Designing Effective Numeracy Programs in Low- and Middle-Income Countries

Introduction

Numeracy has received limited attention in discussions around school quality in the early grades, despite evidence that strong early math skills not only predict later academic achievement[1] but may also predict literacy skills,[2] making a case for spending equal time on both math and literacy skills from the very beginning.[3] Early math achievement is linked to later life outcomes, including secondary school graduation and earning potential.[4] With the proliferation of technology and STEM-related careers, and the need for strong skills in data interpretation and problem solving, having a strong early math foundation has implications for 21st-century careers more now than ever before.

PURPOSE

In this guide, we offer suggestions to policy makers, donors, and implementers for how to effectively design and manage foundational numeracy at scale. We present evidence from high income countries and low- and middle-income countries (LMICs), where available and relevant, and discuss potential solutions for challenges across three areas: curriculum and standards, teachers and teaching, and teaching and learning materials. The guide concludes with a discussion of areas where more research is needed.

KEY DEFINITIONS

Numeracy: the knowledge, skills, behaviors, and dispositions that students need in order to use mathematics in a wide range of situations.

Conceptual knowledge: the comprehension of mathematical concepts, operations, and relations.

Procedural fluency: the ability to apply procedures accurately, efficiently, and flexibly.

Process skills: processes students use to build and apply mathematical proficiency, including problem solving, using reasoning and proof, clearly communicating ideas, making connections between different concepts, and using models to represent mathematical ideas or concepts.

Number sense: the ability to think flexibly and fluently about numbers, including taking numbers apart and putting them together in different ways, computing mentally, and relating numbers to real-life problems.

Manipulatives: physical objects that can be moved and touched, which are used in such a way that a learner can perceive some mathematical concept by manipulating them. Examples include counters, sticks, geometric shapes, etc.

IMPORTANCE OF NUMERACY AND STARTING EARLY

Humans are born with an innate number sense, and young children's foundational numeracy skills are further developed as they interact with caregivers, everyday objects, and their environment.[5] When children enter school, they begin to formalize the informal math knowledge that they have developed since birth. For example, they learn that the sticks they count and play with can be represented by written symbols such as "3" and "5," and that putting objects together is called addition and is associated with the "+" symbol. Starting children on their math journey early—beginning at birth, continuing through preschool, and with sufficient attention in the early grades—can close achievement gaps between children from different socioeconomic backgrounds[6] and provide time for mastery in foundational areas such as counting and number recognition, principles that have shown to impact later success in mathematics.[7] Starting math education early also fosters positive attitudes toward math, which research has shown to improve outcomes in primary school and beyond.[8] Additionally, research shows that children who struggle with foundational math skills tend to fall further behind and have a difficult time catching up,[9] pointing to the importance of developing a sound foundation early on. **Starting girls on their math journey early and exposing them to careers that require math, like engineering and medicine, can have long-lasting impacts into adulthood; has the potential to close the gender pay gap; and helps reduce inequities between boys and girls.**[10]

STATUS OF NUMERACY OUTCOMES IN LMICS

The prioritization of literacy skills in LMICs has translated to a dearth of research on early-grade mathematics in LMICs. Of the limited evidence that exists, much of it focuses on assessing learning outcomes and children's progress in acquiring foundational skills. The data are not encouraging and suggest that children are not mastering the early, basic skills that are needed for developing more complex mathematical concepts.

- Early-grade math assessment trends from Ghana in 2013 and 2015 show students doing poorly beyond the most procedural and recall items (number identification and simple addition/subtraction facts).[11]
- In one district in the province of Punjab, Pakistan, the People Action Network used the International Common Assessment of Numeracy tool to assess students in grades 2–3. Only 32.2% of these students were able to do a set of foundational numeracy tasks (one task each on spatial orientation, shape recognition, measurement, and number recognition, as well as at least three simple number operations).[12]
- In Tanzania in 2013, only 7.9% of second grade students met the addition and subtraction benchmark at the end of the school year, with girls lagging behind boys (7.3% vs 8.5%).[13]
- According to SACMEQ III conducted in 2007 in 15 sub-Saharan African countries, 31% of grade six students are classified as innumerate; in Zambia, this proportion rises to 67.3%.[14]

NUMERACY AND GENDER

Math assessment results for girls and boys in the early grades show them performing equally well from kindergarten through second grade. However, starting in third grade, boys outperform girls in math across many countries.[15] As students enter the upper grades, the trend continues. For example, Pratham's Annual Status of Education Report from 2018 shows boys aged 14–16 outperforming girls in basic division skills.[16] These results do not indicate that boys are better suited to math than girls. Instead, gendered stereotypes presenting mathematics as a subject for boys, as well as teachers' attitudes about who can and cannot do math, are linked to girls' attitudes and motivation toward math.[17] These stereotypes and norms run particularly strong in LMICs, where girls are often treated differently and where both teachers and parents have lower expectations of girls compared to their male peers.[18]

Curriculum and Standards

CONTENT AREAS

Recent research on effective math curricula for young children has found that devoting time to core content areas and activities can improve math learning before and at the beginning of elementary school.[19] Early math programs around the world focus on five domains of math knowledge (see Table 1), although they may be organized differently across countries. The Global Proficiency Framework (GPF) for Mathematics—a joint effort between donors, early math experts, and the international development community—defines the minimum proficiency levels that students are expected to obtain at the end of grades 1–9 across these five domains. Mastery of the skills and concepts within these content areas represent the body of knowledge that is needed to be able to flexibly apply mathematics to solve problems encountered in everyday life and further schooling. Given this, **the GPF can serve as a useful tool for countries that are revising their curriculum, as it establishes a shared minimum proficiency level for math skills and concepts.**

TABLE 1. Mathematics domains and constructs in primary school

Domains	Constructs
Number and operations	Whole numbers; fractions; decimals; operations; real-world problems; use of rounding and estimation to solve problems and check answers
Measurement	Length, capacity, volume, area, perimeter; time; currency, use estimation
Statistics and probability	Data management; chance and probability
Geometry and spatial reasoning	Properties of shapes and figures; constructions; position and direction
Algebra	Patterns; relations and functions; variation (ratio, proportion, and percentage)

LEARNING PROGRESSIONS

Math learning can be viewed as proceeding through learning progressions, where children's understanding of a concept develops through predictable levels of sophistication.[20] Curriculum that "spirals" supports children's development through these levels in each domain and helps children build deep understanding in mathematics. In a "spiral" curriculum, learning within a domain is spread out and concepts are revisited repeatedly over months and across grades. Students are scaffolded to higher levels of understanding by revisiting a particular concept on multiple occasions, which leads to better long-term learning and encourages greater sophistication in mathematical thinking.[21] In a review of the literature on the benefits of spiraling as compared to "massing," where one topic is addressed in one extended time period without breaks or later revisiting, Son and Simon[22] find that spiraling leads to better performance. For example, with regard to addition, children first learn how to add single-digit numbers (such as 4 + 5 and 2 + 3) and return to that concept multiple times over the year, each time with increasing complexity (e.g., first adding 4 + 5 with objects, then with symbols, and later with story problems). Basic addition as a foundational skill is key for building the more complex skill of adding multi-digit numbers, such as 34 + 65 and 206 + 329. Through repeated experiences and practice, students attain mastery and fluency in such key foundational skills.

The sequencing of math skills and how each skill is taught in relation to others has been researched extensively.[23] A joint statement by the National Association for the Education of Young Children and the National Council of Teachers of Math states that "articulating goals and standards for young children as a developmental or learning continuum is a particularly useful strategy in ensuring engagement with and mastery of important mathematical ideas."[24] Learning progressions developed by researchers map out the sequence in which young children develop math understanding and skills and are a helpful tool for curriculum planners when setting standards and developing curricula (see Figure 1). A learning progressions framework supports teachers in providing developmentally

FIGURE 1. Learning progression for developing understanding in addition and subtraction[25]

Grade 1 – Students will:	Grade 2 – Students will:	Grade 3 – Students will:
Use addition and subtraction with numbers under 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing (e.g., by using objects, drawings, and equations). Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20 (e.g., by using objects, drawings, and equations). Apply commutative and associative properties of addition to operations as strategies to add and subtract. Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and the relationship between addition and subtraction; relate the strategy to a written method. Subtract multiples of 10 in the range 10–90 from multiples of 10 in the range 10–90, using concrete models or drawings and strategies based on place value, properties of operations, and the relationship between addition and subtraction; relate the strategy to a written method.	Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing (e.g., by using drawings and equations). Fluently add and subtract within 20 using mental strategies. By the end of grade 2, know from memory all sums of two one-digit numbers. Fluently add and subtract within 100 using strategies based on place value, properties of operations, and the relationship between addition and subtraction. Add up to four two-digit numbers using strategies based on place value and properties of operations. Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and the relationship between addition and subtraction; relate the strategy to a written method. Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900. Explain why addition and subtraction strategies work, using place value and the properties of operations.	Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and the relationship between addition and subtraction

appropriate strategies in their classrooms and providing support catered to each child, since not all children will be in the same place in their math learning at the same time. The GPF is an example of content that is sequenced according to documented learning progressions.

BUILDING MATHEMATICAL PROFICIENCY

CONCEPTUAL AND PROCEDURAL KNOWLEDGE

Proficiency in each mathematical domain requires strong conceptual knowledge, procedural fluency, and process skills. Conceptual knowledge, the "comprehension of mathematical concepts, operations, and relations";[26] procedural fluency, "the ability to apply procedures accurately, efficiently, and flexibly";[27] and process skills are intertwined and are all needed for building proficiency. Research shows that **children who use computational strategies grounded in a conceptual understanding of the number system score higher in numeracy assessments compared to students relying on memorization alone.**[28] Each time a new concept is introduced, regardless of the grade, it is essential to build this conceptual understanding. At the same time, procedural fluency is crucial for both reinforcing previous concepts and learning new concepts. Consider, for example, young students learning to add single-digit numbers. As part of learning basic addition, students learn the commutative property—in other words, that 3 + 4 is the same as 4 + 3. This conceptual knowledge then helps strengthen procedural fluency with basic addition by cutting in half the number of problems they need to know quickly and easily. This fluency, in turn, allows them to apply what they know to adding double-digit numbers such as 23 + 24.

USING A 100s CHART

A 100s chart is a simple but powerful learning aid that can help students in the early grades master number and operations. The chart helps students see and understand the patterns and relationships between numbers up to 100 and build both conceptual understanding and procedural fluency by using the chart to support activities such as skip-counting, identifying patterns, and addition and subtraction. For example, when students first begin learning how to solve problems such as 34 + 52, the 100s chart provides a visual scaffold on how to create different strategies to solve the problem—such as by starting in the box for 34 and counting down 5 rows (50) and then to the right 2 columns (2) to get 86. Eventually, students will be able to replicate this process without having to refer to the chart. Thus, the conceptual sense of number patterns that students gain by using the 100s chart will also help them gain fluency.

Numbers 1-100

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

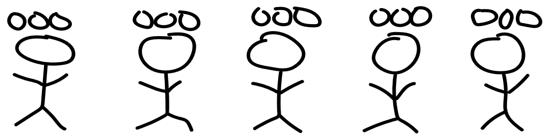
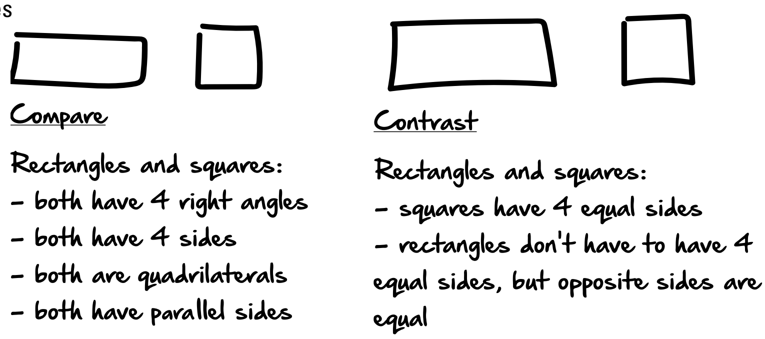
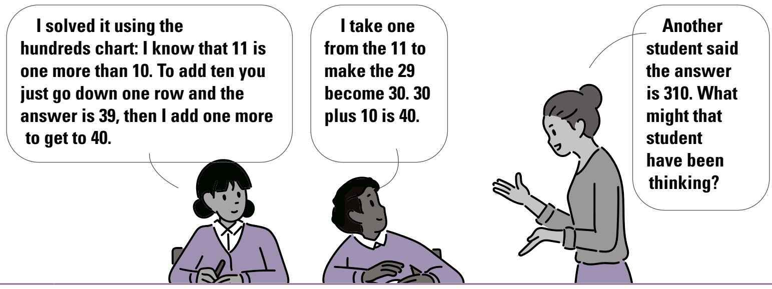
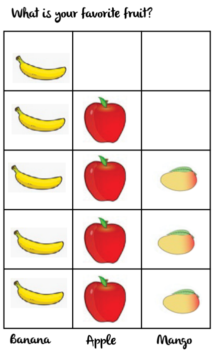
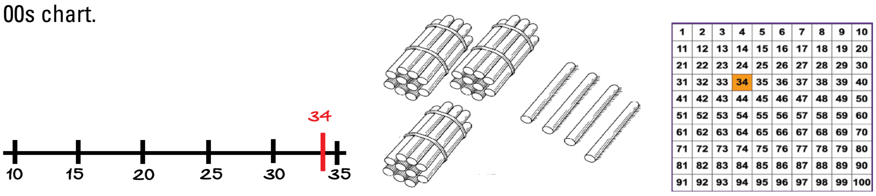
PROCESS SKILLS

Process skills are the processes students use to build and apply mathematical proficiency. As young children develop process skills, they are able to engage in more complex mathematics.[29] These process skills are built and supported by engaging children in diverse and challenging mathematical activities that require higher-order thinking.[30] The National Council of Teachers of Math defines process skills as the following, and they are illustrated in Figure 2:

- solving abstract and real-life problems and building mathematical knowledge through problem solving
- recognizing and using reasoning and proof as fundamental aspects of mathematics
- communicating mathematical ideas clearly and coherently
- recognizing and using connections among mathematical ideas and between mathematics and other disciplines
- using representations to model and interpret mathematical concepts[31]

Supporting the development of process skills in turn helps children build conceptual and procedural fluency, and many of the process skills work together. For example, classroom discussions where students explain their thinking and consider multiple possible strategies to solve a problem provide opportunities for them to practice applying reasoning and proof, as well as to communicate mathematical ideas. This classroom "math talk," which demands rich tasks for children to solve and probing questions from teachers, leads to the development of deeper understanding and proficiency.[33] Figure 3 provides an example of questions that teachers might pose to generate rich "math talk."

FIGURE 2. Core process skills for mathematics

<table>
<tr><th>Process skill</th><th>Illustrations from the classroom</th><th></th></tr>
<tr><td>Solving abstract and real-life problems</td><td>Students are asked to solve a problem about how many bottle caps are needed if there are 15 bottle caps that need to be shared among 5 students. The teacher encourages students to solve the problem in different ways, such as through drawings and using equations.</td><td></td></tr>
<tr><td>Recognizing and using reasoning and proof as fundamental mathematical ideas</td><td>Students are asked to prove that all squares are rectangles using their knowledge of the attributes of rectangles and squares.</td><td>
</td></tr>
<tr><td>Communicating mathematical ideas clearly and coherently</td><td>Students and teachers engage in math talk about the problem 29 + 11, explaining multiple ways to solve the problem and reasoning about incorrect solutions.</td><td>
</td></tr>
<tr><td>Recognizing and using connections between mathematical ideas</td><td>The students in a class take a poll to find out each student's favorite fruit. They then work together to make a pictogram of the results. They come up with questions and answers using the pictogram—such as "What is the most popular fruit in the class? What is the least popular? How many more people like apples than mangoes?" Finally, they discuss the operations they will use to find the answers.</td><td>
</td></tr>
<tr><td>Representing mathematical ideas or concepts with models</td><td>A student shows the number 34 in multiple ways: using a number line, counting sticks, and a 100s chart.</td><td>
</td></tr>
</table>

Strong process skills also allow students to apply the knowledge they are building to solve real-world problems that arise in their lives. Students can see links among mathematical domains and other subject areas, such as the role of data in science, which creates new and powerful understanding of the world.

CURRICULA AND STANDARDS IN LMICS

While the core domain areas of mathematics are often represented in math curricula and standards in LMICs, research on early-grade math curricula shows that curricular content is often not in line with learning progressions and that there is a heavy emphasis on procedural knowledge over conceptual knowledge and process skills.[34]

FIGURE 3. Using questions to support rich "math talk"[32]

How did you get that answer?
How do you know?
What do you notice about…?
How is this the same or different from…?
How else could you have solved that problem?

For example:

- **Progressions between grades**: The cognitive jump from preprimary, where math is often hardly discussed, to first grade can be huge. In primary, math curricula tend to be overloaded, and the speed at which content is covered is not apace with student learning.[35] In Pakistan, a national assessment conducted in 2015 found that only three-fifths of third grade students in urban areas could correctly perform a subtraction problem (54 – 25); in rural areas, only two-fifths could.[36]
- **Progression and connections between topics:** Progressions and connections across math topics may not be well aligned. For example, in Mozambique, textbooks were revised to include more real-life problems, but content related to measuring objects using centimeters and meters, which wasn't taught until second grade, was present in activities in the first-grade textbook.[37]
- **Unrealistic expectations**: In many contexts, curriculum is overloaded and not apace with students' learning. For example, in Uganda, students are expected to know the multiplication table up to 6, division by 1, 2, 3, and 4, and how to add fractions by the end of second grade. This is out of line with most curricula in the region and the developed world, where students are generally taught these skills in third and fourth grade.[38]
- **Emphasis on procedural knowledge over conceptual knowledge**: In many contexts, curriculum and instruction have tended to favor using rote memorization and focus primarily on learning procedures rather than understanding math concepts. For example, in Ghana, a review undertaken by the Ministry of Education into why students score poorly on standardized math test scores revealed a number of factors, including "the nature of the curriculum, an inadequate supply of mathematics teaching/learning materials, weak teacher content knowledge and teachers' use of ineffective mathematics teaching practices."[39] And despite a major curricular reform process, which also drastically modified in-service teacher training, teachers continue to emphasize procedural skills over building conceptual knowledge; one reason for this is because of misalignment between the curriculum for teacher training institutes and school curricula.[40]

SUGGESTIONS

- **Math curricula should include core domains and follow a learning progression framework (such as the GPF).** Children should receive instruction in five core domains: number and operations, measurement, statistics, geometry, and algebra. Learning in these domains should follow a clear learning progression to support engagement and mastery.
- **Math curricula should include a specific focus on building conceptual understanding, procedural fluency, and development of process skills.** A holistic mathematics curriculum helps children understand math concepts and not just memorize a set of rules and procedures. Learning how to solve math problems procedurally, without understanding the underlying concept, leads to a shallow understanding of mathematics and inability to use conceptual and procedural knowledge to solve problems.
- **Schools should adopt a "spiral" curriculum, which supports conceptual understanding by spacing out content and revisiting concepts throughout the year.** This approach leads to greater retention and long-term learning. It also helps children learn complex and challenging material throughout the school year, deepening their understanding of a given concept each time it is taught.

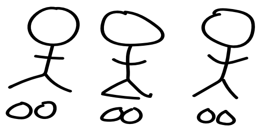
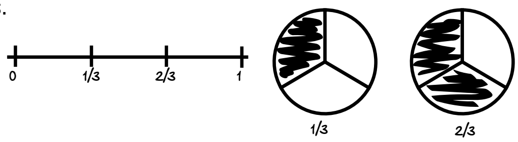
- **To develop new curricula or update current curricula, experts with various specialties are needed: those who have a deep understanding of learning progressions across domains in the country; those who have practical experience in the country; and those who have experience developing and publishing curricula in the country.** Mathematics curriculum development is often caught up in debates about what is considered developmentally appropriate (e.g., Should we teach fractions in the early grades? When should fractions instruction begin?). These types of questions are essential to discuss among experts, who should draw on research and local knowledge to decide what is appropriate.

Teachers and Teaching

HIGH-IMPACT INSTRUCTIONAL STRATEGIES

In the classroom, the instructional approach and strategies used by teachers has more impact on student learning than the type of textbook or curriculum used.[41] The following high-impact strategies[42] illustrated in Table 2 support the development of conceptual and procedural knowledge, as well as process skills, and have been linked to improved math learning outcomes:

TABLE 2. High-impact instructional strategies

Strategy	Example	Benefits to the student
Linking informal and formal mathematics	When introducing the concept of division, the teacher gives students a familiar, real-life problem to work on: Akilah has 6 mangoes. She and her two sisters want to share them equally. How many mangoes does each sister get?  After teaching the concept of division, the teacher asks students to apply their knowledge to solve complex problems, such as: There are 80 watermelons that need to be packed into boxes that hold 10 watermelons each. How many boxes do you need?	• Helps students bridge math they learn outside of school with the knowledge they learn inside of school • Linking informal and formal math provides students with a deeper understanding of math • Knowledge from out of school is formalized and represented with symbols such as + and = • Knowledge from school is given meaning through everyday application
Discussing mathematics	When discussing a problem, the teacher facilitates a discussion where multiple students share the strategies they used to arrive at the same solution. Students use **explanation and justification** to show why they think their solution is correct. The teacher uses **questioning** to ask students to explain why an incorrect solution is incorrect.	• Helps students learn to question their solution • Helps students understand and clarify key concepts • Contributes to developing new and deeper understanding • Supports development of new strategies • Holds students accountable for their own learning
Using appropriate models and representations	The teacher models for students two different ways of representing fractions and then asks them to practice using the models to compare fractions. 	• Enables children to "see" abstract mathematical concepts • Helps students reason concretely with mathematical ideas • Provides meaning to abstract symbols
Using knowledge of students and learning progressions to target instruction	Students are working on using the standard algorithm to solve the problem 28 + 53 Many students think that the answer is 71, not 81. The teacher realizes she needs to review the concept of place value before proceeding with the algorithm.	• Uses learning progressions to provide appropriate instruction • Allows for reteaching of concepts and skills when needed • Provides multiple opportunities and practice to build knowledge

Formative assessment is another important high-impact instructional strategy, though it is not unique to mathematics. **Embedding formative assessment into regular classroom practice, and linking it to the instructional strategies listed above, supports stronger student learning outcomes.**[43] **Learning gains are greatest when information from assessments is used to modify instruction and to identify ways to provide support to students.**[44] Simple and reliable assessment tools that are linked to key skills and associated remediation or enrichment activities should be used by teachers on an ongoing basis. Teacher training efforts must integrate the use of ongoing formative assessment into training plans, including time to practice administering and reviewing them. Teachers will need to be supported in managing this complex task and given concrete guidance on how to integrate it into daily practice, particularly for larger class sizes. See the guide on Assessment Informed Instruction: Classroom Level for a more detailed discussion of formative assessment.

TEACHER CONTENT KNOWLEDGE AND PEDAGOGICAL CONTENT KNOWLEDGE

In order to teach effectively, teachers need to develop both strong content knowledge (knowledge of mathematics) and strong pedagogical content knowledge (knowledge about teaching mathematics).[45] A "connected" understanding of early numeracy content that includes knowledge of how numeracy learning progresses and pedagogic content knowledge leads to stronger conceptual understanding and greater outcomes for children.[46] Hill and Ball found that in-service teachers demonstrated the greatest growth through professional development that focused on mathematical content and problems that they would face as teachers.[47]

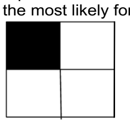
In LMICs, many teachers do not have a solid knowledge of foundational math skills, particularly those that require conceptual understanding. A survey across African countries found that over 90% of teachers were able to solve basic addition problems but that as content became more challenging and less procedural, the numbers declined: only 11% were able to interpret data in a graph and only 15% were able to solve word problems.[48] According to a World Bank study from Latin America and the Caribbean, 84% of teachers in Peru scored below level 2 in math, which is defined by the Peruvian Ministry of Education as "unable to establish mathematical relationships and adapt routine and simple mathematical procedures and strategies."[49]

When it comes to teaching math, many teachers rely on rote memorization techniques that favor procedural fluency over conceptual understanding.[50] This may reflect a struggle with both mathematical content knowledge and pedagogical content knowledge, where teachers may not know how to teach mathematical ideas conceptually and instead focus on the procedure, relying on how they learned math themselves. For example, many teachers know how to solve the problem 41+ 56 using the procedure of column addition, but do not know which strategies and models to use to help a student understand the algorithm. Teachers may know procedures for creating equivalent fractions, but may not understand how to represent equivalent fractions on a number line and area model. In Paraguay, research on teacher math practices revealed that 90% of preschool teachers did not understand the math content that they were teaching to their students,[51] and in South Africa, poor student performance has been tied to teachers' poor subject knowledge.[52] In Kenya, research has indicated that teachers often rely largely on rote memorization, recitation, and lecture to deliver information,[53] and the language of the classroom is formal and devoid of any connection to the real world.[54] **An approach to teacher training that sufficiently includes and connects both content and pedagogical content knowledge would produce teachers better prepared to teach early-grade math concepts.**

TEACHERS' PEDAGOGICAL CONTENT KNOWLEDGE

Problems such as the ones below can be used to understand teachers' existing knowledge and to develop teachers' knowledge during professional development sessions.

To the question "What part of the larger square does the shaded part represent?" the student answered "1/3." Which of the following reasons is the most likely for such answer by the student?



a. This is a correct answer
b. The student does not understand that the denominator must include all equal parts
c. The student made a counting error
d. I have difficulties answering

Almaz added 35 and 36 together and said that the total was 611. What can his teacher do to help Almaz?

a. Provide more similar practice problems
b. Review the topic of addition and subtraction up to 20
c. Review the topic of the place value of numbers
d. I have difficulties answering

TEACHER PROFESSIONAL DEVELOPMENT IN LMICS

Pre-service teacher education across many LMICs faces numerous challenges, as discussed in the how-to guide on Pre-service Teacher Education. A UNESCO report on teacher preparation in Latin America found that pre-service programs did not provide enough time for or focus on content mastery and active learning, with teaching practica in schools sorely limited or entirely missing.[55] While teachers may want to use strategies such as discussions, small group work, and making connections to the outside world in their classrooms, exemplars of this "connected" way of teaching in their contexts are limited.[56] Their default, then, is to fall back on traditional strategies such as rote memorization and recitation.[57]

Teacher pre-service and training programs for mathematics typically employ faculty who may have strong content knowledge but who are not adequately prepared to teach math concepts, particularly in the early primary grades.[59] One study found that mathematics teacher trainers in Ghana struggle to provide appropriate guidance in the use of teaching and learning materials, a critical component of teaching and learning in the early years. Teachers are often not given space to "reflect, analyse, or critique how a specific method would work for teaching their future students in real classroom contexts."[60] Teacher trainers are also not required to have school teaching experience—and when they do have such experience, it is usually from the middle and secondary levels. Teacher preparation programs should ensure that faculty have practical knowledge of the current situations in classrooms and can provide knowledge on how to teach, in addition to what to teach, in these contexts.

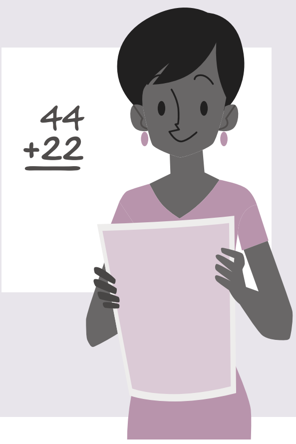
AUTHENTIC PROBLEM SOLVING [58]

In an effort to improve math student learning outcomes in Ghana, a new primary math curriculum was developed that focuses on increasing students' mathematical conceptual understanding, using physical models to represent math concepts, and emphasizing students' mental computation skills. With support from the USAID-funded Ghana Learning program, weekly learning circles helped teachers move away from a focus on memorization of procedures to constructing their own understanding of the math concepts underlying procedures. Teachers engaged in authentic math problem solving.

At the end of the intervention period, a majority of teachers (89%) felt that the in-service program increased their knowledge of mathematics. Approximately three-quarters of the teachers felt that the program was more effective than traditional in-service programs in helping them try out new math instructional practices in the classroom and in helping them understand new models and practices.

Changing teacher practice takes time, and not all teachers will be comfortable implementing new ideas right away. It is important to meet teachers where they are and provide opportunities for growth at all levels. Sitabkhan et al. discuss four different entry points in teachers' professional development pathways and provide exemplars of how to support teachers in understanding and implementing high-impact strategies in their classrooms, depending on their level of skill and knowledge (see Figure 4).[61] Through strengthened and coordinated pre-service, in-service, and ongoing teacher support, teachers can gain the knowledge and skills needed to apply the high-impact instructional strategies in their classrooms.

FIGURE 4. Teacher professional pathway for discussing mathematics



Building teacher knowledge

During professional development, teachers practice solving problems themselves, exploring multiple pathways to the solution, and explaining their solution to one another.

Beginning to integrate use

The teacher presents a problem to the class, and asks at least 3 students to share their answer. The teacher asks one student with the correct answer to explain their solution.

Delving deeper

The teacher presents a problem to the class, asks multiple students to share their answers, and then asks students to explain and justify their answers.

Expanding and refining

The teacher engages students in a mathematical discussion, asking questions of students with both the correct and incorrect solution. The teacher and students listen to one another and provide feedback when needed.

SUGGESTIONS

- **Build a cadre of pre-service faculty, in-service trainers, and coaches with math content and pedagogical knowledge.** New teachers need support, encouragement, and role modeling. They should receive consistent support from a subject-matter specialist who observes lessons and models effective teaching, taking into consideration actual classroom dynamics.
- **Ensure that pre-service and continuing professional development includes and connects both mathematics content knowledge and pedagogical content knowledge.** Teacher training programs must address math concepts across the five domains using flexible problem-solving approaches rooted in conceptual understanding. An emphasis on deep understanding—rather than on only procedural steps to problem solving—must form the foundation of preparation programs, especially for abstract concepts such as place value and the concept of even and odd numbers. Showing teachers how to make connections within and across other domains—for example, showing the link between addition and multiplication, and how the area of a shape is related to its geometric properties—is fundamental to helping teachers develop a "connected" understanding of mathematics. Teachers should engage in authentic problem solving so they can "see" how to solve a problem in multiple ways and learn how to explain and justify their answers. This type of practice will prepare teachers to be able to lead a discussion where students are sharing multiple answers.
- **Pair teacher trainees with master teachers for a practicum experience that exposes trainees to real classroom situations.** The practicum is critical for pre-service teachers and must be of a duration where they can meaningfully apply knowledge and skills learned during their preparation program. Master teachers must recognize their role in modeling strong instruction to trainees and providing them with rich experiences to develop skills. See also the how-to guide on Pre-service Teacher Education for further discussion. It is also critical for master teachers to help teacher trainees change gendered perceptions that math is a subject for boys. Only a small percentage of girls pursue careers in science or engineering, due largely to such perceptions among parents, teachers, and community members. Teachers must be strong advocates of girls' excellence in fields that require a strong math background.
- **Ensure that in-service training, in combination with ongoing support, embraces high-impact strategies and provides scaffolding for teachers to become more skilled over time.** Teachers can participate in training focused on effective teaching practices, but instructional change takes time. Meeting teachers where they are is critical to building confidence and success. **Professional development pathways, with various entry points depending on teacher skill and knowledge, can support teachers in understanding and implementing high-impact instructional strategies in their classrooms at their own pace and comfort level.** Additionally, some teaching methods promote higher levels of achievement to the advantage of girls, including presenting mathematical problems in gender-appropriate contexts; setting mathematical problems that promote deeper understanding; using collaborative methods in the classroom; and, using assessment methods that are not time-stressed.[62] These ideas should be included in pre-service teacher training programs. See the structured pedagogy how-to guides on Teacher Professional Development: Teacher Training and Teacher Professional Development: Ongoing Teacher Support.

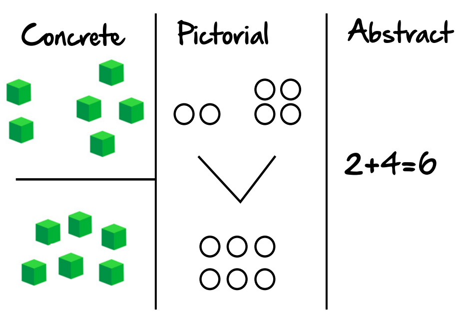
Teaching and Learning Materials

THE IMPORTANCE OF INSTRUCTIONAL MATERIALS

Instructional materials are a critical component of math teaching and learning. Teachers' guides, student workbooks, and textbooks are the most common type of instructional materials. Equally important are physical materials that can be moved and touched; these are often referred to as manipulatives and include counters, fingers, beads, popsicle sticks, geometric shapes, and so forth.

In addition to physical materials, pictorial or representational models (such as drawings, diagrams, and symbols) also help make abstract concepts clearer for young children. In the early primary grades, children may work first with concrete manipulatives, then use drawn or pictorial representations, and finally rely on abstract mathematical symbols to represent mathematical ideas or

FIGURE 5. Learning about addition using manipulatives, pictures, and then symbols



concepts.[63] Figure 5 shows how children learn about the concept of addition: first concretely with manipulatives such as blocks or sticks, then by using pictorial models, and finally by using abstract symbols.

Using materials to represent abstract concepts helps children reason more meaningfully,[64] and the appropriate and systematic integration of materials into the math classroom can have positive outcomes on learning.[65] The accurate use of materials has shown to improve skills ranging from counting to problem solving.[66] Conversely, if used inappropriately, materials may "result in frustration or confusion, and ultimately disrupt student motivation and overall learning opportunities."[67] Providing materials to classrooms and teachers without instruction or guidance has a relatively small impact on learning outcomes.[68]

MATERIALS AND TRAINING

Teachers must be well trained in the appropriate use of materials and should be knowledgeable on how to lead constructive discussions around student observations.[69] The context within which manipulatives are used is what creates meaning, and the talk and interaction between students and the teacher is what leads to understanding.[70] Carefully planning for the use of materials and knowing how different materials highlight important mathematical ideas is critical.[71] For example, in Figure 6, the students are using straws to help understand the concept of place value. They group the straws into bundles of 10, with single straws representing the ones. This manipulative allows students to "see" the structure of our number system and understand that the number 14, for example, is one bundle of 10 and 4 ones. Once students have internalized this concept, they no longer need to use the straws to "see" place value and can instead use place value to solve new problems. Using manipulatives to support understanding involves developing a lesson that includes the math content; identifies the type of material to be used, whether physical (such as counters) or a pictorial or representational model (such as a number line); and explains how the teacher will model using the materials and allow the children to practice. Teachers must then determine how many materials are needed, whether they will be used individually or in groups, how they will be distributed and stored, and how children will be guided on their use. Research shows that when children receive guidance on how to use materials appropriately, they achieve better learning outcomes.[72] To better support teachers in this regard, it is important to provide them with guides that specify how to use certain materials to teach specific math content and that provide support with sequencing the content and the materials.[73][74]

FIGURE 6. Using math manipulatives

Children using concrete manipulatives (straws) to learn about place value concepts.

Materials can be locally sourced and inexpensive (such as bottle tops, seeds, and pebbles as counters). While teachers can gather and create materials themselves, there is some evidence that providing them with the materials they need can help ensure that they are appropriate, ample in number, and are likely to be utilized.[75] It should also be recognized that keeping the diversity of materials to a manageable number helps ease the burden on teachers of collecting different manipulatives and other supplies and reduces the number of new instructional techniques teachers are required to know.[76] Figure 7 shows local counting and place value materials in Malawi and Ghana.

FUNDA WANDE'S STRUCTURED PEDAGOGY APPROACH

In Funda Wande's math program, Bala Wande, teachers receive a teacher's guide that lays out daily and weekly lesson objectives and indicates the type of teaching and learning activities required to meet learning targets, including how to integrate manipulatives into the lesson and what type of assessment to use for the particular topic being taught. Videos of master teachers teaching the content provide additional scaffolding and support for teachers, and a concept development section describes the mathematics concept targeted in the whole class teaching activities, the vocabulary teachers should use, and the specific mathematics-related behaviors they should look out for as pupils progress toward the targeted concepts.[74]

FIGURE 7. **Local math manipulatives in Malawi and and Ghana**

TEACHING AND LEARNING MATERIALS IN LMICS

Teaching and learning materials in early-grade mathematics classrooms are essential. They help young children make concepts like place value, addition, and multiplication more concrete. Using materials to represent abstract concepts helps children reason more meaningfully,[77] and the appropriate and meaningful integration of materials into the math classroom can have positive outcomes on learning.[78] In LMICs, textbooks and learning materials are often limited, with children having to share resources.[79] In sub-Saharan Africa, on average, three students share one mathematics book; in Cameroon specifically, on average, 14 students share one mathematics textbook.[80] In the Global Book Fund's first phase report, which looked at teaching and learning materials in 13 developing countries in the Caribbean, sub-Saharan Africa, the Indian subcontinent, and Southeast Asia, only three countries provided more than just basic textbooks and teachers' guides. Additionally, none of the countries surveyed achieved or maintained their own basic targets, even for the supply of textbooks and teachers' guides. Moreover, data suggest that governments in LMICs often prioritize secondary-grade textbooks over primary-grade ones, and upper-primary books over lower-primary ones.[81]

FIGURE 8. **Numeracy Boost math kit using locally sourced materials**

Most LMIC classrooms have scant materials aside from textbooks. While it is rare to see a classroom with a library, it is even more rare to see math materials in classrooms. Save the Children, as part of its Numeracy Boost intervention, provides a "math kit" to each classroom it works in (see Figure 8). These kits include rocks, bundles of straws, and other objects for sorting and counting, along with guidance for teachers on how to use them. Prior to receiving these kits, schools participating in the Numeracy Boost program in Bangladesh, El Salvador, Malawi, and Pakistan had no math materials.[82]

Research on the use of materials by teachers in LMICs is extremely limited. The evidence that does exist points to a lack of adequate training time and a methodology that does not allow teachers to reflect on practice and discuss the real-life applicability of material use in classrooms. This hinders teachers from using materials in the classroom, leading them to teach in ways that are more familiar to them[83] and to employ materials for demonstration purposes only while children watch passively.[84] Teachers also associate the use of materials as something "extra" or outside the curriculum.[85] In addition, factors like large class sizes and double-shifting can leave many teachers with little time to prepare for classroom teaching, including preparation of materials.[86] An exploratory pilot in Ghana provided sequenced numeracy activities to implement with six- and seven-year-olds.[87] Some of the teachers were given all of the materials needed to implement these activities, while other teachers were asked to create or gather them. Teachers who received all of the materials needed for a lesson were both more likely to use the activities provided to them and more likely to use high-impact teaching strategies compared to the other group.

SUGGESTIONS

- **Ensure that there is a minimum standard for teaching and learning materials in each classroom that includes a textbook for each student and a teacher's guide for each teacher.** The teacher's guide should include instructions linking lessons with materials and explaining how to use the materials. See additional discussion of this in the structured pedagogy how-to guide on Teaching and Learning Materials Development.
- **Provide all classrooms with a kit of manipulatives for student use.** A kit of locally sourced math materials is an essential component of the early-grade math classroom. The kit should focus on a few high-leverage materials that can be used for multiple concepts and topics, and it must include enough materials for all children to use in small groups.
- **Ensure that teachers have sufficient training, modeling, and practice on how to appropriately use the teaching and learning materials.** Materials to be used in the classroom must come with training and support that focuses on how to choose the right materials for the lesson being taught, how materials can be shared in small groups, and how materials should be handled and stored. Trainings should include appropriate modeling and demonstration by a specialized math trainer, and ample opportunity for practice. See the structured pedagogy guide on Teacher Professional Development: Teacher Training for more discussion of incorporating modeling and practice into training.

Conclusion

Policy makers, donors, and implementers should recognize the importance of early literacy and numeracy as core foundational topics, and they should emphasize each of them equally. The focus on early-grade reading has produced scholarship and research on promising practices to support literacy development in LMICs, which has improved the sector's understanding of successful implementation models and approaches to improving literacy outcomes at scale; similar work is necessary to better understand what works to strengthen math skills in these contexts. More early-grade math interventions that are well planned, developed in partnership with host-country governments, and based on rigorous research will help fill gaps in this field related to teacher content knowledge, pedagogy, and the use of teaching and learning materials. The creation of an incubator to test successful math approaches and programs developed in various LMICs could be a starting point for generating more information on what works in early math education.

Governments must do their part to ensure a minimum standard of teaching and learning materials in the classroom, starting with every child having their own textbook. Math trainers and curriculum developers in LMICs should help policy makers and government officials understand that children in the early grades develop their understanding of math through a concrete-pictorial-abstract progression and that teaching and learning materials help deepen students' understanding of abstract concepts. Using locally sourced math materials would not only benefit math learning but also show children that math is something that is all around us.

While not discussed here in detail, **gendered attitudes about who math is "for," as well as teachers' attitudes around who is "good" at math, must be addressed by the global community. Such perceptions hinder girls from making progress in math- and STEM-related careers, and much more must be done to change these norms.**

The following are some topics deserving of further research in LMICs:

- Early math topics that teachers struggle to teach effectively
- Most effective teaching strategies to increase conceptual understanding of math content

- Use of teaching and learning materials in the classroom
- In-service and pre-service math curricula and training methodologies
- Pre-service practicum experiences in early math
- Attitudes toward math among students, teachers, families, and community members

RESOURCES

A desk study by B. Atweh, M. Graven, and H. Venkat that addresses the development of numeracy in the early years of childhood and schooling in the context of low-income countries: https://www.academia.edu/21205934/Teaching_Numeracy_in_Pre_School_and_Early_Grades_in_Low_Income_Countries_GIZ_report_Atweh_Bose_Graven_Subraniam_and_Venkat

Global Reading Network paper for stakeholders involved in designing, implementing, or overseeing early-grade literacy and numeracy programs: https://www.globalreadingnetwork.net/resources/towards-design-and-implementation-comprehensive-primary-grade-literacy-and-numeracy

In-depth discussion of four instructional strategies that are key to effective mathematics instruction: https://shared.rti.org/content/instructional-strategies-mathematics-early-grades

Multiple studies from LMICs on the use of common instructional strategies in early-grades math classrooms: https://files.eric.ed.gov/fulltext/ED586780.pdf

"Bala Wande: Calculating with Confidence," an early-grade math program designed by mathematics experts in South Africa, featuring bilingual content: https://fundawande.org

Pratham's "Teaching at the Right Level," which groups students by learning needs and dedicates time to strengthening basic skills: https://www.teachingattherightlevel.org/

TECHNICAL EXPERTISE NEEDED

- **Early-grade math content and curriculum development expertise** for the alignment of national pre-service curricula with current evidence-based practices in early-grade mathematics and for the development of specific materials to support these curricula
- **Mathematics teacher training pedagogy expertise** for the strengthening of professional development programs for pre-service lecturers focused on modeling evidence-based instructional approaches to early-grade mathematics
- **Expertise related to partnerships between schools and pre-service training programs,** including the development of high-impact practicum programs that include opportunities for teachers to teach math lessons and receive constructive feedback
- **Expertise related to continuing professional development for mathematics instruction,** including the use of technology and blended learning to effectively support teacher development

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