Free Quality School Education

Ministry of Basic and Senior Secondary Education

Pupils' Handbook for

Senior Secondary *Mathematics*

SSS II

Term II

STRICTLY NOT FOR SALE

DOI: 10.5281/zenodo.3745370

Please see final page for further information.

Foreword

These Lesson Plans and the accompanying Pupils' Handbooks are essential educational resources for the promotion of quality education in senior secondary schools in Sierra Leone. As Minister of Basic and Senior Secondary Education, I am pleased with the professional competencies demonstrated by the writers of these educational materials in English Language and Mathematics.

The Lesson Plans give teachers the support they need to cover each element of the national curriculum, as well as prepare pupils for the West African Examinations Council's (WAEC) examinations. The practice activities in the Pupils' Handbooks are designed to support self-study by pupils, and to give them additional opportunities to learn independently. In total, we have produced 516 lesson plans and 516 practice activities – one for each lesson, in each term, in each year, for each class. The production of these materials in a matter of months is a remarkable achievement.

These plans have been written by experienced Sierra Leoneans together with international educators. They have been reviewed by officials of my Ministry to ensure that they meet the specific needs of the Sierra Leonean population. They provide step-by-step guidance for each learning outcome, using a range of recognized techniques to deliver the best teaching.

I call on all teachers and heads of schools across the country to make the best use of these materials. We are supporting our teachers through a detailed training programme designed specifically for these new lesson plans. It is really important that the Lesson Plans and Pupils' Handbooks are used, together with any other materials they may have.

This is just the start of educational transformation in Sierra Leone as pronounced by His Excellency, the President of the Republic of Sierra Leone, Brigadier Rtd Julius Maada Bio. I am committed to continue to strive for the changes that will make our country stronger and better.

I do thank our partners for their continued support. Finally, I also thank the teachers of our country for their hard work in securing our future.

Timbo

Mr. Alpha Osman Timbo

Minister of Basic and Senior Secondary Education

The policy of the Ministry of Basic and Senior Secondary Education, Sierra Leone, on textbooks stipulates that every printed book should have a lifespan of three years.

To achieve thus, <u>DO NOT WRITE IN THE BOOKS</u>.

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Introduction

to the Pupils' Handbook

These practice activities are aligned to the Lesson Plans, and are based on the National Curriculum and the West Africa Examination Council syllabus guidelines. They meet the requirements established by the Ministry of Basic and Senior Secondary Education.

1. The practice activities will not take the whole term, so use any extra time to revise material or re-do activities where you made mistakes.
2. Use other textbooks or resources to help you learn better and practise what you have learned in the lessons.
3. Read the questions carefully before answering them. After completing the practice activities, check your answers using the answer key at the end of the book.
4. Make sure you understand the learning outcomes for the practice activities and check to see that you have achieved them. Each lesson plan shows these using the symbol to the right.

   Learning Outcomes
5. Organise yourself so that you have enough time to complete all of the practice activities. If there is time, quickly revise what you learned in the lesson before starting the practice activities. If it is taking you too long to complete the activities, you may need more practice on that particular topic.
6. Seek help from your teacher or your peers if you are having trouble completing the practice activities independently.
7. Make sure you write the answers in your exercise book in a clear and systematic way so that your teacher can check your work and you can refer back to it when you prepare for examinations.
8. Congratulate yourself when you get questions right! Do not worry if you do not get the right answer – ask for help and continue practising!

KEY TAKEAWAYS FROM SIERRA LEONE'S PERFORMANCE IN WEST AFRICAN SENIOR SCHOOL CERTIFICATE EXAMINATION – GENERAL MATHEMATICS[1]

This section, seeks to outline key takeaways from assessing Sierra Leonean pupils' responses on the West African Senior School Certificate Examination. The common errors pupils make are highlighted below with the intention of giving teachers an insight into areas to focus on, to improve pupil performance on the examination. Suggestions are provided for addressing these issues.

Common errors

1. Errors in applying principles of BODMAS
2. Mistakes in simplifying fractions
3. Errors in application of Maths learned in class to real-life situations, and vis-a-versa.
4. Errors in solving geometric constructions.
5. Mistakes in solving problems on circle theorems.
6. Proofs are often left out from solutions, derivations are often missing from quadratic equations.

Suggested solutions

1. Practice answering questions to the detail requested
2. Practice re-reading questions to make sure all the components are answered.
3. If possible, procure as many geometry sets to practice geometry construction.
4. Check that depth and level of the lesson taught is appropriate for the grade level.

[1] This information is derived from an evaluation of WAEC Examiner Reports, as well as input from WAEC examiners and Sierra Leonean teachers.

Lesson Title: Sequences	Theme: Numbers and Numeration
Practice Activity: PHM2-L049	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to determine the rule that generates a sequence of terms, and extend the sequence.

Overview

A sequence is a list of numbers that follows a certain rule. For example, these are all sequences:

a. 2, 4, 8, 16, 32, …
b. 1, 4, 9, 16, 25, …
c. 1, 3, 5, 7, 9, …

Every sequence has a rule. For example, in sequence a., the rule is that each term is multiplied by 2 to get the next term. Or, we can say that each term is twice the previous term.

The **general term** can be found for a given sequence. The general term describes any term in the sequence, and uses the variable n. n represents a number's place in the sequence.

For example, look at sequence a., 2 has place $n = 1$, 4 has place $n = 2$, 8 has place $n = 3$, and so on.

The general term for a is 2^n. Each number in the sequence can be calculated by raising 2 to the power of its place (its value of n).

The general term can be used to find any term in a sequence. For example, if you want to know the 100$^{\text{th}}$ term of a sequence, substitute $n = 100$ into its general term.

Solved Examples

1. For the sequence 5, 10, 15, 20, 25, …
   a. Find the next 3 terms.
   b. Find the general term.
   Solution:
   a. The sequence counts by 5's. Thus, the next 3 terms are: 30, 35, 40.
   b. The general term is $5n$. We can observe this because each term of the sequence is 5 multiplied by its value of n.

2. Find the 10$^{\text{th}}$ term in the sequence whose general term is $2n + 4$
   Solution:

Substitute $n = 10$ into the general term:

$$\begin{aligned} 2n + 4 &= 2(10) + 4 \\ &= 20 + 4 \\ &= 24 \end{aligned}$$

3. Determine the rule that gives the following sequence and write the next three terms.
   a. $3, 6, 12, 24, 48, \dots$
   b. $1, 4, 7, 10, 13, \dots$
   c. $1, 5, 9, 13, 17, \dots$
   d. $20, 15, 10, 5, 0, \dots$

   Solution:
   a. Each term is multiplied by 2 to get the next term; each term is twice the previous term. The next three terms are: $96, 192$ and 384.
   b. A 3 is added to each term to get the next term. The next 3 terms are: $16, 19$ and 22.
   c. A 4 is added to each term to get the next term. The next 3 terms are: $21, 25$ and 29.
   d. A 5 is subtracted from each term to get the next term. The next 3 terms are: $-5, -10$, and -15.

4. Find the next 4 terms of each sequence. Then write down the general term.
   a. $2, 3, 4, 5, \dots$
   b. $3, 6, 9, 12, \dots$

   Solution:
   a. Next 4 terms: $6, 7, 8, 9$ ...
      General term: $n + 1$
   b. Next 4 terms: $15, 18, 21, 24, \dots$
      General term: $3n$

5. Find the following:
   a. The 7th term in a sequence whose general term is $2n + 4$
   b. The 10th term in a sequence whose n th term is $4n - 1$

   Solution:
   a. Substitute 7 in the general term: $2(7) + 4 = 14 + 4 = 18$
   b. Substitute 10 in the general term: $4(10) - 1 = 40 - 1 = 39$

6. Find the 4th and 9th terms of the sequence whose nth term is:
   a. $3n + 1$
   b. $n^2 + 2$

   Solution:
   a. The 4th term is: $3(4) + 1 = 12 + 1 = 13$

The 9th term is: $3(9) + 1 = 27 + 1 = 28$

b. The 4th term is: $4^2 + 2 = 16 + 2 = 18$

The 9th term is: $9^2 + 2 = 81 + 2 = 83$

Practice

1. For each of the following, find:
    a. The next three terms
    b. The rule that gives the terms
        i. $2, 4, 8, ...$
        ii. $3, 15, 75, 375, ...$
        iii. $10, 7, 4, 1, ...$
2. For the sequence $1, 3, 5, 7, ...$, find:
    a. The next 3 terms
    b. The general term
3. Find the 6th term in sequence whose general term is $4n - 3$
4. Find the 12th term in a sequence whose n^{th} term is $n^2 + 2$

Lesson Title: Arithmetic progressions	**Theme:** Numbers and Numeration
Practice Activity: PHM2-L050	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to define an arithmetic progression in terms of its common difference, d, and first term, a.

Overview

A sequence in which the terms either increase or decrease by a common difference is an **arithmetic progression**. It can be abbreviated to AP.

Consider the sequence: 5, 7, 9, 11, 13, …

The first term is 5 and the **common difference** is 2. The common difference is a difference that is the same between each term and the next term.

The letter a is commonly used to describe the first term, and the letter d is used for the common difference.

The general arithmetic progression is given as:

$$a, a + d, a + 2d, a + 3d, \ldots$$

The first term is a, and a difference of d is added to each subsequent term.

Solved Examples

1. Find a and d for the sequence $5, 10, 15, 20, \ldots$
   Solution:
   The first term is $a = 5$, the common difference is $d = 5$.

2. Write the first 6 terms of an AP where $a = -4$ and $d = 3$.
   Solution:
   Make the first term -4, and add 3 to get each subsequent term:
   $-4, -1, 2, 5, 8, 11, \ldots$

3. Identify the first term and common difference in each sequence below:
   a. $2, 4, 6, 8, \ldots$
   b. $10, 6, 2, -2, \ldots$
   c. $45, 15, -15, -45, \ldots$
   d. $-8, -6, -4, -2, \ldots$
   Solution:
   a. $a = 2, d = 2$
   b. $a = 10, d = -4$

c. $a = 45, d = -30$
d. $a = -8, d = 2$

4. Find a and d for each sequence. Then, find the next 3 terms.
   a. $2, 5, 8, 11, \ldots$
   b. $0, -3, -6, -9, \ldots$
   c. $-3, -7, -11, -15, \ldots$
   d. $6, 10, 14, 18, \ldots$
   e. $42, 38, 34, 30, \ldots$

   Solution:
   a. $a = 2,\ d = 3$; the next 3 terms: $14, 17, 20$
   b. $a = 0,\ d = -3$; the next 3 terms: $-12, -15, -18$
   c. $a = -3,\ d = -4$; the next 3 terms: $-19, -23, -27$
   d. $a = 6,\ d = 4$; the next 3 terms: $22, 26, 30$
   e. $a = 42$; $d = -4$; the next 3 terms: $26, 22, 18$

Practice

1. Find the first term, the common difference, and the next 3 terms in the following sequences:
   a. $-20, -17, -14, -11, \ldots$
   b. $0, -2, -4, -6, \ldots$
   c. $4, 7, 10, 13, \ldots$
   d. $3, 8, 13, 18, \ldots$
   e. $2\frac{1}{2}, 5, 7\frac{1}{2}, 10, \ldots$
2. For each of the following, write the first 4 terms of the A. P.
   a. $a = 3$ and $d = 3$
   b. $a = 2$ and $d = -2$
   c. $a = 10$ and $d = -5$
   d. $a = -2$ and $d = 4$
   e. $a = -5$ and $d = -6$

Lesson Title: Geometric progressions	**Theme:** Numbers and Numeration
Practice Activity: PHM2-L051	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to define a geometric progression in terms of its common ratio, r, and first term, a.

Overview

A sequence in which the terms either increase or decrease by a common ratio is a **geometric progression**. It can be abbreviated to GP.

Consider the sequence: 1, 2, 4, 8, 16, 32, …

The first term is 1 and the **common ratio** is 2. The common ratio is multiplied by each term to get the next term.

The letter a is commonly used to describe the first term, and the letter r is used for the common ratio.

The general geometric progression is given as:

$$a, ar, ar^2, ar^3, \ldots$$

where the first term is a, and the common ratio r is multiplied by each subsequent term.

Note these special cases for GPs:

- If the numbers decrease as the GP progresses, the common ratio must be a fraction. (Example: The sequence $32, 16, 8, 4, \ldots$, where $r = \frac{1}{2}$.)
- If the numbers alternate between positive and negative digits, the common ratio must be a negative value. (Example: The sequence $1, -2, 4, -8, 16, \ldots$, where $r = -2$)

Solved Examples

1. Find a and r for the sequence $2, 6, 18, 54, 162, \ldots$
   Solution:
   The first term is $a = 2$, the common ratio is $r = 3$. We can observe that each term of the sequence is 3 times the previous term.

2. Find a and r for the sequence $2, -8, 32, -128, 512, \ldots$
   Solution:
   Notice that the numbers alternate between positive and negative digits. Therefore, the common ratio must be a negative value.

The first term is $a = 2$, the common ratio is $r = -4$. Each term is -4 times the previous term.

3. Write the first 6 terms of a GP where $a = -8$ and $r = \frac{1}{2}$

   Solution:

   Write the first term, -8. Multiply this by $\frac{1}{2}$ to get the next term, and repeat this until there are 6 terms:

   $-8, -4, -2, -1, -\frac{1}{2}, -\frac{1}{4}, \ldots$

4. Find a and r for each sequence. Then, find the next 3 terms.
   a. $4, 8, 16, 32, \ldots$
   b. $-3, -6, -12, -24, \ldots$
   c. $2, -6, 18, -54, \ldots$
   d. $4096, 1024, 256, 64, \ldots$
   e. $\frac{3}{16384}, \frac{3}{4096}, \frac{3}{1024}, \frac{3}{256}, \ldots$

   Solution:
   a. $a = 4, r = 2$, next 3 terms: $64, 128, 256$
   b. $a = -3, r = 2$, next 3 terms: $-48, -96, -192$
   c. $a = 2, r = -3$, next 3 terms: $162, -486, 1458$
   d. $a = 4096, r = \frac{1}{4}$, next 3 terms: $16, 4, 1$
   e. $a = \frac{3}{16384}, r = 4$; next 4 terms: $\frac{3}{64}, \frac{3}{16}, \frac{3}{4},\ 3$

5. For each of the following, write the first 4 terms of the GP.
   a. $a = 2$ and $r = -4$
   b. $a = 729$ and $r = \frac{1}{3}$
   c. $a = \frac{1}{5}$ and $r = 5$
   d. $a = \frac{1}{10}$ and $r = 10$

   Solution:
   a. $2, -8, 32, -128, \ldots$
   b. $729, 243, 81, 27, \ldots$
   c. $\frac{1}{5}, 1, 5, 25, \ldots$
   d. $\frac{1}{10}, 1, 10, 100, \ldots$

Practice

1. For each of the following, find: a, r and the next 4 terms.
   a. $6, 12, 24, 48, \ldots$
   b. $1, -2, 4, -8, \ldots$

c. $\frac{1}{3}, \frac{1}{6}, \frac{1}{12}, \frac{1}{24}, \ldots$
d. $-8, 32, -128, 512, \ldots$
e. $128, 64, 32, 16, \ldots$
f. $\frac{1}{3}, -1, 3, -9, \ldots$
g. $0.05, 0.25, 1.25, 6.25, \ldots$

2. For each of the following, write the first 4 terms of the GP.
   a. $a = 2$ and $r = -3$
   b. $a = \frac{1}{3}$ and $r = \frac{1}{3}$
   c. $a = \frac{1}{9}$ and $r = 9$
   d. $a = 384$ and $r = \frac{1}{2}$
   e. $a = 3$ and $r = 4$
   f. $a = -2$ and $r = 4$
   g. $a = 64$ and $r = \frac{1}{2}$
   h. $a = 1.5$ and $r = 3$

Lesson Title: nth term of an arithmetic sequence	**Theme:** Numbers and Numeration
Practice Activity: PHM2-L052	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to apply the formula to find the nth term of an arithmetic sequence.

Overview

The below formula gives the general term of an AP. That is, it describes every term in the sequence:

$U_n = a + (n-1)d$, where U_n is the nth term of the AP.

The letter n gives the place of a term in the sequence. From previous lessons, a is the first term of the sequence and d is the common difference.

Solved Examples

1. Find the 10th term of the sequence 3, 8, 13, 18, 23, ….
   Solution:
   First, identify the values of a, n, and d. From the given sequence, $a = 3$, and $d = 5$. We want to find the 10^{th} term, so let $n = 10$.

$$\begin{array}{rcll} U_n & = & a + (n-1)d & \text{Formula} \\ U_{10} & = & 3 + (10-1)5 & \text{Substitute for } a, n, \text{ and } d \\ & = & 3 + (9)5 & \text{Simplify} \\ & = & 48 & \end{array}$$

2. Find the 8th term of the sequence 3, 6, 9, 12, ….
   Solution:
   First, identify the values of a, n, and d. From the given sequence, $a = 3$, and $d = 3$. We want to find the 8^{th} term, so let $n = 8$.

$$\begin{array}{rcll} U_n & = & a + (n-1)d & \\ U_8 & = & 3 + (8-1)3 & \text{Substitute for } a, n, \text{ and } d \\ & = & 3 + (7)3 & \text{Simplify} \\ & = & 24 & \end{array}$$

3. In the sequence $5, 11, 17, 23, \ldots.$, find:
   a. The 26^{th} term
   b. The 44^{th} term
   c. The n^{th} term
   Solution:
   First, identify the values of a and d. From the given sequence, $a = 5$, and $d = 6$. Substitute the given value of n for each part of the problem:

a.

$$\begin{aligned} U_n &= a + (n-1)d \\ U_{26} &= 5 + (26-1)6 && \text{Substitute for } a,\ d,\ n = 26 \\ &= 5 + (25)6 && \text{Simplify} \\ &= 155 \end{aligned}$$

b.

$$\begin{aligned} U_n &= a + (n-1)d \\ U_{44} &= 5 + (44-1)6 && \text{Substitute for } a,\ d,\ n = 44 \\ &= 5 + (43)6 && \text{Simplify} \\ &= 263 \end{aligned}$$

c. To find the nth term, substitute the given values of a and d, and simplify.

$$\begin{aligned} U_n &= a + (n-1)d \\ &= 5 + (n-1)6 && \text{Substitute for } a \text{ and } d \\ &= 5 + 6n - 6 && \text{Simplify} \\ &= 6n - 1 \end{aligned}$$

4. For the sequence $71, 70, 69, 68, \ldots,$ use the formula to find:
   a. The 14th term
   b. The 110th term

   Solutions:

   First, identify the values of a and d. From the given sequence, $a = 71$, and $d = -1$. Substitute the given value of n for each part of the problem:

   a.

$$\begin{aligned} U_n &= a + (n-1)d \\ U_{14} &= 71 + (14-1)(-1) && \text{Substitute for } a,\ d,\ n = 14 \\ &= 71 + (13)(-1) && \text{Simplify} \\ &= 71 - 13 \\ &= 58 \end{aligned}$$

   b.

$$\begin{aligned} U_n &= a + (n-1)d \\ U_{110} &= 71 + (110-1)(-1) && \text{Substitute for } a,\ d,\ n = 110 \\ &= 71 + (109)(-1) && \text{Simplify} \\ &= 71 - 109 \\ &= -38 \end{aligned}$$

5. Find the 8th and 17th terms of the AP whose first term is 6 and common difference is 7.

   Solution:

$$\begin{aligned} U_n &= a + (n-1)d \\ U_8 &= 6 + (8-1)7 && \text{Substitute for } a,\ d,\ n = 8 \\ &= 6 + (7)7 && \text{Simplify} \\ &= 55 \end{aligned}$$

$$
\begin{aligned}
U_{17} &= 6+(17-1)7 && \text{Substitute for } a,\ d,\ n=17\\
&= 6+(16)7 && \text{Simplify}\\
&= 118
\end{aligned}
$$

6. How many terms are in an AP if the first term is 15, the common difference is 3, and the last term is 57?

   Solution:

   We are given the values $a = 15$, $d = 3$, and $U_n = 57$. We want to find the value of n. Substitute the given values into the formula, and solve for n.

$$
\begin{aligned}
U_n &= a+(n-1)d\\
57 &= 15+(n-1)3 && \text{Substitute } a,\ d, \text{ and } U_n\\
57 &= 15+3n-3 && \text{Simplify}\\
57 &= 3n+12 && \text{Solve for } n\\
57-12 &= 3n\\
45 &= 3n\\
\frac{45}{3} &= \frac{3n}{3}\\
n &= 15
\end{aligned}
$$

   The number of terms in the AP is 15.

Practice

1. In the sequence $4, 11, 18, 25,$ ..... Find:
   a. The 24th terms
   b. The 99th terms
   c. The n^{th} terms
2. In the sequence $50, 46, 42, 38,$ .... Find:
   a. The 8th term
   b. The 100th term
   c. The n^{th} term
3. Find the 16th and 24th terms of the AP whose first term is 8 and common difference is 6.
4. How many terms does an AP have if first term is 24, the common difference is 4, and the last term is 60?

Lesson Title: nth term of a geometric sequence	**Theme:** Numbers and Numeration
Practice Activity: PHM2-L053	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to apply the formula to find the nth term of a geometric sequence.

Overview

The formula below gives the general term of a GP. That is, it describes every term in the sequence:

$U_n = ar^{n-1}$, where U_n is the nth term of the GP

The letter n gives the place of a term in the sequence. From previous lessons, a is the first term of the sequence and r is the common ratio.

Solved Examples

1. Find the 10th term of the sequence $1, 2, 4, 8, 16, \ldots$
   Solution:
   First, identify the values of a, n, and r. From the given sequence, $a = 1$, and $r = 2$. We want to find the 10th term, so let $n = 10$.

$$\begin{aligned} U_n &= ar^{n-1} && \\ U_{10} &= 1(2^{10-1}) && \text{Substitute for } a, n, \text{ and } r \\ &= 2^9 && \text{Simplify} \\ &= 512 && \end{aligned}$$

2. For the GP $3, 9, 27, 81$ ..., find:
   a. The 5th term
   b. the 15th term
   c. the nth term
   Solution:
   First, identify the values of a and r. From the sequence, $a = 3$ and $r = 3$.
   Substitute the given value of n for each part of the problem:
   a.

$$\begin{aligned} U_n &= ar^{n-1} && \\ U_5 &= 3(3^{5-1}) && \text{Substitute for } a, n, \text{ and } r \\ &= 3(3^4) && \text{Simplify} \\ &= 243 && \end{aligned}$$

   b.

$$\begin{aligned} U_n &= ar^{n-1} && \\ U_{15} &= 3(3^{15-1}) && \text{Substitute for } a, n, \text{ and } r \end{aligned}$$

$$
\begin{aligned}
&= 3(3^{14}) && \text{Simplify} \\
&= 14{,}348{,}907
\end{aligned}
$$

c. To find the nth term, substitute the given values of a and r, and simplify.

$$
\begin{aligned}
U_n &= ar^{n-1} \\
&= 3(3^{n-1}) \\
&= 3^{1+n-1} \\
&= 3^n
\end{aligned}
$$

3. Find the 6th term of the G.P $3, 6, 12, 24, \dots$

Solution:

First, identify the values of a, n, and r. From the given sequence, $a = 3$, and $r = 2$. We want to find the 6th term, so let $n = 6$.

$$
\begin{aligned}
U_n &= ar^{n-1} \\
U_6 &= 3(2^{6-1}) \\
&= 3(2^5) \\
&= 3(32) \\
&= 96
\end{aligned}
$$

4. Find the 10th term of the GP whose sequence is $1{,}024, 512, 256, 128, \dots$

Solution:

First, identify the values of a, n, and r. From the given sequence, $a = 1024$, and $r = \frac{1}{2}$. Remember that when the terms of a GP decrease, the value of r is a fraction. We want to find the 10th term, so let $n = 10$.

$$
\begin{aligned}
U_n &= ar^{n-1} \\
U_{10} &= 1{,}024\left(\frac{1}{2}\right)^{10-1} \\
&= 1{,}024\left(\frac{1}{2}\right)^{9} \\
&= 1{,}024\left(\frac{1^9}{2^9}\right) \\
&= 1{,}024\left(\frac{1}{512}\right) \\
&= \frac{1{,}024}{512} = 2
\end{aligned}
$$

5. The 5th term of a GP is 2,500. Find the first term if its common ratio is 5.

Solution:

We are given the values $r = 5$ and $U_5 = 2{,}500$. We want to find the value of a. Substitute the given values into the formula, and solve for a.

$$
\begin{aligned}
U_n &= ar^{n-1} \\
2{,}500 &= a(5^{5-1}) && \text{Substitute for } r, n, \text{ and } U_n \\
2{,}500 &= a(5^4) && \text{Simplify} \\
2{,}500 &= 625a && \text{Solve for } a \\
\frac{2{,}500}{625} &= a \\
a &= 4
\end{aligned}
$$

6. The 4th term of a GP is 108. If the first term is 4, find:
    a. The common ratio
    b. The 8th term

    Solutions:

    a. We are given the values $a = 4$ and $U_4 = 108$. We want to find the value of r. Substitute the given values into the formula, and solve for r.

$$\begin{aligned} U_n &= ar^{n-1} & \\ 108 &= 4r^{4-1} & \text{Substitute for } a, n, \text{ and } U_n \\ 108 &= 4r^3 & \text{Simplify} \\ \frac{108}{4} &= \frac{4r^3}{4} & \text{Solve for } r \\ 27 &= r^3 & \\ \sqrt[3]{27} &= r & \\ r &= 3 & \end{aligned}$$

    b. Use the formula to find the 8th term, U_8:

$$\begin{aligned} U_8 &= ar^{n-1} \\ &= 4(3^{8-1}) \\ &= 4(3^7) \\ &= 4(2187) \\ &= 8{,}748 \end{aligned}$$

Practice

1. The first term of a GP is 7 and the common ratio is 4. Find:
    a. The 3rd term
    b. The 10th term
2. For the GP $4, 8, 16, 32, \ldots$, find:
    a. The 8th term
    b. The 14th term
    c. The n^{th} term
3. The 7th term of a GP is 12,288. Find the first term if its common ratio is 4.
4. The 4th term of a GP is $1{,}728$. If the first term is 8, find the common ratio and the 5th term.
5. The following are the first 3 terms of a GP: $5, D, 1{,}125$. Find the value of D.

Lesson Title: Series	**Theme:** Numbers and Numeration
Practice Activity: PHM2-L054	**Class:** SSS 2

Learning Outcomes

By the end of the lesson, you will be able to:

1. Distinguish between a sequence and a series.
2. Find the sum of the terms of a series by adding.

Overview

When the terms of a sequence are added together, the result is a series.

Some series carry on forever. These are called **infinite series**. It is often impossible to find the sum of an infinite series. The following are examples of an infinite series:

$$2 + 4 + 6 + 8 + \ldots$$
$$50 + 100 + 150 + 200 + \ldots$$

Some series have a certain number of terms. They do not carry on forever, but end at a certain point. It is always possible to find the sum of a finite series. The following are examples of a finite series:

$$1 + 2 + 3 + 4 + 5$$
$$5 + 10 + 15 + \ldots + 100$$

Solved Examples

1. Find the sum of the series: $1 + 2 + 3 + 4 + 5$
   Solution:
   Simply add the terms together: $1 + 2 + 3 + 4 + 5 = 15$

2. Find the sum of the first 5 terms of an AP where $a = 4$ and $d = -2$.
   Solution:
   Step 1. List the first 5 terms of the AP: $4, 2, 0, -2, -4$.
   Step 2. Add the terms in a series:
   $$4 + 2 + 0 + (-2) + (-4) = 4 + 2 + 0 - 2 - 4 = 0$$

3. For the AP where $a = 4$ and $d = 10$:
   a. Write the first 6 terms of the sequence.
   b. Write the first 6 terms of the series.
   c. Find the sum of the first 4 terms.
   Solution:
   a. Write the sequence: $4, 14, 24, 34, 44, 54, \ldots$
   b. Write the series: $4 + 14 + 24 + 34 + 44 + 54 + \cdots$
   c. Find the sum of the first 4 terms: $4 + 14 + 24 + 34 = 76$

4. Find the sum of the finite series where $a = 3$, $d = 2$, and the last term is 15.
   Solution:
   Step 1. Write out the series: $3 + 5 + 7 + 9 + 11 + 13 + 15$
   Step 2. Add the terms: $3 + 5 + 7 + 9 + 11 + 13 + 15 = 63$

5. Find the sum of the first 5 terms of the AP where $a = -3$ and $d = -4$
   Solution:
   $-3 + (-7) + (-11) + (-15) + (-19) = -3 - 7 - 11 - 15 - 19 = -55$

6. For the AP where $a = -4$ and $d = -6$:
   a. Write the first 8 terms of the sequence.
   b. Write the first 8 terms of the series.
   c. Find the sum of the first 8 terms.

   Solution:
   a. Write the sequence: $-4, -10, -16, -22, -28, -34, -40, -46, \ldots$
   b. Write the series: $-4 + (-10) + (-16) + (-22) + (-28) + (-34) + (-40) + (-46)$
   c. Find the sum of the first 8 terms:
   $-4 - 10 - 16 - 22 - 28 - 34 - 40 - 46 = -200$

7. Find the sum of the first 6 terms of an AP where $a = 5$ and $d = -3$
   Solution:
   Step 1. List the first 6 terms of the AP: $5, 2, -1, -4, -7, -10$
   Step 2. Add the terms in the series:
   $$5 + 2 + (-1) + (-4) + (-7) + (-10)$$
   $$= 5 + 2 - 1 - 4 - 7 - 10 = -15$$

Practice

1. For the AP where $a = 3$ and $d = 5$:
   a. Write the first 7 terms of the sequence.
   b. Write the first 7 terms of the series.
   c. Find the sum of the first 7 terms.
2. Find the sum of the finite series: $5 + 8 + 11 + 14 + 17 + 20 + 23 + 26$.
3. Find the sum of the first 8 terms of the AP where $a = -2$ and $d = -5$.
4. Find the sum of the first 6 terms of the AP where $a = -10$ and $d = 4$.
5. For the AP where $a = -2$ and $d = -5$,
   a. Write the first 9 terms of the sequence.
   b. Write the first 9 terms of the series.
   c. Find the sum of the first 9 terms.
6. Find the sum of the first 5 terms of an AP where $a = 4$ and $d = -6$.

Lesson Title: The sum of an arithmetic series	**Theme:** Numbers and Numeration
Practice Activity: PHM2-L055	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to calculate the sum of the first n terms of an arithmetic series.

Overview

When we have a few terms of an AP, it is simple to add them together. In most cases there are more terms than can easily be added. For these, we use the following formula:

$$S_n = \frac{1}{2}n[2a + (n-1)d]$$

This is the formula for the sum of n terms of an AP. Remember that the variable n gives the number of terms, a gives the first term, and d gives the common difference.

Note that this formula can only be used for arithmetic progressions. It cannot be used for geometric progressions.

Solved Examples

1. Find the sum of the first 12 terms of the AP 10, 8, 6, 4.….
   Solution:
   First, identify the values of a, n, and d. From the given sequence, $a = 10$, and $d = -2$. We want to find the sum of the first 12 terms, so let $n = 12$.

$$\begin{aligned} S_n &= \frac{1}{2}n[2a + (n-1)d] && \\ S_{12} &= \frac{1}{2}(12)[2(10) + (12-1)(-2)] && \text{Substitute for } n, a, \text{ and } d \\ &= (6)[20 - 22] && \text{Simplify} \\ &= 6(-2) = -12 && \end{aligned}$$

2. An AP with 14 terms has a first term of 20, and a sum of 7. What is the common difference?
   Solution:
   We are given the values $a = 20$, $S = 7$, and $n = 14$. We want to find the value of d. Substitute the given values into the formula, and solve for d.

$$\begin{aligned} S_n &= \frac{1}{2}n[2a + (n-1)d] && \\ 7 &= \frac{1}{2}(14)[2(20) + (14-1)d] && \text{Substitute for } S, n, \text{ and } a \\ 7 &= 7[40 + 13d] && \text{Simplify} \\ 1 &= 40 + 13d && \text{Divide throughout by 7} \\ 1 - 40 &= 13d && \text{Transpose 40} \end{aligned}$$

$$-39 = 13d$$

$$\frac{-39}{13} \quad \frac{13d}{13} \qquad \text{Divide throughout by 13}$$

$$-3 = d$$

3. Find the sum of the first 11 terms of the sequence: $12, 10\frac{1}{2}, 9, \ldots$

   Solution:

   Identify the values of a, n, and d: $a = 12, n = 11, d = 12 - 10\frac{1}{2} = -1\frac{1}{2}$.

   Substitute and simplify:

$$\begin{aligned}
S_n &= \tfrac{1}{2}n[2a + (n-1)d] \\
S_{11} &= \tfrac{1}{2}(11)\left[2(12) + (11-1)\left(-1\tfrac{1}{2}\right)\right] \\
&= 5\tfrac{1}{2}\left[24 + (10)\left(-1\tfrac{1}{2}\right)\right] \\
&= 5\tfrac{1}{2}(24 - 15) \\
&= 5\tfrac{1}{2}(9) \\
&= 49\tfrac{1}{2}
\end{aligned}$$

4. Find the sum of the first 28 terms of the series $3 + 10 + 17 + \cdots$

   Solution:

   Here, $a = 3$, $d = 7$ and $n = 28$

$$\begin{aligned}
S_n &= \tfrac{1}{2}n[2a + (n-1)d] \\
S_{28} &= \tfrac{1}{2}(28)[2(3) + (28-1)7] \\
&= 14[6 + (27)7] \\
&= 14(6 + 189) \\
&= 14(195) \\
&= 2{,}730
\end{aligned}$$

5. If the first term of an AP is 4 and the common difference is 6, find the sum of the:
   a. First n terms.
   b. First fourteen terms.

   Solution:

   Note that $a = 4$ and $d = 6$.

   a. To find the nth term, substitute the given values of a and d, and simplify:

$$\begin{aligned}
S_n &= \tfrac{1}{2}n[2a + (n-1)d] \\
&= \tfrac{1}{2}n[2(4) + (n-1)6] \\
&= \tfrac{1}{2}n[8 + 6n - 6] \\
&= \tfrac{1}{2}n(6n + 2) \\
&= n(3n + 1)
\end{aligned}$$

   b. The sum of the first fourteen terms of the AP is:

$$
\begin{aligned}
S_{14} &= 14[3(14)+1] \\
&= 14(42+1) \\
&= 14(43) \\
&= 602
\end{aligned}
$$

6. Find the sum of the AP: $1, 6, 11, 16, \ldots, 186$

   Solution:

   First, find the number of terms, n, using the formula for the nth term (from lesson 52).

$$
\begin{aligned}
U_n & \quad a+(n-1)d & & \\
186 &= 1+(n-1)5 & & \text{Substitute } a=1,\ d=5, U_n=186 \\
186 &= 1+5n-5 & & \text{Simplify} \\
186 &= 5n-4 & & \text{Solve for } n \\
190 &= 5n & & \\
n &= \frac{190}{5} = 38 & &
\end{aligned}
$$

   Now we know that the AP has 38 terms, and can apply the formula to find its sum:

$$
\begin{aligned}
S_n &= \tfrac{1}{2}n[2a+(n-1)d] \\
S_{38} &= \tfrac{1}{2}(38)[2(1)+(38-1)(5)] \\
&= 19[2+(37)(5)] \\
&= 19(187) \\
&= 3{,}553
\end{aligned}
$$

Practice

1. Find the sum of the first 18 terms of an AP with first term 4, and common difference 3.
2. Find the sum of the first 20 terms of a series where $a = 22$ and $d = -2$.
3. A series has 10 terms, and the first term is -5. If the sum is 400, what is the common difference?
4. Find the sum of the sequence; $18, 15\frac{1}{2}, 13, 10\frac{1}{2}, \ldots$ to 84 terms.
5. Find the sum of the first 40 terms of the series: $2 + 5 + 8 + 11 \ldots$
6. If the first term of an AP is 8 and the common difference is 3, find the sum of:
   a. The first n terms
   b. The first twelve terms
7. Given the AP 2, 5, 8, 11, …, 89, find:
   a. The number of terms, n.
   b. The sum of the series.
8. An AP has 26 terms, and a sum of $1{,}040$, if the common difference is 2. What is the first term?

Lesson Title: Numerical and real-life problems involving sequences and series	Theme: Numbers and Numeration
Practice Activity: PHM2-L056	**Class:** SSS 2

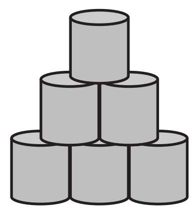
Learning Outcome
By the end of the lesson, you will be able to apply sequences and series to numerical and real-life problems.

Overview

This lesson applies the content on sequence and series from previous lessons to solve real-life problems.

Solved Examples

1. Mr. Bangura sells cans of fish in the market. He decided to display them in a nice way to attract more customers. He arranged them in a stack so that 1 can is in the top row, 2 cans are in the next row, 3 cans are in the third row, and so on. The bottom row has 14 cans of fish. The top 3 rows are shown below.
   a. Write a sequence based on this story.
   b. What is the total number of cans in the stack?



   Solutions:

   a. 1, 2, 3, 4, 5, …, 14.
   b. Calculate the sum of the series using $a = 1$, the number of cans in the first row; $n = 14$, the number of rows; and $d = 1$, the difference between each row and the next row.

$$
\begin{aligned}
S_n &= \tfrac{1}{2}n[2a + (n-1)d] \\
S_{14} &= \tfrac{1}{2}(14)[2(1) + (14-1)1] && \text{Substitute for } n, a, \text{ and } d \\
&= 7(2+13) && \text{Simplify} \\
&= 7(15) = 105
\end{aligned}
$$

2. A trader borrows ₦$4{,}000.00$ and agrees to pay with an interest of ₦116.00 in 12 monthly installments, each installment being less than the preceding installment of by ₦10.00. What should be his first installment?
   Solution:
   The symbol ₦ represents Naira, the currency of Nigeria. Treat the numbers as usual and label the answer with the correct symbol.

   Since the difference each month is 10 Naira, let $d = -10$. Let $n = 12$, because there are 12 monthly installments. The total sum S_{12} is the sum of the borrowed amount and interest. Therefore, $S_{12} = 4{,}000 + 116 =$ ₦$4{,}116.00$.

Substitute $n = 12$, $d = -10$ and $S_{12} = 4{,}116$ into the formula for the sum of the series, and solve for a to find the first installment.

$$\begin{aligned}
S_n &= \tfrac{1}{2}n[2a + (n-1)d] \\
4{,}116 &= \tfrac{12}{2}[2a + (12-1)(-10)] \\
4{,}116 &= 6[2a - (11)10] \\
4{,}116 &= 6[2a - 110] \\
\frac{4{,}116}{6} &= \frac{6[2a-110]}{6} \\
686 &= 2a - 110 \\
686 + 110 &= 2a \\
796 &= 2a \\
a &= 398
\end{aligned}$$

His first payment will be ₦398.00.

3. Mr. Williams decides to save part of his salary each month. In the first month, he saved ₦30.00. In the second month, he saved ₦50.00 and in the third month, he saved ₦70.00. He continued in this pattern for an entire year.
   a. Write a sequence for the word problem.
   b. How much did he save during the 8th month?
   c. How much did he save in total during the 12 months?

   Solutions:
   a. Sequence: $30, 50, 70, 90, \ldots$
   b. To find how much was saved during a certain month, use the formula for nth term of an AP:

$$\begin{aligned}
U_n &= a + (n-1)d \\
U_8 &= 30 + (8-1)20 \\
&= 30 + (7)20 \\
&= 30 + 140 \\
U_8 &= ₦170.00
\end{aligned}$$

   He was able to save ₦170.00 in the 8th month.

   c. Use the formula to find the sum of his savings during the 12 months:

$$\begin{aligned}
S_n &= \tfrac{1}{2}n[2a + (n-1)d] \\
S_{12} &= \tfrac{12}{2}[2(30) + (12-1)20] \\
&= 6(60 + 220) \\
&= 6(280) \\
&= ₦1{,}680.00
\end{aligned}$$

4. Mr. Edward, a house owner, decides to rent his house monthly. In January, he rented the house for Le 100,000.00, and he increased his rent by Le 20,000.00 each month.
   a. How much did his renter pay in October?
   b. What was his total income from rent for the year?

   Solutions:

a. Apply the formula to find the 10th term of the arithmetic sequence. Note that $a = Le\ 100{,}000.00$, $d = Le\ 20{,}000.00$ and $n = 10$.

$$\begin{aligned} U_n &= a + (n-1)d \\ U_{10} &= 100{,}000 + (10-1)20{,}000 \\ &= 100{,}000 + 180{,}000 \\ &= Le\ 280{,}000.00.00 \end{aligned}$$

The rent in October was $Le\ 280{,}000.00$

b. Apply the formula for finding the sum of a sequence. Note that $n = 12$.

$$\begin{aligned} S &= \tfrac{n}{2}[2a + (n-1)d] \\ &= \tfrac{12}{2}[2(100{,}000) + (12-1)20{,}000] \\ &= 6[200{,}000 + 220{,}000] \\ &= 6[420{,}000] \\ &= Le\ 2{,}520{,}000 \end{aligned}$$

His income from rent for the year was $Le\ 2{,}520{,}000.00$

Practice

1. Michael took a loan of ₦$2{,}250.00$ from his brother, who doesn't charge him interest. He pays a certain amount the first month and then decreases the payment by ₦5.00 every month. If it takes him 20 months to pay the full loan, how much was the first payment?
2. Agnes, a pupil with five credits in WASSCE, decides to save some money for a university course. In the first month she saved $Le\ 20.00$. In the second month, she saved $Le\ 40.00$. She continued with this pattern for 1 year.
   a. Write a sequence for the word problem.
   b. How much did she save during the 6th month?
   c. How much did she save in total during 12 months?
3. Musa's mother paid $Le\ 1{,}200.00$ in school fees for his first year of school. The tuition increases by $Le\ 300.00$ per year each year that he is in school.
   a. How much does his mother pay for his 5th year of school?
   b. What does she pay in total for the first 6 years?

Lesson Title: Characteristics of Quadrilaterals	**Theme:** Geometry
Practice Activity: PHM2-L057	**Class:** SSS 2

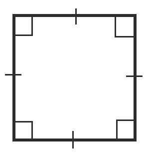
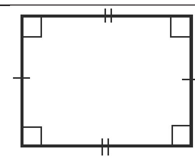
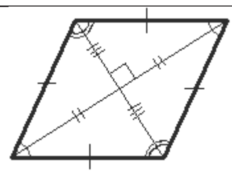
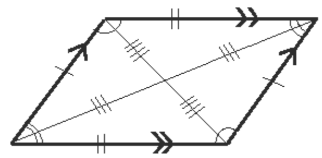
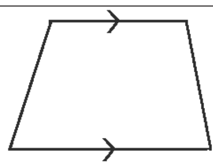
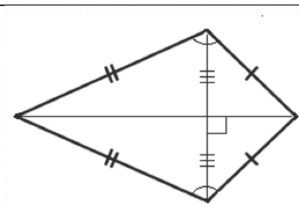
Learning Outcomes

By the end of the lesson, you will be able to:

1. Identify and describe characteristics of quadrilaterals: square, rectangle, rhombus, parallelogram, kite and trapezium.
2. Differentiate between types of quadrilaterals.

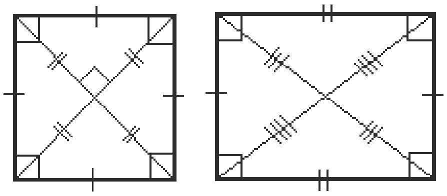
Overview

The following are common quadrilaterals and some of their characteristics:

 **Square** • All sides are the same length. • Opposite sides are parallel. • It has 4 right angles.	 **Rectangle** • Opposite sides are the same length. • Opposite sides are parallel. • It has 4 right angles.
 **Rhombus** • All sides are the same length. • Opposite sides are parallel. • Opposite angles are equal. • Diagonals form a right angle. • Diagonals bisect each other.	 **Parallelogram** • Opposite sides are the same length. • Opposite sides are parallel. • Opposite angles are equal. • Diagonals bisect each other.
 **Trapezium** • One pair of opposite sides are parallel.	 **Kite** • Has two sets of sides with equal lengths. • The angles where the two pairs of equal sides meet are equal. • Diagonals form a right angle. • One diagonal bisects the other.

Note the following:

- Any shape with opposite sides parallel are called parallelograms. Squares, rectangles, and rhombi are parallelograms.
- Because rectangles and squares are types of parallelograms, their diagonals bisect each other.
- A square is actually a type of rhombus because its sides are equal. This means that the diagonals of a square bisect each other at right angles.



Solved Examples

1. Answer the following questions about quadrilaterals:
    a. Which quadrilaterals have 4 right angles?
    b. Which quadrilaterals have all 4 sides of equal length?
    c. Which quadrilaterals have at least 1 set of parallel sides?
    d. Which quadrilaterals have at least 2 angles that are equal?
    e. Which quadrilaterals have diagonals that bisect each other at a right angle?

    Solutions:

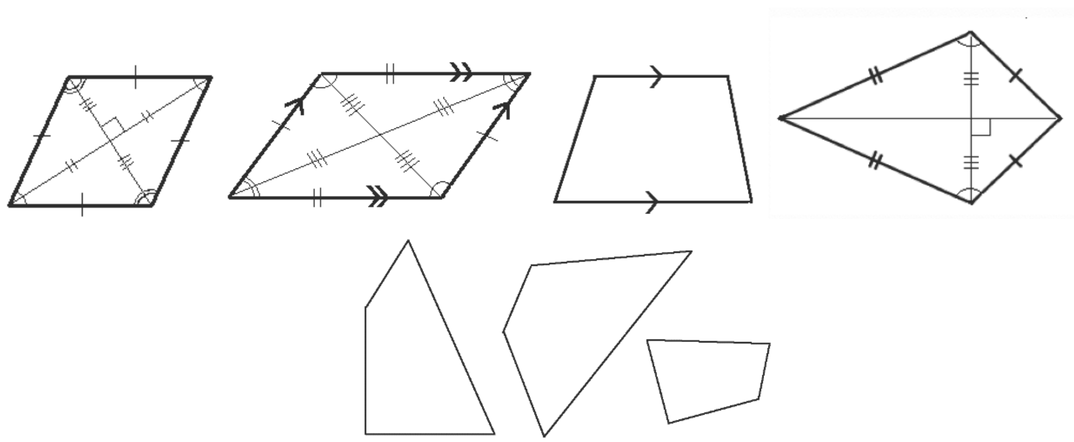
    a. Square, rectangle
    b. Square, rhombus
    c. Square, rectangle, rhombus, parallelogram, trapezium
    d. Square, rectangle, rhombus, parallelogram, kite
    e. Square, rhombus, kite

2. Draw as many different types of quadrilaterals as you can think of that have at least 1 acute angle.

    Solution:

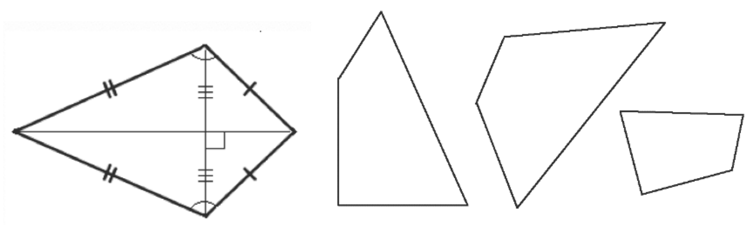
    Quadrilaterals that have at least 1 acute angle are: rhombus, parallelogram, trapezium, and kite. There are many other quadrilaterals that are not classified as a specific type but do have acute angles.

Example answers:



3. Draw as many different types of quadrilaterals as you can think of that do not have any parallel sides.
   Solution:
   Kite is the only quadrilateral classified above that does not have parallel sides. However, there are many quadrilaterals that are not classified as a specific type but do not have parallel sides.

   Example answers:



Practice

1. Draw a parallelogram and list its properties.
2. Draw a rectangle and list the properties.
3. Which quadrilaterals must have at least 2 sides of equal length?
4. Which quadrilaterals have only 2 sides that are parallel?
5. Which quadrilaterals have diagonals that both bisect each other?

Lesson Title: Interior angles of quadrilaterals	**Theme:** Geometry
Practice Activity: PHM2-L058	**Class:** SSS 2

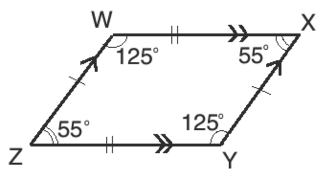
Learning Outcome
By the end of the lesson, you will be able to calculate the measurement of interior angles of quadrilaterals.

Overview

This lesson focuses on finding missing angles in quadrilaterals. There are several characteristics of quadrilaterals that can be used to solve for missing angles. The most important to note is that the 4 angles in any quadrilateral sum to 360°.

The angles of parallelograms have special characteristics. Recall that square, rectangle, and rhombus are actually parallelograms – so these characteristics apply to them too. The opposite angles in a parallelogram are equal. The co-interior angles (the angles next to each other) sum to 180°.

For example, consider parallelogram WXYZ:



The following angles are opposite, so are equal: W and Y; X and Z.

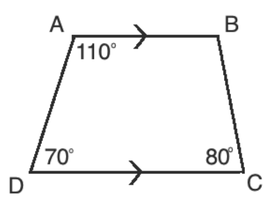
The following angles are co-interior, so sum to 180°: W and X; X and Y; Y and Z; W and Z.

Use additional characteristics from the previous lesson to find missing angles. For example:

- All 4 angles of a square or rectangle are 90°.
- A kite has opposite angles that are equal.

Solved Examples

1. Find the measure of missing angle B:



Solution:

Method 1. Subtract the known angles from 360° to find B:

$$\begin{aligned} A + B + C + D &= 360° \\ 110° + B + 80° + 70° &= 360° \\ B + 260° &= 360° \\ B &= 360° - 260° \\ B &= 100° \end{aligned}$$

Method 2. Use C, the co-interior angle, to find B: $B = 180° - 80° = 100°$

2. Quadrilateral $WXYZ$ has four angles which are in the ratio $2:4:4:5$. Find the degree measure of the smallest angle of quadrilateral $WXYZ$.

   Solution:

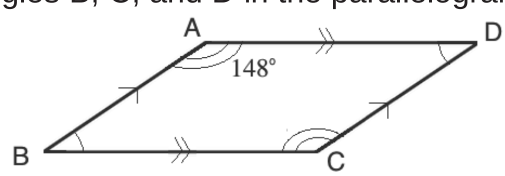
   Let x be their common factor that we need to find.

   Step 1. Set the sum of the angles equal to $360°$ and solve for x:

$$\begin{aligned} 2x + 4x + 4x + 5x &= 360° \\ 15x &= 360° \\ x &= \frac{360°}{15} \qquad \text{Divide both sides by 15} \\ x &= 24 \end{aligned}$$

   Step 2. $2x$ represents the smallest angle. Multiply: $2x = 2(24) = 48°$

3. Find the measures of angles B, C, and D in the parallelogram:



   Solution:

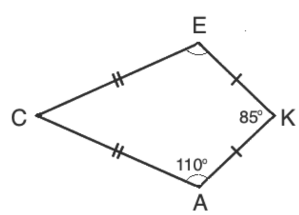
$$C = A = 148°$$

$$\begin{aligned} A + B + C + D &= 360° \\ 148° + B + 148° + D &= 360° \\ B + D + 296° &= 360° \\ B + D &= 360° - 296° \\ B + D &= 64° \end{aligned}$$

   Since $B = D$, divide $64°$ by 2 to find the measure of each:

   $B = D = 64° \div 2 = 31°$

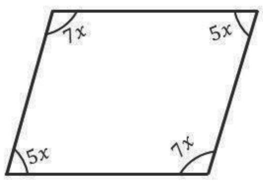
4. Find the measures of angles C and E in the kite CAKE:



Solution:
Find E: $E = A = 110°$.
Solve for angle C:

$$\begin{aligned} C + A + K + E &= 360° \\ C + 110° + 85° + 110° &= 360° \\ C + 305° &= 360° \\ C &= 360° - 305° \\ C &= 55° \end{aligned}$$

5. Find the value of x in the diagram:

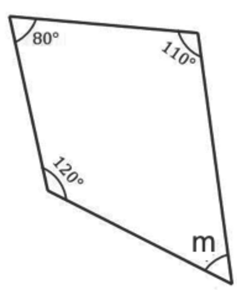


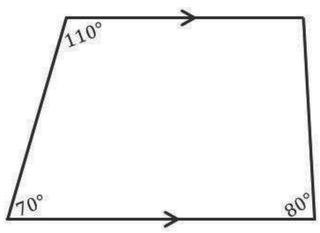
Solution:
Set the sum of the angles equal to 360°, and solve for x:

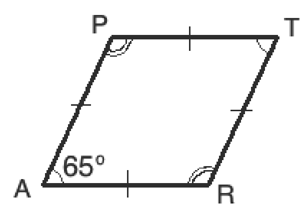
$$\begin{aligned} 7x + 7x + 5x + 5x &= 360° \\ 24x &= 360° \qquad \text{Divide both sides by 24} \\ x &= 15° \end{aligned}$$

Practice

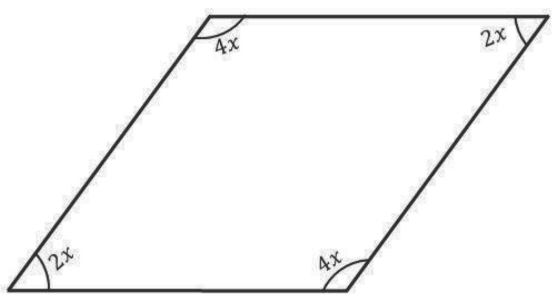
1. Find the measures of the unknown angles:

a. 

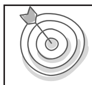
b. 

c. 

2. Find the value of x:



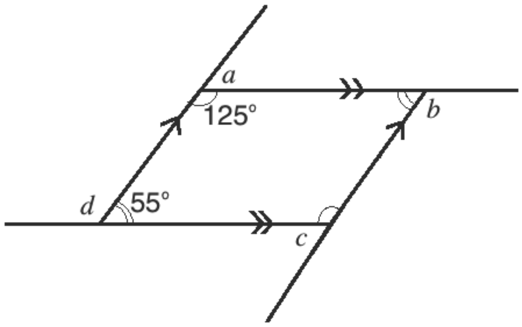
Lesson Title: Exterior angles of quadrilaterals	Theme: Geometry
Practice Activity: PHM2-L059	**Class:** SSS 2



Learning Outcome
By the end of the lesson, you will be able to calculate the measurement of exterior angles of quadrilaterals.

Overview

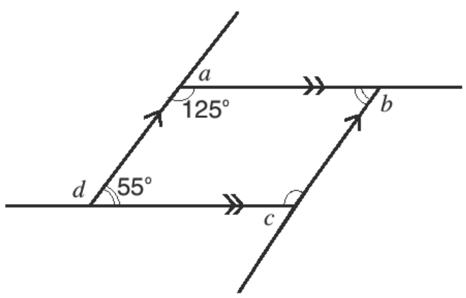
This lesson focuses on the external angles of quadrilaterals. For example, in the diagram below, the angles a, b, c, and d are called **exterior angles**. Exterior angles are adjacent to interior angles. Together they form a straight line. We can find the measure of an exterior angle by subtracting the interior angle from 180°.



The sum of the exterior angles of any polygon is 360°. We can use this fact to calculate exterior angles, or to check our work after we find all of the exterior angles. For squares and rectangles, the exterior angles are always right angles, the same as the interior angles.

Solved Examples

1. Find the unknown angles in the shape:



Solution:
Subtract each interior angle from 180° to find the exterior angles:

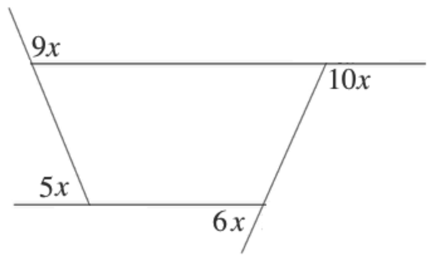
Angle a: $a = 180° - 125° = 55°$
Angle b: Note that the interior angle is $55°$
$b = 180° - 55° = 125°$

Angle c: Note that the interior angle is $125°$

$c = 180° - 125° = 55°$

Angle d: $d = 180° - 55° = 125°$

2. Find the value of x in the figure below:

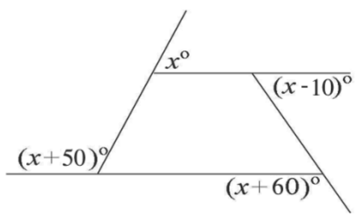


Solution:

The exterior angles sum to 360°. Set the sum equal to 360° and solve for x.

$$\begin{aligned} 9x + 10x + 6x + 5x &= 360° \\ 30x &= 360° \\ \frac{30x}{30} &= \frac{360°}{30} \\ x &= 12° \end{aligned}$$

3. The exterior angles of the quadrilateral below are x^{o}, $(x - 10)^{o}$, $(x + 60)^{o}$ and $(x + 50)^{o}$. Find the value of x.



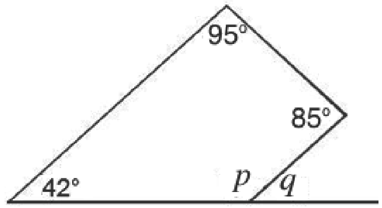
Solution:

Set the sum of the angles equal to $360°$ and solve for x:

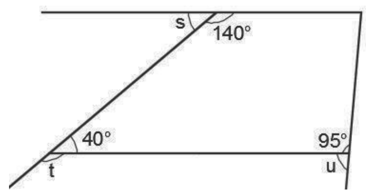
$$\begin{aligned} x^{\circ} + (x^{\circ} - 10)^{\circ} + (x + 60)^{\circ} + (x + 50)^{\circ} &= 360° \\ 4x + 100^{\circ} &= 360° \\ 4x &= 360° - 100^{\circ} \\ 4x &= 260^{\circ} \\ \frac{4x}{4} &= \frac{260^{\circ}}{4} \\ x &= 65^{\circ} \end{aligned}$$

Practice

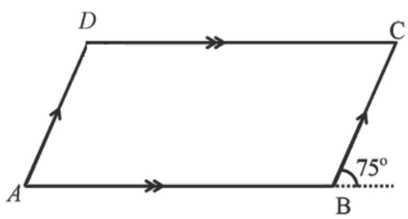
1. Find the measures of p and q in the figure below:



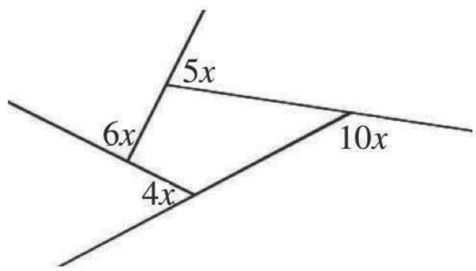
2. Find the values of s, t and u in the figure below:



3. Find the measures of interior angles A, B, C and D in the figure below:



4. Find the value of x in the figure below:



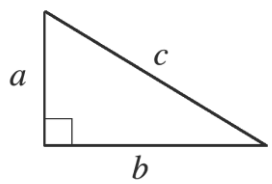
Lesson Title: Solving triangles	**Theme:** Geometry
Practice Activity: PHM2-L060	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to identify how to solve various types of triangles by finding side and angle measures (review).

Overview

Triangles are “solved” by finding the measures of any missing sides and angles.

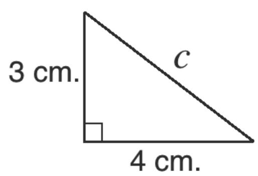
Missing sides of a right-angled triangle can be found using **Pythagoras’ theorem**. For the right-angled triangle below, the theorem states that $a^2 + b^2 = c^2$, where c is the hypotenuse, and a and b are the other 2 sides.



The sum of the exterior angles of any triangle is 180°. We can use this fact to calculate its angles.

Solved Examples

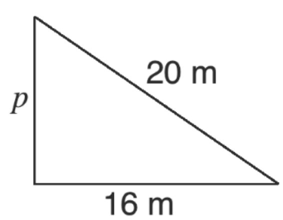
1. Find the measure of side c:



Solution:

$$\begin{aligned} 3^2 + 4^2 &= c^2 && \text{Substitute for 3 and 4 into the formula} \\ 9 + 16 &= c^2 && \text{Simplify} \\ 25 &= c^2 && \\ \sqrt{25} &= \sqrt{c^2} && \text{Take the square root of both sides} \\ c &= 5 && \end{aligned}$$

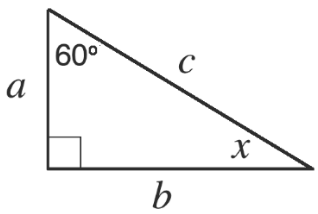
2. Find the measure of side p:



Solution:

$$
\begin{aligned}
p^2 + 16^2 &= 20^2 && \text{Substitute for 16 and 20 into the formula} \\
p^2 + 256 &= 400 && \text{Simplify} \\
p^2 &= 400 - 256 \\
p^2 &= 144 && \text{Take the square root of both sides} \\
\sqrt{p^2} &= \sqrt{144} \\
p &= 12
\end{aligned}
$$

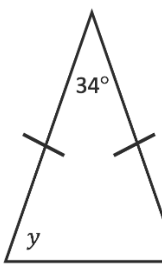
3. Find the measure of angle x:



Solution:
Subtract each known angle from 180° to find the missing angle:

$$x = 180° - 90° - 60° = 30°$$

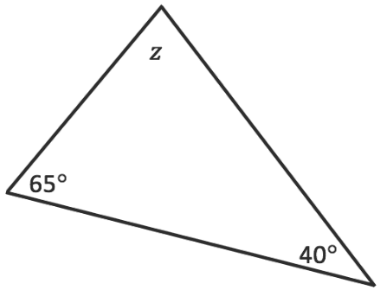
4. Find the missing angle y:



Solution:
This is an isosceles triangle. The bottom 2 angles are equal to each other. The measure of either can be found by subtracting 34° from 180°, and dividing the result by 2.

$$180° - 34° = 146°$$
$$y = 146° \div 2 = 73°$$

5. Find the missing angle z:



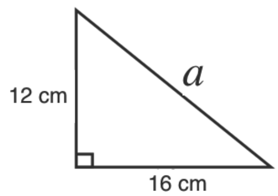
Solution:

Subtract each known angle from 180° to find the missing angle:

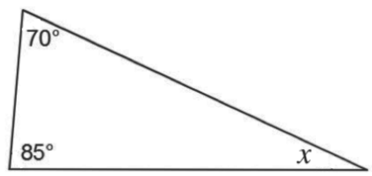
$$z = 180° - 40° - 65° = 75°$$

Practice

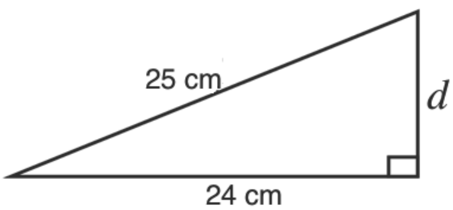
1. Calculate the length of a in the triangle below:



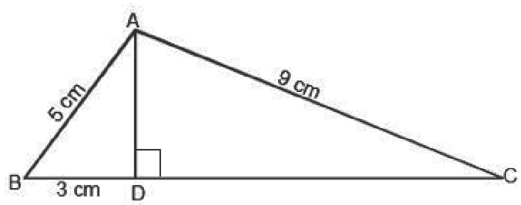
2. Find the unknown angle in the figure:



3. Calculate the length of side d:



4. Find the height AD of triangle ABC below. Use Pythagoras' theorem.

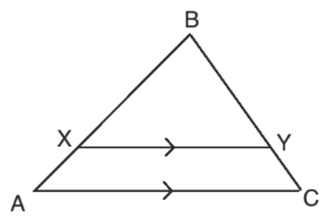


Lesson Title: Proportional division of the side of a triangle	Theme: Geometry
Practice Activity: PHM2-L061	**Class:** SSS 2

Learning Outcomes
By the end of the lesson, you will be able to:
1. Apply ratios to find missing lengths when a line parallel to one side divides a triangle.
2. Apply the mid-point theorem.

Overview

This lesson is on proportional division of the sides of a triangle. For example, consider the triangle:

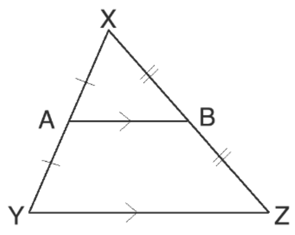


The line XY divides triangle ABC. It is parallel to side AC. When a line is drawn parallel to one side of a triangle, it divides the other 2 sides in the same ratio.

The ratios formed by the sides are: $\frac{BX}{XA} = \frac{BY}{YC}$. We can use this to solve for the unknown sides.

A special case is when the line parallel to one side joins the mid-points of the other two sides. In this case we can apply the mid-point theorem.

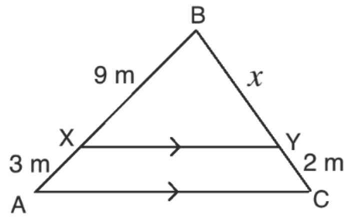
Consider the triangle:



In triangle XYZ, line AB joins the mid-points of two of its sides, XY and XZ. The lines AB and YZ are parallel to each other, and line AB has exactly half the length of the line YZ. Thus, we have the formula $|AB| = \frac{1}{2}|YZ|$.

Solved Examples

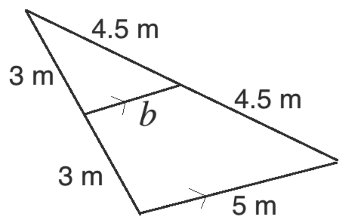
1. Find the measure of x:



Solution:

$\frac{BX}{XA}$	$=$	$\frac{BY}{YC}$	Use the ratios
$\frac{9}{3}$	$=$	$\frac{x}{2}$	Substitute for known sides
3	$=$	$\frac{x}{2}$	Cross multiply
3×2	$=$	x	Solve for x
x	$=$	6	

2. Find the length of b in the triangle:

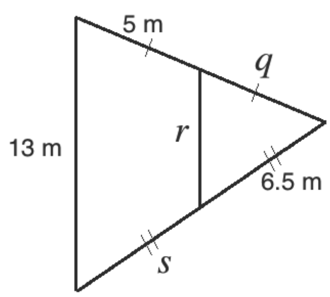


Solution:

b is half the length of the side that it is parallel to:

$b = \frac{1}{2} \times 5 = \frac{5}{2} = 2.5\ \mathrm{m}$

3. Find the measures of q, r, and s in the triangle below:

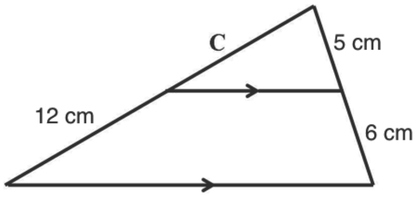


Solution:

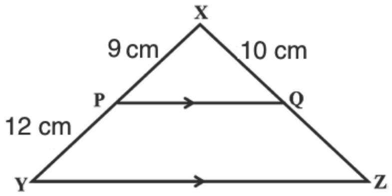
$q = 5\ \mathrm{m}$ because q is equal to the other segment on that side.
$r = \frac{1}{2} \times 13\ \mathrm{m.} = 6.5\ \mathrm{m}$ because r is half of the line it is parallel to.
$s = 6.5\ \mathrm{m}$ because s is equal to the other segment on that side.

Practice

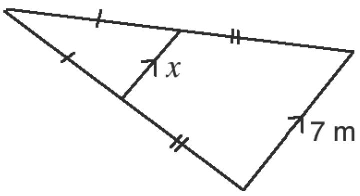
1. Find the length of C in the triangle:



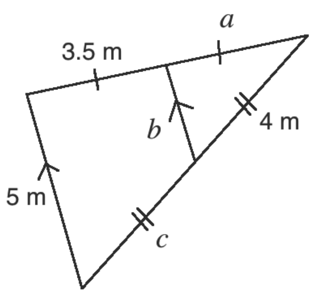
2. In the diagram below, $PQ||YZ$. Find the length of QZ.



3. Find the length of x in the triangle:



4. Find the measurement of $a,\ b$ and c in the triangle below:

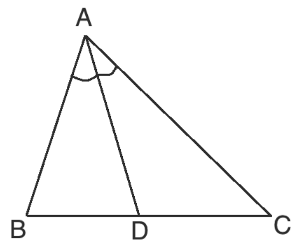


Lesson Title: Bisector of an angle in a triangle	**Theme:** Geometry
Practice Activity: PHM2-L062	**Class:** SSS 4

Learning Outcome
By the end of the lesson, you will be able to you will be able to apply the angle bisector theorem.

Overview

This lesson is on using the angle bisector theorem to find missing sides. Consider the triangle below:

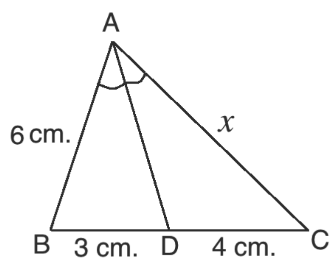


The line AD bisects angle A. If a line bisects an angle, it divides it into 2 equal parts. The angle bisector theorem states that an angle bisector of a triangle divides the opposite side into two segments that are proportional to the other 2 sides of the triangle. In this case, AD divides BC into two segments (BD and DC) that are proportional to the other sides, AC and AB.

This gives the proportion: $\frac{|BD|}{|DC|} = \frac{|AB|}{|AC|}$. This can be used to find missing side lengths.

Solved Examples

1. Find the measure of x:



Solution:

$$\frac{|BD|}{|DC|} = \frac{|AB|}{|AC|}$$

$$\frac{3}{4} = \frac{6}{x}$$ Substitute for the known sides

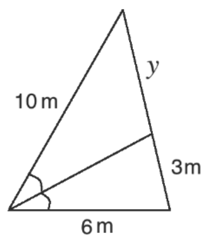
$$3x = 4 \times 6$$ Cross-multiply

$$3x = 24$$ Simplify

$$x = \frac{24}{3}$$

$$x = 8 \text{ cm}$$

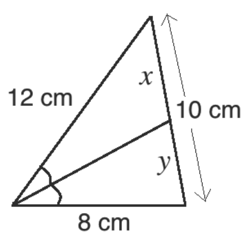
2. Find the length of y in the triangle:



Solution:

$$\begin{aligned} \frac{|BD|}{|DC|} &= \frac{|AB|}{|AC|} && \\ \frac{3}{y} &= \frac{6}{10} && \text{Substitute for the known sides} \\ 3 \times 10 &= 6y && \text{Cross-multiply} \\ 30 &= 6y && \text{Simplify} \\ \frac{30}{6} &= y && \\ y &= 5\text{ m.} && \end{aligned}$$

3. Find the measures of x and y in the triangle below:



Solution:

This problem is different because there are 2 missing segments. However, we know that their sum is 10 cm. We can use the ratios to find what fraction of the full length x and y are. We can use that fraction to find their lengths.

Write the ratios: $\frac{y}{x} = \frac{8}{12} = \mathbf{\frac{2}{3}}$

Therefore, $y = \frac{2}{\mathbf{5}}$ of 10 cm

We know the ratio of y to x is $\frac{2}{3}$. To find the ratio of either part to the whole length of 10, add the individual parts (the numerator and denominator, $2 + 3$). This gives us y is $\frac{2}{5}$ of the whole side, and x is $\frac{3}{5}$ of the whole side. We only need to use the fraction to find x or y. We can find the other by subtracting from 10.

Calculate y:

$$\begin{aligned} y &= \frac{2}{5} \text{ of } 10\text{ cm} \\ &= \frac{2}{5} \times 10\text{ cm} \end{aligned}$$

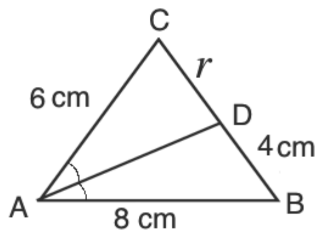
$$= \quad 2 \times 2 \text{ cm}$$
$$= \quad 4 \text{ cm}$$

Calculate x:

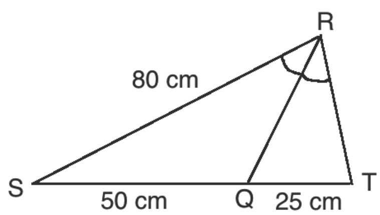
$$x = 10 - y = 10 - 4 = 6 \text{ cm}$$

Practice

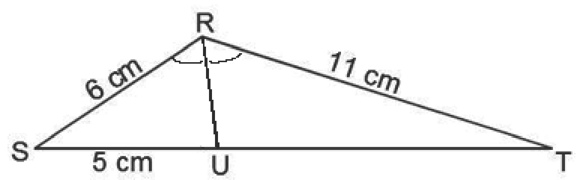
1. Find the length of r in the triangle:



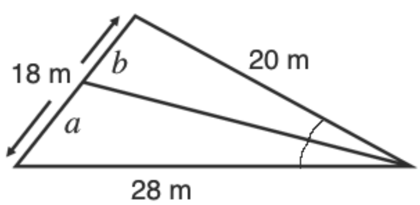
2. Find the length of RT in the triangle:



3. In the triangle RST, RU is a bisector of angle R, $|RT| = 11$ cm, $|RS| = 6$ cm, and $|SU| = 5$ cm. Find $|UT|$.



4. Find the ratio $a : b$ and the values of a and b in the figure below:



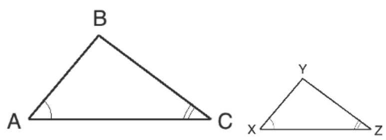
5. In the triangle ABC, $|AB| = 30$ cm, $|BC| = 32$ cm and $|CA| = 18$ cm. The bisector of angle A meets $\overline{\text{BC}}$ to form two missing segments, x and y. Find the measures of x and y.

Lesson Title: Similar triangles	**Theme:** Geometry
Practice Activity: PHM2-L063	**Class:** SSS 4

Learning Outcome
By the end of the lesson, you will be able to use the properties of similar triangles to deduce lengths in similar shapes.

Overview

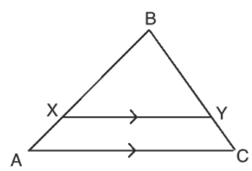
Similar triangles have the same shape but are of different sizes. Triangles are similar if their respective angles are equal. In the diagram below, triangle ABC is similar to triangle XYZ.



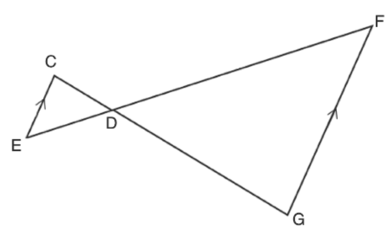
If two triangles are similar, their corresponding sides are in the same ratio. From the triangles above, we have: $\frac{|AB|}{|XY|} = \frac{|BC|}{|YZ|} = \frac{|CA|}{|ZX|}$.

To find the unknown side lengths, we substitute the known sides into the ratio formula and solve. You do not need to use all 3 ratios at once. Choose 2 convenient ratios from among the 3. There should only be 1 unknown in the ratios you choose.

The following are examples of similar triangles:



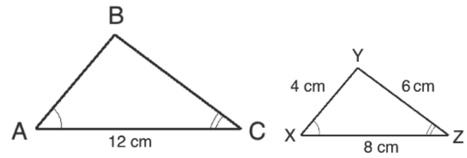
When a line that is parallel to one of the sides is drawn through a triangle, it forms a second triangle. The 2 triangles are similar, because all 3 of their angles are the same. Triangle XBY is similar to ABC. This means that ratios can be applied to find the missing sides.



When two lines intersect between parallel lines, they form 2 similar triangles. Triangle CDE is similar to triangle GDF. The triangle sides that share a line are in the same ratio. For example, DE and DF.

Solved Examples

1. Find the measures of AB and BC.



Solution:

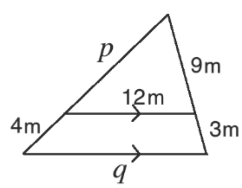
Find side AB:

$$\begin{aligned} \frac{|AB|}{|XY|} &= \frac{|CA|}{|ZX|} \\ \frac{|AB|}{4\text{ cm}} &= \frac{12\text{ cm}}{8\text{ cm}} \\ 8|AB| &= 4 \times 12 \\ 8|AB| &= 48 \\ |AB| &= \frac{48}{8} \\ |AB| &= 6\text{ cm} \end{aligned}$$

Find side BC:

$$\begin{aligned} \frac{|BC|}{|YZ|} &= \frac{|CA|}{|ZX|} \\ \frac{|BC|}{6\text{ cm}} &= \frac{12\text{ cm}}{8\text{ cm}} \\ 8|AB| &= 6 \times 12 \\ 8|AB| &= 72 \\ |AB| &= \frac{72}{8} \\ |AB| &= 9\text{ cm} \end{aligned}$$

2. Solve for p and q:



Solution:

Write the ratios: $\frac{9}{9+3} = \frac{9}{12} = \frac{12}{q} = \frac{p}{p+4}$

Remember to include the entire side in your ratio. The left side of the big triangle is the 2 smaller lengths combined, so we have $p + 4$.

When working with larger numbers, it is easier if you simplify fractions where possible. Therefore, use $\frac{3}{4}$ in place of $\frac{9}{12}$.

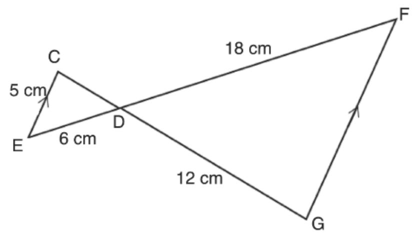
Find side q:

$$\begin{aligned} \frac{3}{4} &= \frac{12}{q} \\ 3q &= 4 \times 12 \\ 3q &= 48 \\ q &= \frac{48}{3} \\ q &= 16\text{ m} \end{aligned}$$

Find side p:

$$\begin{aligned} \frac{3}{4} &= \frac{p}{p+4} \\ 3(p+4) &= 4p \\ 3p + 12 &= 4p \\ 12 &= 4p - 3p \\ 12 &= p \\ p &= 12\text{ m} \end{aligned}$$

3. Find the measures of sides CD and FG:



Solution:

Write the ratios: $\frac{|CD|}{|DG|} = \frac{|DE|}{|DF|} = \frac{|CE|}{|FG|}$ or $\frac{|CD|}{12\text{ cm}} = \frac{6\text{ cm}}{18\text{ cm}} = \frac{5\text{ cm}}{|FG|}$

Simplify $\frac{6\text{ cm}}{18\text{ cm}}$ and use $\frac{1\text{ cm}}{3\text{ cm}}$

Find side CD:

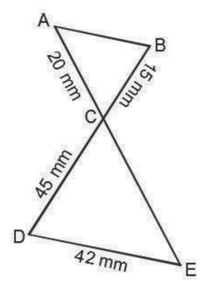
$$\begin{aligned} \frac{|CD|}{|DG|} &= \frac{|DE|}{|DF|} \\ \frac{|CD|}{12\text{ cm}} &= \frac{1\text{ cm}}{3\text{ cm}} \\ 3|CD| &= 12 \times 1 \\ |CD| &= \frac{12}{3} \\ |CD| &= 4\text{ cm} \end{aligned}$$

Find side FG:

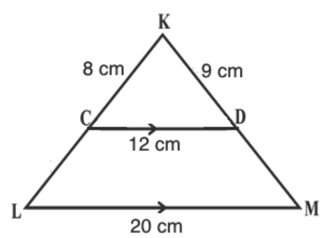
$$\begin{aligned} \frac{|CE|}{|FG|} &= \frac{|DE|}{|DF|} \\ \frac{5\text{ cm}}{|FG|} &= \frac{1\text{ cm}}{3\text{ cm}} \\ 5 \times 3 &= |FG| \\ 15\text{ cm} &= |FG| \end{aligned}$$

Practice

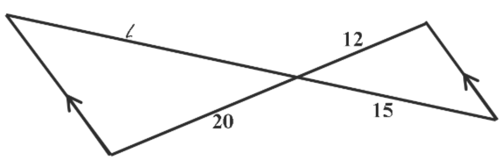
1. Find the measurements of the unknown sides in the figure:



2. In the diagram, find the lengths of CL and DM.



3. Find the value of l in the figure.



Lesson Title: Triangle problem solving	Theme: Geometry
Practice Activity: PHM2-L064	**Class:** SSS 4

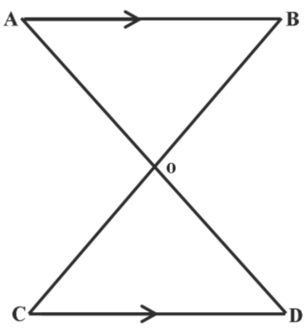
Learning Outcome
By the end of the lesson, you will be able to apply various theorems and properties of triangles to solve for angles and lengths.

Overview

This lesson is on solving various triangle problems. It revises the content of previous lessons, including: Pythagoras' theorem, finding missing angles, proportional division of sides, the mid-point theorem, angle bisector theorem, and similar triangles.

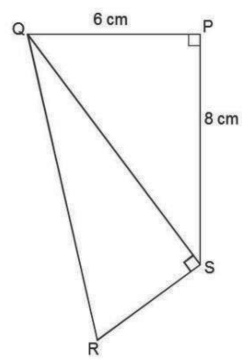
Solved Examples

1. In the diagram below, $AB \parallel CD$. If $|AB| = 12$ cm, $|CD| = 8$ cm and $|BO| = 15$ cm, calculate $|CO|$.
   Solution:



$\frac{\lvert AB\rvert}{\lvert CD\rvert}$	$=$	$\frac{\lvert BO\rvert}{\lvert CO\rvert}$	By similar triangle
$\frac{12}{8}$	$=$	$\frac{15}{\lvert CO\rvert}$	Substitute
$12 \times \lvert CO\rvert$	$=$	8×15	Cross multiply
$\lvert CO\rvert$	$=$	$\frac{8\times15}{12}$	Divide both sides by 12
$\lvert CO\rvert$	$=$	$\frac{120}{12}$	Simplify
$\lvert CO\rvert$	$=$	10 cm.	

2. In the diagram, $PQRS$ is a quadrilateral. Given that $\angle QPS = \angle QSR = 90^\circ$, $|PQ| = 6$ cm, $|PS| = 12$ cm and $|RS| = 4$ cm, Calculate $|QR|$, correct to one decimal place.
   Solution:
   Step 1. Use Pythagoras' theorem to find $|QS|$:



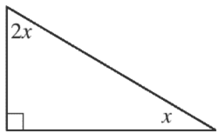
$\lvert QS\rvert^2$	$=$	$\lvert PQ\rvert^2 + \lvert PS\rvert^2$	Pythagoras theorem
$\lvert QS\rvert^2$	$=$	$6^2 + 8^2$	Substitute
$\lvert QS\rvert^2$	$=$	$36 + 64$	Simplify
$\lvert QS\rvert^2$	$=$	100	
$\lvert QS\rvert$	$=$	$\sqrt{100} = 10$ cm	

   Step 2. Use Pythagoras' theorem to find $|QR|$:

$\lvert QR\rvert^2$	$=$	$\lvert QS\rvert^2 + \lvert RS\rvert^2$	Pythagoras theorem
$\lvert QR\rvert^2$	$=$	$10^2 + 4^2$	Substitute

$$
\begin{aligned}
|QR|^2 &= 100 + 16 && \text{Simplify} \\
|QR|^2 &= 116 \\
|QR| &= \sqrt{116} \\
|QR| &= 10.8
\end{aligned}
$$

3. In the triangle below, find the value of x.



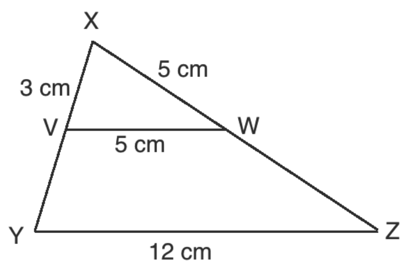
Solution:

Use the fact that the angles of a triangle sum to 180°. Add the given angles, and solve for x.

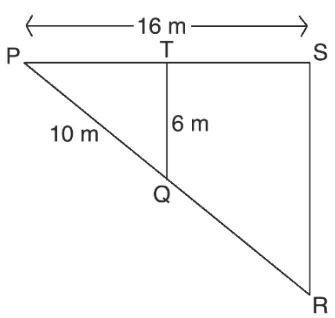
$$
\begin{aligned}
180° &= 90° + 2x + x && \text{Set up equation} \\
180° &= 90° + 3x && \text{Solve for } x \\
180° - 90° &= 3x \\
90° &= 3x \\
30° &= x
\end{aligned}
$$

Practice

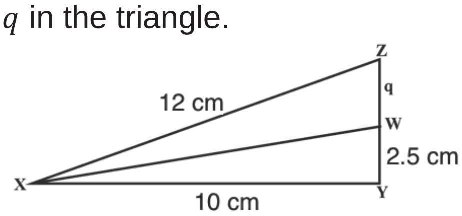
1. In the diagram below, find the length of WZ.



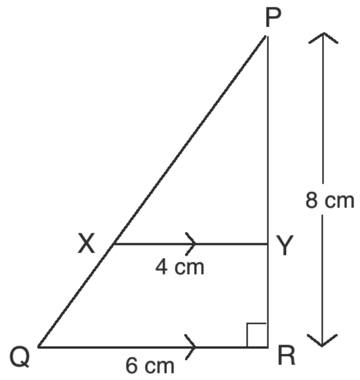
2. In the diagram, $\angle PTQ = \angle PSR = 90^{\circ}$. Find the measurement of $|PT|$ and $|RS|$.



3. Find the length of q in the triangle.



4. Find the following measurements in the diagram below. Give your answers to 1 decimal place: a. $|PQ|$ b. $|PY|$



Lesson Title: Conversion of units: smaller to larger	**Theme:** Mensuration
Practice Activity: PHM2-L065	**Class:** SSS 4

Learning Outcome
By the end of the lesson, you will be able to convert from smaller units to larger units using common units of measurement.

Overview

This lesson is on converting various common units of measurement. To convert from a smaller unit to a larger unit, **divide** by the conversion factor.

Some common conversion factors are listed below. Use these for this lesson and the following lesson.

Length / Distance		
10 decimetres	=	1 metre
100 centimetres	=	1 metre
1,000 millimetres	=	1 metre
1 kilometre	=	1,000 metres

Mass / Weight		
1 gramme	=	1,000 milligrammes
1 kilogrammes	=	1,000 grammes
1 tonne	=	1,000 kilogrammes

Volume / Capacity		
1 litre	=	1,000 millilitres
1 litre	=	1,000 cm^3

Solved Examples

1. Fatu walked a total of 3,000 metres in one day. How much did she walk in kilometres?
   Solution:
   Use the conversion factor $1{,}000 \text{ m} = 1 \text{ km}$. Metres are smaller than kilometres, so we divide by 1,000: $3{,}000 \div 1{,}000 = 3$ km.

2. Convert 32,000 cm to km.
   Solution:
   When you do not have an easy conversion factor, it is possible to do 2 conversions. We can convert centimetres to metres, then metres to kilometres.
   Step 1. Centimetres to metres:
   $$32{,}000 \div 100 = 320 \text{ metres}$$

Step 2. Metres to kilometres:

$$320 \div 1000 = 0.32 \text{ kilometres}$$

3. Gabriel was asked to supply water for lunch. At first, he supplied $3{,}400$ ml and later $630\ \text{cm}^3$. How much does he supply in all? Give your answer in litres.

Solution:

This is an example of a word problem that requires us to convert units before solving. Recall that quantities must be in the same units before we can apply operations to them. Use the conversation factors 1 litre = $1{,}000$ ml, and 1 litre = $1{,}000\ \text{cm}^3$.

Step 1. Convert each quantity to litres:

$$3{,}400\ \text{ml} \div 1{,}000 = 3.4 \text{ litres}$$
$$630\ \text{cm}^3 \div 1{,}000 = 0.63 \text{ litres}$$

Step 2. Add the two quantities:

$$3.4 + 0.63 = 4.03 \text{ litres}$$

4. Amadu has two textbooks that weigh 200 g and 0.8 kg respectively; how much do they weigh altogether? Give your answer in kilogrammes.

Solution:

Use the conversion factor 1,000 g = 1 kg.

Step 1. Convert grammes into kilogrammes:

$$200\ \text{g} \div 1{,}000 = 0.2\ \text{kg}$$

Step 2. Add the quantities in kilogrammes:

$$0.2 + 0.8 = 1.0\ \text{kg}$$

Practice

1. John measures his classroom desk to be 63 cm. Convert 63 cm into metres.
2. Convert 360 cm into metres.
3. Fanta wants to make some drawings on the board. She did the first drawing that measures 3.6 metres wide and the second drawing that measures 60 cm wide. What is the total width of the board used for her drawings? Give your answer in metres.
4. Abu walked a distance of $5{,}426$ metres and Jane a distance of $4{,}276$ metres. What is the difference between Abu and Jane? Give your answer in kilometres.
5. The weight of Alpha is $8{,}800$ grammes. Last year, he weighed $7{,}600$ grammes. What is the difference in weight between last year and this year? Gives your answer in kilogrammes.
6. The Guma valley water company allocated $9{,}548{,}300$ ml of water to the Education Ministry and $8{,}825{,}200$ ml to the Ministry of Agriculture. How much was allocated altogether? Give your answer in litres.

Lesson Title: Conversion of units: larger to smaller	**Theme:** Mensuration
Practice Activity: PHM2-L066	**Class:** SSS 4

Learning Outcome
By the end of the lesson, you will be able to convert from large units to smaller units using common units of measurement.

Overview

This lesson is on converting various common units of measurement. To convert from a larger unit to a smaller unit, **multiply** by the conversion factor.

Use the conversion factors listed in the previous lesson if needed.

Solved Examples

1. Foday travels 1.5 kilometres to school each day. How far is that in metres?
   Solution:
   Use the conversion factor $1{,}000 \text{ m} = 1 \text{ km}$. Kilometres are larger than metres, so we multiply by 1,000:
   $$1.5 \times 1{,}000 = 1500 \text{ m}$$

2. Convert 6.8 litres into millilitres.
   Solution:
   Use the conversion factor $1000 \text{ ml} = 1 \text{ l}$. Liters are larger than milliters, so we multiply by $1{,}000$:
   $$6.8 \times 1{,}000 = 6{,}800 \text{ ml}$$

3. Convert 10.84 grams into milligrams.
   Solution:
   Use the conversion factor $1{,}000 \text{ mg} = 1 \text{ g}$. Grammes are larger than milligrammes, so we multiply by $1{,}000$:
   $$10.84 \times 1{,}000 = 10{,}840 \text{ mg}$$

4. Convert 0.25 kg to mg.
   Solution:
   When you do not have an easy conversion factor, it is possible to do 2 conversions. We can convert kilogrammes to grammes, then grammes to milligrammes.

   Step 1. Kilogrammes to grammes:
   $$0.25 \times 1{,}000 = 250 \text{ g}$$
   Step 2. Grammes to milligrammes:
   $$250 \times 1{,}000 = 250{,}000 \text{ mg}$$

5. Jusu and Agnes traveled on bicycle to different locations. They traveled 3.6 and 2.8 kilometres respectively. Calculate their total distance in metres.
   Solution:
   Use the conversion factor $1{,}000 \text{ m} = 1 \text{ km}$ for each quantity, then add to find the total.

   For Jusu: $3.6 \times 1{,}000 = 3{,}600$ m
   For Agnes: $2.8 \times 1{,}000 = 2{,}800$ m
   Their total distance: $3{,}600 + 2{,}800 = 6{,}400$ m

6. Convert 9.75 tonnes into grammes.
   Solution:
   Here you do not have an easy conversion factor, so it is possible to do 2 conversions. We can convert tonnes to kilogrammes, then kilogrammes to grammes.

   Step 1. Tonnes to kilogrammes:
   $9.75 \times 1000 = 9{,}750$ kilogrammes
   Step 2. Kilogrammes to grammes:
   $9{,}750 \times 1{,}000 = 9{,}750{,}000$ grammes

7. Peter took two days to complete his journey to a neighbouring town. On the first day, he walked for 8.674 km and for the second day, he walked for 7.236 km. How much did he walk altogether? Give your answer in metres.
   Solution:
   Use the conversion factor $1{,}000 \text{ m} = 1 \text{ km}$. Convert the amount that Peter walked on each day.

   For day one: $8.674 \times 1{,}000 = 8{,}674$ m
   For day two: $7.236 \times 1{,}000 = 7{,}236$ m
   Total amount walked: $8{,}674 + 7{,}236 = 15{,}910$ m

8. Convert 3.36 kg into milligrams.
   Solution:
   Here, it is possible to do two 2 conversions. You can convert kilogrammes to grammes, and then from grammes to milligrammes.

   Step 1. Kilogrammes to grammes:
   $3.36 \times 1{,}000 = 3360$ grammes
   Step 2. Grammes to milligrammes:
   $3360 \times 1{,}000 = 3{,}360{,}000$ milligrammes

Practice

1. Convert 63 grammes to milligrammes.
2. Convert 0.09 kilometres to centimetres.
3. Mr. Johnny has 3.9 metres of mat to cover a floor in his room. If he used 210 cm to cover the floor, how much does he have left? Give your answer in centimetres.
4. Mariam’s house is 1.4 kilometres from the centre market. How far from the market is Mariam’s house in metres?
5. George, who used to have three meals a day, cut down his diet to two meals a day. After some months, he found that his weight was reduced by 2.36 kg. How much was his weight reduced in milligrammes?
6. Abu fetches 26 litres of water every day for domestic work. How much is this in millilitres?

Lesson Title: Perimeter and area of a square and rectangle	Theme: Mensuration
Practice Activity: PHM2-L067	Class: SSS 4

Learning Outcome
By the end of the lesson, you will be able to calculate the perimeter and area of a square and rectangle.

Overview

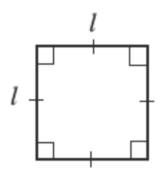
In Maths, "perimeter" is the total length around a shape. "Area" is the size of the space inside a shape.

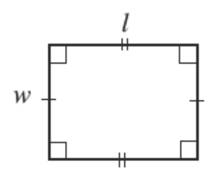
To find the **perimeter** of a rectangle or square, add the lengths of all 4 sides together. To find the **area** of a square or rectangle, multiply the measurements of the two sides, length and width. For a square, the sides are the same length, so the area will be length squared. Area is always given in square units.

The formulae are:

Shape	Perimeter	Area
Square	$P = l + l + l + l = 4l$	$A = l \times l = l^2$
Rectangle	$P = l + l + w + w = 2l + 2w$	$A = l \times w$

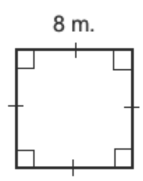
For the shapes:





Solved Examples

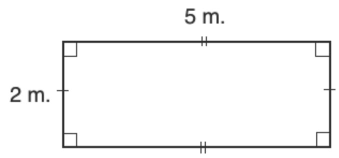
1. Find the perimeter and area of the square:



Solution:

$$P = 4l = 4 \times 8 \text{ m} = 32 \text{ m}$$
$$A = l^2 = 8 \text{ m} \times 8 \text{ m} = 64 \text{ m}^2$$

2. Find the perimeter and area of the rectangle:



Solution:

$$P = 2l + 2w = 2 \times 5 \text{ m} + 2 \times 2 \text{ m} = 10 \text{ m} + 4 \text{ m} = 14 \text{ m}$$
$$A = l \times w = 5 \text{ m} \times 2 \text{ m} = 10 \text{ m}^2$$

3. Find the perimeter and area of a square with sides 15 cm.
Solution:
Substitute $l = 15 \text{ cm}$ into each formula:
$$P = 4l = 4 \times 15 \text{ cm} = 60 \text{ cm}$$
$$A = l^2 = 15 \text{ cm} \times 15 \text{ cm} = 225 \text{ cm}^2$$

4. If the perimeter of a square is 64 m, find the area of the square.
Solution:
This problem involves 2 steps. First, use the perimeter to find the length l of the square. Use l to calculate the area.

Step 1. Find l using the perimeter:
$$\begin{aligned} P = 64 \text{ m} &= 4l \\ \tfrac{64}{4} \text{ m} &= l \\ 16 \text{ m} &= l \end{aligned}$$
Step 2. Find the area:
$$A = l^2 = 16 \text{ cm} \times 16 \text{ cm} = 256 \text{ m}^2$$

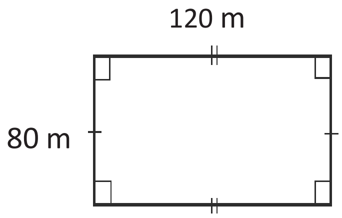
5. The area of square is 324 cm^2, find its perimeter.
Solution:
This problem involves 2 steps. First, use the area to find the length l of the square. Use l to calculate the perimeter.
Step 1. Find l using the area:
$$\begin{aligned} A = 324 \text{ cm}^2 &= l^2 \\ \sqrt{324 \text{ cm}^2} &= l \\ 18 \text{ cm} &= l \end{aligned}$$
Step 2. Find the perimeter:
$$P = 4l = 4 \times 18 \text{ m} = 72 \text{ cm}$$

6. Bright Secondary school has a football field that measures 120 metres on one side and 80 metres on the other side.
   a. Draw a diagram of the football field.

b. A gardener is hired to plant a carpet grass on the field, and he needs to know the area of the field. Calculate the area.
c. Find the cost of planting carpet grass at a rate of Le200.00 per square metre.

Solution:

a. Diagram:



b. Calculate the area:

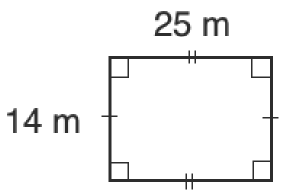
$$\begin{aligned} A &= l \times w \\ &= 120 \text{ m} \times 80 \text{ m} \\ &= 9{,}600 \text{ m}^2 \end{aligned}$$

c. Multiply the rate per square metre by the number of square metres:
Cost: $9{,}600 \times 200 = \text{Le } 1{,}920{,}000.00$

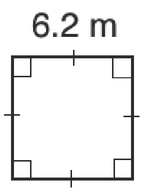
Practice

1. Find the perimeter and area of shapes a. and b.

a.



b.



2. Find the perimeter of a square with sides of 14 cm.
3. The perimeter of a square is 104 metres. Find the area of the square.
4. If the area of a square is 144 m^2, find its perimeter.
5. A school principal has a rectangular office with a length 10 metres, and an area of 70 square metres.
   a. Draw a diagram of the principal's office.
   b. Find the width of her office.
   c. Find the perimeter of her office.
   d. If $\frac{1}{2}$ of the office is occupied by office equipment, what is the area that is occupied by office equipment?
6. The perimeter of a square courtyard is 196 m. Find the cost of cementing it at the rate of $\text{Le } 10.00$ per m^2.

Lesson Title: Perimeter and area of a parallelogram	Theme: Mensuration
Practice Activity: PHM2-L068	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to calculate the perimeter and area of a parallelogram.

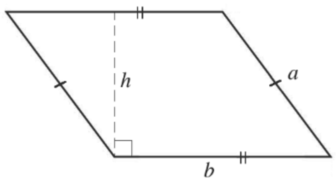
Overview

We calculate the perimeter in the same way for any shape: by adding all of the sides. We can use the same formula for the perimeter of a parallelogram that we used for a rectangle, using its two sides a and b as length and width. To find the area of a parallelogram, multiply its base times its height. The height of a parallelogram forms a right angle with its base.

The formulae are:

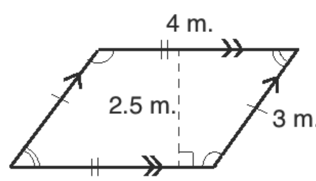
Shape	Perimeter	Area
Parallelogram	$P = a + a + b + b = 2a + 2b$	$A = b \times h$

For the shape:



Solved Examples

1. Find the perimeter and area of the parallelogram:

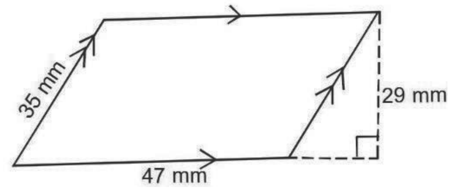


Solution:

$$\begin{aligned} P &= 2a + 2b \\ &= 2 \times 4 \text{ m} + 2 \times 3 \text{ m} \\ &= 8 \text{ m} + 6 \text{ m} \\ &= 14 \text{ m} \end{aligned}$$

$$\begin{aligned} A &= b \times h \\ &= 4 \text{ m.} \times 2.5 \text{ m} \\ &= 10 \text{ m}^2 \end{aligned}$$

2. Find the perimeter and area of the parallelogram below:



Solution:

$$\begin{aligned} P &= 2a + 2b \\ &= 2 \times 47 \text{ mm} + 2 \times 35 \text{ mm} \\ &= 94 \text{ mm} + 70 \text{ mm} \\ &= 164 \text{ mm} \end{aligned}$$

$$\begin{aligned} A &= b \times h \\ &= 47 \text{ mm} \times 29 \text{ mm} \\ &= 1363 \text{ mm}^2 \end{aligned}$$

3. A parallelogram has base 12 cm and area 192 cm^2. What is the height of the parallelogram?

   Solution:

   Substitute the known values in the area formula, and solve for the height:

$$\begin{aligned} A &= b \times h \\ 192 \text{ cm}^2 &= 12 \text{ cm} \times h \\ \frac{192 \text{ cm}^2}{12 \text{ cm}} &= h && \text{Divide throughout by } 12 \text{ cm} \\ 16 \text{ cm} &= h \end{aligned}$$

4. A parallelogram has 2 sides of length 7 cm and perimeter 32 cm. What is the length of its other two sides?

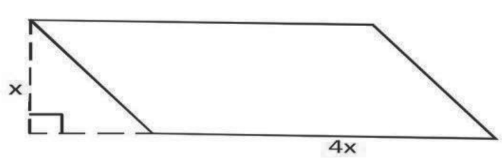
   Solution:

$$\begin{aligned} P &= 2a + 2b \\ 32 \text{ cm} &= 2(7 \text{ cm}) + 2b \\ 32 \text{ cm} &= 14 \text{ cm} + 2b && \text{Subtract } 14 \text{ cm from both sides} \\ 32 - 14 \text{ cm} &= 2b \\ 18 \text{ cm} &= 2b \\ \frac{18 \text{ cm}}{2} &= b && \text{Divide throughout by 2} \\ 9 \text{ cm} &= b \end{aligned}$$

5. The base of a parallelogram is four times its height. If its area is 676 cm^2, find the base and the height.

   Solution:

   Step 1. Draw a diagram:



   Step 2. Substitute the area, x, and 4x into the area formula, and solve for x:

$$
\begin{aligned}
A &= b \times h \\
676 \text{ cm}^2 &= 4x \times x \\
676 \text{ cm}^2 &= 4x^2 \\
\frac{676 \text{ cm}^2}{4} &= x^2 && \text{Divide throughout by } 4 \\
169 &= x^2 \\
x^2 &= 169 \\
\sqrt{x^2} &= \sqrt{169} && \text{Take the square root of both sides} \\
x &= 13 \text{ cm}
\end{aligned}
$$

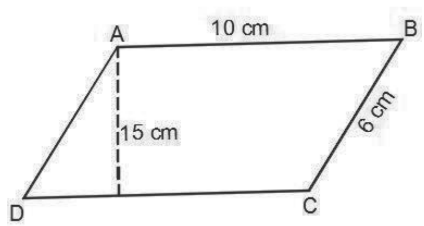
Step 3. Find the lengths of the base and height:

$b = 4x = 4 \times 13 = 52 \text{ cm}$

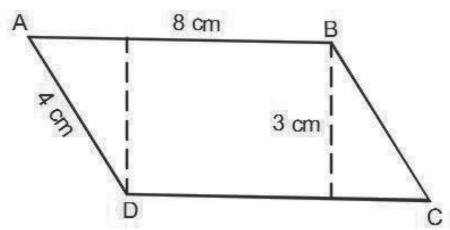
$h = x = 13 \text{ cm}$

Practice

1. Find the area and perimeter of the parallelograms below:
   a.



   b.



2. The base of a parallelogram is five times its height. If its area is 180 cm^2, find the base and the height.
3. A parallelogram has a base of 13 cm and an area of 208 cm^2. What is the height of the parallelogram?
4. A parallelogram has a height of 4 cm and an area of 32 cm^2. What the base of the parallelogram?

Lesson Title: Perimeter and area of a trapezium	**Theme:** Mensuration and Geometry
Practice Activity: PHM2-L069	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to calculate the perimeter and area of a trapezium.

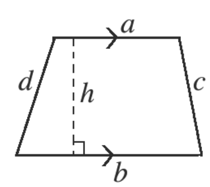
Overview

We calculate the perimeter of a trapezium in the same way for any shape: by adding all of the sides. To calculate the area of a trapezium, we use a special formula.

The formulae for the perimeter and area are:

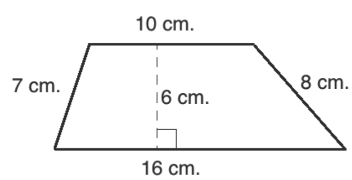
Shape	Perimeter	Area
Trapezium	$P = a + b + c + d$	$A = \frac{1}{2}(a + b)h$

For the shape:



Solved Examples

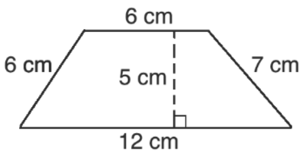
1. Find the perimeter and area of the trapezium:



Solution:

$$\begin{aligned} P &= 10 + 16 + 8 + 7 \\ &= 41 \text{ cm} \\ A &= \tfrac{1}{2}(a + b)h \\ &= \tfrac{1}{2}(10 + 16)6 \\ &= \tfrac{1}{2}(26)6 \\ &= 78 \text{ cm}^2 \end{aligned}$$

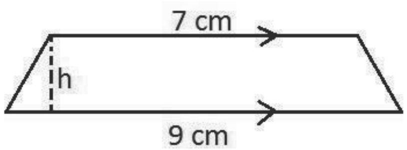
2. Calculate the perimeter and area of the trapezium below.



Solution:

$$
\begin{aligned}
\text{P} &= 6 + 6 + 7 + 12 \\
&= 31 \text{ cm} \\
A &= \tfrac{1}{2}(a + b)h \\
&= \tfrac{1}{2}(6 + 12)5 \\
&= \tfrac{1}{2}(18)5 \\
&= 45 \text{ cm}^2
\end{aligned}
$$

3. Below is a diagram of a trapezium with area 14 cm^2. Find the height h of the trapezium.



Solution:

Substitute known values in the area formula and solve for h.

$$
\begin{aligned}
A &= \tfrac{1}{2}(a + b)h \\
14 \text{ cm}^2 &= \tfrac{1}{2}(7 \text{ cm} + 9 \text{ cm})h \\
14 \text{ cm}^2 &= \tfrac{1}{2}(16 \text{ cm})h \\
14 \text{ cm}^2 &= (8 \text{ cm})h \\
\frac{14 \text{ cm}^2}{8 \text{ cm}} &= \frac{(8 \text{ cm})h}{8 \text{ cm}} \\
\tfrac{7}{4} \text{ cm} &= h \\
h &= 1.75 \text{ cm}
\end{aligned}
$$

4. Find the area of a trapezium whose parallel sides are 75mm and 82mm long, and whose vertical height is 39 mm.

Solution:

$$
\begin{aligned}
A &= \tfrac{1}{2}(a + b)h \\
&= \tfrac{1}{2}(75 + 82)39 \\
&= (78.5)39 \\
&= 3{,}061.5 \text{ mm}^2
\end{aligned}
$$

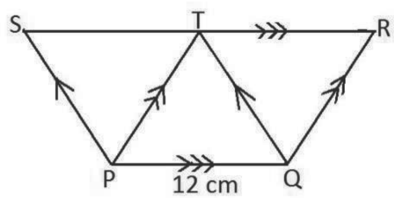
5. A trapezium has parallel sides that are 12 m and 17 m. If its area 290 m^2, what is its height?

Solution:

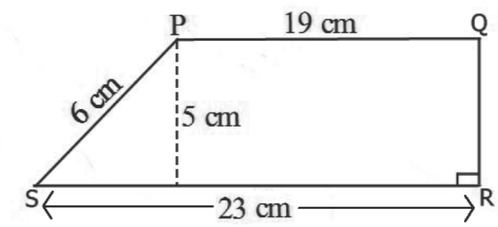
$$
\begin{aligned}
A &= \tfrac{1}{2}(a+b)h \\
290\ \mathrm{m}^2 &= \tfrac{1}{2}(12\ \mathrm{m} + 17\ \mathrm{m})h \\
290\ \mathrm{m}^2 &= \tfrac{1}{2}(29\ \mathrm{m})h \\
2 \times 290\ \mathrm{m}^2 &= (29\ \mathrm{m})h \\
\frac{580\ \mathrm{m}^2}{29\ \mathrm{m}} &= h \\
20\ \mathrm{m} &= h
\end{aligned}
$$

Practice

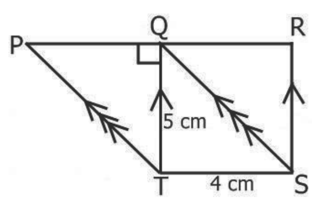
1. The parallel sides PQ and RS of a trapezium are 7.6 and 9.9 cm. If the distance between them is 6 cm, calculate the area of the trapezium.
2. In the diagram below, PQRS is a trapezium formed by 3 equilateral triangles, with PQ||SR. If $|PQ| = 12$ cm and the height of the trapezium is $6\sqrt{3}$ cm, find its area.



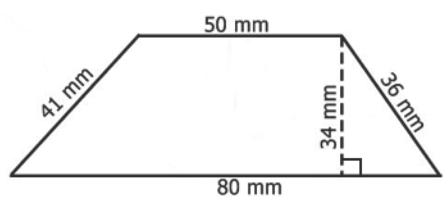
3. Calculate the perimeter and area of the trapezium PQRS.



4. In the diagram below, PQST and QRST are parallelograms. Calculate the area of the trapezium PRST.



5. The area of trapezium PQRS is $120\mathrm{cm}^2$. $PQ||RS,\ |PQ| = 10$ cm, and $|RS| = 20$ cm. Calculate the perpendicular height of $PQRS$.
6. Find the perimeter in mm and area of the shape below:



Lesson Title: Perimeter and area of a rhombus	**Theme:** Mensuration
Practice Activity: PHM2-L070	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to calculate the perimeter and area of a rhombus.

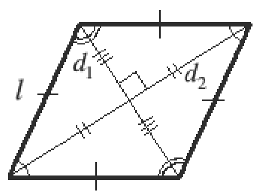
Overview

We calculate the perimeter in the same way for any shape: by adding all of the sides. Because the rhombus has 4 equal sides, we can simply multiply the length by 4. Area involves a specific formula, which rhombus and kite share. Multiply the diagonals by $\frac{1}{2}$.

The formulae are:

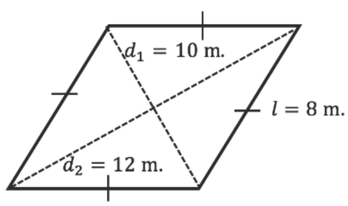
Shape	Perimeter	Area
Rhombus	$P = l + l + l + l = 4l$	$A = \frac{1}{2}d_1 \times d_2$

For the shape:



Solved Examples

1. Find the perimeter and area of the rhombus:

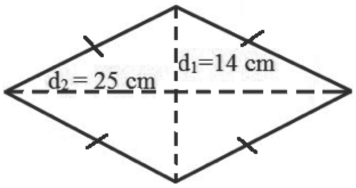


Solution:

$$\begin{aligned} P &= 4l \\ &= 4(8) \\ &= 32 \text{ m.} \\ A &= \tfrac{1}{2}d_1 \times d_2 \\ &= \tfrac{1}{2}(10 \times 12) \\ &= \tfrac{1}{2}(120) \\ &= 60 \text{ m.}^2 \end{aligned}$$

2. The rhombus below has a perimeter of 36 cm, and diagonals of 25 cm and 14 cm.

Find: a. The length of the sides.
b. The area of the rhombus.



a.

$$\begin{aligned} P &= 4l \\ 36 \text{ cm} &= 4l \\ \frac{36 \text{ cm}}{4} &= l \qquad \text{Divide throughout by } 4 \\ 9 \text{ cm} &= l \end{aligned}$$

b.

$$\begin{aligned} A &= \tfrac{1}{2} d_1 \times d_2 \\ &= \tfrac{1}{2}(25 \times 14) \qquad \text{Substitute values} \\ &= \tfrac{1}{2}(350) \\ &= 175 \text{ cm}^2 \end{aligned}$$

3. A rhombus has sides 17 cm and diagonals 28 cm and 10 cm. Find:
   a. The area of the rhombus
   b. The perimeter of the rhombus

Solutions:

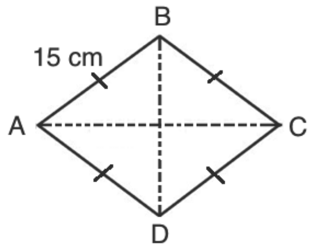
a.

$$\begin{aligned} A &= \tfrac{1}{2} d_1 \times d_2 \\ &= \tfrac{1}{2}(28 \times 10) \qquad \text{Substitute values} \\ &= \tfrac{1}{2}(280) \\ &= 140 \text{ cm}^2 \end{aligned}$$

b.

$$\begin{aligned} P &= 4l \\ &= 4 \times 17 \text{ cm} \\ &= 68 \text{ cm} \end{aligned}$$

4. Find the area of the rhombus below, which has sides of length 15 cm and diagonal $|AC| = 24$ cm



Solution:

We can apply Pythagoras' Theorem to find the length of the other diagonal. Use right triangle AOD to find the length of half of the diagonal. Since $|AC| = 24$, the side of the triangle is $|AO| = \frac{1}{2}|AC| = 12 \text{ cm}$.

$$\begin{aligned} AD^2 &= AO^2 \times OD^2 && \text{Apply Pythagoras theorem} \\ 15^2 &= 12^2 \times OD^2 && \text{Substitute values} \\ 225 &= 144 + OD^2 \\ 225 - 144 &= \mathrm{OD}^2 && \text{Transpose 144} \\ 81 &= OD^2 \\ OD &= 9 \text{ cm} \end{aligned}$$

Since OD is only half of the diagonal, multiply by 2 to find the diagonal:

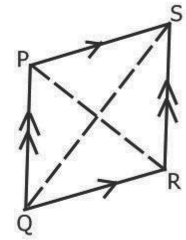
$$\begin{aligned} \text{Therefore, } BD &= 2\mathrm{OD} \\ &= 2 \times 9 \\ &= 18 \text{ cm} \end{aligned}$$

We now have enough information to apply the formula for the area:

$$\begin{aligned} A &= \tfrac{1}{2} d_1 \times d_2 \\ &= \tfrac{1}{2}(24 \times 18) \\ &= \frac{432}{2} \\ &= 216 \text{ cm}^2 \end{aligned}$$

Practice

1. Calculate the perimeter and area of the rhombus with sides of length 8.3 m, and diagonals 9 m and 14 m.
2. Find the lengths of the sides of a rhombus with perimeter 160 cm.
3. The floor of a building consists of $3{,}000$ tiles which are rhombus shaped and its diagonals are 30 cm and 15 cm in length.
    a. Find the area of one tile in square centimetres.
    b. Find the area of the floor in square metres (use $1 \text{ m}^2 = 10{,}000 \text{ cm}^2$)
    c. Find the total cost of polishing the floor, if the cost per m^2 is $\mathrm{Le}200.00$.
4. In the diagram, $PQRS$ is a rhombus with diagonals $|PR| = 10 \text{ cm}$ and $|QS| = 24 \text{ cm}$. Calculate the area and perimeter of the rhombus.



5. A rhombus has one diagonal that is 9 m. If its area is 108 m^2, what is the length of the other diagonal?

Lesson Title: Perimeter and area of a kite	Theme: Mensuration
Practice Activity: PHM2-L071	**Class:** SSS 4

Learning Outcome
By the end of the lesson, you will be able to calculate the perimeter and area of a kite.

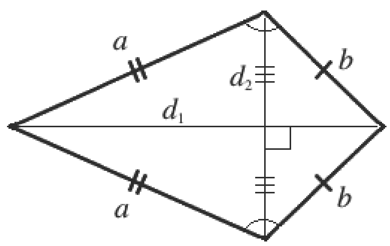
Overview

We calculate the perimeter in the same way for any shape: by adding all the sides. Because the kite has 2 sets of 2 equal sides, we can simply multiply both lengths by 2 before adding them together. The area is found using the same formula we used for a rhombus. Multiply the diagonals by $\frac{1}{2}$.

The formulae are:

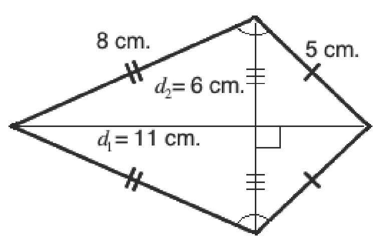
Shape	Perimeter	Area
Kite	$P = a + a + b + b = 2a + 2b$	$A = \frac{1}{2} d_1 \times d_2$

For the shape:



Solved Examples

1. Find the perimeter and area of the kite:

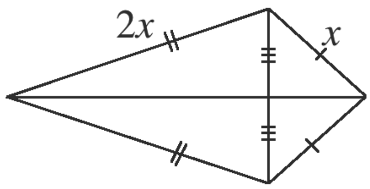


Solution:

$$\begin{aligned} P &= 2a + 2b \\ &= 2(8) + 2(5) && \text{Substitute values} \\ &= 16 + 10 \\ &= 26 \text{ cm} \end{aligned}$$

$$
\begin{aligned}
A &= \tfrac{1}{2} d_1 \times d_2 \\
&= \tfrac{1}{2}(11 \times 6) \\
&= \tfrac{1}{2}(66) \\
&= 33 \text{ cm.}^2
\end{aligned}
$$

2. The kite below has a perimeter of 48 cm. Find the lengths of the sides.



Step 1. Find the value of x:

P	$=$	$2a + 2b$	Use the perimeter formula.
48	$=$	$2(x) + 2(2x)$	Substitute $P = 48, a = x$ and $b = 2x$
48	$=$	$2x + 4x$	Simplify
48	$=$	$6x$	
8 cm	$=$	x	Divide both sides by 6

Step 2. Find the length of the sides. Call them a and b

$a = x = 8$ cm

$b = 2x = 2(8) = 16$ cm

3. The centre stage of a school is in the shape of a kite. The diagonals are 23 m and 32 m. What is the area of the school's centre stage?

Solution:

$$
\begin{aligned}
A &= \tfrac{1}{2} d_1 \times d_2 \\
&= \tfrac{1}{2}(23 \times 32) \\
&= \tfrac{1}{2}(736) \\
&= 368 \text{ m}^2
\end{aligned}
$$

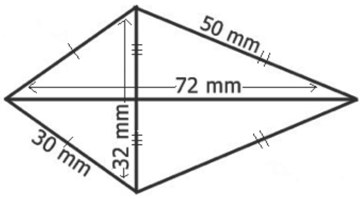
4. A kite has one diagonal that is 10 cm. If the area of the kite is 125 cm^2, what is the length of the other diagonal?

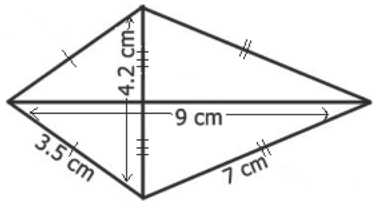
Solution:

A	$= \frac{1}{2} d_1 \times d_2$	
125 cm^2	$= \frac{1}{2}(10 \text{ cm}) \times d_2$	Substitute the given values
125 cm^2	$= 5 \text{ cm} \times d_2$	Simplify
$\frac{125 \text{ cm}^2}{5}$	$= d_2$	Divide both sides by 5
25 cm	$= d_2$	

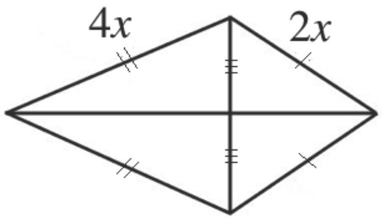
Practice

1. Find the area and perimeter of the following kites:

a. 

b. 

2. An agricultural bed prepared for corn planting has the shape of a kite. The diagonals are 45 m and 35 m. Find the area of the bed.
3. The kite below has a perimeter 120 mm. Find the lengths of the sides.



4. A kite has one diagonal that is $30\ \mathrm{mm}$. If its area is $1{,}500\ \mathrm{mm}^2$, what is the length of the other diagonal?

Lesson Title: Perimeter and area of a triangle	**Theme:** Mensuration
Practice Activity: PHM2-L072	**Class:** SSS 4

Learning Outcome
By the end of the lesson, you will be able to calculate the perimeter and area of a triangle.

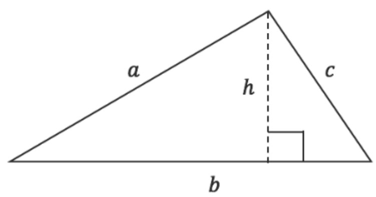
Overview

We calculate the perimeter in the same way for any shape: by adding all of the sides. To find the perimeter of a triangle, simply add the 3 sides. The area is found using a formula that is specifically for triangles. Multiply the base and height by $\frac{1}{2}$.

The formulae are:

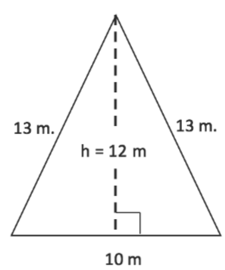
Shape	Perimeter	Area
Triangle	$P = a + b + c$	$A = \frac{1}{2}b \times h$

For the shape:



Solved Examples

1. Find the perimeter and area of the triangle:



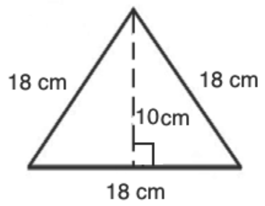
Solution:

$$\begin{aligned} P &= 13 + 13 + 10 \\ &= 36\text{m} \\ A &= \frac{1}{2}\ b \times h \\ &= \frac{1}{2}(10 \times 12) \qquad \text{Substitute the values} \end{aligned}$$

$= \frac{1}{2}(120)$ Simplify

$= 60 \text{ cm}^2$

2. The equilateral triangle shown below has sides of length 18 cm, and a height of 10 cm. Find the area and perimeter of the triangle.



Solution:

$A = \frac{1}{2}\ b \times h$

$= \frac{1}{2} \times 18 \times 10$ Substitute values

$= \frac{1}{2}(180)$ Simplify

$= 90 \text{ cm}^2$

$P = 18 + 18 + 18$

$= 54 \text{ cm}$

3. The area of the triangle is 108 cm^2. If its height is 18 cm, find the length of its base.

Solution:

$A = \frac{1}{2}\ b \times h$

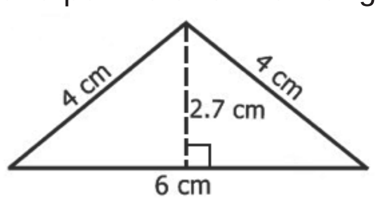
$108 \text{ cm}^2 = \frac{1}{2} \times b \times 18 \text{ cm}$ Substitute values

$2 \times 108 \text{ cm}^2 = 18b$ Simplify

$\frac{216}{18} = b$ Divide throughout by 18

$12 \text{ cm} = b$

4. The height of an isosceles triangle is 2.7 cm, its base is 6 cm, and the equal sides are 4 cm long. Find the area and perimeter of the triangle.



Solution:

$A = \frac{1}{2}\ b \times h$

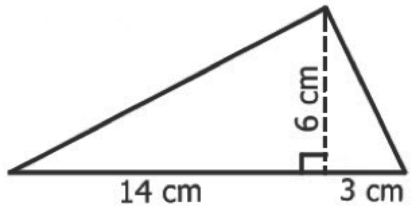
$= \frac{1}{2} \times 6 \times 2.7$

$= \frac{1}{2}(16.2)$

$= 8.1 \text{ cm}^2$

$$\begin{aligned} P &= 4 + 4 + 6 \\ &= 14 \text{ cm} \end{aligned}$$

5. Find the area of the triangle:



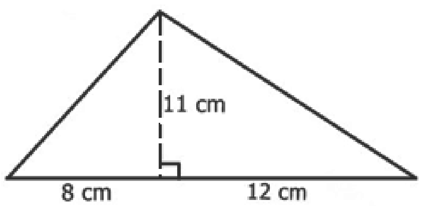
Solution:

$$\begin{aligned} A &= \tfrac{1}{2}\ b \times h \\ &= \tfrac{1}{2}\ \times 17 \times 6 && \text{Substitute values} \\ &= \tfrac{1}{2} \times (102) && \text{Simplify} \\ &= 51 \text{ cm}^2 \end{aligned}$$

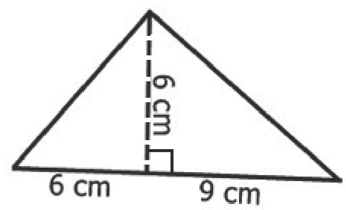
Practice

1. Find the area of the triangles below:

a.

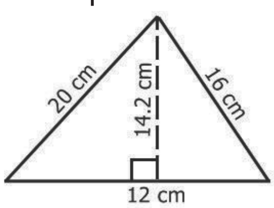


b.

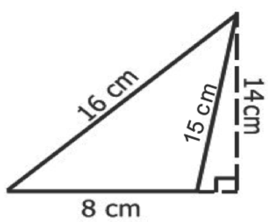


2. Calculate the perimeter and area of the following triangles:

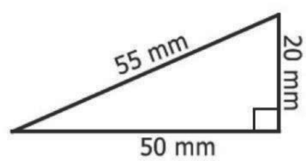
a.



b.

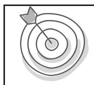


c.



3. A plot of land to be cultivated is in the shape of a triangle which has sides of length 24 m, 20 m and 18 m. Find the perimeter of the plot of land.

Lesson Title: Circumference and area of a circle	**Theme:** Mensuration
Practice Activity: PHM2-L073	**Class:** SSS 4



Learning Outcome
By the end of the lesson, you will be able to calculate the circumference and area of a circle.

Overview

The perimeter of a circle is known as circumference. To calculate the circumference and area of a circle, use the formulae.

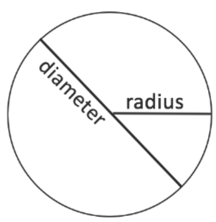
Each formula involves radius, which is the distance from the centre of the circle to its circumference. The diameter is twice the radius ($d = 2r$), and the radius is half of the diameter ($r = \frac{1}{2}d$).

The formulae also use pi (π). Pi is a decimal number that stretches on forever. It can be estimated with numbers such as 3.14 and $\frac{22}{7}$. We will use these numbers in our calculations.

The formulae are:

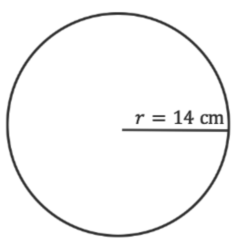
Shape	Circumference	Area
Circle	$C = 2\pi r$	$A = \pi r^2$

For the shape:



Solved Examples

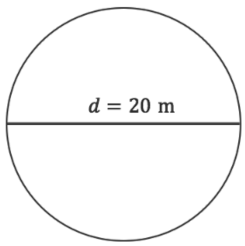
1. Find the circumference and area of the circle. Use $\pi = \frac{22}{7}$.



Solution:

$$C = 2\pi r = 2\left(\frac{22}{7}\right)(14\text{ cm}) = 2(22)(2\text{ cm}) = 88\text{ cm}.$$
$$A = \pi r^2 = \left(\frac{22}{7}\right)14^2 = \left(\frac{22}{7}\right)196 = 22 \times 28 = 616\text{ cm}^2$$

2. Find the circumference and area of the circle. Use $\pi = 3.14$.



Solution:

First find the radius. Divide diameter by 2:

$$r = \frac{1}{2}d = \frac{1}{2}(20\text{ m}) = 10\text{ m}$$

Find the circumference:

$$C = 2\pi r = 2(3.14)(10) = 2(31.4) = 62.8\text{ m}$$

Find the area:

$$A = \pi r^2 = (3.14)(10^2) = (3.14)(100) = 314\text{ m}^2$$

3. Find the circumference of the circle whose radius is 3.5 mm. Use $\pi = \frac{22}{7}$.

Solution:

$$C = 2\pi r = 2 \times \frac{22}{7} \times 3.5 = \frac{154}{7} = 22\text{ mm}$$

4. Calculate the circumference and area of the circle with radius 10.5 cm. Use $\pi = \frac{22}{7}$.

Solution:

$$C = 2\pi r = 2 \times \frac{22}{7} \times 10.5 = \frac{462}{7} = 66\text{ cm}$$
$$A = \pi r^2 = \frac{22}{7}(10.5)^2 = \frac{2425.5}{7} = 346.5\text{ cm}^2$$

5. Find the radius of a circle whose area is 61.6 cm^2. Use $\pi = \frac{22}{7}$. Give your answer to 2 decimal places.

Solution:

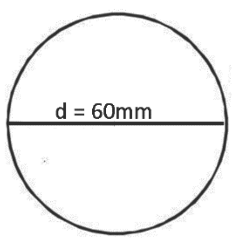
A	$=$	πr^2	
61.6	$=$	$\frac{22}{7} \times r^2$	Substitute given values
61.6×7	$=$	$22 \times r^2$	Multiply throughout by 7
431.2	$=$	$22r^2$	

$$\begin{aligned} \frac{431.2}{22} &= r^2 && \text{Divide throughout by 22} \\ 19.6 &= r^2 \\ \sqrt{19.6} &= r \\ 4.43 \text{ cm} &= r \end{aligned}$$

6. Find the diameter of the circle whose circumference is 31.4 mm. Use $\pi = 3.14$.
   Solution:
   Step 1. Use the formula for circumference to find the radius:
   $$\begin{aligned} C &= 2\pi r \\ 31.4 &= 2 \times 3.14 \times r && \text{Substitute given values} \\ 31.4 &= 6.28r \\ \frac{31.4}{6.28} &= r && \text{Divide throughout by 6.28} \\ 5 \text{ mm} &= r \end{aligned}$$
   Step 2. Use the radius to find the diameter:
   We know that the diameter is twice the radius ($d = 2r$).
   $$d = 2(5 \text{ mm})$$
   $$d = 10 \text{ mm}$$

Practice

1. Find the circumference of a circle whose radius is 13.8 metres, to the nearest 1 decimal place. (Take $\pi = 3.14$)
2. Find the diameter of a circle whose circumference is 18.9 mm. Give your answer to the nearest whole number. (Take $\pi = 3.14$)
3. The area of a circle is 154 cm^2. Find its radius and circumference to the nearest whole number. (Take $\pi = 3.14$)
4. Find the circumference and area of the circle below, to 2 decimal places. (Take $\pi = \frac{22}{7}$)



5. The circumference of a circle is 88 metres. Find its radius and area to 2 decimal places. (Take $\pi = 3.14$)
6. A goat is tied to a pole with a rope 15 metres long. He can eat all the grass around him, the area of which forms a perfect circle. For the questions below, take $\pi = \frac{22}{7}$ and give your answers to the nearest tenth.
   a. What is the area of the grass from which the goat can eat?
   b. What is the circumference of the area he can eat?

Lesson Title: Perimeter and area of compound shapes	**Theme:** Mensuration
Practice Activity: PHM2-L074	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to calculate the perimeter and area of a compound shape.

Overview

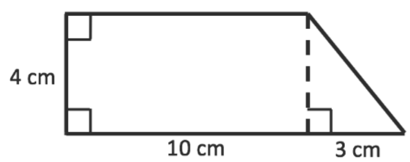
To find the area or perimeter of a compound shape, first divide it into its individual parts. Find any missing side lengths.

To find the **perimeter**, add the sides of the shape together. Do not count any shared sides that are inside of the compound shape toward the perimeter.

Find the **area** of the compound shape by finding the area of each of the shapes it is made from, and adding them together.

Solved Examples

1. Find the perimeter and area of the shape.

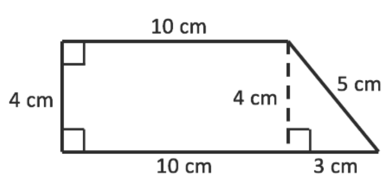


Solution:

Step 1. Find and label the unknown sides:

- Label the top of the rectangle 10 cm. Label the shared side 4 cm.
- Calculate the missing side of the triangle using Pythagoras' theorem:

$3^2 + 4^2$	$=$	c^2	Substitute 3 and 4 into the formula
$9 + 16$	$=$	c^2	Simplify
25	$=$	c^2	
$\sqrt{25}$	$=$	$\sqrt{c^2}$	Take the square root of both sides
c	$=$	5	



Step 2. Find perimeter by adding the lengths of the sides:

$P = 4 + 10 + 5 + 3 + 10 = 32$ cm

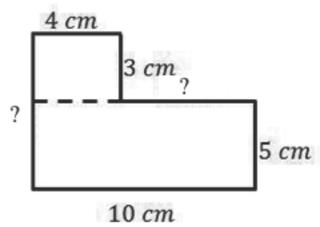
Step 3. Find the area by finding the areas of the rectangle and triangle, then adding them together:

Area of the rectangle: $A = l \times w = 10 \times 4 = 40 \text{ cm}^2$

Area of the triangle: $A = \frac{1}{2} b \times h = \frac{1}{2}(3)(4) = 3 \times 2 = 6 \text{ cm}^2$

Total area: $40 \text{ cm}^2 + 6 \text{ cm}^2 = 46 \text{ cm}^2$

2. Find the perimeter and area of the shape.

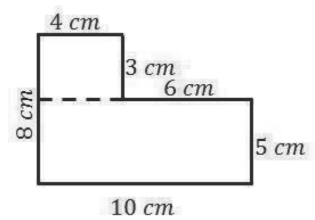


Solution:

Step 1. Find the missing side lengths and label them:

$3\ cm + 5\ cm = 8\ cm$

$10\ cm - 4\ cm = 6\ cm$



Step 2. To find the perimeter, add all of the sides together

$4 + 3 + 6 + 5 + 10 + 8 = 36\ cm$

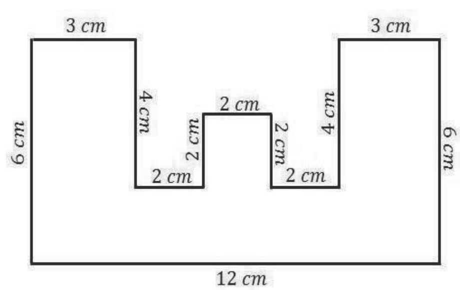
Step 3. To find the total area, find the areas of both rectangles, then add them together:

Area of rectangle 1: $A = l \times w = 10 \times 5 = 50\ cm^2$

Area of rectangle 2: $A = l \times w = 4 \times 3 = 12\ cm^2$

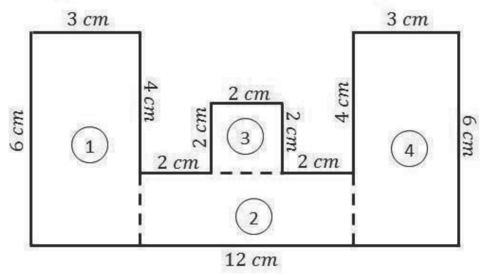
Total Area: $A = 50\ cm^2 + 12\ cm^2 = 62\ cm^2$

3. Find the area of the shape:



Solution:

Step 1. Divide the composite shape into smaller shapes:



Step 2. Each shape is now much easier to evaluate. Calculate the area of each:

Shape 1: A $= 6 \times 3 = 18\ cm^2$

Shape 2: A $= 6 \times 2 = 12\ cm^2$

Shape 3: A $= 2 \times 2 = 4\ cm^2$

Shape 4: A $= 6 \times 3 = 18\ cm^2$

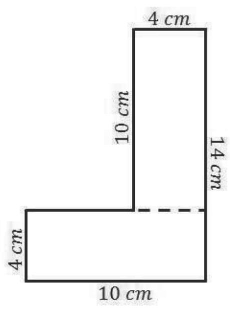
Step 3. Add to find the area of the whole shape:

$A = 18\ cm^2 + 12\ cm^2 + 4\ cm^2 + 18\ cm^2 = 52\ cm^2$

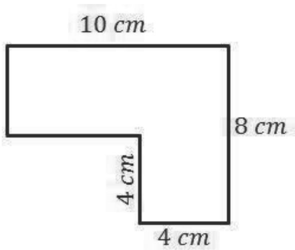
Practice

Find the perimeter and area of each shape.

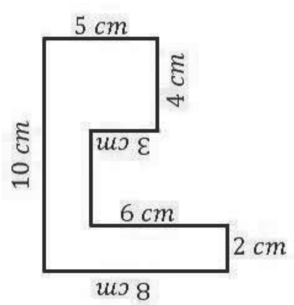
1.



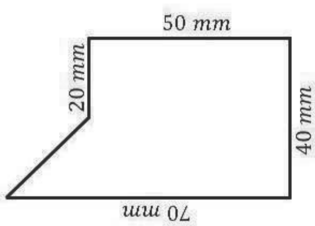
2.



3.



4.

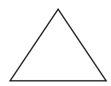
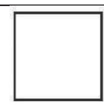
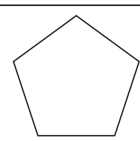
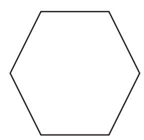
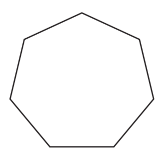
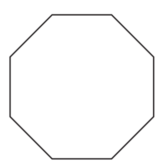
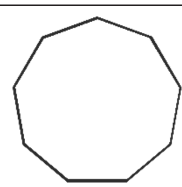


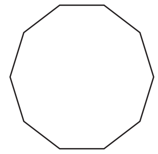
Lesson Title: Properties of polygons	**Theme:** Geometry
Practice Activity: PHM2-L075	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to identify and describe properties of polygons (pentagon to decagon).

Overview

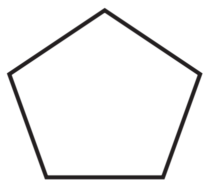
This lesson is on the properties of the polygons in the table below.

Sides	Name	Picture
3	Triangle	
4	Quadrilateral	
5	Pentagon	
6	Hexagon	
7	Heptagon	
8	Octagon	
9	Nonagon	

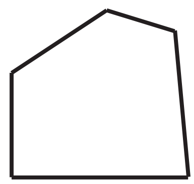
10	Decagon	

The name of each shape tells us how many sides and angles it has. “Tri” means 3, which is the number of sides and angles in a triangle. “Quad” means 4, which is the number of sides and angles in a quadrilateral. “Penta” means 5, “hexa” means 6, “hepta” means 7, and so on.

A **regular** polygon is one with equal angles and equal sides. For example, an equilateral triangle is a regular polygon. A square is also a regular polygon. A regular pentagon is shown below, alongside one that is not regular:

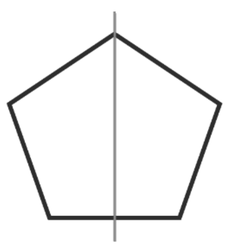


regular pentagon

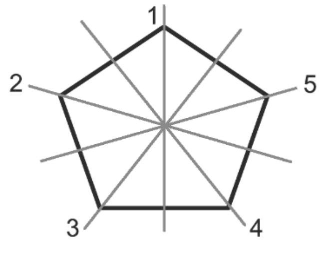


pentagon

A regular polygon with a given number of sides has the same number of **lines of symmetry**. Recall that a line of symmetry is like a mirror. The polygon is exactly the same shape on both sides of the line of symmetry. A line of symmetry of a pentagon is shown below:

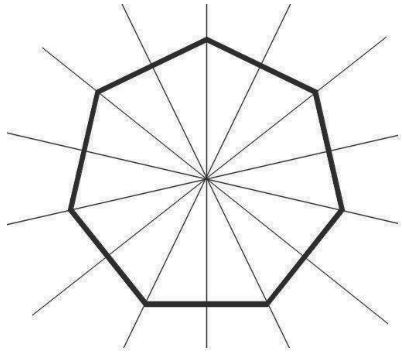


A regular pentagon has 5 lines of symmetry, because it has 5 sides. All 5 are shown below:

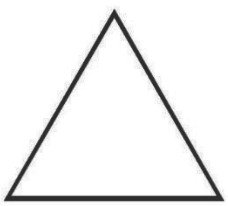
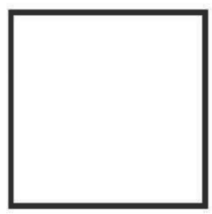
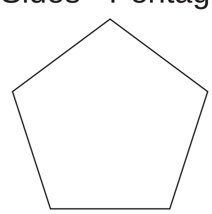
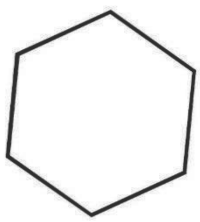
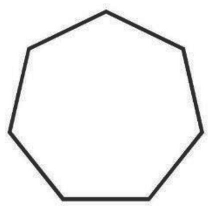
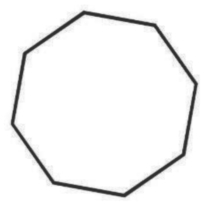


Solved Examples

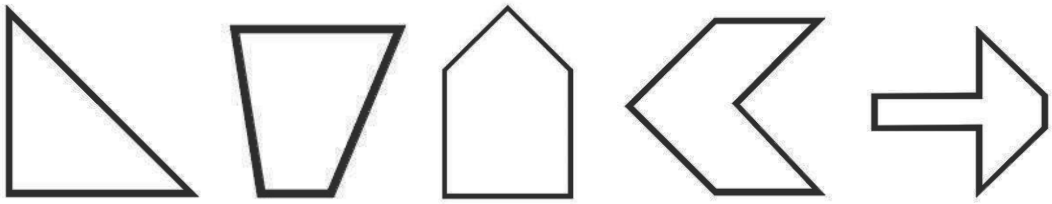
1. Sketch a regular heptagon. Sketch its lines of symmetry. How many are there?



2. List and draw six types of regular polygons.
   Solution:
   Regular polygons are polygons that have all sides equal, and all internal angles are equal. 6 example answers are given below.

3 Sides - Triangle	4 Sides - Quadrilateral	5 Sides - Pentagon
		
6 Sides - Hexagon	7 Sides - Heptagon	8 Sides - Octagon
		

3. Draw some examples of irregular polygons.
   Solution:
   There are many different types of irregular polygons. These are examples of polygons that are not regular:



Practice

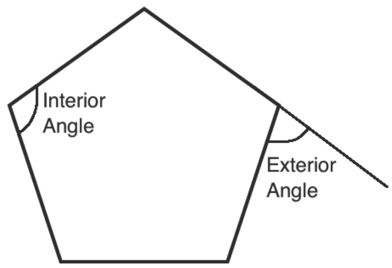
1. Draw a regular octagon. Draw its lines of symmetry. How many are there?
2. Draw a regular decagon.

Lesson Title: Sum of interior angles of polygons	Theme: Geometry
Practice Activity: PHM2-L076	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to calculate the sum of the interior angles of polygons.

Overview

This lesson is on the interior angles of a polygon. The next lessons are also on interior and exterior angles. The diagram below shows these on a pentagon.



Interior angles are the angles that are on the inside of a polygon. **Exterior angles** are on the outside. They are formed by extending the sides of the polygon. Every polygon has the same number of interior and exterior angles as it has sides. For example, a pentagon has 5 interior angles and 5 exterior angles.

Note that each exterior angle forms a straight line with an interior angle. This means that the exterior and interior angles sum to 180°.

This is the formula for finding the **sum** of the interior angles in a polygon:

$(n-2)\times 180°$ where n is the number of sides.

The sum of the interior angles up to a decagon are given in the table below. It is important that you understand how these are calculated as well.

Sides	Name	Sum of Interior Angles
3	Triangle	180°
4	Quadrilateral	360°
5	Pentagon	540°
6	Hexagon	720°
7	Heptagon	900°
8	Octagon	1,080°
9	Nonagon	1,260°
10	Decagon	1,440°

Solved Examples

1. Calculate the sum of the interior angles of a pentagon.
   Solution:
   A pentagon has 5 sides, so substitute $n = 5$ in the formula and solve:

$$\begin{aligned} \text{Sum of angles} &= (n-2)\times 180° \\ &= (5-2)\times 180° \\ &= 3\times 180° \\ &= 540° \end{aligned}$$

2. Calculate the sum of the interior angles of a polygon with 15 sides.
   Solution:
   Substitute $n = 15$ in the formula and solve:

$$\begin{aligned} \text{Sum of angles} &= (n-2)\times 180° \\ &= (15-2)\times 180° \\ &= 13\times 180° \\ &= 2{,}340° \end{aligned}$$

3. The sum of the interior angles of a polygon is 1,440°. Calculate the number of sides.
   Solution:
   This can be solved by substituting 1,440° into the interior angle formula, then solving for n.

$$\begin{aligned} \text{Sum of angles} &= (n-2)\times 180° & & \\ 1{,}440° &= (n-2)\times 180° & & \text{Substitute the sum} \\ 8 &= n-2 & & \text{Divide throughout by 180°} \\ 8+2 &= n & & \text{Transpose } -2 \\ n &= 10 & & \end{aligned}$$

   The polygon has 10 sides. It is a decagon.

4. The sum of the interior angles of a regular polygon is $900°$. How many sides does it have?
   Solution:

$$\begin{aligned} \text{Sum of angles} &= (n-2)\times 180° & & \\ 900° &= (n-2)\times 180° & & \text{Substitute the values} \\ 900° &= 180°n - 360° & & \text{Clear bracket} \\ 900° + 360° &= 180°n & & \text{Transpose } -360° \\ 1260° &= 180°n & & \text{Divide throughout by } 180° \\ \frac{1260°}{180°} &= n & & \text{Simplify} \\ n &= 7 & & \end{aligned}$$

   The polygon has 7 sides. It is a heptagon.

Practice

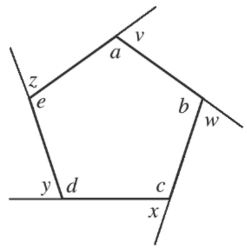
1. Calculate the sum of the interior angles of polygons with the following number of sides:
    a. 20
    b. 25
    c. 30
    d. 50
2. The sums of the interior angles of certain polygons are given below. Find the number of sides that each polygon has.
    a. $2,880°$
    b. $4,680°$
    c. $1,620°$
    d. $1,260$

Lesson Title: Interior and exterior angles of polygons	**Theme:** Geometry
Practice Activity: PHM2-L077	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to calculate the measurement of interior and exterior angles of polygons.

Overview

This lesson is on calculating the measures of interior and exterior angles of a polygon. In the diagram below, the following angles are interior angles: a, b, c, d, e. The following angles are exterior angles: v, w, x, y, z.



For regular polygons, all of the interior angles are equal. There is a formula for finding the measure of each interior angles of a regular polygon: $\frac{(n-2)\times 180^\circ}{n}$ where n is the number of sides.

For polygons that are not regular, missing angles can be found by subtracting known angles from the sum of the angles for that type of polygon. The sum of interior angles for polygons up to a decagon were given in the previous lesson.

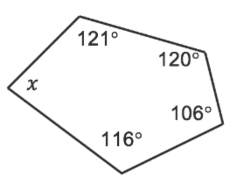
Exterior angles can be found if you know the corresponding interior angle. Recall that exterior angles form a straight line with interior angles. This means that each exterior angle and the adjacent interior angle sum to 180°.

Exterior angles can also be found if you are given known exterior angles. All of the exterior angles of **any** polygon sum to 360°.

The exterior angles of a regular polygon are all equal. We can use the following formula to find the measure of each exterior angle: $\frac{360^\circ}{n}$ where n is the number of sides.

Solved Examples

1. Find the missing angle x:



Solution:
From the previous lesson, we know that the sum of the angles of a pentagon is $540°$. Therefore, subtract the known angles from $540°$ to find x:

$$x = 540° - 121° - 120° - 106° - 116° = 77°$$

2. Find the interior angle of a regular hexagon.
   Solution:
   Apply the formula and substitute $n = 6$, because a hexagon has 6 sides.

$$\begin{aligned}\text{Interior angle} &= \frac{(n-2)\times 180°}{n} \\ &= \frac{(6-2)\times 180°}{6} \\ &= \frac{4\times 180°}{6} \\ &= \frac{720°}{6} \\ &= 120°\end{aligned}$$

3. The interior angle of a regular polygon is $108°$. Find the number of sides of the polygon.
   Solution:
   Apply the formula for interior angle, and solve for n:

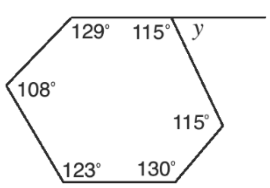
$$\begin{aligned}108° &= \frac{(n-2)\times 180°}{n} \\ 108°n &= (n-2)\times 180° \\ 108°n &= 180°n - 360° \\ 108°n - 180°n &= -360° \\ -72°n &= -360° \\ \frac{-72°n}{-72°} &= \frac{-360°}{-72°} \\ n &= 5\end{aligned}$$

   The polygon has 5 sides. It is a pentagon.

4. Find the exterior angle of a regular hexagon.
   Solution:
   Apply the formula for finding the exterior angle of a regular polygon.

$$\begin{aligned}\text{Exterior angle} &= \frac{360°}{n} \\ &= \frac{360°}{6} \\ &= 60°\end{aligned}$$

5. Find angle y in the diagram.



Solution:

Angle y corresponds to interior angle 115°. Subtract from 180° to find its measure: $y = 180° - 115° = 65°$

6. A regular polygon has 20 sides. Find using the formula:
   a. The measure of each interior angle.
   b. The measure of each exterior angle.

   Solution:

   a. Interior angle

$$\begin{aligned} &= \frac{(n-2)\times 180°}{n} \\ &= \frac{(20-2)\times 180°}{20} \\ &= \frac{18\times 180°}{20} \\ &= \frac{3240}{20} \\ &= 162° \end{aligned}$$

   b. The Exterior angle can be calculated using the formula, or by using the measure of the interior angle. Both methods are shown:

   Exterior angle

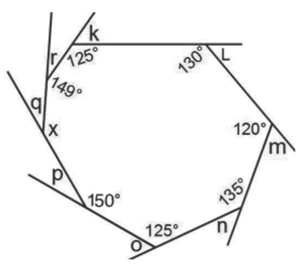
$$\begin{aligned} &= \frac{360°}{n} \\ &= \frac{360°}{20} \\ &= 18° \end{aligned}$$

   Exterior angle

$$\begin{aligned} &= 180° - \text{interior angle} \\ &= 180° - 162° \\ &= 18° \end{aligned}$$

Practice

1. Find the missing interior and exterior angles in the octagon below.



2. Each interior angle of a regular polygon is $140°$. Find the number of sides of the polygon.
3. A regular polygon has 15 sides. Find using the formulae:
   a. The measure of each exterior angle.
   b. The measure of each interior angle.
   c. The sum of the interior angles.

Lesson Title: Polygon problem solving	**Theme:** Geometry
Practice Activity: PHM2-L078	**Class:** SSS 2

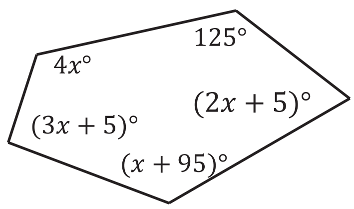
Learning Outcome
By the end of the lesson, you will be able to solve problems involving polygons.

Overview

This lesson uses information on the angles of polygons from previous lessons to solve various problems.

Solved Examples

1. The interior angles are given in the pentagon below. Solve for x:



Solution:
The angles are labelled in this pentagon but there is an unknown variable. Recall that the angles of a pentagon add up to 540°. We can add the interior angles and set them equal to 540°. Then, solve for x.

$$\begin{aligned} 540^\circ &= 125^\circ + (2x+5)^\circ + (x+95)^\circ + (3x+5)^\circ + 4x^\circ && \text{Add the angles} \\ &= (125^\circ + 5^\circ + 95^\circ + 5^\circ) + (2x + x + 3x + 4x)^\circ && \text{Combine like terms} \\ &= 230^\circ + 10x^\circ && \\ 540^\circ - 230^\circ &= 10x^\circ && \\ 310^\circ &= 10x^\circ && \text{Transpose } 230^\circ \\ 31^\circ &= x && \text{Divide by } 10^\circ \end{aligned}$$

2. The sum of the interior angles of a regular polygon is 3,600° . Calculate:
   a. The number of sides.
   b. The value of one exterior angle.

   Solution:
   a. The number of sides can be found from the formula:

$$\begin{aligned} \text{Sum of angles} &= (n-2) \times 180^\circ && \\ 3{,}600^\circ &= (n-2) \times 180^\circ && \text{Substitute the sum} \\ 20 &= n - 2 && \text{Divide throughout by } 180^\circ \\ 20 + 2 &= n && \text{Transpose } -2 \\ n &= 22 && \end{aligned}$$

   The polygon has 22 sides.

   b. The measure of one exterior angle can be found from the formula:

$$\begin{aligned} \text{Exterior angle} &= \frac{360°}{n} \\ &= \frac{360°}{22} \\ &= \frac{180°}{11} \\ &= 16\frac{4}{11}° \end{aligned}$$

Each interior angle is $16\frac{4}{11}°$. This can also be calculated to be $16.\overline{36}°$ using a calculator.

3. A regular polygon has exterior angles measuring 40°. How many sides does the polygon have?

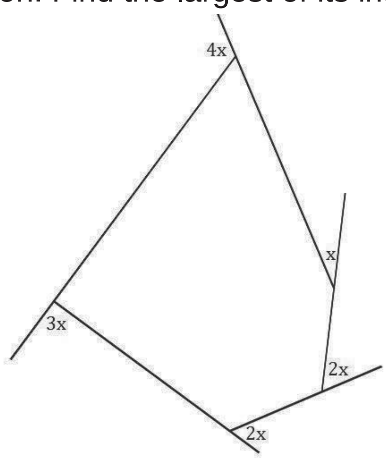
   Solution:

   Use the formula:

$$\begin{aligned} \text{Exterior angle} &= \frac{360°}{n} & & \\ 40° &= \frac{360°}{n} & & \text{Substitute } 40° \\ 40° \times n &= 360° & & \text{Multiply throughout by } n \\ n &= \frac{360°}{40°} & & \text{Divide throughout by } 40° \\ n &= 9 & & \end{aligned}$$

   The polygon has 9 sides. It is a nonagon.

4. The diagram shows a polygon. Find the largest of its interior angles.



   Solution:

   First, note that the largest of its interior angles will correspond the smallest of its exterior angles. The smallest exterior angle is x, which forms a straight line ($180°$) with the largest interior angle.

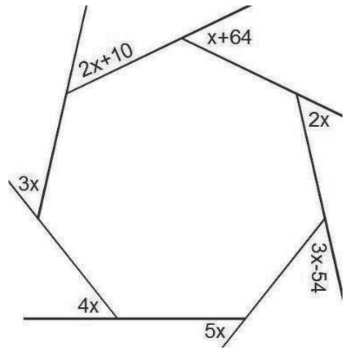
   Step 1. Find the value of x:

$$\begin{aligned} 4x + x + 2x + 2x + 3x &= 360° & & \\ 12x &= 360° & & \\ x &= \frac{360°}{12} & & \text{Divide both side by 12} \\ x &= 30° & & \end{aligned}$$

   Step 2. Since the largest interior angle forms a line with x, subtract from $180°$:

The largest interior angle $= 180° - x = 180° - 30° = 150°$.

5. Find the value of x in the diagram:



Solution:

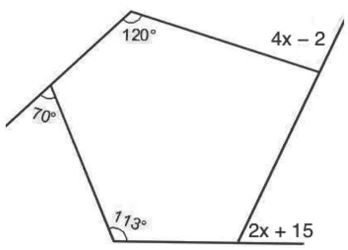
Set the sum of the exterior angles equal to $360°$, and solve for x.

$$\begin{aligned}
(2x + 10) + (x + 64) + 2x + (3x - 54) + 5x + 4x + 3x &= 360° \\
(2x + x + 2x + 3x + 5x + 4x + 3x) + (10 + 64 - 54) &= 360° \\
20x + 20 &= 360° \\
20x &= 360° - 20° \\
20x &= 340° \\
x &= \frac{340°}{20} \\
x &= 17°
\end{aligned}$$

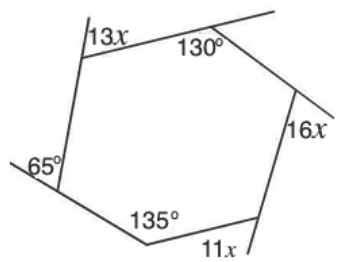
Practice

1. A heptagon has the following interior angles: $x°, (x + 4)°, (x + 8)°, (x + 14)°, (x + 26)°, (x + 34)°$, and $(x + 51)°$. Find the value of x.
2. Find the value of x in each of the diagrams below.

a.



b.



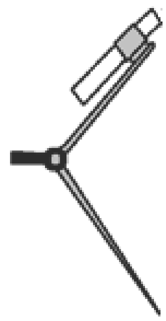
3. The interior angle of a regular polygon is $120°$. How many sides does it have?
4. A hexagon has one exterior angle with a measure of $40°$, two exterior angles with a measure of $y°$ and three exterior angles with a measure of $(20 + 2y)°$. Find the value of y.

Lesson Title: Bisect a given line segment	Theme: Geometry
Practice Activity: PHM2-L079	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to use a pair of compasses to construct a perpendicular bisection of a line.

Overview

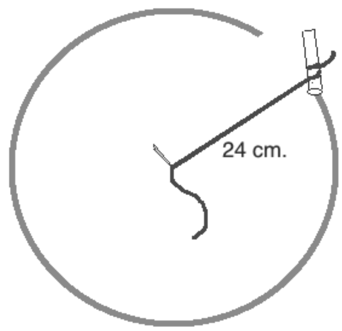
The tool pictured at right is a pair of compasses. It makes circles, and can be used to construct many different angles and shapes in geometry. It will be used for the next 18 lessons.



If you do not have a pair of compasses, you can make one using string or a strip of paper.

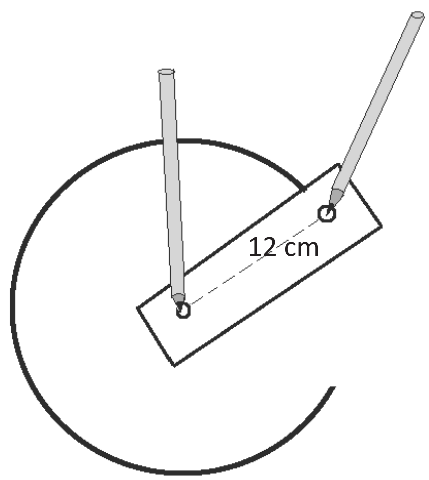
Using **string** to make a pair of compasses:

1. Cut a piece of string longer than the radius of the circle you will make.
2. Tie one end of the string to a pencil.
3. Hold the string to your paper. The distance between the place you hold and the pencil will be the **radius** of the circle. In the diagram, the radius of the circle is 24 cm.
4. Use one hand to hold the string to the same place on the paper. Use the other hand to move the pencil around and draw a circle.



Using **paper** to make a pair of compasses:

1. Cut or tear any piece of paper, longer than the radius of the circle you want to make.
2. Make two small holes in the paper. The distance between the two holes will be the radius of your circle. In the diagram below, the radius of the circle is 12 cm.
3. Put something sharp (a pen or pencil will work) through one hole. Place this in your exercise book, at the centre of the circle you want to draw.
4. Put your pen or pencil through the other hole, and move it in a circle on your paper.

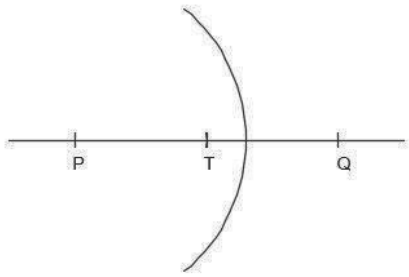


This lesson uses a pair of compasses to construct a **perpendicular bisector** of a line. To **bisect** something means to divide it into 2 equal parts. A perpendicular bisector is a line that is perpendicular to a given line, and divides it equally.

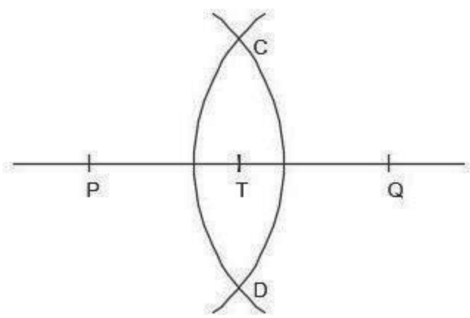
In the diagram below, $\overline{CD}$ is the perpendicular bisector of line segment $\overline{PQ}$. It is perpendicular to $\overline{PQ}$ at point T.

To construct this perpendicular bisector, follow these steps:

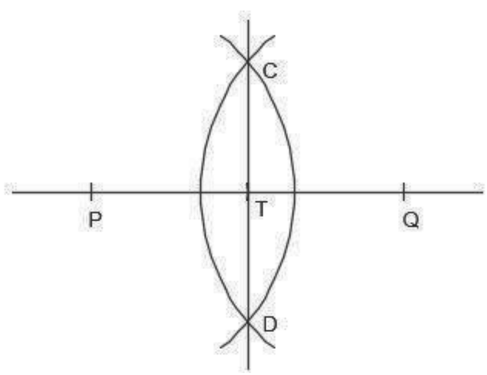
1. Draw a line, and mark point T around the middle.
2. Draw arcs to cut the line segment at 2 points the same distance from T, and label these points P and Q (It is important that $\overline{PT} = \overline{TQ}$).
3. With point P as centre, open your compass more than half way to point Q. Then draw an arc that intersects $\overline{PQ}$.



4. Using the same radius and point Q as centre, draw an arc that intersects the first arc. Label the points where the 2 arcs intersect as C and D.



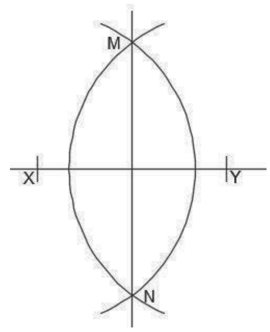
5. Draw $\overline{CD}$.



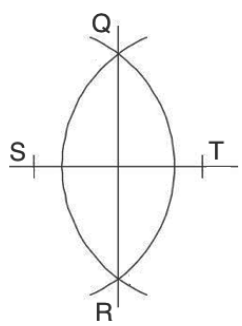
Solved Examples

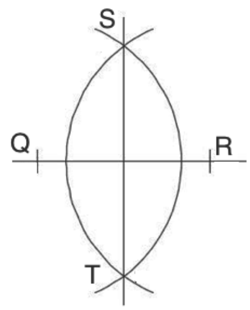
1. Construct line XY with perpendicular bisector MN.

Solution:



2. Line ST and QR are perpendicular lines. Construct the 2 lines.
   Solution:
   Either construction is correct:





3. Line AB is 10 cm long. Line CD is its perpendicular bisector, and the 2 lines intersect at point O.
   a. What is the length of AO?
   b. What is the length of OB?

   Solutions:

   This problem does not require a construction. We know that a perpendicular bisector divides a line into 2 equal segments. Therefore, $AO = OB$. Divide the length of AB by 2 to get the length of each.

   a. $|AO| = 10 \text{ cm } \div 2 = 5 \text{ cm}$
   b. $|OB| = 10 \text{ cm } \div 2 = 5 \text{ cm}$

Practice

1. Construct line BO with perpendicular bisector SL.
2. Line MN is 24 metres long. If it is bisected at point O, what is:
   a. The length of MO?
   b. The length of ON?

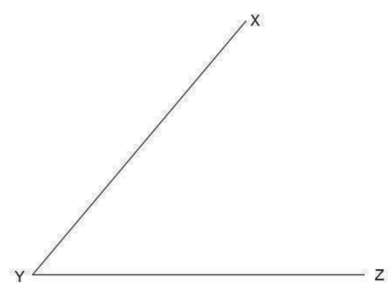
Lesson Title: Bisect a given angle	Theme: Geometry
Practice Activity: PHM2-L080	**Class:** SSS 2

Learning Outcomes
By the end of the lesson, you will be able to:
1. Use a pair of compasses to bisect an angle.
2. Use a protractor to measure a given angle and its bisected parts.

Overview

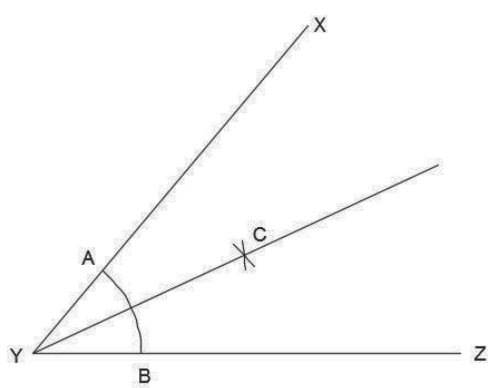
This lesson focuses on bisecting angles. To bisect an angle means to divide it into 2 equal parts. For example, if an angle is 60°, we can bisect it to find an angle of 30°.

Consider angle XYZ:

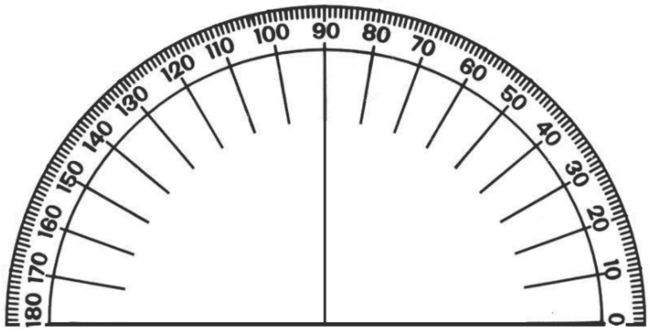


We can divide it into 2 equal angles. To bisect $\angle XYZ$, follow these steps:

1. With point Y as the centre, open your pair of compasses to any convenient radius. Draw an arc AB to cut XY at A and YZ at B.
2. With point A as centre, draw an arc using any convenient radius.
3. With the same radius as the step above, use point B as the centre and draw another arc to intersect the first one at C.
4. Label point C.
5. Join Y to C to get the angle bisector as shown.



You can use a protractor to check the measure of the angles you draw. If you do not have a protractor, you can make one with paper. Trace this protractor with a pen onto another piece of paper.



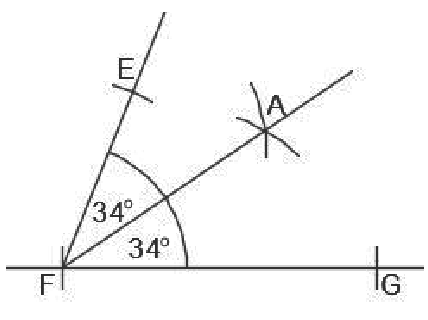
Let's check the bisection of $\angle XYZ$:

1. Hold the protractor to $\angle XYZ$, and measure the entire angle (it is $50°$).
2. Write down the angle measure.
3. Hold the protractor up again, and measure angles CYZ and XYC.
4. Write the measure of each bisection. (In this case, $\angle CYZ = 25°$ and $\angle XYC = 25°$)

Solved Examples

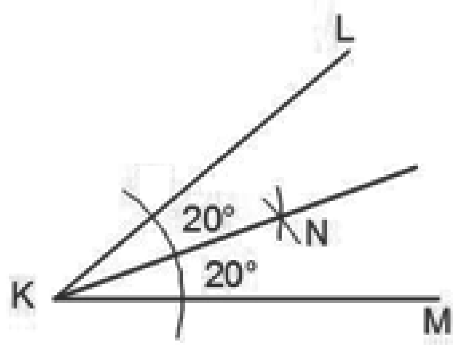
1. Draw any angle and label it EFG. On your angle:
    a. Construct a bisection, line FA.
    b. Measure each bisection and label it with its value.

    Solutions:
    Note that this is an example. The angle you draw could have a different measure.

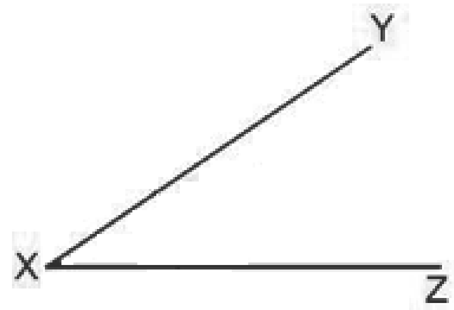


2. Draw any angle with letter labels of your choice. On your angle:
    a. Construct its bisector using any letter.
    b. Check your bisector using a protractor and label each angle with its measurement.

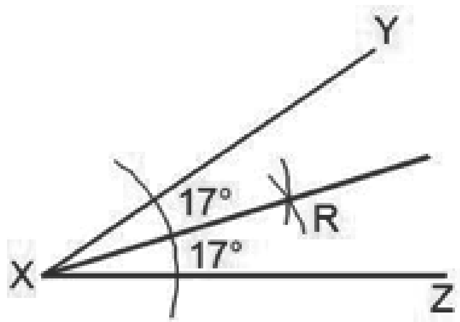
    Solutions:
    Note that this is an example. The angle you draw could have a different measure and labels.



3. Given the angle YXZ below, construct a line bisecting YXZ with X as the centre. Label the bisector R. Label each angle with its measurement.



Solution:



Practice

1. Draw any angle and label it PQR. On your angle:
    a. Construct an angle bisector QU.
    b. Measure the bisector using a protractor and label each angle with its measurement.
2. Draw an angle labeled with three letters of your choice.
    a. Bisect the angle.
    b. Check your bisection using a protractor. Label each angle with its measurement.

Lesson Title: Construct 90°, 60°, and 120° angles	Theme: Geometry
Practice Activity: PHM2-L081	**Class:** SSS 2

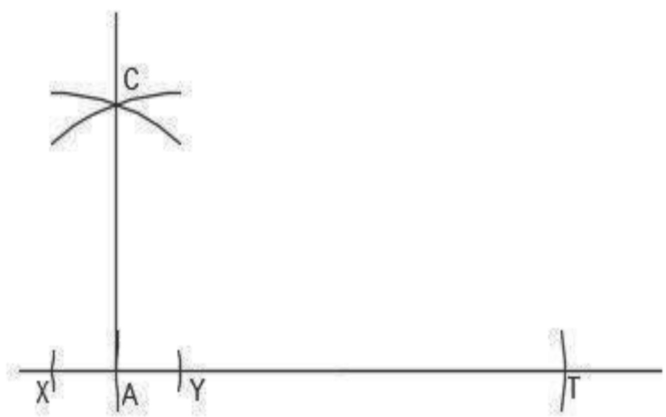
Learning Outcome
By the end of the lesson, you will be able to use a pair of compasses to construct 90°, 60° and 120° angles.

Overview

There are some angles that can be constructed without using a protractor. This lesson uses **only** a straight edge and a pair of compasses to construct 90°, 60°, and 120° angles.

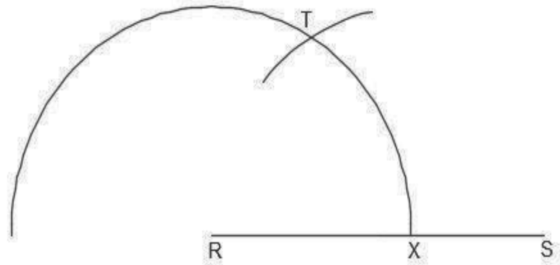
To construct a 90° angle, follow these steps:

1. Draw a horizontal line and label it AT.
2. Extend the straight line outwards from A.
3. With A as the centre, open your compass to a convenient radius and draw an arc to cut line AT at X and Y.
4. Use X and Y as centres. Using any convenient radius, draw arcs to intersect at C.
5. Draw a line from A to C.

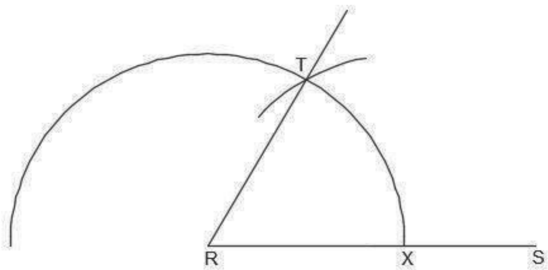


To construct a 60° angle, follow these steps:

1. Draw the line RS.
2. With centre R, open your compass to any convenient radius and draw a semi-circle that cuts RS at X.
3. With centre X and the **same radius**, draw another arc on the semi-circle. Label this point T.

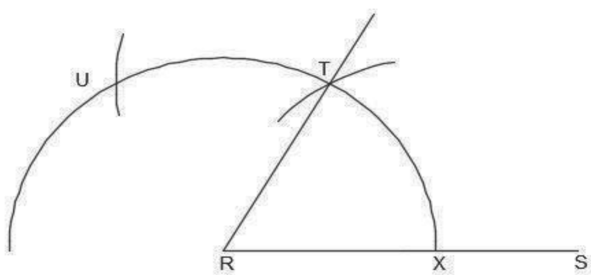


4. Draw a line from R to T. This gives $\angle TRS = 60°$.

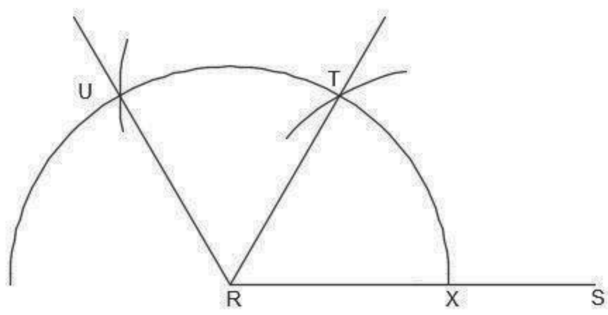


To construct a 120° angle, continue on the same construction for 60°. Follow these steps:

a. Use the same radius that we used to create the semi-circle.
b. Use T as the centre, and draw another arc on the semi-circle. Label this point U.



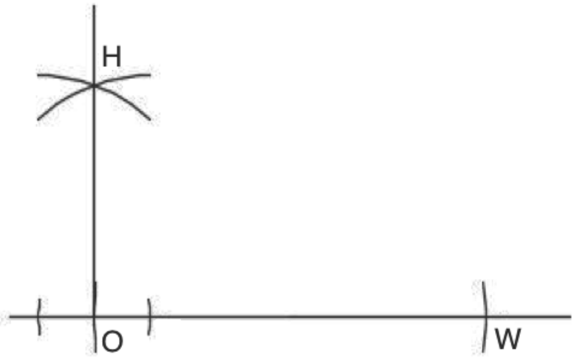
c. Draw a line from R to U. This gives $\angle URS = 120°$.



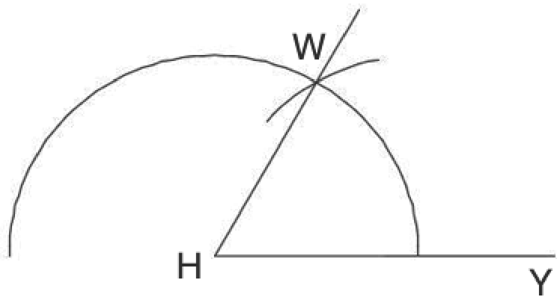
Solved Examples

1. Construct an angle of 90°. Label it $\angle HOW$.

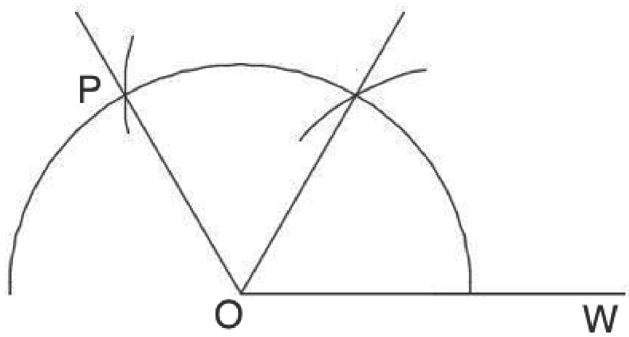
Solution:



2. Construct an angle of 60°. Label it $\angle WHY$.
   Solution:



3. Construct an angle of 120°. Label it $\angle POW$.
   Solution:



Practice

1. Construct a 90-degree angle. Label it $\angle CUP$.
2. Construct an angle of 60°. Label it $\angle BIT$.
3. On the same construction as problem 2, construct a 120-degree angle. Label it $\angle LIT$.

Lesson Title:	Theme:
Lesson Title: Construct 45°, 30° and 15° angles	**Theme:** Geometry
Practice Activity: PHM2-L082	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to use a pair of compasses to construct 45°, 30°, and 15° angles using bisection of 90° and 60°.

Overview

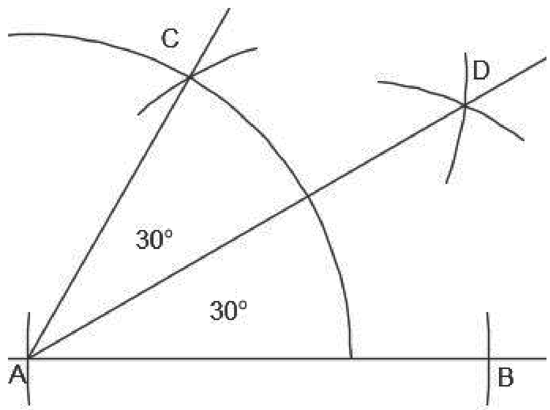
This lesson is on constructing angles using bisection. The previous lesson covered construction of 60° and 90° angles. These angles can be bisected to construct of 45°, 30°, and 15° angles.

For 15°, the 60° angle will be bisected twice. The first bisection gives a 30° angle. Bisection of the 30°angle gives a 15° angle.

For example, these are the steps to construct a 30° angle:

1. Construct a 60° angle using steps from the previous lesson. Label it $\angle CAB$.
2. Centre your pair of compasses at the points where the semi-circle intersects CA and AB. Draw arcs from each point, using a convenient radius.
3. Label the point where the arcs intersect as D.
4. Join A to D to get the angle bisector.

In this construction, $\angle CAD$ and $\angle DAB$ are both 30°.

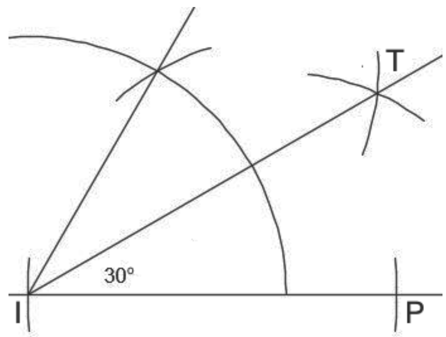


Bisection of any other angle follows the same steps as constructing 30° listed above.

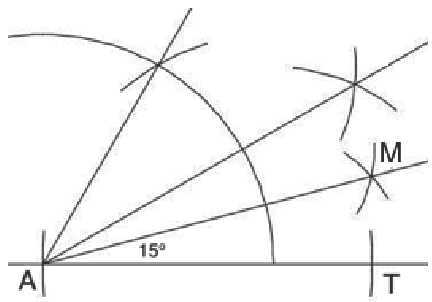
Solved Examples

1. Construct an angle of 30°. Label it $\angle TIP$.

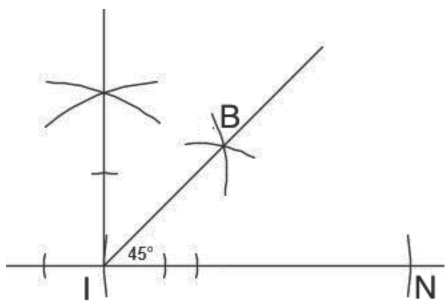
Solution:



2. Construct an angle of 15°. Label it $\angle MAT$.
   Solution:



3. Construct an angle of 45°. Label it $\angle BIN$.
   Solution:



4. If you bisected a 45-degree angle, what would be the measure of the result?
   Solution:
   Bisecting an angle gives a result that is half of the original angle. Therefore, you would have $\frac{45°}{2} = 22.5°$

Practice

1. Construct 45-degree angle. Label it $\angle PAT$.
2. Construct an angle of 30°. Label it $\angle BAT$.
3. On the same construction as problem 2, construction a 15-degree angle. Label it $\angle RAT$.
4. If you bisected a 15-degree angle, what would be the measure of the result?

Lesson Title: Construct 75°, 105° and 150° angles	Theme: Geometry
Practice Activity: PHM2-L083	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to use a pair of compasses to construct 75°, 105°, and 150° angles.

Overview

This lesson is on constructing angles using bisection. The 75°, 105°, and 150° angles can be constructed using bisection of other angles you already know.

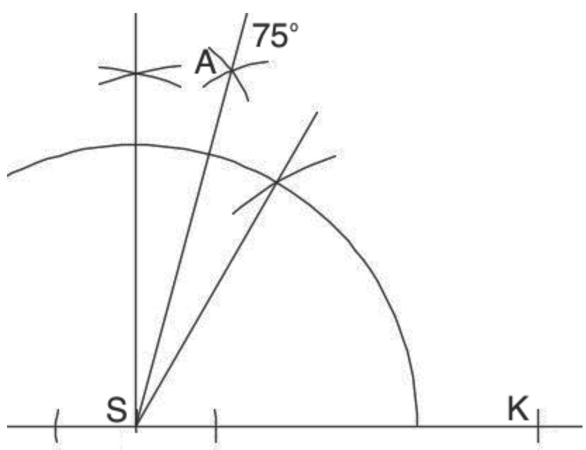
Note that 75° is halfway between 60° and 90°. It is 60° plus 15°. To construct a 75° angle, draw 60° and 90° on the same construction, then bisect the angle between them.

Similarly, 105° is halfway between 90° and 120°. It is 90° plus 15°. To construct a 105° angle, draw 90° and 120° on the same construction, then bisect the angle between them.

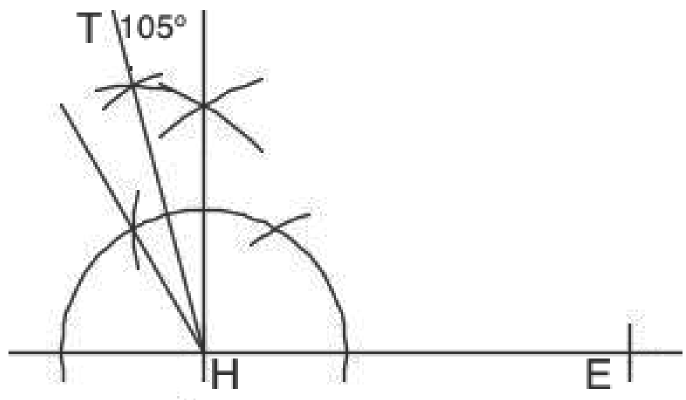
Finally, 150° is halfway between 120° and 180°. It is 120° plus 30°. Recall that 180° is a straight line. To construct a 150° angle, draw 120° and extend the straight line on the same construction. Then, bisect the angle between them.

Solved Examples

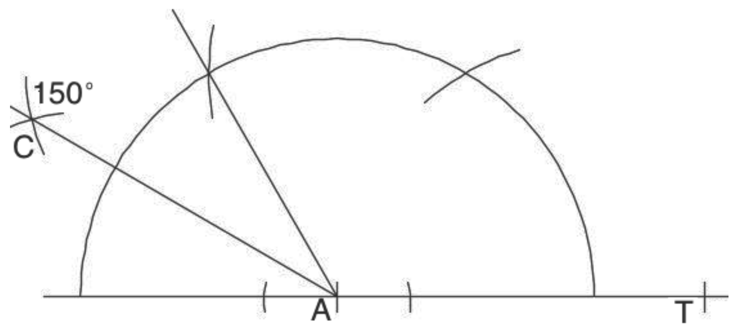
1. Construct an angle of 75°. Label it $\angle ASK$.
   Solution:



2. Construct an angle of 105°. Label it $\angle THE$.
   Solution:



3. Construct an angle of 150°. Label it $\angle CAT$.
   Solution:



4. How could you construct an angle of 165°? Explain in words.
   Solution:
   175° is halfway between 150° and 180°. It could be constructed by constructing an angle of 150°, then bisecting the angle between 150° and 180°.

Practice

1. Construct an angle of 75°. Label it $\angle MAP$.
2. Construct an angle of 105°. Label it $\angle FUN$.
3. Construct an angle of 150°. Label it $\angle RUN$.
4. How could you construct an angle of 135°? Explain in words.

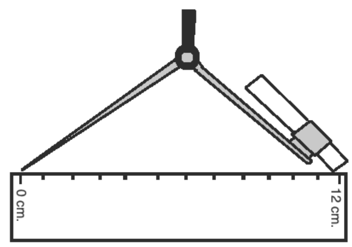
Lesson Title: Construction of triangles – Part 1	**Theme:** Geometry
Practice Activity: PHM2-L084	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to construct triangles using the given lengths of three sides (SSS).

Overview

This is the first of 3 lessons on constructing triangles. This lesson uses the lengths of the triangle's 3 sides.

To construct a triangle given 3 sides, you will need to set the radius of your compass equal to the lengths of the sides of the triangle. Place the tip at zero, and open the pencil's point to the distance you want.



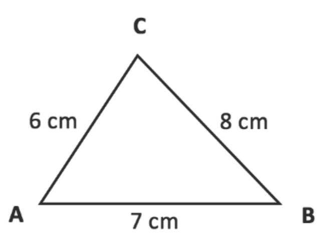
See the diagram at right on how to do this:

If you have a ruler, use it in class and at home to do construction. If you do not have a real ruler, use the printed one below. This ruler is not exactly to scale (1 cm is not exactly 1 cm). However, it can be used for the purpose of learning construction.

Consider an example problem: Construct a triangle ABC with sides 6 cm, 7 cm, and 8 cm.

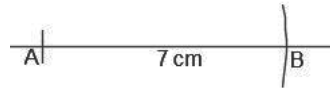
Before starting your construction, it would be helpful to quickly sketch a picture of what the constructed triangle will look like. Don't worry about sketching it to scale, this sketch is just to guide your construction.

Sketch:

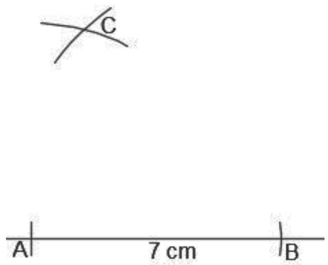


To construct ABC, follow these steps:

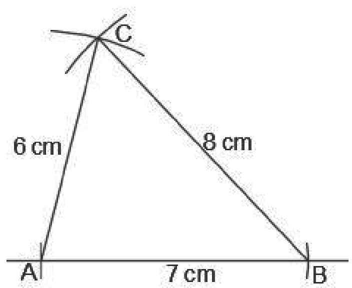
- Draw a line and label point A on one end.
- Open your compass to the length of 7 cm. Use it to mark point B 7 cm from point A. This gives line segment $\overline{AB} = 7 \text{ cm}$.



- Open your compass to the length of 6 cm. Use A as centre, and draw an arc of 6 cm above $\overline{AB}$.
- Open your compass to the length of 8cm. With the point B as centre, draw an arc that intersects with the arc you drew from point A. Label the point of intersection C.

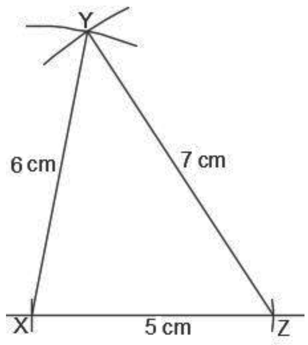


- Join $\overline{AC}$ and $\overline{BC}$. This is the required triangle ABC.
- Label the sides with the correct lengths:

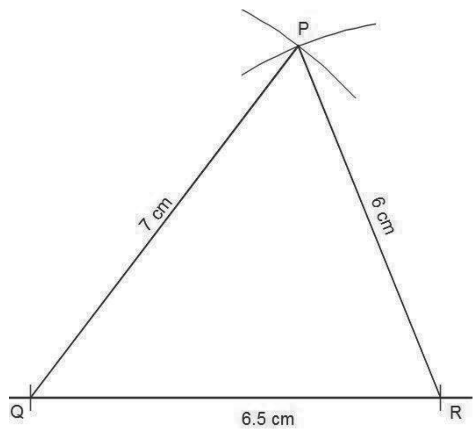


Solved Examples

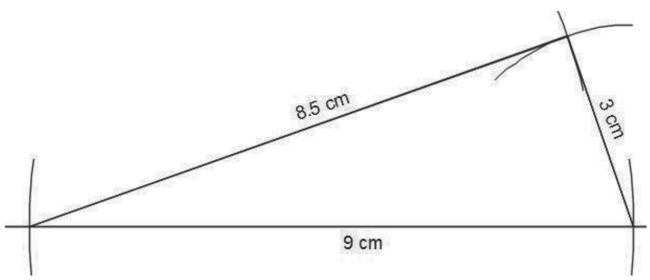
1. Construct triangle XYZ with sides of length 5 cm, 6 cm, and 7 cm.
   Solution:
   (Note that your triangles could be turned in any direction. For example, line XY or YZ could be the base.)



2. Using a ruler and a pair of compasses only, construct a triangle PQR with $|PR| = 6$ cm $|PQ| = 7$ cm and $|QR| = 6.5$ cm.
   Solution:



3. Construct a triangle with sides of length 3 cm, 9 cm and 8.5 cm.
   Solution:



Practice

1. Construct a triangle ABC such that $|AB| = 5$ cm, $|BC| = 6$ cm and $|AC| = 4$ cm.
2. Construct triangle JKL where $|JK|$ is 9 cm, $|KL|$ is 7 cm and $|LJ|$ is 6 cm.
3. Using a ruler and a pair of compasses, construct a triangle ABC such that $|AB| = 6.0$ cm, $|BC| = 8.0$ cm and $|AC| = 7.0$ cm.

Lesson Title: Construction of triangles – Part 2	**Theme:** Geometry
Practice Activity: PHM2-L085	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to construct triangles using two given sides and an angle (SAS).

Overview

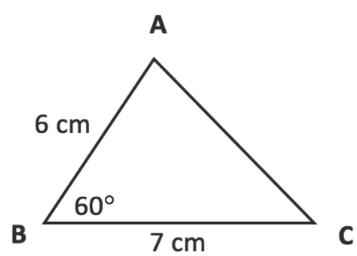
This is the second of 3 lessons on constructing triangles. This lesson uses the lengths of two sides, and the angle between them. These are SAS (side-angle-side) triangles.

To construct an SAS triangle, draw one side with the first known length. Then, construct the angle from this. Extend the constructed line so that it is the second known length. Connect the 2 known sides to make the third side.

It is always useful to draw a sketch of the triangle first.

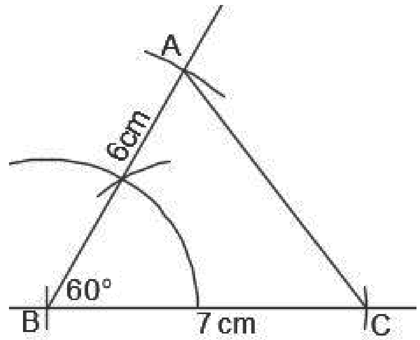
Consider an example problem: Construct triangle ABC where $|AB|$ is 6 cm, $|BC|$ is 7 cm, and angle B is 60°.

Sketch:



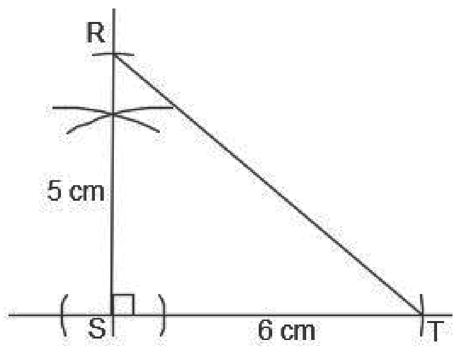
To construct ABC, follow these steps:

- Draw the side $|BC| = 7 \text{ cm}$ and label it 7 cm.
- From $\overline{BC}$, construct an angle of 60° at B, and label it 60°.
- Open your compass to the length of 6 cm. Use B as the centre, and draw an arc of 6 cm on the 60° line. Label this point A.
- Join $\overline{AB}$ and $\overline{BC}$. This is the required triangle ABC.

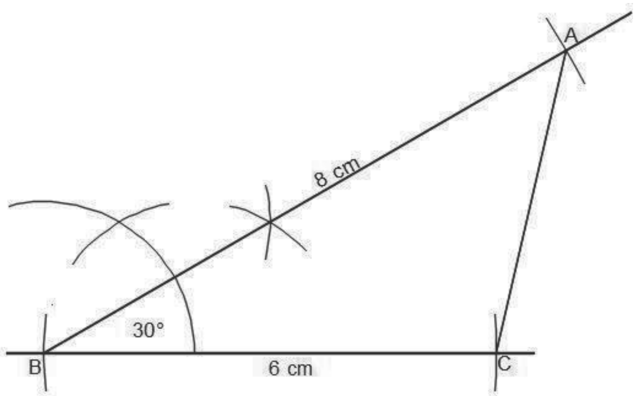


Solved Examples

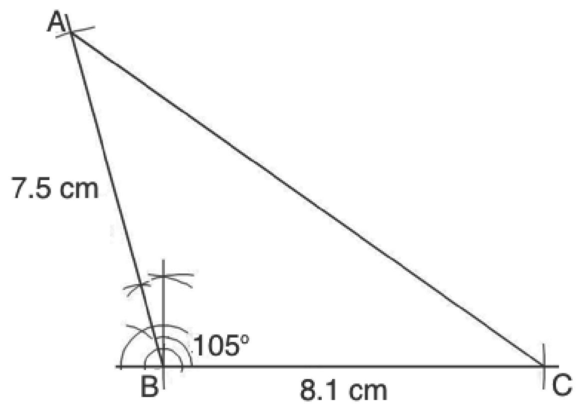
1. Construct triangle RST where $|RS|$ is 5 cm, $|ST|$ is 6 cm, and $\angle S$ is 90°.
   Solution:



2. Using a pair of compasses and ruler only, construct ΔABC, where $|AB| = 8$ cm, $|BC| = 6$ cm and $\angle ABC = 30°$.
   Solution:



3. Using a ruler and a pair of compasses only, construct triangle ABC with $|AB| = 7.5$ cm, $|BC| = 8.1$ cm and $\angle ABC = 105°$.
   Solution:



Practice

Using a ruler and a pair of compasses only, construct the following triangles:

1. ΔABC in which $|AC| = 4$ cm, $|BC| = 2$ cm and $\angle C = 75°$.
2. ΔABC in which $|AB| = 7$ cm, $|BC| = 8$ cm and $\angle B = 120°$.

Lesson Title: Construction of triangles – Part 3	**Theme:** Geometry
Practice Activity: PHM2-L086	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to construct triangles using two given angles and a side (ASA).

Overview

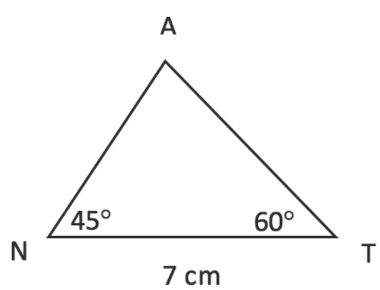
This is the third of 3 lessons on constructing triangles. This lesson uses the measures of 2 angles, and the length of the side between them. These are ASA (angle-side-angle) triangles.

To construct an ASA triangle, draw one side with the known length. Then, construct the 2 given angles on the 2 sides of this. Extend the constructed lines so that they intersect.

It is always useful to draw a sketch of the triangle first.

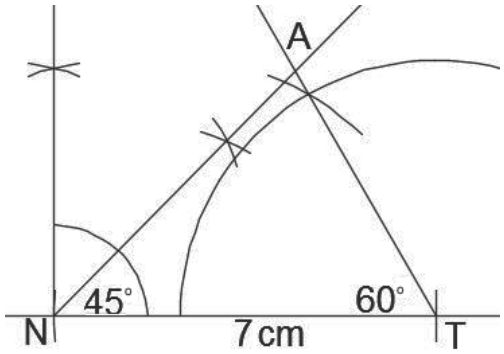
Consider an example problem: Construct triangle ANT where $\angle N = 45°$, $\angle T = 60°$ and $\overline{NT} = 7$ cm.

Sketch:



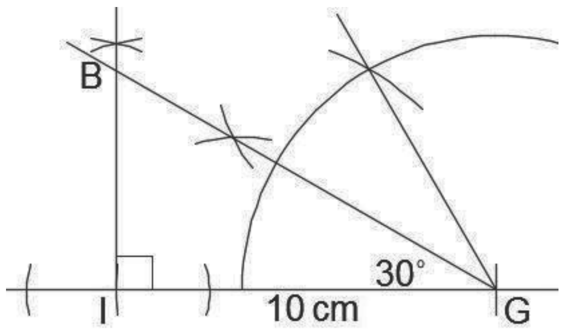
To construct ANT, follow these steps:

- Draw the side $|NT| = 7\text{ cm}$ and label it 7 cm.
- From $\overline{NT}$, construct an angle of 45° at N, and label it 45°.
- From $\overline{NT}$, construct an angle of 60° at T, and label it 60°.
- Extend the 2 angle constructions until they meet. Label this point A. This is the required triangle ANT.

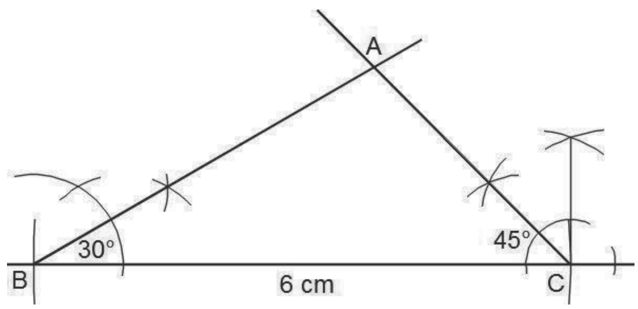


Solved Examples

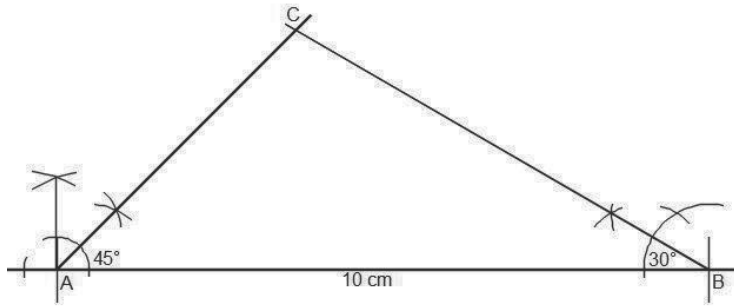
1. Construct triangle BIG where $\angle I = 90°$, $\angle G = 30°$ and $\overline{IG} = 10$ cm.
   Solution:



2. Using a ruler and a pair of compasses only, construct ΔABC with $|BC| = 6$ cm, $\angle ABC = 30°$ and $\angle ACB = 45°$.
   Solution:

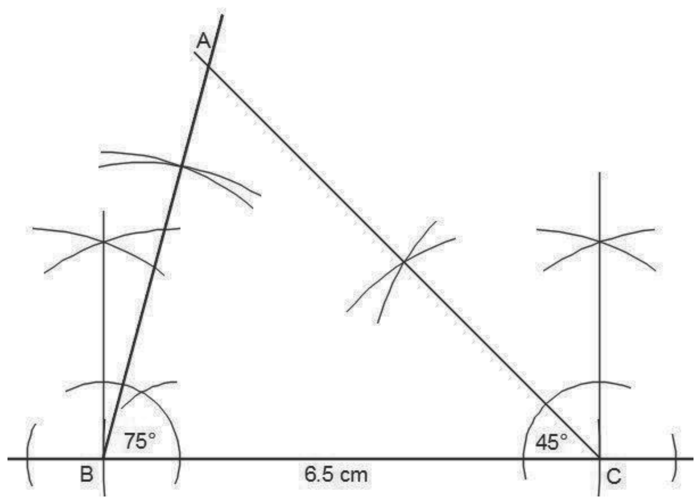


3. Using a ruler and a pair of compasses only, construct ΔABC with $|AB| = 10$ cm, $\angle ABC = 30°$ and $\angle BAC = 45°$.
   Solution:



4. Using a ruler and a pair of compasses only, construct ΔABC with $|BC| = 6.5$ cm, $\angle ABC = 75°$ and $\angle ACB = 45°$.

   Solution:



Practice

1. Using a ruler and a pair of compasses only, construct ΔCRS with $|RS| = 7$ cm, $\angle R = 45°$ and $\angle S = 90°$.
2. Using a ruler and a pair of compasses only, construct ΔXYZ with $|XY| = 8$ cm, $\angle X = 60°$ and $\angle Y = 30°$.
3. Construct ΔPQR with $|PQ| = 8$ cm, $\angle Q = 90°$ and $\angle P = 30°$.
4. Using a ruler and a pair of compasses only, construct ΔKLM with $|LM| = 7$ cm, $\angle KLM = 105°$ and $\angle KML = 30°$.

Lesson Title: Construction of quadrilaterals - Part 1	**Theme:** Geometry
Practice Activity: PHM2-L087	**Class:** SSS 2

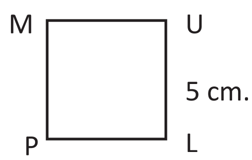
Learning Outcome
By the end of the lesson, you will be able to construct squares and rectangles using given sides.

Overview

Squares and rectangles can be constructed using their characteristics. The angles are all right angles, but we only need to construct one of them. After constructing 1 right angle, we can complete the square or rectangle using a pair of compasses to mark the lengths of the sides.

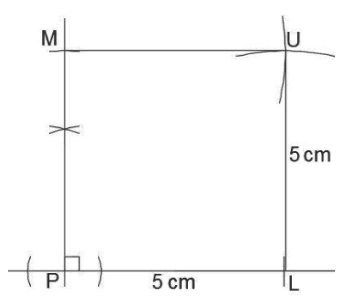
Consider an example problem: Construct a square $PLUM$ with sides of length 5 cm.

Sketch:



To construct $PLUM$, follow these steps:

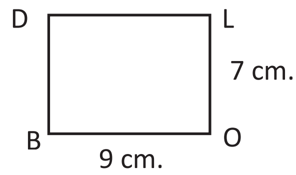
- Draw the side $|PL| = 5 \text{ cm}$ and label it 5 cm.
- From $\overline{PL}$, construct an angle of 90° at P.
- Open your pair of compasses to 5 cm. With P as the centre, draw an arc on the 90° line. Label the intersection M.
- With L as the centre, draw an arc above the line PL. With M as the centre, draw an arc to the right, above L. Label the intersection of these 2 arcs U.
- Draw lines to join M with U, and U with L.



Rectangle construction follows a similar process, but the pair of compasses should be opened to the appropriate distance to mark the given length and width.

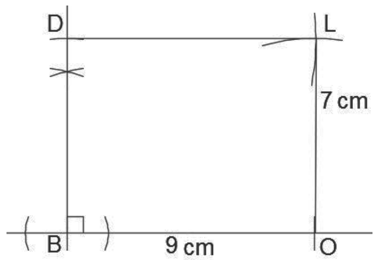
Consider an example: Construct rectangle $BOLD$ where $l = 9$ cm and $w = 7$ cm

Draw a rough sketch of the rectangle to be constructed:



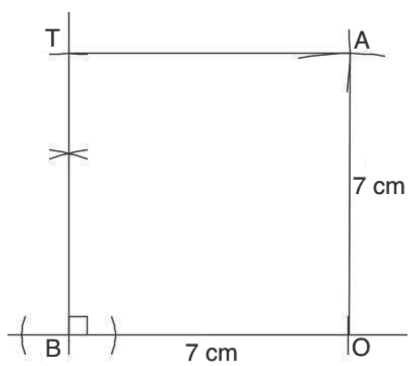
To construct $BOLD$, follow these steps:

a. Draw the side $\overline{BO} = 9 \text{ cm}$ and label it 9 cm.
b. From $\overline{BO}$, construct an angle of 90° at B.
c. Open your pair of compasses to 7 cm. With B as the centre, draw an arc on the 90° line. Label the intersection D.
d. Keep the radius of your pair of compasses at 7 cm. With O as the centre, draw an arc above the line BO.
e. Change the radius of your pair of compasses to 9 cm. With D as the centre, draw an arc to the right, above O. Label the intersection of these 2 arcs L.
f. Draw lines to join D with L, and O with L.



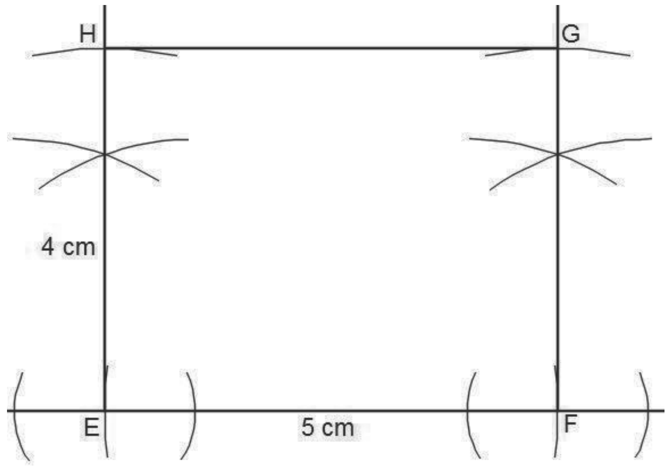
Solved Examples

1. Construct square $BOAT$ with sides of length 7 cm.
   Solution:



2. Construct a rectangle $EFGH$ given the sides $EF = 5$ cm and $EH = 4$ cm.

Solution:



Practice

1. Construct the following squares:
    a. $EACH$ where $|EA| = 5$ cm and $|EH| = 5$ cm
    b. $RATE$ with sides of length 6 cm
2. Construct the following rectangles:
    a. $PINT$ where $|PI| = 6$ cm and $|PT| = 4$ cm
    b. $COKE$ where $|CO|$ = 7 cm and $|CE| = 2.5$ cm

Lesson Title: Construction of quadrilaterals - Part 2	**Theme:** Geometry
Practice Activity: PHM2-L088	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to construct rhombi and parallelograms using two sides and an angle.

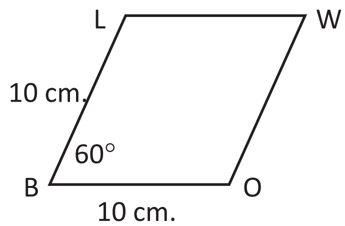
Overview

We construct a parallelogram or a rhombus using their characteristics. The constructions are carried out in the same way for parallelogram and rhombus.

Remember that rhombus is a type of parallelogram. In parallelogram construction problems, we are given the measure of an angle, and the lengths of 2 sides. We construct the given angle and extend the sides to the correct lengths.

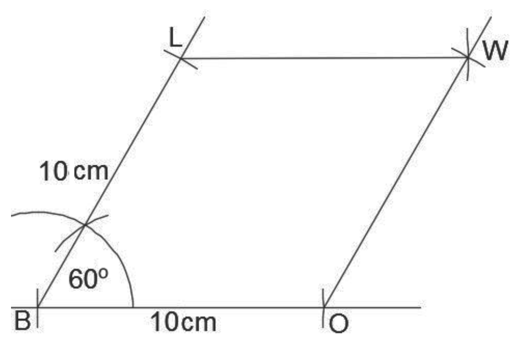
Consider an example problem: Construct a rhombus $BOWL$ with sides of length 10 cm, and angle $B = 60°$.

Sketch:



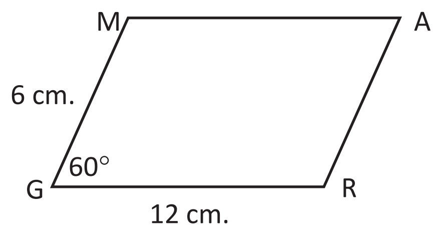
To construct $BOWL$, follow these steps:

- Draw the side $|BO| = 10\text{ cm}$ and label it 10 cm.
- From $\overline{BO}$, construct an angle of 60° at B.
- Open your pair of compasses to 10 cm. With B as the centre, draw an arc on the 60° line. Label the intersection L.
- With O as the centre, draw an arc above the line BO. With L as the centre, draw an arc to the right, above O. Label the intersection of these 2 arcs W.
- Draw lines to connect L with W, and W with O.



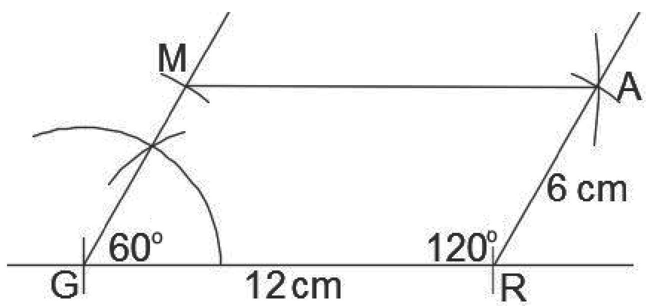
Parallelogram construction follows a similar process, but the pair of compasses should be opened to the appropriate distance to mark the sides.
Consider the problem: Construct parallelogram $GRAM$ where $|GR| = 12\text{ cm}$, $|GM| = 6\text{ cm}$, and angle $G = 60°$.

Draw a rough sketch of the parallelogram to be constructed:



To construct $GRAM$, follow these steps:

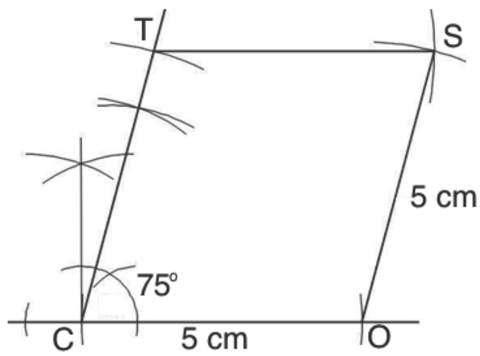
a. Draw the side $|GR| = 12\text{ cm}$ and label it 12 cm.
b. From $\overline{GR}$, construct an angle of 60° at G.
c. Open your pair of compasses to 6 cm. With G as the centre, draw an arc on the 60° line. Label the intersection M.
d. Keep the radius of your pair of compasses at 6 cm. With R as the centre, draw an arc above the line GR.
e. Change the radius of your pair of compasses to 12 cm. With M as the centre, draw an arc to the right, above R. Label the intersection of these 2 arcs A.
f. Draw lines to join M with A, and A with R.



Solved Examples

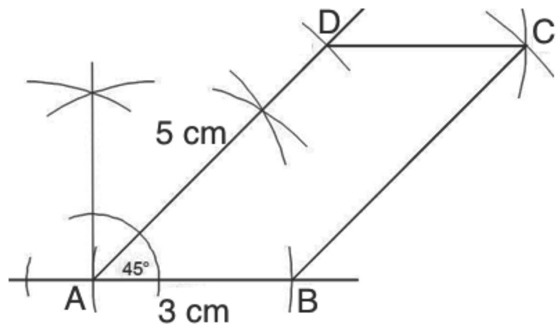
1. Construct a rhombus $COST$ with sides of length 5 cm. and angle $C = 75°$

Solution:



2. Construct a parallelogram, $ABCD$ with $|AB| = 3$ cm, $|AD| = 5$ cm and angle $DAB = 45°$

   Solution:



Practice

1. Construct a rhombus $GIRL$ with sides of length 7 cm. and angle $I = 60°$
2. Construct a rhombus $CITE$ with sides of length 5.5 cm. and angle $C = 45°$
3. Construct a parallelogram $PQRS$ such that $|PQ| = 7$ cm, $|QR| = 4.5$ cm. and $\angle SPQ = 135°$.
4. Construct a parallelogram $ABCD$ such that $|AB| = 6$ cm, $\angle B = 105°$ and $|AD| = 4$ cm.

Lesson Title: Construction of quadrilaterals - Part 3	**Theme:** Geometry
Practice Activity: PHM2-L089	**Class:** SSS 2

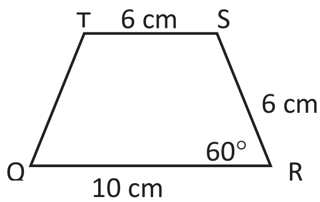
Learning Outcomes
By the end of the lesson, you will be able to:
1. Construct trapeziums using the lengths of 3 sides and an angle.
2. Construct other quadrilaterals given side and angle measures.

Overview

We construct a trapezium using their characteristics. In trapezium construction problems, we are given the lengths of 3 sides and the measure of at least 1 angle. We are told which 2 sides are parallel.

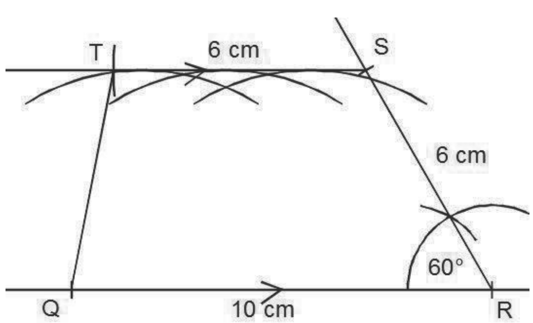
Consider an example problem: Construct a trapezium $QRST$ such that $|QR| = 10\text{ cm}$, $|RS| = 6\text{ cm}$, $|ST| = 6\text{ cm}$, and $\angle QRS = 60°$ and line $\overline{QR}$ is parallel to line $\overline{ST}$.

Sketch:



To construct $QRST$, follow these steps:

- Draw the side $|QR| = 10\text{ cm}$ and label it 10 cm.
- From $\overline{QR}$, construct an angle of 60° at R.
- Open your pair of compasses to 6 cm. With R as the centre, draw an arc on the 60° line. Label the intersection S.
- Construct a line parallel to QR:
  - Centre your pair of compasses at S, and open them to the distance between point S and the line QR.
  - Choose any 3 points on line QR. Keep your compass open to the distance between S and QR, and draw 3 arcs above QR.
  - Place your ruler on the highest points of these 3 arcs, and connect them to draw a line parallel to QR.
- Open your compass to 6 cm. With S as the centre, draw an arc through the parallel line you constructed. Label the intersection T.
- Draw a line to join T with Q.

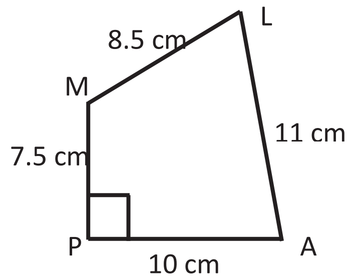


There are other types of quadrilaterals that do not fit into a category such as parallelogram or trapezium. These quadrilaterals can also be constructed if enough information is given.

Consider the example: Construct quadrilateral $PALM$ where $|PA| = 10$ cm, $|AL| = 11$ cm, $|LM| = 8.5$ cm, and $|MP| = 7.5$ cm. Angle P is a right angle.

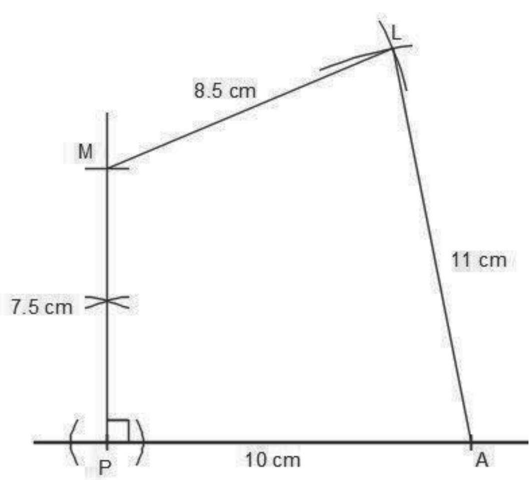
There is enough information to construct the quadrilateral.

Sketch:



To construct $PALM$, follow these steps:

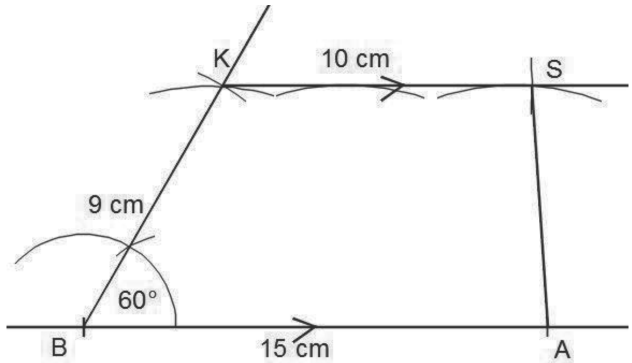
- Draw the side $|PA| = 10$ cm and label it 10 cm.
- From $\overline{PA}$, construct an angle of 90° at P.
- Open your pair of compasses to 7.5 cm. With P as the centre, draw an arc on the 90° line. Label the intersection M.
- Open your compass to 8.5 cm. With M as the centre, draw an arc to the right. Open your compass to 11 cm. With A as the centre, draw an arc above PA. Label the intersection of the two arcs L.
- Draw a line to connect M with L, and a line to connect A with L.



Solved Examples

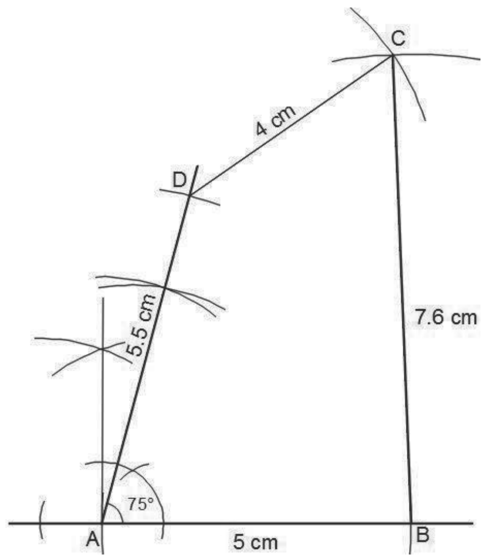
1. Construct trapezium $BASK$ where $|BA| = 15$ cm, $|BK| = 9$ cm, $|SK| = 10$ cm, and $\angle B = 60°$ and line $\overline{KS}$ is parallel to line $\overline{BA}$.

Solution:



2. Using a ruler and a pair of compasses only, construct a quadrilateral $ABCD$ such that $|AB| = 5 \text{ cm}$, $|BC| = 7.6 \text{ cm}$, $|CD| = 4 \text{ cm}$ and $|DA| = 5.5 \text{ cm}$ and $\angle A = 75°$.

Solution:



Practice

1. Using a ruler and a pair of compasses only, construct a quadrilateral $ZINC$ such that $|ZI| = 6 \text{ cm}$, $|IN| = 5 \text{ cm}$, $|NC| = 4 \text{ cm}$, and $\angle I = 45°$.
2. Using a ruler and a pair of compasses only, construct a trapezium $PQRS$ such that $|PQ| = 8 \text{ cm}$, $|PS| = 6 \text{ cm}$, $|RS| = 6 \text{ cm}$, and $\angle P = 75°$, and $\overline{SR}$ is parallel to $\overline{PQ}$.
3. Using a ruler and a pair of compasses only, construct a trapezium $BUST$ such that $|BU| = 12 \text{ cm}$, $|BT| = 6 \text{ cm}$, $|ST| = 5 \text{ cm}$, and $\angle B = 30°$, and $\overline{TS}$ is parallel to $\overline{BU}$.
4. Using a ruler and a pair of compasses only, construct a quadrilateral $WXYZ$ such that $|WX| = 5.5 \text{ cm}$, $|XY| = 5.8 \text{ cm}$, $|YZ| = 4.2 \text{ cm}$, $|ZW| = 7.8 \text{ cm}$ and angle $\angle W = 45°$.
5. Using ruler and a pair of compasses only, construct a quadrilateral $RICE$ where $|RI| = 6 \text{ cm}$, $|IC| = |RE| = 5 \text{ cm}$, $|CE| = 4 \text{ cm}$, $\angle R = 60°$.

Lesson Title: Construction word problems – Part 1	**Theme:** Geometry
Practice Activity: PHM2-L090	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to construct angles and triangles based on information in word problems.

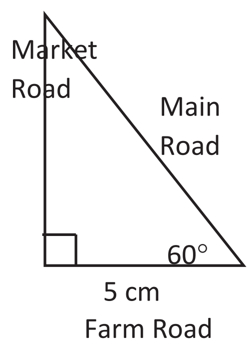
Overview

Construction can be used in many real-life situations. This lesson uses information from previous lessons to construct angles and triangles from word problems.

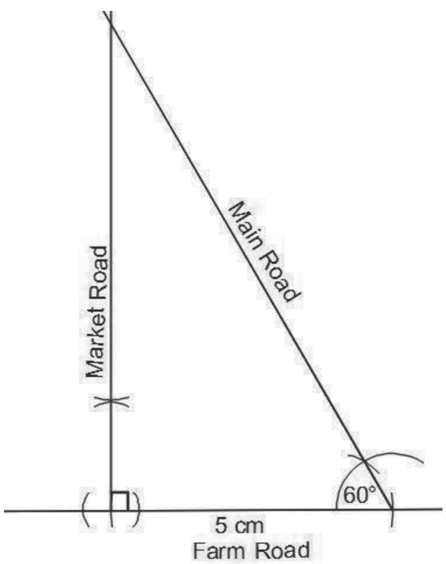
Solved Examples

1. Three roads intersect each other to form a triangle. Market Road and Farm Road intersect at a 90° angle. Five kilometres from that intersection, Farm Road intersects with Main Road at a 60° angle. Construct the triangle formed by the 3 roads. Use 1 cm for each kilometre.

   Solution:
   Two angles (90°, 60°) and a side (5 km) are known. We will construct an ASA triangle. It is helpful to first draw a sketch:

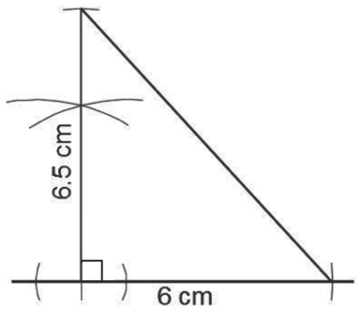
   

   Then, use a pair of compasses to construct the triangle:

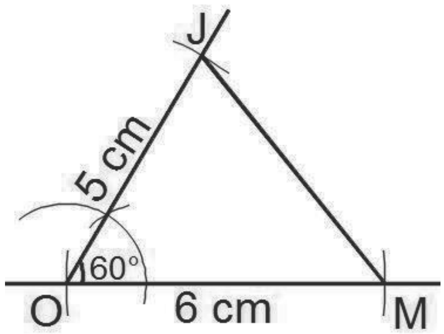
   

2. Jim, a contractor, wants to construct a floor in a triangular shape. It will have a base of length 6 metres, and a height of 6.5 metres. The base and height will meet at a 90-degree angle. Construct the shape of the floor, using 1 cm for each metre.

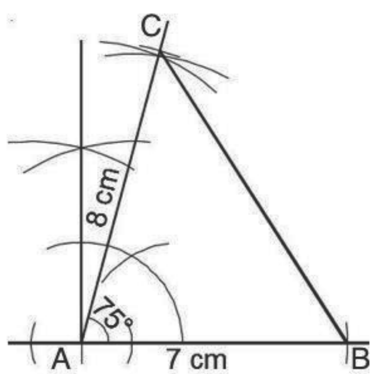
Solution:



3. Mary and Jane live in different houses in a city. On a map, their roads form a triangle, intersecting at a $60°$ angle. On their way to school, they meet at that intersection. If the distance covered by Mary is 60 metres and Jane is 50 metres, construct the triangle. Use 1 centimetre for 10 metres.
Solution:



4. Two vehicles leave town A at the same time. One vehicle travels 70 km from town A to town B. The other vehicle travels 80 km to town C. Their paths form a $75°$ angle. Construct a triangle showing towns A, B, and C. Use a scale of 1 cm to 10 km.
Solution:



Practice

1. Two helicopters leave from airport K. The first helicopter travels 400 kilometres to airport M. The second helicopter travels 800 kilometres to airport L. Their paths form an angle of 60°. Construct a triangle to show the relationship between the 3 airports. Use 1 cm for 100 km.
2. A carpenter cuts a triangular piece of wood. It has sides of length 90 cm and 65 cm, which form a 45° angle. Construct the triangle, using a scale of 1 cm for 10 cm.
3. Abu went to the farm for an agricultural practical. He was to construct a seed bed, which took the shape of an equilateral triangle with sides 8 metres in length. Construct the shape of the seed bed using a scale of 1 cm for each metre.

Lesson Title: Construction word problems – Part 2	Theme: Geometry
Practice Activity: PHM2-L091	**Class:** SSS 2

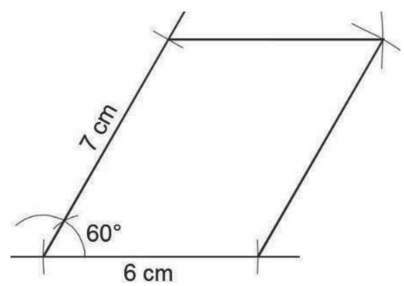
Learning Outcome
By the end of the lesson, you will be able to construct quadrilaterals and compound shapes based on information given in word problems.

Overview

Construction can be used in many real-life situations. This lesson uses information from previous lessons to construct quadrilaterals and compound shapes.

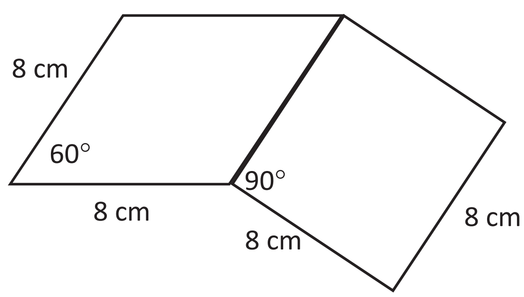
Solved Examples

1. A farmer went to build a pottery house in the shape of a quadrilateral. The building has adjacent sides of 6 and 7 metres, which join at an angle of $60°$. Construct the shape of the building using a scale of 1 cm for every metre.
   Solution:

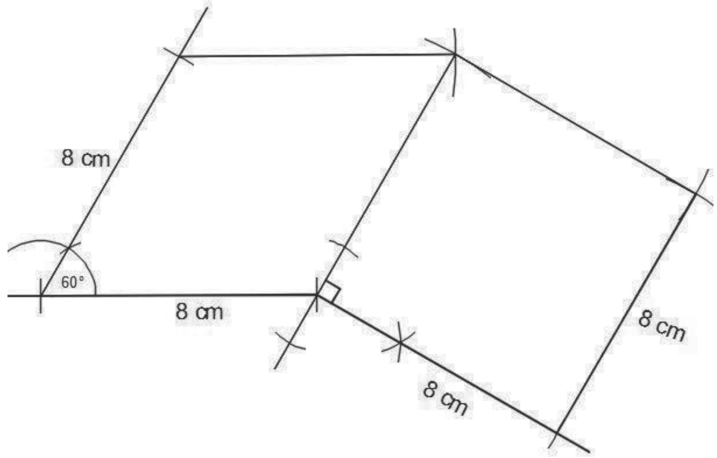


2. Mohamed has a large piece of land. He planted a garden in the shape of a rhombus with sides of length 80 m and one angle of 60°. He wants to plant more crops. He decides to plant an additional plot in the shape of a square. The square shares a side with the rhombus. Construct the shape of Mohamed's garden. Use 1 cm for each 10 m.
   Solution:
   Using 1 cm for each 10 m, each side of the rhombus will be 8 cm long. Since the square shares a side with the rhombus, its sides will also be 8 cm long.

   Draw a sketch of the shape:

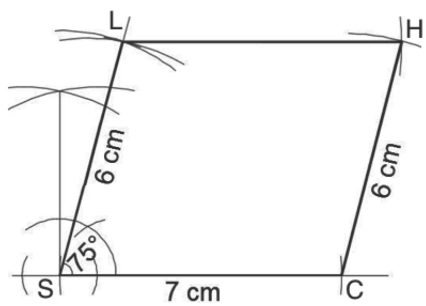


Construct the shape. Start with the rhombus, then construct the square using any side of the rhombus.



Note that the diagram may look different depending on which side the square is drawn on.

3. The principal of Happiness Secondary School wants to draw an accurate map of the school. The school's land is in the shape of a parallelogram. He labels the corners $SCHL$, and measures side SC to be 70 metres and CH to be 60 metres. He also knows that the measure of corner S is $75°$. Construct the boundary of the school's land, using a scale of 1 cm for 10 metres.
Solution:



Practice

1. Alpha, a carpenter, is to make a table top that is rectangular in shape and measures 80 centimetres long and 40 metres wide. Construct the shape of the table top using 1 cm for each 10 cm.
2. Ansu, a contractor, constructed a building. Its front is a rectangle 10 metres long and 5 metres tall. He further constructed a roof in the shape of a triangle that forms a $30°$ angle at the top of each side of the rectangle. Construct the shape of the face of the house. Use 1 cm for each metre.

Lesson Title: Construction of loci – Part 1	Theme: Geometry
Practice Activity: PHM2-L092	**Class:** SSS 2

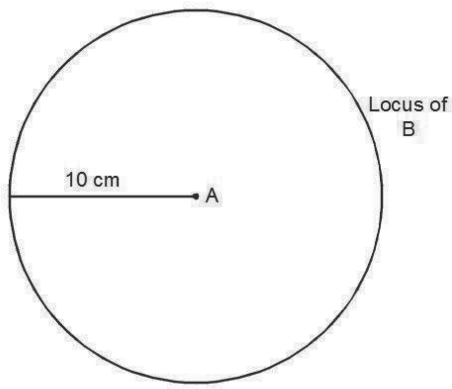
Learning Outcome
By the end of the lesson, you will be able to construct points at a given distance from a given point.

Overview

This is the first lesson on constructing loci. Loci is the plural of locus. A locus is a path. It is a specific path that a point moves through. The point obeys certain rules as it moves through the locus.

For example, consider the locus of point B 10 cm from point A. The locus of point B is all of the possible points where B could exist. The only information that we know about point B is that it is 10 cm away from point A. Therefore, the locus of points where B could exist is a circle with a radius of 10 cm and centre A.

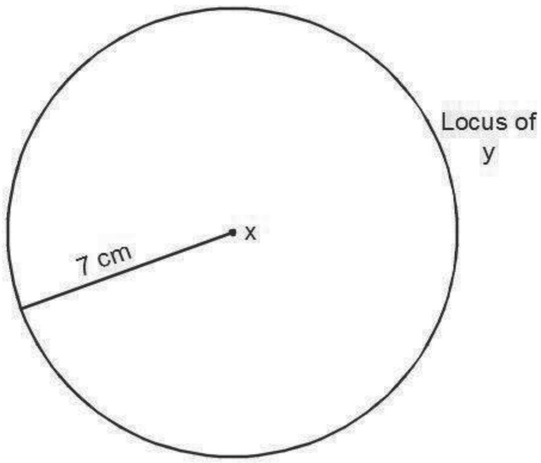
The locus of B is shown below:



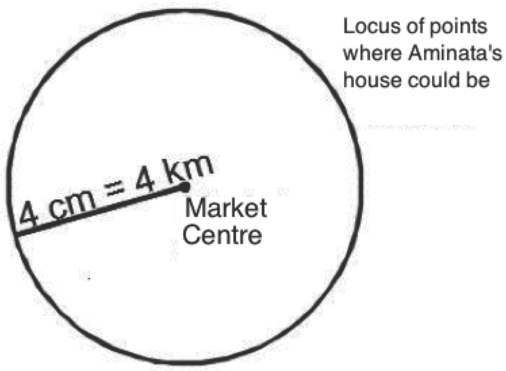
Note that the diagram is not drawn to scale.

Solved Examples

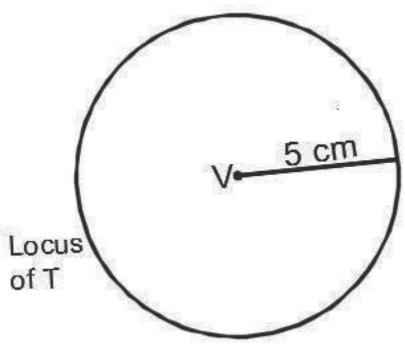
1. The locus of a point (y) moves so that it is 7 cm away from a fixed point (x). Draw the locus of y.
   Solution:
   The solution is a circle with a radius of 7 cm and centre x (not shown to scale).



2. Aminata, a trader, lives exactly 4 km from the market centre. Some business partners are looking for her house. Draw the locus of all possible locations of Aminata's house. Use 1 m for each kilometre.
   Solution:
   The solution is a circle with a radius of 4 cm and the market in the centre (not drawn to scale).



3. T is a point such that $\overline{VT} = 5$ cm. With V as the centre, draw the locus of T.
   Solution:
   The solution is a circle with a radius of 5 cm (not drawn to scale).



Practice

1. A goat is tied to a rope that measures 30 metres in length. It is tied to a pole and feeding on the grass. Construct the locus of points where the goat feeds. Use a scale of 1 cm to 10 metres.
2. Y is a point such that $\overline{XY} = 4.5$ cm. With X as the centre, draw the locus of Y.
3. A is a point 4 cm from x. Draw the locus of A.
4. A dog is tied to a chain that measures 3 metres long, and keeps on moving around wishing for freedom. It walks around in a circle. Draw the locus of the dog. Use a scale of 1 cm for each metre.

Lesson Title: Construction of loci – Part 2	**Theme:** Geometry
Practice Activity: PHM2-L093	**Class:** SSS 2

Learning Outcome By the end of the lesson, you will be able to construct points equidistant from two given points.

Overview

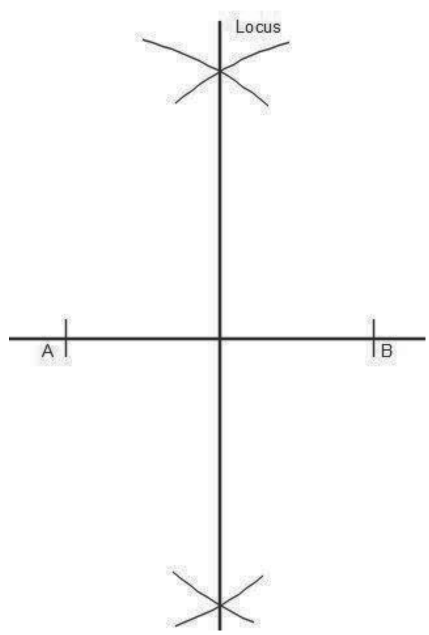
The locus of points that is equidistant from 2 given points is the perpendicular bisector of the line that connects the 2 given points.

For example, consider two points A and B:

A ●   ● B

The locus of points that is equidistant from these 2 points is a vertical line between them. This line is the perpendicular bisector of line AB.

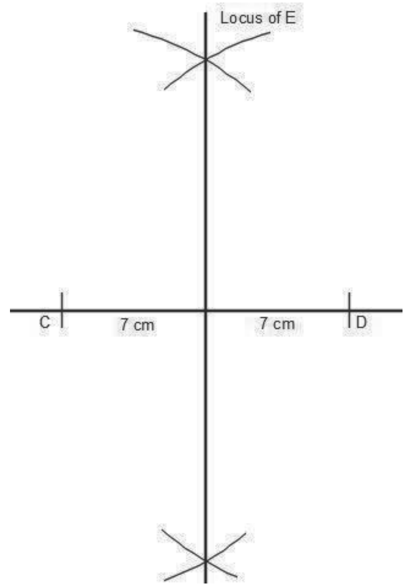
Construct the perpendicular bisector to show the locus of points equidistant from A and B:



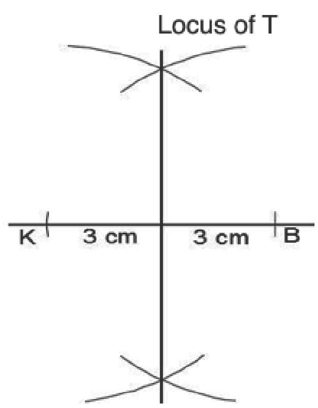
Solved Examples

1. Points C and D are 14 cm. from one another. Point E is equidistant from C and D. Draw the locus of point E.

Solution:
The solution is the perpendicular bisector of line CD, where $\overline{CD} = 14$.

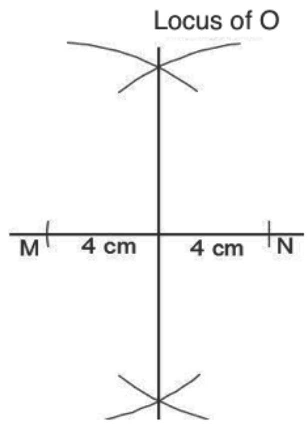


2. The distance between Mr. Kamara's house and Mr. Banguara's house is 60 km. Mr. Turay's house is equidistant from Mr. Kamara and Mr. Bangura's house. Draw the locus of Mr. Turay's house. Use a scale of 1 cm to 10 km.
   Solution:



3. Point N is 8 cm from point M. Point O is equidistant from M and N. Draw the locus of point O.

Solution:



Practice

1. Musa and Idrissa are standing 40 metres from one another. Their classmate, John, is walking in a straight line. He maintains the same distance from Musa and Idrissa. Construct John's path. Use a scale of 1 cm to 10 metres.
2. T is a point that is equidistant from 2 points X and Y. Draw X and Y a distance of 5 cm from each other. Construct the locus of T.
3. Two villages are 70 km a part, and they decided to create a boundary between the two villages. Draw the locus of points where the boundary should be placed if it is equidistant from both villages. Use 1 cm to represent 10 km.

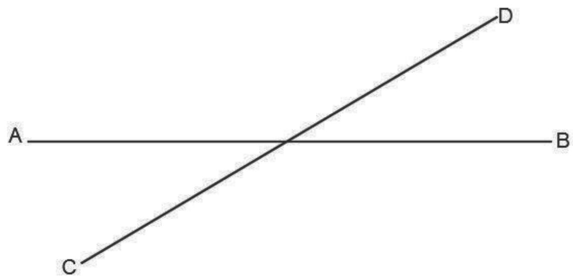
Lesson Title: Construction of loci – Part 3	**Theme:** Geometry
Practice Activity: PHM2-L094	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to construct points equidistant from two straight lines.

Overview

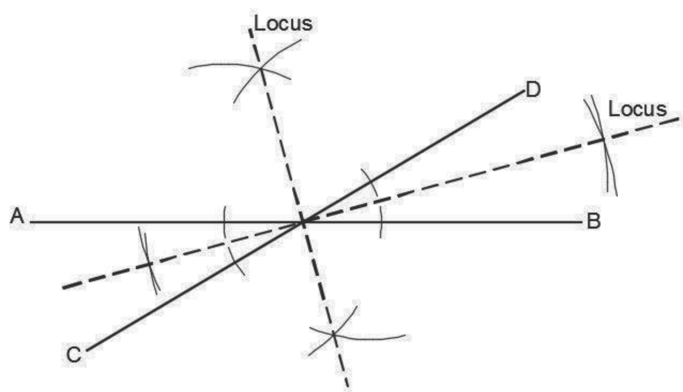
The locus of points that is equidistant from 2 given intersecting lines is found by bisecting the angles formed by the lines.

For example, consider two lines AB and CD:



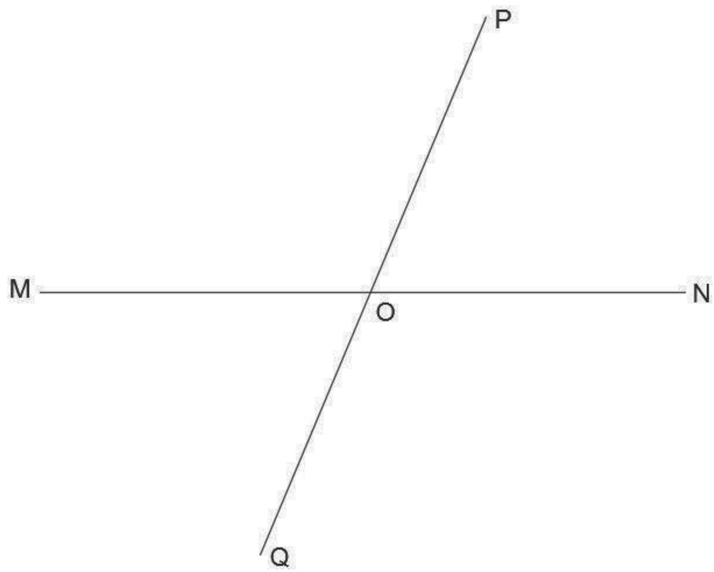
The locus of points that is equidistant from these 2 lines is the set of angle bisectors.

Construct each angle bisector to show the locus of points equidistant from AB and CD:



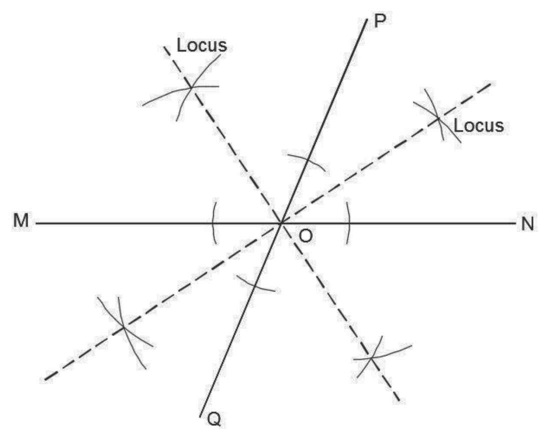
Solved Examples

1. On the lines MN and PQ below, construct the locus of the point equidistant from the 2 lines.



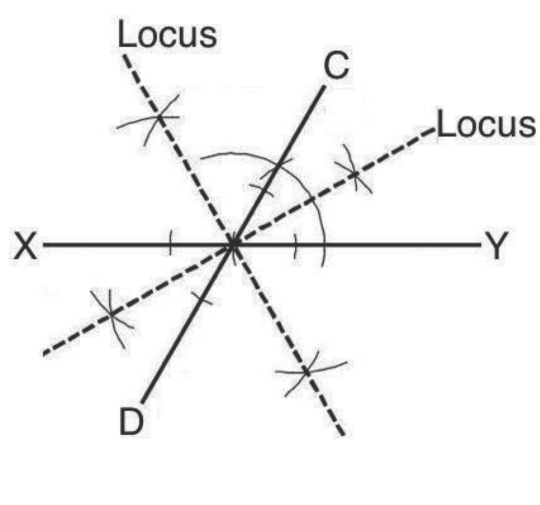
Solution:

Your construction should look like the construction shown below:



2. Lines XY and CD intersect at point O such that $\angle COY = 60°$. Construct the lines, then construct the locus of point P that is equidistant from the 2 lines.

   Solution:

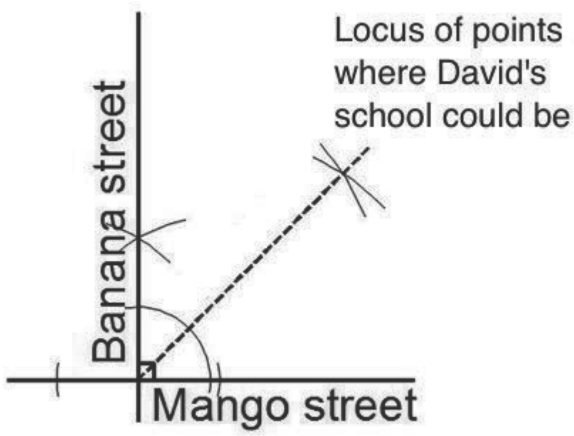


3. David's school is situated at a junction of two streets, Banana Street and Mango Street. The intersection of the streets forms a $90°$ angle, and the school is equidistant from the 2 streets.
   a. Draw the intersection of the two streets.

b. Construct the locus of points where the school could be.

Solutions:

a. and b.:



Practice

1. Lines WX and YZ intersect at point O such that $\angle YOX = 45°$. Construct the lines, then construct the locus of a point P that is equidistant from the 2 lines.
2. Draw any 2 intersecting lines and label them AB and CD. Construct the locus of a point P that is equidistant from the 2 lines.
3. Andrew owns a piece of land with sides that form a $60°$ angle. He wants to erect a fence equidistant from the 2 lines.
    a. Construct 2 sides of the land.
    b. Construct the locus of points where he could erect the fence.

Lesson Title: Construction of loci – Part 4	Theme: Geometry
Practice Activity: PHM2-L095	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to construct points at a given distance from a given straight line.

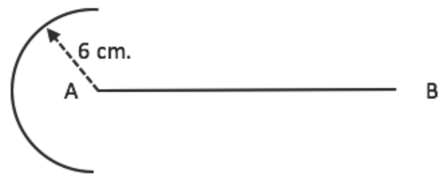
Overview

The locus of points that is a given distance from a given line segment is an oblong shape around the line.

For example, consider the problem: Construct the locus of points 6 cm from line segment AB:

Follow these steps to draw the construction:

a. Open your pair of compasses to a radius of 6 cm.
b. With A as the centre, draw a semi-circle to the left of the line:



c. With B as the centre, draw another semi-circle to the right of the line:

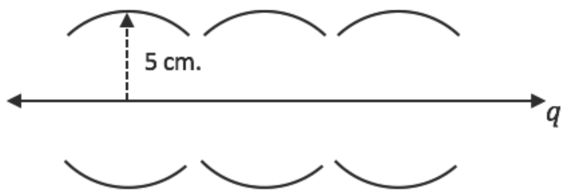
d. Use a straight edge to connect the semi-circles above and below the line segment.

Recall that a line can extend in 2 directions forever. This is shown with arrows at the ends of the line. For such lines, the locus of points a given distance from the line is 2 additional, parallel lines.

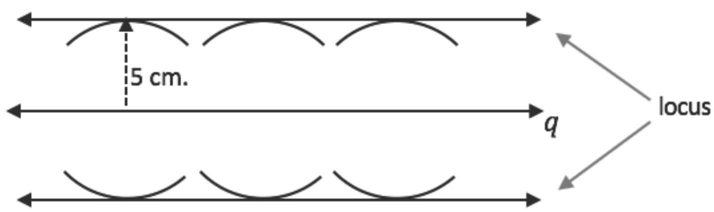
For example, consider the problem: Construct the locus of points 5 cm from line q:

Follow these steps to draw the construction:

a. Open your pair of compasses to a radius of 5 cm.
b. Choose several points on q, and centre your compass at each. From each point, draw an arc directly above and below line q.

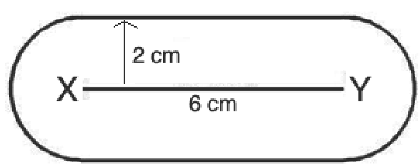


c. Hold a straight edge along the points of the arcs farthest from line q. Connect these points.
d. Draw arrows to show that the locus extends forever in both directions.

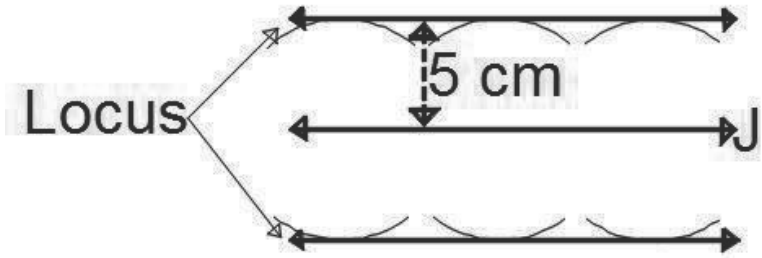


Solved Examples

1. Draw line $XY = 6$ cm. Construct the locus of a point P that is 2 cm from the line.
   Solution:

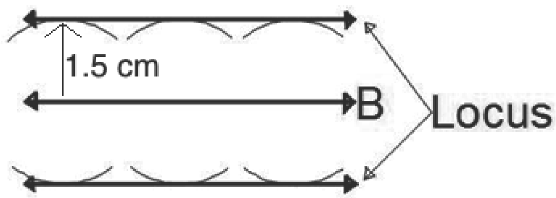


2. Draw a line J that extends forever in both directions. Construct the locus of a point if it is 5 cm from J.
   Solution:



3. Draw a line B that extends in both directions. Construct the locus of points 1.5 cm from the line.

   Solution:



Practice

1. Draw a line segment $PQ = 7$ cm. Construct the locus of points 2.5 cm from the line segment.
2. Draw a line W that extends forever in both directions. Construct the locus of point 2 cm from the line.
3. Draw a line segment $\overline{ST} = 9$ cm, and construct the locus of points 1.5 cm from the line.
4. Draw a line V that extends in both directions. Construct the locus of points 10 mm from the line.

Lesson Title: Construction practice	**Theme:** Geometry
Practice Activity: PHM2-L096	**Class:** SSS 2

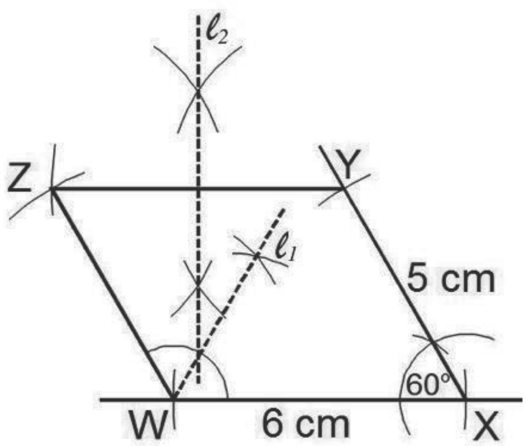
Learning Outcome
By the end of the lesson, you will be able to apply construction techniques to construct various figures.

Overview

This lesson combines the information you learned in the previous 17 lessons on geometry construction. You will be constructing shapes and loci in the same diagram.

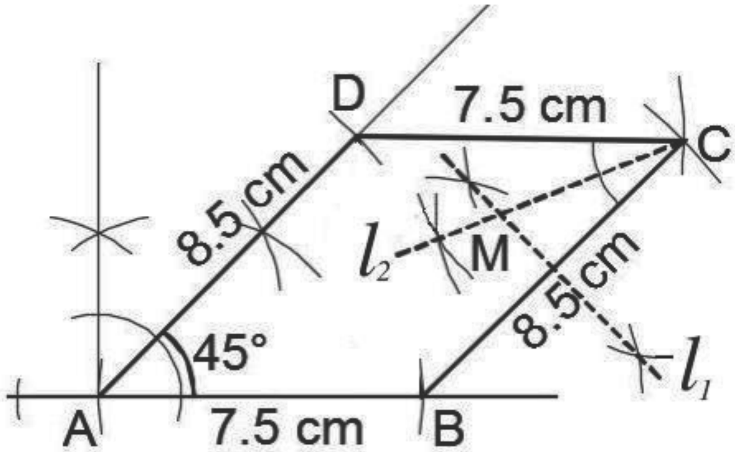
Solved Examples

1. Using a ruler and a pair of compasses only, construct:
   a. Parallelogram $WXYZ$ such that $\angle X = 60°$, $|WX| = 6$ cm and $|XY| = 5$ cm.
   b. The locus l_1 of points equidistant from XW and WZ.
   c. The locus l_2 of points equidistant from Y and Z.
   Solutions:



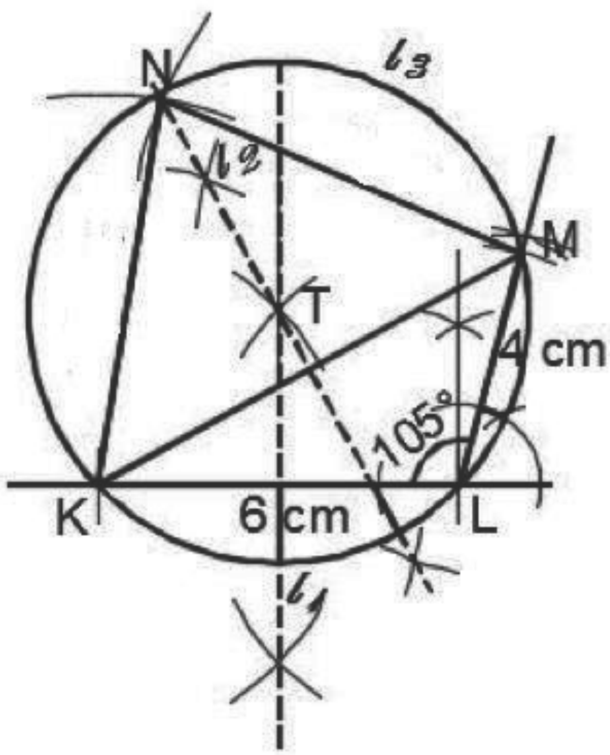
2. Using a ruler and a pair of compasses only:
   a. Construct a parallelogram $ABCD$ such that $|AB| = 7.5$ cm, $|AD| = 8.5$ cm, $\angle DAB = 45°$ and $AD||BC$.
   b. Construct:
      i. Locus l_1 of points equidistant from B and C
      ii. Locus l_2 of points equidistant from CD and CB.
   c. Locate M, the point of intersection of l_1 and l_2.

Solutions:



3. Using a ruler and a pair of compasses only, construct:
   a. $\Delta\, KLM$ such that $|KL| = 6$ cm, $|LM| = 4$ cm and $\angle KLM = 105°$.
   b. Locus l_1 of points equidistant from K and L.
   c. Locus l_2 of points equidistant from K and M.
   d. Label the point T where l_1 and l_2 intersect.
   e. With centre T and radius $|TK|$, construct a circle l_3.
   f. Complete quadrilateral $KLMN$ such that N lies on the circle and $|KN| = |MN|$.

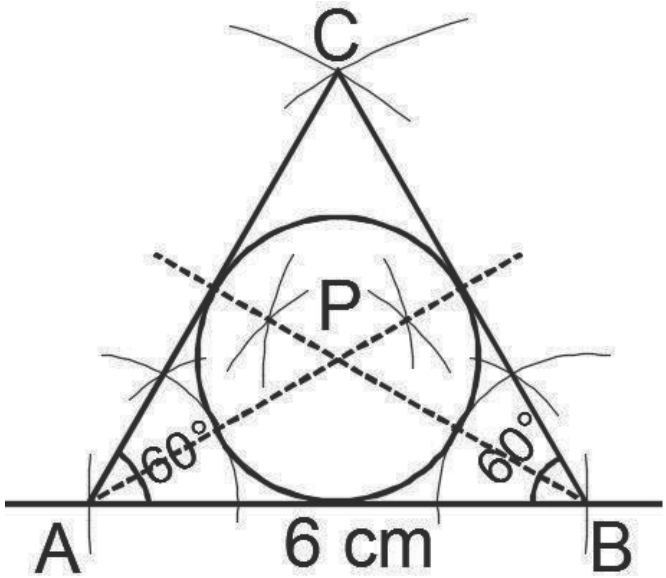
   Solutions:



4. Using a ruler and a pair of compasses only,

a. Construct $\Delta\, ABC$ such that $|AB| = 6$ cm and $\angle A = \angle B = \angle C = 60°$
b. Locate a point P inside the triangle equidistant from AB and AC, and also equidistant from BA and BC.
c. Construct a circle with centre at P.

Solutions:



Practice

1. Using a ruler and a pair of compasses only, construct:
   a. A quadrilateral $WXYZ$ with $|WX| = 5.5$ cm, $\angle YXW = 90°$, $|YX| = 7$ cm, $|YZ| = 8$ cm and $|WZ| = 6$ cm.
   b. The bisectors of $\angle W$ and $\angle X$ to meet at O.
2. Using a ruler and a pair of compasses only, construct:
   a. Quadrilateral $ABCD$ such that $|AB| = 8$ cm, $|AD| = 7$ cm, $|BC| = 5$ cm, $\angle DAB = 60°$ and $\angle ABC = 75°$.
   b. The locus l_1 of points equidistant from AD and CD.
   c. The locus l_2 of points equidistant from C and D
3. Using a ruler and a pair of compasses only:
   a. Construct triangle XYZ such that $|XY| = 7$ cm, $|XZ| = 6$ cm and $\angle YXZ = 75°$.
   b. Construct the locus l_1 of points equidistant from X and Z.
   c. Construct the locus l_2 of points equidistant from X and Y.
   d. Locate the point O equidistant from X, Y and Z.
   e. With O as centre, draw the circle l_3 that circumscribes the triangle XYZ.

Answer Key – Term 2

Lesson Title:	Sequences
Practice Activity:	PHM2-L049

1. a. i. $16, 32, 64$; ii. Each term is 2 raised to the nth power, 2^n;
   b. i. $1{,}875, 9{,}375, 46{,}875$; ii. Each term is 5 times the previous term;
   c. i. $-2, -5, -8$; ii. Subtract 3 to get the next term;
2. a. $9, 11, 13$; b. $2n - 1$
3. 21
4. 146

Lesson Title:	Arithmetic progressions
Practice Activity:	PHM2-L050

1. a. $a = -20,\ d = 3$; next 3 terms: $-8, -5, -2$
   b. $a = 0,\ d = -2$; next 3 terms: $-8, -10, -12$
   c. $a = 4,\ d = 3$; next 3 terms: $16, 19, 22$
   d. $a = 3,\ d = 5$; next 3 terms: $23, 28, 33$
   e. $a = 2\frac{1}{2},\ d = 2\frac{1}{2}$; next 3 terms: $12\frac{1}{2}, 15, 17\frac{1}{2}$
2. a. $3, 6, 9, 12, \ldots$
   b. $2, 0, -2, -4, \ldots$
   c. $10, 5, 0, -5, \ldots$
   d. $-2, 2, 6, 10, \ldots$
   e. $-5, -11, -17, -23, \ldots$

Lesson Title:	Geometric progressions
Practice Activity:	PHM2-L051

1.
   a. $a = 6, r = 2$; next 4 terms: $96, 192, 384, 768$
   b. $a = 1, r = -2$; next 4 terms: $16, -32, 64, -128$
   c. $a = \frac{1}{3}, r = \frac{1}{2}$; next 4 terms: $\frac{1}{48}, \frac{1}{96}, \frac{1}{192}, \frac{1}{384}$
   d. $a = -8, r = -4$; next 4 terms: $-2048, 8192, -32768, 131072$
   e. $a = 128, r = \frac{1}{2}$; next 4 terms: $8, 4, 2, 1$
   f. $a = \frac{1}{3}, r = -3$; next 4 terms: $27, -81, 243, -729$
   g. $a = 0.05, r = 5$; next 4 terms: $31.25, 156.25, 781.25, 3906.25$
2.
   a. $2, -6, 18, -54, \ldots$
   b. $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \frac{1}{81}, \ldots$

c. $\frac{1}{9}, 1, 9, 81, \ldots$
d. $384, 192, 96, 48, \ldots$
e. $3, 12, 48, 192, \ldots$
f. $-2, -8, -32, -128, \ldots$
g. $64, 32,16, 8, \ldots$
h. $1.5, 4.5, 13.5, 40.5$ ...

Lesson Title:	nth term of an arithmetic sequence
Practice Activity:	PHM2-L052

1. a. $U_{24} = 165$
   b. $U_{99} = 690$
   c. $U_n = 7n - 3$
2. a. $U_8 = 22$
   b. $U_{100} = -346$
   c. $U_8 = 54 - 4n = 2(27 - 2n)$
3. $U_{16} = 98$ and $U_{24} = 146$
4. $n = 10$

Lesson Title:	nth term of a geometric sequence
Practice Activity:	PHM2-L053

1. a. $U_3 = 112$; b. $U_{10} = 1{,}835{,}008$
2. a. $U_8 = 512$; b. $U_{14} = 32{,}768$; c. $U_n = 4(2^{n-1})\ or\ 2^{n+1}$
3. $a = 3$
4. $r = 6$, $U_5 = 10{,}368$
5. $r = 15, D = 75$

Lesson Title:	Series
Practice Activity:	PHM2-L054

1.
   a. Sequence: $3, 8, 13, 18, 23, 28, 33, \ldots$
   b. Series: $3 + 8 + 13 + 18 + 23 + 28 + 33 + 38 +$ ...
   c. Sum: 126
2. 124
3. -156
4. 0
5.
   a. Sequence: $-2, -7, -12, -17, -22, -27, -32, -37, -42, \ldots$
   b. Series: $-2 + (-7) + (-12) + (-17) + (-22) + (-27) + (-32) + (-37) + (-42)$

c. Sum: -198
6. -40

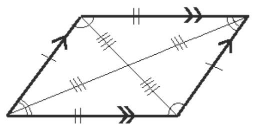
Lesson Title:	The sum of an arithmetic series
Practice Activity:	PHM2-L055

1. $S = 531$
2. $S = 60$
3. $d = 10$
4. $S = -7203$
5. $S = 2420$
6. (a) $S_n = \frac{n}{2}(3n + 13)$
   (b) $S_{12} = 294$
7. $n = 30,\ S_{30} = 1{,}365$
8. $a = 15$

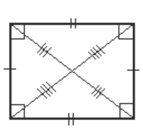
Lesson Title:	Numerical and real-life problems involving sequences and series
Practice Activity:	PHM2-L056

1. ₦ 160.00
2. a. Sequence: $20, 40, 60, \ldots$
   b. $Le120.00$
   c. $Le\ 1{,}560.00$
3. a. $Le\ 2{,}400.00$
   b. $Le\ 11{,}700.00$

Lesson Title:	Characteristics of Quadrilaterals
Practice Activity:	PHM2-L057

1. 

Properties
- Opposite sides are the same length.
- Opposite sides are parallel.
- Opposite angles are equal.
- Diagonals bisect each other.
- If one of the angles of a parallelogram is a right angle, then all other angles are right and it is a rectangle.

2. 

Properties
- Opposite sides are the same length.
- Opposite sides are parallel.
- It has 4 right angles.

3. Square, rectangle, rhombus, parallelogram, kite

4. Trapezium
5. Square, rectangle, rhombus, parallelogram

Lesson Title:	Interior angles of quadrilaterals
Practice Activity:	PHM2-L058

1. a. $m = 50^{o}$; b. 100^{o}; c. $\mathrm{A} = \mathrm{T} = 65^{o}$; $\mathrm{P} = \mathrm{R} = 115^{o}$
2. $x = 30^{o}$

Lesson Title:	Exterior angles of quadrilaterals
Practice Activity:	PHM2-L059

1. $p = 138^{\circ},\ q = 42^{\circ}$
2. $s = 40^{\circ},\ t = 140^{\circ}, u = 85^{\circ}$
3. $A = C = 75^{\circ}; B = D = 105^{\circ}$
4. $x = 14.4^{\circ}$

Lesson Title:	Solving triangles
Practice Activity:	PHM2-L060

1. $a = 20$
2. $x = 25^{\circ}$
3. $d = 7$ cm
4. $|AD| = 4$ cm

Lesson Title:	Proportional division of the side of a triangle
Practice Activity:	PHM2-L061

1. $C = 10$ cm
2. $|QZ| = 13\frac{1}{3}$ cm
3. $x = 3.5$ m
4. $a = 3.5$ m; $b = 2.5$ m; $c = 4$ m

Lesson Title:	Bisector of an angle in a triangle
Practice Activity:	PHM2-L062

1. $r = 3$ cm
2. $|RT| = 40$ cm
3. $9\frac{1}{6}$ cm or 9.2 cm
4. Ratio: $28{:}20 \rightarrow 7{:}5$; $a = 10.5$ m; $b = 7.5$ m
5. The values of x and y are 20 cm and 12 cm

Lesson Title:	Similar triangles
Practice Activity:	PHM2-L063

1. $|AB| = 14$ mm, $|CE| = 60$ mm
2. $|CL| = 5\frac{1}{3}$ cm and $|DM| = 6$ cm
3. $l = 25$

Lesson Title:	Triangle problem solving
Practice Activity:	PHM2-L064

1. $WZ = 7$ cm
2. $|PT| = 8$ cm, $|RS| = 12$ cm
3. $q = 3$ cm
4. a. $|PQ| = 10.0$ cm; b.$|PY| = 5.3$ cm

Lesson Title:	Conversion of units: smaller to larger
Practice Activity:	PHM2-L065

1. 0.63 m
2. 3.60 m
3. 4.2 m
4. 1.15 km
5. 1.2 kg
6. 18,373.5 litres

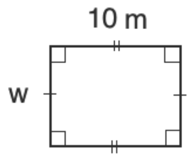
Lesson Title:	Conversion of units: larger to smaller
Practice Activity:	PHM2-L066

1. 63,000 mg
2. 9,000 cm
3. 180 cm
4. 1,400 m
5. 2,360,000 mg
6. 26,000 ml

Lesson Title:	Perimeter and area of a square and rectangle
Practice Activity:	PHM2-L067

1. a. $P = 78\text{ m}$, $A = 350\text{ m}^2$
   b. $P = 24.8\text{ m}$, $A = 38.44\text{ m}^2$
2. $P = 56\text{ cm}$, $A = 196\text{ cm}^2$
3. $A = 676\text{ m}^2$

4. $P = 48$ m
5. a. Diagram:



b. Width: $w = 7$ m
c. Perimeter: $P = 34$ m
d. 35 m^2

6. Le 24,010.00

Lesson Title:	Perimeter and area of a parallelogram
Practice Activity:	PHM2-L068

1. a. $A = 150 \text{ cm}^2$; $P = 32$ cm
   b. $A = 24 \text{ cm}^2$; $P = 24$ cm
2. $h = 6$ cm; $b = 30$ cm
3. $h = 16$ cm
4. $b = 8$ cm

Lesson Title:	Perimeter and area of a trapezium
Practice Activity:	PHM2-L069

1. $A = 52.5 \text{ cm}^2$
2. $A = 108\sqrt{3}\ cm^2$
3. $P = 53$ cm; $A = 105 \text{ cm}^2$
4. $A = 30 \text{ cm}^2$
5. $h = 8$ cm
6. $P = 207$ mm; $A = 2{,}210 \text{ mm}^2$

Lesson Title:	Perimeter and area of a rhombus
Practice Activity:	PHM2-L070

1. $P = 33.2$; $A = 63 \text{ m}^2$
2. $l = 40$ cm
3. a. 225 cm^2; b. 67.5 m^2; c. Le 13,500.00
4. $A = 120 \text{ cm}^2$, $P = 52$ cm
5. The other diagonal has length 24 m

Lesson Title:	Perimeter and area of a kite
Practice Activity:	PHM2-L071

1. a. $A = 1{,}152 \text{ mm}^2$; $P = 160 \text{ mm}$
   b. $A = 18.9 \text{ cm}^2$; $P = 21.0 \text{ cm}$
2. A= 787.5 m^2
3. The sides have lengths 40 mm and 20 mm
4. The other diagonal is 100 mm

Lesson Title:	Perimeter and area of a triangle
Practice Activity:	PHM2-L072

1. a. $A = 110 \text{ cm}^2$
   b. $A = 45 \text{ cm}^2$
2. a. $P = 48 \text{ cm}$, $A = 85.2 \text{ cm}^2$
   b. $P = 39 \text{ cm}$, $A = 56 \text{ cm}^2$
   c. $P = 125 \text{ mm}$, $A = 500 \text{ mm}^2$
3. $P = 62 \text{ m}$

Lesson Title:	Circumference and area of a circle
Practice Activity:	PHM2-L073

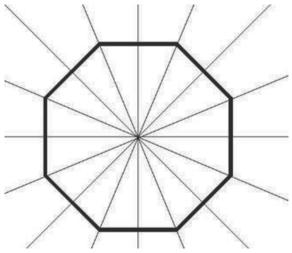
1. $C = 86.7 \text{ m}$
2. $d = 6 \text{ mm}$
3. $r = 7$; $C = 44 \text{ cm}$
4. $C = 188.57 \text{ mm}$; $A = 2{,}828.57 \text{ mm}^2$
5. $r = 14.01 \text{ m}$; $A = 616.32 \text{ m}^2$
6. a. $A = 707.1 \text{ m}^2$; b. $C = 94.3 \text{ m}$

Lesson Title:	Perimeter and area of compound shapes
Practice Activity:	PHM2-L074

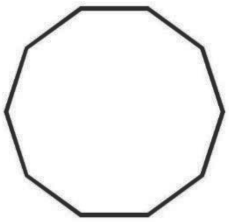
1. $P = 48 \text{ cm}$; $A = 80 \text{ cm}^2$
2. $P = 36 \text{ cm}$; $A = 56 \text{ cm}^2$
3. $P = 42 \text{ cm}$; $A = 44 \text{ cm}^2$
4. $P = 208.3 \text{ mm}$; $A = 2{,}200 \text{ mm}^2$

Lesson Title:	Properties of polygons
Practice Activity:	PHM2-L075

1. There are 8 lines of symmetry:



2.



Lesson Title:	Sum of interior angles of polygons
Practice Activity:	PHM2-L076

1. a. $3{,}240°$; b. $4{,}140°$; c. $5{,}040°$; d. $8{,}640°$
2. *a.* $n = 18$*; b.* $n = 28$*; c.* $n = 11$*;* d. $n = 9$

Lesson Title:	Interior and exterior angles of polygons
Practice Activity:	PHM2-L077

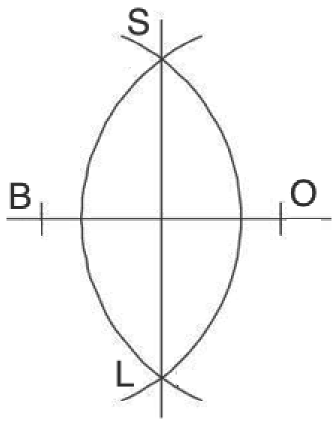
1. Interior angle: $x = 146°$; Exterior angles: $k = 55°$; $l = 50°$, $m = 60°$; $n = 45°$; $o = 55°$; $p = 30°$; $q = 34°$; $r = 31°$.
2. The polygon has 9 sides
3. a. $24°$; b. $156°$; c. $2340°$

Lesson Title:	Polygon problem solving
Practice Activity:	PHM2-L078

1. $x = 109°$
2. a. $x = 25°$; b. $x = 5°$
3. 6 sides
4. $y = 32.5°$

Lesson Title:	Bisect a given line segment
Practice Activity:	PHM2-L079

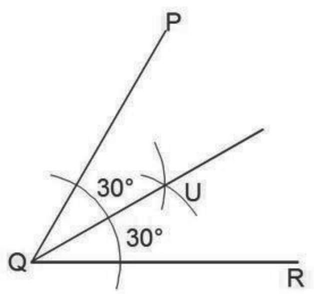
1.



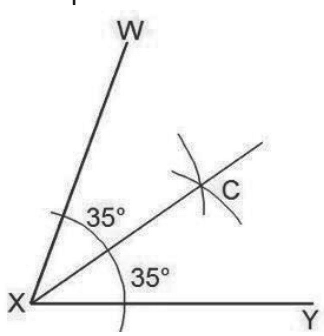
2.
   a. 12 metres
   b. 12 metres

Lesson Title:	Bisect a given angle
Practice Activity:	PHM2-L080

1.

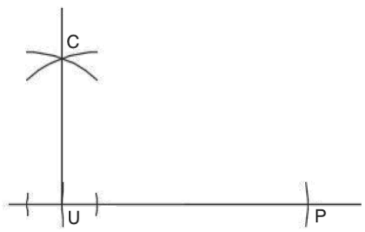


2. Example answer:

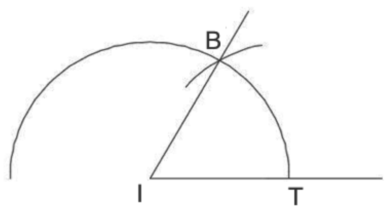


Lesson Title: Construct 90°, 60°, and 120° angles	
Practice Activity: PHM2-L081	

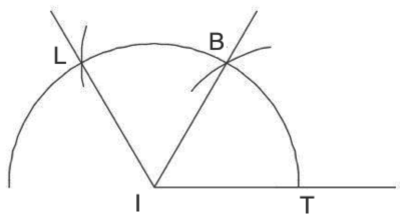
1.



2.

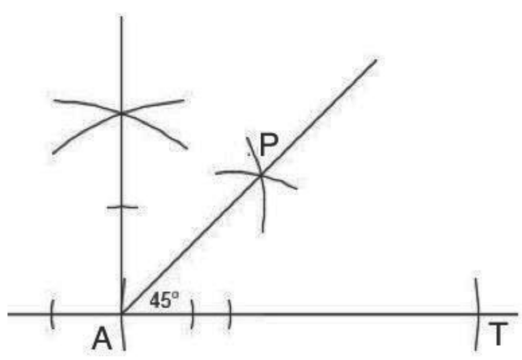


3.

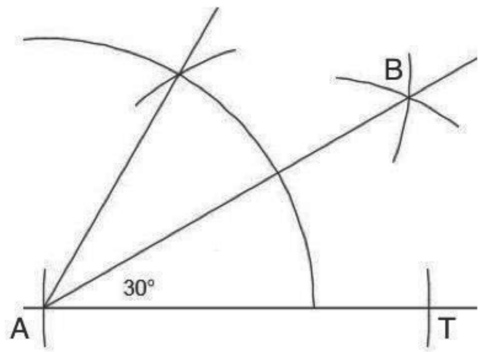


Lesson Title: Construct 45°, 30° and 15° angles	
Practice Activity: PHM2-L082	

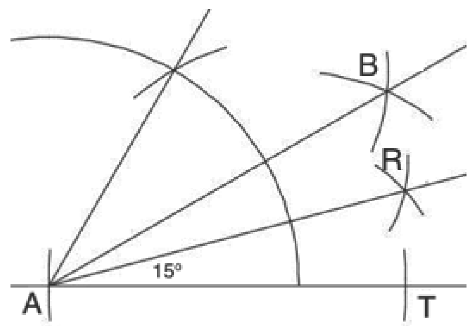
1.



2.



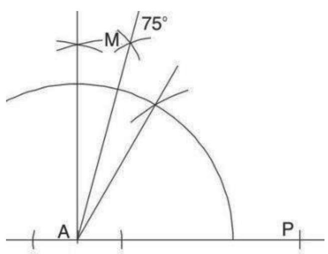
3.



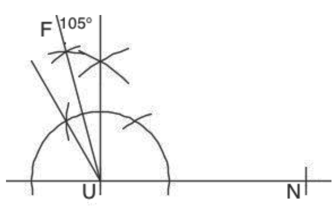
4. 7.5°

Lesson Title:	Construct 75°, 105° and 150° angles
Practice Activity:	PHM2-L083

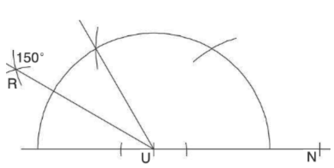
1.



2.



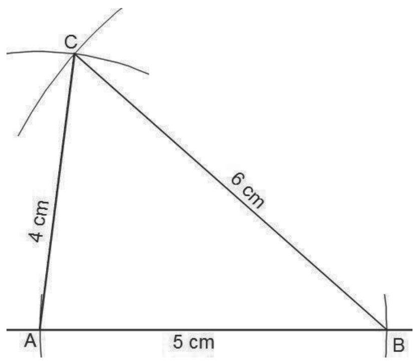
3.



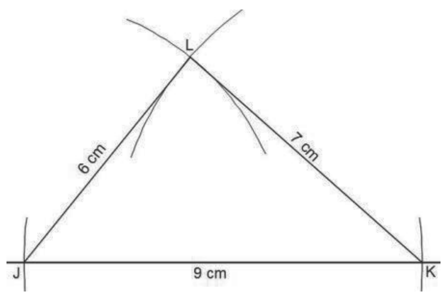
4. 135° is halfway between 90° and 180°. To construct 135°, first construct 90° and bisect the angle between 90° and 180°.

Lesson Title:	Construction of triangles – Part 1
Practice Activity:	PHM2-L084

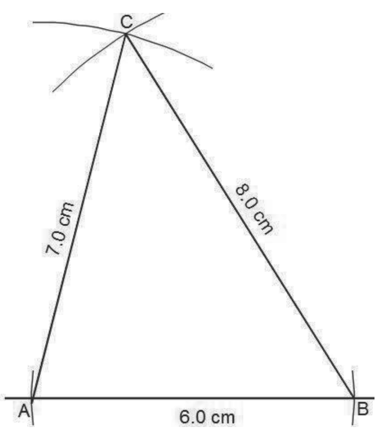
1.



2.

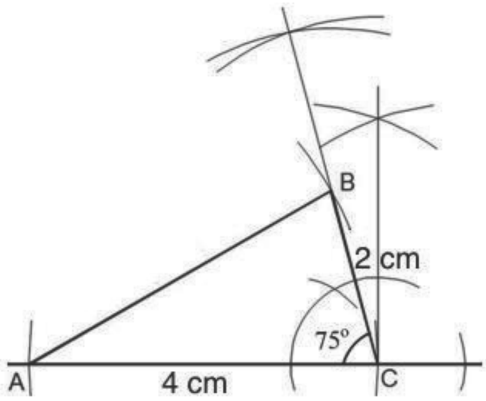


3.

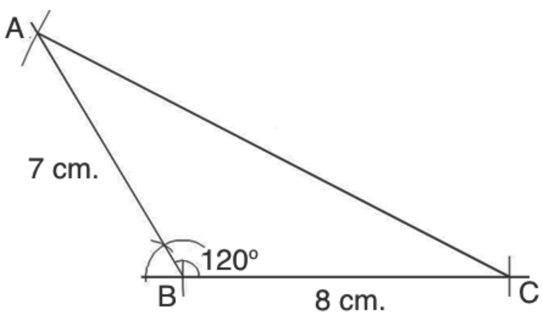


Lesson Title:	Construction of triangles – Part 2
Practice Activity:	PHM2-L085

1.

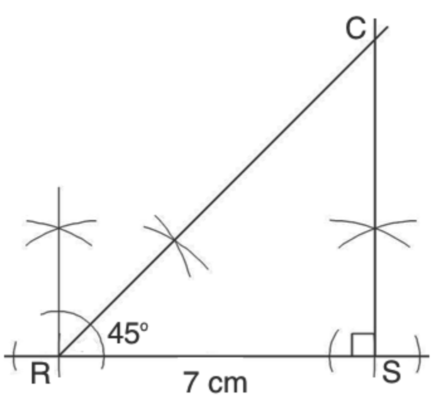


2.

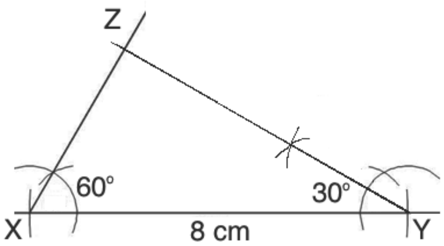


Lesson Title:	Construction of triangles – Part 3
Practice Activity:	PHM2-L086

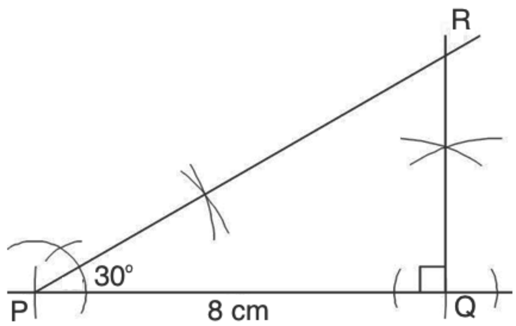
1.



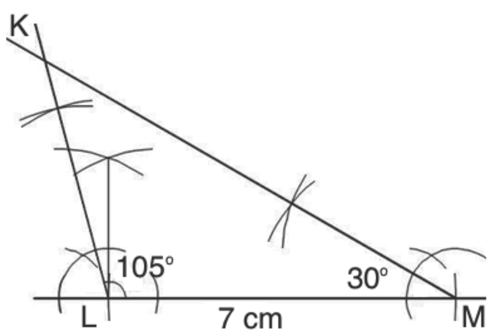
2.



3.

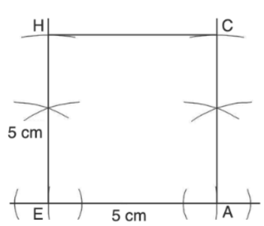


4.

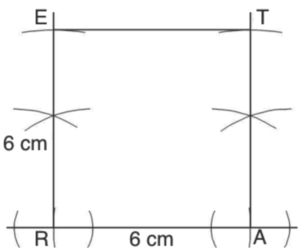


Lesson Title:	Construction of quadrilaterals - Part 1
Practice Activity:	PHM2-L087

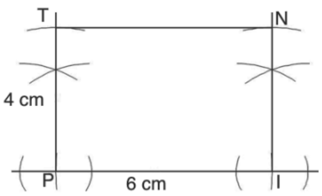
1. a.



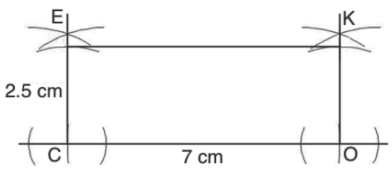
b.



2. a.

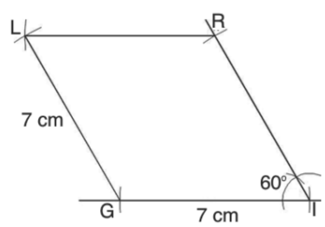


b.

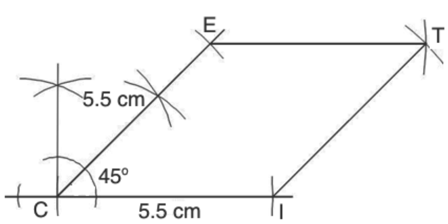


Lesson Title:	Construction of quadrilaterals - Part 2
Practice Activity:	PHM2-L088

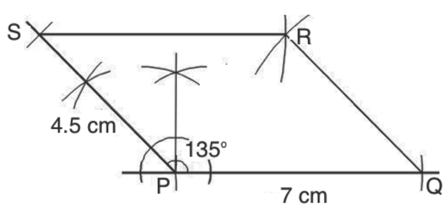
1.



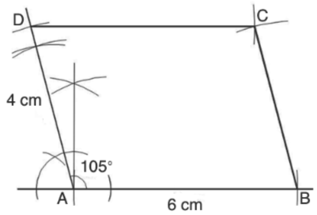
2.



3.

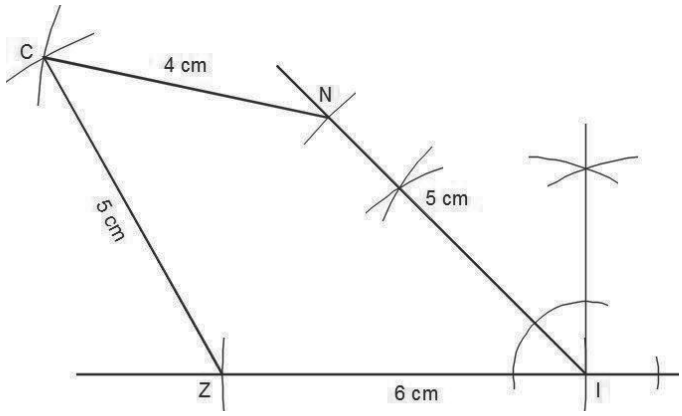


4.

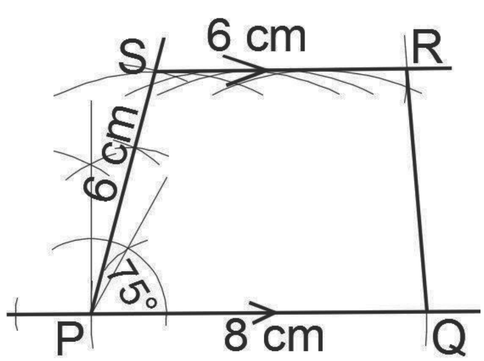


Lesson Title:	Construction of quadrilaterals - Part 3
Practice Activity:	PHM2-L089

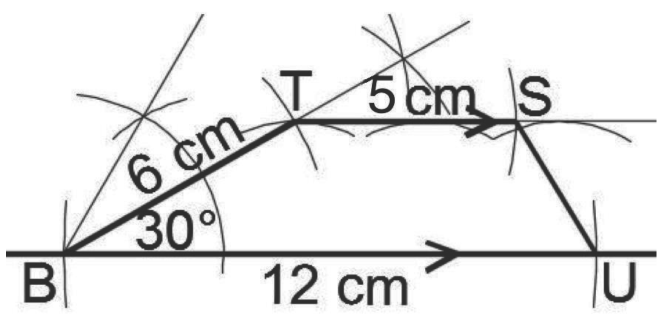
1.



2.



3.



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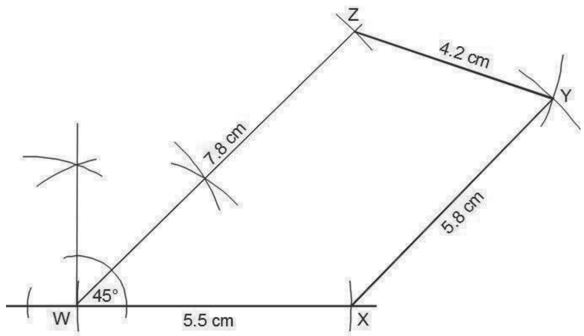

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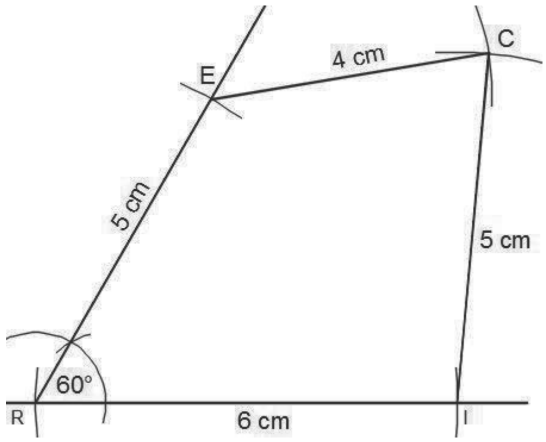
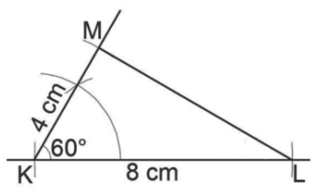
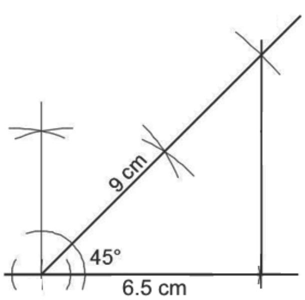


Lesson Title:	Construction word problems – Part 1
Practice Activity:	PHM2-L090

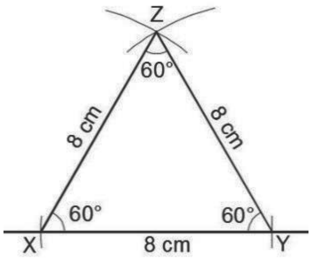
1.



2.

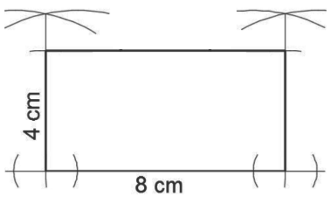


3.

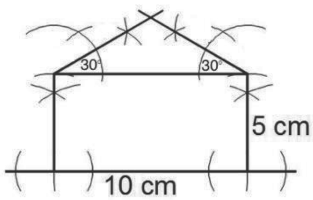


Lesson Title:	Construction word problems – Part 2
Practice Activity:	PHM2-L091

1.

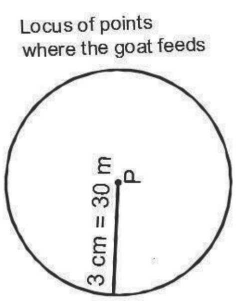


2.



Lesson Title:	Construction of loci – Part 1
Practice Activity:	PHM2-L092

1.



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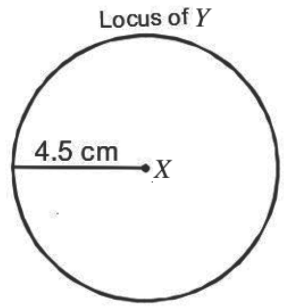

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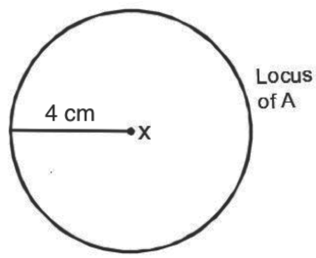

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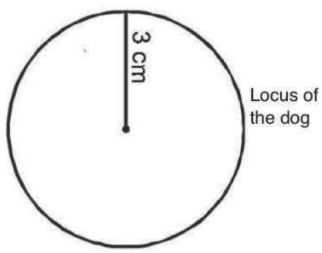
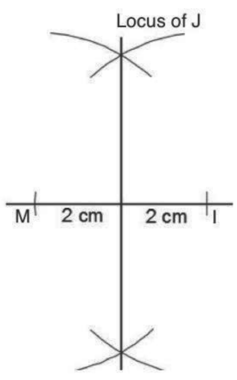
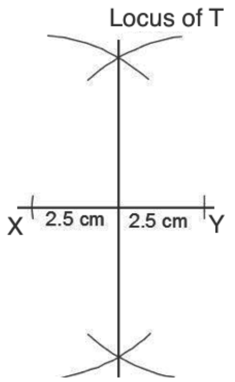


Lesson Title:	Construction of loci – Part 2
Practice Activity:	PHM2-L093

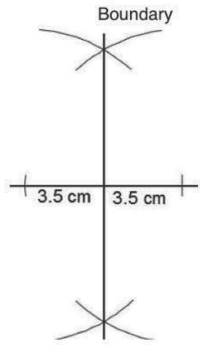
1.



2.

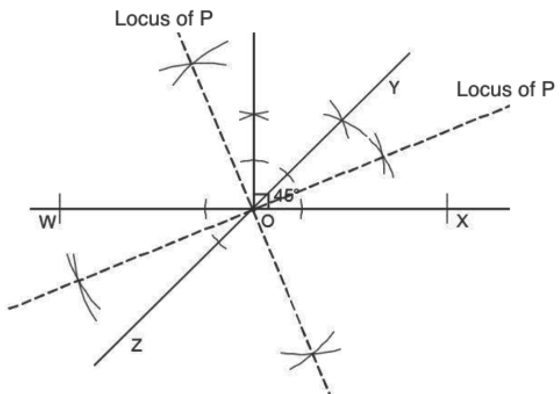


3.

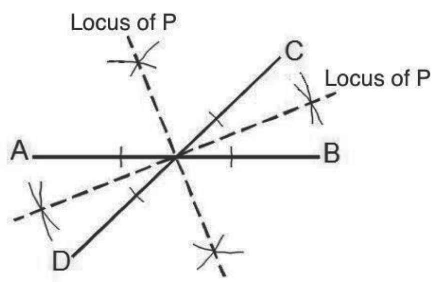


Lesson Title:	Construction of loci – Part 3
Practice Activity:	PHM2-L094

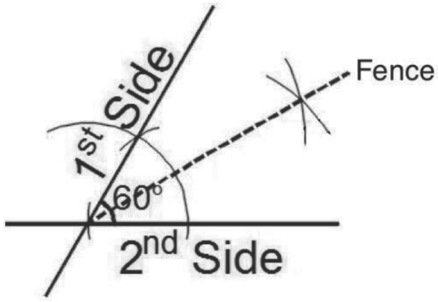
1.



2. Example (lines may form any angle):

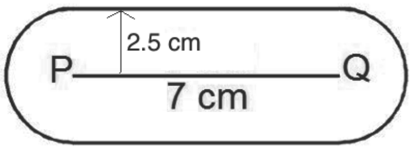


3. Solution to a and b:

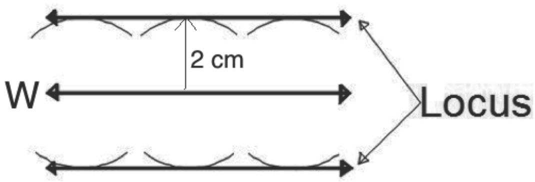


Lesson Title:	Construction of loci – Part 4
Practice Activity:	PHM2-L095

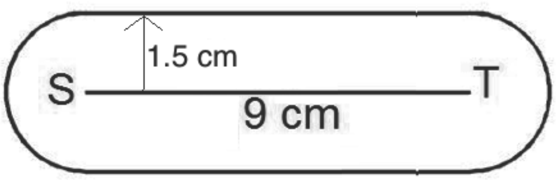
1.



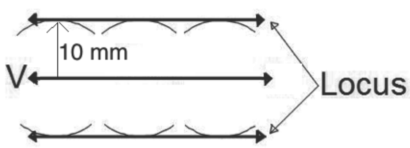
2.



3.

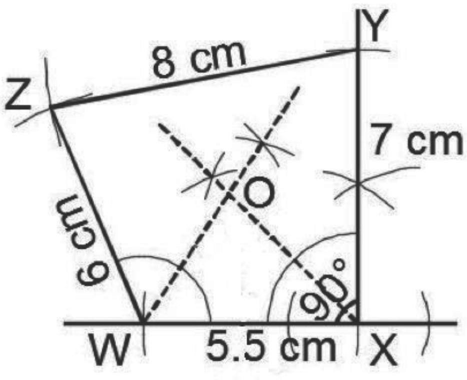


4.

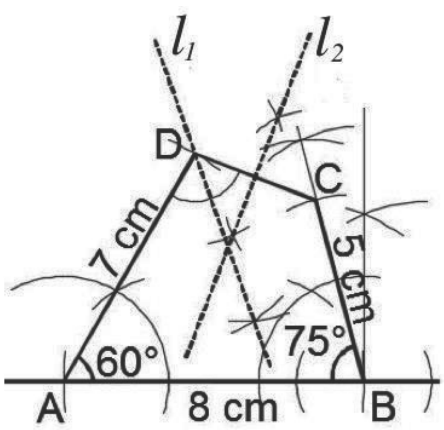


Lesson Title:	Construction practice
Practice Activity:	PHM2-L096

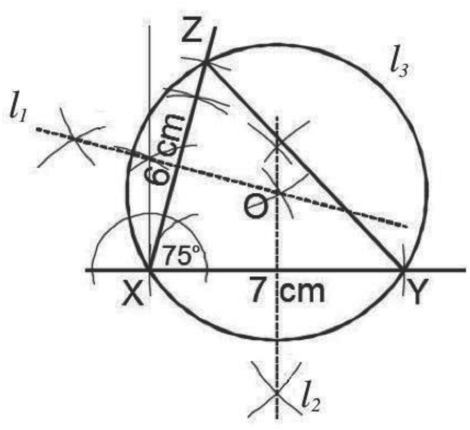
1.



2. .

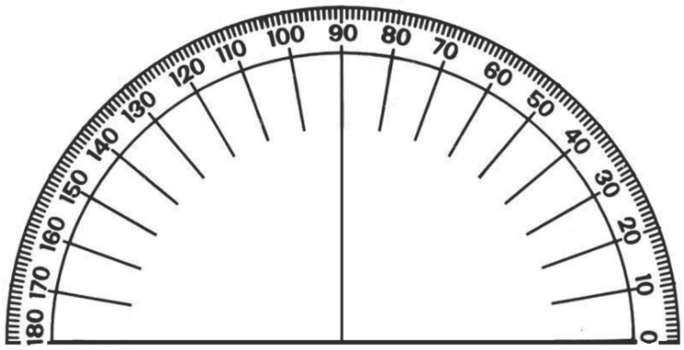


3. .



Appendix I: Protractor

You can use a protractor to measure angles. If you do not have a protractor, you can make one with paper. Trace this protractor with a pen onto another piece of paper. Then, cut out the semi-circle using scissors.



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