Ministry of Basic and Senior Secondary Education

Lesson Plans for

Senior Secondary *Mathematics Revision*

PART I

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Please see final page for further information.

Foreword

These Lesson Plans and the accompanying Pupils' Handbooks are essential educational resources for the promotion of quality education in senior secondary schools in Sierra Leone. As Minister of Basic and Senior Secondary Education, I am pleased with the professional competencies demonstrated by the writers of these educational materials in English Language and Mathematics.

The Lesson Plans give teachers the support they need to cover each element of the national curriculum, as well as prepare pupils for the West African Examinations Council's (WAEC) examinations. The practice activities in the Pupils' Handbooks are designed to support self-study by pupils, and to give them additional opportunities to learn independently. In total, we have produced 516 lesson plans and 516 practice activities – one for each lesson, in each term, in each year, for each class. The production of these materials in a matter of months is a remarkable achievement.

These plans have been written by experienced Sierra Leoneans together with international educators. They have been reviewed by officials of my Ministry to ensure that they meet the specific needs of the Sierra Leonean population. They provide step-by-step guidance for each learning outcome, using a range of recognized techniques to deliver the best teaching.

I call on all teachers and heads of schools across the country to make the best use of these materials. We are supporting our teachers through a detailed training programme designed specifically for these new lesson plans. It is really important that the Lesson Plans and Pupils' Handbooks are used, together with any other materials they may have.

This is just the start of educational transformation in Sierra Leone as pronounced by His Excellency, the President of the Republic of Sierra Leone, Brigadier Rtd Julius Maada Bio. I am committed to continue to strive for the changes that will make our country stronger and better.

I do thank our partners for their continued support. Finally, I also thank the teachers of our country for their hard work in securing our future.

Mr. Alpha Osman Timbo

Minister of Basic and Senior Secondary Education

The policy of the Ministry of Basic and Senior Secondary Education, Sierra Leone, on textbooks stipulates that every printed book should have a lifespan of three years.

To achieve thus, DO NOT WRITE IN THE BOOKS.

Table of Contents

Introduction
to the Lesson Plans

These lesson plans are based on the National Curriculum and the West Africa Examination Council syllabus guidelines, and meet the requirements established by the Ministry of Basic and Senior Secondary Education.

1. The lesson plans will not take the whole term, so use spare time to review material or prepare for examinations.
2. Teachers can use other textbooks alongside or instead of these lesson plans.
3. Read the lesson plan before you start the lesson. Look ahead to the next lesson, and see if you need to tell pupils to bring materials for next time.
4. Make sure you understand the learning outcomes, and have teaching aids and other preparation ready – each lesson plan shows these using the symbols on the right.
5. If there is time, quickly review what you taught last time before starting each lesson.
6. Follow the suggested time allocations for each part of the lesson. If time permits, extend practice with additional work.
7. Lesson plans have a mix of activities for the whole class and for individuals or in pairs.
8. Use the board and other visual aids as you teach.
9. Interact with all pupils in the class – including the quiet ones.
10. Congratulate pupils when they get questions right! Offer solutions when they don't, and thank them for trying.

Learning outcomes

Preparation

KEY TAKEAWAYS FROM SIERRA LEONE'S PERFORMANCE IN WEST AFRICAN SENIOR SCHOOL CERTIFICATE EXAMINATION – GENERAL MATHEMATICS[1]

This section, seeks to outline key takeaways from assessing Sierra Leonean pupils' responses on the West African Senior School Certificate Examination. The common errors pupils make are highlighted below with the intention of giving teachers an insight into areas to focus on, to improve pupil performance on the examination. Suggestions are provided for addressing these issues.

Common errors

1. Errors in applying principles of BODMAS
2. Mistakes in simplifying fractions
3. Errors in application of Maths learned in class to real-life situations, and vis-a-versa.
4. Errors in solving geometric constructions.
5. Mistakes in solving problems on circle theorems.
6. Proofs are often left out from solutions, derivations are often missing from quadratic equations.

Suggested solutions

1. Practice answering questions to the detail requested
2. Practice re-reading questions to make sure all the components are answered.
3. If possible, procure as many geometry sets to practice geometry construction.
4. Check that depth and level of the lesson taught is appropriate for the grade level.

[1] This information is derived from an evaluation of WAEC Examiners' Reports, as well as input from their examiners and Sierra Leonean teachers.

Using this book

The purpose of this SSS4 Lesson Plans is to review the material that pupils had learned in previous years of schooling, and prepare them for the West African Senior Secondary Certificate Examination (WASSCE).

This book has enough materials for 2 full terms. Depending on your school schedule, you may not have time to teach each lesson and complete each mock exam. Plan your time accordingly, and teach the topics that your pupils need the most review of. It is helpful to assess your pupils at the beginning of the academic year to understand which topics they need to review. This can be done by giving them a short exam similar to the WASSCE exam, with various topics from the syllabus.

There are 8 mock exams provided at the end of this book, in lessons 89 through 96. These are designed to be used in a 40-minute lesson. The exams are shortened so that pupils will have enough time to complete each problem that is similar to the time they will have in the exam. Each lesson plan includes tips for administering the mock exam and preparing pupils to sit the WASSCE exam. It is not necessary to administer the mock exams consecutively, or to wait until the end of the academic year to administer them. You may choose to administer mock exams throughout the year. If your school has additional time for mock exams, design your own exams similar in style to the mock exams in this book, using topics from across the curriculum.

Using the lesson plans

At the SSS level, it is generally better to keep explanations of content brief. Pupils have an overview of each topic in the Pupil Handbook. Spend most of the class time allowing pupils to solve problems in the Teaching and Learning and Practice sections of the lesson plans. If pupils have a good understanding of the topic, ask them to work independently. You may also ask them to solve some problems with seatmates before working independently to solve the rest.

Preparing pupils for the exam

It is important that candidates understand what to expect on the day of the WASSCE exam. Details of the exam are given below. Make sure this is clear to your pupils, and that they are well prepared.

Content of the WASSCE Exam

The WASSCE Mathematics exam consists of 3 sections as described below:

Paper 1 – Multiple Choice

- Paper 1 is 1.5 hours, consists of 50 multiple choice questions, and is worth 50 marks. This gives 1.8 minutes per problem, so time must be planned accordingly.
- The questions are drawn from all topics on the WASSCE syllabus.

Paper 2 – Essay Questions

- Paper 2 consists of 13 essay questions in 2 sections – 2A and 2B.
- Paper 2 is worth 100 marks in total.

- Pupils will be required to answer 10 essay questions in all, across the 2 sections.
- This is an average of 15 minutes per question. However, keep in mind that section 2B is more complicated, so plan your time accordingly.

Paper 2A – Compulsory Questions

- Paper 2A is worth 40 marks.
- There are 5 **compulsory** essay questions in paper 2A.
- Compulsory questions often have multiple parts (a, b, c, …). The questions may not be related to each other. Each part of the question should be completed.
- The questions in 2A are simpler than in 2B, generally requiring fewer steps.
- The questions in 2A are drawn from the common area of the WASSCE syllabus (they are not on topics that are specific to certain countries).

Paper 2B – Advanced Questions

- Paper 2B is worth 60 marks. There are 8 essay questions in paper 2B, and candidates are expected to answer 5 of them.
- Questions in section 2B are of a greater length and difficulty than section 2A.
- A maximum of 2 questions (from among the 8) may be drawn from parts of the WASSCE syllabus that are not meant for Sierra Leone. These topics are not in the Sierra Leone national curriculum for secondary schools. Candidates from Sierra Leone may choose to answer such questions, but it is not required.
- Choose 5 questions on topics that you are more comfortable with.

Exam Day

- Candidates should bring a pencil, geometry set, and scientific calculator to the WASSCE exam.
- Candidates are allowed to use log books (logarithm and trigonometry tables), which are provided in the exam room.

Exam-taking skills and strategies

- Candidates should read and follow the instructions carefully. For example, it may be stated that a trigonometry table should be used. In this case, it is important that a table is used and not a calculator.
- Plan your time. Do not spend too much time on one problem.
- For essay questions, show all of your working on the exam paper. Examiners can give some credit for rough working. Do not cross out working.
- If you complete the exam, take the time to check your solutions. If you notice an incorrect answer, double check it before changing it.
- For section 2B, it is a good idea to skim read the questions quickly before getting started. If you see a topic that you are familiar and comfortable with, work on those problems. Try not to spend a lot of time deciding which problems to solve, or thinking about problems you will not solve.

FACILITATION STRATEGIES

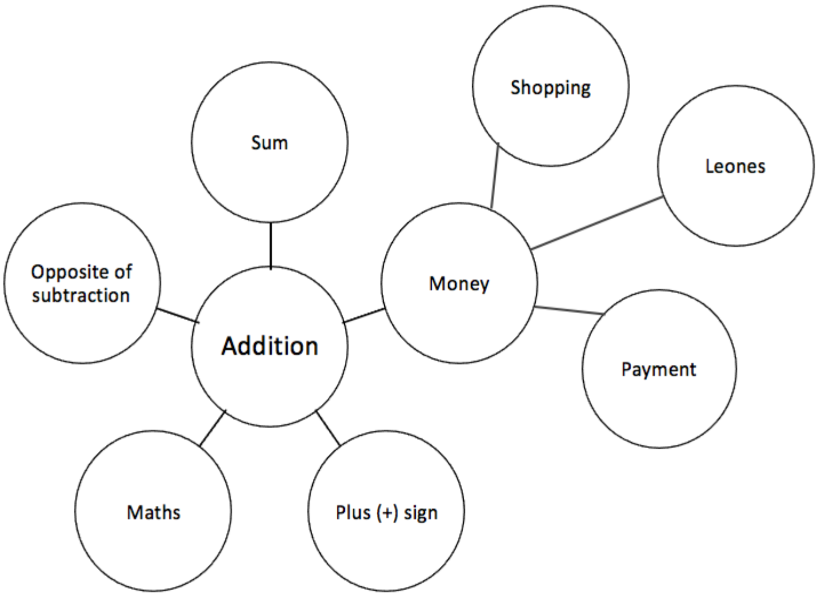
This section includes a list of suggested strategies for facilitating specific classroom and evaluation activities. These strategies were developed with input from national experts and international consultants during the materials development process for the Lesson Plans and Pupils' Handbooks for Senior Secondary Schools in Sierra Leone.

Strategies for introducing a new concept

- **Unpack prior knowledge:** Find out what pupils know about the topic before introducing new concepts, through questions and discussion. This will activate the relevant information in pupils' minds and give the teacher a good starting point for teaching, based on pupils' knowledge of the topic.
- **Relate to real-life experiences:** Ask questions or discuss real-life situations where the topic of the lesson can be applied. This will make the lesson relevant for pupils.
- **K-W-L:** Briefly tell pupils about the topic of the lesson, and ask them to discuss 'What I **k**now' and 'What I **w**ant to know' about the topic. At the end of the lesson have pupils share 'What I **l**earned' about the topic. This strategy activates prior knowledge, gives the teacher a sense of what pupils already know and gets pupils to think about how the lesson is relevant to what they want to learn.
- **Use teaching aids from the environment:** Use everyday objects available in the classroom or home as examples or tools to explain a concept. Being able to relate concepts to tangible examples will aid pupils' understanding and retention.
- **Brainstorming:** Freestyle brainstorming, where the teacher writes the topic on the board and pupils call out words or phrases related that topic, can be used to activate prior knowledge and engage pupils in the content which is going to be taught in the lesson.

Strategies for reviewing a concept in 3-5 minutes

- **Mind-mapping:** Write the name of the topic on the board. Ask pupils to identify words or phrases related to the topic. Draw lines from the topic to other related words. This will create a 'mind-map', showing pupils how the topic of the lesson can be mapped out to relate to other themes. Example below:



- **Ask questions:** Ask short questions to review key concepts. Questions that ask pupils to summarise the main idea or recall what was taught is an effective way to review a concept quickly. Remember to pick volunteers from all parts of the classroom to answer the questions.
- **Brainstorming:** Freestyle brainstorming, where the teacher writes the topic on the board and pupils call out words or phrases related that topic, is an effective way to review concepts as a whole group.
- **Matching:** Write the main concepts in one column and a word or a phrase related to each concept in the second column, in a jumbled order. Ask pupils to match the concept in the first column with the words or phrases that relate to in the second column.

Strategies for assessing learning without writing

- **Raise your hand:** Ask a question with multiple-choice answers. Give pupils time to think about the answer and then go through the multiple-choice options one by one, asking pupils to raise their hand if they agree with the option being presented. Then give the correct answer and explain why the other answers are incorrect.
- **Ask questions:** Ask short questions about the core concepts. Questions which require pupils to recall concepts and key information from the lesson are an effective way to assess understanding. Remember to pick volunteers from all parts of the classroom to answer the questions.
- **Think-pair-share:** Give pupils a question or topic and ask them to turn to seatmates to discuss it. Then, have pupils volunteer to share their ideas with the rest of the class.
- **Oral evaluation:** Invite volunteers to share their answers with the class to assess their work.

Strategies for assessing learning with writing

- **Exit ticket:** At the end of the lesson, assign a short 2-3 minute task to assess how much pupils have understood from the lesson. Pupils must hand in their answers on a sheet of paper before the end of the lesson.
- **Answer on the board:** Ask pupils to volunteer to come up to the board and answer a question. In order to keep all pupils engaged, the rest of the class can also answer the question in their exercise books. Check the answers together. If needed, correct the answer on the board and ask pupils to correct their own work.
- **Continuous assessment of written work:** Collect a set number of exercise books per day/per week to review pupils' written work in order to get a sense of their level of understanding. This is a useful way to review all the exercise books in a class which may have a large number of pupils.
- **Write and share:** Have pupils answer a question in their exercise books and then invite volunteers to read their answers aloud. Answer the question on the board at the end for the benefit of all pupils.
- **Paired check:** After pupils have completed a given activity, ask them to exchange their exercise books with someone sitting near them. Provide the answers, and ask pupils to check their partner's work.
- **Move around:** If there is enough space, move around the classroom and check pupils' work as they are working on a given task or after they have completed a given task and are working on a different activity.

Strategies for engaging different kinds of learners

- For pupils who progress faster than others:
  - Plan extension activities in the lesson.
  - Plan a small writing project which they can work on independently.
  - Plan more challenging tasks than the ones assigned to the rest of the class.
  - Pair them with pupils who need more support.

- For pupils who need more time or support:
  - Pair them with pupils who are progressing faster, and have the latter support the former.
  - Set aside time to revise previously taught concepts while other pupils are working independently.
  - Organise extra lessons or private meetings to learn more about their progress and provide support.
  - Plan revision activities to be completed in the class or for homework.
  - Pay special attention to them in class, to observe their participation and engagement.

Lesson Title: Basic numeration	**Theme:** Numbers and Numeration	
Lesson Number: M4-L001	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcomes By the end of the lesson, pupils will be able to: 1. Apply the principles of BODMAS to operations on rational numbers. 2. Approximate answers to a given number of decimal places and significant figures. 3. Calculate the percentage error using rounded values.	**Preparation** Review the content of this lesson and be prepared to explain the solutions.	

Opening *(1 minute)*

1. Discuss and allow pupils to share ideas:
    - What does BODMAS stand for? (Answer: Bracket, Orders, Division Multiplication, Addition and Subtraction)
    - What is BODMAS? Why is it important? (Answer: It tells us the order of operations in which to solve a problem.)
2. Explain that this lesson is on numeration. Pupils will be calculating and approximating values, and calculating percentage error.

Teaching and Learning *(20 minutes)*

1. Write on the board:
    a. Calculate and give your answer to 1 decimal place: $1.2 + 3.95 \times (2 - 7)$
    b. Evaluate $\left(1\frac{1}{2} + 6\right) \div 4\frac{1}{2}$
2. Ask volunteers to explain in their own words how to solve the problems. Allow discussion. Example answers:
    a. Apply the order of operations (BODMAS) to find the answer, then round to 1 decimal place.
    b. Convert each mixed fraction to an improper fraction, and apply the order of operations.
3. Ask pupils to work with seatmates to solve the problems.
4. Walk around to check for understanding and clear misconceptions.
5. Invite a volunteer to write the solution to problem a. on the board. They should explain each step.
   Solution:

$1.2 + 3.95 \times (2 - 7)$	$=$	$1.2 + 3.95 \times (-5)$	Brackets
	$=$	$1.2 + (-19.75)$	Multiplication
	$=$	-18.55	Addition
	$=$	-18.6	Round to 1 d.p.

6. Revise the operations on negative numbers if needed. (For example, remind pupils that a negative times a positive is a negative, and adding a negative is the same as subtracting.)
7. Discuss rounding:
   - When do we round up? (Answer: When the next digit in the number is 5-9.)
   - When do we round down? (Answer: When the next digit in the number is less than 5.)
8. Invite a volunteer to write the solution to problem b. on the board. They should explain each step.

$$\begin{aligned} \left(1\tfrac{1}{2}+6\right) \div 4\tfrac{1}{2} &= 7\tfrac{1}{2} \div 4\tfrac{1}{2} && \text{Brackets} \\ &= \tfrac{15}{2} \div \tfrac{9}{2} && \text{Convert to improper fractions} \\ &= \tfrac{15}{2} \times \tfrac{2}{9} && \text{Divide} \\ &= \tfrac{15}{9} = \tfrac{5}{3} = 1\tfrac{2}{3} && \text{Simplify and convert to mixed fraction} \end{aligned}$$

9. Revise operations on fractions if needed. (For example, remind pupils that to divide we multiply by the reciprocal of the divisor.)
10. Write the following the board: Round the numbers to 2 significant figures:
    a. 0.00038561
    b. 4,245
    c. 2.0014
    d. 230.56
11. Ask pupils to work with seatmates to find the answers. Tell them to look at the significant figures table in their Pupil Handbook if needed.
12. Ask pupils to give the answers and explain. (Answers: a. 0.00039; b. 4,200; c. 2.0; d. 230)
13. Revise identifying and rounding significant figures if needed.
14. Write the following problem on the board: Sam measured a wall to be 1.5 metres long. However, the actual length is 1.65 metres. What is the percentage error?
15. Discuss: How do we solve percentage error problems? (Answer: Calculate the error, and use the formula for percentage error.)
16. Write on the board: $\text{Percentage error} = \frac{\text{error}}{\text{actual value}} \times 100\%$, where $\text{error} = \text{measured value} - \text{actual value}$
17. Solve the problem on the board. Involve pupils by asking them to give the steps.
    Solution:

$$\begin{aligned} \text{Percentage error} &= \frac{\text{error}}{\text{actual value}} \times 100\% \\ &= \frac{(1.5-1.65)\text{ m.}}{1.65\text{ m.}} \times 100\% \\ &= -0.091 \times 100\% \\ &= -9.1\% \end{aligned}$$

18. Write the following problem on the board: The length of a chalkboard is measured and is found to be 2.8 metres to 2 significant figures. What is the percentage error? Give your answer to 1 decimal place.

19. Explain:
    - Since we know the length has been rounded to 2 significant figures, we can find the range of lengths that it could actually have. We use this to find percentage error.
    - If the width has been rounded to 2.8, then the range of values that the width could actually fall into is 2.75 – 2.85.
    - In this case we do not have the actual value for the denominator of our percentage error calculation. We will use the measured value.
20. Solve the problem on the board, explaining each step:

$$\begin{aligned} \text{error} &= 2.75 - 2.8 = -0.05 \text{ m} \\ \text{or error} &= 2.85 - 2.8 = +0.05 \text{ m} \\ \text{Percentage error} &= \frac{\text{error}}{\text{measurement}} \times 100\% \\ &= \frac{\pm 0.05 \text{ m}}{2.8 \text{ m}} \times 100\% \\ &= \pm 1.8\% \end{aligned}$$

Practice *(18 minutes)*

1. Write the following problems on the board:
    a. Evaluate and round to 1 d.p.: $(3.24 + 1.923) \times 3 - 2.4$
    b. Evaluate: $\frac{1}{2}\left(2\frac{1}{2} + 4\frac{1}{3}\right) \times 4\frac{1}{2} + 2$
    c. Round to 4 significant figures: i. 23,405 ii. 0.0310025 iii. 4,235.01
    d. The weight of a puppy is measured, and is found to be 3.2 kg, correct to 1 decimal place. What is the percentage error?
2. Ask pupils to solve the problems either independently or with seatmates.
3. Encourage pupils to look at the solved examples in their Pupil Handbook if they need guidance.
4. Invite volunteers to come to the board at the same time to write the solutions. All other pupils should check their work.
    Solutions:
    a. $(3.24 + 1.923) \times 3 - 2.4 = 5.163 \times 3 - 2.4 = 15.489 - 2.4 = 13.089 \approx 13.1$
    b. $\frac{1}{2}\left(2\frac{1}{2} + 4\frac{1}{3}\right) \times 4\frac{1}{2} + 2 = \frac{1}{2}\left(6\frac{5}{6}\right) \times 4\frac{1}{2} + 2 = 3\frac{5}{12} \times 4\frac{1}{2} + 2 = \frac{41}{12} \times \frac{9}{2} + 2 = \frac{123}{8} + 2 = 15\frac{3}{8} + 2 = 17\frac{3}{8}$
    c. i. 23,410; ii. 0.0310; iii. 4,235
    d. Error: $3.2 - 3.25 = -0.05$ kg or $3.2 - 3.15 = +0.05$ kg
    Percentage error= $\frac{\pm 0.05 \text{ m}}{3.2 \text{ m}} \times 100\% = \pm 15.6\%$

Closing *(1 minute)*
1. For homework, have pupils do the practice activity of PHM4-L001 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L002 in the Pupil Handbook before the next class.

<table>
<tr><td>Lesson Title: Sequences</td><td colspan="2">Theme: Numbers and Numeration</td></tr>
<tr><td>Lesson Number: M4-L002</td><td>Class: SSS 4</td><td>Time: 40 minutes</td></tr>
<tr><td>Learning Outcomes
By the end of the lesson, pupils will be able to:
1. Identify arithmetic and geometric sequences.
2. Apply the formulae to find the nth term of a sequence.</td><td colspan="2">Preparation
Review the content of this lesson and be prepared to explain the solutions.</td></tr>
</table>

Opening *(1 minute)*

1. Discuss and allow pupils to share ideas:
    - What is a sequence? Explain in your own words. (Example answer: A list of numbers that increases or decreases; a sequence of numbers that follows a certain rule.)
    - What types of sequences do you know about? (Example answers: Arithmetic and geometric sequences.)
2. Explain that this lesson is on various sequence problems.

Teaching and Learning *(10 minutes)*

1. Write on the board:
    a. 5, 8, 11, 14, …
    b. 4, 8, 16, 32, …
2. Ask volunteers to identify whether each sequence is an arithmetic or geometric sequence. Ask them to explain.
   Answers:
    a. An arithmetic sequence, because it increases by a common difference (3 is added to each term).
    b. A geometric sequence, because it increases by a common ratio (each term is multiplied by 2).
3. Review **arithmetic sequence:**
    - A sequence in which the terms either increase or decrease by a common difference is an arithmetic sequence, or arithmetic progression. It can be abbreviated to AP.
    - Arithmetic progressions have a **common difference** that is the same between each term and the next term.
4. Write on the board: $U_n = a + (n-1)d$, where U_n is the nth term of the AP, a is the first term, and d is the common difference.
5. Review **geometric sequence:**
    - A sequence in which the terms either increase or decrease by a common ratio is a geometric sequence, or geometric progression. It can be abbreviated to GP.

- Geometric progressions have a **common ratio** that is multiplied by each term to get the next term.

6. Write on the board: $U_n = ar^{n-1}$, where U_n is the nth term of the GP, a is the first term, and r is the common ratio.
7. Ask pupils to work with seatmates to find the 10th term of each sequence on the board.
8. Walk around to check for understanding and clear misconceptions.
9. Invite volunteers to write the solutions on the board.

   Solutions:

   a.

$$\begin{aligned} U_n &= ar^{n-1} && \\ U_{10} &= 4 \times 2^{10-1} && \text{Substitute } a = 4, n = 10, r = 2 \\ &= 4 \times 2^9 && \text{Simplify} \\ &= 4 \times 512 && \\ &= 2{,}048 && \end{aligned}$$

   b.

$$\begin{aligned} U_n &= a + (n-1)d && \\ U_{10} &= 5 + (10-1)3 && \text{Substitute } a = 5, d = 3 \text{ and } n = 10 \\ &= 5 + 9 \times 3 && \text{Clear the brackets} \\ &= 5 + 27 && \text{Simplify} \\ &= 32 && \end{aligned}$$

10. Allow pupils to ask any questions they have about sequences.

Practice *(28 minutes)*

1. Write the following problems on the board:
   a. Find the 8th term of the sequence $4, -2, -8, -14, \ldots$
   b. Find the number of terms in the arithmetic sequence $4, 8, 12, 16, \ldots, 64$.
   c. Find the 9th term of the sequence $4, -8, 16, -32, \ldots$
   d. Find the number of terms in the geometric sequence $3, 9, 27, \ldots, 6{,}561$
   e. The nth term of a sequence is $P_n = 100 - (n+1)^2$. Evaluate $P_7 - P_4$.
2. Ask pupils to solve the problems either independently or with seatmates.
3. Encourage pupils to look at the solved examples in their Pupil Handbook if they need guidance.
4. Invite volunteers to come to the board at the same time to write the solutions. All other pupils should check their work.

   Solutions:

   a.

$$\begin{aligned} U_n &= a + (n-1)d && \\ U_8 &= 4 + (8-1)(-6) && \text{Substitute for } a, d \text{ and } n \\ &= 4 + (7)(-6) && \\ &= 4 - 42 && \text{Simplify} \end{aligned}$$

$= \quad -38$

b.

$$\begin{aligned}
U_n &= a + (n-1)d \\
64 &= 4 + (n-1)4 && \text{Substitute } U_n = 64, a = 4, d = 4 \\
64 - 4 &= (n-1)4 && \text{Solve for } n \\
60 &= 4n - 4 \\
60 + 4 &= 4n \\
\frac{64}{4} &= n \\
n &= 21
\end{aligned}$$

There are 21 terms in the progression.

c.

$$\begin{aligned}
U_n &= ar^{n-1} \\
U_9 &= 4 \times (-2)^{9-1} && \text{Substitute } a = 4, n = 9, r = -2 \\
&= 4 \times (-2)^8 && \text{Simplify} \\
&= 4 \times 256 \\
&= 1{,}024
\end{aligned}$$

The ninth term is 1024.

d.

$$\begin{aligned}
U_n &= ar^{n-1} \\
6{,}561 &= 3 \times 3^{n-1} && \text{Substitute } U_n = 6561, a = 3, r = 3 \\
3^8 &= 3^n && \text{Substitute } 6561 = 3^8 \\
n &= 8
\end{aligned}$$

There are 8 terms.

e.

Step 1. Find P_7 and P_4:

$$\begin{aligned}
P_7 &= 100 - (7+1)^2 && \text{Substitute } n = 7 \\
P_7 &= 100 - 8^2 && \text{Simplify} \\
P_7 &= 100 - 64 = 36 \\
P_4 &= 100 - (4+1)^2 && \text{Substitute } n = 4 \\
P_4 &= 100 - 5^2 && \text{Simplify} \\
P_4 &= 100 - 25 = 75
\end{aligned}$$

Step 2. Subtract: $P_7 - P_4 = 36 - 75 = -39$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L002 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L003 in the Pupil Handbook before the next class.

Lesson Title: Series	Theme: Numbers and Numeration	
Lesson Number: M4-L003	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcomes By the end of the lesson, pupils will be able to: 1. Distinguish between sequence and series. 2. Calculate the sum of the first n terms of an arithmetic and a geometric series.	**Preparation** Review the content of this lesson and be prepared to explain the solutions.	

Opening *(1 minute)*

1. Discuss and allow pupils to share ideas: What is a series? Explain in your own words. (Example answer: A series is like a sequence, but the numbers are added to find a sum.)
2. Explain that this lesson is on various series problems.

Teaching and Learning *(10 minutes)*

1. Write on the board:
   a. $3 + 7 + 11 + 15 + 19 + \cdots$
   b. $2 + 8 + 32 + 128 + 512$
2. Ask volunteers to identify whether each series is an arithmetic or geometric series. Ask them to explain.
   Answers:
   c. An arithmetic series, because the numbers increase by a common difference (4 is added to each term).
   d. A geometric series, because each term increases by a common ratio (each term is multiplied by 4).
3. Explain:
   a. Series a. has 3 points after it (⋯), because it continues on forever. This is an **infinite series**. It is often impossible to find the sum of infinite series, but we can find the sum of the first n terms.
   b. Series b. stops at 512. It is a finite series, which ends at a certain point.
4. Write on the board:
   a. Find the sum of the first 16 terms of $3 + 7 + 11 + 15 + 19 + \cdots$.
   b. Find the sum of $2 + 8 + 32 + 128 + 512$.
5. Write the formulae on the board:
   Sum of the first n terms of an AP: $S_n = \frac{1}{2}n[2a + (n - 1)d]$
   Sum of the first n terms of a GP if r is a fraction $|r| < 1$: $S_n = \frac{a(1-r^n)}{1-r}$
   Sum of the first n terms of a GP if $|r| > 1$: $S_n = \frac{a(r^n-1)}{r-1}$
   Where a is the first term, d is the common difference, and r is the common ratio.

6. Ask pupils to look at problem a., and identify the values of a, n, and d. Write them on the board. (Answer: $a = 3$, $n = 16$, $d = 4$)
7. Ask pupils to look at problem b., and identify the values of a, n, and r. Write them on the board. (Answer: $a = 2$, $n = 5$, $r = 4$)
8. Ask pupils to work with seatmates to solve the problems on the board.
9. Walk around to check for understanding and clear misconceptions.
10. Make sure pupils are using the correct formula for problem b. Since $|r| > 1$, they should be using $S_n = \frac{a(r^n-1)}{r-1}$.
11. Invite volunteers to write the solutions on the board.

Solutions:

a.

$$\begin{aligned} S_n &= \tfrac{1}{2}n[2a + (n-1)d] \\ S_{16} &= \tfrac{1}{2}(16)[2(3) + (16-1)4] && \text{Substitute } n, a, \text{ and } d \\ &= (8)[6 + 60] && \text{Simplify} \\ &= 8(66) = 528 \end{aligned}$$

b.

$$\begin{aligned} S_n &= \frac{a(r^n-1)}{r-1} \\ S_5 &= \frac{2(4^5-1)}{4-1} && \text{Substitute } n, a, \text{ and } r \\ &= \frac{2(1024-1)}{3} && \text{Simplify} \\ &= \frac{2046}{3} \\ &= 682 \end{aligned}$$

12. Allow pupils to ask any questions they have about series.

Practice *(28 minutes)*

1. Write the following problems on the board:
   a. Find the sum of the first 12 terms of the AP 10, 8, 6, 4, ….
   b. An AP with 14 terms has a first term of 20, and a sum of 7. What is the common difference?
   c. An AP has 10 terms, and a sum of 240. If the common difference is 2, what is the first term?
   d. Find the sum of the first 10 terms of the series: $2, -4, +8, -16, + \cdots$
   e. The sum of the first five terms of a GP is 242. If the common ratio is 3, find the sixth term.
2. Ask pupils to solve the problems either independently or with seatmates.
3. Encourage pupils to look at the solved examples in their Pupil Handbook if they need guidance.
4. Invite volunteers to come to the board at the same time to write the solutions. All other pupils should check their work.

Solutions:

a.

$$S_n = \tfrac{1}{2}n[2a + (n-1)d]$$

	$=$	$\frac{1}{2}(12)[2(10)+(12-1)(-2)]$	Substitute n, a, and d
	$=$	$(6)[20-22]$	Simplify
	$=$	$6(-2)=-12$	

b.

S_n	$=$	$\frac{1}{2}n[2a+(n-1)d]$	
7	$=$	$\frac{1}{2}(14)[2(20)+(14-1)d]$	Substitute n, a, and d
7	$=$	$7[40+13d]$	Simplify
1	$=$	$40+13d$	Divide throughout by 7
$1-40$	$=$	$13d$	Transpose 40
-39	$=$	$13d$	
-3	$=$	d	Divide throughout by 13

c.

S_n	$=$	$\frac{1}{2}n[2a+(n-1)d]$	
240	$=$	$\frac{1}{2}(10)[2a+(10-1)2]$	Substitute n, a, and d
240	$=$	$5[2a+18]$	Simplify
48	$=$	$2a+18$	Divide throughout by 5
$48-18$	$=$	$2a$	Transpose 18
30	$=$	$2a$	
15	$=$	a	Divide throughout by 2

d.

S_{10}	$=$	$\frac{4((-2)^{10}-1)}{-2-1}$	Substitute $n=10$, $a=2$, and $r=-2$
	$=$	$\frac{4(1023)}{-3}$	
	$=$	$-4(341)$	Simplify
	$=$	-1364	

e.

S_n	$=$	$\frac{a(r^n-1)}{r-1}$	Since $\lvert r\rvert > 1$
242	$=$	$\frac{a(3^5-1)}{3-1}$	Substitute S_5,n, and r
	$=$	$\frac{a(242)}{2}$	Simplify
$2(242)$	$=$	$a(242)$	
a	$=$	2	Solve for a

To find the sixth term, use the formula for the n^{th} term of a GP.

U_n	$=$	ar^{n-1}
U_6	$=$	$2(3)^{6-1}$
	$=$	486

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L003 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L004 in the Pupil Handbook before the next class.

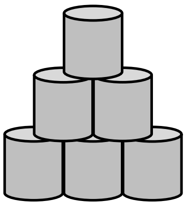
<table>
<tr><td>Lesson Title: Problem solving using sequences and series</td><td colspan="2">Theme: Numbers and Numeration</td></tr>
<tr><td>Lesson Number: M4-L004</td><td>Class: SSS 4</td><td>Time: 40 minutes</td></tr>
<tr><td>Learning Outcome
By the end of the lesson, pupils will be able to apply sequences and series to numerical and real-life problems.</td><td colspan="2">Preparation
Review the content of this lesson and be prepared to explain the solutions.</td></tr>
</table>

Opening *(1 minute)*

1. Discuss and allow pupils to share ideas: Have you observed any sequences or series in real life? (Example answer: Goods in a shop can be arranged in a sequence; a person's salary or payment can follow a sequence.)
2. Explain that this lesson is on real-life problems involving sequences and series.

Teaching and Learning *(10 minutes)*

1. Tell a story: Mr. Bangura sells cans of fish in the market. He decided to display them in a nice way to attract more customers. He arranged them in a stack so that 1 can is in the top row, 2 cans are in the next row, 3 cans are in the third row, and so on. The bottom row has 14 cans of fish.
2. Draw a picture of the top 3 rows on the board:



3. Discuss: Can you write a sequence based on this story? What would it be?
4. Allow pupils to share their ideas, then explain: The number of cans in each row forms a sequence.
5. Write the sequence on the board, as shown: 1, 2, 3, 4, …, 14
6. Discuss: Can you calculate the total number of cans in the stack? What steps would you take?
7. Allow pupils to share their ideas, then explain: The stack forms an arithmetic sequence, so we can use the formula for finding the sum of a series.
8. Ask pupils to give the values of a, n, and d for this problem, and explain. (Answer: $a = 1$, the number of cans in the first row; $n = 14$, the number of rows; $d = 1$, the difference between each row and the next row.)
9. Ask pupils to work with seatmates to calculate the total number of cans.
10. Invite a volunteer to write the solution on the board.

Solution:

$$S = \frac{1}{2}n[2a + (n-1)d]$$

$$= \frac{1}{2}(14)[2(1) + (14-1)1] \quad \text{Substitute } n, a, \text{ and } d$$
$$= 7(2 + 13) \quad \text{Simplify}$$
$$= 7(15) = 105$$

11. Explain: There are 105 cans of fish in Mr. Bangura's display.

Practice *(28 minutes)*

1. Write the following problems on the board:
   a. Fatu picks mangos from her tree each day. On the first day, she picked 2 mangos, on the second day she picked 4 mangos, and on the third day she picked 6 mangos. She continued picking 2 more mangos for each day of mango season.
      i. Write a sequence for the word problem.
      ii. How many mangos did she pick on the 10^{th} day?
      iii. How many mangoes does she pick in total in 30 days?
   b. Mrs. Jalloh's business profit rose steadily each month of last year. In January, her profit was Le 200,000.00, and it increased by Le 50,000.00 each month.
      i. What was her profit in December?
      ii. What was her total profit for the year?
   c. A sum of money is shared among 14 people so that the first person receives Le 5,000.00, the next person receives Le 15,000.00, the next Le 25,000.00, and so on.
      i. How much money does the 14^{th} person receive?
      ii. How much money is shared in total?
   d. A principal divided all of the pupils in a school into groups. There were 2 pupils in the first group, 4 in the second group, 8 in the third group, and so on. If there are 8 groups in total, how many pupils are there in the school?
2. Ask pupils to solve the problems either independently or with seatmates.
3. Encourage pupils to look at the solved examples in their Pupil Handbook if they need guidance.
4. Invite volunteers to come to the board at the same time to write the solutions. All other pupils should check their work.

   Solutions:

   a. i. 2, 4, 6, 8, …

   ii.

$$U_{10} = a + (n-1)d$$
$$= 2 + (10-1)2 \quad \text{Substitute } a, n, \text{ and } d$$
$$= 2 + (9)2 \quad \text{Simplify}$$
$$= 20$$

   She picked 20 mangos on the 10^{th} day.

   iii.

$$S = \frac{1}{2}n[2a + (n-1)d]$$
$$= \frac{1}{2}(30)[2(2) + (30-1)2] \quad \text{Substitute } n, a, \text{ and } d$$

$= (15)[4 + 58]$ Simplify

$= 15(62) = 930$

She picked a total of 930 mangos in 30 days.

b. Note that $a = 200{,}000$, $d = 50{,}000$, and $n = 12$.

i.

$U_{12} = a + (n-1)d$

$= 200{,}000 + (12-1)50{,}000$ Substitute a, n, and d

$= 200{,}000 + 550{,}000$ Simplify

$= 750{,}000$

Her profit in December was Le 750,000.00.

ii.

$S = \frac{1}{2}n[2a + (n-1)d]$

$= \frac{1}{2}(12)[2(200{,}000) + (12-1)50{,}000]$ Substitute n, a, and d

$= (6)[400{,}000 + 550{,}000]$ Simplify

$= 6(950{,}000)$

$= 5{,}700{,}000$

Her profit for the year was Le 5,700,000.00.

c. Note that $a = 5{,}000$, $d = 10{,}000$, and $n = 14$.

i.

$U_{14} = a + (n-1)d$

$= 5{,}000 + (14-1)10{,}000$ Substitute a, n, and d

$= 5{,}000 + 130{,}000$ Simplify

$= 135{,}000$

The 14th person receives Le 135,000.00.

ii.

$S = \frac{1}{2}n[2a + (n-1)d]$

$= \frac{1}{2}(14)[2(5{,}000) + (14-1)10{,}000]$ Substitute n, a, and d

$= (7)[10{,}000 + 130{,}000]$ Simplify

$= 7(140{,}000)$

$= 980{,}000$

The total amount shared is Le 980,000.00.

d. This is a geometric series where $a = 2$, $r = 2$, and $n = 8$

$S_n = \frac{a(r^n - 1)}{r-1}$ Since $r > 1$

$S_8 = \frac{2(2^8 - 1)}{2-1}$ Substitute a, n, and r

$= 2(256 - 1)$ Simplify

$= 510$

There are 510 pupils.

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L004 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L005 in the Pupil Handbook before the next class.

<table>
<tr><td>Lesson Title: Ratios</td><td colspan="2">Theme: Numbers and Numeration</td></tr>
<tr><td>Lesson Number: M4-L005</td><td>Class: SSS 4</td><td>Time: 40 minutes</td></tr>
<tr><td>Learning Outcomes
By the end of the lesson, pupils will be able to:
1. Increase and decrease quantities in a given ratio.
2. Solve real-life problems involving ratio.</td><td colspan="2">Preparation
Review the content of this lesson and be prepared to explain the solutions.</td></tr>
</table>

Opening *(2 minutes)*

1. Ask pupils to discuss with seatmates everything they know about ratios.
2. After 1 minute, invite volunteers to tell the class what they know. (Example answers: A ratio is used to compare 2 or more quantities. The quantities must be measured in the same unit. We can write a ratio using a colon **:** or as a fraction).
3. Explain that this lesson is on solving problems involving ratios.

Teaching and Learning *(18 minutes)*

1. Explain:
   - We use ratios to compare quantities of the same type, e.g. length, weight, people, money and many more.
   - For example, consider an SSS 4 class with 24 girls and 36 boys.
2. Invite volunteers to give the ratio of girls to boys. (Answer: 24 : 36 or $\frac{24}{36}$)
3. Explain:
   - The ratio can be written with a colon or as a fraction, and is read "24 to 36".
   - Always simplify ratios to lowest terms by dividing by common factors.
4. Ask pupils to simplify the ratio on the board in their exercise books.
5. Invite a volunteer to give the answer and explain what it means. (Answer: 2 : 3, $\frac{2}{3}$. This means that for every 2 girls there are 3 boys).
6. Write on the board: To increase or decrease a quantity Q by a ratio $m : n$, use the formula: $\frac{m}{n} \times Q$
7. Explain: We are sometimes required to increase or decrease quantities by a given ratio to find the new amounts.
8. Write on the board: The cost of a school uniform increased from Le 60,000.00 in the ratio 8**:**5. What is the new price?
9. Explain:
   - This means that every Le5 is increased to Le8.
   - We know it is an increase because the first part of the ratio is larger than the second part of the ratio.
10. Solve on the board, involving pupils: New amount = $\frac{8}{5} \times 60{,}000 = \text{Le } 96{,}000.00$
11. Write another problem on the board: Decrease 350 g in the ratio 2**:**7.
12. Ask pupils to work with seatmates to solve the problem.

13. Invite volunteers to give the solution. (Answer: New amount= $\frac{2}{7} \times 350 = 100$ kg)
14. Explain:
    - We are often asked to compare 2 or more ratios to find out which is biggest or smallest relative to the others.
    - One way we can compare ratios is by writing them as unit ratios. A unit ratio has 1 in either the numerator or the denominator.
    - Once we have converted the given ratios to unit fractions, we then determine which ratio is greatest or smallest in relation to the others.
    - A second way to compare ratios is to use LCM of the denominators to convert each ratio into an equivalent fraction.
    - We then inspect the numerators and determine which ratio is greatest or smallest in relation to the others.
15. Write on the board: Express the ratios $3 : 8$ and $4 : 15$ in the form $m : 1$. Which ratio is greater? Use LCM to confirm your result.

    Solution:

    Method 1.

    Simplify each ratio independently to a unit fraction

    $\frac{3}{8} = \frac{0.375}{1}$ divide the numerator and denominator by 8

    $\frac{4}{15} = \frac{0.267}{1}$ divide the numerator and denominator by 15

    Answer: $3 : 8 > 4 : 15$ because $0.375 > 0.267$

    Method 2.

    Write each fraction using the LCM of the denominators:

    $\frac{3}{8} = \frac{45}{120}$ since the LCM of 8 and 15 is 120

    $\frac{4}{15} = \frac{32}{120}$

    Answer: $3 : 8 > 4 : 15$ because $45 > 32$
16. Explain: We found that $3 : 8$ is the greater ratio using both methods.
17. Write on the board: Find which ratio is greater: $9 : 12$ or $8 : 10$.
18. Ask pupils to work with seatmates to solve.
19. Invite a volunteer to show the solution on the board. If another set of seatmates used the other method, ask them to write the solution as well.

    Solution:

    Method 1.

    Simplify each ratio independently to a unit fraction:

    $\frac{9}{12} = \frac{0.75}{1}$ divide numerator and denominator by 12

    $\frac{8}{10} = \frac{0.8}{1}$ divide numerator and denominator by 10

    Answer: $8 : 10 > 9 : 12$ because $0.8 > 0.75$

    Method 2.

    Write each fraction using the LCM of the denominators:

    $\frac{9}{12} = \frac{45}{60}$ since the LCM of 12 and 10 is 60

    $\frac{8}{10} = \frac{48}{60}$

    Answer: $8 : 10 > 9 : 12$ because $48 > 45$

Practice *(19 minutes)*

1. Write the following problems on the board:
   a. A market contains 32 stalls selling vegetables and 8 stalls selling fish. Express the ratio of vegetable stalls to fish stalls in the lowest terms.
   b. A fish monger increases the price of her fish in the ratio 8 **:** 5. What is the new price of fish she used to sell at Le 35,000?
   c. A cassava farmer noticed that her harvest decreased in the ratio 3 : 4 from last year's harvest. If she harvested 500 kg of cassava last year, what was her harvest this year?
   d. Compare the following ratios. Which is greater?
      i. $2:5$ and $5:12$
      ii. $10:3$ and $40:8$
      iii. $2.5:100$ and $6:150$
2. Ask pupils to solve the problems either independently or with seatmates.
3. Encourage pupils to look at the solved examples in their Pupil Handbook if they need guidance.
4. Invite volunteers to come to the board to write the solutions at the same time. All other pupils should check their work.

   Solutions:

   a. $\frac{32}{8}=\frac{4}{1}$; The ratio is $4:1$.

   b. To increase Le35,000 in the ratio $8:\ 5$, multiply by the fraction form of the ratio: $\frac{8}{5}\times 35{,}000=\text{Le } 56{,}000.00$.

   c. To decrease 500 kg in the ratio 3 : 4, multiply by the fraction form of the ratio: $\frac{3}{4}\times 500=375$ kg.

   d. Pupils may use either method to compare the ratios.
      i. Method 1: $\frac{2}{5}=\frac{0.4}{1}$ and $\frac{5}{12}=\frac{0.42}{1}$; $5:12>2:5$ because $0.42>0.4$.
      Method 2: $\frac{2}{5}=\frac{24}{60}$ and $\frac{5}{12}=\frac{25}{60}$; $5:12>2:5$ because $25>24$.
      ii. Method 1: $\frac{10}{3}=\frac{3.33}{1}$ and $\frac{40}{8}=\frac{5}{1}$; $40:8>10:3$ because $5>3.3$.
      Method 2: $\frac{10}{3}=\frac{80}{24}$ and $\frac{40}{8}=\frac{120}{24}$; $40:8>10:3$ because $120>80$.
      iii. Method 1: $\frac{2.5}{100}=\frac{0.025}{1}$ and $\frac{6}{150}=\frac{0.04}{1}$; $6:150>2.5:100$ because $0.04>0.025$.
      Method 2: $\frac{2.5}{100}=\frac{7.5}{300}$ and $\frac{6}{150}=\frac{12}{300}$; $6:150>2.5:100$ because $12>7.5$.

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L005 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4- L006 in the Pupil Handbook before the next class.

Lesson Title: Rates	**Theme:** Numbers and Numeration	
Lesson Number: M4-L006	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to solve problems related to rate, including real-life applications (e.g. rates of pay, travel rates, currency exchange rates).	**Preparation** Review the content of this lesson and be prepared to explain the solutions.	

Opening *(1 minute)*

1. Discuss: What are some common rates that you use in everyday life? (Example answer: km/hr, which is often used for the speed of a car.)
2. Explain that this lesson is on solving problems involving rates.

Teaching and Learning *(19 minutes)*

1. Explain:
   - We use ratios to compare two or more quantities with the same units.
   - We use rates when we want to compare quantities of different kinds.
   - Examples of rates are: how far a car travels per hour, or how much a person is paid per month.
2. Explain: The units in a rate take on the unit from the numerator and the unit from the denominator, e.g. $\frac{\text{distance}}{\text{time}} = \frac{\text{km}}{\text{hr}}$.
3. Write on the board: A car travels a distance of 240 km in 3 hours.
   a. What is the average speed in kilometres per hour (km/hr)?
   b. How far will it travel in 5 hours?

4. Solve the problem on the board. Involve pupils by asking them to give the steps.
   - Write speed as a fraction with kilometres in the numerator and time in the denominator, then simplify.

rate	$=$	$\frac{240 \text{ km}}{3 \text{ hrs}}$	Write as a fraction
	$=$	$\frac{80 \text{ km}}{1 \text{ hr}}$	Simplify by dividing numerator and denominator by 3
	$=$	80 km/hr	Write as a rate in km/hr.

   - Multiply the rate by the number of hours traveled:

Distance travelled	$=$	$\frac{80 \text{ km}}{1 \text{ hr}} \times 5$ hrs	Multiply by 5 hours
	$=$	400 km	The hours cancel each other

The distance travelled in 5 hours is 400 km.

4. Explain:

- This method is called the unitary method.
- We first find the unit rate at which the car is travelling. This is the distance travelled for every 1 hour.
- A unit rate is a ratio that tells us how many units of one quantity there are for every one unit of the second quantity.
- In our example, this is “80” kilometres for every “1” hour, i.e. 80 km/hr.
- We use the unit rate to find the distance covered in a given time.

5. Write the following problem on the board: A water tank contains 500,000 litres of water. It takes 2 days to empty the tank. If the tank empties at a constant rate
    a. Calculate the rate the tank empties in litres per hour.
    b. How long will it take to empty another tank with 750,000 litres of water if it empties at the same rate?
6. Discuss: What steps would you take to solve this problem? (Example answer: Convert days to hours, then divide to find the rate.)
7. Ask pupils to work with seatmates to solve the problem.
8. Walk around to check for understanding and clear misconceptions.
9. Invite volunteers to come to the board to write the solution.

    Solution:

    a. First, convert days to hours: 2 days = 48 hours
    Divide to find the rate:

$$\text{rate} = \frac{500{,}000 \text{ litres}}{48 \text{ hrs}} \quad \text{Write as a fraction}$$

$$= 10{,}416.67 \text{ l/hr} \quad \text{Divide}$$

The tank empties at a rate of 10,417 litres/hr to the nearest litre.

    b. Since the tanks empty at the same rate, set the rates equal to each other:

$$\frac{10{,}416.667 \text{ litres}}{1 \text{ hr}} = \frac{750{,}000 \text{ litres}}{x}$$

where x is the number of hours it takes the second tank to empty

$$x = \frac{750{,}000 \text{ litres}}{10{,}416.667 \text{ litres}} \times 1 \text{ hr}$$

$$= 71.999 \text{ hrs}$$

The litres cancel each other

$$= 72 \text{ hrs or } 3 \text{ days}$$

The 750,000 litre tank empties in 3 days.

Practice *(19 minutes)*

1. Write the following problems on the board:
    a. A team of 3 workers charged a house owner Le 720,000.00 to paint her house. How much is it costing her per worker for the job?
    b. A car uses 6 litres of fuel for a journey of 150 km. What is the average rate of fuel use in km per litre?
    c. A man’s heart rate was tested at 142 beats in 2 minutes.
        i. What is the rate of heart beats per minute?

ii. How many minutes will it take his heart to beat 852 times?

2. Ask pupils to solve the problems either independently or with seatmates.
3. Encourage pupils to look at the solved examples in their Pupil Handbook if they need guidance.
4. Invite volunteers to come to the board at the same time to write the solutions. All other pupils should check their work.

Solutions:

a. Divide to find the cost per worker, then simplify:

$$\text{rate} = \frac{\text{Le } 720{,}000}{3 \text{ workers}} = \frac{\text{Le } 240{,}000}{1 \text{ worker}}$$

The men worked at a rate of Le 240,000.00/worker.

b. Divide to find the rate, then simplify:

$$\text{rate} = \frac{150 \text{ km}}{6 \text{ l}} = \frac{25 \text{ km}}{1 \text{ l}}$$

The car travels at the rate of 25 km/litre.

c. i. Divide to find the rate:

$$\text{rate} = \frac{142 \text{ beats}}{2 \text{ minutes}} = \frac{71 \text{ beats}}{1 \text{ minute}}$$

The man's heart beats at a rate of 71 beats/minutes.

ii. Set the unitary rate equal to the rate with 852 beats in the numerator, then solve for the number of minutes:

$$\text{rate} = \frac{71}{1}$$
$$\frac{71}{1} = \frac{852}{x}$$
$$x = \frac{852}{71}$$
$$x = 12 \text{ minutes}$$

The man's heart beats 852 times in 12 minutes.

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L006 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L007 in the Pupil Handbook before the next class.

Lesson Title: Proportional division	**Theme:** Numbers and Numeration	
Lesson Number: M4-L007	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to divide quantities into given proportions, and solve real-life applications.	**Preparation** 1. Review the content of this lesson and be prepared to explain the solutions. 2. Write the problem in Opening on the board.	

Opening *(1 minute)*

1. Write the following problem on the board: Compare the results of the 2 calculations below:
   a. Share Le 750,000 equally between 2 children. How much will each child receive?
   b. Share Le 750,000 between 2 children in the ratio 8 : 7. How much will each child receive?
2. Ask pupils to find the answer to part a. in their exercise books.
3. Invite a volunteer to write the solution on the board. (Answer: $\frac{750,000}{2} =$ Le $375,000$ each)
4. Explain that this lesson is on proportional division.

Teaching and Learning *(19 minutes)*

1. Explain:
   - In question a., we shared a quantity equally.
   - Suppose we are asked instead to share the same amount of money according to the ages of the children, as in question b.
   - We perform **proportional division** according to the given ratio.
   - To share something in a given ratio, find the total number of parts in that ratio. This is done by adding all of the numbers in the ratio. Then, find one part of the whole by multiplying by a fraction with the total number of parts in the denominator.
2. Solve the problem on the board. Explain each step carefully:
   Step 1. Find the total number of parts to the ratio: $8 + 7 = 15$
   This ratio means that for every Le15 of the amount to be shared, Le8 will go to Child 1 and Le7 will go to Child 2.
   Step 2. Find what proportion (fraction) of the total is given to each part:

Child 1 receives: $\frac{8}{15} \times 750,000 \quad = \quad \text{Le } 400,000.00$

Child 2 receives: $\frac{7}{15} \times 750,000 \quad = \quad \text{Le } 350,000.00$

Step 3. Check whether the answer is reasonable. If child 1 receives Le 400,000 and child 2 receives Le 350,000, then the sum is Le 750,000. This is the original quantity, so it is reasonable.

3. Explain:
   - A quantity shared equally will result in the same amount per share as in question a.
   - A quantity shared in different proportions will result in different amounts per share as in question b.
4. Write another problem on the board: Divide the quantities below in the given ratio:
   a. 500 g in the ratio 2 : 3
   b. Le 520,000.00 in the ratio 10 : 9 : 7
5. Ask pupils to work with seatmates to solve the problem.
6. Walk around to check for understanding and clear misconceptions.
7. Invite volunteers to write their solutions on the board.
   Solutions:
   a. Total number of parts: $2+3=5$

      2 parts give: $\frac{2}{5}\times 500 = 200$ g

      3 parts give: $\frac{3}{5}\times 500 = 300$ g

      This answer is reasonable as it adds up to 500 g.

   b. Total number of parts: $10+9+7=26$

      10 parts give: $\frac{10}{26}\times 520{,}000 =$ Le $200{,}000.00$

      9 parts give: $\frac{9}{26}\times 520{,}000 =$ Le $180{,}000.00$

      7 parts give: $\frac{7}{26}\times 520{,}000 =$ Le $140{,}000.00$

      This answer is reasonable as it adds up to Le 520,000.00.

Practice *(19 minutes)*

1. Write the following problems on the board:
   a. To start a small business, Femi, Kemi and Yemi contributed money in the ratio 7 : 6 : 5 respectively. If the total amount contributed is Le 900,000.00, how much did each person contribute?
   b. In a chemistry laboratory, acid and water were mixed together in a ratio 1 : 5 to give 216 ml of a mixture. How much acid and how much water was mixed together?
   c. After the first year in business, Le 200,000.00 profit was shared between 3 partners. Ramatu received Le 40,000.00, Mohammed received Le 120,000.00 and Isata received the rest. After the second year, they shared out a profit of Le 500,000.00 in the same ratio. How much did each person receive in the second year?
2. Ask pupils to solve the problems either independently or with seatmates.

3. Encourage pupils to look at the solved examples in their Pupil Handbook if they need guidance.
4. Invite volunteers to come to the board at the same time to write the solutions. All other pupils should check their work.

Solutions:

a. Total number of parts: $7+6+5=18$

$$\text{Femi's share} = \frac{7}{18}\times 900{,}000 = \text{Le } 350{,}000.00$$
$$\text{Kemi's share} = \frac{6}{18}\times 900{,}000 = \text{Le } 300{,}000.00$$
$$\text{Yemi's share} = \frac{5}{18}\times 900{,}000 = \text{Le } 250{,}000.00$$

The answer is reasonable as it adds up to Le 900,000.00.

b. Total number of parts: $1+5=6$

$$\text{acid} = \frac{1}{6}\times 216 = 36 \text{ ml}$$
$$\text{water} = \frac{5}{6}\times 216 = 180 \text{ ml}$$

The answer is reasonable as it adds up to 216 ml.

c. 1st year:

$$\text{Ramatu} = \text{Le } 40{,}000$$
$$\text{Mohamed} = \text{Le } 120{,}000$$
$$\text{Isatu} = 200{,}000-(40{,}000+120{,}000)$$
$$= \text{Le } 40{,}000$$
$$\text{Ramatu's share} = \frac{40{,}000}{200{,}000} = \frac{1}{5}$$
$$\text{Mohamed's share} = \frac{120{,}000}{200{,}000} = \frac{3}{5}$$
$$\text{Isatu's share} = \frac{40{,}000}{200{,}000} = \frac{1}{5} \text{ (same as Ramatu's share)}$$

2nd year

$$\text{Ramatu's share} = \frac{1}{5}\times 500{,}000 = \text{Le } 100{,}000.00$$
$$\text{Mohamed's share} = \frac{3}{5}\times 500{,}000 = \text{Le } 300{,}000.00$$
$$\text{Isatu's share} = \frac{1}{5}\times 500{,}000 = \text{Le } 100{,}000.00$$

The answer is reasonable as it adds up to Le 500,000.00.

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L007 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L008 in the Pupil Handbook before the next class.

Lesson Title: Speed	Theme: Numbers and Numeration	
Lesson Number: M4-L008	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to solve problems involving speed, time, and distance.	**Preparation** Review the content of this lesson and be prepared to explain the solutions.	

Opening *(1 minute)*

1. Ask volunteers to write any formula they know for connecting speed, distance and time on the board. (Answers: $d = st$; $s = \frac{d}{t}$; $t = \frac{d}{s}$)
2. Write any of the 3 formulae they did not give on the board.
3. Explain: These formulae are the same, but are rearranged. They give the relationship between distance, speed and time.
4. Explain that this lesson is on solving problems related to speed.

Teaching and Learning *(19 minutes)*

1. Write on the board: average speed $= \frac{\text{total distance travelled}}{\text{total time taken}}$
2. Explain: This is from the formula $s = \frac{d}{t}$. It can be used if you are asked to find the average speed for a journey.
3. Write the following problem on the board: A lorry takes $3\frac{1}{2}$ hours to travel a distance of 126 km. What is its average speed?
4. Ask pupils to solve the problem with seatmates using the formula on the board.
5. Walk around to check for understanding and clear misconceptions.
6. Ask a volunteer to write the solution on the board.
   Solution:
   $$\text{average speed} = \frac{\text{total distance travelled}}{\text{total time taken}} = \frac{126}{3.5} = 36 \text{ km/hr}$$
7. Write the following problem on the board: A van travels for 3 hours at 48 km/hr. It then travels for 2 hours at 53 km/hr.
   a. What is the total distance travelled by the van?
   b. What is the average speed for the whole journey?
8. Discuss:
   - What steps would you take to solve part a.? (Answer: Use the formula $d = st$ twice, once for each piece of travel; add the results.)
   - What steps would you take to solve part b.? (Answer: Apply the formula $s = \frac{d}{t}$; use the distance from part a, and the total time, 5 hours.)
9. Ask pupils to work with seatmates to solve the problem.
10. Walk around to check for understanding and clear misconceptions.
11. Invite volunteers to write the solutions on the board.
   Solutions:
   a. distance = speed × time

1st part: distance $= 48 \times 3 = 144$ km
2nd part: distance $= 53 \times 2 = 106$ km
total distance travelled $= 144 + 106$
$= 250$ km

b. average speed $= \frac{\text{total distance travelled}}{\text{total time taken}}$
$= \frac{250}{5}$ km/hr since total time $= 3 + 2$ hrs
$= 50$ km/hr

The average speed for the journey is 50 km/hr.

12. Explain:
    - There are times when the connection between distance, speed and time leads to more complex equations.
    - We will look at a case that leads to simultaneous linear equations.
13. Write the following problem on the board: A bus and a poda-poda both left the bus terminal at the same time heading in the same direction. The average speed of the poda-poda is 30 km/hr slower than twice the speed of the bus. In two hours, the poda-poda is 20 miles ahead of the bus. Find:
    a. the speed of the bus.
    b. the speed of the poda-poda.
14. Solve the problem on the board, explaining each step:
    Let $d =$ distance of bus and $s =$ speed of bus. Then we have:

distance of poda-poda $= d + 20$
speed of poda-poda $= 2s - 30$

a. Set up the equations.

distance = speed × time
$d = 2s$ (1) $t = 2$ hours
$d + 20 = 2(2s - 30)$ (2) same time

We now have 2 linear equations in d and s
Solve simultaneously by substitution
Substitute equation (1) into equation (2) and simplify

$2s + 20 = 2(2s - 30)$
$= 4s - 60$
$80 = 2s$
$s = 40$ km/hr

The speed of the bus is 40 km/hr.

b. Find the speed of the poda-poda.

speed of poda-poda $= 2s - 30$
$= (2 \times 40) - 30$
$= 80 - 30 = 50$ km/hr

The speed of the poda-poda is 50 km/hr.

Practice *(19 minutes)*

1. Write the following problems on the board:
    a. How long does it take to travel 300 km at an average speed of 40 km/hr?

b. Adama lives 2 km away from her grandmother. Her speed on the way to visit her is 6 km/hr and her speed on the way back is 4 km/hr. Find:
   i. The total time she took to get to her grandmother's house and back.
   ii. The average speed for the whole journey.

c. Amadu walks for x hours at 5 km/hr and runs for y hours at 10 km/hr. He travels a total of 35 km and his average speed is 7 km/hr. Find x and y.

2. Ask pupils to solve the problems either independently or with seatmates.
3. Invite volunteers to come to the board simultaneously to write the solutions. All other pupils should check their work.

Solutions:

a. $\text{t} = \frac{\text{d}}{\text{s}} = \frac{300}{40} = 7.5$ hrs

b. i. 1st part: $t = \frac{2}{6} = \frac{1}{3}$ hr

2nd part: $t = \frac{2}{4} = \frac{1}{2}$ hr

Total time taken: $\frac{1}{3} + \frac{1}{2} = \frac{5}{6}$ hr or 50 minutes

ii. $s = \frac{\text{d}}{\text{t}} = \frac{4}{\frac{5}{6}} = 4.8$ km/hr

c. 1st part: $d = 5x$; 2nd part: $d = 10y$

$$\begin{aligned} 5x + 10y &= \text{total distance travelled} \\ 5x + 10y &= 35 && (1) \\ \text{average speed} &= \frac{\text{total distance travelled}}{\text{total time taken}} \\ 7 &= \frac{35}{x+y} \\ 7(x+y) &= 35 && \text{divide throughout by 7} \\ x + y &= 5 && (2) \end{aligned}$$

We now have 2 linear equations in x and y

$$\begin{aligned} 5x + 10y &= 35 \\ x + y &= 5 \end{aligned}$$

Solve the two equations simultaneously by elimination

$$\begin{aligned} 5x + 10y &= 35 && (1) \\ 5x + 5y &= 25 && \text{(3) Multiply equation (2) by 5} \\ \hline 5y &= 10 && \text{subtract (3) from (1)} \\ y &= 2 \\ x + y &= 5 && (2) \\ x + 2 &= 5 && \text{substitute } y = 2 \text{ in equation (2)} \\ x &= 5 - 2 = 3 && \text{solve for } x \end{aligned}$$

Amadu walks for $x = 3$ hours, and runs for $y = 2$ hours.

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L008 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L009 in the Pupil Handbook before the next class.

Lesson Title: Applications of percentages – Part 1	**Theme:** Numbers and Numeration	
Lesson Number: M4-L009	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to solve problems involving commission, income taxes, simple interest and compound interest.	**Preparation** Review the content of this lesson and be prepared to explain the solutions.	

Opening *(1 minute)*

1. Explain that this lesson is on solving problems related to percentage.

Teaching and Learning *(19 minutes)*

1. Explain **commission:**
    - Some employees, particularly sales people, are given commission on top of (or instead of) their wages or salaries.
    - The value of the commission is usually worked out as a percentage of the amount they sold during the month or year.
    - The value of the amount sold is taken as 100%.
2. Write on the board: Commission $= \frac{x}{100} \times$ sales amount
3. Explain **income taxes:**
    - Income tax is deducted every month by the government from the money people earn. It is used to provide services to the country such as education and health.
    - Sierra Leone's tax rates are given in your Pupil Handbook. To calculate income tax, find the appropriate percentage of a person's income.
4. Explain **simple interest:**
    - When someone deposits money in a bank, the bank pays them interest on the money deposited.
    - When a bank lends money to its customers, it charges them interest on the money borrowed.
    - Simple interest, I, is the amount earned or charged on the initial amount or principal (P) at a given rate (R), for a given period of time, T (in years).
5. Write on the board: Interest: $I = \frac{PRT}{100}$

    Amount at the end of period: $A = P + I$, where P is the principal.
6. Explain **compound interest:**
    - Compound interest is the interest calculated at given intervals over the loan period and added to the principal.
    - This new amount becomes the principal and changes every time the interest is calculated.
    - Each time we do the calculation, we compound the principal by adding the interest calculated for a given period to the previous principal.

7. Write on the board: $CI = A - P$ where A is the amount at the end of the period and P is the principal.
8. Write the following problems on the board:
   a. A sales manager sold goods worth Le 600 million in her shop in one year. If she was paid a commission of 5.2% on her sales,
      i. How much commission was she paid that year?
      ii. What was her average monthly income?
   b. Mariama earns Le 6,600,000.00 per year. How much tax does she pay each month if her tax rate is 15%?
   c. Memuna invests some money in a savings account. Interest is paid at the rate of 6% per annum. After 1 year there is Le 371,000.00 in the account. How much did she invest initially?
9. Ask volunteers to briefly describe how to explain each problem. Allow discussion.
10. Ask pupils to solve the problems with seatmates. Remind them to use the formulae on the board and their Pupil Handbooks for guidance.
11. Walk around to check for understanding and clear misconceptions.
12. Invite volunteers to write the solutions on the board.

Solutions:

a. i. Calculate commission: $\frac{5.2}{100} \times 600{,}000{,}000 = \text{Le } 31{,}200{,}000.00$

ii. Calculate monthly income: $\frac{31{,}200{,}000}{12} = \text{Le } 2{,}600{,}000.00$

b. Calculate her monthly pay, then her monthly income tax using the appropriate rate, 15%.

$$\begin{aligned} \text{monthly pay} &= \frac{6{,}600{,}000}{12} &&= \text{Le } 550{,}000.00 \\ \text{taxable income} &= 550{,}000 - 500{,}000 &&= \text{Le } 50{,}000.00 \\ \text{income tax} &= \frac{15}{100} \times 50{,}000 &&= \text{Le } 7{,}500.00 \end{aligned}$$

Mariama's monthly tax is Le $7{,}500.00$.

c. Apply the formula for simple interest:

$$\begin{aligned} I &= \frac{PRT}{100} && \text{Interest formula} \\ I &= \frac{P\times 6\times 1}{100} && \text{Substitute given values} \\ I &= \frac{6P}{100} = 0.06P && \text{Evaluate} \\ 371{,}000 &= P + 0.06P && \text{Substitute in } A = P + I \\ 371{,}000 &= P(1 + 0.06)1.06P \\ 371{,}000 &= 1.06P \\ P &= \frac{371{,}000}{1.06} \\ &= \text{Le } 350{,}000.00 \end{aligned}$$

Memuna invested Le $350{,}000.00$ initially.

Practice *(19 minutes)*

1. Write the following problems on the board:

a. An estate agency sold a house for Le 500 million. The agreed commission was 2% of the first $200,000,000$ of the sales price and 3% on the remainder. How much commission did the agency make on the sale?
b. Kasho invested Le 5,000,000.00 at 4% interest for 5 years. He then invested the amount at the end of the 5 years for a further 2 years at 5.5% interest. What was the total amount after the 7 years?
c. Abdul deposited Le 1,500,000.00 in a bank at 3% compound interest rate per annum for 3 years. Find the interest at the end of the period.

2. Ask pupils to solve the problems either independently or with seatmates.
3. Invite volunteers to come to the board to write the solutions at the same time. All other pupils should check their work.

Solutions:

a. commission on the first $200,000,000$ $= \frac{2}{100} \times 200,000,000 =$ Le $4,000,000.00$

remainder $= 500,000,000 - 200,000,000$
$=$ Le$300,000,000.00$

commission on remainder $= \frac{3}{100} \times 300,000,000 =$ Le $9,000,000.00$

total commission $= 4,000,000 + 9,000,000 =$ Le $13,000,000.00$

The estate agency made Le$13,000,000.00$ commission on the house sale.

b. Apply the formula $I = \frac{PRT}{100}$:

After 5 years: $I = \frac{5,000,000 \times 4 \times 5}{100} =$ Le $1,000,000.00$

$A = 5,000,000 + 1,000,000 =$ Le $6,000,000.00$

After 2 years: $I = \frac{6,000,000 \times 5.5 \times 2}{100} =$ Le $660,000.00$

$A = 6,000,000 + 660,000 =$ Le $6,660,000.00$

The total amount after 7 years = Le $6,660,000.00$

c. Use the formula to find the amount after each year.

Year	Principal at start of year (Le)	Interest (Le)	Amount at end of year (Le)
1	$1,500,000$	$\frac{4}{100} \times 1500000 = 60000$	$60000 + 1500000 = 1,560,000$
2	$1,545,000$	$\frac{4}{100} \times 1560000 = 62400$	$62400 + 1560000 = 1,622,400$
3	$1,591,350$	$\frac{4}{100} \times 1622400 = 64896$	$64896 + 1622400 = 1,687,296$

compound interest after 3 years = $1,687,296 - 1,500,000 =$ Le $187,296.00$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L009 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L010 in the Pupil Handbook before the next class.

Lesson Title: Applications of percentages – Part 2	**Theme:** Numbers and Numeration	
Lesson Number: M4-L010	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to solve problems involving profit, loss, hire purchase, and discount.	**Preparation** Review the content of this lesson and be prepared to explain the solutions.	

Opening *(1 minute)*

1. Explain that this is the second lesson on solving problems related to percentage.

Teaching and Learning *(19 minutes)*

1. Explain **profit and loss:**
   - An item is sold at a profit when the selling price is greater than the cost price of the item. If, however the cost price of the item is greater than the selling price, the item is sold at a loss.
   - The profit or loss is calculated by taking the difference between the cost price (*CP)* and selling price (*SP),* and both are always positive.
2. Write the formulae on the board:

   profit $= SP - CP$ loss $= CP - SP$

   Percentage profit $= \frac{SP-CP}{CP} \times 100$ Percentage loss $= \frac{CP-SP}{CP} \times 100$
3. Explain **hire purchase:**
   - There are instances when an item is bought and the full amount is paid in regular instalments over several months or years. Paying an item over time usually costs more than the cash price when bought outright because interest is added.
   - The interest charged can be calculated using the simple interest rate based on the length of the loan.
4. Explain **discount:**
   - Discount is given in shops when customers buy in bulk or when there is a special offer. The discount is usually given as a percentage of the original price.
   - The original price is 100% or 1 $\left(\frac{100}{100}\right)$. We use a multiplier which is given by $1 - \frac{R}{100}$ where R is the percentage discount.
5. Write on the board: Multiplier: $1 - \frac{R}{100}$ where R is the percentage discount.
6. Write the following problems on the board:
   a. A man bought a car for Le 15,000,000.00. He later sold it for Le 12,000,000.00. What was his percentage loss on the sale of the car?
   b. The cash price for a refrigerator is Le 2,000,000.00. Miss Koroma paid a deposit of Le 500,000.00 for the refrigerator. If the balance was paid in 12 monthly instalments of Le 150,000.00, find:
      i. The total amount paid for the refrigerator.

ii. The interest charged.
iii. The approximate rate of interest to 1 decimal place.

c. A shop decided to reduce all its prices by 5% for a month. What is the new price of each of the items below?
   i. Bag of flour, Le 55,000.00.
   ii. Bag of onions, Le 75,000.00.
   iii. Box of tomato puree, Le 60,000.00.

7. Ask volunteers to briefly describe how to explain each problem. Allow discussion.
8. Ask pupils to solve the problems with seatmates. Remind them to use the formulae on the board and their Pupil Handbooks for guidance.
9. Walk around to check for understanding and clear misconceptions.
10. Invite volunteers to write the solutions on the board.

Solutions:

a. percentage loss $= \frac{SP-CP}{CP} \times 100$

percentage loss $= \frac{15{,}000{,}000-12{,}000{,}000}{15{,}000{,}000} \times 100$

$= \frac{3{,}000{,}000}{15{,}000{,}000} \times 100 = 20\%$

b. i. total for instalment $= 150{,}000 \times 12 =$ Le $1{,}800{,}000.00$

total cost $= 500{,}000 + 1{,}800{,}000 =$ Le $2{,}300{,}000.00$

ii. interest paid $= 2{,}300{,}000 - 2{,}000{,}000 =$ Le $300{,}000.00$

iii. **average time,** $T = \frac{1+12}{2}$

$= \frac{13}{2}$ = 6.5 months $= \frac{6.5}{12}$ years

$R = \frac{I \times 100}{PT} = \frac{300{,}000 \times 100 \times 12}{2{,}000{,}000 \times 6.5} = 27.69\%$

The total cost paid by Miss Koroma is Le $2{,}300{,}000.00$; interest paid is Le$300{,}000.00$ and approximate interest rate is 27.7% to 1 d.p.

c. Multiplier: $1 - \frac{5}{100} = 1 - 0.05 = 0.95$

i. Bag of flour at Le55,000.00:

new price $= 0.95 \times 55{,}000 = \text{Le } 52{,}250.00.00$

ii. Bag of onions at Le75,000.00

new price $= 0.95 \times 75{,}000 = \text{Le } 71{,}250.00.00$

iii. Box of tomato puree at Le60,000.00

new price $= 0.95 \times 60{,}000 = \text{Le } 57{,}000.00.00$

Practice *(19 minutes)*

1. Write the following problems on the board:
   a. A motor bike was sold for Le 4,250,000.00 at a loss of 15%. Find the cost price.
   b. Mrs. Mansaray bought an oven on hire purchase for Le 1,687,500.00. She paid 12.5% more than if she had paid cash for the oven. If she made an

initial deposit of 20% of the cash price and then paid the rest in 6 monthly instalments, find:

   i. The initial deposit.
   ii. The amount of each instalment.
   iii. The approximate rate of interest to 1 decimal place.

c. Mrs. Davies bought a television set reduced by 20% What was the original price if the discounted price was Le 2,800,000.00?

2. Ask pupils to solve the problems either independently or with seatmates.
3. Invite volunteers to come to the board to write the solutions at the same time. All other pupils should check their work.

Solutions:

a. multiplier $= 1 - \frac{15}{100} = 0.85$ — subtract the percentage loss from 100% of the cost price

$SP = 0.85 \times CP$

$5{,}000{,}000 = 0.85CP$

$CP = \frac{4{,}250{,}000}{0.85}$

$=$ Le $5{,}000{,}000.00$

The cost price of the motor bike was Le $5{,}000{,}000.00$.

b. i. $\left(1 + \frac{12.5}{100}\right) \times$ cash price of oven $= 1{,}687{,}500$

$1.125 \times$ cash price of oven $= 1{,}687{,}500$

cash price of oven $= \frac{1{,}687{,}500}{1.125}$ = Le $1{,}500{,}000.00$

initial deposit $= \frac{20}{100} \times 1{,}500{,}000$ = Le $300{,}000.00$

ii. **remainder to be paid** $= 1{,}500{,}000 - 300{,}000$ = Le $1{,}200{,}000.00$

amount per instalment $= \frac{1{,}200{,}000}{6}$ = Le $200{,}000.00$

iii. **average time,** $T = \frac{1+6}{2}$

$= \frac{7}{2} = 3.5$ months $= \frac{3.5}{12}$ years

interest, $I = 1{,}687{,}500 - 1{,}500{,}000$ = Le $187{,}500.00$

$R = \frac{I \times 100}{PT}$

$R = \frac{187{,}500 \times 100 \times 12}{1{,}200{,}000 \times 3.5}$

$= 53.57\%$

Mrs. Mansaray paid an initial deposit of Le $300{,}000.00$; monthly instalment is Le $200{,}000.00$ and approximate interest rate is 53.6% to 1 d.p.

c. Let original price $= x$

multiplier $= 1 - \frac{20}{100} = 1 - 0.2$

$= 0.8$

$0.8 \times x = 2{,}800{,}000$

$x = \frac{2{,}800{,}000}{0.8} = \text{Le } 3{,}500{,}000.00$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L010 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L011 in the Pupil Handbook before the next class.

Lesson Title: Applications of percentages – Part 3	**Theme:** Numbers and Numeration	
Lesson Number: M4-L011	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to solve problems involving depreciation, financial partnerships, and foreign exchange.	**Preparation** Review the content of this lesson and be prepared to explain the solutions.	

Opening *(1 minute)*

1. Explain that this is the second lesson on solving problems related to percentage.

Teaching and Learning *(19 minutes)*

1. Explain **depreciation:** Many goods lose their value over time as they get older. Examples are cars, computers and mobile phones. This decrease in value is called depreciation.
2. Write the formulae on the board:
   The value at the end of a particular time period is given by:
   $$V = P\left(1 - \frac{R}{100}\right)^n$$
   where $V =$ Value at the end of the period
   $R =$ rate of depreciation
   $P =$ Original price
   $n =$ Period
   The rate of depreciation can be found using the formula $R = \frac{P-V}{P} \times 100$.
3. Explain **financial partnership:**
   - When 2 or more people come together and invest money for the purpose of providing goods or services at a profit, it is called a financial or business partnership.
   - In many instances, the partners pay out profit in proportion to the money or capital invested.
4. Explain **foreign exchange:**
   - Every country has its own currency which it uses for its money. The exchange rate is the rate at which one unit of a particular currency is converted to another currency.
   - There are usually two rates – the **buying** and the **selling** rate.
   - There is a table in the Pupil Handbook with examples of buying and selling rates, which is also shown.

Currency	Buying	Selling
€ 1.00	Le 8,600	Le 8,900
GH¢ 1.00	Le 1,500	Le 1,560
GMD 1.00	Le 150	Le 156
₦ 1.00	Le 20	Le20.80
£ 1.00	Le 9,500	Le 9,800
$ 1.00	Le 7,600	Le 7,900

5. Write the problems on the board:
   - A car costs Le 50,000,000.00. It depreciates at 15% per annum. Find its value after: i. 1 year ii. 3 years
   - Mustapha and Ahmed invested Le 15,000,000.00 in a business in the ratio 3 : 2 respectively.

i. How much did each partner invest?
ii. If they shared Le 6,000,000.00 profit in the same ratio as their investments, how much did each receive?

- On a certain day, the selling rate for US dollars is Le 7,900.00. How much will Le 9,000,000.00 give you?
- On a certain day, the buying rate for Nigerian naira is Le20.00. How much naira can a bank buy for 1 million Leones?

6. Ask volunteers to briefly describe how to explain each problem. Allow discussion.
7. Ask pupils to solve the problems with seatmates. Remind them to use the formulae on the board and their Pupil Handbooks for guidance.
8. Walk around to check for understanding and clear misconceptions.
9. Invite volunteers to write the solutions on the board.

Solutions:

- Calculate the value at the end of each period.

i. 1 year: $V = 50{,}000{,}000\left(1-\frac{15}{100}\right)^1$
$= \text{Le } 42{,}500{,}000.00$

ii. 3 years: $V = 50{,}000{,}000\left(1-\frac{15}{100}\right)^3$
$= \text{Le } 30{,}706{,}250.00$

b. i. total number of parts $= 3+2 = 5$
Mustapha's investment $= \frac{3}{5}\times 15{,}000{,}000 = \text{Le } 9{,}000{,}000.00$
Ahmed's investment $= \frac{2}{5}\times 15{,}000{,}000 = \text{Le } 6{,}000{,}000.00$

Step 2: Calculate each partner's profit.

ii. Mustapha's profit $= \frac{3}{5}\times 6{,}000{,}000 = \text{Le } 3{,}600{,}000.00$
Ahmed's profit $= \frac{2}{5}\times 6{,}000{,}000 = \text{Le } 2{,}400{,}000.00$

Mustapha invested Le 9,000,000.00 and made a profit of Le 3,600,000.00.
Ahmed invested Le 6,000,000.00 and made a profit of Le 2,400,000.00.

- Write the selling rate as a fraction $\left(\frac{\$1.00}{\text{Le } 7900}\right)$ and use it to find dollars:

$$\text{Le } 9{,}000{,}000.00 = \frac{\$1.00}{\text{Le } 7{,}900}\times \text{Le } 9{,}000{,}000.00 = \$1{,}139.24$$

- Write the buying rate as a fraction $\left(\frac{₦\,1}{\text{Le } 20}\right)$ and use it to find naira:

$$\text{Le } 1{,}000{,}000.00 = \frac{₦1}{\text{Le } 20}\times \text{Le } 1{,}000{,}000.00 = ₦50{,}000.00$$

Practice *(19 minutes)*

1. Write the following problems on the board:
   a. A laptop costs Le 1,500,000.00. Its value depreciates by 20% in the first year and 15% in the second and subsequent years.
      i. What is its value at the end of 5 years? Give your answer to 2 decimal places.

ii. What was the average rate of depreciation over the 5 years?

b. Sia and Fatu invested Le 40,000,000.00 in a business in the ratio 3 : 5 respectively.

iii. How much did each partner invest?

iv. If they shared profit of Le 12,000,000.00 in the same ratio as their investments, how much did each receive?

c. On a certain day, the dollar buying price is Le 7,600.00, and the selling price is Le 7,900.00. How much profit will a bank make if they buy then sell $1,000?

2. Ask pupils to solve the problems either independently or with seatmates.
3. Ask volunteers to come to the board to write the solutions at the same time. All other pupils should check their work.

Solutions:

a. i. After 1 year, $V = 1{,}500{,}000\left(1-\frac{20}{100}\right)^1 = \text{Le } 1{,}200{,}000.00$

After 4 more years, $V = 1{,}200{,}000\left(1-\frac{15}{100}\right)^4 = \text{Le } 626{,}407.50.00$

ii. average rate $= \frac{20+4(15)}{5} = 16\%$

b. i. Total number of parts $= 3+5 = 8$

Sia's investment $= \frac{3}{8}\times 40{,}000{,}000 = \text{Le } 15{,}000{,}000.00$

Fatu's investment $= \frac{5}{8}\times 40{,}000{,}000 = \text{Le } 25{,}000{,}000.00$

Calculate each partner's profit:

ii. Sia's profit $= \frac{3}{8}\times 12{,}000{,}000 = \text{Le } 4{,}500{,}000.00$

Fatu's profit $= \frac{5}{8}\times 12{,}000{,}000 = \text{Le } 7{,}500{,}000.00$

Sia invested Le$15{,}000{,}000.00$ and made a profit of Le $4{,}500{,}000.00$. Fatu invested Le$25{,}000{,}000.00$ and made a profit of Le $7{,}500{,}000.00$.

c. Calculate the amount the bank paid, then the amount that they made from the sale. Subtract to find the profit.

Bought (buying rate): $\$1{,}000 = \$1{,}000\left(\frac{\text{Le } 7{,}600}{\$1.00}\right) = \text{Le } 7{,}600{,}000.00$

Sold (selling rate): $\$1{,}000 = \$1{,}000\left(\frac{\text{Le } 7{,}900}{\$1.00}\right) = \text{Le } 7{,}900{,}000.00$

Profit: $\text{Le } 7{,}900{,}000.00 - \text{Le } 7{,}600{,}000.00 = \text{Le } 300{,}000.00$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L011 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L012 in the Pupil Handbook before the next class.

Lesson Title: Indices	**Theme:** Numbers and Numeration	
Lesson Number: M4-L012	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcomes By the end of the lesson, pupils will be able to: 1. Apply the laws of indices to simplify expressions. 2. Solve equations that involve indices.	**Preparation** 1. Review the content of this lesson and be prepared to explain the solutions. 2. Write the laws of indices in the Opening on the board.	

Opening *(1 minute)*

1. Write the following on the board:
   Laws of indices:
   1. $a^m \times a^n = a^{m+n}$
   2. $a^m \div a^n = a^{m-n}$
   3. $a^0 = 1$
   4. $(a^x)^y = a^{xy}$
2. Explain that these laws will be used throughout this lesson to simplify expressions involving indices.

Teaching and Learning *(19 minutes)*

1. Ask pupils what else they know about **indices**. Encourage them to look at the Overview in the Pupil Handbook. Write their ideas on the board as they list them.
2. Write the following on the board if pupils do not identify them. Make sure pupils understand each.
   - $a^{-n} = \frac{1}{a^n}$
   - $\sqrt[n]{x} = x^{\frac{1}{n}}$
   - $\left(\frac{\mathrm{a}}{\mathrm{b}}\right)^{\mathrm{n}} = \frac{a^n}{b^n}$
   - $\left(\frac{\mathrm{a}}{\mathrm{b}}\right)^{-\mathrm{n}} = \left(\frac{\mathrm{b}}{\mathrm{a}}\right)^{\mathrm{n}}$
3. Explain:
   - We will now use these various rules to solve problems. Refer to the rules in your Pupil Handbook or on the board any time you need.
   - In addition to these rules, apply the normal order of operations (BODMAS) to evaluate expressions involving indices.
4. Write the following problems on the board:
   a. $(4b^2)^3 \div 8b^2a^3$
   b. Solve for x: $2^{x-1} = 16$
5. Ask volunteers to give the steps to solve each. Solve the problems on the board as they give them.
   Solutions:
   a. $(4b^2)^3 \div 8b^2a^3 = 4^3(b^2)^3 \div 8b^2a^3$ Distribute the power
   $= 64b^{2\times3} \div 8b^2a^3$ Multiply powers on b

$= 64b^6 \div 8b^2a^3$

$= (64 \div 8)b^{6-2}a^{0-3}$ Divide (subtract powers)

$= 8b^4a^{-3}$ Simplify

$= \frac{8b^4}{a^3}$ Move negative powers to the denominator

b.

$2^{x-1} = 16$

$2^{x-1} = 2^4$ Write 16 with base 2

$x - 1 = 4$ Set powers equal

$x = 4 + 1$ Solve for x

$x = 5$

6. Write the following problems on the board: Simplify the following expressions:
    a. $\frac{75a^2b^{-2}}{5a^3b^{-3}}$
    b. $(12y^3 \times 2y^2) \div (4y^5 \times 2y^2)$

    Solve for the variable in the following expressions:
    c. $3^{2n+1} = 81$
    d. $4^{2x} = 8^{x-1} \times 4$
7. Ask pupils to work with seatmates to solve the problems.
8. Walk around to check for understanding and clear misconceptions.
9. Invite volunteers to write the solutions on the board.

Solutions:

a.

$\frac{75a^2b^{-2}}{5a^3b^{-3}} = 75a^2b^{-2} \div 5a^3b^{-3}$

$= (75 \div 5)(a^2 \div a^3)(b^{-2} \div b^{-3})$

$= 15a^{2-3}b^{-2-(-3)}$

$= 15a^{-1}b^{-2+3}$

$= 15a^{-1}b^1$

$= \frac{15b}{a}$

b.

$(12y^3 \times 2y^2) \div (4y^5 \times 2y^2) = (12 \times 2 \times y^3 \times y^2) \div (4 \times 2 \times y^5 \times y^2)$

$= (24y^{3+2}) \div (8y^{5+2})$

$= 24y^5 \div 8y^7$

$= (24 \div 8)(y^5 \div y^7)$

$= 3y^{5-7}$

$= 3y^{-2} = \frac{3}{y^2}$

c.

$3^{2n+1} = 81$

$3^{2n+1} = 3^4$

$2n + 1 = 4$

$2n = 4 - 1$

$2n = 3$

$n = \frac{3}{2} = 1\frac{1}{2}$

d.

$4^{2x} = 8^{x-1} \times 4$

$2^{2(2x)} = 2^{3(x-1)} \times 2^2$

$2^{4x} = 2^{3x-3+2}$

$2^{4x} = 2^{3x-1}$

$4x = 3x - 1$

$4x - 3x = -1$

$x = -1$

Practice *(19 minutes)*

1. Write the following problems on the board:
   Simplify the following expressions:
   a. $(x^{-2})^3 \times x^{10}$
   b. $\frac{8a^5 \times 3a^4}{12a^3}$
   c. $36a^9b^5 \div (9a^5b^6 \div 3a^2b^2)^2$

   Solve for the variable in the following:
   d. $10^{2x-1} = \frac{1}{1,000}$
   e. $5^{x+3} = 25^{x+1} \div 125$
2. Ask pupils to solve the problems either independently or with seatmates.
3. Invite volunteers to come to the board to write the solutions at the same time. All other pupils should check their work.

   Solutions:

a. $(x^{-2})^3 \times x^{10} = x^{-2\times 3} \times x^{10}$
$= x^{-6} \times x^{10}$
$= x^{-6+10}$
$= x^4$

b. $\frac{8a^5 \times 3a^4}{12a^3} = \frac{(8\times 3)a^{5+4}}{12a^3}$
$= \frac{24a^9}{12a^3}$
$= 24a^9 \div 12a^3$
$= (24 \div 12)a^{9-3}$
$= 2a^6$

c. $(9a^5b^6 \div 3a^2b^2)^2 = [(9 \div 3)a^{5-2}b^{6-2}]^2$
$= (3a^3b^4)^2$
$= 3^2a^{3\times 2}b^{4\times 2}$
$= 9a^6b^8$

Substitute this into the original expression:

$36a^9b^5 \div (9a^5b^6 \div 3a^2b^2)^2 = 36a^9b^5 \div 9a^6b^8$
$= (36 \div 9)a^{9-6}b^{5-8}$
$= 4a^3b^{-3} = \frac{4a^3}{b^3}$

c.
$$\begin{aligned} 10^{2x-1} &= \frac{1}{1,000} \\ 10^{2x-1} &= \frac{1}{10^3} \\ 10^{2x-1} &= 10^{-3} \\ 2x - 1 &= -3 \\ 2x &= -2 \\ x &= \frac{-2}{2} = -1 \end{aligned}$$

d.
$$\begin{aligned} 5^{x+3} &= 25^{x+1} \div 125 \\ 5^{x+3} &= 5^{2(x+1)} \div 5^3 \\ 5^{x+3} &= 5^{2x+2-3} \\ 5^{x+3} &= 5^{2x-1} \\ x + 3 &= 2x - 1 \\ 3 + 1 &= 2x - x \\ x &= 4 \end{aligned}$$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L012 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L013 in the Pupil Handbook before the next class.

Lesson Title: Indices	**Theme:** Numbers and Numeration	
Lesson Number: M4-L013	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcomes By the end of the lesson, pupils will be able to: 1. Identify the relationship between logarithms and indices, and use it to solve logarithms. 2. Use logarithm tables to solve problems involving logarithms and antilogarithms.	**Preparation** 1. Review the content of this lesson and be prepared to explain the solutions. 2. Bring log tables to class, and ask pupils to do the same.	

Opening *(1 minute)*

1. Discuss and allow pupils to share their ideas: What are logarithms? How are logarithms related to indices?
2. Explain that this lesson is on solving logarithms and antilogarithms.

Teaching and Learning *(23 minutes)*

1. Write on the board:

2. Explain: Any logarithm can be written as an equivalent index.
3. Write on the board: Solve $x = \log_3 9$
4. Ask a volunteer to give an index that is equivalent to the logarithm.
5. Write the answer on the board: $9 = 3^x$
6. Remind pupils that this equation can be solved for x using the laws of indices from the previous lesson.
7. Solve on the board and explain:

$$\begin{aligned} x &= \log_3 9 \\ 9 &= 3^x && \text{Change to index form} \\ 3^2 &= 3^x && \text{Substitute } 9 = 3^2 \\ 2 &= x \end{aligned}$$

8. Write on the board: $\log_{10} 76.83 = 1.8856$
9. Explain:
   - We can find logarithms of numbers to 4 decimal places using log tables.
   - Log tables are for logarithms in base 10. If a base is not given on a log, assume it is in base 10.
   - The whole number part of the result (in this example, 1) is called the **characteristic**.
   - The decimal part (in this example, .8573) is called the **mantissa**.
10. Draw on the board:

11. Explain how to find the characteristic: The characteristic can be found by expressing the number you are finding the logarithm of (in this case 76.83) in standard form. The power on the 10 is the characteristic.
12. Ask a volunteer to express 76.83 in standard form. (Answer: 7.683×10^1)
13. Explain: The whole number part (characteristic) is 1, because the power on 10 is 1.
14. Explain: We will now find the mantissa of $\log_{10} 76.83$ from the logarithm table.
15. Hold up the logarithm table and explain the steps for finding the mantissa:
    - $\log 76.83 = 1.$"something". We are looking for the decimal part.
    - To find the decimal part, go along the row beginning with 76 and under 8, which gives $8{,}854$.
    - Now find the number in the differences column headed 3. This number is 2.
    - Add 2 to $8{,}854$ to get $8{,}856$. Therefore $\log 76.83 = 1.8856$.
12. Explain: Antilogarithms are the opposite of logarithms. They "undo" logarithms.
16. Write on the board: If $\log 673.4 = 2.8283$, then $\text{antilog}\ 2.8283 = 673.4$.
17. Explain:
    - We use tables of antilogarithms to solve antilog problems. We must use a different table. Antilogarithms and logarithms each have their own table.
    - When finding antilog, look up the fractional part only. Using the example on the board, we would drop the 2 and look for 0.8283.
18. Write on the board: Find the antilog of 0.5768.
19. Hold up the antilog table. Explain the steps for finding the antilog:
    - Put the finger under .57, run it along to the column headed 6. Here the number is 3,767.
    - Run your finger to the number under 8 in the differences (It is 7).
    - Add the number under the columns for 6 and difference 8: $3{,}767 + 7 = 3{,}774$.
    - We know that there is 1 integer digits in the antilog, because the integer in the problem (the characteristic) is 0. We get 3.774
20. Explain **how to determine the decimal's placement**:
    - Move the decimal point one more space than the value of the characteristic in the problem.
    - In the example, the characteristic was 0. Therefore, we moved 1 space from the left side, and changed $3{,}774$ to 3.774
    - Recall that when we calculated logs, we gave the characteristic the number of digits before the decimal place minus 1. For antilogs, give the result **one more** integer digit than the characteristic given in the problem.
21. Write the following problems on the board: Find the logarithms of: a. $4{,}137$ b. 8.403
    Find the antilogarithms of: c. 2.7547 d. 5.3914

22. Ask pupils to work with seatmates to solve the problems. Make sure each group of seatmates has a set of log tables, and remind them to look at the Pupil Handbook for guidance.
23. Walk around to check for understanding and clear misconceptions.
24. Invite volunteers to write the solutions on the board and explain using log tables.

    Solutions:

    i. Standard form: 4.137×10^3, which gives characteristic 3; from the log table, 4.137 gives mantissa 0.6167. Therefore, $\log 4137 = 3.6167$.

    ii. Standard form: 8.403×10^0, which gives characteristic 0; from the log table, 8.403 gives mantissa 0.9245. Therefore, $\log 8.403 = 0.9245$.

    iii. The fractional part of 2.7547 is 0.7547, which gives 5684 in the antilog tables. The integer part of 2.7547 is 2, so there are three digits before the decimal point. Therefore $\text{antilog}\, 2.7547 = 568.4$.

    iv. The fractional part of 5.3914 is 0.3914, which gives 2462 in the antilog tables. The integer 5 shows that there are six digits before the decimal point. Therefore, $\text{antilog}\, 5.3914 = 246{,}200$.

Practice *(15 minutes)*

1. Write the following problems on the board:
   a. Find the logarithm of: i. 208.5 ii. 40.02
   b. Find the antilogarithm of: i. 2.1814 ii. 4.2105
   c. If $\log n = 2.3572$, find n.
2. Ask pupils to solve the problems either independently or with seatmates.
3. Ask volunteers to come to the board simultaneously to write the solutions. All other pupils should check their work.

   Solutions:

   a. i. Standard form: 2.085×10^2, which gives characteristic 2; from the log table, 2.085 gives mantissa 0.3192. Therefore, $\log 208.5 = 2.3192$.
   ii. Standard form: 4.002×10^1, which gives characteristic 1; from the log table, 4.002 gives mantissa 0.6023. Therefore, $\log 40.02 = 1.6023$.

   b. i. The fractional part gives 1518 in the antilog tables; the integer 2 tells us there are 3 digits before the decimal place. Thus, $\text{antilog}\, 2.1814 = 151.8$.
   ii. The fractional part gives 1624 in the antilog tables; the integer 4 tells us there are 5 digits before the decimal place. Thus, $\text{antilog}\, 4.2105 = 16{,}240$

   c. Rewrite the equation: $n = \text{antilog}\,(2.3572)$
   The antilog of 0.3572 is 2276. Since the characteristic is 2, put the decimal 3 places from the left side. Therefore $n = 227.6$.

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L013 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L014 in the Pupil Handbook before the next class.

Lesson Title: Logarithms	**Theme:** Numbers and Numeration	
Lesson Number: M4-L014	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to apply the laws of logarithms to solve problems.	**Preparation** 1. Review the content of this lesson and be prepared to explain the solutions. 2. Write the laws of logarithms in the Opening on the board.	

Opening *(1 minute)*

1. Write the following on the board:

   Laws of logarithms:
   1. $\log_{10} pq = \log_{10} p + \log_{10} q$
   2. $\log_{10}\left(\frac{p}{q}\right) = \log_{10} p - \log_{10} q$
   3. $\log_{10}(P)^n = n \log_{10} P$
2. Explain that these laws will be used throughout this lesson to simplify expressions involving logarithms.

Teaching and Learning *(19 minutes)*

1. Ask pupils what else they know about **logarithms**. Encourage them to look at the Overview in the Pupil Handbook. Write their ideas on the board as they list them.
2. Write the following on the board if pupils do not identify them. Make sure pupils understand each.
   - $\log_a a = 1$
   - $\log_a 1 = 0$
3. Explain: We will now use these various rules to solve problems. Refer to the rules in your Pupil Handbook or on the board any time you need.
4. Write the problems on the board:
   a. Simplify: $\frac{\log_2 6^2}{\log_2 6}$
   b. Simplify: $2\log_5\left(\frac{4}{5}\right)$
5. Ask volunteers to give the steps to solve each. Solve the problems on the board as they give them.

   Solutions:

   a.
   $$\frac{\log_2 6^2}{\log_2 6} = \frac{2(\log_2 6)}{(\log_2 6)}$$ Rewrite the numerator
   $$= 2$$ $(\log_2 6)$ cancels

   b.
   $$2\log_5\left(\frac{4}{5}\right) = \log_5\left(\frac{4}{5}\right)^2$$ Law of logarithms 3
   $$= \log_5 \frac{4^2}{5^2}$$
   $$= \log_5 2^4 - \log_5 5^2$$ Law of logarithms 2
   $$= 4\log_5 2 - 2\log_5 5$$ Law of logarithms 3

$= \quad 4\log_5 2 - 2$ Because $\log_a a = 1$

6. Write the following problems on the board: Simplify the following expressions:
   a. $\log_2 8 + \log_2 6 - \log_2 2$
   b. $\frac{\log_a 8 + \log_a 6 - \log_a 3}{\log_a 4}$
   c. $\frac{\log_x 64}{\log_x 4}$
7. Ask pupils to work with seatmates to solve the problems.
8. Walk around to check for understanding and clear misconceptions.
9. Invite volunteers to write the solutions on the board.

   Solutions:

   a.
   $$\begin{aligned} \log_2 8 + \log_2 4 - \log_2 2 &= \log_2\left(\frac{8\times4}{2}\right) \\ &= \log_2 16 \\ &= \log_2 2^4 \\ &= 4(\log_2 2) \\ &= 4\times1 = 4 \end{aligned}$$

   b.
   $$\begin{aligned} \frac{\log_a 8 + \log_a 6 - \log_a 3}{\log_a 4} &= \frac{\log_a\left(\frac{8\times6}{3}\right)}{\log_a 4} \\ &= \frac{\log_a 16}{\log_a 4} \\ &= \frac{\log_a 2^4}{\log_a 2^2} \\ &= \frac{4(\log_a 2)}{2(\log_a 2)} \\ &= \frac{4}{2} = 2 \end{aligned}$$

   c. $\frac{\log_x 64}{\log_x 4} = \frac{\log_x 4^3}{\log_x 4} = \frac{3\cancel{\log_x 4}}{\cancel{\log_x 4}} = 3$

10. Write on the board: Simplify $\log 0.9$ if $\log 3 = 0.4771$
11. Discuss: How do you think we would solve this problem? (Example answer: Rewrite 0.9 as a fraction, then try to change the expression so it has a $\log 3$. We can then use substitution.)
12. Solve the solution on the board, explaining each step:

$\log 0.9$	$=$	$\log\frac{9}{10}$	Convert to a fraction
	$=$	$\log 9 - \log 10$	Apply the division rule
	$=$	$\log 3^2 - \log 10$	Substitute $9 = 3^2$
	$=$	$2\log 3 - \log 10$	Apply the power rule
	$=$	$2(0.4771) - 1$	Substitute $\log 3 = 0.4771$ and $\log 10 = 1$
	$=$	$0.9542 - 1$	
	$=$	-0.0458	

Practice *(19 minutes)*

1. Write the following problems on the board:
   a. Simplify the following expressions: i. $\log_3 9$ ii. $2\log_5\left(\frac{4}{5}\right)$
   b. Given that $\log_{10} 2 = 0.3010$ and $\log_{10} 7 = 0.8451$, calculate $\log_{10} 2.8$.
   c. Given that $\log_{10} 7 = 0.8451$ and $\log_{10} 3 = 0.4771$, find $\log_{10}\left(\frac{9}{7}\right)$.
   d. Find y if $y = \log_3 6 + \log_3 6 - \log_3 3$.
2. Ask pupils to solve the problems either independently or with seatmates.

3. Invite volunteers to come to the board to write the solutions at the same time. All other pupils should check their work.

Solutions:

a. i.

$$\begin{aligned}\log_3 9 &= \log_3 3^2\\ &= 2\log_3 3\\ &= 2\times 1\\ &= 2\end{aligned}$$

ii.

$$\begin{aligned}2\log_5\left(\frac{4}{5}\right) &= \log_5\left(\frac{4}{5}\right)^2\\ &= \log_5\frac{4^2}{5^2}\\ &= \log_5 2^4 - \log_5 5^2\\ &= 4\log_5 2 - 2\log_5 5\\ &= 4\log_5 2 - 2\end{aligned}$$

b.

$$\begin{aligned}\log_{10} 2.8 &= \log_{10}\left(\frac{28}{10}\right)\\ &= \log_{10}\left(\frac{7\times 4}{10}\right)\\ &= \log_{10} 7 + \log_{10} 4 - \log_{10} 10\\ &= \log_{10} 7 + \log_{10} 2^2 - \log_{10} 10\\ &= \log_{10} 7 + 2\log_{10} 2 - \log_{10} 10\\ &= 0.8451 + 2(0.3010) - 1\\ &= 0.8451 + 0.6020 - 1\\ &= 0.4771\end{aligned}$$

c.

$$\begin{aligned}\log_{10}\left(\frac{9}{7}\right) &= \log_{10} 9 - \log_{10} 7\\ &= \log_{10}(3\times 3) - \log_{10} 7\\ &= \log_{10} 3 + \log_{10} 3 - \log_{10} 7\\ &= 2\log_{10} 3 - \log_{10} 7\\ &= 2(0.4771) - 0.8451\\ &= 0.9542 - 0.8451\\ &= 0.1091\end{aligned}$$

d.

$$\begin{aligned}y &= \log_3 6 + \log_3 6 - \log_3 4\\ &= \log_3\frac{6\times 6}{4}\\ &= \log_3\frac{36}{4}\\ y &= \log_3 9\end{aligned}$$

Index form: $3^y = 9$

$$\begin{aligned}3^y &= 3^2\\ \therefore y &= 2\end{aligned}$$

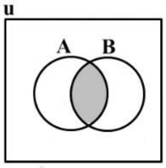
Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L014 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L015 in the Pupil Handbook before the next class.

Lesson Title: Representing sets with diagrams and symbols	**Theme:** Numbers and Numeration	
Lesson Number: M4-L015	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to describe and represent sets using diagrams and symbols (including subsets, the intersection of 2 or 3 sets, disjoint sets, the union of 2 sets, and the complement of a set).	**Preparation** 1. Review the content of this lesson and be prepared to explain the solutions. 2. Draw the diagrams in the Opening on the board.	

Opening *(2 minutes)*

1. Draw the following diagrams on the board:

a.


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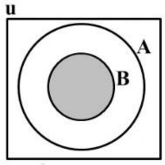

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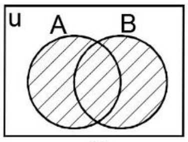

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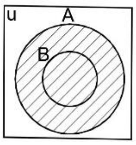
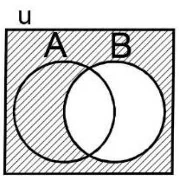


2. Discuss and allow pupils to share their ideas: What are these diagrams? What do they represent?
3. Explain that this lesson is on representing sets with Venn diagrams and symbols.

Teaching and Learning *(18 minutes)*

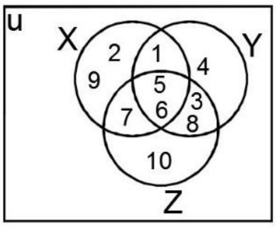
1. Explain:
   a. These are Venn diagrams. Each one shows a different relationship between 2 sets, A and B.
   b. Diagrams a. and b. show the **intersection** of 2 sets. The intersection contains the elements of the sets that are common to both of them.
   c. Diagrams c. and d. show the **union** of 2 sets. The union of two sets A and B is the set formed by putting the two sets together.
2. Label each diagram on the board and explain the symbols:
   a. Diagrams a. and b.: $A \cap B$ (read as "A intersection B")
   b. Diagrams c. and d.: $\mathrm{A} \cup \mathrm{B}$ (read as "A union B")
3. Draw the diagram on the board:



4. Discuss: What does this diagram represent? (Answer: The complement of set B.)
5. Explain:

a. The complement of a set is equal to elements in the universal set (all of the elements of concern) minus elements in the set.
b. There are other diagrams that show complements in the Pupil Handbook. Refer pupils to these for a better understanding.

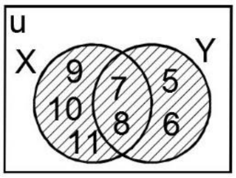
6. Write the following problem on the board: Consider the following sets: $\mathrm{X} = \{1, 2, 5, 6, 7, 9\}$; $\mathrm{Y} = \{1, 3, 4, 5, 6, 8\}$ and $\mathrm{Z} = \{3, 5, 6, 7, 8, 10\}$.
   i. Find $\mathrm{X} \cap \mathrm{Y} \cap \mathrm{Z}$.
   ii. Draw a Venn diagram to illustrate the 3 sets.
7. Invite any volunteer to write down the elements that are common in set X, Y, and Z on the board. (Answer: $\mathrm{X} \cap \mathrm{Y} \cap \mathrm{Z} = \{5, 6\}$).
8. Invite a volunteer to write down the elements in $\mathrm{X} \cap \mathrm{Y}$. (Answer: $\mathrm{X} \cap \mathrm{Y} = \{1, 5, 6\}$).
9. Invite a volunteer to write down the elements in $\mathrm{Y} \cap \mathrm{Z}$. (Answer: $\mathrm{Y} \cap \mathrm{Z} = \{3, 5, 6, 8\}$).
10. Invite another volunteer to write down the elements in $\mathrm{X} \cap \mathrm{Z}$. (Answer: $\mathrm{X} \cap \mathrm{Z} = \{5, 6, 7\}$).
11. Explain: To draw a Venn diagram, start by filling in the elements in the intersection first. Work outwards, filling in the elements that are in only one set last.



12. Draw the Venn diagram for part b. on the board as a class. Invite volunteers to come to the board and identify where each element belongs.
13. Write the following problem on the board: If $\mathrm{U} = \{-5 < x \leq 5\}$ and $\mathrm{A} = \{-2 < x \leq 3\}$. Find A'.
14. Invite a volunteer to write down the elements of the universal set U on the board. (Answer: $\mathrm{U} = \{-4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$).
15. Invite another volunteer to write down the elements of set A on the board. (Answer: $\mathrm{A} = \{-1, 0, 1, 2, 3\}$)
16. Ask pupils to work with seatmates to write the complement of A in their exercise books.
17. Invite a volunteer to write the elements of A' on the board. (Answer: $\mathrm{A}' = \{-4, -3, -2, 4, 5\}$).
18. Write the following problem on the board. $\mathrm{X} = \{6 < x < 12\}$ and $\mathrm{Y} = \{4 < x < 9\}$, where x is an integer.
    a. List the elements of sets X and Y.
    b. Find $\mathrm{X} \cup \mathrm{Y}$.
    c. Draw a Venn diagram to illustrate the relationship between sets X and Y.
19. Ask pupils to solve the problem with seatmates.
20. Walk around and check for understanding and clear misconceptions.
21. Invite volunteers to write the solutions on the board.

Solutions:

a. $X = \{7, 8, 9, 10, 11\}$ and $Y = \{5, 6, 7, 8, \}$
b. $X \cup Y = \{5, 6, 7, 8, 9, 10, 11\}$
c. Diagram →

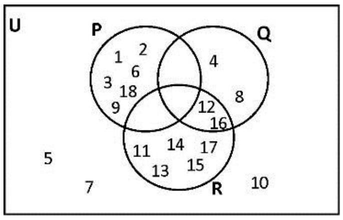


Practice *(19 minutes)*

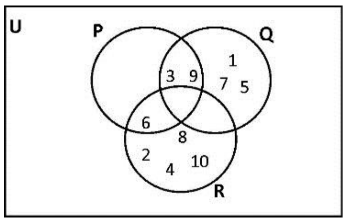
1. Write the following problems on the board:
   a. U is the universal set consisting of all positive integers x such that $\{1 \leq x \leq 18\}$. P, Q and R are subsets of U such that $P = \{x: x$ is a factor of $18\}$, $Q = \{x: x$ is a multiple of 4 less than $18\}$ and $R = \{10 < x < 18\}$. Draw a Venn diagram to illustrate U, P, Q and R.
   b. P, Q and R are subsets of the universal set U, where $U = \{n: 1 \leq n \leq 10\}$ and n is an integer. P consists of multiples of 3; Q consists of odd numbers and R consists of even numbers. Draw a Venn diagram to illustrate U, P, Q and R.
   c. If $A = \{11, 12, 13, 14\}$ and $B = \{13, 14, 15, 16, 17\}$ are subsets of $U = \{10, 11, 12, 13, 14, 15, 16, 17, 18\}$, list the elements of: i. A'; ii. B'; iii. $A' \cap B'$.
2. Ask pupils to solve the problems either independently or with seatmates.
3. Invite volunteers to come to the board to write the solutions at the same time. All other pupils should check their work.

   Solutions:

   a. List the sets: $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\ 16, 17, 18\}$; $P = \{1, 2, 3, 6, 9, 18\}$; $Q = \{4, 8, 12, 16\}$;
   $R = \{11, 12, 13, 14, 15, 16, 17\}$
   Look at the listed sets and identify the intersections: $P \cap Q = \{\ \}$, $P \cap Q = \{\ \}$, $Q \cap R = \{12, 16\}$, $P \cap Q \cap R = \{\ \}$
   Draw the diagram →



   b. List the sets: $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $P = \{3, 6, 9\}$, $Q = \{1, 3, 5, 7, 9\}$, and $R = \{2, 4, 6, 8, 10\}$
   Identify the intersections: $P \cap Q = \{3, 9\}$; $Q \cap R = \{\ \}$; $P \cap R = \{6\}$; $P \cap Q \cap R = \{\ \}$
   Draw the diagram →



   c. i. $A' = \{10, 15, 16, 17, 18\}$; ii. $B' = \{10, 11, 12, 18\}$;
   iii. $A' \cap B' = \{10, 18\}$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L015 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L016 in the Pupil Handbook before the next class.

Lesson Title: Solving problems involving sets	**Theme:** Numbers and Numeration	
Lesson Number: M4-L016	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to diagram and solve real life problems involving 2 or 3 sets.	**Preparation** 1. Review the content of this lesson and be prepared to explain the solutions. 2. Write the laws of logarithms in the Opening on the board.	

Opening *(2 minute)*

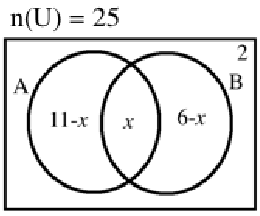
1. Write on the board: $n(\mathrm{A} \cup \mathrm{B}) = 7$
2. Ask volunteers to describe this expression in their own words.
3. Explain:
   - This expression means that the number of elements in the union of sets A and B is 7.
   - The lowercase letter n gives the cardinality of a set, or the number of elements it has.
4. Explain that this lesson is on solving problems involving sets.

Teaching and Learning *(20 minutes)*

1. Write the following question on the board: If A and B are two finite sets such that $n(\mathrm{A}) = 20$, $n(\mathrm{B}) = 28$ and $n(\mathrm{A} \cup \mathrm{B}) = 36$, find $n(\mathrm{A} \cap \mathrm{B})$.
2. Discuss: How can you interpret this question? What is it asking? (Example answer: It is asking for the number of elements that are in both A and B, or the intersection of A and B.)
3. Explain:
   a. We are given the number of elements in both A and B, and we are given the number of elements in their union.
   b. We can use this information to find the number of elements in their intersection.
4. Write on the board: For two sets A and B: $n(\mathrm{A} \cup \mathrm{B}) = n(\mathrm{A}) + n(\mathrm{B}) - n(\mathrm{A} \cap \mathrm{B})$
5. Explain: This formula states that we add the total number of elements in A and B, and subtract the number of elements in their intersection. This is because we do not want to count the elements in their intersection twice.
6. Solve the problem on the board using the formula:

$$\begin{aligned} n(\mathrm{A} \cap \mathrm{B}) &= n(\mathrm{A}) + n(\mathrm{B}) - n(\mathrm{A} \cup \mathrm{B}) \\ &= 20 + 28 - 36 \\ &= 48 - 36 \\ &= 12 \end{aligned}$$

7. Write the following problem and diagram on the board: In the diagram, two sets, A and B, are subsets of the universal set U. Given that $n(\mathrm{U}) = 25$, find:
   a. The value of x.
   b. The value of $n(\mathrm{A} \cup \mathrm{B})$.



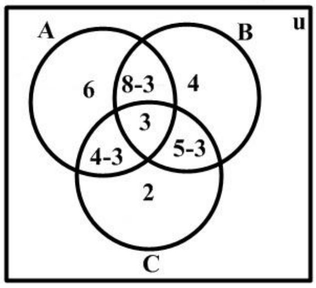
8. Ask volunteers to give the steps to solve the problem. Write the solution on the board as they explain.
   Solutions:
   a. To find x, set up an equation that sum up to $n(\mathrm{U}) = 25$. Notice the 2 in the universal set outside of A and B. This must be included too:

$$\begin{aligned}(11 - x) + x + (6 - x) + 2 &= 25 \\ 11 - x + x + 6 - x + 2 &= 25 \\ 11 + 6 + 2 - x &= 25 \\ 19 + x &= 25 \\ x &= 25 - 19 = 6\end{aligned}$$

   b. To find $n(\mathrm{A} \cup \mathrm{B})$, add the number of elements in both A and B. Substitute $x = 6$ and evaluate:

$$\begin{aligned}n(\mathrm{A} \cup \mathrm{B}) &= 11 - x + x + 6 - x \\ &= 11 + 6 - x \\ &= 11 + 6 - (6) \\ &= 11\end{aligned}$$

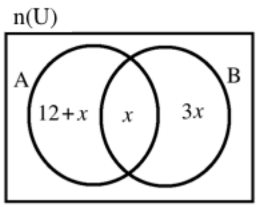
9. Write the following problem on the board: Draw a Venn diagram of the universal set U containing three sets A, B and C. Given that $n(\mathrm{A} \cap \mathrm{B} \cap \mathrm{C}) = 3$, $n(\mathrm{A} \cap \mathrm{B}) = 8$, $n(\mathrm{A} \cap \mathrm{C}) = 4$, $n(\mathrm{B} \cap \mathrm{C}) = 5$, $n(\mathrm{A} \cap \mathrm{B}' \cap \mathrm{C}') = 6$, $n(\mathrm{A}' \cap \mathrm{B}' \cap \mathrm{C}) = 2$ and $n(\mathrm{A}' \cap \mathrm{B} \cap \mathrm{C}') = 4$. Find $n(\mathrm{A})$, $n(\mathrm{B})$ and $n(\mathrm{C})$.
10. Ask pupils to work with seatmates to solve the problem.
11. Walk around to check for understanding and clear misconceptions.
12. Invite volunteers to write the solution on the board.
   Solution:
   Venn diagram →



   $n(A) = 6 + (8 - 3) + 3 + (4 - 3) = 6 + 5 + 3 + 1 = 15$
   $n(B) = 4 + (8 - 3) + (5 - 3) + 3 = 4 + 5 + 2 + 3 = 14$
   $n(C) = 2 + (4 - 3) + (5 - 3) + 3 = 2 + 1 + 2 + 3 = 8$

Practice *(17 minutes)*

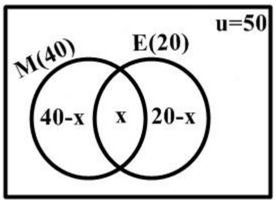
1. Write the following problems on the board:
   a. In a class of 50 pupils, 40 read Mathematics and 20 read English. Each pupil reads at least one subject. How many pupils read both Mathematics and English?
   b. A and B are two sets and the number of elements are shown in the Venn diagram. Given that $n(\mathrm{A}) = n(\mathrm{B})$, find:
   i. x ii. $n(\mathrm{A} \cup \mathrm{B})$



2. Ask pupils to solve the problems either independently or with seatmates.
3. Invite volunteers to come to the board simultaneously to write the solutions. All other pupils should check their work.

Solutions:

a. 50 pupils gives the universal set $n(\mathrm{U}) = 50$. Let M represent pupils who read Mathematics, and E represent pupils who read English. Then $n(\mathrm{M}) = 40,\ n(\mathrm{E}) = 20$. Represented in a Venn diagram where x is the intersection, $n(\mathrm{M} \cap \mathrm{E}) = x$, this is →



From the Venn diagram, we have: $(40 - x) + x + (20 - x) = 50$.
Solve the equation for x:

$$\begin{aligned}(40 - x) + x + (20 - x) &= 50\\ 40 - x + x + 20 - x &= 50\\ 40 + 20 - x &= 50\\ x &= 60 - 50\\ x &= 10\end{aligned}$$

Therefore, 10 pupils read both Mathematics and English.

b. i. We have $n(\mathrm{A}) = n(\mathrm{B})$. Set the number of elements in the 2 sets equal and solve for x.

$$\begin{aligned}12 + x + x &= 3x + x\\ 12 + 2x &= 4x\\ 12 &= 4x - 2x\\ 12 &= 2x\\ 6 &= x\end{aligned}$$

ii. To find $n(\mathrm{A} \cup \mathrm{B})$, find the sum of all of the parts inside A and B. Substitute $x = 6$ and evaluate:

$$\begin{aligned}n(\mathrm{A} \cup \mathrm{B}) &= 12 + x + x + 3x\\ &= 12 + 5x\\ &= 12 + 5(6)\\ &= 12 + 30 = 42\end{aligned}$$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L016 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L017 in the Pupil Handbook before the next class.

Lesson Title: Operations on surds	**Theme:** Numbers and Numeration	
Lesson Number: M4-L017	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to perform operations on surds (addition, subtraction, multiplication).	**Preparation** 1. Review the content of this lesson and be prepared to explain the solutions. 2. Write the properties of surds (at the start of Teaching and Learning) on the board.	

Opening *(1 minute)*

1. Discuss: What are surds? (Answer: Numbers that we cannot find a whole number square root of, and are left in square root form to express their exact values.)
2. Ask volunteers to write some examples of surds on the board. (Example answers: $\sqrt{2}, 2\sqrt{3}, -4\sqrt{5}$)
3. Explain that this lesson is on performing operations on surds.

Teaching and Learning *(19 minutes)*

1. Write the following properties of surds on the board:
   1. $\sqrt{a \times b} = \sqrt{a} \times \sqrt{b}$
   2. $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$
   3. $\frac{b}{\sqrt{a}} = \frac{b}{\sqrt{a}} \times \frac{\sqrt{a}}{\sqrt{a}} = \frac{b\sqrt{a}}{a}$ (This is known as rationalising the denominator, and it will be covered in the next lesson.)
   4. $a\sqrt{c} \pm b\sqrt{c} = (a \pm b)\sqrt{c}$
   5. $\sqrt{a^2} = \sqrt{a} \times \sqrt{a} = a$
   6. $a\sqrt{b} \times \sqrt{c} = a\sqrt{bc}$
   7. $a \times \sqrt{b} = a\sqrt{b}$
   8. $a \times b\sqrt{c} = ab\sqrt{c}$
2. Explain: These properties are used to simplify surds and to apply operations to them. You should be familiar with these properties.
3. Write the following problems on the board: Simplify: a. $-2\sqrt{3} + 6\sqrt{3}$ b. $3\sqrt{2} + 5\sqrt{3}$
4. Ask volunteers to explain how to simplify each expression. Solve them on the board as they explain.
   Solutions:
   a. $-2\sqrt{3} + 6\sqrt{3} = (-2 + 6)\sqrt{3} = 4\sqrt{3}$
   b. Cannot be simplified.
5. Explain:
   - Only like surds can be added or subtracted. When surds are like, simply add or subtract the coefficients in front of them.

- This is property 4 on the board.

6. Write the following problem on the board: Simplify $4\sqrt{2} \times 2\sqrt{3}$
7. Ask volunteers to explain how to simplify. Solve on the board as they explain. (Solution: $4\sqrt{2} \times 2\sqrt{3} = (4 \times 2)\sqrt{2 \times 3} = 8\sqrt{6}$)
8. Explain:
   - When multiplying surds, we multiply the coefficient parts together, and the surd parts together. This is given in properties 1, 6, 7, and 8.
   - When performing any operations on surds, it is best to simplify them first if possible. After performing the operation, simplify the answer if possible.
9. Write the following problem on the board: Simplify $\left(4\sqrt{2} - 2\sqrt{2}\right) \times \sqrt{8}$
10. Discuss: How would you solve this problem? (Answer: Simplify $\sqrt{8}$. Apply the order of operations, BODMAS. Remove brackets before multiplying.)
11. Remind pupils that to simplify a surd, perfect squares are factored out of the surd part. This is property 1 on the board.
12. Simplify $\sqrt{8}$ on the board, and make sure pupils understand. (Answer: $\sqrt{8} = \sqrt{4 \times 2} = \sqrt{4} \times \sqrt{2} = 2\sqrt{2}$)
13. Solve the problem on the board as a class:

$\left(4\sqrt{2} - 2\sqrt{2}\right) \times \sqrt{8}$	$=$	$\left(4\sqrt{2} - 2\sqrt{2}\right) \times 2\sqrt{2}$	Simplify
	$=$	$\left((4-2)\sqrt{2}\right) \times 2\sqrt{2}$	Remove brackets
	$=$	$2\sqrt{2} \times 2\sqrt{2}$	
	$=$	$(2 \times 2)\sqrt{2 \times 2}$	Multiply
	$=$	$4\sqrt{4}$	
	$=$	4×2	Simplify
	$=$	8	

14. Write the following problems on the board: Simplify: a. $\left(2\sqrt{3}\right)^2$
    b. $\sqrt{24} - 3\sqrt{2} \times 2\sqrt{3}$
15. Ask pupils to work with seatmates to solve the problems.
16. Walk around to check for understanding and clear misconceptions.
17. Invite volunteers to write their solutions on the board and explain.

    Solutions:

    a. $\left(2\sqrt{3}\right)^2 = 2\sqrt{3} \times 2\sqrt{3} = (2 \times 2)\sqrt{3 \times 3} = 4\sqrt{9} = 4 \times 3 = 12$

    b. Simplify, then apply BODMAS:

$\sqrt{24} - 3\sqrt{2} \times 2\sqrt{3}$	$=$	$2\sqrt{6} - 3\sqrt{2} \times 2\sqrt{3}$	Simplify
	$=$	$2\sqrt{6} - 3 \times 2\sqrt{2 \times 3}$	Multiply
	$=$	$2\sqrt{6} - 6\sqrt{6}$	
	$=$	$(2-6)\sqrt{6}$	Subtract
	$=$	$-4\sqrt{6}$	

Practice *(19 minutes)*

1. Write the following problems on the board:
    a. Simplify $\sqrt{32}+3\sqrt{8}$
    b. If $a\sqrt{28}+\sqrt{63}-\sqrt{7}=0$, find a.
    c. If $\sqrt{5}=2.2361$, evaluate $\sqrt{245}$, correct to 3 decimal places.
2. Ask pupils to solve the problems independently or with seatmates.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to come to the board at the same time to write the solutions. All other pupils should check their work.

Solutions:

a.

$\sqrt{32}+3\sqrt{8}$	$=$	$\sqrt{16\times 2}+3\sqrt{4\times 2}$	Simplify
	$=$	$\sqrt{16\times 2}+3\times\sqrt{4}\times\sqrt{2}$	
	$=$	$4\sqrt{2}+3\times 2\sqrt{2}$	
	$=$	$4\sqrt{2}+6\sqrt{2}$	Add
	$=$	$10\sqrt{2}$	

b. Note that there are multiple ways to solve. Accept any correct solution.

$a\sqrt{28}+\sqrt{63}-\sqrt{7}$	$=$	0	
$a\sqrt{4\times 7}+\sqrt{9\times 7}-\sqrt{7}$	$=$	0	Simplify surds
$a\times 2\sqrt{7}+3\sqrt{7}-\sqrt{7}$	$=$	0	
$2a\sqrt{7}+2\sqrt{7}$	$=$	0	Combine like terms
$2a\sqrt{7}$	$=$	$-2\sqrt{7}$	Subtract $2\sqrt{7}$ from both sides
$\frac{2a\sqrt{7}}{2\sqrt{7}}$	$=$	$\frac{-2\sqrt{7}}{2\sqrt{7}}$	Divide throughout by $2\sqrt{7}$
a	$=$	-1	

c. Simplify $\sqrt{245}$: $\sqrt{245}=\sqrt{49\times 5}=7\sqrt{5}$.
Substitute $\sqrt{5}=2.2361$: $7\sqrt{5}=7(2.2361)=15.653$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L017 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L018 in the Pupil Handbook before the next class.

Lesson Title: Simplifying surds	**Theme:** Numbers and Numeration	
Lesson Number: M4-L018	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcomes By the end of the lesson, pupils will be able to: 1. Rationalise the denominator of surds. 2. Expand and simplify expressions involving surds.	**Preparation** 1. Review the content of this lesson and be prepared to explain the solutions. 2. Write the properties of surds (at the start of Teaching and Learning) on the board.	

Opening *(1 minute)*

1. Write the following problem on the board: Simplify $\frac{2}{\sqrt{3}}$.
2. Discuss and allow pupils to share their ideas: Can you simplify this expression?
3. Explain:
   a. A surd is an irrational number, and cannot be in the denominator of a fraction.
   b. We can have a surd in the numerator of a fraction. We will change this fraction using a process called rationalising the denominator.
4. Explain that this lesson is on simplifying expressions that contain surds.

Teaching and Learning *(19 minutes)*

1. Explain:
   - "Rationalising the denominator" means to change an irrational number (a surd) in the denominator to a rational one.
   - To rationalise a surd in the denominator, multiply it by itself. We must also multiply the numerator by the surd.
2. Solve the problem on the board, explaining each step: $\frac{2}{\sqrt{3}} = \frac{2\times\sqrt{3}}{\sqrt{3}\times\sqrt{3}} = \frac{2\sqrt{3}}{3} = \frac{2}{3}\sqrt{3}$
3. Explain that either $\frac{2\sqrt{3}}{3}$ or $\frac{2}{3}\sqrt{3}$ is an acceptable answer.
4. Write another problem on the board: Simplify $\frac{1}{2+\sqrt{3}}$.
5. Explain:
   - There is a binomial that contains a surd in the denominator. In such cases, we must multiply the numerator and denominator by the **conjugate** of the denominator.
   - To find the conjugate, simply change the sign in the middle of the binomial.
6. Write on the board: The conjugate of $2+\sqrt{3}$ is $2-\sqrt{3}$.
7. Solve the problem on the board, explaining each step:

$$\frac{1}{2+\sqrt{3}} = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} \qquad \text{Multiply top and bottom by } 2-\sqrt{3}$$

$$= \frac{2-\sqrt{3}}{2^2-\sqrt{3}^2}$$

$$= \frac{2-\sqrt{3}}{4-3} \quad \text{Simplify}$$
$$= \frac{2-\sqrt{3}}{1}$$
$$= 2-\sqrt{3}$$

8. Write the following problem on the board: Simplify $(1+\sqrt{5})(1-\sqrt{5})$
9. Explain: The distributive property applies to expressions involving surds. Distribute the terms as you would in algebraic expressions involving variables.
10. Solve the problem on the board, explaining each step:

$$(1+\sqrt{5})(1-\sqrt{5}) = 1(1-\sqrt{5})+\sqrt{5}(1-\sqrt{5}) \quad \text{Expand (distribute)}$$
$$= 1-\sqrt{5}+\sqrt{5}-(\sqrt{5})^2 \quad \text{Simplify}$$
$$= 1-\sqrt{5}+\sqrt{5}-5 \quad \text{Note that } (\sqrt{5})^2=\sqrt{5}\times\sqrt{5}=5$$
$$= 1-5-\sqrt{5}+\sqrt{5} \quad \text{Collect like terms}$$
$$= -4$$

11. Write the following problem on the board: Simplify $\sqrt{5}\left(\sqrt{5}-\frac{8}{\sqrt{20}}\right)$
12. Ask volunteers to give the steps needed to solve. Solve it on the board as they give the steps:

$$\sqrt{5}\left(\sqrt{5}-\frac{8}{\sqrt{20}}\right) = \sqrt{5}\left(\sqrt{5}-\frac{8}{2\sqrt{5}}\right) \quad \text{Simplify. Note that } \sqrt{20}=2\sqrt{5}$$
$$= (\sqrt{5})^2-\frac{8\sqrt{5}}{2\sqrt{5}} \quad \text{Expand (distribute the } \sqrt{5}\text{)}$$
$$= 5-\frac{8}{2} \quad \text{Simplify}$$
$$= 5-4$$
$$= 1$$

13. Write the following problems on the board: Simplify: a. $\frac{2}{3+\sqrt{7}}$ b. $\sqrt{2}\left(\sqrt{50}+\frac{20}{\sqrt{72}}\right)$
14. Ask pupils to work with seatmates to solve the problems.
15. Walk around to check for understanding and clear misconceptions.
16. Invite volunteers to write their solutions on the board and explain.

Solutions:

a. Rationalise the denominator:

$$\frac{2}{3+\sqrt{7}} = \frac{2}{3+\sqrt{7}}\times\frac{3-\sqrt{7}}{3-\sqrt{7}}$$
$$= \frac{2(3-\sqrt{7})}{3^2-\sqrt{7}^2}$$
$$= \frac{6-2\sqrt{7}}{9-7}$$
$$= \frac{6-2\sqrt{7}}{2}$$
$$= \frac{6}{2}-\frac{2}{2}\sqrt{7}$$
$$= 3-\sqrt{7}$$

b. Distribute and simplify:

$$\sqrt{2}\left(\sqrt{50}+\frac{20}{\sqrt{72}}\right) = \sqrt{2\times 50}+\frac{20\sqrt{2}}{\sqrt{72}}$$
$$= \sqrt{100}+\frac{20\sqrt{2}}{\sqrt{36}\sqrt{2}}$$
$$= 10+\frac{20}{\sqrt{36}}$$
$$= 10+\frac{20}{6}$$
$$= 10+\frac{10}{3}$$
$$= \frac{30+10}{3}$$
$$= \frac{40}{3} = 13\frac{1}{3}$$

Practice *(19 minutes)*

1. Write the following problems on the board:
   a. Simplify: $\frac{10}{\sqrt{12}}$
   b. Express $\frac{\sqrt{3}+\sqrt{7}}{\sqrt{21}}$ in the form $a\sqrt{3}+b\sqrt{7}$, where a and b are rational numbers.
   c. Expand and simplify: $(1+\sqrt{2})(3+\sqrt{2})$
2. Ask pupils to solve the problems independently or with seatmates.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to come to the board at the same time to write the solutions. All other pupils should check their work.

Solutions:

a. Rationalise the denominator:

$$\begin{aligned}\frac{10}{\sqrt{12}} &= \frac{10}{2\sqrt{3}}\\ &= \frac{10}{2\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}\\ &= \frac{10\sqrt{3}}{2\times\sqrt{3}\times\sqrt{3}}\\ &= \frac{10\sqrt{3}}{2\times 3}\\ &= \frac{10\sqrt{3}}{6}\\ &= \frac{5\sqrt{3}}{3}\end{aligned}$$

b. Rationalise the denominator and simplify:

$$\begin{aligned}\frac{\sqrt{3}+\sqrt{7}}{\sqrt{21}} &= \frac{\sqrt{3}+\sqrt{7}}{\sqrt{21}}\times\frac{\sqrt{21}}{\sqrt{21}}\\ &= \frac{\sqrt{21}(\sqrt{3}+\sqrt{7})}{\sqrt{21}\times\sqrt{21}}\\ &= \frac{\sqrt{63}+\sqrt{147}}{21}\\ &= \frac{\sqrt{9\times 7}+\sqrt{49\times 3}}{21}\\ &= \frac{3\sqrt{7}+7\sqrt{3}}{21}\\ &= \frac{7\sqrt{3}+3\sqrt{7}}{21}\\ &= \frac{7\sqrt{3}}{21}+\frac{3\sqrt{7}}{21}\\ &= \frac{1}{3}\sqrt{3}+\frac{1}{7}\sqrt{7}\end{aligned}$$

Where $a=\frac{1}{3}$ and $b=\frac{1}{7}$

c. Expand and simplify:

$$\begin{aligned}(1+\sqrt{2})(3+\sqrt{2}) &= 1(3+\sqrt{2})+\sqrt{2}(3+\sqrt{2})\\ &= 3+\sqrt{2}+3\sqrt{2}+(\sqrt{2})^2\\ &= 3+\sqrt{2}+3\sqrt{2}+2\\ &= 3+2+\sqrt{2}+3\sqrt{2}\\ &= 5+4\sqrt{2}\end{aligned}$$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L018 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L019 in the Pupil Handbook before the next class.

Lesson Title: Simplification and factorisation	Theme: Algebra	
Lesson Number: M4-L019	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to simplify and factor algebraic expressions.	**Preparation** 1. Review the content of this lesson and be prepared to explain the solutions. 2. Write the problem in Opening on the board.	

Opening *(2 minutes)*

1. Write on the board: Simplify $4x - 2(x - 3)$.
2. Ask pupils to solve the problem in their exercise books. Give them less than 1 minute.
3. Invite a volunteer to write the solution on the board.
   Solution:
   $4x - 2(x - 3) = 4x - 2x + 6$ Remove the brackets
   $= 2x + 6$ Combine like terms
4. Explain that this lesson is on simplifying and factorising expressions.

Teaching and Learning *(18 minutes)*

1. Explain:
   - To simplify means to remove the brackets and combine any like terms.
   - Factorisation is the opposite of simplification.
2. Discuss: What are like terms? (Answer: Like terms have the same variable, and the variables have the same power.)
3. Write the problems on the board: Simplify:
   a. $(x - 2)^2$
   b. $(2x + y)(x - y) + (2x - y)(x + y)$
4. Ask pupils to work with seatmates to simplify the expressions.
5. Walk around to check for understanding and clear misconceptions
6. Invite volunteers to write the solutions on the board.
   Solutions:
   a. $(x - 2)^2 = (x - 2)(x - 2)$ Expand
   $= x(x - 2) - 2(x - 2)$ Multiply
   $= x^2 - 2x - 2x + 4$ Remove the brackets
   $= x^2 - 4x + 4$ Combine like terms
   b. The problem can be broken down to parts. Multiply the first set of brackets:
   $(2x + y)(x - y) = 2x(x - y) + y(x - y)$
   $= 2x^2 - 2xy + xy - y^2$
   $= 2x^2 - xy - y^2$

Multiply the second set of brackets:

$$(2x - y)(x + y) = 2x(x + y) - y(x + y)$$
$$= 2x^2 + 2xy - xy - y^2$$
$$= 2x^2 + xy - y^2$$

Add the first and second expressions, combining any like terms:

$$(2x + y)(x - y) + (2x - y)(x + y) = (2x^2 - xy - y^2) + (2x^2 + xy - y^2)$$
$$= 2x^2 + 2x^2 - xy + xy - y^2 - y^2$$
$$= 4x^2 - 2y^2$$

7. Write the following problem on the board: Factorise $8x^2 + 4x$
8. Ask a volunteer to explain how to factorise the expression. (Answer: Find common factors and take them outside of brackets.)
9. Ask pupils to write the factorisation in their exercise books.
10. Invite a volunteer to write the solution on the board. (Solution: $8x^2 + 4x = 4x(2x + 1)$)
11. Write another problem on the board: Factorise $x^3 + 2x^2 - 3x - 6$
12. Ask volunteers to explain how to factorise the expression. As they explain, write the solution on the board:

$x^3 + 2x^2 - 3x - 6 = (x^3 + 2x^2) + (-3x - 6)$ Create groups

$= x^2(x + 2) - 3(x + 2)$ Factorise each group

$= (x^2 - 3)(x + 2)$

13. Explain:
    - This type of factorisation is usually used when there are 4 terms.
    - Break them into smaller groups, and factor each group.
    - If you have the same expression left in brackets, it can then be factored to give an expression that is the multiplication of 2 binomials.
14. Write the problems on the board: Factorise:
    a. $x^3y^2 - x^2y^2$
    b. $x^3 - 3x^2 + 5x - 15$
    c. $3p^2 + 2qt + 6qp + pt$
15. Ask pupils to work with seatmates to solve the problems.
16. Walk around to check for understanding and clear misconceptions.
17. Invite volunteers to write their solutions on the board and explain.

Solutions:

a. $x^3y^2 - x^2y^2 = x^2y^2(x - 1)$

b. $x^3 - 3x^2 + 5x - 15 = (x^3 - 3x^2) + (5x - 15)$ Create groups

$= x^2(x - 3) + 5(x - 3)$ Factorise each group

$= (x^2 + 5)(x - 3)$

a. $3p^2 + 2qt + 6qp + pt = (3p^2 + 6qp) + (pt + 2qt)$ Create groups

$= 3p(p + 2q) + t(p + 2q)$ Factorise each group

$= (3p + t)(p + 2q)$

Practice *(19 minutes)*

1. Write the following problems on the board:
   Simplify:
   a. $12b-(5+2b)-7$
   b. $(x+5)(x+10)$
   c. $(9+8x)(x-4)+(2+x)(x+1)$
   Factorise:
   d. $9x-27y+7x^2-21xy$
   e. $ad+bc+ac+bd$
   f. $4b^2-27ac-6ab+18bc$
2. Ask pupils to solve the problems independently.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to come to the board to write the solutions at the same time. All other pupils should check their work.
   Solutions:
   a. $12b-(5+2b)-7 = 12b-5-2b-7$
   $= 12b-2b-5-7$
   $= 10b-12$
   b. $(x+5)(x+10) = x(x+10)+5(x+10)$
   $= x^2+10x+5x+50$
   $= x^2+15x+50$
   c. $(9+8x)(x-4)+(2+x)(x+1) = 9(x-4)+8x(x-4)+2(x+1)+x(x+1)$
   $= 9x-36+8x^2-32x+2x+2+x^2+x$
   $= 8x^2+x^2+9x-32x+2x+x-36+2$
   $= (8+1)x^2+(9-32+2+1)x-34$
   $9x^2-20x-34$
   d. $9x-27y+7x^2-21xy = 9(x-3y)+7x(x-3y)$
   $= (9+7x)(x-3y)$

   e. $ad+bc+ac+bd = ac+ad+bc+bd$
   $= a(c+d)+b(c+d)$
   $= (a+b)(c+d)$
   f. $4b^2-27ac-6ab+18bc = 4b^2-6ab+18bc-27ac$
   $= 2b(2b-3a)+9c(2b-3a)$
   $= (2b+9c)(2b-3a)$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L019 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L020 in the Pupil Handbook before the next class.

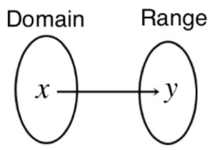
Lesson Title: Functions	**Theme:** Algebra	
Lesson Number: M4-L020	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcomes By the end of the lesson, pupils will be able to: 1. Identify and describe functions, and their domain and range. 2. Use function notation.	**Preparation** 1. Review the content of this lesson and be prepared to explain the solutions. 2. Write the problem in Opening on the board.	

Opening *(2 minutes)*

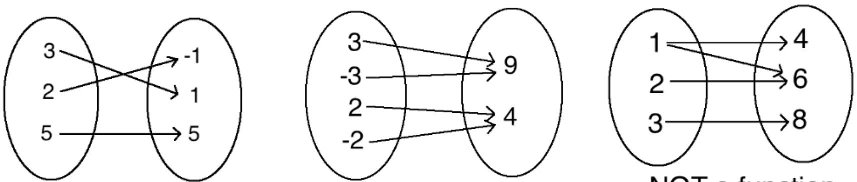
1. Discuss and allow pupils to share ideas: What is a function? Explain in your own words.
2. Explain that this lesson is on identifying and describing functions, and using function notation.

Teaching and Learning *(18 minutes)*

1. Explain:
   - A function is a relation between inputs and outputs.
   - The domain is the set of elements that can be put into the function, and the range is the set of outputs that the function produces.
   - In algebra, we typically represent inputs with x and outputs with y.
2. Draw on the board:



3. Explain:
   - A function maps each element in the domain onto one and only one member of the range.
   - However, note that a member of the range could correspond to more than one member of the range.
4. Draw the following on the board and make sure pupils understand:

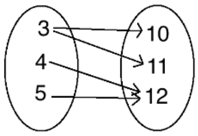


Function Function NOT a function

5. Write on the board: $f(x) = 2x + 1$
6. Ask volunteers to read the expression aloud. (Answer: "f of x equals $2x$ plus 1")
7. Explain: This is a linear equation in function notation. It is the same as $y = 2x + 1$.
8. Write on the board. Are the following functions?
   a. $\{(1, -3), (2,5), (3,10)\}$

b. $\{(2,1),(0,-1),(2,-3)\}$

c.



9. Ask pupils to solve the problem with seatmates.
10. Ask volunteers to share their answers with the class. Allow discussion.

Answers:

a. Yes; each member of the domain (1, 2, 3) maps to exactly 1 member of the range.
b. No; a member of the domain (2) maps to 2 members of the range.
c. No; a member of the domain (3) maps to 2 members of the range.

11. Write the following problem on the board: If $f(x) = 3x + 4$, find $f(3)$ and $f(0)$.
12. Ask pupils to work with seatmates to solve the problems.
13. Walk around to check for understanding and clear misconceptions.
14. Invite volunteers to write the solutions on the board.

Solutions:

$f(x)$	$=$	$3x + 4$	Function
$f(3)$	$=$	$3(3) + 4$	Substitute $x = 3$
	$=$	$9 + 4$	Simplify
	$=$	13	
$f(0)$	$=$	$3(0) + 4$	Substitute $x = 0$
	$=$	$0 + 4$	
	$=$	4	

15. Write the following problem on the board: A function is defined by $f: x \longrightarrow 4x + 1$ on the domain $\{-1, 0, 1, 2\}$, find the range of the function.
16. Explain: For this problem, we need to substitute each value from the domain to find the corresponding value in the range.
17. Find the first 2 values of the range on the board (for $x = -1$ and $x = 0$)

$$f(-1) = 4(-1) + 1 = -4 + 1 = -3$$
$$f(0) = 4(0) + 1 = 0 + 1 = 1$$

18. Ask pupils to complete the problem with seatmates.
19. Walk around to check for understanding and clear misconceptions.
20. Invite three volunteers, one at a time, to write the solutions on the board.

Solutions:

$$f(1) = 4(1) + 1 = 4 + 1 = 5$$
$$f(2) = 4(2) + 1 = 8 + 1 = 9$$

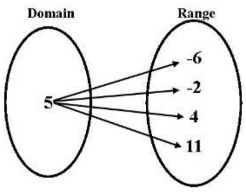
Therefore, range = $\{-3, 1, 5, 9\}$

Practice *(19 minutes)*

1. Write the following problems on the board:

a. If $f(x) = 2x + 3$, find: i. $f(1)$ ii. $f(2)$ iii. $f(-3)$

b. A function $f: x \longrightarrow x^2 + 1$ is defined on the domain $\{1, 2, 3\}$. Find the range.
c. A function $f: x \longrightarrow x^2 + 5$, is defined on the domain $\{0, 1, 2, 3\}$. Find the range.
d. Are the relations expressed in the mapping diagrams functions? Give your reasons.
   i. $\{(-3,7), (-1,5), (0, -2), (5, 9), (5, 3)\}$
   ii. $\{(-2, 0), (-1, -2), (0, 3), (4, -1), (5, -3)\}$
   iii.



2. Ask pupils to solve the problems independently.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to come to the board to write the solutions at the same time. All other pupils should check their work.

Solutions:

a. i. $f(1) = 2(1) + 3 = 2 + 3 = 5$
   ii. $f(2) = 2(2) + 3 = 4 + 3 = 7$
   iii. $f(-3) = 2(-3) + 3 = -6 + 3 = -3$
b. Substitute each value of the domain into $f(x) = x^2 + 1$:
   $f(1) = (1)^2 + 1 = 1 + 1 = 2$
   $f(2) = (2)^2 + 1 = 4 + 1 = 5$
   $f(3) = (3)^2 + 1 = 9 + 1 = 10$
   Therefore, range = $\{2, 5, 10\}$
c. Substitute each value of the domain into $f(x) = x^2 + 5$:
   $f(0) = (0)^2 + 5 = 0 + 5 = 5$
   $f(1) = (1)^2 + 5 = 1 + 5 = 6$
   $f(2) = (2)^2 + 5 = 4 + 5 = 9$
   $f(3) = (3)^2 + 5 = 9 + 5 = 14$
   Therefore, the range is $\{5, 6, 9, 14\}$.
d. i. No, since we have duplicates of x-values with different y-values, then this relation **is not** a function.; ii. Yes, relation **is** a function because every x-value is unique and is associated to only one value of y; iii. No, it is not a function because a single element in the domain is being paired to four elements in the range.

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L019 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L020 in the Pupil Handbook before the next class.

Lesson Title: Graphing linear functions	Theme: Algebraic Processes	
Lesson Number: M4-L021	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to graph linear functions, and identify the solutions and gradient.	**Preparation** 1. Review the content of this lesson and be prepared to explain the solutions. 2. Write the problem in Opening on the board.	

Opening *(1 minute)*

1. Write on the board: Graph $6x - 2y = 4$ on a Cartesian plane.
2. Discuss: How would you solve this problem? (Example answer: Solve for y, then use the equation to fill a table of values and graph the line on the plane.)
3. Explain that this lesson is on graphing linear functions, and identifying the solutions and gradient.

Teaching and Learning *(18 minutes)*

1. Ask pupils to solve the problem on the board for y with seatmates.
2. Ask a volunteer to write the solution on the board:

$$\begin{aligned} 6x - 2y &= 4 \\ -2y &= -6x + 4 && \text{Transpose } 6x \\ \frac{-2y}{-2} &= \frac{-6x}{-2} + \frac{4}{-2} && \text{Divide throughout by } -2 \\ y &= 3x - 2 \end{aligned}$$

3. Graph the equation on the board as a class. Involve pupils by asking volunteers to fill the table and plot the points.

x	-2	-1	0	1	2
y	-8	-5	-2	1	4

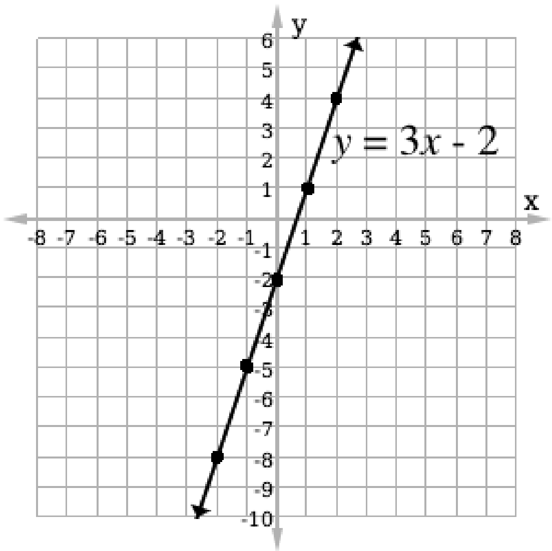
$$\begin{aligned} y &= 3(-2) - 2 \\ &= -6 - 2 \\ &= -8 \end{aligned}$$

$$\begin{aligned} y &= 3(-1) - 2 \\ &= -3 - 2 \\ &= -5 \end{aligned}$$

$$\begin{aligned} y &= 3(0) - 2 \\ &= 0 - 2 \\ &= -2 \end{aligned}$$

$$\begin{aligned} y &= 3(1) - 2 \\ &= 3 - 2 \\ &= 1 \end{aligned}$$

$$\begin{aligned} y &= 3(2) - 2 \\ &= 6 - 2 \end{aligned}$$



$$= \quad 4$$

4. Discuss: What is the gradient of the line? How do you know? (Answer: The gradient is 3.)
5. Write on the board: Slope-intercept form: $y = mx + c$, where m is the gradient and c is the y-intercept of the line.
6. Explain:
   a. When the equation of a line is written in this form, the coefficient of x is the gradient.
   b. Gradient is a number that tells us in which direction a line increases, and how steep it is.
   c. If a line increases as it goes to the right, or in the positive x-direction, the gradient is positive. If a line increases as it goes to the left, or in the negative x-direction, the gradient is negative.
7. Write a problem on the board: Calculate the gradient of the line passing through $(-3, -5)$ and $(5, 11)$.
8. Ask a volunteer to write the formula for calculating gradient on the board. (Answer: $m = \frac{y_2 - y_1}{x_2 - x_1}$)
9. Ask pupils to work with seatmates to solve the problem on the board.
10. Invite a volunteer to write the solution on the board.

Solution:

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{11-(-5)}{5-(-3)} && \text{Substitute } x\text{- and } y\text{-values} \\ &= \frac{11+5}{5+3} && \text{Simplify} \\ &= \frac{16}{8} \\ &= 2 \end{aligned}$$

Practice *(20 minutes)*

1. Write the following problems on the board:
   a. Find the gradient of the line passing through $(1, -2)$ and $(-2, 4)$.
   b. Line AB has a gradient of 2, and line CD has a gradient of -4. Which line is steeper?
   c. Graph the linear equation $y + 2x = 3$.
   d. The gradient of the line joining the points $(a, 4)$ and $(3, -2)$ is 2. Find a.
2. Ask pupils to solve the problems either independently or with seatmates.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to come to the board simultaneously to write the solutions.

Solutions:

a.

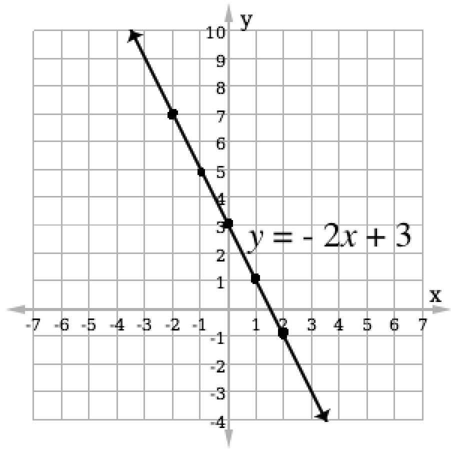
$$\begin{aligned} m &= \frac{4-(-2)}{-2-1} && \text{Substitute } x\text{- and } y\text{-values} \\ &= \frac{4+2}{-3} && \text{Simplify} \\ &= -\frac{6}{3} \\ &= -2 \end{aligned}$$

b. Line CD is steeper, because $|CD| > |BC|$.

c. Solve for y: $y = -2x + 3$

Fill a table of values and graph the line:

x	-2	-1	0	1	2
y	7	5	3	1	-1



d.

$$\begin{aligned} 2 &= \frac{-2-4}{3-a} && \text{Substitute in the gradient formula} \\ 2(3-a) &= -6 && \text{Solve for } a \\ 6-2a &= -6 \\ -2a &= -6-6 \\ \frac{-2a}{-2} &= \frac{-12}{-2} && \text{Simplify} \\ a &= 6 \end{aligned}$$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L021 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L022 in the Pupil Handbook before the next class.

Lesson Title: Applications of linear functions	**Theme:** Algebra	
Lesson Number: M4-L022	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to solve problems involving linear functions.	**Preparation** 1. Review the content of this lesson and be prepared to explain the solutions. 2. Write the equations in Opening on the board.	

Opening *(1 minute)*

1. Write on the board:

$$y = mx + c$$
$$y - y_1 = m(x - x_1)$$

2. Discuss and allow pupils to share their ideas: What are these equations? What do the variables stand for?
3. Explain that this lesson is on solving problems related to linear functions.

Teaching and Learning *(19 minutes)*

1. Explain:
   a. Both equations on the board represent linear equations.
   b. The first equation on the board is in slope-intercept form. We use equations in this form to graph lines in the previous lesson.
   c. The second equation is in point-slope form. m is the gradient, (x_1, y_1) is a specific point on the line, and (x, y) is a general point on the line.
   d. We can use the second equation to find a linear equation if we are given the gradient of a line and a point on the line, or 2 points on the line.
2. Write the following problem on the board: Determine the equation of a straight line whose gradient is -3 and that passes through the point $(1, 4)$.
3. Ask a volunteer to explain how to solve this problem. (Example answer: Substitute the given gradient and point into the point-slope formula and evaluate.)
4. Solve the problem on the board, explaining each step:
   Note that $m = -3$, $(x_1, y_1) = (1, 4)$

$y - y_1$	$=$	$m(x - x_1)$	Formula
$y - 4$	$=$	$-3(x - 1)$	Substitute values for m, x_1, and y_1
$y - 4$	$=$	$-3x + 3$	Simplify
y	$=$	$-3x + 3 + 4$	Transpose -4
y	$=$	$-3x + 7$	

5. Write the following problem on the board: Find the equation of the straight line passing through the points $(-1, -1)$ and $(3, 7)$.
6. Ask a volunteer to explain how to solve this problem. (Example answer: Find the gradient and use the point-slope formula to find the linear equation.)
7. Explain:

a. Given two points on a line, the first step is to find the gradient.
b. The second step is to substitute the gradient and either **one** of the points into the formula $y - y_1 = m(x - x_1)$.

8. Solve the problem on the board, explaining each step:
   Let $(x_1, y_1) = (-1, -1)$ and $(x_2, y_2) = (3, 7)$.
   Find the gradient, then substitute it into the formula:

m	$=$	$\frac{7-(-1)}{3-(-1)} = \frac{7+1}{3+1} = \frac{8}{4} = 2$	Find m
$y - y_1$	$=$	$m(x - x_1)$	Equation of a straight line
$y - 7$	$=$	$2(x - 3)$	Substitute $m = 2$ and one point, $(3, 7)$
$y - 7$	$=$	$2x - 6$	
y	$=$	$2x - 6 + 7$	Transpose -7
y	$=$	$2x + 1$	Equation of the line

9. Write the following problems on the board:
   a. Find the equation of a straight line with a gradient of 4, and which passes through $(-3, 2)$.
   b. Find the equation of a straight line that passes through $(1, 4)$ and $(-1, -2)$.
10. Ask pupils to work with seatmates to solve the problems on the board.
11. Walk around to check for understanding and clear misconceptions.
12. Invite volunteers to write the solutions on the board.
   Solutions:
   a. Apply the point-slope formula and evaluate:

$y - 2$	$=$	$4(x - (-3))$	Substitute values for m, x_1, and y_1
$y - 2$	$=$	$4x + 12$	Simplify
y	$=$	$4x + 12 + 2$	Transpose -2
y	$=$	$4x + 14$	

   b. Find the gradient, then apply the point-slope formula:

m	$=$	$\frac{-2-4}{-1-1} = \frac{-6}{-2} = 3$	Find m
$y - y_1$	$=$	$m(x - x_1)$	Equation of a straight line
$y - 4$	$=$	$3(x - 1)$	Substitute $m = 3$ and one point, $(1, 4)$
$y - 4$	$=$	$3x - 3$	
y	$=$	$3x - 3 + 4$	Transpose -4
y	$=$	$3x + 1$	Equation of the line

Practice *(19 minutes)*

1. Write the following problems on the board:
   a. Determine the equation of a straight line that passes through the point $(0, 5)$ and whose gradient is -1.
   b. Find the equation of the line passing through points $(-2, 3)$ and $(1, 12)$.
   c. Find the gradient of the line joining points $(3, 1)$ and $(3, -5)$.
   d. Determine which line is steeper: line A, which has a gradient of -1, or line B, which passes through points $(-3, 1)$ and $(3, 6)$.

e. Use the table of values below to answer the questions.

x	0	1	2	3
y	-2	1		

i. What is the gradient of the line?
ii. What is the value of y when $x = 3$?

2. Explain that these problems require information from this and previous lessons.
3. Ask pupils to solve the problems either independently or with seatmates.
4. Walk around to check for understanding and clear misconceptions.
5. Invite volunteers to come to the board to write the solutions at the same time.

Solutions:

a. Note that $m = -1$, $x_1 = 0$, $y_1 = 5$.

$y - y_1$	$=$	$m(x - x_1)$	Formula
$y - 5$	$=$	$-1(x - 0)$	Substitute m, x_1, and y_1
$y - 5$	$=$	$-x$	Simplify
y	$=$	$-x + 5$	Transpose -5

b. Note that $x_1 = -2$, $y_1 = 3$, $x_2 = 1$, and $y_2 = 12$.

m	$=$	$\frac{12-3}{1-(-2)} = \frac{9}{3} = 3$	Find m
$y - y_1$	$=$	$m(x - x_1)$	Equation of a straight line
$y - 3$	$=$	$3(x - (-2))$	Substitute $m = 3$ and one point, $(-2, 3)$
$y - 3$	$=$	$3x + 6$	
y	$=$	$3x + 6 + 3$	Transpose -3
y	$=$	$3x + 9$	Equation of the line

c. $m = \frac{-5-1}{3-3} = \frac{-6}{0}$; undefined

d. Find the gradient of B: $m = \frac{6-1}{3-(-3)} = \frac{5}{6}$. Note that $|-1| > \left|\frac{5}{6}\right|$. Therefore, A is steeper.

e. i. Use the 2 given points to find gradient: $m = \frac{1-(-2)}{1-0} = \frac{3}{1} = 3$
ii. Find the linear equation, then use it to find y:

$y - 1$	$=$	$3(x - 1)$	Substitute $m = 3$ and one point, $(1,1)$
$y - 1$	$=$	$3x - 3$	
y	$=$	$3x - 2$	Equation of the line
y	$=$	$3(3) - 2$	Substitute $x = 3$
y	$=$	$9 - 2 = 7$	Answer

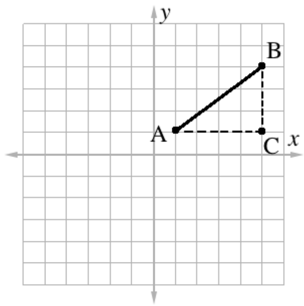
Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L022 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L023 in the Pupil Handbook before the next class.

<table>
<tr><td>Lesson Title: Distance and mid-point formulae</td><td colspan="2">Theme: Algebraic Processes</td></tr>
<tr><td>Lesson Number: M4-L023</td><td>Class: SSS 4</td><td>Time: 40 minutes</td></tr>
<tr><td>Learning Outcomes
By the end of the lesson, pupils will be able to:
1. Identify and apply the distance formula to find the distance between one point and another on a line.
2. Identify and apply the mid-point formula to find the mid-point of a line.</td><td colspan="2">Preparation
1. Review the content of this lesson and be prepared to explain the solutions.
2. Write the problem in Opening on the board.</td></tr>
</table>

Opening *(1 minute)*

1. Write on the board: Find the distance between A(1, 1) and B (5, 4):



2. Discuss: How can you find the distance between A and B? (Example answer: Apply Pythagoras' theorem.)
3. Explain that this lesson is on the distance and mid-point formulae.

Teaching and Learning *(15 minutes)*

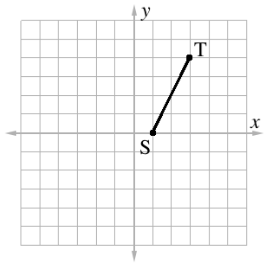
1. Explain the problem on the board:
   a. We can use Pythagoras' theorem to solve this problem, because we can draw a right-angled triangle ABC using points A and B.
   b. In this example, we could count the lengths of the sides and use Pythagoras' theorem to solve. However, in other examples this is not easy.
   c. We have a formula that is based on Pythagoras' theorem, but uses the coordinates of A and B.
2. Write the formula on the board:
   $|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ for any points A(x_1, y_1) and B(x_2, y_2)
3. Use the formula to solve the problem on the board:

$\lvert AB \rvert$	$=$	$\sqrt{(5-1)^2 + (4-1)^2}$	Substitute x- and y-values
	$=$	$\sqrt{4^2 + 3^2}$	Simplify
	$=$	$\sqrt{16 + 9}$	
	$=$	$\sqrt{25}$	
	$=$	5	

4. Write another problem on the board: Find the length of the line joining C$(0,-2)$ and D$(5,10)$.
5. Ask pupils to work with seatmates to solve the problem.
6. Invite a volunteer to write the solution on the board.
   Solution:

$$\begin{aligned} |CD| &= \sqrt{(5-0)^2+(10-(-2))^2} && \text{Substitute } x\text{- and } y\text{-values} \\ &= \sqrt{5^2+(10+2)^2} && \text{Simplify} \\ &= \sqrt{5^2+12^2} \\ &= \sqrt{25+144} \\ &= \sqrt{169} \\ &= 13 \end{aligned}$$

7. Draw the graph at right on the board:



8. Explain: Now we will find the mid-point of a line using a formula. The mid-point is the point that is **exactly** midway, or in the middle, of two other points.
9. Write on the board: The mid-point of two points (x_1, y_1) and (x_2, y_2) is the point M found by the formula: $M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$
10. Find the mid-point M of the line on the board (ST) using the formula.
    a. First, ask pupils to give the coordinates of S and T. Write the coordinates on the board. (Answer: S(1, 0), T(3, 4))
    b. Solve for M, explaining each step:

$$\begin{aligned} M &= \left(\frac{1+3}{2}, \frac{0+4}{2}\right) && \text{Substitute } x\text{- and } y\text{-values} \\ &= \left(\frac{4}{2}, \frac{4}{2}\right) = (2,2) && \text{Simplify} \end{aligned}$$

11. Write the following problem on the board: Find the mid-point between $(-1,4)$ and $(3,6)$.
12. Ask pupils to work with seatmates to solve the problem.
13. Invite a volunteer to write the solution on the board.
    Solution:

$$\begin{aligned} M &= \left(\frac{-1+3}{2}, \frac{4+6}{2}\right) && \text{Substitute } x\text{- and } y\text{-values} \\ &= \left(\frac{2}{2}, \frac{10}{2}\right) = (1,5) && \text{Simplify} \end{aligned}$$

Practice *(21 minutes)*

1. Write the following problems on the board:
   a. Find the length of the line joining (-1, -4) and (-3, -2).
   b. Find the length of the line joining A$(2-3x, 7)$ and B$(x+2, 7-3x)$.
   c. Find the mid-point M of (-8, -9) and (0, -15).
   d. Find the value of p such that $(-3,6)$ is the mid-point between A$(-2,p)$ and B$(-4,4)$.

2. Ask pupils to solve the problems either independently or with seatmates.
3. Invite volunteers to come to the board at the same time to write the solutions.
   Solutions:

a.

d	$=$	$\sqrt{(-3-(-1))^2+(-2-(-4))^2}$	Substitute x- and y-values
	$=$	$\sqrt{(-3+1)^2+(-2+4)^2}$	Simplify
	$=$	$\sqrt{(-2)^2+2^2}$	
	$=$	$\sqrt{4+4}=\sqrt{8}=\sqrt{4\times 2}$	Review surds if needed
	$=$	$2\sqrt{2}$	

b.

$\lvert AB\rvert$	$=$	$\sqrt{((x+2)-(2-3x))^2+((7-3x)-(7))^2}$	Substitute
	$=$	$\sqrt{(x+2-2+3x)^2+(7-3x-7)^2}$	Simplify
	$=$	$\sqrt{(4x)^2+(-3x)^2}$	
	$=$	$\sqrt{16x^2+9x^2}$	
	$=$	$\sqrt{25x^2}$	
	$=$	$\sqrt{25}\,\sqrt{x^2}$	
	$=$	$5x$	

c.

P	$=$	$\left(\frac{-8+0}{2},\frac{-9+(-15)}{2}\right)$	Substitute
	$=$	$\left(\frac{-8}{2},\frac{-24}{2}\right)$	Simplify
	$=$	$(-4,-12)$	

d.

M	$=$	$(-3,6)$	← Use this fact
$(-3,6)$	$=$	$\left(\frac{-2+(-4)}{2},\frac{p+4}{2}\right)$	Apply the mid-point formula
$(-3,6)$	$=$	$\left(\frac{-6}{2},\frac{p+4}{2}\right)$	Simplify
$(-3,6)$	$=$	$\left(-3,\frac{p+4}{2}\right)$	

Note that the x-coordinates match. Set y-coordinates equal and solve for p:

6	$=$	$\frac{p+4}{2}$	
$2(6)$	$=$	$2\left(\frac{p+4}{2}\right)$	Multiply both sides by 2
12	$=$	$p+4$	Transpose 4
$12-4$	$=$	p	
p	$=$	8	

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L023 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L024 in the Pupil Handbook before the next class.

Lesson Title: Graphing and interpreting quadratic functions	**Theme:** Algebraic Processes	
Lesson Number: M4-L024	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to graph quadratic functions, and identify the solutions, and maximum or minimum.	**Preparation** 1. Review the content of this lesson and be prepared to explain the solutions. 2. Write the problem in Opening on the board.	

Opening *(3 minutes)*

1. Write on the board: Graph the quadratic function $y = x^2 + 2x - 3$ for the interval $-3 \leq x \leq 1$. Use it to answer the following questions:
    a. What is the minimum value of $y = x^2 + 2x - 3$?
    b. What is the solution set of the equation $x^2 + 2x - 3 = 0$?
    c. What is the equation of the line of symmetry?
2. Discuss:
    a. How would you graph this equation? (Example answer: Find the y-values for $-3 \leq x \leq 1$ and plot them on the Cartesian plane.)
    b. What is the "solution set" of a quadratic equation? (Answer: The roots of the equation; where the parabola intersects with the x-axis.)
    c. What is a line of symmetry? (Answer: It is the vertical line about which the parabola is symmetric.)
3. Explain that this lesson is on graphing quadratic functions, and answering questions using the graphs.

Teaching and Learning *(12 minutes)*

1. Draw the table of values on the board:

x	-3	-2	-1	0	1
y					

2. Solve for the first value of y, and write it in the table:

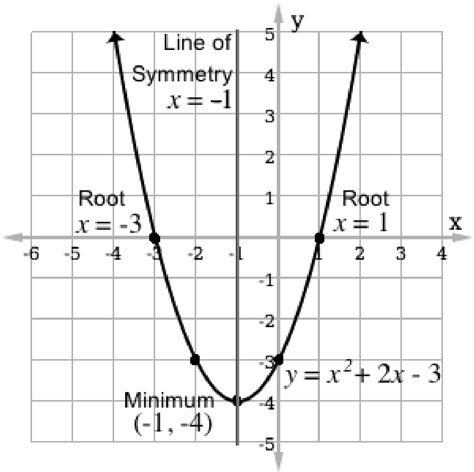
$$\begin{aligned} y &= x^2 + 2x - 3 \\ &= (-3)^2 + 2(-3) - 3 \\ &= 9 - 6 - 3 \\ &= 0 \end{aligned}$$

3. Ask pupils to work with seatmates to complete the table.
4. Invite volunteers to come to the board to fill the table:

x	-3	-2	-1	0	1
y	0	-3	-4	-3	0

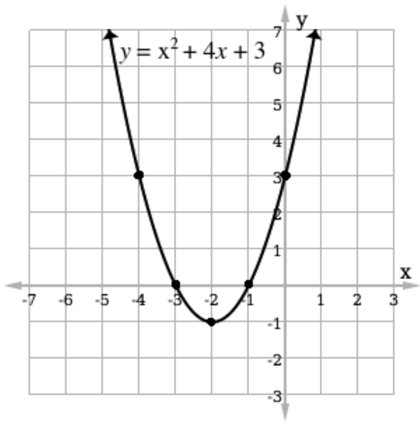
5. Draw an empty Cartesian plane on the board with axes from -5 to 5.
6. Invite volunteers to come to the board and plot the points from the table.

7. Connect the points with a parabola and label it $y = x^2 + 2x - 3$ (see graphed parabola below).
8. Ask a volunteer to answer question a. by identifying the minimum. (Answer: $(-1, -4)$)
9. Ask another volunteer to answer question b. by giving the solution set (roots). (Answer: $x = -3,\ 1$)
10. Ask another volunteer to answer question c. by giving the line of symmetry. (Answer: $x = -1$)
11. Label the parabola as shown:



Practice *(24 minutes)*

1. Write the following problems on the board:
   a. For the graph below, identify for $y = x^2 + 4x + 3$: i. the minimum; ii. the roots; iii. the line of symmetry.



   b. Complete the following:
      i. Complete the table below for the relation $y + 3x^2 + 2x - 9 = 0$.

x	-5	-4	-3	-2	-1	0	1	2	3
y	-56			1			4		

      ii. Using a scale of 2 cm to 1 unit on the x-axis and 2 cm to 10 units on the y-axis, draw the graph of $y = -3x^2 - 2x + 9$.
      iii. Use your graph to find the roots of the equation $-3x^2 - 2x + 9 = 0$.

iv. Find the maximum value of $y = -3x^2 - 2x + 9$.

v. Find the equation of the line of symmetry.

2. Ask pupils to solve the problems either independently or with seatmates.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to come to the board at the same time to write the solutions.

Solutions:

a. i. Minimum: $(-2, -1)$

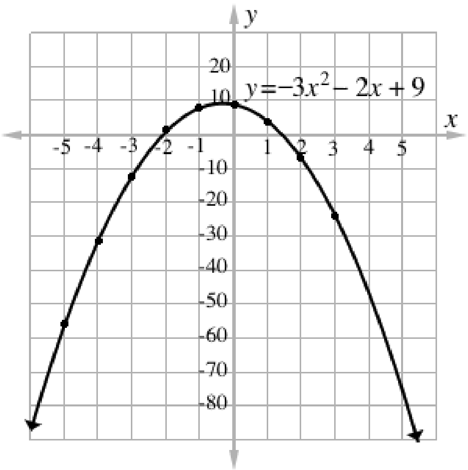
ii. Roots: $x = -3, -1$

iii. Line of symmetry: $x = -2$

b. i. Complete table:

x	-5	-4	-3	-2	-1	0	1	2	3
y	-56	-31	-12	1	8	9	4	-7	-24

ii. Graph (not to scale):



iii. Roots: Accept approximations near $x = -2.1, 1.4$

iv. Maximum: Accept approximations near $(-0.3, 9.3)$

v. Line of symmetry: Accept approximations near $x = -0.3$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L024 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L025 in the Pupil Handbook before the next class.

Lesson Title: Solving quadratic equations algebraically – Part 1	**Theme:** Algebraic Processes	
Lesson Number: M4-L025	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to factorise and solve quadratic equations.	**Preparation** 1. Review the content of this lesson and be prepared to explain the solutions. 2. Write the problem in Opening on the board.	

Opening *(1 minute)*

1. Write the following quadratic equation on the board: Solve for x: $x^2 + 4x + 3 = 0$
2. Discuss: How can you solve this problem? (Example answers: Factorisation, graphing, completing the square, quadratic formula.)
3. Explain that this lesson is on solving quadratic equations using factorisation.

Teaching and Learning *(12 minutes)*

1. Ask volunteers to explain how to factorise the problem on the board in their own words.
2. Review the process of factorisation:
    - Factorisation involves rewriting the quadratic equation in the form $x^2 + 4x + 3 = (x + p)(x + q)$, where p and q are numbers.
    - p and q should sum to the coefficient of the second term of the equation, and multiply to get the third term. (In this case $p + q = 4$ and $p \times q = 3$).
    - After finding p and q, set each binomial equal to 0 and solve to find the value of x. These values are the roots.
3. Solve the quadratic equation by factorisation, explaining each step:

$x^2 + 4x + 3 = (x + p)(x + q)$		Set up the equation
$p + q = 4$	$p \times q = 3$	Note that the values of p and q must be 3 and 1 to satisfy these equations.
$x^2 + 4x + 3 = (x + 1)(x + 3)$		Substitute values of p and q into the equation
$x + 1 = 0$ $x = -1$	$x + 3 = 0$ $x = -3$	Set each binomial equal to 0 and solve for x. These are the roots.

4. Write another problem on the board: Solve the quadratic equation $x^2 - 2x - 8 = 0$ using factorisation.
5. Ask pupils to work with seatmates to solve the problem.
6. Invite a volunteer to write the solution on the board.

Solution:

$x^2 - 2x - 8$	$=$	$(x+a)(x+b)$	Set up the equation
	$=$	$(x-4)(x+2)$	Because $p + q = -2$ and $p \times q = -8$
$x - 4 = 0$		$x + 2 = 0$	Set each binomial equal to 0 and
$x = 4$		$x = -2$	solve for x.

7. Write another problem on the board: Solve using factorisation: $\frac{2}{x} + x = 3$
8. Discuss: How would you solve this problem?
9. Allow pupils to share their ideas, then explain:
   - First multiply both sides of the equation by x to eliminate the fraction.
   - Try to write the result as a quadratic equation in standard form, and solve using factorisation.
10. Ask pupils to work with seatmates to solve the problem.
11. Walk around to check for understanding and clear misconceptions.
12. Invite a volunteer to write the solution on the board.

Solution:

$\frac{2}{x} + x$	$=$	3	
$x\left(\frac{2}{x}\right) + x^2$	$=$	$3x$	Multiply throughout by x
$2 + x^2$	$=$	$3x$	
$x^2 - 3x + 2$	$=$	0	Write in standard form.
$x^2 - 3x + 2$	$=$	$(x-2)(x-1)$	Factorise
$x - 2 = 0$		$x - 1 = 0$	Set each binomial equal to 0 and
$x = 2$		$x = 1$	solve for x. These are the roots.

Practice *(26 minutes)*

1. Write the following problems on the board:
   a. Solve using factorisation: $2x^2 - 5x - 3 = 0$
   b. Find the values of x for which $x^2 - x = 2$.
   c. Solve using factorisation: $3 - \frac{4}{x+1} = x$
   d. Find the equation whose roots are $x = -3$ and $x = 7$.
2. Ask pupils to solve the problems either independently or with seatmates.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to come to the board simultaneously to write the solutions.

Solutions:

a.

$2x^2 - 5x - 3$	$=$	$(2x+p)(x+q)$	Set up the equation
	$=$	$(2x+1)(x-3)$	Factorise
$2x + 1 = 0$		$x - 3 = 0$	Set each binomial equal to 0 and
$x = -\frac{1}{2}$		$x = 3$	solve for x.

b. Rewrite the equation so it is equal to zero: $x^2 - x - 2 = 0$

: $x^2 - x - 2$	$=$	$(x+a)(x+b)$	Set up the equation
	$=$	$(x+1)(x-2)$	Factorise
$x+1=0$		$x-2=0$	Set each binomial equal to 0 and
$x=-1$		$x=2$	solve for x.

c.

$3-\frac{4}{x+1}$	$=$	x	
$3(x+1)-\frac{4(x+1)}{x+1}$	$=$	$x(x+1)$	Multiply throughout by $(x+1)$
$3x+3-4$	$=$	x^2+x	Simplify
$3x-1$	$=$	x^2+x	
0	$=$	x^2-2x+1	Write in standard form.
x^2-2x+1	$=$	$(x-1)(x-1)$	Factorise
	$=$	$(x-1)^2$	
$x-1=0$			Set the binomial equal to 0 and
$x=1$			solve for x.

d. This question requires us to work backwards. If the roots of an equation are $x=-3$ and $x=7$, we know the 2 binomial factors are $(x+3)$ and $(x-7)$. Multiply these to find the equation:

$$(x+3)(x-7)=x^2-4x-21=0$$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L025 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L026 in the Pupil Handbook before the next class.

<table>
<tr><td>Lesson Title: Solving quadratic equations algebraically – Part 2</td><td colspan="2">Theme: Algebra</td></tr>
<tr><td>Lesson Number: M4-L026</td><td>Class: SSS 4</td><td>Time: 40 minutes</td></tr>
<tr><td>Learning Outcomes
By the end of the lesson, pupils will be able to:
1. Solve quadratic equations by completing the square.
2. Solve quadratic equations using the quadratic formula.</td><td colspan="2">Preparation
1. Review the content of this lesson and be prepared to explain the solutions.
2. Write the problem in Opening on the board.</td></tr>
</table>

Opening *(2 minutes)*

1. Write the following problem on the board: Expand: $(x+2)^2$
2. Allow a minute for pupils to answer the question in their exercise books.
3. Invite a volunteer write the answer on the board. (Answer: $(x+2)^2 = x^2 + 4x + 4$)
4. Explain to the pupils that today's lesson is on solving quadratic equations using 2 additional methods: completing the square and the quadratic formula.

Teaching and Learning *(17 minutes)*

1. Write on the board: Factorise $x^2 + 4x + 2$
2. Explain:
   - We cannot use the method we learned to factorise $x^2 + 4x + 2$
   - We can use "completing the square" or the quadratic formula to solve this.
   - We started the lesson by expanding $(x+2)^2$. This expression is called a perfect square because it is a square of a binominal.
3. Write on the board: Compare $x^2 + 4x + 2$ with the perfect square $(x+2)^2 = x^2 + 4x + 4$.
4. Explain:
   - We rewrite the given quadratic as a sum of a perfect square and a number. Since $4 - 2 = 2$, our quadratic remains unchanged.
   - We are able to rewrite our quadratic in terms of a perfect square.
   - Completing the square changes the quadratic to the sum of a perfect square and a number.
5. Write on the board: Let us use $(x+m)^2 = x^2 + 2mx + m^2$, where $x^2 + 2mx + m^2$ is a perfect square trinomial.
6. Write on the board: Find the roots of $x^2 + 4x + 1 = 0$ by completing the square.
7. Ask a volunteer to give the values of a, b and c. (Answer: $\text{a} = 1$, $\text{b} = 4$, $\text{c} = 1$
8. Write the quadratic as the sum of a perfect square and a number on the board:
   $x^2 + 4x + 1 = (x+m)^2 + n$
   $x^2 + 4x + 1 = x^2 + 2mx + m^2 + n$
9. Explain: We need to find the values of m and n to complete the square.
10. Show pupils that in the second line, the value of b in the quadratic equation is 4 on the left-hand side, and $2m$ on the right-hand side.

11. Solve for m on the board:

$$\begin{aligned} 4 &= 2m \\ \frac{4}{2} &= \frac{2m}{2} \\ 2 &= m \end{aligned}$$

12. Show pupils that the value of c in the quadratic equation is 1 on the left-hand side, and $m^2 + n$ on the right-hand side.
13. Solve for n on the board:

$$\begin{aligned} 1 &= m^2 + n \\ 1 &= 2^2 + n \\ 1 &= 4 + n \\ 1 - 4 &= n \\ -3 &= n \end{aligned}$$

14. Substitute the values of m and n on the board, and find the perfect square:

$$\begin{aligned} x^2 + 4x + 1 &= x^2 + 2mx + m^2 + n \\ &= x^2 + 2(2)x + (2)^2 - 3 \\ &= x^2 + 4x + 4 - 3 \\ &= (x + 2)^2 - 3 \end{aligned}$$

15. Solve the perfect square to find the roots of the equation on the board:

$$\begin{aligned} (x + 2)^2 - 3 &= 0 \\ (x + 2)^2 &= 3 \\ \sqrt{(x + 2)^2} &= \sqrt{3} \\ x + 2 &= \pm\sqrt{3} \\ x &= -2 \pm \sqrt{3} \end{aligned}$$

16. Write on the board: Quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
17. Explain:
    - This formula is derived using the completing square method.
    - Identify the values of a, b and c and substitute them to solve the equation.
18. Write the following problem on the board: Use the quadratic formula to solve $x^2 + 4x + 1 = 0$.
19. Write the solution on the board and explain:

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} && \text{Formula} \\ &= \frac{-4 \pm \sqrt{(4)^2 - 4(1)(1)}}{2(1)} && \text{Substitute } a, b, \text{and } c \\ &= \frac{-4 \pm \sqrt{16 - 4}}{2} && \text{Simplify} \\ &= \frac{-4 \pm \sqrt{12}}{2} \\ &= \frac{-4 \pm 2\sqrt{3}}{2} \\ &= \frac{-2 \pm \sqrt{3}}{1} \\ &= -2 + \sqrt{3} \text{ or } -2 - \sqrt{3} \end{aligned}$$

Practice *(20 minutes)*

1. Write the following problems on the board:
   a. Find the roots of $x^2 - 6x + 8 = 0$ by completing the square.
   b. Use the quadratic formula to solve the following equations:
      i. $3x^2 + 2x - 7 = 0$ ii. $x^2 - 8x + 15 = 0$
2. Ask pupils to solve the problems either independently or with seatmates.
3. Invite volunteers to come to the board simultaneously to write the solutions.
   Solutions:
   a. Find the values of m and n:

$$m = \frac{b}{2} = \frac{-6}{2} = -3$$
$$n = c - m^2 = 8 - (-3)^2 = 8 - 9 = -1$$

Write the quadratic equation with a perfect square:

$$\begin{aligned} x^2 - 6x + 8 &= x^2 + 2(-3)x + (-3)^2 + (-1) \\ &= x^2 - 6x + 9 - 1 \\ &= (x-3)^2 - 1 \end{aligned}$$

Find the roots:

$$\begin{aligned} (x-3)^2 - 1 &= 0 \\ (x-3)^2 &= 1 \\ \sqrt{(x-3)^2} &= \sqrt{1} \\ x - 3 &= \pm 1 \\ x &= 3 + 1 \text{ and } 3 - 1 \\ x &= 4 \text{ and } 2 \end{aligned}$$

b. i.

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-2 \pm \sqrt{(2)^2 - 4(3)(-7)}}{2(3)} \\ &= \frac{-2 \pm \sqrt{4 + 84}}{6} \\ &= \frac{-2 \pm \sqrt{88}}{6} \\ &= \frac{-2 \pm 9.38}{6} \\ &= \frac{-2 + 9.38}{6} \text{ or } \frac{-2 - 9.38}{6} \\ &= \frac{7.38}{6} \text{ or } \frac{-11.38}{6} \\ &= 1.23 \text{ or } -1.90 \end{aligned}$$

ii.

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(15)}}{2(1)} \\ &= \frac{8 \pm \sqrt{64 - 60}}{2} \\ &= \frac{8 \pm \sqrt{4}}{2} \\ &= \frac{8 \pm 2}{2} \\ &= \frac{8 + 2}{2} \text{ or } \frac{8 - 2}{2} \\ &= \frac{10}{2} \text{ or } \frac{6}{2} \\ &= 5 \text{ or } 3 \end{aligned}$$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L026 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L027 in the Pupil Handbook before the next class.

Lesson Title: Problem solving with quadratic equations	**Theme:** Algebra	
Lesson Number: M4-L027	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to solve word problems that lead to quadratic equations.	**Preparation** 1. Review the content of this lesson and be prepared to explain the solutions. 2. Write the problem in Opening on the board.	

Opening *(2 minutes)*

1. Write the following problem on the board: The sum of two numbers is 12 and their product is -35. What are the two numbers?
2. Ask pupils to write expressions from the problem in their exercise books.
3. Invite volunteers to write the 2 expressions on the board:
   i. $n + m = 12$
   ii. $n \times m = 35$
4. Explain that this lesson is on solving word problems on quadratic equations.

Teaching and Learning *(19 minutes)*

1. Explain: We have 2 equations in 2 variables, which can be solved using substitution.
2. Solve the problem on the board using substitution:
   Solve equation i. from n: $n = 12 - m$
   Substitute n into equation ii. and evaluate:

$$\begin{aligned}
(12-m)\times m &= 35 && \text{Substitute } n = 12 - m \text{ into (ii)}\\
12m - m^2 &= 35 && \text{Removing bracket}\\
m^2 - 12m + 35 &= 0 && \text{Transpose } -35\\
m^2 - 5m - 7m + 35 &= 0 && \text{Factor}\\
m(m-5) - 7(m-5) &= 0\\
(m-5)(m-7) &= 0
\end{aligned}$$

   Either $m - 5 = 0$ or $m - 7 = 0$

   $m = 0 + 5 \qquad m = 0 + 7$
   $m = 5 \qquad$ or $\quad m = 7$
   Therefore, the 2 numbers are 5 and 7.
3. Ask pupils to look at the process for solving word problems that is given in the Pupil Handbook. Review these steps (it is not necessary to write them on the board):
   - Read through the problem carefully and know what is being asked.
   - Choose variables to represent unknown numbers.
   - Write an algebraic expression for the problem.
   - Solve the problem.

- Check your work.

4. Explain: Once you find the quadratic equation from a problem, you can use any method you like to solve it, including: factorisation, completing the square or the quadratic formula.
5. Write the following problems on the board:
   a. Find two consecutive integers that have a product of 42.
   b. Foday is 4 times older than his son. Five years ago, the product of their ages was 175. Find their present ages.
6. Ask pupils to work with seatmates to solve the problems.
7. Walk around to check for understanding and clear misconceptions.
8. Invite volunteers to write the solutions on the board.

Solutions:

a. Let the first integer be x. Then the second integer is $x+1$. The product of the two integers is $x(x+1)=42$.
   Write the quadratic equation and solve for x:

$$\begin{aligned} x(x+1) &= 42 \\ x^2+x &= 42 \\ x^2+x-42 &= 0 \\ x^2+7x-6x-42 &= 0 \\ x(x+7)-6(x+7) &= 0 \\ (x-6)(x+7) &= 0 \end{aligned}$$

   Either $x-6=0$ or $x+7=0$
   $x=0+6$ $\quad$ $x=0-7$
   $x=6$ $\quad$ or $\quad$ $x=-7$

   Note that this has only given us the first integer, x. We must find the next consecutive integer for each.
   When $x=6,\ x+1=6+1=7$. Therefore the two consecutive numbers are 6 and 7.
   When $x=-7,\ x+1=-7+1=-6$. Therefore, the other consecutive numbers are -7 and -6.
   Answer: The numbers are 6 and 7 or -7 and -6.

b. Let the child's age be p years, then Foday's age be $4p$ years.
   5 years ago, the child's age was $(p-5)$ years and Foday's age was $(4p-5)$ years. The product of their ages was $(p-5)(4p-5)=175$.
   Write the quadratic equation and solve for p:

$$\begin{aligned} (p-5)(4p-5) &= 175 \\ 4p^2-5p-20p+25 &= 175 \\ 4p^2-25p+25 &= 175 \\ 4p^2-25p+25-175 &= 0 \\ 4p^2-25P-150 &= 0 \\ (4p+15)(p-10) &= 0 \end{aligned}$$

   Either $4p+15=0$ $\quad$ or $\quad$ $p-10=0$
   $4p=-15$ $\quad$ or $\quad$ $p=0+10$

$p = \frac{-15}{4}$ or $p = 10$

The child's age is 10 years, because ages cannot be negative. Therefore, Foday's age is: $4p = 4(10) = 40$ years

Practice *(18 minutes)*

1. Write the following problems on the board:
   a. The product of two consecutive integers is 72. What are the integers?
   b. Abass is 7 years older than Juliet. The product of their ages is 18 years. How old is Abass? How old is Juliet?
2. Ask pupils to solve the problems either independently or with seatmates.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to come to the board to write the solutions at the same time.

Solutions:

a. Let the numbers be y and $y + 1$

$$
\begin{aligned}
y(y+1) &= 72 \\
y^2 + y &= 72 \\
y^2 + y - 72 &= 0 \\
y^2 + 9y - 8y - 72 &= 0 \\
y(y+9) - 8(y+9) &= 0 \\
(y+9)y-8) &= 0
\end{aligned}
$$

Either $y + 9 = 0$ or $y - 8 = 0$

$\mathrm{y} = -9$ or 8

If $y = -9$, the other integer is $-9 + 1 = -8$

If $y = 8$, the other integer is $8 + 1 = 9$

b. Let Juliet's age= x years and Abass' age $= (7 + x)$ years

$$
\begin{aligned}
\text{Product } x(7+x) &= 18 \\
7x + x^2 &= 18 \\
x^2 + 7x - 18 &= 0 \\
x^2 + 9x - 2x - 18 &= 0 \\
x(x+9) - 2(x+9) &= 0 \\
(x+9)(x-2) &= 0 \\
x &= -9 \quad or \quad 2
\end{aligned}
$$

Therefore, Juliet's age is 2 years and Abass' age is $7 + 2 = 9$ years.

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L027 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L028 in the Pupil Handbook before the next class.

Lesson Title: Simultaneous linear equations	**Theme:** Algebraic Processes	
Lesson Number: M4-L028	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to solve simultaneous linear equations using elimination, substitution, and graphing.	**Preparation** 1. Review the content of this lesson and be prepared to explain the solutions. 2. Write the problem in Opening on the board.	

Opening *(1 minute)*

1. Write on the board: Solve the simultaneous equations:
$$-x + y = 1$$
$$2x + y = 4$$
2. Discuss:
   a. What does the solution to a set of simultaneous equations look like? (Answer: It has two values, x and y, that satisfy both equations.)
   b. How would you solve these simultaneous equations? (Example answers: Pupils may describe solving by elimination, substitution, or graphing.)
3. Explain that this lesson is on solving simultaneous equations using 3 methods: elimination, substitution, and graphing.

Teaching and Learning *(18 minutes)*

1. Ask volunteers to explain how to factorise the problem on the board in their own words.
2. Review the process for solving by each method, as needed:
   - To solve by **elimination**:
     - Subtract one equation from the other so that one of the variables (x or y) is eliminated.
     - If neither of the variables has the same coefficient, you will need to multiply the equations throughout so that the coefficients are the same before subtracting.
   - To solve by **substitution**:
     - To solve using the method of substitution, we must change the subject.
     - We should choose one of the given equations and make one of the variables the subject of the other one.
     - After changing the subject, we substitute the expression into the other linear equation.
   - To solve by **graphing**:
     - We can solve a set of simultaneous equations by graphing both lines. The solution is the point where the lines intersect.
   - We can check answers by substituting the values for x and y into the original equations.

3. Ask pupils to work with seatmates to solve the problem on the board using all 3 methods. Remind them that they should get the same answer using each method. If this is too challenging, solve the problem as a class on the board.
4. Invite volunteers to write the solutions on the board and explain.

Elimination:

$-2x + 2y = 2$		Equation (1) × 2
$+ \quad 2x + y = 4$		Equation (2)
$0 + 3y = 6$		Add the equations to eliminate $2x$
$\frac{3y}{3} = \frac{6}{3}$		Divide throughout by 3
$y = 2$		
$-x + y = 1$	(1)	
$-x + 2 = 1$		Substitute $y = 2$ in equation (1)
$x = 1$		

Answer: $(1, 2)$

Substitution:

$-x + y = 1$	(1)	
$y = 1 + x$		Change the subject of equation (1) by transposing $-x$
$2x + (1 + x) = 4$	(2)	Substitute equation (1) into equation (2)
$3x + 1 = 4$		Simplify the left-hand side
$3x = 4 - 1$		Transpose 1
$3x = 3$		
$\frac{3x}{3} = \frac{3}{3}$		Divide throughout by 3
$x = 1$		
$y = 1 + x$		
$y = 2$		

Answer: $(1, 2)$

Graphing:

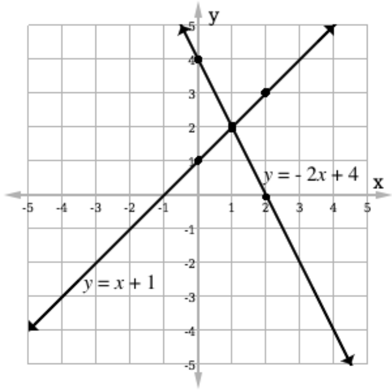
Step 1. Change the subject of the equations:

$$-x + y = 1 \qquad\qquad 2x + y = 4$$
$$y = x + 1 \qquad\qquad y = -2x + 4$$

Step 2. Fill a table of values for each equation:

Equation (1)			
x	0	1	2
y	1	2	3

Equation (2)			
x	0	1	2
y	4	2	0



Step 3. Graph each line (see graph at right).

Step 4. Identify that the point of intersection is (1, 2), which gives the answer.

Practice *(20 minutes)*

1. Write the following problems on the board:
   a. Solve by graphing: $y = \frac{1}{2}x + 4$ and $y = -x + 7$.
   b. Solve using elimination: $3x + 4y = -1$ and $3x + 8y = 4$.
   c. Solve using substitution: $2a - b = 5$ and $3a + 2b = -24$.
2. Ask pupils to solve the problems either independently or with seatmates.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to come to the board at the same time to write the solutions.

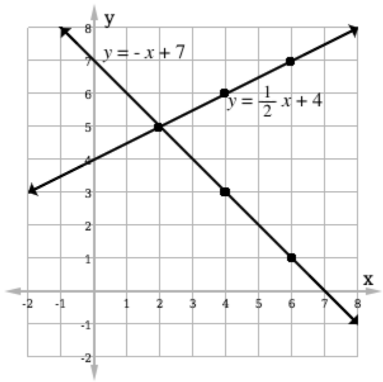
Solutions:

a. Equation (1)

x	2	4	6
y	5	6	7

Equation (2)

x	2	4	6
y	5	3	1



Answer: $(2, 5)$

b.

$$3x + 4y = -1$$
$$3x + 8y = 4$$
$$0x - 4y = -5$$
$$\frac{-4y}{-4} = \frac{-5}{-4}$$
$$y = \frac{5}{4}$$

$$3x + 8y = 4$$
$$3x + 8(\tfrac{5}{4}) = 4$$
$$3x + 10 = 4$$
$$3x = 4 - 10$$
$$3x = -6$$
$$\frac{3x}{3} = \frac{-6}{3}$$

c.

$$2a - b = 5 \quad (1)$$
$$2a - 5 = b$$

$$3a + 2(2a - 5) = -24 \quad (2)$$
$$3a + 4a - 10 = -24$$
$$7a - 10 = -24$$
$$7a = -14$$
$$\frac{7a}{7} = \frac{-14}{7}$$
$$a = -2$$

Substitute a into the formula for b:

$$b = 2(-2) - 5$$
$$b = -4 - 5$$
$$b = -9$$

$$x = -2$$

Answer: $x = -2, y = \frac{5}{4}$

Answer: $a = -2, b = -9$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L028 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L029 in the Pupil Handbook before the next class.

Lesson Title: Applications of simultaneous linear equations	**Theme:** Algebraic Processes	
Lesson Number: M4-L029	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to solve word problems leading to simultaneous linear equations.	**Preparation** 1. Review the content of this lesson and be prepared to explain the solutions. 2. Write the problem in Opening on the board.	

Opening *(1 minute)*

1. Write on the board: One pen and 2 exercise books cost Le 4,500.00. Two pens and 3 exercise books cost Le 7,000.00. How much does each pen and exercise book cost?
2. Discuss: How would you solve this problem? (Answer: Write 2 linear equations using variables for the unknowns, which are the cost of 1 pen and 1 exercise book.)
3. Explain that this lesson is on applications of simultaneous linear equations.

Teaching and Learning *(20 minutes)*

1. Tell pupils that we will let the variable e represent the cost of an exercise book, and p represent the cost of a pen.
2. Write on the board: $p + 2e = 4{,}500$, where p is the cost of a pen, and e is the cost of an exercise book.
3. Explain:
   - This is for the first sentence of the problem.
   - This equation tells us that the total cost of 1 pen and 2 exercise books is 4,500 Leones.
   - We do not yet have enough information to solve this problem. We need another equation.
4. Ask pupils to look at the second sentence in the problem (2 pens and 3 exercise books cost Le 7,000.00). Ask them to work with seatmates to write an equation for this sentence.
5. Invite a volunteer to write the equation on the board. (Answer: $2p + 3e = 7{,}000$)
6. Ask pupils to work with seatmates to solve the system of equations. They may use either substitution or elimination.

$$\begin{aligned} p + 2e &= 4{,}500 \qquad (1) \\ 2p + 3e &= 7{,}000 \qquad (2) \end{aligned}$$

7. Ask one group of seatmates to write their solution on the board (either method).

Solutions:

Elimination:

$$
\begin{array}{rcll}
2(p + 2e & = & 4{,}500) & \text{(1)} \\
-(2p + 3e & = & 7{,}000) & \text{(2)} \\
\hline
\downarrow & & \downarrow & \\
2p + 4e & = & 9{,}000 & \text{(1)x2} \\
-(2p + 3e & = & 7{,}000) & \text{(2)} \\
\hline
0 + e & = & 2{,}000 & \\
e & = & 2{,}000 & \\
 & & & \\
p + 2(2{,}000) & = & 4{,}500 & \text{(1)} \\
p + 4{,}000 & = & 4{,}500 & \\
p & = & 500 &
\end{array}
$$

Substitution:

$$
\begin{array}{rcll}
p + 2e & = & 4{,}500 & \text{(1)} \\
p & = & 4{,}500 - 2e & \\
 & & & \\
2(4{,}500 - 2e) + 3e & = & 7{,}000 & \text{(2)} \\
9{,}000 - 4e + 3e & = & 7{,}000 & \\
9{,}000 - e & = & 7{,}000 & \\
2{,}000 & = & e & \\
 & & & \\
p & = & 4{,}500 - 2(2{,}000) & \text{(1)} \\
p & = & 500 &
\end{array}
$$

Answer: $e = \text{Le } 2{,}000.00$, $p = \text{Le } 500.00$

8. Explain: One exercise book costs Le 2,000.00, and one pen costs Le 500.00.
9. Write the following problem on the board: The sum of the ages of Hawa and Fatu is 32. Hawa is 4 years older than Fatu. What are their ages?
10. Ask pupils to work with seatmates to write equations for the problem.
11. Invite volunteers to write the equations on the board. (Answer: $h + f = 32$ and $h = f + 4$, where h is Hawa's age and f is Fatu's age.)
12. Ask pupils to work with seatmates to solve the problem. They may use either substitution or elimination.
13. Ask one group of seatmates to write their solution on the board.

Solutions:

Elimination:

$$
\begin{array}{rcll}
h + f & = & 32 & \text{(1)} \\
-(h - f & = & 4) & \text{(2)} \\
\hline
0 + 2f & = & 28 & \\
2f & = & 28 & \\
\frac{2f}{2} & = & \frac{28}{2} & \\
f & = & 14 & \\
 & & & \\
h + (14) & = & 32 & \text{(1)} \\
h & = & 18 &
\end{array}
$$

Substitution:

$$
\begin{array}{rcll}
h + f & = & 32 & \text{(1)} \\
h & = & 32 - f & \\
 & & & \\
(32 - f) - f & = & 4 & \text{(2)} \\
32 - 2f & = & 4 & \\
-2f & = & 4 - 32 & \\
\frac{-2f}{-2} & = & \frac{-28}{-2} & \\
f & = & 14 & \\
 & & & \\
h & = & 32 - (14) & \text{(1)} \\
h & = & 18 &
\end{array}
$$

Answer: $f = 14$ years old $h = 18$ years old

Practice *(18 minutes)*

1. Write the following problems on the board:
   a. The total age of 2 brothers is 112. One of the brothers is 14 years older than the other. What are their ages?
   b. The cost of a cup of sugar is x Leones, and the cost of a cup of flour is y Leones. If 1 cup of sugar and 4 cups of flour cost Le 7,000.00, and 3 cups of sugar and 2 cups of flour cost Le 6,000.00, find x and y.
2. Ask pupils to solve the problems either independently or with seatmates.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to come to the board simultaneously to write the solutions.
   Solutions:
   Pupils may use any method to solve. Example solutions are below.

a. Using elimination:

$$\begin{aligned} x+y &= 112 && (1)\\ -(x-y &= 14) && (2)\\ 0+2y &= 98\\ 2y &= 98\\ \frac{2y}{2} &= \frac{98}{2}\\ y &= 49\\ \\ x+(49) &= 112 && (1)\\ x &= 63 \end{aligned}$$

Answer: Their ages are 49 and 63.

b. Using substitution:

$$\begin{aligned} x+4y &= 7{,}000 && (1)\\ x &= 7{,}000-4y\\ \\ 3(7{,}000-4y))+2y &= 6{,}000 && (2)\\ 21{,}000-12y+2y &= 6{,}000\\ 21{,}000-10y &= 6{,}000\\ 21{,}000-6{,}000 &= 10y\\ 15{,}000 &= 10y\\ 1{,}500 &= y\\ \\ x &= 7{,}000-4y && (1)\\ &= 7{,}000-4(1{,}500)\\ &= 7{,}000-6{,}000\\ &= 1{,}000 \end{aligned}$$

Answer: $x = \text{Le } 1{,}000.00, y = \text{Le } 1{,}500.00$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L029 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L030 in the Pupil Handbook before the next class.

Lesson Title: Simultaneous quadratic and linear equations	**Theme:** Algebra	
Lesson Number: M4-L030	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to solve simultaneous quadratic and linear equations using substitution and graphing.	**Preparation** 1. Review the content of this lesson and be prepared to explain the solutions. 2. Write the equations in Opening on the board.	

Opening *(1 minute)*

1. Write on the board:

$$y = x + 2 \quad (1)$$
$$y = x^2 \quad (2)$$

2. Discuss:
    a. What type of equations are these? (Answer: (1) is linear, and (2) is quadratic)
    b. Can we solve this system of equations? How? (Allow pupils to share ideas)
3. Explain that this lesson is on solving simultaneous linear and quadratic equations.

Teaching and Learning *(18 minutes)*

1. Explain:
    - These are simultaneous linear and quadratic equations. They can be solved using substitution or graphing.
    - Simultaneous linear and quadratic equations can have 0, 1 or 2 solutions.
    - The solutions are ordered pairs, (x, y).
2. Explain how to solve using **substitution:**
    - Make y the subject of one equation, then substitute it into the other equation.
    - Simplify until it has the form of a standard quadratic equation ($ax^2 + bx + c = 0$).
    - Solve the quadratic equation using any method. Today we will use factorisation.
3. Solve the simultaneous equations on the board using substitution. Involve pupils by asking volunteers to explain each step.
   Solution:

x^2	$=$	$x + 2$	(1)	Substitute $y = x^2$ for y in equation (1)
$x^2 - x - 2$	$=$	0		Transpose x and 2
$(x-2)(x+1)$	$=$	0		Factorise the quadratic equation
$x - 2 = 0$	or	$x + 1 = 0$		Set each binomial equal to 0

$x = 2$ or $x = -1$ Transpose -2 and 1

$y = (2) + 2$ Substitute $x = 2$ into equation (2)
$y = 4$

$y = (-1) + 2$ Substitute $x = -1$ into equation (2)
$y = 1$

Solutions: $(2, 4)$ and $(-1, 1)$

4. Explain how to solve using **graphing:**
   a. To graph quadratic or linear equations, we need to find points on the curve and the line. We then plot the points and connect them.
   b. The intersection points of the curve and line are the solutions to the simultaneous equations.
5. Draw an empty Cartesian plane and empty tables of values on the board (see below for tables).
6. Ask pupils to work with seatmates to fill the tables of values and graph the line and quadratic function.
7. Ask volunteers to fill the tables of values on the board and graph the parabola and line. Determine the solutions as a class.

Solution:

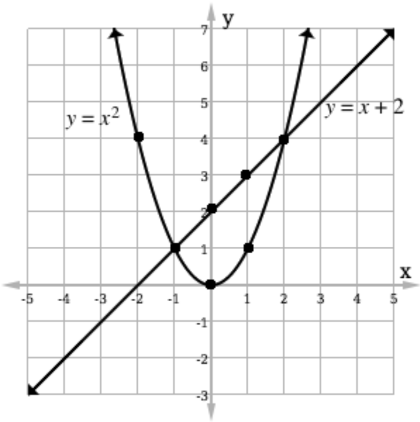
Fill the table of values for $y = x + 2$:

x	-2	-1	0	1	2
y	0	1	2	3	4

Fill the table of values for $y = x^2$:

x	-2	-1	0	1	2
y	4	1	0	1	4

Plot the points from both tables. Draw the line and curve:



Answers: $(-1, 1)$, $(2, 4)$

Practice *(20 minutes)*

1. Write the following problems on the board:
   a. Find the solution(s) to the following equations using substitution: $y = x^2 - 3$ and $x = y - 9$.

   b. Find the solution(s) to the following equations using graphing: $y = x^2 + 1$ and $y = x - 1$
2. Ask pupils to solve the problems either independently or with seatmates.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to come to the board simultaneously to write the solutions.
   Solutions:
   a.

x	$=$	$(x^2 - 3) - 9$	(2)	Substitute equation (1) into equation (2)
x	$=$	$x^2 - 12$		Simplify
0	$=$	$x^2 - x - 12$		Transpose x
$x^2 - x - 12$	$=$	0		
$(x - 4)(x + 3)$	$=$	0		Factorise the quadratic equation
$x - 4 = 0$	or	$x + 3 = 0$		Set each binomial equal to 0
$x = 4$	or	$x = -3$		Transpose -4 and 3
y	$=$	$4^2 - 3$		Substitute $x = 4$ into equation (1)
y	$=$	$16 - 3$		
y	$=$	13		
y	$=$	$(-3)^2 - 3$		Substitute $x = -3$ into equation (1)
y	$=$	$9 - 3$		
y	$=$	6		

Answers: $(4, 13)$ and $(-3, 6)$

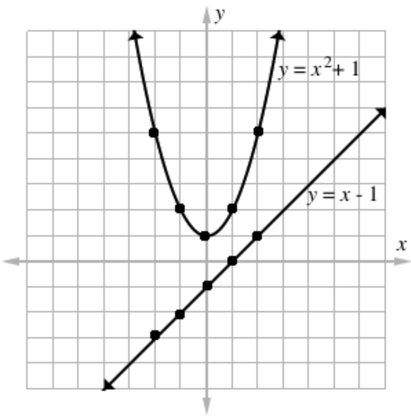
b.

$y = x^2 + 1$

x	-2	-1	0	1	2
y	5	2	1	2	5

$y = x - 1$

x	-2	-1	0	1	2
y	-3	-2	-1	0	1



No solution

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L030 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L031 in the Pupil Handbook before the next class.

<table>
<tr><td>Lesson Title: Tangent to a quadratic function</td><td colspan="2">Theme: Algebra</td></tr>
<tr><td>Lesson Number: M4-L031</td><td>Class: SSS 4</td><td>Time: 40 minutes</td></tr>
<tr><td>Learning Outcome
By the end of the lesson, pupils will be able to solve problems involving the tangent line to a quadratic function.</td><td colspan="2">Preparation
Review the content of this lesson and be prepared to explain the solutions.</td></tr>
</table>

Opening *(2 minutes)*

1. Discuss:
   a. What is the meaning of "gradient" in Maths? (Answer: Gradient is a value that gives the steepness of a line.)
   b. What is a tangent line? (Answer: It is a line that touches a curve at exactly 1 point.)
2. Explain that today's lesson is on finding the gradient of a curve using the tangent line.

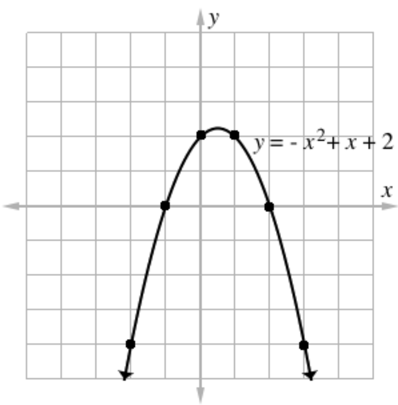
Teaching and Learning *(18 minutes)*

1. Write the problem on the board: Graph $y = -x^2 + x + 2$ using the table of values:

x	-2	-1	0	1	2	3
y						

2. Ask pupils to work with seatmates to graph the parabola. Give them several minutes to work.
3. Invite a volunteer to fill the table, and another volunteer to graph the parabola on the board.

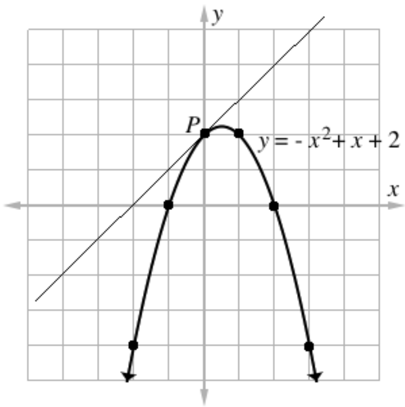
x	-2	-1	0	1	2	3
y	-4	0	2	2	0	-4



4. Explain:
   - The gradient of a line stays the same along the entire line. However, the gradient of a curve changes from point to point.
   - The gradient at any point on a curve is the same as the gradient of the tangent line at that exact point.
   - A tangent line touches the curve at only one point.

- A tangent to a curve at point P can be drawn by placing a straight edge on the curve at P, then drawing a line. The "angles" between the curve and line should be nearly equal.
- Use anything straight for a straight edge, including paper or the side of an exercise book.

5. Label point $(0, 2)$ on the parabola as *P*. Draw a tangent to the parabola at this point, as shown:



6. Explain:
   - The parabola and tangent line must be drawn very accurately and clearly to find the gradient. Use a very exact scale on the x- and y-axes.
   - The gradient found using a tangent is usually only an **approximate** gradient for the curve.
   - We use this tangent line to find the gradient of the curve at $(0, 2)$. We find any 2 points on the line and use them in the gradient formula.
7. Ask pupils to identify two points on the tangent line. (Example answers: $(-2, 0)$ and $(0, 2)$)
8. Assign variables to each of these from the gradient formula, and write them on the board. (Example: $(x_1, y_1) = (-2, 0)$ and $(x_2, y_2) = (0, 2)$)
9. Substitute these coordinates into the gradient formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{2-0}{0-(-2)} \quad \text{Substitute } x\text{- and } y\text{-values.}$$

$$= \frac{2}{2} \quad \text{Simplify}$$

$$m = 1$$

10. Explain: By finding the gradient of the tangent line at point P, we have also found the gradient of the curve $y = -x^2 + x + 2$ at this point.

Practice *(19 minutes)*

1. Write the following problems on the board:
   a. Graph $y = x^2 + 2x - 1$ using the table of values below.

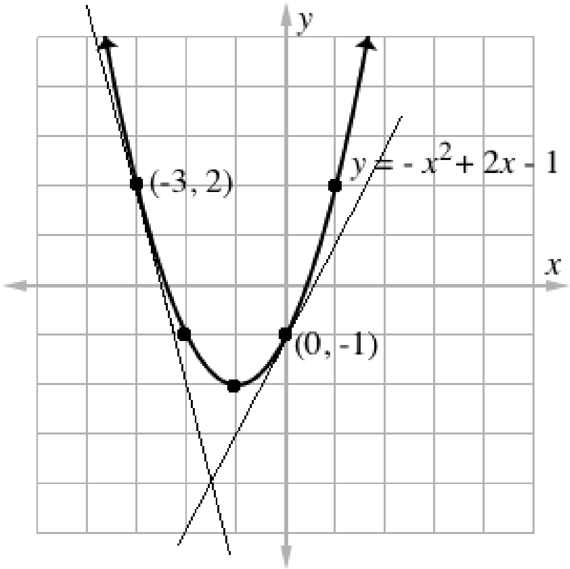
x	-3	-2	-1	0	1
y					

b. Find the gradient of the curve at $(-3, 2)$ by drawing a tangent line.
c. Find the gradient of the curve at $(0, -1)$ by drawing a tangent line.

2. Ask pupils to solve the problems either independently or with seatmates.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to come to the board simultaneously to write the solution.

   Solution:

   a. Graph of the curve (with tangent lines shown):



   b. Draw the tangent line at $(-3, 2)$. Use 2 points on the line at (for example, $(x_1, y_1) = (-3, 2)$ and $(x_2, y_2) = (-2, -2)$) in the slope formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-2}{-2-(-3)} = \frac{-4}{-2+3} = \frac{-4}{1} = -4$$

   c. Draw the tangent line at $(0, -1)$. Use 2 points on the line (for example, $(x_1, y_1) = (0, -1)$ and $(x_2, y_2) = (-1, -3)$) in the slope formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3-(-1)}{-1-0} = \frac{-3+1}{-1} = \frac{-2}{-1} = 2$$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L031 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L032 in the Pupil Handbook before the next class.

Lesson Title: Inequalities	**Theme:** Algebra	
Lesson Number: M4-L032	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to solve inequalities in one variable and represent them on number lines.	**Preparation** 1. Review the content of this lesson and be prepared to explain the solutions. 2. Write the problem in Opening on the board.	

Opening *(1 minute)*

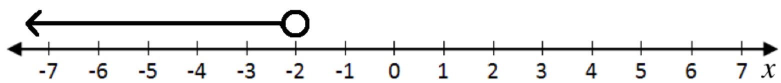
1. Write on the board: Solve and represent your solution on a number line: $-2x + 9 > 13$
2. Discuss and allow pupils to share their ideas:
    a. How can we solve inequalities? (Answer: Follow the same steps as solving an equation; reverse the inequality symbol when multiplying or dividing by a negative number.)
    b. How can we represent inequalities on number lines? (Answer: With arrows showing the solution set.)
3. Explain that this lesson is on solving inequalities and representing the solution sets on number lines.

Teaching and Learning *(22 minutes)*

1. Explain:
    - To solve an inequality for a variable, you may add or subtract a number from both sides of the inequality. The inequality symbol does not change.
    - You may multiply or divide both sides by a positive number. The sign of the inequality is unchanged.
    - You may also multiply or divide both sides by a **negative** number, but the direction of the inequality is **reversed**. Greater than becomes less than, and less than becomes greater than.
    - The solution to an inequality can be called the "truth set" or "solution set".
2. Solve the problem on the board. Involve pupils by asking them to give the steps:

$$\begin{aligned} -2x + 9 &> 13 \\ -2x &> 13 - 9 && \text{Transpose 9} \\ -2x &> 4 \\ \frac{-2x}{-2} &< \frac{4}{-2} && \text{Divide throughout by } -2 \text{ (reverse the direction of the inequality)} \\ x &< -2 \end{aligned}$$

3. Draw the number line on the board:



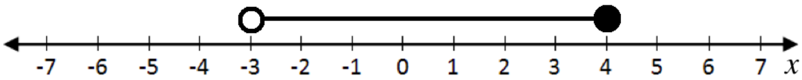
4. Explain:
    - This number line shows $x < -2$.

- The open circle shows that -2 is not included. x cannot be equal to -2.
- When we have $\leq$ or $\geq$ symbols, we use filled circles to show that x can be equal to the number.

5. Write the following problem on the board: Write $-5 < 2x + 1 \leq 9$ in the form $a < x \leq b,$ where a and b are integers, and represent the solution on the number line.
6. Explain:
   - This inequality has 3 parts, but we can still solve for x by applying the same operation throughout.
   - We want to get x by itself in the middle.
7. Solve on the board, involving pupils:

$$\begin{array}{rcccl} -5-1 & < & 2x+1-1 & \leq & 9-1 \quad \text{Subtract 1 throughout} \\ -6 & < & 2x & \leq & 8 \\ \frac{-6}{2} & < & \frac{2x}{2} & \leq & \frac{8}{2} \quad \text{Divide throughout by 2} \\ -3 & < & x & \leq & 4 \end{array}$$

8. Ask pupils to work with seatmates to represent the solution on a number line.
9. Invite a volunteer with the correct answer to write the answer on the board:

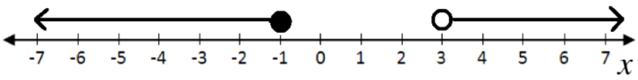


10. Write the following problem on the board: Solve: $3x + 2 > 11$ or $2x + 2 \leq x + 1$ and represent the solution on the number line.
11. Explain:
    - Here we have 2 different inequalities that represent values of x.
    - Solve both of them separately, and represent both on the number line.
12. Ask pupils to work with seatmates to solve the problem.
13. Walk around to check for understanding and clear misconceptions.
14. Invite volunteers to write the solution and the number line on the board.

Solution:

$$\begin{array}{rcl} 3x+2 & > & 11 \\ 3x & > & 9 \\ x & > & 3 \end{array} \quad \begin{array}{c} \text{or} \\ \text{or} \\ \text{or} \end{array} \quad \begin{array}{rcl} 2x+2 & \leq & x+1 \\ 2x-x & \leq & 1-2 \\ x & \leq & -1 \end{array}$$

Number line:



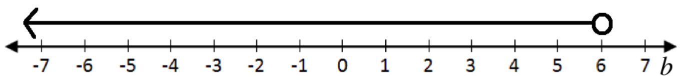
Practice *(16 minutes)*

1. Write the problems on the board: Solve the following inequalities and represent them on number lines:
   a. $b - 9 < -3$
   b. $4x - 2 \geq x - 14$
   c. $-1 < a + 5 \leq 10$
   d. $x - 1 \geq 2x + 6$ or $-2x + 5 < 3$
2. Ask pupils to solve the problems either independently or with seatmates.

3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to come to the board simultaneously to write the solutions.
   Solutions:
   a. Solve for b:

$$\begin{aligned} b-9 &< -3 \\ b &< -3+9 && \text{Transpose } -9 \\ b &< 6 \end{aligned}$$

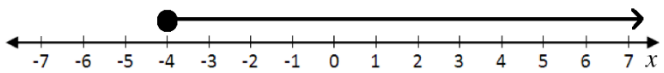
Number line:



   b. Solve for x:

$$\begin{aligned} 4x-2 &\geq x-14 \\ 4x-x &\geq -14+2 && \text{Collect like terms} \\ 3x &\geq -12 \\ \frac{3x}{3} &\geq \frac{-12}{3} && \text{Divide throughout by 3} \\ x &\geq -4 \end{aligned}$$

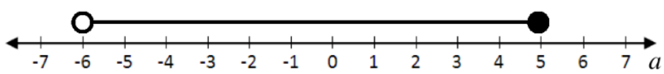
Number line:



   c. Solve for a:

$$\begin{aligned} -1 &< a+5 \leq 10 \\ -1-5 &< a \leq 10-5 && \text{Subtract 5 throughout} \\ -6 &< a \leq 5 && \text{Divide throughout by 2} \end{aligned}$$

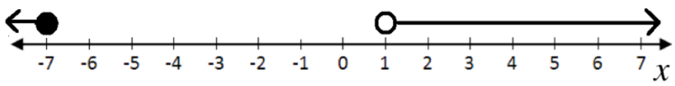
Number line:



   d. Solve for x:

$$\begin{aligned} x-1 &\geq 2x+6 && \text{or} && -2x+5 < 3 \\ -x &\geq 7 && \text{or} && -2x < -2 \\ x &\leq -7 && \text{or} && x > 1 \end{aligned}$$

Number line:



Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L032 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L033 in the Pupil Handbook before the next class.

<table>
<tr><td>Lesson Title: Variation</td><td colspan="2">Theme: Algebra</td></tr>
<tr><td>Lesson Number: M4-L033</td><td>Class: SSS 4</td><td>Time: 40 minutes</td></tr>
<tr><td>Learning Outcomes
By the end of the lesson, pupils will be able to:
1. Identify and differentiate between direct, indirect, joint, and partial variation.
2. Solve variation problems.</td><td colspan="2">Preparation
1. Review the content of this lesson and be prepared to explain the solutions.
2. Write the problems in Step 2 of Teaching and Learning on the board.</td></tr>
</table>

Opening *(2 minutes)*

1. Discuss:
    - What is variation? (Example answers: A change in the value or quantity of something; a change in one variable that is associated with a change in another variable.)
    - What types of variation do you know? (Example answers: direct, indirect, joint, partial)
2. Explain that this lesson is on solving problems on 4 types of variation: direct, indirect, joint, and partial.

Teaching and Learning *(23 minutes)*

1. Explain briefly:
    - **Direct variation** means that two quantities, x and y, are related such that a change in one results in a change in the other in the same ratio.
    - **Indirect variation** means that two quantities x and y are related such that an increase in one results in a decrease in the other.
    - **Joint variation** occurs when a variable varies directly or inversely with multiple variables. For example, a variable x could vary directly with y and indirectly with z.
    - **Partial variation** occurs when a variable is related to two or more other variables added together.
2. Write the following problems on the board:
    a. A car travels 330 km in 5 hours with a uniform speed. In how many hours will it travel 4,290 km?
    b. Juliet is traveling to Freetown. If she drives at the rate of 90 kph it will take her 2 hours. How long will it take her to reach Freetown if she drives at the rate of 60 kph?
    c. The cost of making a dress is partly constant and partly varies with time. The fabric has a constant cost of Le25,000.00. The tailor charges Le10,000.00 per hour of work.
        i. Find the relationship, using C for total cost and t for time.
        ii. If the dress takes 3 hours to make, what is the total cost?

3. Discuss:
   - Problem a. is direct variation, b. is indirect variation, and c. is partial variation.
   - In problem a., distance is directly proportional to time. This is because the car is traveling at a uniform speed.

   a. In problem b., speed is inversely proportional to time. If Juliet drives faster, it will take less time to reach Freetown. If she drives slower, it will take more time.
   b. In problem c., the total cost of a dress varies partly as the amount of time spent in hours.
4. Solve the problems as a class. Ask pupils to give the steps as you solve on the board. Make sure they understand each type of variation.

   Solutions:

   a. A car travels 330 km in 5 hours with a uniform speed. In how many hours will it travel 4,290 km?

   Remind pupils that because distance and time are directly proportional, we have the formula $\mathrm{d} = \mathrm{kt}$, where d and t are time.

   Find the formula:

$$d \propto t$$
$$d = kt$$
$$330 = k(5)$$
$$k = \frac{330}{5} = 66$$
$$d = 66t$$

   Solve for t when $\mathrm{d} = 4{,}290$ km:

$$4{,}290 = 66t$$
$$t = \frac{4290}{66}$$
$$t = 65$$

   b. Write the relationship between speed and time, and solve:

s	$\propto$	$\frac{1}{t}$	Relationship
s	$=$	$\frac{k}{t}$	Equation
90	$=$	$\frac{k}{2}$	Substitute known values for s and t
k	$=$	90×2	Solve for the constant, k
k	$=$	180	
s	$=$	$\frac{180}{t}$	Write the formula
60	$=$	$\frac{180}{t}$	Substitute $s = 60$
t	$=$	$\frac{180}{60}$	
t	$=$	3 hr	

c. Apply the partial variation formula $C = k_1 + kt$, where C is the total cost, t is time in hours, k_1 is the fixed constant (for material), and k is the constant associated with variation, in this case the amount charged per hour of labour.
   i. The relationship is $C = 25{,}000 + 10{,}000t$
   ii. Total cost is Le 55,000.00:

C	$=$	$25{,}000 + 10{,}000(3)$	Substitute $t = 3$
	$=$	$25{,}000 + 30{,}000$	Simplify
	$=$	$55{,}000$	

Practice *(14 minutes)*

1. Write the following problems on the board:
   a. Three pupils can brush the schoolyard in 4 hours. If it needs to be done in 2 hours, how many pupils are needed in total?
   b. z varies jointly with x and y. When $x = 3$, $y = 8$, and $z = 6$. Find z when $x = 6$ and $y = 4$.
2. Discuss:
   a. What type of variation is problem a.? (Answer: Inverse variation)
   b. What type of variation is problem b.? (Answer: Joint variation)
3. Ask pupils to work independently or with seatmates to solve the problems. Remind them to refer to the example problems in the Pupil Handbook if needed.
4. Walk around to check for understanding and clear misconceptions.
5. Invite volunteers to come to the board to write the solutions.
   Solutions:
   a. Let $p =$ pupils and $t =$ time

p	$\propto$	$\frac{1}{t}$	Identify the relationship between p and t
p	$=$	$\frac{k}{t}$	
3	$=$	$\frac{k}{4}$	Substitute the known values for p and t
k	$=$	3×4	Solve for the constant, k
k	$=$	12	
p	$=$	$\frac{12}{2}$	Write the formula
p	$=$	6 pupils	

   Answer: 6 pupils are needed to brush the yard in 2 hours.

   b. When the problem does not say whether joint variation is direct or inverse, assume both variables vary directly.

z	$\propto$	xy	Identify the relationship between z, x and y
z	$=$	kxy	
6	$=$	$k(3)(8)$	Substitute the known values for z, x and y
6	$=$	$24k$	Solve for the constant, k

k	$=$	$\frac{6}{24}$	
k	$=$	$\frac{1}{4}$	
z	$=$	$\frac{1}{4}xy$	Write the formula
z	$=$	$\frac{1}{4}(6)(4)$	Substitute $x = 6$ and $y = 4$ into the formula
z	$=$	$\frac{1}{4}(24)$	Simplify
z	$=$	6	

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L033 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L034 in the Pupil Handbook before the next class.

Lesson Title: Simplification of algebraic fractions	**Theme:** Algebra	
Lesson Number: M4-L034	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to use factorisation to simplify algebraic fractions.	**Preparation** 1. Review the content of this lesson and be prepared to explain the solutions. 2. Write the problem in Opening on the board.	

Opening *(1 minute)*

1. Write on the board: Simplify $\frac{8x^3yz}{10xy^2z}$.
2. Discuss and allow pupils to share their ideas: How would you simplify this algebraic fraction? (Example answer: Cancel factors from the numerator and denominator.)
3. Explain that this lesson is on simplifying algebraic fractions.

Teaching and Learning *(19 minutes)*

1. Explain: Algebraic fractions are fractions with variables in them. These are the steps for simplifying algebraic fractions:
   - Factor the numerator and denominator if possible.
   - Find the highest common factor (HCF) of the numerator and denominator. Remember that the HCF is simply all of the common factors multiplied together.
   - Divide (cancel) the HCF from the numerator and denominator.
2. Ask volunteers to identify the common factors of the numerator and denominator on the board. (Answer: 2, x, y, z)
3. Ask a volunteer to give the GCF. (Answer: $2xyz$)
4. Solve the problem on the board: $\frac{8x^3yz}{10xy^2z} = \frac{8x^3yz \div 2xyz}{10xy^2z \div 2xyz} = \frac{4x^2}{5y}$
5. Explain:
   - Divide both the numerator and denominator by the GCF, $2xyz$.
   - Remember that to divide a term with variables, subtract the powers. (For example, $x^3 \div x = x^{3-1} = x^2$)
6. Write on the board: Simplify $\frac{2x+xy}{2y+y^2}$.
7. Explain: This is a more complicated fraction, but we follow the same process. Factor the numerator and denominator, and divide by the GCF.
8. Solve on the board, explaining each step:
   Step 1. Factor the numerator and denominator:
   $$\frac{2x+xy}{2y+y^2} = \frac{x(2+y)}{y(2+y)}$$

Step 2. Identify that the GCF in the numerator and denominator is $(2+y)$.

Step 3. Divide (cancel) the numerator and denominator by the GCF:

$$\frac{2x+xy}{2y+y^2} = \frac{x}{y}$$

9. Write the following problem on the board: Reduce $\frac{x^2+4x+3}{x^2+3x}$ to its lowest terms.
10. Ask pupils to work with seatmates to factor both the numerator and denominator.
11. Invite volunteers to write the factorisation of each on the board. (Answers: Numerator: $x^2+4x+3=(x+1)(x+3)$; Denominator: $x^2+3x=x(x+3)$)
12. Rewrite the algebraic fraction on the board: $\frac{x^2+4x+3}{x^2+3x}=\frac{(x+1)(x+3)}{x(x+3)}$
13. Ask volunteers to give the common factors. (Answer: $x+3$).
14. Divide the numerator and denominator by $x+3$: $\frac{x^2+4x+3}{x^2+3x}=\frac{(x+1)(x+3)}{x(x+3)}=\frac{(x+1)}{x}$
15. Write on the board: Simplify each expression:
    a. $\frac{16b^4}{12a^2b}$ b. $\frac{4xy^3}{6x^2y}$ c. $\frac{x^2-4x}{x^2-3x-4}$ d. $\frac{x^2-y^2}{(x-y)^2}$ e. $\frac{x^2-xy}{xz}$
16. Ask pupils to work with seatmates to solve the problems. Walk around and make sure they understand.
17. Invite volunteers to write the solutions on the board and explain. All other pupils should check their work.

    Solutions:

    a. $\frac{16b^4}{12a^2b}=\frac{16b^4\div 4b}{12a^2b\div 4b}=\frac{4b^3}{3a^2}$

    b. $\frac{4xy^3}{6x^2y}=\frac{4xy^3\div 2xy}{6x^2y\div 2xy}=\frac{2y^2}{3x}$

    c. $\frac{x^2-4x}{x^2-3x-4}=\frac{x(x-4)}{(x-4)(x+1)}=\frac{x}{x+1}$

    d. $\frac{x^2-y^2}{(x-y)^2}=\frac{(x+y)(x-y)}{(x-y)(x-y)}=\frac{x+y}{x-y}$

    e. $\frac{x^2-xy}{xz}=\frac{x(x-y)}{xz}=\frac{x-y}{z}$

Practice *(19 minutes)*

1. Write the following problems on the board: Simplify the following:
    a. $\frac{3x^2y}{xy^2}$
    b. $\frac{15m^2n^3}{5mn^2}$
    c. $\frac{100ab^2c}{50b^2c^2d}$
    d. $\frac{u^9vw^3}{12u^2v^2w^2}$
    e. $\frac{ab-b^2}{(a-b)^2}$
    f. $\frac{5x^2}{5x-10xy}$
    g. $\frac{a^3b-a^2b^2}{a^3b+a^2b^2}$
    h. $\frac{x^2-2xy+y^2}{x^2-xy}$
2. Ask pupils to solve the problems independently.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to write the solutions on the board and explain.

Solutions:

a. $\frac{3x^2y}{xy^2} = \frac{3x^2y \div xy}{xy^2 \div xy} = \frac{3x}{y}$

b. $\frac{15m^2n^3}{5mn^2} = \frac{15m^2n^3 \div 5mn^2}{5mn^2 \div 5mn^2} = 3mn$

c. $\frac{100ab^2c}{50b^2c^2d} = \frac{100ab^2c \div 50b^2c}{50b^2c^2d \div 50b^2c} = \frac{2a}{cd}$

d. $\frac{u^9vw^3}{12u^2v^2w^2} = \frac{u^9vw^3 \div u^2vw^2}{12u^2v^2w^2 \div u^2vw^2} = \frac{u^7w}{12v}$

e. $\frac{ab-b^2}{(a-b)^2} = \frac{b(a-b)}{(a-b)(a-b)} = \frac{b}{a-b}$

f. $\frac{5x^2}{5x-10xy} = \frac{5x^2}{5x(1-2y)} = \frac{x}{1-2y}$

g. $\frac{a^3b-a^2b^2}{a^3b+a^2b^2} = \frac{a^2b(a-b)}{a^2b(a+b)} = \frac{a-b}{a+b}$

h. $\frac{x^2-2xy+y^2}{x^2-xy} = \frac{(x-y)(x-y)}{x(x-y)} = \frac{x-y}{x}$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L034 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L035 in the Pupil Handbook before the next class.

Lesson Title: Operations on algebraic fractions	**Theme:** Algebra	
Lesson Number: M4-L035	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to apply basic operations (addition, subtraction, multiplication, division) to algebraic fractions and reduce them to their lowest terms.	**Preparation** 1. Review the content of this lesson and be prepared to explain the solutions. 2. Write the problem in Opening on the board.	

Opening *(1 minute)*

1. Write on the board: Simplify $\frac{x^2+x}{x+4} \times \frac{xy+4y}{x+1}$.
2. Discuss and allow pupils to share their ideas: What steps would you take to simplify this expression?
3. Explain that this lesson is on applying operations to algebraic fractions.

Teaching and Learning *(19 minutes)*

1. Explain **multiplication** of algebraic fractions:
   - To multiply algebraic fractions, first factor the numerators and denominators.
   - Identify the common factors in the numerators and denominators.
   - Divide the numerators and denominators by the factors they have in common. In other words, cancel factors that are in both the numerator and denominator.
   - Leave the result as your answer. If there are brackets, you do not need to multiply them out.
2. Solve on the board, explaining each step:
   Step 1. Factor the numerators and denominators:
   $$\frac{x^2+x}{x+4} \times \frac{xy+4y}{x+1} \quad = \quad \frac{x(x+1)}{x+4} \times \frac{y(x+4)}{x+1}$$
   Step 2. Identify that common factors in the numerator and denominator are $(x+1)$ and $(x+4)$
   Step 3. Divide (cancel) the numerator and denominator by the factors:
   $$\frac{x\cancel{(x+1)}}{\cancel{x+4}} \times \frac{y\cancel{(x+4)}}{\cancel{x+1}}$$
   Step 4. Write the result: xy
3. Write the following problem on the board: Simplify $\frac{x^2+2x+1}{x+4} \div \frac{x+1}{5x+15}$
4. Explain **division** of algebraic fractions:
   - To divide by an algebraic fraction, multiply by the reciprocal of the second fraction. This is the same step we take when dividing fractions with numbers.

- Then, follow the steps for multiplication of algebraic fractions.

5. Solve on the board, explaining each step:

   Step 1. Multiply by the reciprocal of the second fraction:

   $$\frac{x^2+2x+1}{x+4} \times \frac{5x+20}{x+1}$$

   Step 1. Factor the numerators and denominators:

   $$\frac{x^2+2x+1}{x+3} \times \frac{5x+15}{x+1} \quad = \quad \frac{(x+1)(x+1)}{x+3} \times \frac{5(x+3)}{x+1}$$

   Step 2. Identify that common factors in the numerator and denominator are $(x+1)$ and $(x+3)$

   Step 3. Divide (cancel) the numerator and denominator by the factors:

   $$\frac{(x+1)\cancel{(x+1)}}{\cancel{x+3}} \times \frac{5\cancel{(x+3)}}{\cancel{x+1}}$$

   Step 4. Write the result: $\frac{5(x+1)}{1} = 5(x+1)$

6. Write on the board: Simplify $\frac{5}{x} + \frac{2}{3y}$.

7. Explain **addition and subtraction** of algebraic fractions:
   - Factor the algebraic fractions if possible.
   - To add or subtract algebraic fractions, the denominators should be the same. Express each fraction with a denominator that is the LCM of the denominators in the problem.
   - Once the fractions have the same denominator, they are **like fractions** and can be added or subtracted.
   - Add the fractions, and combine like terms if possible.
   - If the answer can be simplified, simplify it. This requires factoring the answer.

8. Solve on the board, explaining each step:

   Step 1. Find the LCM of x and $3y$. x and $3y$ do not have common factors, so we find the LCM by multiplying them: $3xy$.

   Step 2. Change both denominators to the LCM:

   $$\frac{5}{x} + \frac{2}{3y} = \frac{5\times3y}{x\times3y} + \frac{2\times x}{3y\times x} = \frac{15y}{3xy} + \frac{2x}{3xy}$$

   Step 3. Add the fractions by adding the numerators:

   $$\frac{15y}{3xy} + \frac{2x}{3xy} = \frac{15y+2x}{3xy}$$

9. Write on the board: Apply the operations and simplify:

   a. $\frac{ab}{a-2b} \times \frac{2a-4b}{2a}$ b. $\frac{x^2+4x-5}{x} \div \frac{x^2+2x-15}{x^2-3x}$ c. $\frac{3x+2}{2} - \frac{x+6}{3}$

10. Ask pupils to work with seatmates to solve the 3 problems. Walk around and make sure they understand.

11. Invite 3 volunteers to write their solutions on the board and explain. All other pupils should check their work.

    Solutions:

    b. $\frac{ab}{a-2b} \times \frac{2a-4b}{2a} = \frac{ab}{a-2b} \times \frac{2(a-2b)}{2a} = b$

c. $\frac{x^2+4x-5}{x} \div \frac{x^2+2x-15}{x^2-3x} = \frac{x^2+4x-5}{x} \times \frac{x^2-3x}{x^2+2x-15} = \frac{(x+5)(x-1)}{x} \times \frac{x(x-3)}{(x+5)(x-3)}$
$= \frac{x-1}{1} = x - 1$

d. $\frac{3x+2}{2} - \frac{x+6}{3} = \frac{(3x+2)\times 3}{2\times 3} - \frac{(x+6)\times 2}{3\times 2} = \frac{9x+6}{6} - \frac{2x+12}{6} = \frac{9x+6-(2x+12)}{6} =$
$\frac{9x+6-2x-12}{6} = \frac{7x-6}{6}$

Practice *(19 minutes)*

1. Write the following problems on the board: Apply the operations and simplify:
   a. $\frac{x^2+xy}{xy+y^2} \times \frac{2xy}{x}$
   b. $\frac{x^2-4}{x^2-3x+2} \times \frac{5x}{x^2+2x}$
   c. $\frac{x^2-xy}{y^2-yz} \div \frac{y^2-xy}{xy-xz}$
   d. $\frac{a^2+3a-10}{4a^2+16a} \div \frac{a^2-25}{a^2-a-20}$
   e. $\frac{x}{2x-4} + \frac{4}{x^2-4}$
2. Ask pupils to solve the problems either independently or with seatmates.
3. Walk around to check for understanding and clear misconceptions.
4. Ask volunteers to come to the board simultaneously to write the solutions.
   Solutions:
   a. $\frac{x^2+xy}{xy+y^2} \times \frac{2xy}{x} = \frac{x(x+y)}{y(x+y)} \times \frac{2xy}{x} = \frac{2x}{1} = 2x$
   b. $\frac{x^2-4}{x^2-3x+2} \times \frac{5x}{x^2+2x} = \frac{(x+2)(x-2)}{(x-2)(x-1)} \times \frac{5x}{x(x+2)} = \frac{5}{x-1}$
   c. $\frac{x^2-xy}{y^2-yz} \div \frac{y^2-xy}{xy-xz} = \frac{x^2-xy}{y^2-yz} \times \frac{xy-xz}{y^2-xy} = \frac{x(x-y)}{y(y-z)} \times \frac{x(y-z)}{y(y-x)} = \frac{x^2(x-y)}{y^2(y-x)}$
   d. $\frac{a^2+3a-10}{4a^2+16a} \div \frac{a^2-25}{a^2-a-20} = \frac{a^2+3a-10}{4a^2+16a} \times \frac{a^2-a-20}{a^2-25} = \frac{(a+5)(a-2)}{4a(a+4)} \times \frac{(a-5)(a+4)}{(a+5)(a-5)} = \frac{a+2}{4a}$
   e. $\frac{x}{2x-4} + \frac{4}{x^2-4} = \frac{x}{2(x-2)} + \frac{4}{(x-2)(x+2)} = \frac{x(x+2)}{2(x-2)(x+2)} + \frac{2(4)}{2(x-2)(x+2)} = \frac{x(x+2)+8}{2(x-2)(x+2)} =$
   $\frac{x^2+2x+8}{2(x-2)(x+2)}$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L035 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L036 in the Pupil Handbook before the next class.

Lesson Title: Logical reasoning – Part 1	**Theme:** Logical Reasoning	
Lesson Number: M4-L036	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcomes By the end of the lesson, pupils will be able to: 1. Distinguish between simple and compound statements. 2. Identify the negation of a statement. 3. Draw conclusions from a given implication. 4. Distinguish between conjunction and disjunction, and represent them on truth tables. 5. Recognise equivalent statements and apply them to arguments.	**Preparation** Review the content of this lesson and be prepared to explain the solutions.	

Opening *(1 minute)*

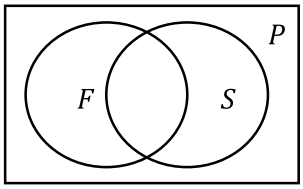
1. Explain that this is the first of 2 lessons on logical reasoning.

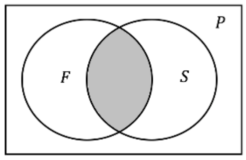
Teaching and Learning *(16 minutes)*

1. Provide a brief explanation of each concept, as given below.
2. Explain **simple** and **compound statements:**
    - **Compound statements** are made from a simple statement and connecting words. Common connecting words are: and, but, or, if and only if, if…then.
    - Consider the compound statement "Sami is a cat and he likes to eat fish". It has two parts: "Sami is a cat" and "he likes to eat fish", which are connected by the word "and".
3. Explain **negation:**
    - The **negation** of a statement has an opposite meaning to the original statement. The negation of a statement is formed by adding the word "not".
    - The negation of a statement p is written as $\sim p$. For example, if p: Francis is a tailor, then $\sim p$: Francis is **not** a tailor.
4. Write the negation on the board with symbols: $\sim p$
5. Explain implication:
    - **Implications** are compound statements that can be written with the connecting words "if…then". The first statement implies that the second is true.
    - Consider the implication, "If Hawa lives in Freetown, then she lives in Sierra Leone." There are 2 simple statements in this implication: A: Hawa lives in Freetown, and B: She lives in Sierra Leone.
6. Write the implication on the board with symbols: $A \Rightarrow B$
7. Explain **conjunction** and **disjunction:**

- A **conjunction** is a compound statement that uses "and". A **disjunction** is a compound statement that uses "or".

8. Demonstrate **conjunction** using a **Venn diagram**:
    - Draw the diagram on the board without shading, and explain that P: People who live in Freetown, F: Females, and S: Students.
    - Shade the middle and explain that this represents the conjunction of F and S, people who are both female and students.
    - Write on the board with symbols: $A_1 = F \cap S$ or $A_1 = F \wedge S$

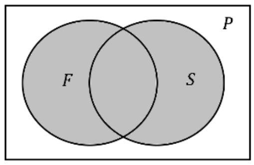




9. Demonstrate **conjunction** using **a truth table:**
    - Draw the truth table on the board:
    - Explain that the truth table says that statement A_1 is only true if both of the sub-statements are true ("people who are female" and "people who are students"). If either of the sub-statements is false, then statement A_1 is also false.

F	S	$F \wedge S$
T	T	T
T	F	F
F	T	F
F	F	F

10. Demonstrate **disjunction** using a **Venn diagram**:
    - Using the same diagram as above, shade all segments in the circles representing F and S.
    - Explain that the two statements "people who are female" and "people who are students" are linked by the words **"either – or – or both"**. In the diagram, this is all of the space inside of the two circles, including where they intersect.
    - Write on the board with symbols: $A_2 = F \cup S$ or $A_2 = F \vee S$



F	S	$F \vee S$
T	T	T
T	F	T
F	T	T
F	F	F

11. Demonstrate **disjunction** using a **truth table:**
    - Draw the truth table on the board:
    - Explain that this truth table says that A_2 is true if either or both of the sub-statements are true. A_2 is false only if both sub-statements are false.
12. Explain **equivalence:**
    - If $X \Rightarrow Y$ and $Y \Rightarrow X$, then X and Y are equivalent and we write $X \Leftrightarrow Y$. The implication $Y \Rightarrow X$ is the converse of $X \Rightarrow Y$.
    - Give an example on the board: consider $x^2 = 16 \Rightarrow x = \pm 4$. Its converse is $x = \pm 4 \Rightarrow x^2 = 16$ is true, and we can also write $x^2 = 16 \Leftrightarrow x = \pm 4$.

Practice *(22 minutes)*

1. Write the following problems on the board:
    a. Write the negation of each sentence:

        r: There are forests in Sierra Leone.

s: x is a number between -5 and 7.

b. Consider 2 statements: B: Fatu lives in Bo. And S: Fatu lives in Sierra Leone. Use these statements to write an implication using words and symbols.

c. Bentu has a farm with many types of animals in many different colours. Consider the following statements:

G: Some animals are goats.

B: Some animals are brown.

Based on this information, prepare truth tables that describe:

i. An animal on the farm that is a brown goat.

ii. An animal on the farm that is either brown, or a goat, or both.

d. Consider the following statements about some people living in a village:

T: Some people are teachers.

F: Some people have farms.

Based on this information, prepare truth tables that describe:

i. A person in the village who is either a teacher, or has a farm, or both.

ii. A person in the village who is both a teacher and has a farm.

2. Ask pupils to solve the problems either independently or with seatmates.
3. Walk around to check for understanding and clear misconceptions.
4. Ask volunteers to come to the board to write the solutions at the same time.

Solutions:

a. $\sim r$: There are **not** forests in Sierra Leone.; $\sim s$: x is **not** a number between -5 and 7.

b. Words: If Fatu lives in Bo, then she lives in Sierra Leone.

Symbols: $B \Rightarrow S$

c. Truth tables:

i.

B	G	$B \wedge G$
T	T	T
T	F	F
F	T	F
F	F	F

ii.

B	G	$B \vee G$
T	T	T
T	F	T
F	T	T
F	F	F

d. Truth tables:

i.

T	F	$T \vee F$
T	T	T
T	F	T
F	T	T
F	F	F

ii.

T	F	$T \wedge F$
T	T	T
T	F	F
F	T	F
F	F	F

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L036 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L037 in the Pupil Handbook before the next class.

Lesson Title: Logical reasoning – Part 2	**Theme:** Logical Reasoning	
Lesson Number: M4-L037	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcomes By the end of the lesson, pupils will be able to: 1. Apply the chain rule. 2. Use Venn diagrams to demonstrate connections between statements. 3. Determine the validity of statements.	**Preparation** Review the content of this lesson and be prepared to explain the solutions.	

Opening *(1 minute)*

1. Explain that this is the second of 2 lessons on logical reasoning.

Teaching and Learning *(24 minutes)*

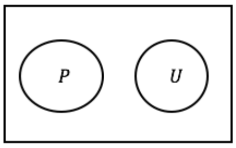
1. Provide a brief explanation of each concept, as given below.
2. Explain the **chain rule:**
   - If X, Y and Z are 3 statements such that $X \Rightarrow Y$ and $Y \Rightarrow Z$, then $X \Rightarrow Z$
   - Recall that if $X \Rightarrow Y$, then $\sim Y \Rightarrow \sim X$ is an equivalent statement. This applies to statements under the chain rule as well.
3. Write an example on the board and explain:
   - Consider the statements, where $X \Rightarrow Y$ and $Y \Rightarrow Z$:
     X: Hawa studies hard.
     Y: Hawa passes exams.
     Z: Hawa graduates secondary school.
   - If $X \Rightarrow Y$ and $Y \Rightarrow Z$, then $X \Rightarrow Z$. In other words, these statements are true:
     $X \Rightarrow Y$: If Hawa studies hard, then she passes exams.
     $Y \Rightarrow Z$: If Hawa passes exams, then she graduates secondary school.
     $X \Rightarrow Z$: If Hawa studies hard, then she graduates secondary school.
4. Write on the board:
   A: The school team practises football.
   B: The team wins the match.
   C: The team qualifies for the championship.
   $A \Rightarrow B$ and $B \Rightarrow C$. Write as many statements as possible with this information.
5. Ask pupils to work with seatmates to write statements in symbols and words. Encourage them to use equivalence and the chain rule.
6. Invite volunteers to write their statements on the board. Allow all other pupils to check if the statements are valid and correct them.
   Possible answers:

$A \Rightarrow B$	If the school team practises football, then the team wins the match.
$B \Rightarrow C$	If the team wins the match, then the team qualifies for the championship.

$A \Rightarrow C$	If the school team practises football, then the team qualifies for the championship.
$\sim B \Rightarrow \sim A$	If the team does not win a match, then the team does not practise football.
$\sim C \Rightarrow \sim B$	If the team does not qualify for the championship, then the team does not win the match.
$\sim C \Rightarrow \sim A$	If the team does not qualify for the championship, then the team does not practise football.

7. Write on the board and briefly revise representing the following relationships between sets with Venn diagrams:

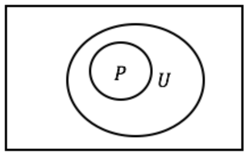
X:



$P \cap U = \emptyset$

P and U are disjoint.

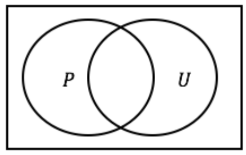
Y:



$P \subset U$

P is a subset of U.

Z:



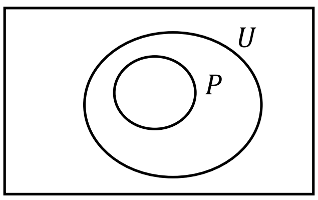
$P \cap U$

P intersects with U.

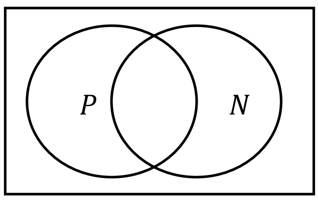
8. Explain using **Venn diagrams:**
   - Venn diagrams can also represent statements.
   - The Venn diagrams on the board show the following statements, where P ={police officers} and U ={people who wear uniforms}:
     - X: No police officers wear uniforms.
     - Y: All police officers wear uniforms.
     - Z: Some police officers wear uniforms.
9. Write the following problem on the board: Draw a Venn diagram for each of the statements:
   a. X: All senior secondary pupils wear a uniform.
   b. Y: Some senior secondary pupils live near the school.
   c. Z: No senior secondary pupils are rich.
10. Ask pupils to work with seatmates to solve the problem.
11. Invite volunteers to draw the Venn diagrams on the board**.**

Answers:

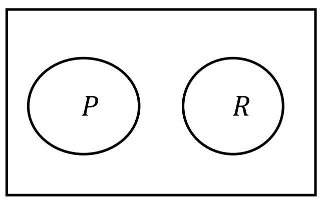
a.



b.



c.



12. Explain **validity:**
    - Venn diagrams and the chain rule are used to determine whether statements are **valid.**
    - Note that the actual **truth does not matter** when determining whether a statement is valid.

- For example, consider two statements, A: Monrovia is in Sierra Leone, and B: Sierra Leone is in Asia. Although these statements are false, a valid conclusion can be drawn. Based on A and B and using the chain rule, the following is valid: C: Monrovia is in Asia.

Practice *(14 minutes)*

1. Write the following problem on the board: Draw one Venn diagram to illustrate both of the following statements:

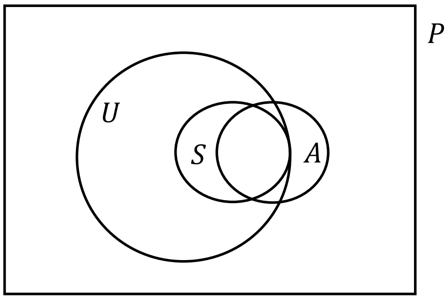
   X: All senior secondary pupils wear uniforms.
   Y: Most senior secondary pupils have high attendance.

   Use your diagram to determine which of the following implications are valid based on X and Y:

   a. Fatu wears a uniform ⇒ Fatu is a senior secondary pupil.
   b. Michael is a senior secondary pupil ⇒ He has high attendance.
   c. Hawa does not wear a uniform ⇒ She is not a senior secondary pupil.

   Ask pupils to solve the problems either independently or with seatmates.
2. Walk around to check for understanding and clear misconceptions.
3. Invite volunteers to come to the board simultaneously to write the solutions.

   Solutions:



   Venn diagram, where $P =$ {all pupils}, $U =${pupils who were uniforms}, $S =${senior secondary pupils}, $A =${pupils with high attendance}**:**

   a. Not valid. All senior secondary pupils wear uniforms. However, all pupils who wear uniforms are not necessarily senior secondary pupils. Fatu may be in U, but outside of S.

   b. Not valid. Although most senior secondary pupils have high attendance, not all of them do. Michael might be in set S, but outside of set A.

   c. Valid. If Hawa does not wear a uniform, she is outside of set U. She must also be outside of set S, which is a subset of U.

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L037 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L038 in the Pupil Handbook before the next class.

Lesson Title: Pie charts and bar charts	**Theme:** Probability and Statistics	
Lesson Number: M4-L038	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcomes By the end of the lesson, pupils will be able to draw and interpret pie charts and bar charts.	**Preparation** Bring a protractor to class. It is best to make a large one on paper, which can be used on the board.	

Opening *(2 minutes)*

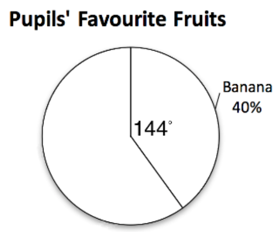
1. Discuss, and allow pupils to explain in their own words:
   a. What is a pie chart? What is it used for? (Example answer: A chart that shows segments of a circle; it is used to represent parts of a whole.)
   b. What is a bar chart? What is it used for? (Example answer: A chart with x- and y-axes, with bars of different heights; it is used to compare quantities.)
2. Explain that this lesson is on drawing and interpreting pie charts and bar charts.

Teaching and Learning *(20 minutes)*

1. Draw the table on the board: The table below shows the favourite fruits of 40 pupils in a class. Use this information to draw a pie chart.

Favourite Fruit	Frequency	Percentage
Banana	16	40%
Mango	10	25%
Orange	6	15%
Pineapple	8	20%
TOTAL	40	100%

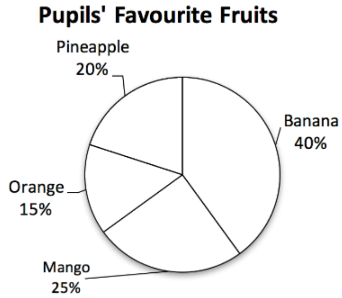
2. Explain:
   - To draw a pie chart accurately, we must use a protractor.
   - Each fruit type is one part of the whole. We must find its measure using the fact that there are 360 degrees in 1 full rotation.
   - To find the degree measure of each segment, write the frequency for each fruit as a fraction of the whole. Multiply this fraction by 360°.
3. Calculate the degree measure for banana on the board:
   $\text{Banana} = \frac{16}{40} \times 360° = 144°$
4. Ask pupils to work with seatmates to calculate the degrees for the other fruits.
5. Invite volunteers to write the solutions on the board. (Answers: Mango $= \frac{10}{40} \times 360° = 90°$; Orange $= \frac{6}{40} \times 360 = 54°$; Pineapple $= \frac{8}{40} \times 360° = 72°$)



6. Draw the pie chart with the segment for "Banana":
   - Place the centre of the protractor on the centre of the pie chart and place the bottom of the protractor exactly along one radius of the circle.

- Find the angle measurement for banana, 144°.
- Use a straight edge to draw another radius at 144°.

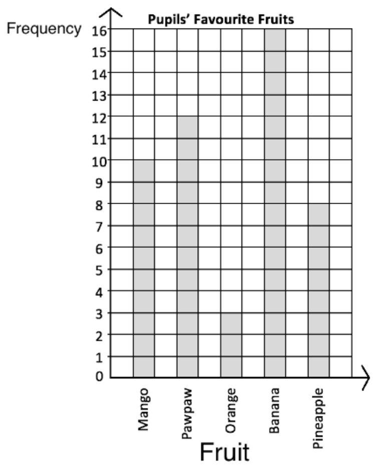
7. Invite volunteers to come to the board to draw the other segments. Solution →



8. Discuss:
   - Which fruit is loved by the most pupils? (Answer: Banana)
   - What percentage of pupils prefer either orange or mango? (Answer: Add the percentages: $15\% + 25\% = 40\%.$)
9. Write the following problem on the board: Draw a bar chart for the data:

Favourite Fruits	
Mango	10
Pawpaw	12
Orange	3
Banana	16
Pineapple	8

10. Explain: Bar charts are used to compare quantities.
11. Draw and label the axes on the board.
12. Draw the bar for "mango". Shade it in with chalk.
13. Invite volunteers to come to the board and draw the bars for the other 4 fruits.
14. Discuss:
    - What is the least favourite fruit among the class? (Answer: orange)
    - How many more pupils chose pawpaw than mango? (Answer: Subtract: $12 - 10 = 2$ pupils)

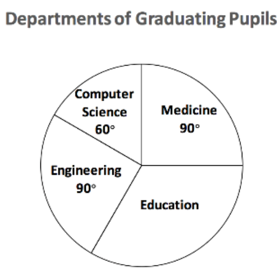


Practice *(17 minutes)*

1. Write the following problems on the board:
   a. A survey was conducted of 1,000 people in Sierra Leone to find out their sources of news. Draw a pie chart for the information in the table. →

Source	Percentage
Radio	35%
Mobile Phone	50%
Television	5%
Other	10%

   b. A total of 1,000 pupils graduated from a certain university. The pie chart at right shows the departments they graduated from. Use it to answer the questions.
      i. How many pupils graduated from the education department?



ii. What percentage of the total graduated from the medicine department?

c. The table below shows the distribution of marks on a test given to a class of pupils. Draw a bar chart and use it to answer the questions below.

Marks	10 %	20 %	30 %	40 %	50 %	60 %	70 %	80 %	90 %	100 %
Frequency	2	1	0	0	5	8	9	3	6	7

i. How many pupils took the test?

ii. If 60% is passing, how many pupils passed the test?

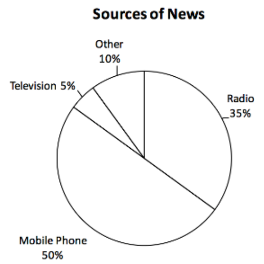
iii. What percentage of pupils passed the test?

iv. How many pupils scored 60% or 70%?

2. Ask pupils to solve the problems either independently or with seatmates.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to come to the board at the same time to write the solutions.

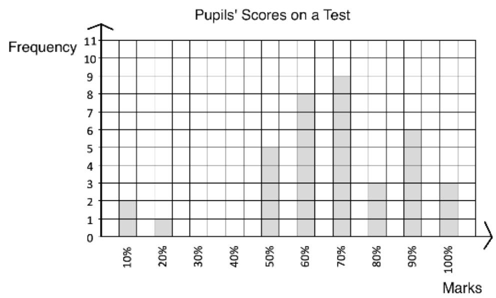
Solutions:

a. Angle calculations: Radio = $\frac{35}{100} \times 360° = 126°$; Mobile Phone = $\frac{50}{100} \times 360° = 180°$; Television = $\frac{5}{100} \times 360° = 18°$; Other = $\frac{10}{100} \times 360° = 36°$.



b. i. Find the degree for Education: $360° - (90° + 60° + 90°) = 120°$
Multiply ratio by the total number of pupils to find those studying Education:
Number in Education= $\frac{120}{360} \times 1{,}000 = \frac{1}{3} \times 1{,}000 = 333.3$
Answer: 333 pupils
ii. Write medicine as a percentage of the whole using its degree measure:
Pupils graduating from medicine = $\frac{90}{360} \times 100\% = 25\%$

c. Bar chart →



i. Add the frequencies: $2 + 1 + 5 + 8 + 9 + 3 + 6 + 7 = 41$

ii. Add frequencies that scored between $60\% - 100\%$: $8 + 9 + 3 + 6 + 7 = 33$ pupils.

iii. Find the answer from part b. as a percentage of the answer from part a.: $\frac{33}{41} \times 100\% = 80.5\%$.

iv. Add the frequencies scoring 60% and 70%: $8 + 9 = 17$ pupils.

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L038 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L039 in the Pupil Handbook before the next class.

Lesson Title: Mean, median, and mode of ungrouped data	**Theme:** Probability and Statistics	
Lesson Number: M4-L039	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to calculate the mean, median, and mode of ungrouped data from lists, tables, and charts.	**Preparation** 1. Review the content of this lesson and be prepared to explain the solutions. 2. Write the problem in Opening on the board.	

Opening *(2 minutes)*

1. Write the problem in the board: 10 pupils received the following scores on their Maths exam: 87, 100, 76, 92, 90, 95, 85, 67, 99 and 95. Calculate the mean, median, and mode of the scores.
2. Discuss:
    a. What is mean? (Example answer: It is the average of a set of numbers.)
    b. What is median? (Example answer: It is the number that falls in the middle when you list a set of numbers in ascending or descending order.)
    c. What is mode? (Example answer: It is the number that appears the greatest number of times in a set of data.)
3. Explain that this lesson is on calculating mean, median, and mode of ungrouped data. Ungrouped data can be listed individually, in a frequency table, or in a bar chart.

Teaching and Learning *(17 minutes)*

1. Ask pupils to work with seatmates to solve the problem on the board.
2. Walk around to check for understanding and clear misconceptions.
3. Invite volunteers to write the solutions on the board and explain.

   Solutions:

   Mean:

   Add the numbers: $87 + 100 + 76 + 92 + 90 + 95 + 85 + 67 + 99 + 95 = 886$

   Divide by the number of pupils: mean $= 886 \div 10 = 88.6$

   Median:

   List the numbers in ascending order: 67, 76, 85, 87, 90, 92, 95, 95, 99, 100

   Identify the middle of the list: 90, 92

   Add the 2 numbers in the middle and divide by 2: median $= \frac{90+92}{2} = 91$

   Mode: Note that 95 is the only number that occurs more than once, so it must be the mode.
4. Write the following problem on the board: The table below shows the distribution of marks on an assignment that a class completed. No one scored below 6 marks. Find the mean, median, and mode of the scores.

Marks	6	7	8	9	10
Frequency	3	9	4	3	1

5. Discuss: How can we find the **mean**? (Answer: Find the sum of the scores of all pupils in the class by multiplying marks by frequencies, then adding them. Then, divide by the number of pupils.)
6. Write on the board: mean= $\frac{\sum fx}{\sum f}$ where x represents each value in the data set, and f represents the corresponding frequency of each value.
7. Explain:
   a. $\sum fx$ can be found by multiplying each frequency by the corresponding value and adding them.
   b. $\sum f$ is the total frequency, or the total number of values in the data set. In this case, $\sum f = 20$.
8. Discuss: How can we find the **median**? (Answer: The median is the mean score of the 2 pupils in the middle, which are the 10^{th} and 11^{th} pupils. Find the scores of the 10^{th} and 11^{th} pupils by counting in the frequency table.)
9. Write on the board: median position: $\frac{(n+1)}{2}$ where n is the total frequency.
10. Explain: This formula gives the position of the median. If you have many values in the data set, this formula is useful to find its middle.
11. Discuss: How can we find the **mode**? (Answer: The mode is the mark with the highest frequency.)
12. Ask pupils to work with seatmates to solve the problem.
13. Invite volunteers to write the solutions on the board and explain.

Solutions:

Mean: $\frac{\sum fx}{\sum f} = \frac{6(3)+7(9)+8(4)+9(3)+10(2)}{20} = \frac{18+63+32+27+10}{20} = \frac{150}{20} = 7.5$ marks

Median:

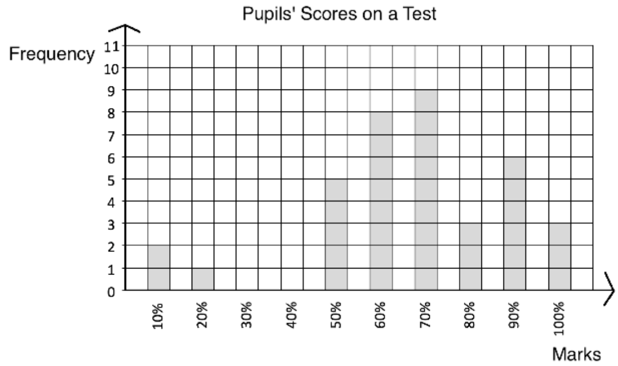
Find the scores of the 10^{th} and 11^{th} pupils by counting in the frequency table. They are both 7, so their mean is 7.

Median = 7 marks

Mode: The mode is 7 marks, because it has the greatest frequency (9).

Practice *(20 minutes)*

1. Write the following problems on the board:
   a. On her exams, Fatu scored x% in Mathematics, 90% in English, 95% in Biology, and 80% in Chemistry. If her mean score for all subjects was 88%, what is the value of x?
   b. The bar chart below shows marks that pupils achieved on a test, as percentages. Find the mean, median, and mode.



c. 10 pupils ran 1 kilometre in a race. Their finishing times are in the table below, to the nearest 30 seconds. Find the mean, median, and mode of their times.

Time (minutes)	4.0	4.5	5.0	5.5	6.0
Frequency	1	2	2	4	1

2. Ask pupils to solve the problems either independently or with seatmates.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to come to the board simultaneously to write the solutions.

Solutions:

a.

88	$=$	$\frac{x+90+95+80}{4}$	Set up the equation
88	$=$	$\frac{x+265}{4}$	Simplify
4×88	$=$	$x + 265$	Multiply throughout by 4
352	$=$	$x + 265$	
$352 - 265$	$=$	x	Subtract 265 from both sides
87	$=$	x	

b. **Mean:** $\frac{\sum fx}{\sum f} = \frac{2(10)+20+5(50)+8(60)+9(70)+3(80)+6(90)+3(100)}{2+1+5+8+9+3+6+3} = \frac{2480}{37} = 67.02\%$

Median: Find the median position: $\frac{(n+1)}{2} = \frac{37+1}{2} = 19$. Locate the 19th pupil in the bar chart by counting the bars, from least to greatest. The 19th pupil is within the 70% bar. Therefore, the median = 70%

Mode: The highest bar is at 70%; therefore, the mode is 70%.

c. **Mean:** $\frac{\sum fx}{\sum f} = \frac{4.0+2(4.5)+2(5.0)+4(5.5)+6.0}{10} = \frac{51}{10} = 5.1$ minutes

Median: Out of 10 pupils, the 5th and 6th pupils are in the middle. Count up to 5 and 6 in the table. They have times of 5.0 and 5.5, respectively. Median = $\frac{5.0+5.5}{2} = 5.25$ minutes.

Mode: 5.5 minutes, because it has the greatest frequency.

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L039 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L040 in the Pupil Handbook before the next class.

Lesson Title: Histograms		**Theme:** Probability and Statistics
Lesson Number: PHM4-L040	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcomes By the end of the lesson, pupils will be able to: 1. Create a frequency distribution table and use it to draw a histogram. 2. Interpret histograms.		**Preparation** Review the content of this lesson and be prepared to explain the solutions.

Opening *(1 minute)*

1. Discuss:
    a. What does a histogram look like? (Example answer: It looks like a bar chart with bars touching.)
    b. What is a histogram used for? (Example answers: To compare grouped data; to display data in class intervals.)
2. Explain that this lesson is on histograms, which are similar to bar charts, but are used for grouped data.

Teaching and Learning *(16 minutes)*

1. Write the following problem on the board: The ages of 25 community members are given: 25, 31, 45, 42, 36, 28, 43, 49, 52, 28, 24, 40, 44, 36, 48, 52, 41, 54, 32, 38, 39, 41, 54, 50, 28.
    a. Create a frequency distribution table using intervals 21-25, 26-30, 31-35, and so on.
    b. Draw a histogram of the distribution.
2. Explain how to create a **frequency table:**
    - The list given in the problem is ungrouped data. We want to create a frequency table using intervals. This means we will divide it into groups (called class intervals) to create grouped data.
    - A class interval is a group of data with a certain range. Class intervals of a set of grouped data should all have the same range.
    - To create a frequency table, the first step is to write the list of numbers in ascending order. Then count the numbers that fall into each class interval. Each class interval is assigned one row of the frequency table.
3. Ask volunteers to write the list in ascending order on the board. (Answer: 24, 25, 28, 28, 28, 31, 32, 36, 36, 38, 39, 40, 41, 41, 42, 43, 44, 45, 48, 49, 50, 52, 52, 54, 54.)
4. Create the frequency table as a class. Draw the table on the board and ask volunteers to fill the rows. Answer →

Marks scored	**Frequency (f)**
21-25	2
26-30	3
31-35	2
36-40	5
41-45	6
46-50	3
51-55	4

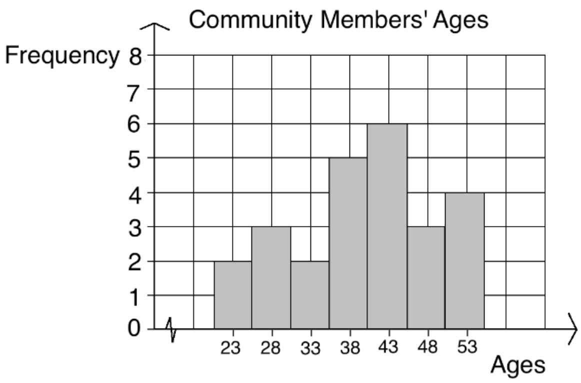
5. Explain **histograms**:

- Like a bar chart, a histogram consists of vertical bars. However, in histograms, the bar does not represent only 1 piece of data, but a range of data. Each bar represents a class interval.
- We will draw each bar centred on a **class mid-point** on the x-axis. Class mid-points are the points that lie exactly in the middle of class intervals.
- The bars of a histogram touch each other, unlike bar charts. Histogram bars touch each other because they represent continuous intervals.

6. Create the histogram as a class. First, add a column to the frequency table for class mid-points. Have volunteers give the mid-points and draw the histogram bars:

Solution:

Ages	**Frequency (f)**	**Class mid-points**
21-25	2	23
26-30	3	28
31-35	2	33
36-40	5	38
41-45	6	43
46-50	3	48
51-55	4	53



7. Discuss:
   - Which class do the greatest number of people fall into? (Answer: Age 41-45)
   - How many people are age 41 or older? (Answer: Add the heights of the last 3 bars: $6 + 3 + 4 = 13$ people)
   - How many people are between 26 and 35 years old? (Answer: $3 + 2 = 5$ people)

Practice *(22 minutes)*

1. Write the following problems on the board:
   a. The following are marks scored by 25 pupils in an examination: 87, 62, 68, 74, 92, 90, 83, 73, 58, 46, 92, 93, 81, 88, 90, 45, 61, 54, 72, 79, 57, 98, 47, 93, 38.
      i. Draw a frequency table using class intervals 31-40, 41-50, 51-60, etc.
      ii. Draw a histogram for the frequency table.
      iii. Which interval do the greatest number of pupils fall into?
      iv. If 71% or higher is passing, how many pupils passed?
      v. How many pupils scored 50% or lower?
   b. A chimpanzee reserve houses 39 chimpanzees that have been rescued from hunters and people who kept them as pets. The weights of the chimpanzees are displayed in the frequency table:

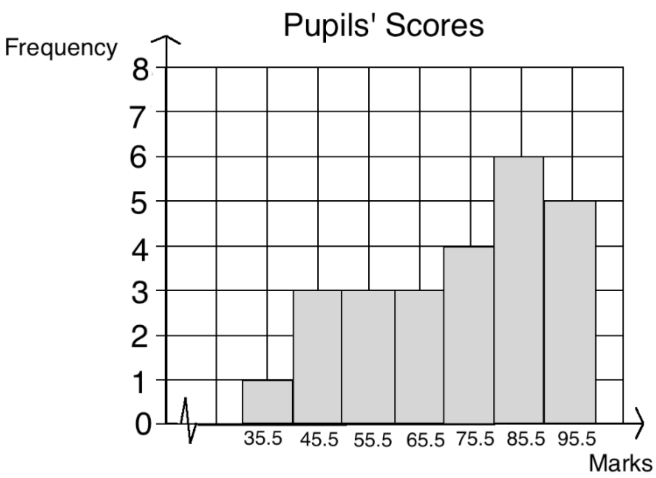
Weight (kg)	10-19	20-29	30-39	40-49	50-59	60-69
Frequency	6	7	12	5	7	2

i. Draw a histogram for the data.
ii. All of the child chimpanzees weigh under 30 kg, and the adults weigh 30 kg or more. How many children and adults are there?
iii. Chimpanzees that weigh 50 kg or more are put on a special diet. How many receive this diet?

2. Ask pupils to solve the problems either independently or with seatmates.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to come to the board at the same time to write the solutions.
Solutions:
a.
i. Write in ascending order: 38, 45, 46, 47, 54, 57, 58, 61, 62, 68, 72, 73, 74, 79, 81, 83, 87, 88, 90, 90, 92, 92, 93, 93, 98.
Frequency table:

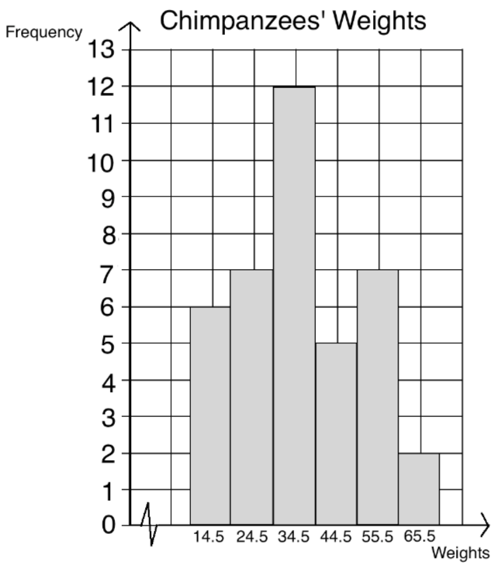
Marks	31-40	41-50	51-60	61-70	71-80	81-90	91-100
Frequency	1	3	3	3	4	6	5

ii. First, find the class mid-points (35.5, 45.5, 55.5, …)
Draw the histogram. →



iii. The greatest number of pupils fall into the interval 81-90 (the tallest bar).
iv. Add the heights of the last 3 bars: $4 + 6 + 5 = 15$ pupils passed.
v. Add the heights of the first 2 bars: $1 + 3 = 4$ pupils scored 50% or lower.

b.
i. First, find the class mid-points (14.5, 24.5, 34.5, …)
Draw the histogram. →



ii. Children: $6 + 7 = 13$
Adults: $12 + 5 + 7 + 2 = 26$
iii. $7 + 2 = 9$ chimpanzees

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-T1-W10-L040 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L041 in the Pupil Handbook before the next class.

Lesson Title: Frequency polygons	Theme: Statistics	
Lesson Number: PHM4-L041	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to present and interpret grouped data in frequency polygons.	**Preparation** 1. Review the content of this lesson and be prepared to explain the solutions. 2. Write the problem in Opening on the board.	

Opening *(3 minutes)*

1. Write on the board: The table below shows 32 pupils' marks on a Maths test.

Marks	51-60	61-70	71-80	81-90	91-100
Frequency	5	7	9	8	3

2. Discuss:
   - What are the class intervals of this data? (Answer: 51-60, 61-70, …)
   - What is the mid-point of the first class interval? (Answer: 55.5)
   - Which class interval do the greatest number of pupils fall into? (Answer: 71-80)
   - How many pupils scored more than 80 marks? (Answer: $8 + 3 = 11$ pupils)
3. Explain that this lesson is on frequency polygons, another way to present grouped data.

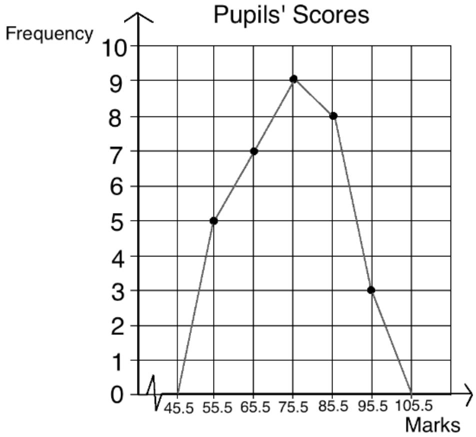
Teaching and Learning *(16 minutes)*

1. Explain:
   - We can create a frequency polygon for the table on the board.
   - Frequency polygons are similar to line graphs, in the same way that histograms are similar to bar charts.
   - Frequency polygons are used to display grouped data, which means that we plot class intervals.
   - We must find each class mid-point on the x-axis, and plot the frequency for the corresponding class interval.
   - Recall that for histograms we can use either the class mid-points or class boundaries to draw the bars. For frequency polygons, we must use the mid-points.
2. Draw a third row on the frequency table and ask volunteers to fill it with the class mid-points:

Marks	51-60	61-70	71-80	81-90	91-100
Frequency	5	7	9	8	3
Class Mid-point	55.5	65.5	75.5	85.5	95.5

3. Draw and label the axes on the board (see frequency polygon below).
4. Plot the first point, using 55.5 on the x-axis, and 5 on the y-axis.
5. Invite volunteers to plot the other points in the table.

6. Connect the points as shown:



7. Explain: Normally we extend the line of the frequency polygon to the mid-point of what would be the next interval, if that interval existed in the data.
8. Explain **modal class** and **median class**:
   - The modal class is the class with the highest frequency, where the mode is likely to occur.
   - The median class is the class where the median occurs, which in this case is the pupil with the median score on the exam.
9. Discuss:
   - What is the modal class in this example? (Answer: 71-80)
   - What is the median class? (Answer: The median class contains the average mark of the 16^{th} and 17^{th} pupils, which is class interval 71-80.)

Practice *(20 minutes)*

1. Write the following problems on the board:
   a. The heights of 20 pupils in centimetres are: 179, 180, 161, 163, 170, 182, 168, 172, 175, 164, 168, 157, 158, 169, 159, 178, 164, 175, 167, 183.
      i. Draw a frequency table using class intervals 156-160, 161-165, 166-170, 171-175, 176-180, 181-185.
      ii. Draw a frequency polygon to display the data.
      iii. What is the modal class?
      iv. What is the median class?
      v. How many pupils are 170 cm or shorter?
      vi. What percentage of the pupils are taller than 175 cm?
   b. A school wants to buy shoes for its football team. They measured the shoe sizes of 25 football players and displayed them in the table below.

Shoe Size	32-34	35-37	38-40	41-43	44-46
Frequency	3	5	7	8	2

      i. Draw a frequency polygon to display the data.
      ii. What is the modal class?

iii. What is the median class?
iv. The market only has football shoes in size 41 and larger this week. How many football players have to wait to receive their shoes?

2. Ask pupils to solve the problems either independently or with seatmates.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to come to the board simultaneously to write the solutions.

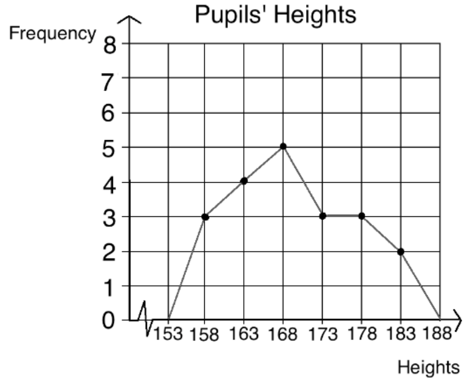
Solutions:

a. Write the numbers in ascending order before counting and grouping them: 157, 158, 159, 161, 163, 164, 164, 167, 168, 168, 169, 170, 172, 175, 175, 178, 179, 180, 182, 183.

i. Table:

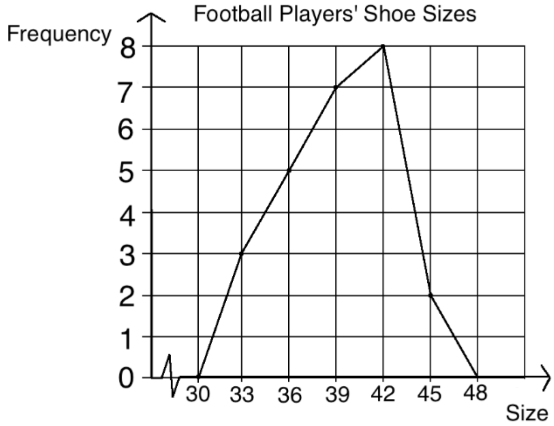
Pupils' Heights	
Heights	**Frequency**
156-160	3
161-165	4
166-170	5
171-175	3
176-180	3
181-185	2

ii. Frequency polygon:



ii. The modal class is 166-170.
iii. The median class is where the 10th and 11th pupils fall, which is 166-170.
iv. Add the frequencies of the first 3 class intervals: $3+4+5=12$ pupils.
v. Find the number of pupils taller than 175 cm as a percentage of 20. Pupils taller than 175: $3+3=5$. As a percentage of 20: $\frac{5}{20}\times 100\% = 25\%$ of pupils.

b. i. Frequency polygon →



ii. The modal class is 41-43.
iii. The median class is where the 13th pupil falls, which is 38-40.
iv. Add the frequencies of classes less than 41: $3+5+7=15$ pupils.

Closing *(1 minute)*

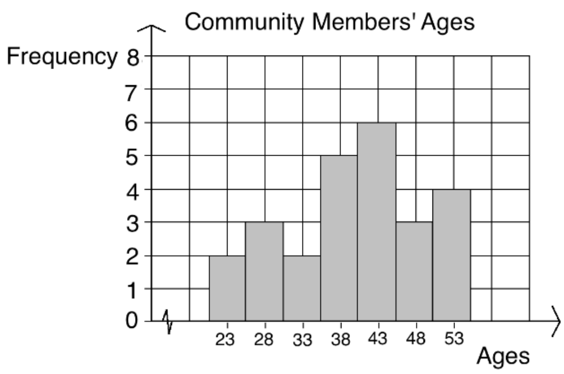
1. For homework, have pupils do the practice activity of PHM4-L041 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L042 in the Pupil Handbook before the next class.

Lesson Title: Mean, median and mode of grouped data	**Theme:** Statistics	
Lesson Number: PHM4-L042	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcome By the end of the lesson, you will be able to estimate the mean, median and mode of grouped data and apply them to problem solving.	**Preparation** 1. Review the content of this lesson and be prepared to explain the solutions. 2. Write the frequency table and histogram in Opening on the board.	

Opening *(3 minutes)*

1. Write the frequency table and histogram from lesson 41 on the board: The ages of 25 community members are represented in the table and chart:

Ages	Frequency (f)	Class mid-points
21-25	2	23
26-30	3	28
31-35	2	33
36-40	5	38
41-45	6	43
46-50	3	48
51-55	4	53



2. Discuss and allow pupils to share their ideas: Can you find the mean, median, or mode of this data? How?
3. Explain that today we will estimate the mean, median, and mode of grouped data.

Teaching and Learning *(18 minutes)*

1. Explain:
   - When data is divided into groups, we cannot determine the value of each piece of data in the set. Therefore, we cannot determine the exact mean, median, or mode. We can **estimate** these values.
   - The mean and median are estimated using formulae. The mode is calculated using a histogram.
2. Write on the board:
   - **Estimated mean** is $\bar{x} = \frac{\sum fx}{\sum f}$, where f is frequency, and x is the corresponding class mid-point.
   - **Estimated median** is $L + \left[\frac{\frac{n}{2} - (\sum f)_L}{f_m}\right] \times c$**,** where L is the lower class boundary of the group containing the median, n is the total frequency of the data, $(\sum f)_L$ is the total frequency for the groups **before** the median group, f_m is the frequency of the median group, and c is the group width.

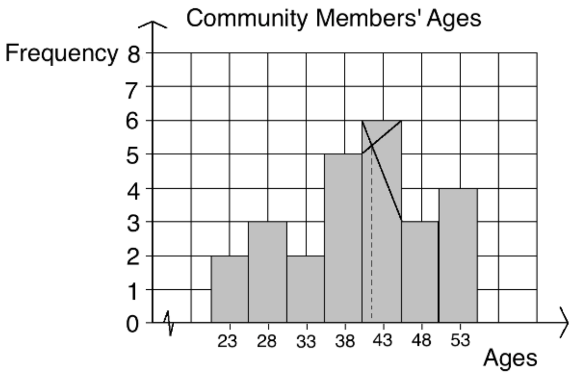
3. Find the estimated mean of the data on the board as a class. Ask volunteers to identify the value of each variable before substituting it.

$$\bar{x} = \frac{\sum fx}{\sum f} = \frac{(2\times23)+(3\times28)+(2\times33)+(5\times38)+(6\times43)+(3\times48)+(4\times53)}{2+3+2+5+6+3+4}$$
$$= \frac{46+84+66+190+258+144+212}{25}$$
$$= \frac{1000}{25} = 40 \text{ years old}$$

4. Ask a volunteer to give the median class. (Answer: It is where the 13th community member falls, which is class interval 41-45.)
5. Find the estimated median of the data on the board as a class. Ask volunteers to identify the value of each variable before substituting it.

$$\text{Median} = L + \left[\frac{\frac{n}{2} - (\sum f)_L}{f_m}\right] \times c = 41 + \left[\frac{\frac{25}{2} - (2+3+2+5)}{6}\right] \times 5 \quad \text{Substitute values}$$
$$= 41 + \left[\frac{13.5-12}{6}\right] \times 5 \quad \text{Simplify}$$
$$= 41 + \left[\frac{1.5}{6}\right] \times 5$$
$$= 41 + 0.25 \times 5$$
$$= 42.25 \text{ years old}$$

6. Explain:
   - We can estimate the mode using the histogram. The class interval that the estimated mode lies in is called the **modal class**.
   - The tallest bar in the histogram is the modal class. We estimate the mode using this bar.
7. On the board, draw intersecting lines using vertices of the tallest bar, and draw a vertical dotted line to the x-axis. →



8. Explain: The point where the lines intersect on the x-axis is the estimated mode.
9. Ask pupils to give the estimated mode. Write it on the board. (Answer: Accept realistic answers, for example in the range 41-42.)

Practice *(18 minutes)*

1. Write the following problems below on the board: Draw the histograms from the Practice section of lesson M4-L040 that correspond to each question:
   a. Use the histogram showing 25 pupils' scores on an examination to estimate the mean, median, and mode scores.
   b. Use the histogram showing 39 chimpanzees' weights to estimate the mean, median, and mode of their weights.
2. Ask pupils to solve the problems either independently or with seatmates.

3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to come to the board simultaneously to write the solutions.
   Solutions:
   a. **Mean:**

$$\bar{x} = \frac{(1\times35.5)+(3\times45.5)+(3\times55.5)+(3\times65.5)+(4\times75.5)+(6\times85.5)+(5\times95.5)}{1+3+3+3+4+6+5}$$
$$= \frac{35.5+136.5+166.5+196.5+302+513+477.5}{25}$$
$$= \frac{1827.5}{25} = 73.1 \text{ marks}$$

Median:

$$\text{Median} = L + \left[\frac{\frac{n}{2} - (\Sigma f)_L}{f_m}\right] \times c = 71 + \left[\frac{\frac{25}{2} - (2+3+3+3)}{4}\right] \times 10$$
$$= 71 + \left[\frac{13.5-11}{4}\right] \times 10$$
$$= 71 + 3.125$$
$$= 74.13 \text{ marks}$$

Mode: Accept approximations near 87 marks (see histogram below).

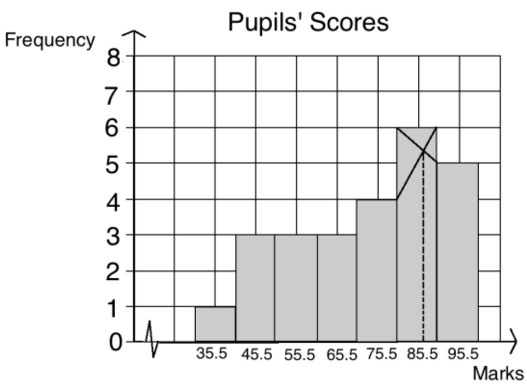
   b. **Mean:**

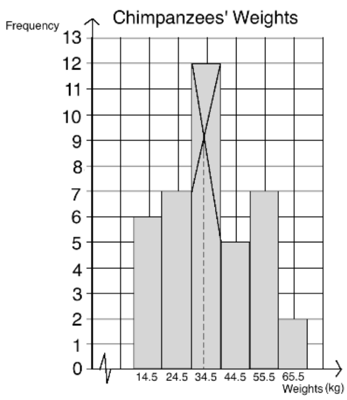
$$\bar{x} = \frac{(6\times14.5)+(7\times24.5)+(12\times34.5)+(5\times44.5)+(7\times54.5)+(2\times64.5)}{6+7+12+5+7+2}$$
$$= \frac{87+171.5+414+222.5+381.5+129}{39}$$
$$= \frac{1405.5}{39} = 36.04 \text{ kg}$$

Median:

$$\text{Median} = L + \left[\frac{\frac{n}{2} - (\Sigma f)_L}{f_m}\right] \times c = 30 + \left[\frac{\frac{39}{2} - (6+7)}{12}\right] \times 10$$
$$= 30 + \left[\frac{19.5-13}{12}\right] \times 10$$
$$= 30 + 5.42$$
$$= 35.42 \text{ kg.}$$

Mode: Accept approximations near 34 kg (see histogram below).





Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L042 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L043 in the Pupil Handbook before the next class.

Lesson Title: Cumulative frequency curves and quartiles	Theme: Probability and Statistics	
Lesson Number: M4-L043	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcomes By the end of the lesson, pupils will be able to: 1. Construct a cumulative frequency curve and estimate quartiles. 2. Calculate inter-quartile range and semi-interquartile range.	**Preparation** 1. Review the content of this lesson and be prepared to explain the solutions. 2. Write the frequency table in Opening on the board.	

Opening *(3 minutes)*

1. Draw the frequency table below on the board, but leave the "Cumulative Frequency" column empty.
2. Invite volunteers to come to the board and find the cumulative frequency of each row. Remind them to add each frequency to the total frequency of the rows above it.
3. Explain that this lesson is a review of drawing and interpreting a cumulative frequency curve.

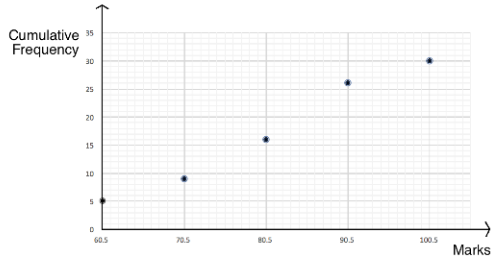
Pupils' Scores on a Maths Test		
Marks	**Frequency**	**Cumulative Frequency**
51 – 60	5	5
61 – 70	4	5+4=9
71 – 80	7	9+7=16
81 – 90	10	16+10=26
91 – 100	4	26+4=30
Total	30	

Teaching and Learning *(18 minutes)*

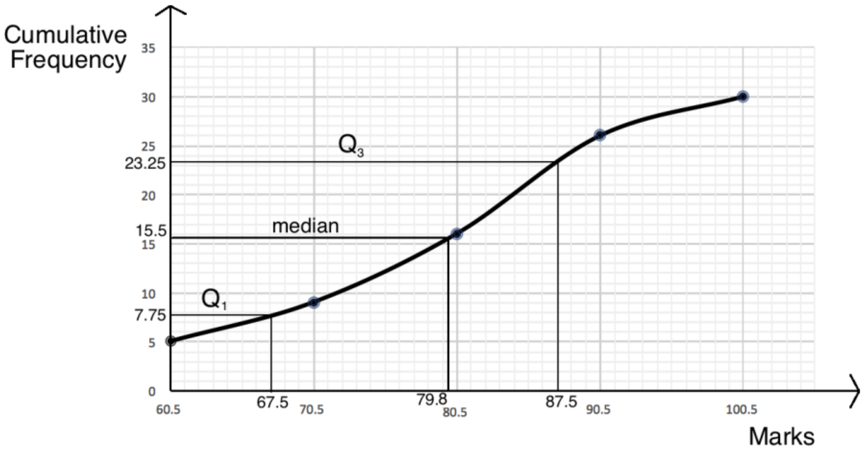
1. Explain: A **cumulative frequency curve** (or **"ogive"**) can be graphed in a similar way to line graphs and frequency polygons.
2. Explain:
   a. For the x-values, plot the upper class boundary of each class interval. This is the highest data point in each class interval.
   b. Where there is a space of 1 unit between each interval, take the middle (for example, the upper class boundary of 51-60 is 60.5, covering the gap to the next interval, which starts at 61.)
   c. For the y-value, plot the cumulative frequency from the table.
3. Draw another column in the table on the board, and write the upper class boundary for each class interval. Make sure pupils understand.

Pupils' Scores on a Maths Test			
Marks	**Frequency**	**Cumulative Frequency**	**Upper Class Interval**
51 – 60	5	5	60.5
61 – 70	4	5 + 4 = 9	70.5
71 – 80	7	7 + 9 = 16	80.5
81 – 90	10	10 + 16 = 26	90.5
91 – 100	4	4 + 26 = 30	100.5
Total	30		

4. Draw the axes on the board, and plot each point from the table (see the larger graph below). → Note that for the sake of time, it is not necessary to draw each minor gridline on the graph.



5. Connect the points with a smooth curve.
6. Explain **quartiles:**
   a. We can estimate the quartiles using the cumulative frequency curve.
   b. Quartiles divide a data set into 4 equal parts. The lower quartile (Q_1) is one-quarter of the way from the bottom of the data. The upper quartile (Q_3) is one-quarter of the way from the top of the data set. The second quartile (Q_2) is the median, or the middle quartile.
   c. We estimate quartiles by finding their placement. We then locate their placement on the x-axis, and find the corresponding value on the y-axis.
7. Write on the board: Positions of quartiles: $Q_1: \frac{1}{4}(n+1)$, $Q_2: \frac{1}{2}(n+1)$ and $Q_3: \frac{3}{4}(n+1)$,
8. Use the formulae to find the position of each quartile as a class on the board. (Answers: $Q_1: \frac{1}{4}(n+1) = \frac{1}{4}(31) = 7.75$; $Q_2: \frac{1}{2}(31) = 15.5$; $Q_3: = \frac{3}{4}(31) = 23.25$)
9. Draw each horizontal line from the y-axis, then draw a vertical line connecting this point on the curve to the x-axis. Identify the marks at the corresponding point.



10. Explain: The lower quartile (Q_1) is 67.5, the median (Q_2) is 79.8, and the upper quartile (Q_3) is 87.5. Remember that these are estimates.
11. Explain **interquartile range:**
    a. Just as we can calculate the range of a data set, we can calculate the interquartile range. Subtract the lower quartile from the upper quartile.
    b. This represents how spread out the middle half of the data is.
12. Calculate interquartile range on the board: $Q_3 - Q_1 = 87.5 - 67.5 = 20$ marks
13. Explain semi-interquartile range: The semi-interquartile range tells us about one quarter of the data set ("semi" means half, so it is half of the interquartile range).
14. Calculate semi-interquartile range: $Q = \frac{Q_3 - Q_1}{2} = \frac{87.5 - 67.5}{2} = \frac{20}{2} = 10$ marks

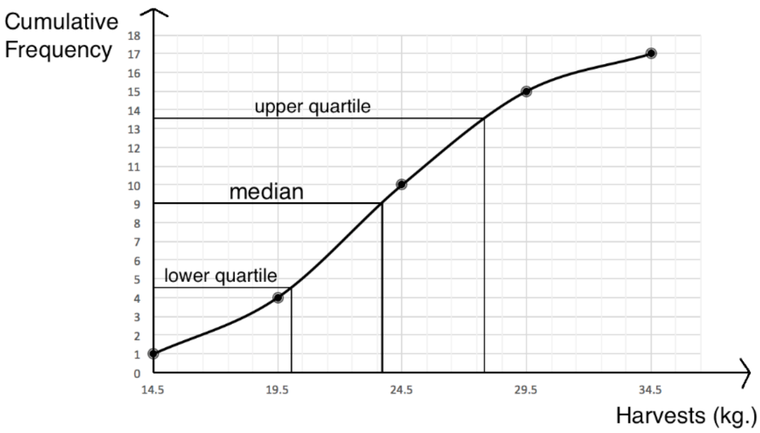
15. Explain: This tells us that about half of the pupils scored within 10 marks of the median score.

Practice *(18 minutes)*

1. Write the following problem on the board: The table below gives the cassava harvests of 17 farmers.
   a. Fill the empty columns.
   b. Draw the cumulative frequency curve.
   c. Use the curve to estimate each quartile of the distribution.
   d. Calculate the interquartile range and semi-interquartile range.

Farmers' Harvests			
Cassava (kg)	**Frequency**	**Upper Class Interval**	**Cumulative Frequency**
10 – 14	1		
15 – 19	3		
20 – 24	6		
25 – 29	5		
30 – 34	2		
Total	17		

2. Ask pupils to work independently or with seatmates.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to write the solutions on the board and explain:
   i. For the table, upper class intervals: 14.5, 19.5, 24.5, 29.5, 34.5; Cumulative frequencies: 1, 4, 10, 15, 17
   ii. Curve →
   iii. Accept approximate answers: $Q_1 = 20.1$ kg; $Q_2 = 23.6$ kg; $Q_3 = 27.7$ kg
   iv. Interquartile range: $Q_3 - Q_1 = 27.7 - 20.1 = 6.6$ kg
   Semi-interquartile range: $Q = \frac{Q_3 - Q_1}{2} = \frac{27.7-20.1}{2} = \frac{6.6}{2} = 3.3$ kg



Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L043 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L044 in the Pupil Handbook before the next class.

Lesson Title: Percentiles	Theme: Probability and Statistics	
Lesson Number: M4-L044	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcomes By the end of the lesson, pupils will be able to: 1. Estimate percentiles of data from the cumulative frequency curve. 2. Apply percentiles to real-life problems.	**Preparation** 1. Review the content of this lesson and be prepared to explain the solutions. 2. Write the table and c.f. curve at the start of Teaching and Learning on the board.	

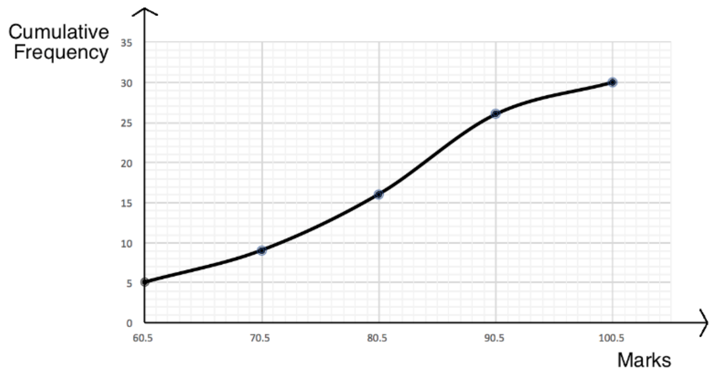
Opening *(3 minutes)*

1. Discuss:
   - What are quartiles? (Answer: Values that divide data into 4 equal parts.)
   - How do we estimate quartiles? (Answer: Use the formulae to find their positions in the data, then estimate the value of each quartile using the cumulative frequency curve.
2. Explain that this lesson is on estimating percentiles. This is similar to estimating quartiles.

Teaching and Learning *(16 minutes)*

1. Write the table and c.f. curve on the board (from the previous lesson):

Pupils' Scores on a Maths Test			
Marks	**Frequency**	**Cumulative Frequency**	**Upper Class Interval**
51 – 60	5	5	60.5
61 – 70	4	5 + 4 = 9	70.5
71 – 80	7	7 + 9 = 16	80.5
81 – 90	10	10 + 16 = 26	90.5
91 – 100	4	4 + 26 = 30	100.5
Total	30		



2. Explain:
   a. In the previous lesson, we used this curve to estimate the quartiles of the data set.
   b. Today we will use the same curve to estimate the percentiles.

c. Percentiles divide the data set into 100 equal parts.
d. For example, the 30th percentile divides off the lowest 30% of the data.

3. Write on the board: The nth percentile is the mark at $\frac{n}{100}\sum f$.
4. Explain:
    a. This formula tells us the position of the percentile.
    b. After using the formula, find the position on the y-axis of the curve. Draw horizontal and vertical lines to estimate the corresponding value on the x-axis. This is the estimated percentile.
5. Write on the board: Using the curve, estimate: a. The 30th percentile; b. The 75th percentile; c. The 90th percentile.
6. Use the formula to find the position of the 30th percentile on the board:
$$\frac{n}{100}\sum f = \frac{30}{100}(30) = \frac{900}{100} = 9$$
7. Estimate the 30th percentile using the curve on the board (see the next page).
8. Write on the board: 30th percentile $= 70.1$ marks
9. Remind pupils that this is an estimated value.
10. Ask pupils to work with seatmates to solve parts b. and c.
11. Walk around to check for understanding and clear misconceptions.
12. Invite volunteers to write the solutions on the board.

Solutions:

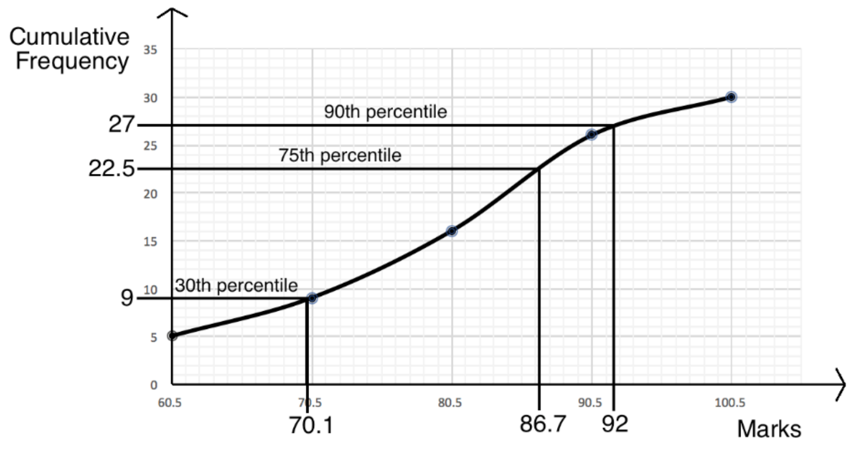
b. 75th percentile: $\frac{n}{100}\sum f = \frac{75}{100}(30) = \frac{225}{10} = 22.5$

Find the 75th percentile on the c.f. curve on the board (see below).

70th percentile $= 86.7$ marks

c. 90th percentile: $\frac{n}{100}\sum f = \frac{90}{100}(30) = \frac{2700}{100} = 27$

Find the 90th percentile on the c.f. curve:



90th percentile $= 92$ marks

13. Write on the board: Find the pass mark if 70% of pupils passed.
14. Explain:
    a. If 70% of pupils passed, then 30% of pupils failed.
    b. If 30% of pupils scored below the passing mark, we can find the 30th percentile to find the passing mark.
    c. We found that 70.1% is the 30th percentile, so that is the pass mark.

Practice *(20 minutes)*

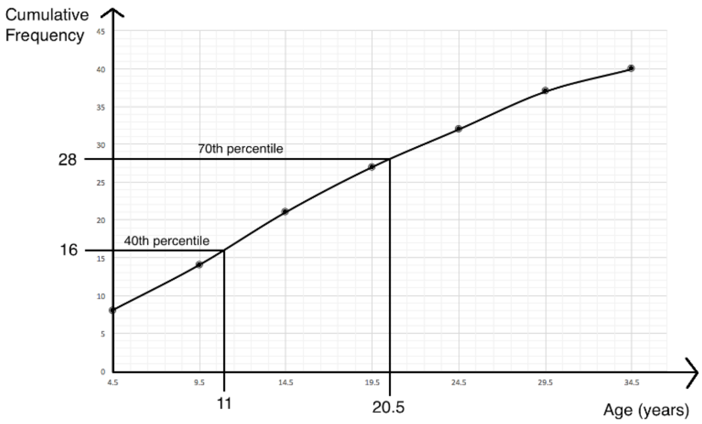
1. Write the following problem on the board: The table below gives the age distribution of all of the people in the village who were under 35 years old on 1 January 2017, when a survey was conducted.

Age (years)	0-4	5-9	10-14	15-19	20-24	25-29	30-34
Frequency	8	6	7	6	5	5	3

   2. Construct a cumulative frequency table.
   3. Use the table to draw the cumulative frequency curve.
   4. Use the curve to estimate the 70th percentile.
   5. If the youngest 40% of the village is eligible for an education programme, estimate how many people are eligible.
   6. Estimate the age of the oldest villager eligible for the programme.
2. Ask pupils to work with seatmates to complete the problem.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to write the solutions on the board and explain:
   a. Cumulative frequency table:

Villagers' ages			
Age (years)	**Frequency**	**Upper Class Boundary**	**Cumulative Frequency**
0 – 4	8	4.5	8
5 – 9	6	9.5	8+6=14
10 – 14	7	14.5	14+7=21
15 – 19	6	19.5	21+6=27
20 – 24	5	24.5	27+5=32
25 – 29	5	29.5	32+5=37
30 – 34	3	34.5	37+3=40
Total	40		

   b. Cumulative frequency curve →



   c. Find the position of the 70th percentile:
   $\frac{n}{100}\sum f = \frac{70}{100}(40) = \frac{2800}{100} = 28$
   Estimate the 70th percentile using the curve: 20.5 years
   d. Find the position of the 40th percentile:
   $\frac{n}{100}\sum f = \frac{40}{100}(40) = \frac{1600}{100} = 16$
   An estimated 16 villagers qualify for the programme.
   e. Using the curve, the 40th percentile is 11 years. Thus, the oldest villager eligible for the programme is 11 years old.

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L044 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L045 in the Pupil Handbook before the next class.

Lesson Title: Measures of dispersion	**Theme:** Probability and Statistics	
Lesson Number: M4-L045	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcomes By the end of the lesson, pupils will be able to: 1. Describe and interpret the dispersion or spread of values in a data set. 2. Calculate the range and variance of a set of ungrouped values.	**Preparation** Review the content of this lesson and be prepared to explain the solutions.	

Opening *(2 minutes)*

1. Discuss and allow pupils to share their ideas:
   - What is the meaning of dispersion? (Answer: Dispersion is related to how spread out a set of data is; higher dispersion means the data is more spread out.)
   - What are some measures of dispersion? (Example answers: range, interquartile range, semi-interquartile range, variance, standard deviation, mean deviation.)
2. Explain that this lesson is on dispersion. This is related to how spread out the data is.

Teaching and Learning *(20 minutes)*

1. Explain:
   a. The interquartile range that we calculated in a previous lesson is one example of a **measure of dispersion**.
   b. The dispersion of a set of data tells us how spread out the data is, and there are many measures for this.
   c. If a measure of dispersion has greater value, the data is more spread out.
   d. Range is another common measure of dispersion that is simple to calculate.
2. Write the following problem on the board: The ages of 15 university pupils are 18, 18, 18, 19, 19, 19, 19, 20, 20, 20, 21, 21, 22, 22, 24. Calculate the range and mean of the data.
3. Ask pupils to work with seatmates to calculate the range and mean.
4. Walk around to check for understanding and clear misconceptions.
5. Invite volunteers to write the solutions on the board and explain.
   Solutions:
   Range = greatest value – least value $= 24 - 18 = 6$ years
   Mean $= \frac{\text{sum of ages}}{\text{number of pupils}} = \frac{3(18)+4(19)+3(20)+2(21)+2(22)+24}{15} = \frac{300}{15} = 20$ years
6. Discuss: How can you interpret the range of this data? (Example answer: It tells us that the pupils' ages are spread out by 6 years. The oldest pupil is 6 years older than the youngest pupil.)

7. Explain **deviation:**
    a. Deviation is another measure of dispersion.
    b. If the mean of a distribution is subtracted from any value in the distribution, the result is called the deviation of the value from the mean.
8. Write on the board:
    a. Calculate the deviation of an 18-year-old pupil.
    b. Calculate the deviation of a 20-year-old pupil.
    c. Calculate the deviation of a 24-year-old pupil.
9. Solve the problems on the board and explain:
    a. Deviation $= 18 - \text{mean} = 18 - 20 = -2$
    b. Deviation $= 20 - \text{mean} = 20 - 20 = 0$
    c. Deviation $= 24 - \text{mean} = 24 - 20 = +4$
10. Explain:
    a. Deviation can be positive or negative.
    b. For any distribution, the sum of all of the deviations from the mean is always zero. In other words, if we calculated the deviation for all 15 pupils in the data set, their sum would be 0.
11. Ask pupils to work with seatmates to calculate the deviation for each of the 15 values on the board, and write them in a list.
12. Invite volunteers to write the list on the board. (Answer: $-2, -2, -2, -1, -1, -1, -1, 0, 0, 0, 1, 1, 2, 2, 4$)
13. Explain **variance:**
    a. Variance is another measure of dispersion.
    b. Variance is calculated using the deviations of the data.
    c. The deviations of the data tell you information about each piece of data, and this information can be used to obtain a single number which indicates the overall dispersion of the data.
    d. To calculate variance, find the sum of the square of each deviation from the mean. Divide by the frequency (in this case, 15).
14. Calculate the variance of the set of data on the board:

$$\begin{aligned}\text{Variance} &= \frac{(-2)^2+(-2)^2+(-2)^2+(-1)^2+(-1)^2+(-1)^2+(-1)^2+(0)^2+(0)^2+(0)^2+(1)^2+(1)^2+(2)^2+(2)^2+(4)^2}{15} \\ &= \frac{4+4+4+1+1+1+1+0+0+0+1+1+4+4+16}{15} \\ &= \frac{42}{15} \\ &= 2.8\end{aligned}$$

15. Explain:
    a. The value 2.8 by itself doesn't tell us a lot of information.
    b. If we had another group of pupils, we could find the variation of that data set too.
    c. A larger variation means that a data set is more spread out. For example, if we found that another group of pupils had a variance of 3, then their ages would be more spread out than the ages of the pupils in our data set.
16. Explain dispersion:

a. Range and variance are 2 important measures of dispersion, but there are others as well.
b. Interquartile range and semi-interquartile range are other measures of dispersion that you have calculated before. They are better measures of dispersion and range, because they are not affected by any extremely low or high values. Remember that they are based on where the middle half of the data is.

Practice *(16 minutes)*

1. Write a problem on the board: Bentu runs an education centre that children attend while their parents are working. The ages of the children that attend are: 1, 1, 2, 2, 2, 3, 4, 4, 5, 5, 5, 6, 6, 7, 7. Calculate:
    a. The range in ages of the children.
    b. The mean age of the children.
    c. The variance in the children's ages.
2. Ask pupils to work with independently or with seatmates to complete the problem.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to write the solutions on the board and explain:
    a. Range $= 7 - 1 = 6$ years
    b. Mean $= \frac{\text{sum of ages}}{\text{number of pupils}} = \frac{2(1)+3(2)+3+2(4)+3(5)+2(6)+2(7)}{15} = \frac{60}{15} = 4$ years
    c. **Step 1.** Calculate the deviation of each value from the mean:

$$-3, -3, -2, -2, -2, -1, 0, 0, 1, 1, 1, 2, 2, 3, 3$$

Step 2. Calculate the variance:

$$\begin{aligned}\text{Variance} &= \frac{(-3)^2+(-3)^2+(-2)^2+(-2)^2+(-2)^2+(-1)^2+(0)^2+(0)^2+(1)^2+(1)^2+(1)^2+(2)^2+(2)^2+(3)^2+(3)^2}{15} \\ &= \frac{9+9+4+4+4+1+0+1+1+1+1+4+4+9+9}{15} \\ &= \frac{60}{15} \\ &= 4\end{aligned}$$

Closing *(2 minutes)*

1. Discuss: Which set of data is more spread out, the data of university pupils or young children? (Answer: The ranges are the same; however, the set of young children have a greater variance, so they are generally more spread out in age.)
2. For homework, have pupils do the practice activity of PHM4-L045 in the Pupil Handbook.
3. Ask pupils to read the overview of the next lesson, PHM4-L046 in the Pupil Handbook before the next class.

Lesson Title: Standard deviation	Theme: Probability and Statistics	
Lesson Number: M4-L046	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to calculate the standard deviation of ungrouped and grouped data.	**Preparation** Review the content of this lesson and be prepared to explain the solutions.	

Opening *(1 minute)*

1. Explain that this lesson is on standard deviation, which is a measure of dispersion.

Teaching and Learning *(19 minutes)*

1. Explain:
    a. Standard deviation is a measure of dispersion that tells us how close data points generally are to the mean.
    b. A low standard deviation indicates that points are generally close to the mean (the deviation is low).
    c. A high standard deviation indicates that points are spread out over a wide range of values, farther from the mean (the deviation is high).
2. Write the following problems on the board:
    a. 10 pupils achieved the following scores on a Maths exam: 80, 82, 88, 89, 84, 79, 81, 82, 85, 80. Calculate the standard deviation of the distribution.
    b. The ages of 20 children are given in the table below. Calculate the mean and standard deviation of their ages.

Age (years)	1	2	3	4	5	6
Frequency (f)	3	4	2	3	6	2

3. Explain:
    a. Problem a. is ungrouped data, and problem b. is grouped data.
    b. We will use different formulae to find the standard deviation of ungrouped and grouped data.
4. Write the formulae on the board:
    a. Ungrouped data: $s = \sqrt{\frac{1}{n}\sum(x-\bar{x})^2}$, where n is the frequency, x is each value in the set, and $\bar{x}$ is the mean.
    b. Grouped data: $s = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2}$, where f is frequency, x is each data point, and $\bar{x}$ is the mean.
5. Explain:
    a. We need to take several steps to solve each problem. We will use a table to organise these calculations.
    b. When drawing a table, it is important to have a column for each term that is needed in the formula.

c. Notice that each formula requires us to find the mean. The mean does not get a column in the table. We will calculate the mean of each data set separately.

6. Solve problem a. as a class. Involve pupils by asking them to give the steps and fill the table on the board.
Solution:
Step 1. Calculate mean: $\bar{x} = \frac{80+82+88+89+84+79+81+82+85+80}{10} = \frac{830}{10} = 83$
Step 2. Fill the table to find $\sum(x-\bar{x})^2$. →
Step 3. Apply the formula:

$s = \sqrt{\frac{1}{n}\sum(x-\bar{x})^2} = \sqrt{\frac{106}{10}} \approx 3.26$

$\boldsymbol{x}$	$\boldsymbol{x-\bar{x}}$	$\boldsymbol{(x-\bar{x})^2}$
80	$80-83=-3$	9
82	$82-83=-1$	1
88	$88-83=+5$	25
89	$89-83=+6$	36
84	$84-83=+1$	1
79	$79-83=-4$	16
81	$81-83=-2$	4
82	$82-83=-1$	1
85	$85-83=+2$	4
80	$80-83=-3$	9
Total	$=\sum(x-\bar{x})^2=$	106

7. Solve problem b. as a class. Involve pupils by asking them to give the steps and fill the table on the board.
Solution:
Remind pupils of the formula for mean of grouped data: $\bar{x} = \frac{\sum fx}{\sum f}$.
Step 1. Draw and fill a table with the values needed for the mean and standard deviation formulae:

$\boldsymbol{x}$	$\boldsymbol{f}$	$\boldsymbol{fx}$	$\boldsymbol{fx^2}$
1	3	$1\times3=3$	$1\times3=3$
2	4	$2\times4=8$	$2\times8=16$
3	2	$3\times2=6$	$3\times6=18$
4	3	$4\times3=12$	$4\times12=48$
5	6	$5\times6=30$	$5\times30=150$
6	2	$6\times2=12$	$6\times12=72$
Totals	$\sum f=20$	$\sum fx=71$	$\sum fx^2=307$

Step 2. Calculate mean: $\bar{x} = \frac{\sum fx}{\sum f} = \frac{71}{20} = 3.55$ years old

Step 3. Calculate standard deviation: $s = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2} = \sqrt{\frac{307}{20} - (3.55)^2} =$
$\sqrt{15.35 - 12.60} = \sqrt{2.75} \approx 1.66$

Practice *(19 minutes)*

1. Write the following problems on the board:

a. Six boys weighed themselves and found their weights in kilogrammes to be 43, 48, 45, 50, 47, and 43. Calculate:
   i. The range of their weights.
   ii. The mean of their weights.
   iii. The standard deviation of their weights.

b. The weights of 20 children are given in the table below. Calculate the standard deviation.

Weight (kg)	10-14	15-19	20-24	25-29	30-34	35-39
Frequency (f)	3	2	4	5	3	3

2. Explain: Problem b. can be solved with the formula for grouped data. Find the mid-point of each class interval and use these for the values of x.
3. Ask pupils to work with seatmates to complete the problems.
4. Draw the solution table for question b on the board to support pupils.
5. Walk around to check for understanding and clear misconceptions.
6. Invite volunteers to write the solutions on the board and explain:

a. i. Range = $50 - 43 = 7$ kg

ii. Mean = $\frac{43+48+45+50+47+43}{6} = \frac{276}{6} = 46$ kg

iii. Standard deviation (see table):

$$\sqrt{\frac{1}{n}\sum(x-\bar{x})^2} = \sqrt{\frac{40}{6}} \approx 2.58$$

$\boldsymbol{x}$	$\boldsymbol{x-\bar{x}}$	$\boldsymbol{(x-\bar{x})^2}$
43	$43-46=-3$	9
48	$48-46=+2$	4
45	$45-46=-1$	1
50	$50-46=+4$	16
47	$47-46=+1$	1
43	$43-46=-3$	9
	$\sum(x-\bar{x})^2=$	40

c. Mean (see table below): $\bar{x} = \frac{\sum fx}{\sum f} = \frac{500}{20}$

Standard deviation: $s = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2} = \sqrt{\frac{13770}{20} - 25^2} = \sqrt{688.5 - 625} = \sqrt{63.5} \approx 7.97$

Interval	**Mid-point (x)**	$\boldsymbol{f}$	$\boldsymbol{fx}$	$\boldsymbol{fx^2}$
10-14	12	3	36	432
15-19	17	2	34	578
20-24	22	4	88	1936
25-29	27	5	135	3645
30-34	32	3	96	3072
35-39	37	3	111	4107
	Totals	$\sum f = 20$	$\sum fx = 500$	$\sum fx^2 = 13{,}770$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L046 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L047 in the Pupil Handbook before the next class.

Lesson Title: Mean deviation	**Theme:** Probability and Statistics	
Lesson Number: M4-L047	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to calculate the mean deviation of ungrouped and grouped data.	**Preparation** Review the content of this lesson and be prepared to explain the solutions.	

Opening *(1 minute)*

1. Explain that this lesson is on mean deviation, another measure of dispersion.

Teaching and Learning *(19 minutes)*

1. Explain:
   - Mean deviation is similar to standard deviation.
   - When we calculate standard deviation, we square the deviations, which makes them positive. For mean deviation, we use absolute value to make the deviations positive.
   - Mean deviation is another measure of the spread (dispersion) of data.
2. Write the following problems on the board:
   a. The goals scored by 10 football players in a season are 3, 1, 4, 7, 3, 8, 12, 15, 8 and 9. Calculate the mean and mean deviation of the data to 2 decimal places.
   b. The ages of young children in one community are given in the table below. Calculate the mean and mean deviation of their ages.

Age (years)	1	2	3	4	5	6
Frequency (f)	7	6	7	3	4	3

3. Explain:
   - Problem a. is ungrouped data, and problem b. is grouped data.
   - We will use different formulae to find the mean deviation of ungrouped and grouped data.
4. Write the formulae on the board:
   - Ungrouped data: $MD = \frac{\sum|x-\bar{x}|}{n}$, where x is each piece of data, $\bar{x}$ is the mean, and n is the total frequency.
   - Grouped data: $MD = \frac{\sum f|x-\bar{x}|}{\sum f}$, where x is each piece of data, $\bar{x}$ is the mean, and f is frequency.
5. Explain: We will use a table to organise these calculations, as we did for standard deviation. Remember to have a column for each term that is needed in the formula.
6. Solve problem a. as a class. Involve pupils by asking them to give the steps and fill the table on the board.

Solution:

Step 1. Calculate mean: $\bar{x} = \frac{3+1+4+7+3+8+12+15+8+9}{10} = \frac{70}{10} = 7$ goals

Step 2. Fill the table to find $\sum |x - \bar{x}|$. →

$\boldsymbol{x}$	$\boldsymbol{x - \bar{x}}$	$\boldsymbol{\|x - \bar{x}\|}$
3	$3 - 7 = -4$	4
1	$1 - 7 = -6$	6
4	$4 - 7 = -3$	3
7	$7 - 7 = 0$	0
3	$3 - 7 = -4$	4
8	$8 - 7 = +1$	1
12	$12 - 7 = +5$	5
15	$15 - 7 = +8$	8
8	$8 - 8 = +1$	1
9	$9 - 7 = +2$	2
	Total $= \sum \|\boldsymbol{x} - \bar{\boldsymbol{x}}\| =$	34

Step 3. Apply the formula:

$\text{MD} = \frac{\sum |x - \bar{x}|}{n} = \frac{34}{10} = 3.4$

7. Solve problem b. as a class. Involve pupils by asking them to give the steps and fill the table on the board.

   Solution:

   Remind pupils of the formula for mean of grouped data: $\bar{x} = \frac{\sum fx}{\sum f}$.

   Step 1. Draw a table and fill the columns needed for the mean formula (columns 1-3).

$\boldsymbol{x}$	$\boldsymbol{f}$	$\boldsymbol{fx}$	$\boldsymbol{x - \bar{x}}$	$\boldsymbol{\|x - \bar{x}\|}$	$\boldsymbol{f\|x - \bar{x}\|}$
1	7	7	$1 - 3 = -2$	2	14
2	6	12	$2 - 3 = -1$	1	6
3	7	21	$3 - 3 = 0$	0	0
4	3	12	$4 - 3 = +1$	1	3
5	4	20	$5 - 3 = +2$	2	8
6	3	18	$6 - 3 = +3$	3	9
Totals:	$\sum f = 30$	$\sum fx = 90$			$\sum f\|x - \bar{x}\| = 40$

   Step 2. Calculate mean: $\bar{x} = \frac{\sum fx}{\sum f} = \frac{90}{30} = 3$ years old

   Step 3. Fill columns 4-6 of the table with the values needed for mean deviation.

   Step 4. Calculate mean deviation: $MD = \frac{\sum f|x - \bar{x}|}{\sum f} = \frac{40}{30} = 1.33$

Practice *(19 minutes)*

1. Write the following problems on the board:
   - If the ages of 10 children are 2, 3, 8, 4, 9, 10, 12, 7, 6 and 9, calculate: i. Range; ii. Mean; iii. Mean deviation.

- The weights of 20 children are given in the table below. Calculate the mean and mean deviation of the data.

Weight (kg)	10-14	15-19	20-24	25-29	30-34	35-39
Frequency (f)	3	2	4	5	3	3

2. Explain: Problem b. can be solved with the formula for grouped data. Find the mid-point of each class interval and use these for the values of x.
3. Ask pupils to work with seatmates to complete the problems.
4. Draw the solution table for question b. on the board to support pupils.
5. Walk around to check for understanding and clear misconceptions.
6. Invite volunteers to write the solutions on the board and explain:
   - i. Range: $12 - 2 = 10$ years
     ii. Mean: $\bar{x} = \frac{2+3+8+4+9+10+12+7+6+9}{10} = \frac{70}{10}$ = 7 years old
     iii. Mean deviation (see table): $\text{MD} = \frac{\sum|x-\bar{x}|}{n} = \frac{26}{10} = 2.6$

x	$x - \bar{x}$	$\lvert x - \bar{x}\rvert$
2	$2 - 7 = -5$	5
3	$3 - 7 = -4$	4
8	$8 - 7 = +1$	1
4	$4 - 7 = -3$	3
9	$9 - 7 = +2$	2
10	$10 - 7 = +3$	3
12	$12 - 7 = +5$	5
7	$7 - 7 = 0$	0
6	$6 - 7 = -1$	1
9	$9 - 7 = +2$	2
	Total $= \sum\lvert x - \bar{x}\rvert =$	26

   - Fill the first 4 columns of the table and calculate mean: Answer: $\bar{x} = \frac{\sum fx}{\sum f} = \frac{500}{20} = 25$ kg
     Fill the remaining columns of the table and calculate mean deviation: $MD = \frac{\sum f|x-\bar{x}|}{\sum f} = \frac{134}{20} = 6.7$

Interval	**Mid-point (x)**	f	fx	$x - \bar{x}$	$\lvert x - \bar{x}\rvert$	$f\lvert x - \bar{x}\rvert$
10-14	12	3	36	$12 - 25 = -13$	13	39
15-19	17	2	34	$17 - 25 = -8$	8	16
20-24	22	4	88	$22 - 25 = -3$	3	12
25-29	27	5	135	$27 - 25 = +2$	2	10
30-34	32	3	96	$32 - 25 = +7$	7	21
35-39	37	3	111	$37 - 25 = +12$	12	36
	Totals:	$\sum f = 20$	$\sum fx = 500$			$\sum f\lvert x - \bar{x}\rvert = 134$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L047 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L048 in the Pupil Handbook before the next class.

Lesson Title: Statistics problem solving	**Theme:** Probability and Statistics	
Lesson Number: M4-L048	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to solve advanced problems involving statistics.	**Preparation** 1. Review the content of this lesson and be prepared to explain the solutions. 2. Write the problems at the start of Teaching and Learning on the board.	

Opening *(1 minute)*

1. Explain that this lesson is solving statistics problems. Pupils will use information from previous lessons, and their problem-solving skills.

Teaching and Learning *(19 minutes)*

1. Write the following problems on the board:
   a. The heights of 20 football players are given in the table below. Calculate the: i. Mean height. ii. Mean deviation. iii. Probability that a player chosen at random will be at least 170 cm tall.

Height (cm)	150-154	155-159	160-164	165-169	170-174	175-179	180-184
Frequency	1	1	2	4	6	5	1

   b. The frequency distribution shows the marks scored by 50 pupils on a Maths exam.

Marks (%)	30-39	40-49	50-59	60-69	70-79	80-89	90-99
Frequency	3	4	7	13	11	8	4

      a. Draw a cumulative frequency curve for the distribution.
      b. Use the graph to find the 60^{th} percentile.
      c. If pupils must score more than 65% to pass, use the graph to find the probability that a pupil chosen at random passed the test.

   c. Ask pupils to work with seatmates to solve the problems. Remind them to look at the appropriate lessons in their Pupil Handbooks if needed.
   d. Walk around to check for understanding and clear misconceptions. Discuss problems as a class if needed.
   e. Invite volunteers to write the solutions on the board and explain.

   Solutions:

   a. i. Organise a table (see below) and apply the formula:
   $\bar{x}=\frac{\sum fx}{\sum f}=\frac{3400}{20}=170$ cm.
   ii. Complete the relevant columns of the table (see below) and apply the formula: $MD=\frac{\sum f|x-\bar{x}|}{\sum f}=\frac{118}{20}=5.9$
   iii. Probability $=\frac{\text{players taller than 170 cm.}}{\text{all players}}=\frac{6+5+1}{20}=\frac{12}{20}=\frac{3}{5}=0.6$

Complete table:

Interval	Mid-point (x)	f	fx	$x - \bar{x}$	$\lvert x - \bar{x}\rvert$	$f\lvert x - \bar{x}\rvert$
150-154	152	1	152	$152 - 170 = -18$	18	18
155-159	157	1	157	$157 - 170 = -13$	13	13
160-164	162	2	324	$162 - 170 = -8$	8	16
165-169	167	4	668	$167 - 170 = -3$	3	12
170-174	172	6	1032	$172 - 170 = +2$	2	12
175-179	177	5	885	$177 - 170 = +7$	7	35
180-184	182	1	182	$182 - 170 = +12$	12	12
	Totals:	$\sum f = 20$	$\sum fx = 3400$			$\sum f\lvert x - \bar{x}\rvert = 118$

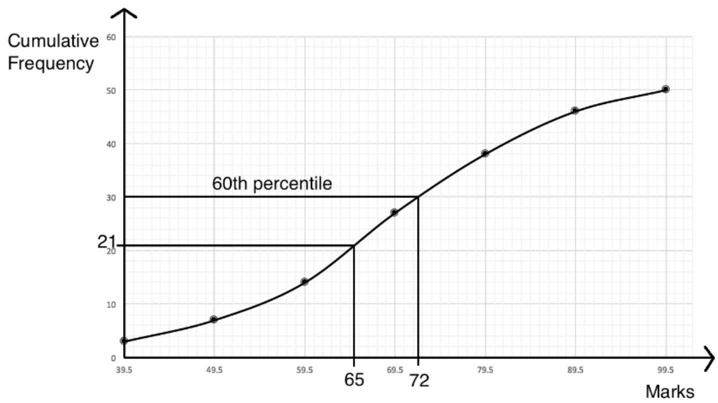
b. i. Before drawing the cumulative frequency curve, complete a cumulative frequency table:

Pupils' Marks			
Marks	**Frequency**	**Upper Class Boundary**	**Cumulative Frequency**
30 – 39	3	39.5	3
40 – 49	4	49.5	3+4=7
50 – 59	7	59.5	7+7=14
60 – 69	13	69.5	14+13=27
70 – 79	11	79.5	27+11=38
80 – 89	8	89.5	38+8=46
90 – 99	4	99.5	46+4=50
Total	50		

See curve below.

c. Find the position of the 60th percentile: $\frac{n}{100}\sum f = \frac{60}{100}(50) = \frac{3000}{100} = 30$
Identify the 60th percentile on the curve as 72 marks. (see curve below).

d. To identify the number of pupils scoring above 65%, first identify 65 marks on the c.f. curve. 65 marks corresponds to a cumulative frequency of 21. If 21 pupils scored 65 or lower, then the number that passed is $50 - 21 = 29$.
Probability that a pupil passed $= \frac{\text{passing pupils}}{\text{all pupils}} = \frac{29}{50} = 0.58$



Practice *(19 minutes)*

1. Write the problems on the board:
   a. The ages of 10 children are 5, 4, 3, 7, 10, 2, 3, 8, 6, and 2. Calculate: i. Mean; ii. Standard deviation; iii. The probability that a child chosen at random will be at least 5 years old.
   b. The ages of 20 pupils are given in the table below. Use the table to calculate the: a. mean; b. mean deviation; c. probability that a child chosen at random is at least 11 years old.

Ages (years)	5-7	8-10	11-13	14-16
Frequency (f)	2	5	4	9

2. Ask pupils to work with independently to solve the problems.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to write the solutions on the board and explain:
   a. i. Mean: $\bar{x} = \frac{5+4+3+7+10+2+3+8+6+2}{10} = \frac{50}{10} = 5$ years old; ii. Standard deviation (see table): $s = \sqrt{\frac{1}{n}\sum(x-\bar{x})^2} = \sqrt{\frac{66}{10}} = \sqrt{6.6} = 2.57$; iii. Probability $= \frac{\text{children 5 and older}}{\text{all children}} = \frac{5}{10} = 0.5$
   b. Organise a table (see below) and apply the formula: $\bar{x} = \frac{\sum fx}{\sum f} = \frac{240}{20} = 12$ years old; ii. Complete the relevant columns of the table (see below) and apply the formula: $MD = \frac{\sum f|x-\bar{x}|}{\sum f} = \frac{54}{20} = 2.7$; iii. $\text{Probability } = \frac{\text{children 11 or older}}{\text{all children}} = \frac{4+9}{20} = \frac{13}{20} = 0.65$

$\boldsymbol{x}$	$\boldsymbol{x-\bar{x}}$	$\boldsymbol{(x-\bar{x})^2}$
5	$5-5=0$	0
4	$4-5=-1$	1
3	$3-5=-2$	4
7	$7-5=+2$	4
10	$10-5=+5$	25
2	$2-5=-3$	9
3	$3-5=-2$	4
8	$8-5=+3$	9
6	$6-5=+1$	1
2	$2-5=-3$	9
Total	$=\sum(x-\bar{x})^2=$	66

Interval	**Mid-point (x)**	$\boldsymbol{f}$	$\boldsymbol{fx}$	$\boldsymbol{x-\bar{x}}$	$\boldsymbol{\lvert x-\bar{x}\rvert}$	$\boldsymbol{f\lvert x-\bar{x}\rvert}$
5-7	6	2	12	$6-12=-6$	6	12
8-10	9	5	45	$9-12=-3$	3	15
11-13	12	4	48	$12-12=0$	0	0
14-16	15	9	135	$15-12=+3$	3	27
	Totals:	$\sum f = 20$	$\sum fx = 240$			$\sum f\lvert x-\bar{x}\rvert = 54$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L048 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L049 in the Pupil Handbook before the next class.

Appendix I: Logarithm Table

COMMON LOGARITHM TABLE

	0	1	2	3	4	5	6	7	8	9	Mean Difference								
											1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

Appendix II: Anti-Logarithm Table

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
·01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
·02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
·03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
·04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	2	2	2	2
·05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
·06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	1	2	2	2
·07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
·08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	3
·09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	3
·10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
·11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
·12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
·13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
·14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
·15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
·16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
·17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
·18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
·19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	3	3	3
·20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	3	3	3
·21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1696	0	1	1	2	2	2	3	3	3
·22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	3	3	3
·23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	4
·24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	3	3	4
·25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4
·26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
·27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
·28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
·29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
·30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4
·31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
·32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
·33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	2	2	3	3	4	4
·34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	3	3	4	4	5
·35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	3	3	4	4	5
·36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
·37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1	1	2	2	3	3	4	4	5
·38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3	3	4	4	5
·39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	3	3	4	5	5
·40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	3	4	4	5	5
·41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3	4	4	5	5
·42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1	2	2	3	4	4	5	6
·43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	3	3	4	4	5	6
·44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	2	3	3	4	4	5	6
·45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	3	3	4	5	5	6
·46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
·47	2951	2958	2965	2972	2970	2985	2992	2999	3006	3013	1	1	2	3	3	4	5	5	6
·48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	3	4	4	5	6	6
·49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	3	4	4	5	6	6

FRITHLOGARTAIM ANTI-LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
·51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
·52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
·53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
·54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
·55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
·56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
·57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
·58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
·59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
·60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	6	7	8
·61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
·62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
·63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
·64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
·65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
·66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
·67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
·68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
·69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
·70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
·71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
·72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
·73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
·74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
·75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
·76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
·77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
·78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
·79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
·80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
·81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
·82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
·83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
·84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
·85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
·86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
·87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
·88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
·89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	13	14	16
·90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
·91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
·92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
·93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
·94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
·95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
·96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
·97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
·98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
·99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20

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