combination of the 4 simple operations.

Let $*$ and Δ represent other operations. Then we can describe the following properties:

- Commutative property: $a * b = b * a$
- Associative property : $a * (b * c) = (a * b) * c$
- Distributive property : $a * (b\Delta c) = (a * b)\Delta(a * c)$

The commutative property means that the order does not matter; a or b can come first. Addition and multiplication are examples of commutative operations. The associative property means that it does not matter which operation is performed first when combining 3 or more terms. Addition and multiplication are also associative. The distributive property is used when there are 2 different operations that combine 3 different terms.

Solved Examples

1. Evaluate both sides of the equation in the following, then identify the property that is shown.
   a. $4 \times 8 = 8 \times 4$
   b. $2 + (7 + 8) = (2 + 7) + 8$
   c. $3 \times (5 + 2) = (3 \times 5) + (3 \times 2)$

   Solutions:
   a. $4 \times 8 = 32$
      $8 \times 4 = 32$
      Commutative property
   b. $2 + (7 + 8) = 2 + 15 = 17$
      $(2 + 7) + 8 = 9 + 8 = 17$
      Associative property
   c. $3 \times (5 + 2) = 3 \times 7 = 21$
      $(3 \times 5) + (3 \times 2) = 15 + 6 = 21$
      Distributive property

2. The operation $*$ is defined as $m * n = m + n + mn$. Evaluate $3 * 4$.
Solution:
In this case the operation is not simply to add, subtract, divide or multiply. The operation given by the star means that we should perform the right side of the equation, $m + n + mn$. We can do this if we are given the values for m and n. In this case, we are given 3 and 4.

$3 * 4$	$= 3 + 4 + 3(4)$	Substitute the values of m and n
	$= 3 + 4 + 12$	Remove bracket
	$= 19$	Add

3. Using the same operation as in problem 2 above, evaluate 4 $*$ 3.
Solution:

$4 * 3$	$= 4 + 3 + 4(3)$	Subtitle the values of m and n
	$= 4 + 3 + 12$	Remove bracket
	$= 19$	Add

In the two (2) examples above, you will notice the commutative property.
$3 * 4 = 4 * 3$

4. The operation $*$ is defined by $m * n = m + n + mn$. Show that $6 * (4 * 5) = (6 * 4) * 5$.
Solution:
Show that the left-hand side and the right-hand side of the equation have the same result:

$6 * (4 * 5)$	$= 6 * [4 + 5 + 4(5)]$	Do the operations in the bracket first
	$= 6 * (9 + 20)$	
	$= 6 * 29$	
	$= 6 + 29 + 6(29)$	
	$= 35 + 174$	
	$= 209$	
$(6 * 4)5$	$= [6 + 4 + 6(4)] * 5$	
	$= (10 + 24) * 5$	
	$= 34 * 5$	
	$= 34 + 5 + 34(5)$	
	$= 39 + 170$	
	$= 209$	

5. The operation $*$ is defined on the set of real numbers by $m * n = \frac{m-n}{n}, n \neq 0$.
Evaluate $9 * (7 * 5)$
Solution:
$9 * (7 * 5) \quad = 9 * \left(\frac{7-5}{5}\right)$

$$= 9 * \frac{2}{5}$$
$$= \frac{9-\frac{2}{5}}{\frac{2}{5}}$$
$$= \frac{\frac{43}{5}}{\frac{2}{5}}$$
$$= \frac{43}{5} \div \frac{2}{5}$$
$$= \frac{43}{5} \times \frac{5}{2}$$
$$= \frac{43}{2}$$
$$= 21\frac{1}{2}$$

Practice

1. The operation $*$ is defined by $m + n + mn$. Find the value of $6 * 7$.
2. If the operation $*$ is defined by $p * q = \frac{p+q}{p-q}$, find the value of $6 * (4 * 3)$
3. The operation $*$ is defined over the set of real numbers by $u * v = \frac{u-v}{u-v}$. Find the value of:
    a) $8 * 2$
    b) $5 * -3$
4. The operation $*$ is defined over the set of real numbers by $m * n = \frac{m+n}{m-n}$. Evaluate $7 * (5 * 4)$.
5. The operation $*$ is defined by $a * b = a + b + ab$. Show that $8 * (3 * 4) = (8 * 3) * 4$.

Lesson Title: Order of operations (BODMAS)	**Theme:** Numbers and Numeration
Practice Activity: PHM1-L031	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to apply the order of operations (BODMAS) to solve mathematical problems.

Overview

The letters BODMAS stand for: Bracket, Of, Division, Multiplication, Addition and Subtraction. When working problems which have more than one operation (of, ×, +, –, ÷), we use BODMAS. The letters of the word (BODMAS) tell us the order in which we should work the operations in a Math problem. The term "of" represents the multiplication sign, which includes powers.

Solved Examples

1. Simplify: $7 + (6 + 5^2) \times 3$

   Solution:

   $7 + (6 + 5^2) \times 3 \quad = 7 + (6 + 25) \times 3$ Start inside the brackets, using "of" first

   $= 7 + (31) \times 3$ Add inside the brackets

   $= 7 + 93$ Multiply

   $= 100$ Add

2. Evaluate: $3 + 6 \times (5 + 4) \div 3 - 7$

   Solution:

   $3 + 6 \times (5 + 4) \div 3 - 7 \quad = 3 + 6 \times 9 \div 3 - 7$ Bracket

   $= 3 + 6 \times 3 - 7$ Division

   $= 3 + 18 - 7$ Multiplication

   $= 21 - 7$ Addition

   $= 14$ Subtraction

3. Evaluate: $1\frac{2}{3} - (1\frac{3}{4} \div 2\frac{5}{8})$

   Solution:

   $1\frac{2}{3} - (1\frac{3}{4} \div 2\frac{5}{8}) \quad = \frac{5}{3} - \left(\frac{7}{4} \div \frac{21}{8}\right)$ Convert mixed fraction to improper fraction

   $= \frac{5}{3} - \left(\frac{7}{4} \times \frac{8}{21}\right)$ Solve inside the brackets

   $= \frac{5}{3} - \left(\frac{1}{1} \times \frac{2}{3}\right)$

   $= \frac{5}{3} - \frac{2}{3}$ Subtract

   $= \frac{3}{3}$

$= 1$

4. Simplify: $\frac{1}{6} \div \left(\frac{3}{5} \times \frac{5}{7}\right) - \frac{1}{3}$

   Solution:

   $= \frac{1}{6} \div \left(\frac{3}{5} \times \frac{5}{7}\right) - \frac{1}{3}$ $= \frac{1}{6} \div \left(\frac{15}{35}\right) - \frac{1}{3}$ Solve inside the brackets

   $= \frac{1}{6} \div \frac{3}{7} - \frac{1}{3}$ Simplify the fraction

   $= \frac{1}{6} \times \frac{7}{3} - \frac{1}{3}$ Divide

   $= \frac{7}{18} - \frac{1}{3}$ Subtract

   $= \frac{7-6}{18}$

   $= \frac{1}{18}$

5. Simplify: $\frac{3\frac{3}{5} \times 1\frac{5}{9}}{2\frac{4}{5}}$

   Solution:

   When you have operations in the numerator or denominator of fractions, treat them as though they are in brackets and work them first. Note that $\frac{3\frac{3}{5} \times 1\frac{5}{9}}{2\frac{4}{5}}$ is the same as $(3\frac{3}{5} \times 1\frac{5}{9}) \div 2\frac{4}{5}$.

   $\frac{3\frac{3}{5} \times 1\frac{5}{9}}{2\frac{4}{5}} = \frac{\frac{18}{5} \times \frac{14}{9}}{\frac{14}{5}}$ Convert to improper fractions

   $= \frac{\frac{2}{5} \times \frac{14}{1}}{\frac{14}{5}}$ Do the multiplication in the numerator

   $= \frac{\frac{28}{5}}{\frac{14}{5}}$

   $= \frac{28}{5} \div \frac{14}{5}$ Divide

   $= \frac{28}{5} \times \frac{5}{14}$

   $= 2$

Practice

1. Simplify: $10 + (4^2 - 3) \times 2$
2. Evaluate: $120 \div (9 \times 3 - 10) + 4$
3. Simplify: $\frac{1}{3} + \frac{5}{12} - \frac{5}{8} \times \frac{4}{10}$
4. Simplify: $7\frac{2}{3} - (1\frac{5}{6} + 2\frac{3}{8})$
5. Simplify: $\frac{2\frac{1}{10} \times 3\frac{1}{7}}{1\frac{3}{8}}$
6. Evaluate: $(2\frac{1}{3} + 1\frac{1}{2}) \div (3\frac{1}{5} - 2\frac{1}{3})$
7. $7\frac{1}{2} \times (\frac{7}{8} - \frac{5}{24})$

Lesson Title: Index notation	**Theme:** Numbers and Numeration
Practice Activity: PHM1-L032	**Class:** SSS 1

Learning Outcomes

By the end of the lesson, you will be able to:

1. Identify the index and base in index notation.
2. Identify that the index indicates the number of times the base is multiplied by itself.

Overview

Numbers multiplied by themselves can be written in a form such that the number is raised to a power. This power gives the number of times it is multiplied by itself. For example, consider squares: $3 \times 3 = 3^2$ and $7 \times 7 = 7^2$.

Numbers written with a base and a power are written in **index form.** For example, 7^3 is in index form, and can be written as $7^3 = 7 \times 7 \times 7$. Seven is the **base** and three is the **power** or **index**.

$$\text{base} \rightarrow 7^{3 \leftarrow \text{power/index}}$$

When numbers are raised to negative powers, the expression can be written as a fraction. The expression is moved to the denominator and the index is changed to a positive power. The numerator is 1. For example: $3^{-2} = \frac{1}{3^2}$ and $7^{-3} = \frac{1}{7^3}$.

From the trend in the examples, you can observe a general rule for numbers with negative indices as follows: a number (a) raised to a negative index ($-n$) can be written as: $a^{-n} = \frac{1}{a^n}$.

Indices that have an operation between them can be expanded and simplified, but if their bases are different then they cannot be combined.

For example, this is the expanded form: $2 \times 2 \times 2 \times 3 \times 3$

This is the simplified form of the same expression: $2^3 \times 3^2$

Solved Examples

1. Write the following numbers in index form:
   a. $7 \times 7 \times 7 \times 7 \times 7$
   b. $29 \times 29 \times 29 \times 29$
   c. $100 \times 100 \times 100$

   Solutions:
   a. $7 \times 7 \times 7 \times 7 \times 7 = 7^5$
   b. $29 \times 29 \times 29 \times 29 = 29^4$
   c. $100 \times 100 \times 100 = 100^3$
2. Write the following in expanded form:

a. 8^3
b. 2^{-5}
c. 17^4

Solutions:

a. $8^3 = 8 \times 8 \times 8$
b. $2^{-5} = \frac{1}{2^5} = \frac{1}{2\times2\times2\times2\times2}$
c. $17^4 = 17 \times 17 \times 17 \times 17$

3. Write the following numbers in index form:
   a. $7 \times 7 \times 7 \times 7 \times 7 \times 2 \times 2$
   b. $3 \times 3 \times 5 \times 5 \times 5 \times 6 \times 6$

   Solutions:

   a. $7 \times 7 \times 7 \times 7 \times 7 \times 2 \times 2 = 7^5 \times 2^2$
   b. $3 \times 3 \times 5 \times 5 \times 5 \times 6 \times 6 = 3^2 \times 5^3 \times 6^2$

4. Write the following numbers with negative indices as fractions:
   a. 2^{-18}
   b. 3^{-8}

   Solutions:

   a. $2^{-18} = \frac{1}{2^{18}}$
   b. $3^{-8} = \frac{1}{3^8}$

5. Simplify the following:
   a. $3 \times 3 \times 3 + 5 \times 5$
   b. $2 \times 2 \times 2 - 3 \times 3 \times 3 + 4 \times 4$

   Solutions:

   a. $3 \times 3 \times 3 + 5 \times 5 = 3^3 + 5^2$
   b. $2 \times 2 \times 2 - 3 \times 3 \times 3 + 4 \times 4 = 2^3 - 3^3 + 4^2$

6. Expand the following:
   a. $8^4 + 9^2$
   b. $3^5 \times 4^2 \times 8^{-2}$

   Solutions:

   a. $8^4 + 9^2 = 8 \times 8 \times 8 \times 8 + 9 \times 9$
   b. $3^5 \times 4^2 \times 8^{-2} = 3 \times 3 \times 3 \times 3 \times 3 \times 4 \times 4 \times \frac{1}{8^2} = 3 \times 3 \times 3 \times 3 \times 3 \times 4 \times 4 \times \frac{1}{8\times8}$

7. Expand the following:
   a. $2^{-3} + 3^2 - 5^{-1}$
   b. $2^2 - 2^{-3} + 3^{-2}$

   Solutions:

a. $2^{-3} + 3^2 - 5^{-1} = \frac{1}{2^3} + 3 \times 3 - \frac{1}{5} = \frac{1}{2\times2\times2} + 3 \times 3 - \frac{1}{5}$

b. $2^2 - 2^{-3} + 3^{-2} = 2 \times 2 - \frac{1}{2^3} + \frac{1}{3^2} = 2 \times 2 - \frac{1}{2\times2\times2} + \frac{1}{3\times3}$

Practice

1. Write the following numbers in index form:
   a. $5 \times 5 \times 6 \times 6 \times 6 \times 9 \times 9$
   b. $2 \times 2 \times 4 \times 4 \times 4 \times 7 \times 7$
2. Write the following numbers with negative indices as fractions:
   a. 3^{-10}
   b. 8^{-3}
   c. 6^{-4}
3. Simplify the following:
   a. $2 \times 2 \times 3 \times 3 + 4 \times 4 \times 4 \times 5 \times 5$
   b. $3 \times 3 \times 3 + \times 4 \times 4 - 2 \times 2 \times 2$
4. Expand the following:
   a. 9^3
   b. 6^5
   c. $5^3 \times 2^4 \times 3^{-4}$
5. Expand the following:
   a. $3^{-2} + 2^3 - 4^{-2}$
   b. $2^{-2} + 3^2 - 5^{-3}$

Lesson Title: First and second laws of indices	**Theme:** Numbers and Numeration
Practice Activity: PHM1-L033	**Class:** SSS 1

Learning Outcomes

By the end of the lesson, you will be able to:

1. Identify the first law of indices ($a^m \times a^n = a^{m+n}$) and multiply two or more indices.
2. Identify the second law of indices ($a^m \div a^n = a^{m-n}$) and divide two or more indices.

Overview

To multiply two or more indices with the same base, add the powers. Example $3^2 \times 3^5 = 3^{2+5} = 3^7$.

This is what we refer to as the first law of indices, which has the general formula $a^m \times a^n = a^{m+n}$. Here m and n are the powers of a, and a is the base. This rule only works if the bases are the same.

To divide two or more indices, we subtract the powers to get the answer. For example, $2^5 \div 2^2 = 2^{5-2} = 2^3$. We use the general formula $a^m \div a^n = a^{m-n}$. This is referred to as the second law of indices. This rule only works if the bases are the same.

Solved Examples

1. Simplify: $7^3 \times 7^4$

   Solution:

   Add the powers: $7^3 \times 7^4 = 7^{3+4} = 7^7$

2. Simplify: $r^3 \times r^4 \times r^5$

   Solution:

   $r^3 \times r^4 \times r^5 = r^{3+4+5} = r^{12}$

3. Simplify: $10x^4y \times 2x^3y^2$

   Solution:

   When there are coefficients (numbers) before the variables, multiply those together. Also, when we have multiple variables to multiply, add the powers of each of them separately.

$$\begin{aligned} 10x^4y \times 2x^3y^2 &= 10 \times x^4 \times y \times 2 \times x^3 \times y^2 \\ &= (10 \times 2) \times (x^4 \times x^3) \times (y \times y^2) \\ &= 20 \times x^{4+3} \times y^{1+2} \\ &= 20x^7y^3 \end{aligned}$$

4. Simplify: $3^6 \div 3^2$
Solution:
Subtract the powers: $3^6 \div 3^2 = 3^{6-2} = 3^4$

5. Simplify: $8b^5a^3 \div 4b^3a$
Solution:
When there are coefficients (numbers) before the variables, divide them. When there are multiple variables, subtract the powers of each of them separately.
$$\begin{aligned} 8b^5a^3 \div 4b^3a &= (8 \div 4)b^{5-3}a^{3-1} \\ &= 2b^2a^2 \end{aligned}$$

6. Simplify: $\frac{p^7}{p^5}$
Solution:
Fractions are the same as division. We simply follow the second law of indices and subtract the indices.
$$\frac{p^7}{p^5} = p^7 \div p^5 = p^{7-5} = p^2$$

7. Simplify: $\frac{24xy^5}{6xy^4}$
Solution:
$$\begin{aligned} \frac{24xy^5}{6xy^4} &= 24xy^5 \div 6xy^4 \\ &= (24 \div 6)x^{1-1}y^{5-4} \\ &= 4x^0y^1 \\ &= 4y \end{aligned}$$

8. Simplify: $2^8 \times 2^{-5}$
Solution:
This problem can be solved in 2 ways. One method applies division, and the other applies multiplication to indices with negative powers.
Method 1.
$2^8 \times 2^{-5} = 2^8 \times \frac{1}{2^5}$ Change the negative power to a fraction
$= \frac{2^8}{2^5}$ Apply the second law of indices
$= 2^8 \div 2^5$
$= 2^{8-5}$
$= 2^3$
Method 2.
$2^8 \times 2^{-5} = 2^{8+(-5)}$ Apply the first law of indices
$= 2^{8-5}$
$= 2^3$

9. Simplify: $15p^7q^4 \div 3p^{-3}q^2$

Solution:

$$\begin{aligned} 15p^7q^4 \div 3p^{-3}q^2 &= (15 \div 3)p^{7-(-3)}q^{4-2} \\ &= 5p^{7+3}q^2 \\ &= 5p^{10}q^2 \end{aligned}$$

10. Simplify: $4x^6y^3 \times 3x^{-4}y$

Solution:

$$\begin{aligned} 4x^6y^3 \times 3x^{-4}y &= (4 \times 3)x^{6+(-4)}y^{3+1} \\ &= 12x^{6-4}y^4 \\ &= 12x^2y^4 \end{aligned}$$

Practice

Simplify the following:

1. $a^{12} \times a^3$
2. $5^9 \div 5^3$
3. $u^4 \times u^3 \times u^2$
4. $14^8 \times 14^{-3}$
5. $9a^3b^4 \times 3ab^2$
6. $32f^4g^7 \div 8f^2g$
7. $\frac{45mn^3}{9mn^2}$
8. $5x^5y^4 \times 8x^{-2}y^3 \times xy$
9. $\frac{42m^9n^4}{7m^{-1}n^3}$
10. $25u^4v^3 \div 5u^{-2}v^{-1}$

Lesson Title: Third and fourth laws of indices	**Theme:** Numbers and Numeration
Practice Activity: PHM1-L034	**Class:** SSS 1

Learning Outcomes
By the end of the lesson, you will be able to:
1. Identify and apply the third law of indices $(a^0 = 1)$.
2. Identify and apply the fourth law of indices $[(a^x)^y = a^{xy}]$.

Overview

Any non-zero number raised to the power zero is equal to 1. For example, $3^0 = 1$. This is true for any base $(x^0 = 1)$ and is known as the **third law of indices.**

Zero to power of zero is often said to be "an indeterminate form", because it could have several different values. We could also think of 0^0 having the value 0, because zero to any power (other than zero power) is zero. Therefore $0^0 = 0$.

"Powers of powers" involves expressions with brackets. It's important to remember that everything inside the bracket is raised to the outside power. If a power is raised to another power, multiply the indices.

Solved Examples

1. Simplify: 8^0
   Solution:
   Apply the third law of indices: $8^0 = 1$

2. Simplify: $\left(\frac{1}{5}\right)^0$
   Solution:
   The power applies to both the numerator and denominator:

$$\left(\frac{1}{5}\right)^0 = \frac{1^0}{5^0} = \frac{1}{1} = 1$$

3. Simplify: $(2^2)^3$
   Solution:
   Apply the fourth law of indices: $(2^2)^3 = 2^{2\times3} = 2^6$

4. Simplify: $m^5 \div m^5$
   Solution:

$m^5 \div m^5 = m^{5-5}$ Second law of indices
$= m^0$ Third law of indices
$= 1$

5. Simplify: $(3a^2b^3)^3$

Solution:

$(3a^2b^3)^3 = 3^3a^{2\times3}b^{3\times3}$ Apply the power to everything in the bracket

$= 27a^6b^9$

6. Simplify: $(p^3)^2 \div p^6$

Solution:

$(p^3)^2 \div p^6 = p^{3\times2} \div p^6$

$= p^6 \div p^6$

$= p^{6-6}$ Second law of indices

$= p^0$ Third law of indices

$= 1$

7. Simplify: $(n^{-3})^4 \times n^7$

Solution:

$(n^{-3})^4 \times n^7 = n^{-3\times4} \times n^7$ Fourth law of indices

$= n^{-12} \times n^7$

$= n^{-12+7}$ First law of indices

$= n^{-5}$

$= \frac{1}{n^5}$

8. Simplify: $(2u^5)^2 \div 2u^6$

Solution:

$(2u^5)^2 \div 2u^6 = 2^2u^{5\times2} \div 2u^6$ Fourth law of indices

$= 4u^{10} \div 2u^6$

$= (4 \div 2)u^{10-6}$ Second Law of indices

$= 2u^4$

9. Simplify: $3p^4q^{-3} \times 2p^2q^3$

Solution:

$3p^4q^{-3} \times 2p^2q^3 = 6p^{4+2} \times q^{-3+3}$

$= 6p^6 \times q^0$

$= 6p^6 \times 1$

$= 6p^6$

10. Simplify: $c^{-4} \div c^{-4}$

Solution:

$c^{-4} \div c^{-4} = c^{-4-(-4)}$

$= c^{-4+4}$ Second Law

$= c^0$ Third Law

$= 1$

11. Simplify: $(4a^2b)^2 \times (2ab^3)^2$

Solution:

$$(4a^2b)^2 \times (2ab^3)^2 = 16a^{2\times2}b^{1\times2} \times 4a^{1\times2}b^{3\times2}$$
$$= 16a^4b^2 \times 4a^2b^6$$
$$= (16 \times 4)a^{4+2}b^{2+6}$$
$$= 64a^6b^8$$

12. Simplify: $(3m^2 \times 2m)^2$

Solution:

$$(3m^2 \times 2m)^2 = [(2 \times 3)m^{2+1}]^2$$ Simplify inside the bracket first
$$= (6m^3)^2$$
$$= 36m^{3\times2}$$
$$= 36m^6$$

Practice

Simplify the following:

1. 0.154^0
2. $\left(\frac{17}{20}\right)^0$
3. $4^5 \div 4^5$
4. $(9^7)^3$
5. $(a^4)^{-2}$
6. $(2m^3n^4)^2$
7. $(u^4)^2 \div u^8$
8. $v^{-7} \div v^{-7}$
9. $(4x^3y)^2 \div (2xy^2)^2$
10. $(4a^3 \times 5a^2)^2$

Lesson Title: Simplifying indices	**Theme:** Numbers and Numeration
Practice Activity: PHM1-L035	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to apply multiple laws of indices to simplify expressions that contain indices.

Overview

To get started, let's look at how to simplify the following expressions from the previous lessons.

- $(a^4)^5 = a^{4\times5} = a^{20}$
- $2^4 \times 2 \times 2^3 = 2^{4+1+3} = 2^8$
- $y^6 \div y^6 = y^{6-6} = y^0 = 1$

Now remember that when you multiply two or more indices with the same base, simply add the powers ($a^m \times a^n = a^{m+n}$). When you divide two or more indices, subtract the powers ($a^m \div a^n = a^{m-n}$). Any non-zero number raised to the power of zero is equal to 1 ($x^0 = 1$). If an index is raised to another power, multiply the powers $(a^x)^y = a^{xy}$.

Solved Examples

1. Simplify: $(Z^4)^3 \div Z^{12}$

   Solution:

$(Z^4)^3 \div Z^{12}$	$= Z^{4\times3} \div Z^{12}$	Apply the fourth law of indices
	$= Z^{12} \div Z^{12}$	
	$= Z^{12-12}$	Apply the second law of indices
	$= Z^0$	
	$= 1$	

2. Simplify: $\frac{75a^2b^{-2}}{5a^3b^{-3}}$

   Solution:

   $$\begin{aligned}\frac{75a^2b^{-2}}{5a^3b^{-3}} &= 75a^2b^{-2} \div 5a^3b^{-3}\\ &= (75 \div 5)(a^2 \div a^3)(b^{-2} \div b^{-3})\\ &= 15a^{2-3}b^{-2-(-3)}\\ &= 15a^{-1}b^{-2+3}\\ &= 15a^{-1}b^1\\ &= \frac{15b}{a}\end{aligned}$$

3. Simplify: $(12y^3 \times 2y^2) \div (4y^5 \times 2y^2)$

Solution:

Simplify inside the bracket first.

$$
\begin{aligned}
(12y^3 \times 2y^2) \div (4y^5 \times 2y^2) &= (12 \times 2 \times y^3 \times y^2) \div (4 \times 2 \times y^5 \times y^2) \\
&= (24y^{3+2}) \div (8y^{5+2}) \\
&= 24y^5 \div 8y^7 \\
&= (24 \div 8)(y^5 \div y^7) \\
&= 3y^{5-7} \\
&= 3y^{-2} = \frac{3}{y^2}
\end{aligned}
$$

4. Simplify: $(6p^2 \times 7p^4) \div 14p^6$

Solution:

$$
\begin{aligned}
(6p^2 \times 7p^4) \div 14p^6 &= (6 \times 7)p^{2+4} \div 14p^6 \\
&= 42p^6 \div 14p^6 \\
&= (42 \div 14)p^{6-6} \\
&= 3p^0 \\
&= 3 \times 1 \\
&= 3
\end{aligned}
$$

5. Simplify: $36a^9b^5 \div (9a^5b^6 \div 3a^2b^2)^2$

Solution:

Simplify the expression inside the bracket first:

$$
\begin{aligned}
(9a^5b^6 \div 3a^2b^2)^2 &= [(9 \div 3)a^{5-2}b^{6-2}]^2 \\
&= (3a^3b^4)^2 \\
&= 3^2a^{3\times 2}b^{4\times 2} \\
&= 9a^6b^8
\end{aligned}
$$

Substitute this into the original expression:

$$
\begin{aligned}
36a^9b^5 \div (9a^5b^6 \div 3a^2b^2)^2 &= 36a^9b^5 \div 9a^6b^8 \\
&= (36 \div 9)a^{9-6}b^{5-8} \\
&= 4a^3b^{-3} \\
&= \frac{4a^3}{b^3}
\end{aligned}
$$

6. Simplify: $\frac{8a^5 \times 3a^4}{12a^3}$

Solution:

$$
\begin{aligned}
\frac{8a^5 \times 3a^4}{12a^3} &= \frac{(8\times 3)a^{5+4}}{12a^3} \\
&= \frac{24a^9}{12a^3} \\
&= 24a^9 \div 12a^3 \\
&= (24 \div 12)a^{9-3} \\
&= 2a^6
\end{aligned}
$$

Practice

Simplify the following:

1. $(m^3 \times n^2)^2 \div m^6 \div n^3$
2. $\frac{64x^4y^{-3}}{8x^{-2}y^2}$
3. $(9m^4 \times 2m^2) \div (2m^5 \times 3m^2)$
4. $24x^7y^8 \div (2x^2y \times 4x^3y^4)$
5. $\frac{15q^4 \times 3q^5}{5q^2}$
6. $75a^{12}b^7 \div (15a^4b^3 \div 3a^3b^5)^2$
7. $(12a^8 \times 2a^2) \div (5a^5 \times 6a^3)$

Lesson Title: Fractional indices- Part 1	**Theme:** Numbers and Numeration
Practice Activity: PHM1-L036	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to simplify expressions that contain fractional indices.

Overview

Consider how we take the square root of a number, for example $\sqrt{4} = 2$. The square root of a number can be rewritten as a number raised to a fractional index. In this case, the power is $\frac{1}{2}$. So we have $\sqrt{4} = 4^{\frac{1}{2}}$.

Generally, if we need the nth root of any number, say x, we can write it as $\sqrt[n]{x}$. This when translated into index notation becomes $\sqrt[n]{x} = x^{\frac{1}{n}}$. This can be interpreted as, "what number can you multiply by itself n times to get x?" For example: $\sqrt{9} = 9^{\frac{1}{2}} = 3$ means if you multiply 3 by itself two times you get 9 ($3 \times 3 = 9$).

A number can be raised to a fractional power with an integer numerator other than 1, such as $5^{\frac{2}{3}}$. The numerator is treated as a power, and the denominator is the root. This can be written as $5^{\frac{2}{3}} = \sqrt[3]{5^2}$.

Generally, for any number x raised to a fractional power $\frac{m}{n}$ such that m is not 1, we can write the number in the index notation as $\sqrt[n]{x^m} = x^{\frac{m}{n}}$.

There are different ways to simplify the fractional indices whose numerator is not 1. You can either apply the numerator or denominator of the index first. Or, you can substitute the base for an index that makes it easy to simplify. You can use the method you prefer. For example, consider $8^{\frac{2}{3}}$:

Apply the numerator (2) first: $8^{\frac{2}{3}} = \sqrt[3]{8^2} = \sqrt[3]{64} = (64)^{\frac{1}{3}} = (4^3)^{\frac{1}{3}} = 4^{3\times\frac{1}{3}} = 4^1 = 4$

Apply the denominator (3) first: $8^{\frac{2}{3}} = \sqrt[3]{8^2} = (\sqrt[3]{8})^2 = 2^2 = 4$

Substitute 2^3 for 8: $8^{\frac{2}{3}} = (2^3)^{\frac{2}{3}} = 2^{3\times\frac{2}{3}} = 2^2 = 4$

Note that the numbers you are working with may get very large if you apply the numerator first. In many cases, it is better to apply the denominator first because you will have smaller numbers to work with.

The solutions below are examples. You may find another way to solve some of the problems.

Solved Examples

1. Write the following as fractional indices:
   a. $\sqrt{13}$
   b. $\sqrt[7]{x^4}$
   c. $\sqrt[b]{2^a}$

   Solutions:
   a. $13^{\frac{1}{2}}$
   b. $x^{\frac{4}{7}}$
   c. $2^{\frac{a}{b}}$

2. Simplify:
   a. $7^{\frac{1}{2}} \times 7^{\frac{1}{2}}$
   b. $12^{\frac{1}{3}} \times 12^{\frac{1}{3}} \times 12^{\frac{1}{3}}$

   Solutions:
   a. $7^{\frac{1}{2}} \times 7^{\frac{1}{2}} = 7^{\frac{1}{2}+\frac{1}{2}} = 7^1 = 7$
   b. $12^{\frac{1}{3}} \times 12^{\frac{1}{3}} \times 12^{\frac{1}{3}} = 12^{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}} = 12^{\frac{3}{3}} = 12^1 = 12$

3. Simplify:
   a. $81^{\frac{1}{4}}$
   b. $27^{\frac{1}{3}}$

   Solutions:
   a. $81^{\frac{1}{4}} = (3^4)^{\frac{1}{4}} = 3^{4\times\frac{1}{4}} = 3^1 = 3$
   b. $27^{\frac{1}{3}} = (3^3)^{\frac{1}{3}} = 3^{3\times\frac{1}{3}} = 3^1 = 3$

4. Simplify:
   a. $32^{\frac{2}{5}}$
   b. $64^{\frac{2}{3}}$
   c. $125^{\frac{2}{3}}$

   Solutions:
   a. $32^{\frac{2}{5}} = (2^5)^{\frac{2}{5}} = 2^{5^{\times\frac{2}{5}}} = 2^2 = 2 \times 2 = 4$
   or: $32^{\frac{2}{5}} = \sqrt[5]{32^2} = (\sqrt[5]{32})^2 = 2^2 = 4$
   b. $64^{\frac{2}{3}} = (4^3)^{\frac{2}{3}} = 4^{3\times\frac{2}{3}} = 4^2 = 4 \times 4 = 16$
   or: $64^{\frac{2}{3}} = \sqrt[3]{64^2} = (\sqrt[3]{64})^2 = 4^2 = 16$
   c. $125^{\frac{2}{3}} = (5^3)^{\frac{2}{3}} = 5^{3^{\times\frac{2}{3}}} = 5^2 = 5 \times 5 = 25$
   or: $125^{\frac{2}{3}} = \sqrt[3]{125^2} = (\sqrt[3]{125})^2 = 5^2 = 25$

Practice

1. Write as fractional indices:

   a. $\sqrt{43}$ b. $\sqrt[8]{11^2}$

2. Simplify:

   a. $1{,}000^{\frac{1}{3}}$ b. $625^{\frac{1}{2}}$

3. Simplify:

   a. $2^{\frac{1}{3}} \times 2^{\frac{1}{3}} \times 2^{\frac{1}{3}}$

   b. $3^{\frac{1}{4}} \times 3^{\frac{1}{4}} \times 3^{\frac{1}{4}} \times 3^{\frac{1}{4}}$

   c. $8^{\frac{1}{3}} \times 8^{\frac{1}{3}}$

4. Simplify:

   a. $125^{\frac{4}{3}}$ b. $216^{\frac{2}{3}}$

5. Simplify:

   a. $125^{\frac{1}{3}}$ b. $729^{\frac{2}{3}}$

Lesson Title: Fractional indices - Part 2	Theme: Numbers and Numeration
Practice Activity: PHM1-L037	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to simplify more complicated expressions that contain fractional indices.

Overview

This lesson is on simplifying more complicated expressions involving fractional indices, such as fractions raised to a positive or negative fractional power.

When fractions are raised to powers, remember to distribute the power both to the numerator and denominator. Then, simplify them separately. The general rule is: $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$.

For example, you would rewrite $(\frac{16}{25})^{\frac{1}{2}}$ as $\frac{16^{\frac{1}{2}}}{25^{\frac{1}{2}}}$ before solving.

If there is a negative power on a fraction, take the reciprocal of the fraction and raise it to the same **positive** index. The general rule is $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$ provided $a \neq 0$ and $b \neq 0$.

For example, you would rewrite $\left(\frac{8}{125}\right)^{-\frac{1}{3}}$ as $\left(\frac{125}{8}\right)^{\frac{1}{3}}$ before solving.

Solved Examples

1. Simplify: $(\frac{16}{25})^{\frac{1}{2}}$

$(\frac{16}{25})^{\frac{1}{2}} = \frac{16^{\frac{1}{2}}}{25^{\frac{1}{2}}}$ Apply the exponent to both the numerator and denominator

$= \frac{(4^2)^{\frac{1}{2}}}{(5^2)^{\frac{1}{2}}}$ Simplify the numerator and denominator separately

$= \frac{4^{2\times\frac{1}{2}}}{5^{2\times\frac{1}{2}}}$

$= \frac{4^1}{5^1} = \frac{4}{5}$

2. Simplify: $(\frac{8}{125})^{-\frac{1}{3}}$

$\left(\frac{8}{125}\right)^{-\frac{1}{3}} = \left(\frac{125}{8}\right)^{\frac{1}{3}}$ Reciprocal of the base raised to the positive index

$= \frac{(125)^{\frac{1}{3}}}{(8)^{\frac{1}{3}}}$ Apply the power to the numerator and denominator

$= \frac{(5)^{3\times\frac{1}{3}}}{(2)^{3\times\frac{1}{3}}}$ Find numbers that can be multiply by itself three times to have 125 and 8 respectively

$= \frac{5^1}{2^1}$ Simplify

$= \frac{5}{2}$

$= 2\frac{1}{2}$

3. Simplify:
   a. $625^{\frac{3}{4}} \times 5^{\frac{1}{2}} \div 25$
   b. $\left(\frac{27}{125}\right)^{-\frac{1}{3}} \times \left(\frac{4}{9}\right)^{\frac{1}{2}}$

   Solutions:

   a.

$625^{\frac{3}{4}} \times 5^{\frac{1}{2}} \div 25 = (5^4)^{\frac{3}{4}} \times 5^{\frac{1}{2}} \div 5^2$ Simplify each term separately

$= 5^{4\times\frac{3}{4}} \times 5^{\frac{1}{2}} \div 5^2$

$= 5^3 \times 5^{\frac{1}{2}} \div 5^2$

$= 5^{\frac{3}{1}} \times 5^{\frac{1}{2}-\frac{2}{1}}$ Apply BODMAS

$= 5^3 \times 5^{-\frac{3}{2}}$

$= 5^{3+(-\frac{3}{2})} = 5^{\frac{3}{1}-\frac{3}{2}} = 5^{\frac{6-3}{2}}$

$= 5^{\frac{3}{2}} = 5^{1\frac{1}{2}}$

   b.

$\left(\frac{27}{125}\right)^{-\frac{1}{3}} \times \left(\frac{4}{9}\right)^{\frac{1}{2}} = \left(\frac{125}{27}\right)^{\frac{1}{3}} \times \left(\frac{4}{9}\right)^{\frac{1}{2}}$ Invert the fraction and change the negative exponent to positive

$= \frac{125^{\frac{1}{3}}}{27^{\frac{1}{3}}} \times \frac{4^{\frac{1}{2}}}{9^{\frac{1}{2}}}$ Apply exponents separately to numerators and denominators

$= \frac{(5^3)^{\frac{1}{3}}}{(3^3)^{\frac{1}{3}}} \times \frac{(2^2)^{\frac{1}{2}}}{(3^2)^{\frac{1}{2}}}$ Simplify

$= \frac{5^{3\times\frac{1}{3}}}{3^{3\times\frac{1}{3}}} \times \frac{2^{2\times\frac{1}{2}}}{3^{2\times\frac{1}{2}}}$

$= \frac{5}{3} \times \frac{2}{3}$

$= \frac{10}{9} = 1\frac{1}{9}$

4. Simplify: $27^{\frac{2}{3}} \times 64^{\frac{1}{3}} \div 81^{\frac{1}{4}}$

Solution:

$27^{\frac{2}{3}} \times 64^{\frac{1}{3}} \div 81^{\frac{1}{4}}$
$= (3^3)^{\frac{2}{3}} \times (4^3)^{\frac{1}{3}} \div (3^4)^{\frac{1}{4}}$
$= 3^{3\times\frac{2}{3}} \times 4^{3\times\frac{1}{3}} \div 3^{4\times\frac{1}{4}}$
$= 3^2 \times 4 \div 3$
$= 3^{2-1} \times 4$
$= 3 \times 4 = 12$

5. Simplify: $(0.36)^{\frac{1}{2}}$

Solution:

Remember that decimals can easily be converted to fractions. Convert to a fraction and solve.

$(0.36)^{\frac{1}{2}} = (\frac{36}{100})^{\frac{1}{2}}$ Convert the base to a fraction

$= \frac{36^{\frac{1}{2}}}{100^{\frac{1}{2}}}$ Distribute exponent

$= \frac{(6^2)^{\frac{1}{2}}}{(10^2)^{\frac{1}{2}}}$

$= \frac{6^{2\times\frac{1}{2}}}{10^{2\times\frac{1}{2}}}$

$= \frac{6}{10} = \frac{3}{5}$

Practice

1. Simplify:
   a. $25^{\frac{1}{2}} \times 5^{\frac{1}{2}} \times 625^{\frac{1}{4}}$
   b. $(\frac{125}{27})^{\frac{1}{3}} \times (\frac{4}{9})^{-\frac{1}{2}}$
2. Simplify:
   a. $81^{\frac{1}{4}} \times 64^{\frac{1}{3}}$
   b. $\left(\frac{81}{16}\right)^{\frac{1}{4}} \times \left(\frac{2}{3}\right)^{0} \times \left(\frac{4}{9}\right)^{\frac{1}{2}}$
3. Simplify: $(1\frac{9}{16})^{\frac{1}{2}} \div (15\frac{5}{8})^{-\frac{1}{3}}$
4. Simplify:
   a. $(0.81)^{\frac{1}{2}}$
   b. $(0.16)^{-\frac{1}{2}}$

Lesson Title: Simple equations using indices- Part 1	**Theme:** Numbers and Numeration
Practice Activity: PHM1-L038	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to solve simple equations that involve indices.

Overview

Equations that have a variable or unknown quantity as an index or power are referred to as **exponential equations**. For example, $5^x = 5^3$ is an exponential equation. To solve exponent equations, you need to have equations with comparable exponential expressions on either sides of the “equals” sign, so you can compare the powers and solve.

We can solve exponential equations using the following rule:

If $a^x = a^y$, then $x = y$, provided a is not -1, 0 or 1.

We also have exponential equations that are not given with the same base on either side of the “equals” sign. In such cases, we first convert one or both sides of the equation to the same base before we can set the powers equal to each other.

Solved Examples

1. Solve: $5^x = 5^3$

   Solution:

5^x	$= 5^3$	Bases are the same
x	$= 3$	Set the exponents equal

   Your answer can always be checked by substituting into the original exponential equation:

   $5^3 = 5^3$

2. Solve: $10^{1-x} = 10^4$

   Solution:

10^{1-x}	$= 10^4$	Bases are the same
$1 - x$	$= 4$	Set the exponents equal
$1 - 4$	$= x$	Collect like terms
-3	$= x$	

   Answer: $x = -3$

Check:

$$10^{1-(-3)} = 10^4$$
$$10^{1+3} = 10^4$$
$$10^4 = 10^4$$

3. Solve: $5^y = 25$

Solution:

5^y	$= 25$	
5^y	$= 5^2$	Make the bases the same
y	$= 2$	Set the exponents equal

4. Solve: $3^{x+1} = 81$

Solution:

3^{x+1}	$= 81$	
3^{x+1}	$= 3^4$	Make the bases the same
$x + 1$	$= 4$	Set the exponents equal
x	$= 4 - 1$	Transpose 1
x	$= 3$	

5. Solve: $6^y = \frac{1}{216}$

Solution:

6^y	$= \frac{1}{216}$	
6^y	$= (216)^{-1}$	Write as a negative exponent
6^y	$= (6^3)^{-1}$	Substitute $216 = 6^3$
6^y	$= 6^{-3}$	Transpose 1
y	$= -3$	Set the exponents equal

6. Solve: $2^{2x-1} = 32$

Solution:

2^{2x-1}	$= 32$	
2^{2x-1}	$= 2^5$	Make the bases the same
$2x - 1$	$= 5$	Set the exponents equal
$2x$	$= 5 + 1$	Solve for x
$2x$	$= 6$	
x	$= 3$	

7. Solve: $10^{2x-2} = \frac{1}{10{,}000}$

Solution:

10^{2x-2}	$= \frac{1}{10{,}000}$	
10^{2x-2}	$= (10{,}000)^{-1}$	Write as a negative exponent

$$
\begin{aligned}
10^{2x-2} &= (10^4)^{-1} && \text{Substitute } 10{,}000 = 10^4 \\
10^{2x-2} &= 10^{-4} && \text{Simplify} \\
2x-2 &= -4 && \text{Set the exponents equal} \\
2x &= -4+2 && \text{Solve for } x \\
2x &= -2 \\
x &= -1
\end{aligned}
$$

8. Solve: $10^x = 0.001$

Solution:

$$
\begin{aligned}
10^x &= \frac{1}{1{,}000} && \text{Convert the decimal to a fraction} \\
10^x &= (1{,}000)^{-1} && \text{Write as a negative exponent} \\
10^x &= (10^3)^{-1} && \text{Substitute } 1{,}000 = 10^3 \\
10^x &= 10^{-3} && \text{Simplify} \\
x &= -3 && \text{Set the exponents equal}
\end{aligned}
$$

9. Solve: $9^{x-1} = \frac{1}{81}$

Solution:

$$
\begin{aligned}
9^{x-1} &= \frac{1}{81} \\
3^{2(x-1)} &= \frac{1}{3^4} && \text{Make the bases the same} \\
3^{2(x-1)} &= 3^{-4} && \text{Write as a negative exponent} \\
2x-2 &= -4 && \text{Set the exponents equal} \\
2x &= -4+2 && \text{Solve for } x \\
2x &= -2 \\
x &= -1
\end{aligned}
$$

Practice

Solve the following exponential equations:

1. $2^{x+1} = 2^5$
2. $2^x = 128$
3. $10^{2x-1} = 10^3$
4. $4^x = 16$
5. $16^x = 0.25$
6. $5^{x-1} = 625$
7. $32^{x+1} = 64$
8. $10^x = 0.0001$
9. $4^x - 32 = 0$
10. $5^x = \frac{1}{125}$

Lesson Title: Simple equations using indices - Part 2	**Theme:** Numbers and Numeration
Practice Activity: PHM1-L039	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to solve simple equations that involve indices.

Overview

In the previous lesson, you learned how to solve simple exponential equations. Exponential equations are equations in which variables occur as exponents. If two indices are set equal and the bases are equal, then the powers must also be equal. This lesson handles problems that are more challenging than in the previous lesson.

Solved Examples

1. Solve: $3^{2n+1} = 81$

   Solution:

3^{2n+1}	$= 81$	
3^{2n+1}	$= 3^4$	Make the bases the same
$2n + 1$	$= 4$	Set the exponents equal
$2n$	$= 4 - 1$	Solve for n
$2n$	$= 3$	
n	$= \frac{3}{2} = 1\frac{1}{2}$	

   Check by substituting for n in the equation:

3^{2n+1}	$= 81$	
$3^{2\left(\frac{3}{2}\right)+1}$	$= 81$	Make the bases the same
3^{3+1}	$= 81$	Set the exponents equal
3^4	$= 81$	Solve for n

2. Solve: $4^{2x} = 8^{x-1} \times 4$

   Solution:

4^{2x}	$= 8^{x-1} \times 4$	
$2^{2(2x)}$	$= 2^{3(x-1)} \times 2^2$	Make the bases the same
2^{4x}	$= 2^{3x-3+2}$	Simplify powers
2^{4x}	$= 2^{3x-1}$	
$4x$	$= 3x - 1$	Set the exponents equal
$4x - 3x$	$= -1$	Solve for x
x	$= -1$	

3. Solve: $5^{x+3} = 25^{x+1} \div 125$

Solution:

5^{x+3}	$= 25^{x+1} \div 125$	
5^{x+3}	$= 5^{2(x+1)} \div 5^3$	Make the bases the same
5^{x+3}	$= 5^{2x+2-3}$	Apply the second law of indices to RHS
5^{x+3}	$= 5^{2x-1}$	Simplify powers
$x+3$	$= 2x-1$	Set the exponents equal
$3+1$	$= 2x-x$	Solve for x
x	$= 4$	

4. Solve: $4^{2x-1} = \frac{1}{16}$

Solution:

4^{2x-1}	$= \frac{1}{16}$	
$2^{2(2x-1)}$	$= \frac{1}{2^4}$	Convert to the same base
2^{4x-2}	$= 2^{-4}$	Write as a negative exponent
$4x-2$	$= -4$	Set the exponents equal
$4x$	$= -4+2$	Set the exponents equal
$4x$	$= -2$	
x	$= -\frac{2}{4} = -\frac{1}{2}$	

5. Find the value of x if $8^{2x-1} \div 4^{x+1} = 128$.

Solution:

$8^{2x-1} \div 4^{x+1}$	$= 128$	
$2^{3(2x-1)} \div 2^{2(x+1)}$	$= 2^7$	Make the bases the same
$2^{6x-3} \div 2^{2x+2}$	$= 2^7$	Simplify powers
$2^{6x-3-(2x+2)}$	$= 2^7$	Apply the second law of indices to LHS
$2^{6x-3-2x-2}$	$= 2^7$	Simplify powers
2^{4x-5}	$= 2^7$	
$4x-5$	$= 7$	Set the exponents equal
$4x$	$= 7+5$	Solve for x
$4x$	$= 12$	
x	$= 3$	

6. Solve: $3^{2x-1} \times 9 = \frac{1}{27}$

Solution:

$3^{2x-1} \times 9$	$= \frac{1}{27}$	
$3^{2x-1} \times 3^2$	$= \frac{1}{3^3}$	Make the bases the same
$3^{2x-1} \times 3^2$	$= 3^{-3}$	Change to negative exponent
3^{2x-1+2}	$= 3^{-3}$	Apply the first law of indices to LHS

3^{2x+1}	$= 3^{-3}$	Simplify powers
$2x + 1$	$= -3$	Set the exponents equal
$2x$	$= -3 - 1$	Solve for x
$2x$	$= -4$	
x	$= -2$	

Practice

Solve the following exponential equations:

1. $2^{3x-1} = 4^{x+3}$
2. $25^{x+2} = 125^{x+1}$
3. $3^{2x+1} \times 3 = 27$
4. $32^{x+1} \div 4^{2x} = 8$
5. $9^{2x-1} = \frac{1}{81}$
6. $64 = 4^{2-x} \times 16^{x+1}$
7. $\frac{1}{9} = 81^{x+1} \div 3^{2x+2}$

Lesson Title: Introduction to standard form	**Theme:** Numbers and Numeration
Practice Activity: PHM1-L040	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to express and interpret number in standard form.

Overview

Consider how the following numbers are written in index form.

Multiples of ten	Index form
1,000	$10 \times 10 \times 10 = 10^3$
100	$10 \times 10 = 10^2$
10	10^1
1	10^0
0.1	$\frac{1}{10} = 10^{-1}$
0.01	$\frac{1}{100} = \frac{1}{10^2} = 10^{-2}$
0.001	$\frac{1}{1{,}000} = \frac{1}{10^3} = 10^{-3}$

Every number can be expressed and interpreted as a product of the number and a power of ten. Numbers written in this form are said to be in **standard form,** which is the focus of this lesson.

“Standard form” or standard index form is a system of working with very large or very small numbers. For example, 0.00000024 is a very small number, and 5,400,000 is a very large number.

Working with very small or very large numbers can be difficult because of the number of zeroes you have to write, as in the examples above. Expressing numbers in standard form gives us a shorter way out of the difficulty.

Every number can be written in standard form using the notation $a \times 10^n$ where $1 \leq a < 10$ and n is an integer.

Standard form is a number between 1 and 10, which is multiplied by a power of 10. Often, the number being multiplied (a) is a decimal number.

When **very small numbers** are changed to standard form, they have a **negative power.** When **very large numbers** are changed to standard form, they have a **positive power.**

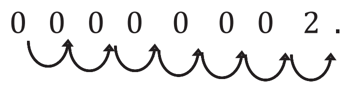
To change a number to standard form, first write it as a number between 1 and 10. Then, count the number of spaces you need to move the decimal point to get the new decimal number. The number of spaces you move the decimal point tells you what power to use on the 10.

Solved Examples

1. Write the following numbers in standard form:
   a. 0.00000024
   b. 5,400,000

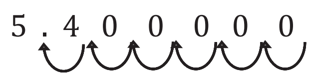
   Solutions:

   a. To get a number between 1 and 10, the decimal point needs to move 7 places to the right, to come after the first non-zero integer.



   Thus, we have $0.00000024 = 2.4 \times 10^{-7}$. The power on the 10 is 7 because that's how many places we moved the decimal point. It is negative because we are dealing with a very small number.

   b. To get a number between 1 and 10, the decimal point needs to be moved 6 places to the left, to come after the first integer.



   Thus, we have $5{,}400{,}000 \quad = 5.4 \times 10^{6}$. The power on the 10 is 6 because that's how many places we moved the decimal point. It is positive because we are dealing with a very big number.

2. Write the numbers in standard form:
   a. 87.2×10^{6}
   b. 0.257×10^{-4}

   Solutions:

   a. The whole number part (87) is not between 1 and 10, so the number is not in standard form. To write it in standard form, shift the decimal point to the left to come after 8 (the first non-zero digit) and multiply by 10^{+1}. This adds one more to the power on the 10, making it 7.

$$\begin{aligned} 87.2 \times 10^{6} &= 8.72 \times 10^{1+6} \\ &= 8.72 \times 10^{7} \end{aligned}$$

   b. The whole number part is 0, which is not between 1 and 10. To change the number to standard form, shift the decimal point to the right to come after 2 (the first non-zero digit) and multiply by 10^{-1}.

$$\begin{aligned} 0.257 \times 10^{-4} &= 2.57 \times 10^{-1} \times 10^{-4} \\ &= 2.57 \times 10^{-1+(-4)} \\ &= 2.57 \times 10^{-5} \end{aligned}$$

3. Convert these numbers from standard form to ordinary form:
    a. 3.4×10^{-5}
    b. 2.69×10^{7}

    Solutions:

    a. Shift the decimal point 5 places to the left, and fill the empty slots with zeros: $3.4 \times 10^{-5} = 0.000034$
    b. Shift the decimal point 7 places to the right, and fill the empty slots with zeros: $2.69 \times 10^{7} = 26{,}900{,}000$

4. Write in standard form:
    a. 7102.3
    b. 0.00012

    Solutions:

    a. Shift the decimal point 3 spaces to the left: $7102.3 = 7.1023 \times 10^{3}$
    b. Shift the decimal point 4 spaces to the right: $0.00012 = 1.2 \times 10^{-4}$

Practice

1. Write the following numbers in standard form:
    a. 0.00000513
    b. 99,895,600
2. Write the following standard form in an ordinary form:
    a. 2.752×10^{-2}
    b. 6.75×10^{4}
3. Write the following standard form numbers in an ordinary form:
    a. 1.90×10^{6}
    b. 4.51×10^{-3}
4. Write the following numbers in standard form:
    a. 3,201.7
    b. 0.0000002

Lesson Title: Standard form addition and subtraction	**Theme:** Numbers and Numeration
Practice Activity: PHM1-L041	**Class:** SSS 1
Learning Outcome By the end of the lesson, you will be able to add and subtract numbers in standard form.	

Overview

To add and subtract standard form numbers with the same power of 10, simply add or subtract their number parts and then multiply this by the same power of 10. Make sure the result is a number in a valid standard form. If the answer is not in standard form, convert it.

When two numbers have different powers of 10 in their standard form, first convert into decimal form or ordinary numbers before adding or subtracting. Then, convert your answer back into standard form.

Solved Examples

1. Evaluate $3.4 \times 10^5 + 2.5 \times 10^5$
   Solution:

$$\begin{array}{r} 3.4 \times 10^5 \\ +2.5 \times 10^5 \\ \hline 5.9 \times 10^5 \\ \hline \end{array}$$

2. Evaluate $2.9 \times 10^{-4} \times 2.2 \times 10^{-4}$
   Solution:

$$\begin{array}{r} 2.9 \times 10^{-4} \\ -2.2 \times 10^{-4} \\ \hline 0.7 \times 10^{-4} \\ \hline \end{array}$$

   Note: This result is not in standard form. Remember standard forms are written in this form $a \times 10^n$, where our a should be between 1 and 10. Therefore convert your answer to standard form.

$$0.7 \times 10^{-4} = (7 \times 10^{-1}) \times 10^{-4} = 7 \times 10^{-1+(-4)} = 7 \times 10^{-5}$$

3. Calculate $3.5 \times 10^4 + 2.45 \times 10^5$
   Solution:
   The powers of 10 are not the same. Therefore convert to ordinary numbers and add, then convert your answer to standard form.
   Step 1. Convert to ordinary numbers:
   $3.5 \times 10^4 = 35{,}000$
   $2.45 \times 10^5 = 245{,}000$

Step 2. Add:

$$\begin{array}{r} 35000 \\ \underline{+245000} \\ \underline{280000} \end{array}$$

Step 3. Convert to standard form: $280{,}000 = 2.8 \times 10^5$

4. Calculate $7.45 \times 10^{-4} + 1.3 \times 10^{-5}$
   Solution:

   Convert to ordinary form:
   $7.45 \times 10^{-4} = 0.000745$
   $1.3 \times 10^{-5} = 0.000013$
   Add:

$$\begin{array}{r} 0.000745 \\ \underline{+\ 0.000013} \\ \underline{0.000758} \end{array}$$

   Convert to standard form: $0.000758 = 7.58 \times 10^{-4}$

5. Evaluate $3.48 \times 10^{-2} - 2.1 \times 10^{-3}$
   Solution:

   Convert to ordinary form:
   $3.48 \times 10^{-2} = 0.0348$
   $2.1 \times 10^{-3} = 0.0021$
   Subtract:

$$\begin{array}{r} 0.0348 \\ \underline{-\ 0.0021} \\ \underline{0.0327} \end{array}$$

   Convert to standard form: $0.0327 = 3.27 \times 10^{-2}$

6. Simplify $8.47 \times 10^4 + 9.65 \times 10^3$
   Solution:

   Convert to ordinary form:
   $8.47 \times 10^4 = 84{,}700$
   $9.65 \times 10^3 = 9{,}650$
   Add:

$$\begin{array}{r} 84700 \\ \underline{+\ 9650} \\ \underline{94{,}350} \end{array}$$

   Convert to standard form:$94{,}350 = 9.435 \times 10^4$

7. Evaluate $5.4 \times 10^{-3} - 2.5 \times 10^{-4}$

Solution:

Convert to ordinary form:

$5.4 \times 10^{-3} = 0.0054$

$2.5 \times 10^{-4} = 0.00025$

Subtract:

```
  0.00540
- 0.00025
  0.00515
```

Convert to standard form: $0.00515 = 5.15 \times 10^{-3}$

Practice

Evaluate the following and give your answers in standard form:

1. $6.8 \times 10^{5} + 2.4 \times 10^{5}$
2. $5.4 \times 10^{-3} + 1.8 \times 10^{-3}$
3. $5.7 \times 10^{-6} - 3.4 \times 10^{-6}$
4. $8.4 \times 10^{3} - 5.3 \times 10^{3}$
5. $9.14 \times 10^{5} + 6.2 \times 10^{4}$
6. $4.72 \times 10^{-4} + 3.48 \times 10^{-5}$
7. $8.74 \times 10^{-3} - 4.81 \times 10^{-4}$
8. $7.64 \times 10^{4} - 3.87 \times 10^{3}$

Lesson Title: Standard form multiplication and division	**Theme:** Numbers and numeration
Practice Activity: PHM1-L042	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to multiply and divide numbers in standard form.

Overview

To multiply numbers in standard form, multiply the number parts and multiply the powers of 10. Apply the laws of indices in multiplying the powers of 10, that is, add the powers.

To divide in standard form, divide the number parts and divide the powers of 10. Apply the laws of indices in dividing the powers of 10, that is, subtract the powers.

Solved Examples

1. Simplify: $(9.1 \times 10^5) \times (2 \times 10^3)$. Give your answer in standard form.
   Solution:

$(9.1 \times 10^5) \times (2 \times 10^3)$	$=$	$(9.1 \times 2) \times (10^5 \times 10^3)$	Group numbers, powers of 10
	$=$	$18.2 \times 10^{5+3}$	Multiply
	$=$	18.2×10^8	Apply law of indices
	$=$	$(1.82 \times 10^1) \times 10^8$	Change to standard form
	$=$	1.82×10^9	

2. Simplify $(6 \times 10^{-3}) \div (2 \times 10^3)$ and leave your answer in standard form.
   Solution:

$(6 \times 10^{-3}) \div (2 \times 10^3)$	$=$	$(6 \div 2) \times (10^{-3} \div 10^3)$	Group parts
	$=$	$3 \times 10^{-3-3}$	Divide
	$=$	3×10^{-6}	Apply law of indices

3. Simplify $(6.4 \times 10^{-5}) \times (3.2 \times 10^{-3})$. Give your answer in standard form.
   Solution:

$(6.4 \times 10^{-5}) \times (32 \times 10^{-3})$	$= (6.4 \times 3.2) \times (10^{-5} \times 10^{-3})$	Group parts
	$= 20.48 \times 10^{-5+(-3)}$	Multiply
	$= 20.48 \times 10^{-5-3}$	Apply law of indices
	$= 20.48 \times 10^{-8}$	
	$= (2.048 \times 10^1) \times 10^{-8}$	Change to standard form
	$= 2.048 \times 10^{1-8}$	
	$= 2.048 \times 10^{-7}$	

4. Simplify $\frac{4.75\times10^{-8}}{2.5\times10^{-7}}$. Give your answer in standard form.

Solution:

$$\begin{aligned}\frac{4.75\times10^{-8}}{2.5\times10^{-7}} &= (4.75\times10^{-8})\div(2.5\times10^{-7})\\ &= (4.75\div2.5)\times(10^{-8}\div10^{-7})\\ &= 1.9\times10^{-8-(-7)}\\ &= 1.9\times10^{-8+7}\\ &= 1.9\times10^{-1}\end{aligned}$$

5. Simplify $\frac{8.6\times10^{17}}{2.5\times10^{7}}$, and leave your answer in standard form.

Solution:

$$\begin{aligned}\frac{8.6\times10^{17}}{2.5\times10^{7}} &= (8.6\div2.5)\times(10^{17}\div10^{7})\\ &= 3.44\times10^{17-7}\\ &= 3.44\times10^{10}\end{aligned}$$

Practice

1. Simplify $(5.7\times10^{8})\times(4\times10^{3})$ and give your answer in standard form.
2. Simplify $(7.46\times10^{-3})\times(1.14\times10^{-1})$. Give your answer in standard form.
3. Simplify $(6.4\times10^{-5})\div(1.8\times10^{-4})$ and give the answer in standard form.
4. Calculate $(8.63\times10^{5})\div(2.6\times10^{4})$ and give the answer in standard form.
5. Simplify $\frac{9.43\times10^{15}}{2.3\times10^{4}}$ and give the answer in standard form.

Lesson Title: Practice application of standard form	**Theme:** Numbers and Numeration
Practice Activity: PHM1-L043	**Class:** SSS 1

Learning Outcome

By the end of the lesson, you will be able to apply operations on numbers in standard form to real-life problems.

Overview

You are going to learn how to solve real life problems using standard form. Many real-life situations involve either very large or very small numbers.

In science, astronomy, or engineering many measurements are given in very small or very large numbers. Examples include:

- The size of very small particles at the atomic level.
- Very large amounts of money in a bank or government budget, we normally deal with millions (1×10^6 units).
- Populations of different species of living things in biology and demography.
- Inter-planetary/celestial distances that can be measured in millions and billions of light years.
- In the health profession, the purity of water and other substances can be measured with so many parts per million.

The use of standard form could be a convenient way of representing such real-life quantitative data.

Solved Examples

1. The population of a certain country is estimated to be 5.7×10^6 people and the land area is $3 \times 10^4\ \text{km}^2$. Calculate the number of people per square kilometer of that country (population density).
   Solution:
   The number of people living per square kilometer of land in that country also known as population density is given by:

$$\begin{aligned}\text{Population density} &= \frac{\text{number of people occuping space}}{\text{area of space occupied}}\\ &= (5.4 \times 10^6) \div (3 \times 10^4)\\ &= (5.7 \div 3) \times (10^6 \div 10^4)\\ &= 1.9 \times 10^{6-4}\\ &= 1.9 \times 10^2 \text{ people per km}^2\end{aligned}$$

2. The following allocations were made in the budget of a local government for the 1st quarter of a year:

Telecommunications	Le 294,000,000.00
Education	Le 301,000,000.00
Health and sanitation	Le 409,500,000.00
Infrastructure	Le 510,000,000.00

a. Find the total budgeted amount
b. Express the sum in standard form

Solutions:

a.	Telecommunications	294,000,000.00
	Education	301,000,000.00
	Health and sanitation	409,500,000.00
	Infrastructure	+ 510,000,000.00
	Total	Le 1,514,500,000.00

b. 1,514,500,000.00 $= \text{Le } 1.5145 \times 10^9$

3. The distance between the earth and the two nearest stars (proxima centauri and alpha centauri) are 3.97×10^7 km and 4.16×10^7 km, respectively. Find how much further from earth alpha centauri is than proxima centauri. Express your result in standard form.

Solution:

Difference = distance to alpha centauri - distance to proxima centauri
$= (4.16 \times 10^7) - (3.97 \times 10^7)$
$= (4.16 - 3.97) \times 10^7$
$= 0.19 \times 10^7$ km
$= 1.9 \times 10^6$ km

4. If the mass of an atom is made up of the mass of the protons and neutrons found in the nucleus, find the mass of a helium atom if it has two protons and two neutrons.

Mass of a proton $= 1.67 \times 10^{-27}$kg
Mass of a neutron $= 1.67 \times 10^{-27}$kg

Solution:

Mass of helium atom = mass of 2 protons + mass of 2 neutrons
$= 2 \times 1.67 \times 10^{-27} + 2 \times 1.67 \times 10^{-27}$
$= 3.34 \times 10^{-27} + 3.34 \times 10^{-27}$
$= (3.34 + 3.34) \times 10^{-27}$
$= 6.68 \times 10^{-27} kg$

Practice

1. According to Statistics Sierra Leone (SSL), the approximate population of Sierra Leone according to the 2016 national census is 7.396×10^6 people. If the area of

the country is 71,740 km^3, estimate the population density of Sierra Leone in standard form.

2. According to the 2016 National budget, the following allocations were made to education by the government:

Primary Education	Le 90,000,000.00
Secondary Education	Le 200,000,000.00
Tertiary education	Le 400,000,000.00

Find the total budget allocated to education in that year in standard form.

3. The masses of four of the moons of the planet Jupiter are 2.45×10^{10} kg, 4.0×10^{10} kg, 3.3×10^{10} kg and 2.0×10^{10} kg. Determine the total mass of the four moons.
4. If the mass of an atom is made up of the mass of the protons and neutrons found in the nucleus, find the mass of a carbon atom if it has 4 protons and 2 neutrons. The mass of one proton or neutron is 1.67×10^{-27} kg.

Lesson Title: Relationship between logarithms and indices	**Theme:** Numbers and Numeration
Practice Activity: PHM1-L044	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to identify the relationship between logarithms and indices (e.g. $y = 10^k$ implies $\log_{10} y = k$).

Overview

This lesson is on the relationship between logarithms and indices. Logarithmic form is the opposite of index form, just as subtraction is the opposite of addition and division is the opposite of multiplication.

Logarithms have the form $\log_b y = x$, where b is the base. $\log_b y = x$ can be read "log to base b of y equals x". Each equation written as a logarithm has an equivalent equation written as an index.

This relationship is shown below:

Solved Examples

1. Convert $6^3 = 216$ to its equivalent logarithm.
   Solution:
   The base (6) is in the position next to "log". The 3 and 216 switch positions, so the exponent is on the right-hand side of the equation:
   $\log_6(216) = 3$

2. Convert $4^5 = 1024$ to its equivalent logarithm.
   Solution:
   The base remains the same but 5 and 1024 switch positions in the logarithmic notation:
   $\log_4(1024) = 5$

3. Convert $\log_a p = b$ to index form.
   Solution:
   The base a is raised to the power b, and set equal to p:
   $a^b = p$

4. Convert $\log_3(xy) = 2$ to index form.

Solution:
The base 3 is raised to the power 2, and set equal to xy:
$3^2 = xy$

5. Convert $10^5 = 100{,}000$ to its equivalent logarithm.
Solution:
The base remains the same but 5 and 100,000 switch positions in the logarithm notation:
$\log_{10}(100{,}000) = 5$

6. Complete the following table. Convert equations given in index form to logarithm form. Convert equations given in logarithm form to index form.

Index form	Logarithm form
$10^3 = 1000$	
	$\log_q N = k$
$7^x = y$	
	$\log_a(MN) = b$
$4^3 = 64$	

Solution:

Index form	Logarithm form
$10^3 = 1000$	$\log_{10} 1000 = 3$
$q^k = N$	$\log_q N = k$
$7^x = y$	$\log_7 y = x$
$a^b = MN$	$\log_a(MN) = b$
$4^3 = 64$	$\log_4 64 = 3$

Practice

1. Write the following in their equivalent logarithmic form:
   a. $2^4 = 16$
   b. $9^2 = 81$
2. Write the following in their equivalent index form:
   a. $\log_5 125 = 3$
   b. $\log_3 81 = m$
3. Convert the following to their equivalent logarithmic form:
   a. $5^{-2} = 0.04$
   b. $b^a = 34$
4. Convert the following to their equivalent index form:
   a. $\log_5 625 = 4$

b. $\log_3 \sqrt{27} = \frac{3}{2}$

5. Complete the following table:

Index form	Logarithm form
$5^{-1} = 0.2$	
	$\log_a X = y$
$6^r = s$	
	$\log_a(XY) = b$
$7^3 = 343$	

Lesson Title: Solving logarithms using indices	**Theme:** Numbers and Numeration
Practice Activity: PHM1-L045	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to solve logarithms using the relationships to indices.

Overview

In the last lesson, you learned how to write exponential expressions in logarithm form, for example:

$4^4 = 256 \rightarrow \log_4 256 = 4$

$10^{-4} = 0.0001 \rightarrow \log_{10} 0.0001 = -4$

This lesson is on solving logarithms. For example, consider $x = \log_3 9$. The logarithm is equal to an unknown value, x. We can use the relationship to indices to solve for x. Convert the equation to one with an index, then apply the rules for solving equations involving indices.

Solved Examples

1. Solve $x = \log_3 9$

   Solution:

   $x = \log_3 9$

   $9 = 3^x$ Change to index form

   $3^2 = 3^x$ Substitute $9 = 3^2$

   $2 = x$

2. Find the value of $\log_3 27$

   Solution:

   This logarithm is not equal to anything, but we can still write it in index form. Set it equal to x, then solve for x to get its value.

   $\log_3 27 = x$

   $27 = 3^x$ Change to index form

   $3^3 = 3^x$ Substitute $27 = 3^3$

   $3 = x$

3. Find the value of y if $\log_5 125 = y$.

   Solution:

   $\log_5 125 = y$

   $125 = 5^y$ Change to index form

   $5^3 = 5^y$ Substitute $125 = 5^3$

   $3 = y$

4. Find the value of p if $\log_{10} 0.0001 = p$

Solution:

Let $\log_{10} 0.0001 = p$

$0.0001 = 10^p$ Change to index form

$\frac{1}{10000} = 10^p$ Convert decimal to fraction

$\frac{1}{10^4} = 10^p$ Substitute $10{,}000 = 10^4$

$10^{-4} = 10^p$

$-4 = p$

5. Find the value of $\log_3 \frac{1}{81}$.

Solution:

Let $\log_3 \frac{1}{81} = x$

$\frac{1}{81} = 3^x$ Change to index form

$\frac{1}{3^4} = 3^x$ Substitute $81 = 3^4$

$3^{-4} = 3^x$

$-4 = x$

6. Find the value of $\log_3 \sqrt{27}$.

Solution:

Let $\log_3 \sqrt{27} = y$

$\sqrt{27} = 3^y$ Change to index form

$(3^3)^{\frac{1}{2}} = 3^y$ Substitute $27 = 3^3$

$3^{\frac{3}{2}} = 3^y$

$y = \frac{3}{2} = 1\frac{1}{2}$

7. If $\log_9 3 = n$, find the value of n.

Solution:

$\log_9 3 = n$

$3 = 9^n$

$3^1 = 3^{2n}$ Substitute $9 = 3^2$

$1 = 2n$ Equate powers

$n = \frac{1}{2}$ Divide both sides by 2

8. If $\log_{10} p = 4$, what is the value of p?

Solution:

$\log_{10} p = 4$

$p = 10^4$ Change to index form

$p = 10{,}000$

Practice

1. Find the value of $\log_3 27$.
2. Find the value of $\log_5 625$.
3. Find the value of $\log_2 0.25$.
4. Find the value of $\log_2 \sqrt{8}$.
5. Solve $\log_3 \frac{1}{243} = x$.
6. Find the value of $\log_5 0.04$.
7. If $\log_2 x = 5$, find the value of x.
8. Find the value of $\log_2 128$.
9. Find the value of $\log_{10} 100{,}000$.

Lesson Title: Logarithms – Numbers greater than 1	Theme: Numbers and Numeration
Practice Activity: PHM1-L046	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to find the logarithms of numbers greater than 1 using logarithm tables.

Overview

We can write the logarithm of each number as a decimal number. Logarithms of numbers can be found either using calculators or the logarithm table. The logarithm of a number is given in two parts, an integer (or whole number) before the decimal point, and a fractional part after the decimal point. The whole number part is called the characteristic and the decimal part is called the mantissa.

The **characteristic** can be found by expressing the number you are finding the logarithm of in standard form. The power on the 10 is the characteristic. For example, numbers in standard form are written as $a \times 10^n,$ where n is the characteristic of the number. As a general note, if the number is greater than 1, then the characteristic is the number of digits before the decimal point, minus one. For example, the characteristic of 104.6 is 2, since the whole number part has 3 digits.

The **mantissa** is found using a logarithm table. In the table, look for each digit of the number you are taking the logarithm of. The first 2 digits are in the far-left column. If there is a 3rd digit, it will correspond to one of the large columns marked 1-9. If there is a 4th digit, find it in the "mean differences" columns, and add the number found there to the 4-digit number that you found from the first 3 digits.

Note that we will only be working with logarithms of numbers in base 10. The logarithm tables are in the Appendix.

Solved Examples

1. Find $\log_{10} 76.83$.
   Solution:
   The characteristic is 1, since we have two digits in the whole number part (7 and 6). Now look for the decimal part in the log table. Move along the row beginning with 76 and under 8, which gives 8854. Next find the number in the difference column headed 3. The number is 2. Add the 2 to 8854 to get 8856.

   Therefore $\log 76.83 = 1.8856$.

   Note, when using the log tables, always make sure that the number taken from the difference column is in the same row as the rest of the figures

2. Find $\log 37$.
   Solution:
   Logarithms can be solved by converting to standard form, and rewriting the decimal part using the logarithm table:

37	$=$	3.7×10^1	Write in standard form
	$=$	$10^{0.5682} \times 10^1$	From log table; $3.7 = 10^{0.5682}$
	$=$	$10^{0.5682+1}$	Apply law of indices
	$=$	$10^{1.5682}$	

   Hence $\log 37 = 1.5682$

3. Find the logarithm of 432.5.
4. **Solution:**

432.5	$= 4.325 \times 10^2$	Express in standard form
	$= 10^{0.6360} \times 10^2$	From log table, look for row 43 and the column '2', and difference 5 gives 6360
	$= 10^{0.6360+2}$	
	$= 10^{2.6360}$	

   Hence $\log 432.5 = 2.6360$

5. Find the logarithm 3.82.
   Solution:
   $3.82 = 3.82 \times 10^0$
   $= 10^{0.5821} \times 10^0$
   $= 10^{0.5821+0}$
   $= 10^{0.5821}$
   Hence $\log 3.82 = 0.5821$

Practice

Use the logarithm table on the next page to find the logarithm of the following numbers.

1. 38
2. 4.643
3. 32.81
4. 601.4
5. 5843
6. 59.01

Lesson Title: Antilogarithms – numbers greater than 0	**Theme:** Numbers and Numeration
Practice Activity: PHM1-L047	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to find the antilogarithm of numbers greater than 0 using antilogarithm tables.

Overview

Antilogarithms are the opposite of logarithms. They "undo" logarithms. They are called "antilog" for short. We use tables of antilogarithms to solve antilog problems. Antilogarithms and logarithms each have their own table. We look for antilog in a table the same way as logarithms. Remember that the logarithm of numbers is made up of two parts: the characteristics and the mantissa. When finding antilog, we look up for the fractional part only (mantissa) in the antilog table. The characteristic (integer) tells us where to move the decimal point in the result. We should always add 1 to the characteristic and move the decimal point from left to right that number of spaces. Remember, when finding logarithm, we minus 1 from the integer part, so we do the opposite here in getting the antilog.

The antilogarithm tables are in the Appendix.

Solved Examples

1. Find the antilog of 0.5768.
   Solution:
   Step 1. Go the antilog table. Look at the row marked .57, under the column headed by 6. The number here is 3767.
   Step 2. Look under column 8 in the "difference" section. The number there is 7.
   Step 3. Add the numbers you got in step 1 to 7, that is: $3{,}767+7=3{,}774$
   Step 4. The characteristic in the number 0.5768 is 0. Add 1 to this and move that number of spaces to the right. 3,774 → 3.774
   Answer: $\text{antilog}(0.5768) = 3.774$

2. If $\log n = 2.3572$, find n.
   Solution:
   Take antilog of both sides: $\text{antilog}(\log n) = \text{antilog}(2.3572)$
   This gives: $n = \text{antilog}\ (2.3572)$
   Find the antilog of 2.3572:
   Step 1. Keep the 2 for now, and consider the decimal part. Go to the antilog table, look in the .35 row, under column 7. That is 2,275.
   Step 2. Look under column 2 in the "difference" section, which gives 1.
   Step 3. Add the difference to what you got in Step 1. That is, 2,275 + 1 = 2,276.

Step 4. Since the characteristic is 2, add 1 to it and move that number of decimal places to the right. 2276 → 227.6.
Answer: $n = 227.6$

3. Find the antilog of 3.7068.
   Solution:
   Step 1. Using the table, .706 → 5,082
   Step 2. Difference from the "8" column: 9
   Step 3. Add: $5{,}082 + 9 = 5{,}091$
   Step 4. Move 4 decimal places: 5,091 → 5,091.0
   Answer: $\text{antilog}(3.7068) = 5{,}091$

4. If $\log x = 1.8113$, find the value of x.
   Solution:
   Take the antilog of both sides: $\text{antilog}(\log x) = \text{antilog}(1.8113)$
   This gives: $x = \text{antilog}(1.8113)$
   Find the antilogarithm of 1.8113:
   Step 1. Using the table, 0.811 → 6,471
   Step 2. Difference from the "3" column: 5
   Step 3. Add: $6{,}471 + 5 = 6{,}476$
   Step 4. Move 2 decimal places: $6{,}476$ → 64.76
   Answer: $x = 64.76$

Practice

1. Find the antilog of the following:
   a. 1.2462
   b. 3.1893
   c. 2.8193
   d. 3.4776
2. If $\log m = 2.1415$, find the value of m.
3. If $\log x = 1.2325$, find the value of x.

Lesson Title: Multiplication and division of logarithms – numbers greater than 1	**Theme:** Numbers and Numeration
Practice Activity: PHM1-L048	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to multiply and divide numbers greater than 1 using logarithms.

Overview

This lesson is on multiplying and dividing numbers using logarithms. For example, this problem would be difficult to work using traditional multiplication: 34.83×5.427

Numbers can be **multiplied** using the following steps:

1. Find the logarithms of the numbers.
2. **Add** the logarithms.
3. Find the antilogarithm of the result.

Numbers can be **divided** using the following steps:

1. Find the logarithms of the numbers.
2. **Subtract** the logarithm of the denominator from the numerator.
3. Find the antilogarithm of the result.

Note that for multiplication, the second step is to add the logarithms. For division, the second step is to subtract the logarithms.

Solved Examples

1. Evaluate: 34.83×5.427

   Solution:

   Step 1. Find the logarithms of the numbers (use the table).

   $$\log 34.83 = 1.5420$$
   $$\log 5.427 = 0.7346$$

   Step 2. Add the logarithms.

   $$\begin{array}{r} 1.5420 \\ +\ 0.7346 \\ \hline 2.2766 \end{array}$$

   Step 3. Find the antilog of 2.2766.
   From the table, we have $1{,}891$. The integer part should be 3 digits, since the characteristic is 2. Therefore, the solution is 189.1.

   The problem 34.83×5.427 can also be solved using a table. Write the numbers you are multiplying in the left column, and their logarithms in the right column. After finding the logarithms, add them with vertical addition. Find the antilog of the result, and write it in the next row:

Numbers	Logarithms
34.83 5.427	1.5420 +0.7346
Add the logarithms for multiplication	2.2766
Antilog of 2.2766	189.1

2. Evaluate: $85.73 \div 39.63$
   Solution:
   Using the table:

Numbers	Logarithms
85.73 39.63	1.9332 –1.5980
Subtract the logs for division	0.3352
Antilog of 0.3352	2.164

3. Evaluate $5.932 \times 8.164 \div 18.51$
   Solution:
   This can be solved in a table as shown below. Work the multiplication, then use additional rows to work the division. Remember to convert all 3 numbers to the logarithm.

Numbers	Logarithms
5.932 8.164	0.7732 +0.9119
Add for multiplication	1.6851
18.51	–1.2674
Subtract for division	0.4177
Antilog of 0.4177	2.616

4. Evaluate $\frac{256.2 \times 17.83}{246.9 \times 8.26}$
   Solution:
   Apply the normal order of operations. Work the numerator and denominator separately before dividing.
   Step 1. Multiply the numerator:

Number	Logarithm
256.2 17.83	2.4086 +1.2511
Add for multiplication	3.6567

Step 2. Multiply the denominator:

246.9 8.26	2.3925 +0.9170
Add for multiplication	3.3095

Step 3. Divide the numerator by the denominator:

Subtract for division	3.6597 –3.3095
	0.3502
Find the antilog of 0.3502	2.240

Answer: 2.2398

Practice

1. Evaluate using logarithm tables:
   a. 7256×2929
   b. 4.561×2.222
2. Evaluate using logarithm tables:
   a. $9292 \div 2567$
   b. $6.451 \div 2.333$
3. Evaluate using logarithm tables:
   a. $\frac{7,652 \times 5}{632}$
   b. $\frac{4 \times 321}{895}$
4. Evaluate using logarithm tables:
   a. $\frac{236.7 \times 521}{312 \times 311}$
5. Evaluate using logarithm tables:
   a. $\frac{325 \div 56}{2.432 \times 2}$
   b. $\frac{5634 \times 2}{2,358 \div 110}$

Answer Key – Term 1

Lesson Title: Review of Numbers and Numeration	
Practice Activity: PHM1-L001	

1. 27, 51, 57, 81, 87
2. a. $54 = 2 \times 3 \times 3 \times 3$ b. $64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$
3. 6
4. 180
5. HCF: 12, LCM: 144

Lesson Title: Addition and Subtraction of fractions	
Practice Activity: PHM1-L002	

1. $\frac{5}{8}$
2. $6\frac{1}{6}$
3. $4\frac{7}{12}$
4. $\frac{1}{2}$
5. $1\frac{1}{8}$ miles
6. $\frac{3}{8}$ mile

Lesson Title: Multiplication and division of fractions	
Practice Activity: PHM1-L003	

1. $\frac{3}{26}$
2. 3
3. $3\frac{1}{2}$
4. $\frac{2}{3}$
5. 6 skirts
6. 35 kg
7. $\frac{2}{3}$

Lesson Title: Addition and Subtraction of decimals	
Practice Activity: PHM1-L004	

1. 37.64
2. 0.778
3. 1.803
4. 2.054
5. 113.9 kg
6. 3.95 kg

Lesson Title: Multiplication and Division of decimals	
Practice Activity: PHM1-L005	

1. 94870
2. 4.7362
3. 0.30504
4. 9
5. 5.98 kg
6. 2.3 kg
7. 30 pages

Lesson Title: Conversion of fractions, percentages and decimals	
Practice Activity: PHM1-L006	

1. a. 25%, b. 36%
2. a. $\frac{16}{25}$, b. $\frac{1}{4}$
3. a. 0.3, b. 0.875
4. a. $\frac{13}{20}$, b. $\frac{1}{8}$
5. a. 0.95, b. 0.07
6. a. 18%, b. 4%

Lesson Title: Finding the percentage of a quantity	
Practice Activity: PHM1-L007	

1. 42 mangoes
2. 135 oranges
3. 600 women
4. 285 newspapers

Lesson Title: Express one quantity as a percentage of another	
Practice Activity: PHM1-L008	

1. 10%
2. 20%
3. 15%
4. a. 80%, b. 20%
5. a. 6.25%, b. 31.25%, c.50%

Lesson Title: Percentage change	
Practice Activity: PHM1-L009	

1. Le 66,500.00
2. 8% increase in shoe production
3. 9.2 seconds
4. a. 52 acres, b. 273 acres
5. Le 12,000.00, Le 72,000.00

Lesson Title: Real world use of fractions
Practice Activity: PHM1-L010

1. 22 hours
2. $\frac{1}{4}$ of his land
3. a.12 pupils; b.16 pupils
4. $3\frac{1}{12}$ yards

Lesson Title: Real world use of decimals
Practice Activity: PHM1-L011

1. 24.9 kg
2. 2.2°C
3. 1.7 kg
4. 65.7

Lesson Title: Approximation of Whole Numbers
Practice Activity: PHM1-L012

1. a. 8,752,590; b. 8,752,600; c. 8,753,000; d. 8,750,000; e. 8,800,000, f. 9,000,000.
2. 2,600,000,000
3. 6,000,000,000
4. See the table:

Number	To the nearest ten	To the nearest hundred	To the nearest thousand	To the nearest million	To the nearest billion	To the nearest trillion
98	100	X	X	X	X	X
568	570	600	X	X	X	X
1,115	1,120	1,100	1,000	X	X	X
3,756,235	3,756,240	3,756,200	3,756,000	4,000,000	X	X
9,567,815,395	9,567,815,400	9,567,815,400	9,567,815,000	9,568,000,000	10,000,000,000	X
2,886,711,231,121	2,886,711,231,120	2,886,711,231,100	2,886,711,231,000	2,886,711,000,000	2,887,000,000,000	3,000,000,000,000

Lesson Title: Approximation in everyday life
Practice Activity: PHM1-L013

1. 8,700,000 votes
2. Le12,200,000.00
3. Le3,200,000.00
4. 16 m.

Lesson Title: Conversion from any other base to base ten	
Practice Activity: PHM1-L014	

1. 61
2. $7\frac{5}{8}$
3. 1232
4. $23\frac{57}{125}$
5. $5{,}770$

Lesson Title: Conversion from base ten to any other base	
Practice Activity: PHM1-L015	

1. 1111_{three}
2. 1156_{eight}
3. 111110101_{two}
4. a. 421_{eight} b. 11222_{three}
5. a. 1011100_{two} b. 4414_{five} c. 12121_{four}

Lesson Title: Conversion between bases	
Practice Activity: PHM1-L016	

1. $11{,}012_{three}$
2. $2{,}031_{six}$
3. 233_{five}
4. $1{,}103_{six}$
5. $14{,}515_{eight}$

Lesson Title: Addition and subtraction of number bases	
Practice Activity: PHM1-L017	

1. a. $1{,}441_{five}$; b. $1{,}011{,}010_{two}$
2. a. 342_{five}; b. $10{,}101_{two}$
3. $1{,}532_{seven}$

Lesson Title: Multiplication of number bases	
Practice Activity: PHM1-L018	

1. 452_{six}
2. $4{,}403_{five}$
3. $1{,}011{,}011_{two}$
4. $P = 11{,}224$
5. 540_{eight}
6. $M = 4{,}460$

Lesson Title: Division of number bases	
Practice Activity: PHM1-L019	

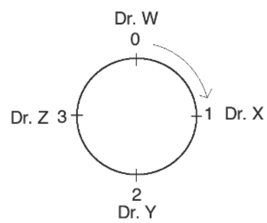
1. 12_{five}
2. 42_{eight}
3. 111_{two}
4. 146_{seven}
5. 101_{two}

Lesson Title: Basic equations involving number bases	
Practice Activity: PHM1-L020	

1. $n = 5$
2. $x = 8$
3. $y = 100$
4. $x = 16$
5. $m = 7$

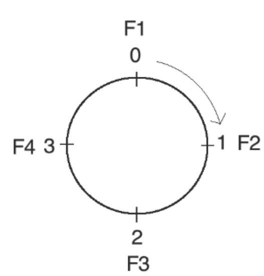
Lesson Title: Introduction to Modular Arithmetic	
Practice Activity: PHM1-L021	

1. a. See the diagram below; b. i. Dr. W, ii. Dr. Z; c. i. Dr. Z, ii. Dr. X, iii. Dr Z



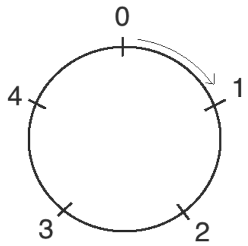
2. See the table and diagram below; b. i. Ferry F3, ii. Ferry F1, iii. Ferry F4, iv. Ferry F1.

Departure schedule/ Hour number	Ferry number
8:00 am / 0	F1
9:00 am / 1	F2
10:00 am / 2	F3
11:00 am / 3	F4



Lesson Title: Simplest form of a given moduli
Practice Activity: PHM1-L022

1. The answer is a cycle labeled 0-4:



2. a. 0 (use diagram above) b. 0
3. a. 1 b. 1 c. 5
4. a. 3 b. 7 c.4

Lesson Title: Operations in various moduli
Practice Activity: PHM1-L023

1. a. $2 (\text{mod } 3)$ b. $3\ (\text{mod } 6)$

2. a. $4\ (\text{mod } 5)$ b. $10\ (\text{mod } 12)$

3. a. $3\ (\text{mod } 9)$ b. $6\ (\text{mod } 10)$

4. See tables below.

a.

Modulo 9				
⊕	2	4	6	8
2	4	6	8	1
4	6	8	1	3
6	8	1	3	5
8	1	3	5	7

b.

Modulo 9				
⊗	2	4	6	8
2	4	8	3	7
4	8	7	6	5
6	3	6	0	3
8	7	5	3	1

Lesson Title: Modular arithmetic in real-life situations
Practice Activity: PHM1-L024

1. a. 7 b. 3
2. a. Wednesday b. Sunday
3. a. 11 b. 5

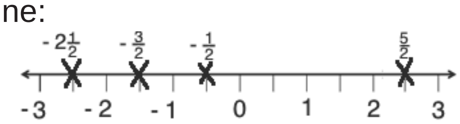
Lesson Title: Rational and irrational numbers	
Practice Activity: PHM1-L025	

1. a. Irrational b. Irrational c. Rational d. Irrational e. Irrational
2. a. Irrational b. Rational c. Rational d. Rational e. Irrational
3. a. Irrational b. Rational c. Rational d. Irrational e. Rational

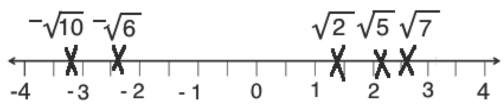
Lesson Title: Real numbers on a number line	
Practice Activity: PHM1-L026	

1. Number line:

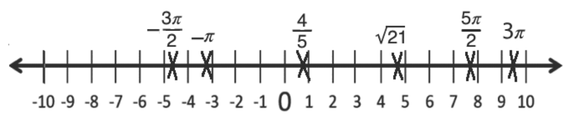
2. Number line:



3. Number line:



4. Number line:



Lesson Title: Comparing and Ordering of rational numbers	
Practice Activity: PHM1-L027	

1. a. $\frac{1}{7} < \frac{3}{7} < \frac{8}{7} < \frac{9}{7}$; b. $-\frac{6}{12} <, -\frac{5}{12} <, -\frac{3}{12} <, -\frac{1}{12}$; c. $-\frac{9}{9} < -\frac{4}{9} < \frac{1}{9} < \frac{8}{9}$
2. a. $\frac{1}{2} > \frac{1}{3} > \frac{1}{4} > \frac{1}{5}$; b. $\frac{6}{7} > \frac{4}{5} > \frac{3}{4} > \frac{2}{3}$
3. a. $3.02 < 3.12 < 3.2 < 3.21$; b. $-2.51 < -2.50 < -2.45 < -2.22$
4. a. $60\% > 0.3 > \frac{1}{5} > 0.14$; b. $1 > \frac{5}{6} > 0.1 > -\frac{4}{5}$; c. $\frac{3}{4} > \frac{1}{2} > 40\% > 0.25$

Lesson Title: Approximating of decimals	
Practice Activity: PHM1-L028	

1. a. 5.2 b. 5.25
2. 6.9
3. 8.20
4. a. 0.102 b. 0.1015
5. a. 15.5 b. 15.50 c. 15.495

Lesson Title: Recurring decimals as common fractions	
Practice Activity: PHM1-L029	

1. $\frac{1}{3}$
2. $\frac{11}{90}$
3. $\frac{4}{33}$
4. $\frac{8}{11}$
5. a. $2\frac{6}{111}$ b. $\frac{25}{99}$
6. $\frac{10}{111}$

Lesson Title: Operations on real numbers	
Practice Activity: PHM1-L030	

1. 55
2. –13
3. a. $\frac{3}{5}$ b. 4
4. –8
5. Evaluate both sides of the equation to find that they are both equivalent to 179.

Lesson Title: Order of operation BODMAS	
Practice Activity: PHM1-L031	

1. 36
2. 11.06 or $11\frac{1}{17}$
3. $\frac{1}{2}$
4. $3\frac{11}{24}$
5. $4\frac{4}{5}$
6. $4\frac{11}{26}$
7. 5

Lesson Title: Index Notation	
Practice Activity: PHM1-L032	

1. a. $5^2 \times 6^3 \times 9^2$ b. $2^2 \times 4^3 \times 7^2$

2. a. $\frac{1}{3^{10}}$ b. $\frac{1}{8^3}$ c. $\frac{1}{6^4}$

3. a. $2^2 \times 3^2 + 4^2 \times 5^2$ b. $3^3 + 4^2 - 2^3$

4. a. $9 \times 9 \times 9$ b. $6 \times 6 \times 6 \times 6 \times 6$ c. $5 \times 5 \times 5 \times 2 \times 2 \times 2 \times 2 \times \frac{1}{3 \times 3 \times 3 \times 3}$

5 a. $\frac{1}{3 \times 3} + 2 \times 2 \times 2 - \frac{1}{4 \times 4}$ b. $\frac{1}{2 \times 2} + 3 \times 3 - \frac{1}{5 \times 5 \times 5}$

Lesson Title: First and Second law of indices	
Practice Activity: PHM1-L033	

1. a^{15}
2. 5^6
3. u^9
4. 14^5
5. $27a^4b^6$
6. $4f^2g^6$
7. $5n$
8. $40x^4y^8$
9. $6m^{10}n$
10. $5u^6v^4$

Lesson Title: Third and Fourth law of indices	
Practice Activity: PHM1-L034	

1. 1
2. 1
3. 1
4. 9^{21}
5. $\frac{1}{a^8}$
6. $4m^6n^8$
7. 1
8. 1
9. $\frac{4x^4}{y^2}$
10. $400a^{10}$

Lesson Title: Simplifying indices	
Practice Activity: PHM1-L035	

1. n
2. $8\frac{x^6}{y^5}$
3. $\frac{3}{m}$
4. $3x^2y^3$
5. $9q^7$

6. $3a^{10}b^{11}$

7. $\frac{4}{5}a^2$

Lesson Title: Fractional indices – Part 1	
Practice Activity: PHM1-L036	

1. a. $43^{\frac{1}{2}}$ b. $11^{\frac{2}{8}} = 11^{\frac{1}{4}}$
2. a. 10 b.25
3. a. 2 b. 3 c. 4
4. a. 625 b. 36
5. a. 5 b. 81

Lesson Title: Fractional indices – Part 2	
Practice Activity: PHM1-L037	

1. a. $5^{2\frac{1}{2}}$ b. $2\frac{1}{2}$
2. a. 12 b. 1
3. $3\frac{1}{8}$
4. a. $\frac{9}{10}$ b. $2\frac{1}{2}$

Lesson Title: Simple equations using indices – Part 1	
Practice Activity: PHM1-L038	

1. 4
2. 7
3. 2
4. 2
5. $-\frac{1}{2}$
6. 5
7. $\frac{1}{5}$
8. –4
9. $2\frac{1}{2}$
10. –3

Lesson Title: Simple equations using indices – Part 2	
Practice Activity: PHM1-L039	

1. 7
2. 1
3. $\frac{1}{2}$
4. –2
5. $-\frac{1}{2}$
6. –1
7. –2

Lesson Title: Introduction to standard form
Practice Activity: PHM1-L040

1. a. 5.13×10^{-6} b. 9.98956×10^{7}
2. a. 0.02752 b. $67{,}500$
3. a. 1,900,000 b. 0.00451
4. a. 3.2017×10^{3} b. 2×10^{-7}

Lesson Title: Standard form addition and subtraction
Practice Activity: PHM1-L041

1. 9.2×10^{5}
2. 7.2×10^{-3}
3. 2.3×10^{-6}
4. 3.1×10^{3}
5. 9.76×10^{5}
6. 5.068×10^{-4}
7. 8.259×10^{-3}
8. 7.253×10^{4}

Lesson Title: Standard form multiplication and division
Practice Activity: PHM1-L042

1. 2.28×10^{12}
2. 8.5044×10^{-4}
3. 3.556×10^{-1} or $3.\overline{5} \times 10^{-1}$
4. 3.32×10^{1} or 3.32×10
5. 4.1×10^{11}

Lesson Title: Practice application of standard form
Practice Activity: PHM1-L043

1. 1. 1.03×10^{2} people per km^2
2. 2. Le 6.9×10^{8}
3. 3. 1.175×10^{11} kg
4. 4 1.002×10^{-26} kg

Lesson Title: Relationships between logarithms and indices
Practice Activity: PHM1-L044

1. a. $\log_2 16 = 4$ b. $\log_9 81 = 2$
2. a. $5^3 = 125$ b. $3^m = 81$
3. a. $\log_5 0.04 = -2$ b. $\log_b 34 = a$
4. a. $5^4 = 625$ b. $3^{\frac{3}{2}} = \sqrt{27}$
5. See table below.

Index form	Logarithm form
$5^{-1} = 0.2$	$\log_5 0.2 = -1$
$a^y = X$	$\log_a X = y$
$6^r = s$	$\log_6 S = r$
$a^b = XY$	$\log_a(XY) = b$
$7^3 = 343$	$\log_7 343 = 3$

Lesson Title: Solving logarithms using indices	
Practice Activity: PHM1-L045	

1. 3
2. 4
3. – 2
4. $\frac{3}{2}$
5. – 5
6. – 2
7. 32
8. 7
9. 5

Lesson Title: Logarithms- Numbers greater than 1	
Practice Activity: PHM1-L046	

1. 1.5798
2. 0.6668
3. 1.5160
4. 2.7792
5. 3.7666
6. 1.7710

Lesson Title: Antilogarithms – Numbers greater than 1	
Practice Activity: PHM1-L047	

1. a. 17.63 b. 1546 c. 659.7 d. 3003
2. 138.6
3. 17.08

Lesson Title: Multiplication and division of logarithm – Number greater than 1	
Practice Activity: PHM1-L048	

1. a. 21,252009.43 b. 10.1344
2. a. 3.6199 b. 2.7650
3. a. 60.5480 b. 1.4348
4. 1.2706
5. a. 1.1929 b.525.6540

Appendix I: Logarithm Table

COMMON LOGARITHM TABLE

	0	1	2	3	4	5	6	7	8	9	Mean Difference								
											1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

Appendix II: Anti-Logarithm Table

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
·01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
·02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
·03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
·04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	2	2	2	2
·05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
·06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	1	2	2	2
·07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
·08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	3
·09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	3
·10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
·11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
·12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
·13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
·14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
·15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
·16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
·17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
·18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
·19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	3	3	3
·20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	3	3	3
·21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1696	0	1	1	2	2	2	3	3	3
·22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	3	3	3
·23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	4
·24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	3	3	4
·25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4
·26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
·27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
·28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
·29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
·30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4
·31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
·32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
·33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	2	2	3	3	4	4
·34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	3	3	4	4	5
·35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	3	3	4	4	5
·36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
·37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1	1	2	2	3	3	4	4	5
·38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3	3	4	4	5
·39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	3	3	4	5	5
·40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	3	4	4	5	5
·41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3	4	4	5	5
·42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1	2	2	3	4	4	5	6
·43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	3	3	4	4	5	6
·44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	2	3	3	4	4	5	6
·45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	3	3	4	5	5	6
·46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
·47	2951	2958	2965	2972	2970	2985	2992	2999	3006	3013	1	1	2	3	3	4	5	5	6
·48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	3	4	4	5	6	6
·49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	3	4	4	5	6	6

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
·51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
·52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
·53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
·54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
·55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
·56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
·57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
·58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
·59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
·60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	6	7	8
·61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
·62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
·63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
·64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
·65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
·66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
·67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
·68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
·69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
·70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
·71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
·72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
·73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
·74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
·75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
·76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
·77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
·78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
·79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
·80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
·81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
·82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
·83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
·84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
·85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
·86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
·87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
·88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
·89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	13	14	16
·90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
·91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
·92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
·93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
·94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
·95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
·96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
·97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
·98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
·99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20

GOVERNMENT OF SIERRA LEONE

FUNDED BY

UKaid
from the British people

IN PARTNERSHIP WITH

INTERNATIONAL RESCUE COMMITTEE

M M
CAMBRIDGE EDUCATION

STRICTLY NOT FOR SALE

Document information:

Leh Wi Learn (2019). *"Maths, SeniorSecondarySchool Year 1 Term 01 Full, pupil handbook."* A resource produced by the Sierra Leone Secondary Education Improvement Programme (SSEIP). DOI: 10.5281/zenodo.3745362.

Uploaded by the EdTech Hub, https://edtechhub.org.
For more information, see https://edtechhub.org/oer.
Archived on Zenodo: April 2020.
DOI: 10.5281/zenodo.3745362