THE PRESIDENT'S RECOVERY PRIORITIES
Education

Ministry of Education, Science and Technology

Lesson plans for JSS Mathematics

JSS 3

TERM 2

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Please see final page for further information.

NOT FOR SALE

Foreword

Our country's future lies in the education of our children. The Government of Sierra Leone is committed to doing whatever it takes to secure this future.

As Minister of Education, Science and Technology since 2007, I have worked every day to improve our country's education. We have faced challenges, not least the Ebola epidemic which as we all know hit our sector hard. The Government's response to this crisis – led by our President – showed first-hand how we acted decisively in the face of those challenges, to make things better than they were in the first place.

One great success in our response was the publication of the Accelerated Teaching Syllabi in August 2015. This gave teachers the tools they needed to make up for lost time whilst ensuring pupils received an adequate level of knowledge across each part of the curriculum. The Accelerated Teaching syllabi also provided the pedagogical resource and impetus for the successful national radio and TV teaching programs during the Ebola epidemic.

It is now time to build on this success. I am pleased to issue new lesson plans across all primary and JSS school grades in Language Arts and Mathematics. These plans give teachers the support they need to cover each element of the national curriculum. In total, we are producing 2,700 lesson plans – one for each lesson, in each term, in each year for each class. This is a remarkable achievement in a matter of months.

These plans have been written by experienced Sierra Leonean educators together with international experts. They have been reviewed by officials of my Ministry to ensure they meet the specific needs of the Sierra Leonean population. They provide step-by-step guidance for each learning outcome, using a range of recognised techniques to deliver the best teaching.

I call on all teachers and heads of schools across the country to make best use of these materials. We are supporting our teachers through a detailed training programme designed specifically for these new plans. It is really important that these Lesson Plans are used, together with any other materials you may have.

This is just the start of education transformation in Sierra Leone. I am committed to continue to strive for the changes that will make our country stronger.

I want to thank our partners for their continued support. Finally, I also want to thank you – the teachers of our country – for your hard work in securing our future.

Dr. Minkailu Bah

Minister of Education, Science and Technology

__Table of Contents__

JSS
Mathematics

Introduction

to the Lesson Plan Manual

These lesson plans are based on the National Curriculum
and meet the requirements established
by the Ministry of Education, Science and Technology.

1. The lesson plans will not take the whole term, so use spare time to review material or prepare for exams
2. Teachers can use other textbooks alongside or instead of these lesson plans.
3. Read the lesson plan before you start the lesson. Look ahead to the next lesson, and see if you need to tell pupils to bring materials for next time.
4. Make sure you understand the learning outcomes, and have teaching aids and other preparation ready – each lesson plan shows these using the symbols on the right.
5. Quickly review what you taught last time before starting each lesson.
6. Follow the suggested time allocations for each part of the lesson. If time permits, extend practice with additional work.
7. Lesson plans have a mix of activities for the whole class and for individuals or in pairs.
8. Use the board and other visual aids as you teach.
9. Interact with all pupils in the class – including the quiet ones.
10. Congratulate pupils when they get questions right! Offer solutions when they don't, and thank them for trying.

Learning outcomes

Teaching aids

Preparation

Lesson Title: Review of Transformations	**Theme**: Geometry	
Lesson Number: M-09-046	**Class/Level**: JSS 3	**Time**: 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to identify and perform translation, reflection, and rotation.	Poster (see Opening)	1. Draw the diagrams for the different sections of the lesson from the end of this lesson plan on the board. 2. Create the poster shown in Opening on vanguard paper (if available). This can be put on the classroom wall for future use. If the material to make a poster is not available, draw it on the board.

Opening (*3 minutes*)

1. **Say**: Write down one sentence for each word in Question i.
2. Allow pupils time to write down their explanation for each word.
3. Have pupils from around the classroom volunteer to give their explanations.
4. Put up the poster on the board/point to the information on the board. Go through the explanation for each word.
5. **Ask:** Which of the transformations changes the size of the object? Raise your hand. (Answer: enlargement)
6. Ask: Which of the transformations does not change the size of the object? Raise your hand. (Answer: translation, reflection and rotation)
7. **Say**: Today we are going to identify and perform translation, reflection, and rotation.

Transformations

Transformation
Changes the position or size of objects. Transformations are: translation, reflection, rotation and enlargement.

Translation
Moves an object up, down, left or right without changing its size or shape.

Reflection
Creates an image of an object of the same size and shape, across a mirror line or line of symmetry.

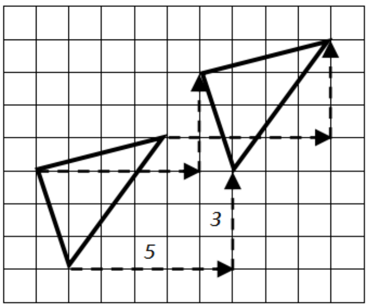
Rotation
Turns an object around a fixed point, called the centre of rotation, without changing its size or shape.

Enlargement
Creates an object of the same shape, but a different size.

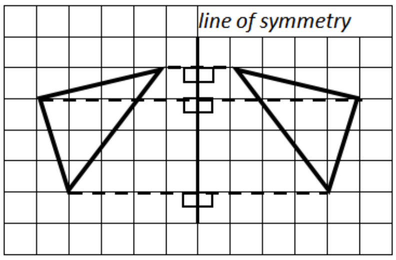
Introduction to the New Material (*20 minutes*)

Note: Due to the nature of this topic, 20 minutes has been allocated to this section. Pupils will consequently do 10 minutes of Guided Practice and no Independent Practice.

1. **Say:** Let us start with translating objects. Let us translate the object in Question ii. First, draw the object in your exercise books.
2. Allow time for pupils to draw the object in Question ii. in their exercise books.
3. Show how to translate the object.
   Count the units, mark the points, and draw the lines for the new position as you speak.
   Pupils should follow the instructions and do the translation in their exercise books.

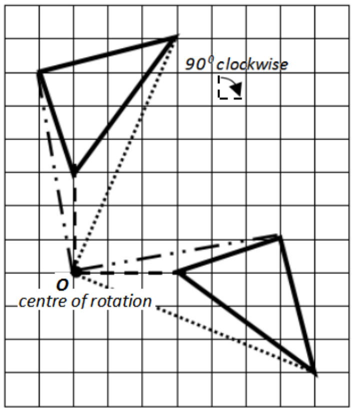


4. Stop from time to time to ensure the pupils are keeping up with the instructions. Clarify any confusion on the instructions.
    - **Say**: We want to move the object 5 units right and 3 units up.
    - Use vector notation, $\binom{5}{3}$, if appropriate.
    - **Say**: When we translate an object, every part of the object moves the same amount in the same direction.
    - **Say**: We select a point on the object.
    - **Say**: Count from that point 5 units right and 3 units up.
    - **Say:** Mark the point for the translated object.
    - **Say**: Do the same for the other points of the object.
    - **Say**: Join the points to draw the object at its new position after translation.
5. Allow time for pupils to complete the translation.
6. **Say**: We can translate any object in a similar way by counting the units left, right, up or down as required.
7. **Say**: Let us now reflect the object as requested in Question iii.
8. Ask pupils to copy the object into their exercise books.
9. Ask them to follow the instructions as you perform the reflection on the board.
10. Stop from time to time to ensure the pupils are keeping up with the instructions.
    - **Say:** We want to reflect the object in the required mirror line or line of symmetry without using a mirror.
    - **Say:** We use the fact that every point in the image is the same distance from the line of symmetry as the original object.
    - **Say:** Draw a line at 90 ° to the line of symmetry from a point on the original object to the other side of the line of symmetry.
    - **Say:** Mark on the 90 ° line the same distance from the line of symmetry as the original object.
    - **Say:** That is the new point of the reflected object.
    - **Say**: Do the same for the other points of the object.
    - **Say**: Make sure the line from the original object is at 90 ° for each point.
    - **Say**: Join the points to draw the shape at its new position after reflection.



11. Allow time for the pupils to complete the reflection.
12. Walk around, if possible, to check pupils' work and correct any misconceptions.
13. Ask pupils to copy the object in Question iv. in their exercise books.
14. **Say**: Let us now rotate the object.
15. Instructions to rotate an object are shown below for pupils to follow in their exercise books. Separate instructions have been provided for tracing paper (or clear plastic) if available.
    - Point to each of the items on the board as you say the next line.
    - **Say:** We are given 3 pieces of information in order to rotate our shape – the centre of rotation, the angle of rotation and the direction of rotation.

- **Ask**: What is the centre of rotation? (Answer: point O)
- **Ask**: What is the angle of rotation? (Answer: 90 °)
- **Ask**: What is the direction of rotation? (Answer: clockwise)
- **Say**: Draw a straight line from one of the points to the centre of rotation. Measure the line.
- **Say**: Draw a line at an angle of 90 ° to the first line.
- **Say**: Mark the measurement on the second line. That is the new point of the object after rotation.
- **Say**: Do the same for the other points.
- **Say**: Join the points to draw the shape at its new position.
- Allow time for the pupils to complete the rotation.
- If tracing paper or clear plastic is available:
  - **Say**: Put the tracing paper over the shape.
  - **Say**: Trace the shape and mark the centre of rotation.
  - **Say**: Hold down the tracing paper with a pencil on the centre of rotation.
  - **Say**: Rotate the tracing paper and copy the image.



Guided Practice (*10 minutes*)

1. Ask pupils to work in pairs to answer the questions for Guided Practice.
2. They should each draw their own shapes and they can discuss and share ideas with each other.
3. Walk around, if possible, to check answers and correct any misconceptions.
4. Have pupils from around the classroom volunteer to give their answers to the questions.
5. **Write** the correct answers and steps on the board. Ask pupils to check their work. (Answers: shown below the questions at the end of this lesson plan)

Closing (*2 minutes*)

1. **Say**: Copy the explanations for the different transformations into your exercise books.

[QUESTIONS FOR OPENING ACTIVITY]

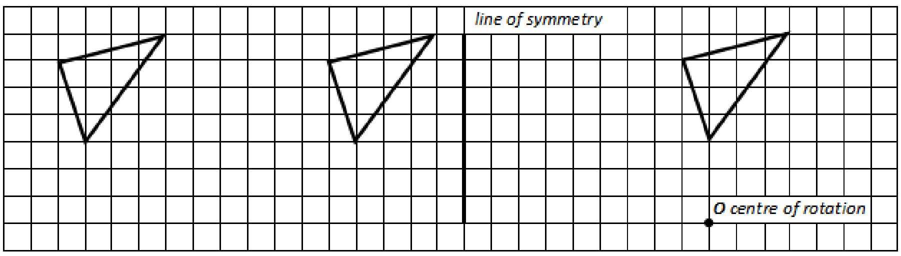
i. Explain what these words mean in <u>Maths</u>:
transformation, translation, reflection, rotation, enlargement

[QUESTIONS FOR INTRODUCTION TO THE NEW MATERIAL]
Leave space around each shape to carry out the transformation.

ii. Translate the shape 5 units right and 3 units up

iii. Reflect the shape in the line of symmetry shown

iv. Rotate the shape 90° clockwise about the point shown.



[QUESTIONS FOR GUIDED PRACTICE]

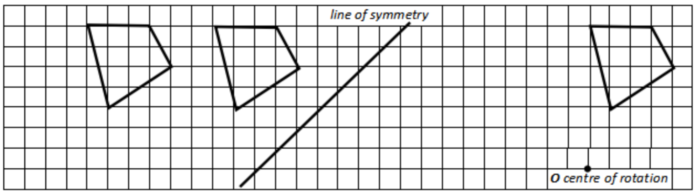
Leave space around each shape to carry out the transformation.

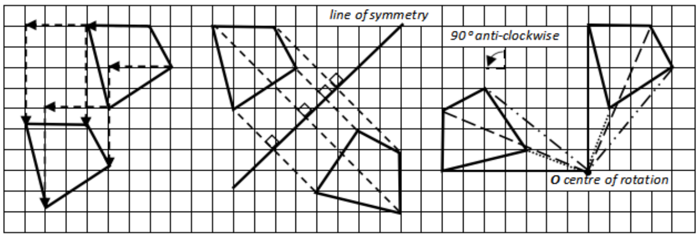
Answers are shown below the questions

v. Translate the shape 3 units left and 5 units down

vi. Reflect the shape in the line of symmetry shown

vii. Rotate the shape 90° anti-clockwise about the point shown.





Lesson Title: Combining Transformations	**Theme**: Geometry	
Lesson Number: M-09-047	**Class/Level**: JSS 3	**Time**: 35 minutes

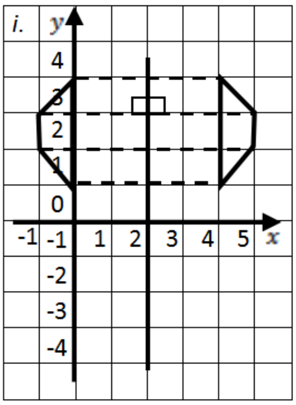
Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to: 1. Carry out combinations of translation, reflection, and rotation. 2. Describe and compare the 3 transformations.	None	Draw the diagrams for the different sections of the lesson from the end of this lesson plan on the board.

Opening (*3 minutes*)

1. **Ask**: Who can remind the class what we did last lesson? Raise your hand.
2. Have one pupil from the back and one from the front of the classroom volunteer to answer. (Example answers: transformations of shapes; translation, reflection and rotation of shapes)
3. **Ask**: Which of the transformations do not change the position of the object? Why? Raise your hand. (Example answer: Reflection, because it creates a mirror image of the object which is the same shape and size as the object; Enlargement, because it creates an object of the same shape but different size.)
4. **Say**: Today we are going to carry out combinations of translation, reflection, and rotation. We will also describe and compare the 3 transformations.

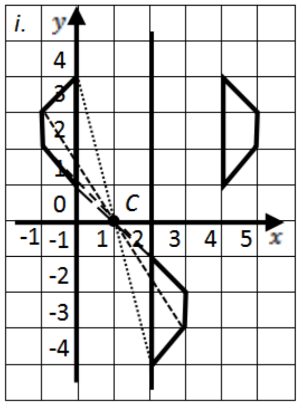
Introduction to the New Material (*10 minutes*)

1. **Say:** The 3 transformations we looked at last lesson can be combined by doing one transformation and then another. For instance, we can translate an object, then reflect the translated object. Or we can rotate, then reflect or translate the resultant shape. When combining transformation, it is often easier to see what has taken place if we use a Cartesian plane to show the transformations.
2. Ask a pupil to read Question a. on the board.
3. Ask pupils to copy the shape into their exercise books.
4. **Say**: Let us first do the reflection as requested in our books. Remember the lines you use to guide you in drawing the reflection are at 90° angle to the line of symmetry.
5. Allow time for the pupils to do the reflection. Ask them to raise their hands once they are done.
6. Select a pupil with raised hand to complete the reflection on the board.
7. Ask pupils if they agree. Correct any errors and ask pupils to do the same in their exercise books. (Answer: shown right)

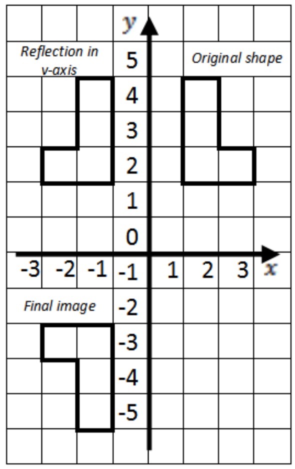


8. **Ask**: We want to rotate the reflected shape 180 °. What is missing from this instruction? Raise your hand.
9. Guide a pupil to say: ‘we have not been told whether it is clockwise or anti-clockwise.’
10. **Say**: Work with your neighbour. One of you go clockwise, the other anti-clockwise.
11. Allow time for the pupils to do the rotation.

12. **Ask**: What do you notice about the rotation? Raise your hand.
13. Guide a pupil to say: we get to the same position after rotation. (Answer: shown right)



14. Ask the class if their pairs also arrived at the same position after rotation.
15. **Say**: When rotating by 180 º, it does not matter if we go clockwise or anti-clockwise, we always get to the same position. This is because no matter which direction we go, we are doing half a turn ($180^{o} = \frac{1}{2}$ of 360). This leaves us facing the opposite direction from where we started. The centre of rotation or direction we turn does not change the end position.
16. Ask a pupil to read Question b.
17. **Say**: Draw the shape in Question b. in your exercise books. Follow the instructions to do the combined transformation.
18. Allow time for the pupils to answer the question.
19. Have a pupil volunteer to come to the board to show the combined transformation.
20. **Ask**: What single transformation could we do to get to the same position as the 2 transformations? Raise your hand.
21. Guide a pupil to say: 'the shape has been rotated 180 º about the origin.'
22. **Say**: It is often the case that a single transformation can move an object to the same position as a combined transformation.



Guided Practice (*10 minutes*)

1. Ask pupils to work in pairs to answer the questions for Guided Practice.
2. They should each draw their own shapes and they can discuss and share ideas with each other.
3. Walk around, if possible, to check answers and correct any misconceptions.
4. Have pupils from around the classroom volunteer to give their answers to the questions.
5. **Write** the correct answers on the board. Ask pupils to check their work. (Answers: shown below the questions at the end of this lesson plan)

Independent Practice (*10 minutes*)

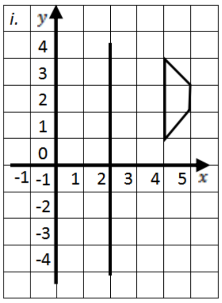
1. Ask pupils to work in pairs to answer the questions for Independent Practice.
2. Walk around, if possible, to check answers and correct any misconceptions.
3. Have pupils from around the classroom volunteer to give their answers to the questions.
4. **Write** the correct answers on the board. Ask pupils to check their work. (Answers: shown below the questions at the end of this lesson plan)

Closing (*2 minutes*)

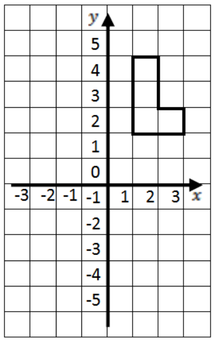
1. **Say**: Write in your own words what you understand a combined transformation to mean.
2. Allow time for pupils to answer.
3. Have a pupil volunteer to give their answer. (Example answer: A transformation where 2 or more individual transformations are done one after the other)

[QUESTIONS FOR INTRODUCTION TO THE NEW MATERIAL]

a. Reflect the shape in the line of symmetry shown. Then rotate it 180 ° about the point (1,0).

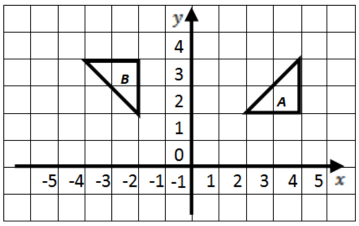


b. Reflect the shape shown in the y-axis. Then reflect it in the x-axis.

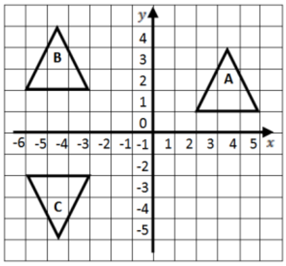


[QUESTIONS FOR GUIDED PRACTICE]

c. What is the name of the transformation that takes shape A to position B?

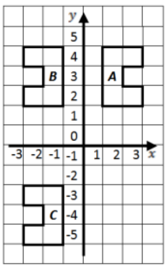


d. Describe the transformation that takes the shape from A to B to C.

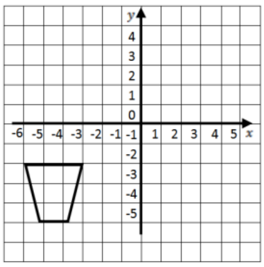


[QUESTIONS FOR INDEPENDENT PRACTICE]

e. Describe the transformation that takes shape A to B to C.
What single transformation has the same effect?



f. Reflect the shape shown in the x-axis then translate it 8 units right and 6 units down.

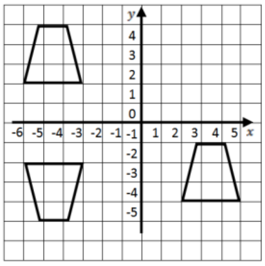


Answers:

c. Rotation

d. Translation 8 units left, I unit up, then reflect in the x-axis

e. A reflection in the y-axis. Then a reflection in the x-axis.
Rotation by 180 ° about the origin.

f. 

Lesson Title: Congruency	**Theme**: Geometry	
Lesson Number: M-09-048	**Class/Level**: JSS 3	**Time**: 35 minutes

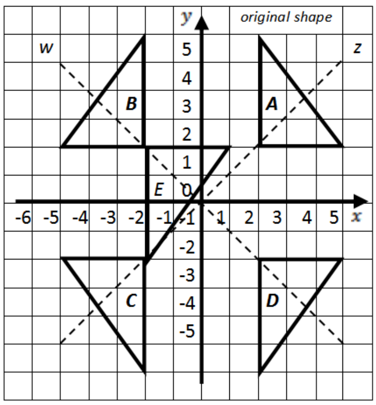
Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to compare 2 shapes that have undergone reflection, rotation and translation and identify them as congruent.	None	1. Draw the diagrams for the different sections of the lesson found at the end of this lesson plan on the board. 2. Write the vocabulary list on the board: congruent, congruency.

Opening (3 minutes)

1. **Ask**: Who can remind the class what we did last lesson? Raise your hand.
2. Have pupils from around the classroom volunteer to give their answers.
   (Example answers: Performed combinations of transformations; described combined transformations)
3. **Say**: Today we are going to compare 2 shapes that have undergone reflection, rotation and translation and identify them as congruent.

Introduction to the New Material (10 minutes)

1. Ask a pupil to read Question a.
2. **Say:** Think about the transformations we have been doing. Describe which ones you can see in the diagram. For example, A reflected in the y-axis gives B.
3. Allow time for pupils to answer.
4. **Say**: Compare your answers with your neighbour. Do you agree with each other's answers?
5. Allow time for pupils to compare their answers.
6. **Say**: Who would like to share what they discussed with the class?
7. Have pupils from around the classroom volunteer to share their answers.
8. Ask other pupils if they agree with the answers being given.
   (Example answers: Reflection in the x–axis takes A and B to D and C respectively; Reflection in the y–axis takes A and D to B and C respectively; Reflection in line w takes A to C, reflection in line z takes B to D, rotation 180 ^{o} about the origin takes A to C and similarly for the other shapes)
9. **Ask**: Who would like to show how to translate D, 4 units left and 4 units up? Call the new shape E.
10. Have a pupil volunteer to answer on the board (Answer: shown right)
11. **Say**: What do you notice about the original shape A, compared to B, C, D and E? Raise your hand.
12. Guide a pupil to say 'the shapes and the sizes are all the same.'
13. **Say:** When objects have the same shape and size after reflection, rotation and translation then they are called 'congruent.' Congruent shapes are identical in every way. We can cut up shapes A, B, C D and E and put them one on top of the other, they will make an exact fit.



Guided Practice (*10 minutes*)

1. Ask pupils to continue to work in pairs to answer Questions b. and c.
2. Walk around, if possible, to check answers and correct any misconceptions.
3. Have pupils from around the classroom volunteer to give their answers to the questions.
4. **Write** the correct answers and steps on the board. Ask pupils to check their work. (Answers: shown below the questions at the end of this lesson plan)

Independent Practice (*10 minutes*)

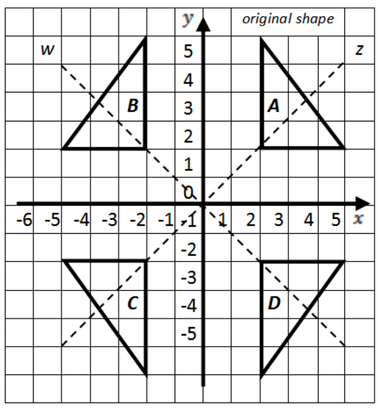
1. Ask the pupils to work independently to answer Question d.
2. Walk around, if possible, to check answers and clear misconceptions.
3. Have pupils from around the classroom volunteer to give their answers to the questions.
4. **Write** the correct answers on the board. Ask pupils to check their work. (Answers: shown below the questions at the end of this lesson plan).
5. Ask pupils to draw their own simple shapes in a grid.
6. Ask them to perform chosen combination of translations, reflections and rotations.
7. Ask them if their transformations verify that shapes remain congruent after translations, reflections and rotations.
8. Correct any mistakes. (Answers: various answers depending on the shapes that pupils have drawn)

Closing (*2 minutes*)

1. **Ask**: How you would explain congruent shapes to a friend who does not understand?
2. **Say:** Write down your explanation in your exercise books.
3. Allow time for pupils to write down their answers.
4. Have pupils from around the classroom volunteer to give their answers. (Example answers: Congruent shapes are identical in every way; they have exactly the same shape and size; they leave no gaps or overlaps when put on top of one another)

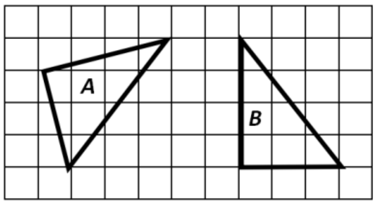
[QUESTIONS FOR INTRODUCTION TO THE NEW MATERIAL]

ii. Name all the transformations you can see in the diagram shown right.

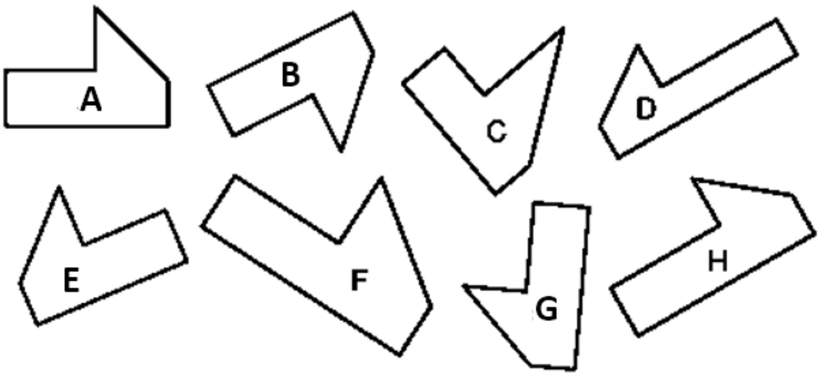


[QUESTIONS FOR GUIDED PRACTICE]

iii. Triangle A has undergone reflection in the x-axis. Triangle B has undergone rotation about 180 °.
The result of transforming triangles A and B are shown on the right.
Are triangles A and B congruent?
Give a reason for your answer.



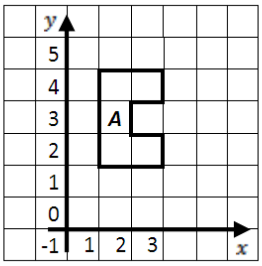
iv. Which of the shapes below are congruent?



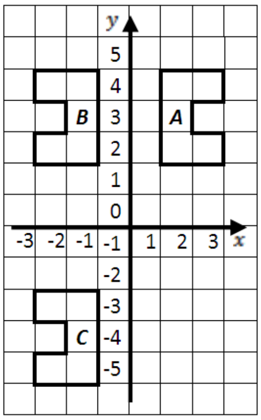
(Answers:
b. Triangles A and B are congruent. They are the same shape and size.
c. A, B, E and G, they are the same shape and size, Note F is the same shape, but not the same size).

[QUESTIONS FOR INDEPENDENT PRACTICE]

d. Reflect the shape shown in the x-axis.
Reflect the resulting shape in the y-axis.
What do you notice about the shapes resulting from both transformations?



(Answers: transformations shown below. the shapes resulting from both transformations are congruent).



Lesson Title: Practice with Congruency	Theme: Geometry	
Lesson Number: M-09-049	**Class/Level:** JSS 3	**Time:** 35 minutes

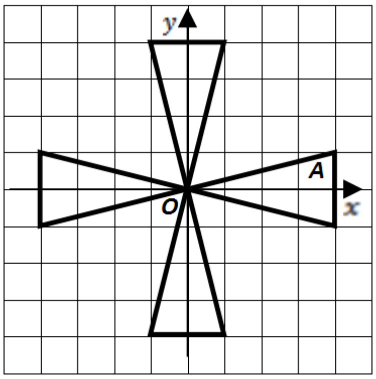
Learning Outcomes	Teaching Aids	Preparation
By the end of the lesson pupils will be able to create congruent shapes by performing transformations.	None	Draw the diagrams for the different sections of the lesson found at the end of this lesson plan on the board.

Opening (*3 minutes*)

1. Ask the following questions in preparation to creating congruent shapes.
2. Wait a few moments after each question. Have pupils from around the classroom volunteer to answer.
3. **Ask**: Who can remind the class what translation does to a shape or object? (Example answer: It moves a shape left, right, up or down without changing its size.)
4. **Ask**: What about a reflection? What does that do to a shape? (Example answer: It creates an image of the same shape and size in a line of symmetry.)
5. **Ask:** The last transformation we have studied so far is a rotation. What happens when you rotate a shape? (Example answer: It turns an object around a centre of rotation without changing its shape or size.)
6. **Say**: Today we are going to create congruent shapes by performing transformations.

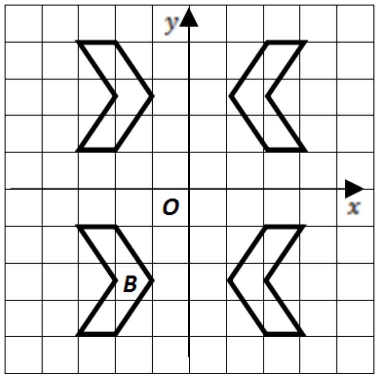
Introduction to the New Material (*10 minutes*)

1. Ask a pupil to read Question a.
2. **Say**: We are going to do a series of transformations to shape A. Copy shape A into your exercise books. Follow the instructions and complete the transformation as we do them on the board.
3. Allow time for pupils to copy the diagram into their exercise books.
4. **Ask:** Who would like to do the first: rotation 90 ° clockwise about O? Raise your hand.
5. Select a pupil to do the rotation on the board.
6. Remind the pupil to use the vertical line as a guide for the rotation.
7. **Say**: We need 2 more volunteers to complete the remaining transformations. Raise your hand.
8. Select 2 more pupils to come to the board one at a time to complete the transformation. (Answer: shown right)



9. **Ask**: We have made a pretty pattern with our shape. But are the shapes we get after each transformation congruent to A? Raise your hand. (Answer: yes)
10. **Ask**: How do we know they are congruent? Raise your hand.
11. Guide a pupil to say: 'Each of them are the same size and shape as A; when we count the matching squares of each of the transformations they are the same as A.'
12. **Ask**: Do you notice anything else? Raise your hand.

13. Guide a pupil to say: 'we get shapes in all the quadrants by performing different transformations on a shape.'
14. Ask pupils to copy the diagram for Question b. into their exercise books.
15. **Say**: Work with your neighbour to do the first transformation. Raise your hand when you are finished.
16. Allow time for pupils to copy and complete the first transformation.
17. Have a pupil volunteer to show the transformation on the board.



18. Correct any errors in the solution on the board. Ask pupils to check their work.
19. **Say**: Let us finish the rest of the transformations for shape B.
20. Allow time for pupils to do this.
21. Have pupils volunteer to complete the transformations on the board (Answer: shown right.)
22. Correct any errors in the solution on the board. Ask pupils to check their work.
23. **Say**: Just like before, we get shapes in all the quadrants by performing different transformations on a shape.

Guided Practice (*10 minutes*)

1. Ask pupils to continue to work in pairs to answer Question c.
2. Walk around, if possible, to check answers and correct any misconceptions.
3. Have pupils from around the classroom volunteer to show their answers on the board.
4. Ask the class if they agree with the answer.
5. Correct any errors in the solution on the board. Ask pupils to check their work. (Answers: shown below the question at the end of this lesson plan)

Independent Practice (*10 minutes*)

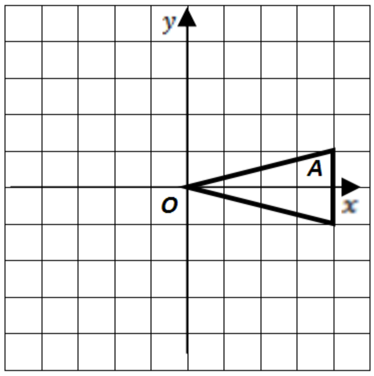
1. Ask the pupils to work independently to answer Questions d. and e.
2. Walk around, if possible, to check answers and clear misconceptions.
3. Have pupils from around the classroom volunteer to show their answers on the board.
4. Ask the class if they agree with the answer.
5. Correct any errors in the solution on the board. Ask pupils to check their work. (Answers: shown below the question at the end of this lesson plan)

Closing (*2 minutes*)

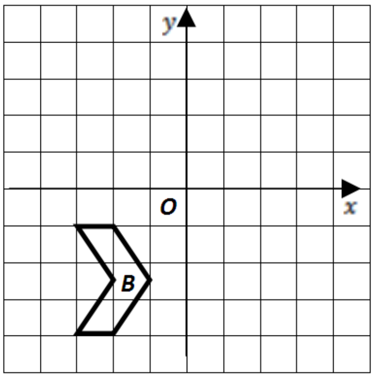
1. **Ask**: Write down one thing you learnt in today's lesson.
2. Allow time for pupils to write down their answers.
3. Have pupils from around the classroom volunteer to give their answers. (Example answers: We can get shapes in all the quadrants performing different transformations on the shape; congruent shapes can be used to make patterns (if a pupil gives this answer, tell them that the patterns are called tessellations))

[QUESTIONS FOR INTRODUCTION TO THE NEW MATERIAL]

v. Copy the isosceles triangle and complete the following transformations:
   - rotate A 90 ^{o} clockwise about the origin
   - rotate A 90 ^{o} anti-clockwise about the origin
   - reflect A in the y-axis.



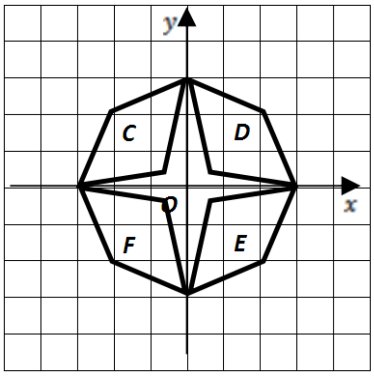
vi. Copy the hexagon and complete the following transformations:
   - rotate B 180 ^{o} around the origin
   - translate B 5 units up
   - reflect B in the y-axis



[QUESTIONS FOR GUIDED PRACTICE]

vii. Describe the transformations to shape C to get the other shapes D, E and F.

(Answer: c. to get shape D: reflect C in the y-axis; E: rotate 180 ^{o} around the origin; F: reflect in the x-axis.
Accept all reasonable (and correct) answers.)

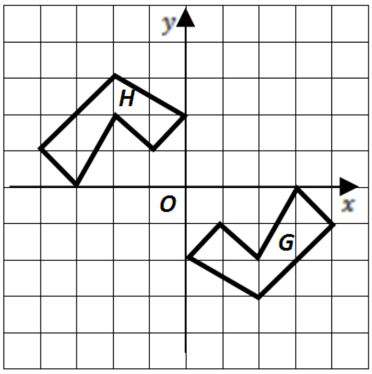


[QUESTIONS FOR INDEPENDENT PRACTICE]

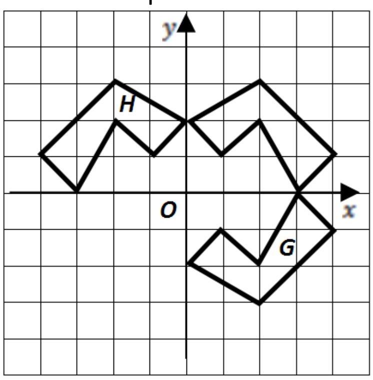
viii. Describe the transformation to shape G to get shape H.

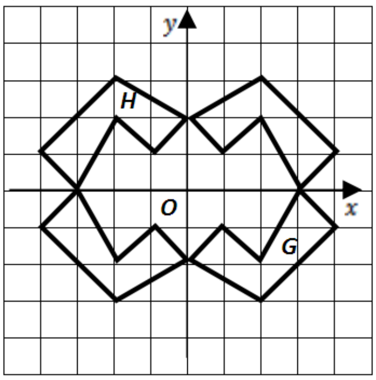
ix. Copy the shapes and complete the following transformations:
   - reflect G in the x-axis
   - reflect H in the y-axis

What do you notice?
Describe Any transformation that will complete the shape.



(Answer: d. to get shape H: rotate G 180 ^{o} around the origin; e. transformations shown below; both G and H end up in the same position after transformation, reflecting G in the x-axis gives the same result as rotating it 180 ^{o} around the origin, then reflecting the resultant shape in the y-axis.
To complete the shape: reflect G in the y-axis OR reflect H in the x-axis OR more.
Accept all reasonable answers.)





Lesson Title: Length Measurement of 2 Congruent Shapes	**Theme:** Geometry	
Lesson Number: M-09-050	**Class/Level:** JSS 3	**Time:** 35 minutes

Learning Outcomes By the end of the lesson, pupils will be able to recognise that length measurements of congruent shapes are maintained.	**Teaching Aids** None	**Preparation** 1. Draw the diagrams for the different sections of the lesson found at the end of this lesson plan on the board. 2. Write the vocabulary list on the board: corresponding sides.

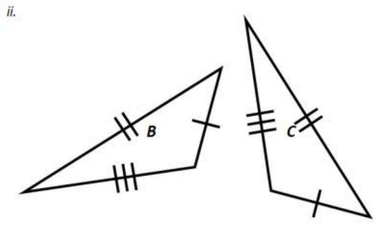
Opening (*3 minutes*)

1. **Say**: Look at the right-angled triangle labelled A on the board. Match 2 congruent triangles of shape A by the same edge to make a new shape.
   An example has been done for you.
2. Allow enough time for pupils to draw one new shape.
3. Have pupils from around the classroom volunteer to present their shapes on the board. (Answer: shown below the question at the end of the lesson plan)
4. **Say**: Today we are going to learn that length measurements of congruent shapes are maintained.

Introduction to the New Material (*10 minutes*)

1. **Ask:** What do the shapes we drew just now tell us about congruent shapes? Raise your hand.
2. Guide pupils to say: 'They are the same shape and size. They have sides which have the same length measurement.'
3. **Say**: Up to now, we have been relying on the fact that we are transforming the same shape to make other congruent shapes.
4. **Ask**: Suppose we have 2 shapes and we want to find out if they are congruent. What can we do? Raise your hand.
5. Guide a pupil to say: 'We can measure the sides to see if they are the same length; we can put the shapes on top of each other to see if they are the same shape.'
6. **Say**: Look at the triangles for Question b.
7. **Ask**: Who can read the lengths of the sides of triangle B? Raise your hand. (Answer: 3 cm, 5 cm, 7 cm)
8. **Ask**: Who can read the lengths of the sides of triangle C? Raise your hand. (Answer: 3 cm, 5 cm, 7 cm)
9. **Ask**: What can you say about the 2 triangles? Raise your hand. (Example answers: The 3 sides in each triangle have the same length measurements. They are congruent to each other.)
10. **Say:** This is one of the 4 tests we use to check if a triangle is congruent. If the lengths of all 3 sides of 2 triangles are equal, then the 2 triangles are congruent.
11. **Write** this on the board (or dictate) for pupils to copy: If the lengths of all 3 sides of 2 triangles are equal, then the 2 triangles are congruent.
12. **Say:** We refer to the sides as corresponding sides because we can match them with each other.

13. **Ask:** Who can show on the board which sides are the corresponding sides? Raise your hand.
14. Select a pupil to point out corresponding sides on each triangle.
15. **Say:** Corresponding sides are sides in the same position in the triangles. We use markings on the shape to show that the corresponding sides are the same length.
16. Draw the markings on the triangles to show how to indicate this.
17. **Say**: We use the same mark for each of the corresponding sides. This does not mean that the sides with 1 mark are shorter than the sides with 2 or 3 marks. It just means they are equal to each other. Note that if corresponding lengths of any shape, such as squares, are equal, the shapes are said to be congruent.



18. Allow time for pupils to copy the information on the board in their exercise books.
19. **Say**: Let us look at another test for congruency which applies only to right-angled triangles. If we have 2 right-angled triangles, we do not need to measure all the sides. If the lengths of the hypotenuse and a corresponding side of 2 right-angled triangles are equal, then the triangles are congruent.
20. **Ask:** How can we tell if the 2 right-angled triangles shown in Question b. are congruent? Raise your hand.
21. Guide a pupil to say: 'we apply the test for right angle, hypotenuse and one side; since the hypotenuse in both triangles are equal to each other and corresponding sides are also equal to each other, the 2 right-angled triangles are congruent.'
22. Repeat this, pointing to the appropriate sides in the 2 right-angled triangles.
23. **Say**: This is the second of our 4 tests we use to check if 2 triangles are congruent. We use it very often because right-angled triangles are found in many other shapes, for example rectangles or shapes made up of a combination of other shapes.
24. **Say:** The lengths of the third corresponding sides of the right-angled triangles are also equal. We can check this by measuring the lengths of both sides in the 2 triangles.
25. Mark the third pair of sides in the right-angled triangles to show they are also equal to each other.
26. **Write** this on the board (or dictate) for pupils to copy: If the lengths of the hypotenuse and a corresponding side of 2 right-angled triangles are equal, then the triangles are congruent. The lengths of the third corresponding sides of the right-angled triangles are also equal.

Guided Practice (*10 minutes*)

1. Ask pupils to work in pairs to answer Question d.
2. Walk around, if possible, to check answers and correct any misconceptions.
3. Have pupils from around the classroom volunteer to give their answers to the questions.
4. **Write** the correct answers on the board. Ask pupils to check their work. (Answers: shown below the questions at the end of this lesson plan)

Independent Practice (*10 minutes*)

1. Ask the pupils to work independently to answer Question e.
2. Walk around, if possible, to check answers and clear misconceptions.
3. Have pupils from around the classroom volunteer to give their answers to the questions.

4. **Write** the correct answers on the board. Ask pupils to check their work. (Answers: shown below the questions at the end of this lesson plan)

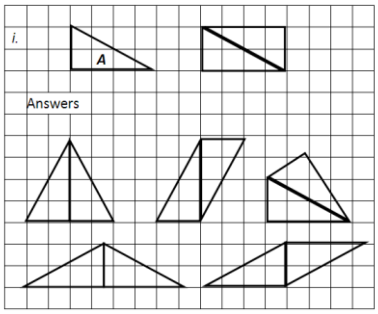
Closing (*2 minutes*)

1. **Ask**: What have we learnt today about congruent triangles? Raise your hand.
2. Allow time for pupils to discuss and share ideas. (Example answers: The lengths of corresponding sides in congruent triangles are equal; if the lengths of the hypotenuse of 2 right-angled triangles are equal, and any 2 of the corresponding sides are also equal, then the 2 right-angled triangles are congruent)
3. **Sa**y: Next lesson, we will look at angles within triangles and see how they can help us decide whether triangles and other objects are congruent.

[QUESTION FOR OPENING]

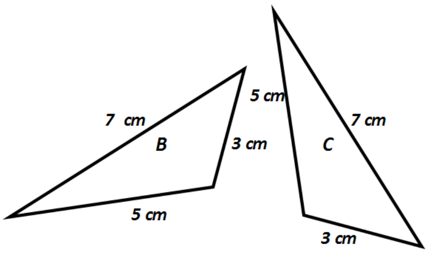
a. Match 2 triangles congruent to shape A by the same edge to make a new shape.

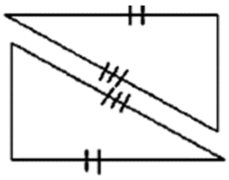
An example has been done for you.
(Answers: shown right).



[QUESTIONS FOR INTRODUCTION TO THE NEW MATERIAL]

b. What do you notice about triangles B and C?
c. Show that the 2 right-angled triangles shown below are congruent.





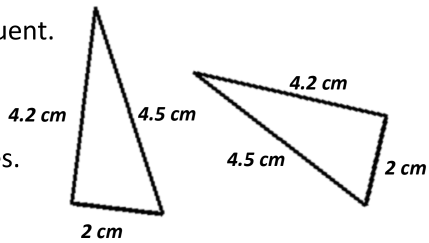
(Note: The third side will be marked during the lesson.)

[QUESTIONS FOR GUIDED PRACTICE]

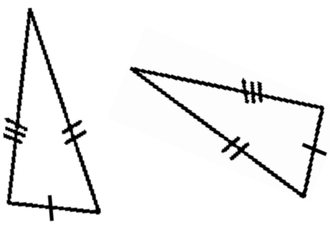
d. i. State whether the 2 triangles shown right are congruent.

ii. Give a reason for your answer.

ii. Copy the triangles and mark any corresponding sides.

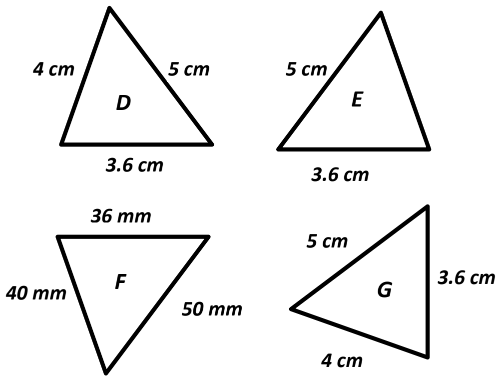


(Answer: the 2 triangles are congruent. All 3 sides are equal, corresponding sides shown marked right.



[QUESTIONS FOR INDEPENDENT PRACTICE]

e. Which of the shapes shown right are congruent to triangle D?
(Answer: triangles F and G are congruent to triangle D; only 2 sides are given for triangle E so we do not have enough information to know whether triangle E is also congruent).
Note: It is important pupils do not assume congruency. They must check the shapes using the 2 tests learnt in this lesson and the 2 they will learn about in the next lesson.



Lesson Title: Angles of Congruent Shapes	**Theme**: Geometry	
Lesson Number: M-09-051	**Class/Level**: JSS 3	**Time**: 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to recognise that angle measurements of congruent shapes are maintained.	None	1. Draw the diagrams for the different sections of the lesson found at the end of this lesson plan on the board.

Opening (_3 minutes_)

1. **Ask**: Who can remind the class what we did last lesson? Raise your hand.
2. Select a pupil to answer: (Example answers: We looked at congruent shapes and found out that length measurements are maintained; If 3 sides of a triangle are equal the triangle is congruent. Accept all reasonable answers)
3. **Ask:** How do you know that 2 right-angled triangles are congruent? Raise your hand. (Example answer: The hypotenuses of both right-angled triangles are the same lengths and the lengths of either one of the shorter sides are also the same.
4. **Say**: Today we are going to learn that angle measurements of congruent shapes are maintained.

Introduction to the New Material (_10 minutes_)

1. **Say:** Let us do a little experiment. We are each going to draw a triangle from these instructions. Find a clean page in your exercise books. Draw a line 4 cm long. It can be at any angle but it must not be horizontal or vertical.
2. Draw a few lines using different slopes (positive and negative) as guidance on the board.
3. **Say**: Draw another line 6 cm long at any angle from one end of your first line. Again, no horizontal or vertical line.
4. **Say**: Draw a third line to complete the triangle. Measure this line.
5. Allow time for pupils to follow these instructions. Repeat them if necessary.
6. **Say**: We are going to list the measurements of the third line we used to complete our triangles.
7. Have 4-5 pupils volunteer to give their measurements for the third side.
8. Make sure there is a good range of measurements.
9. **Say**: Compare your diagram with that of your neighbours on either side of you.
10. **Ask**: What do you notice about your diagrams? Raise your hand.
11. Guide pupils to say: The shapes are different.
12. **Say**: We all started with 2 sides the same lengths, but ended with triangles which had different shapes.
13. **Ask**: Can anyone think of a reason why this happened? Raise your hand.
14. Allow pupils time to discuss and share ideas.
15. Guide a pupil to say: ‘The shapes were different because the angles between the 2 sides we first drew were all different.’
16. **Say**: This gives us a very important test to add to the 2 we found out about last lesson. If the lengths of 2 sides and the angle between them in 2 triangles are equal, then the triangles are congruent.

17. **Say**: The angle between them is called the included angle. We do not need to know the length of the third pair of corresponding sides if we know 2 sides and the included angle for each triangle.
18. **Ask:** Look at Question a. on the board. How can we tell if the triangles are congruent? Raise your hand.
19. Guide a pupil to say: we apply the test for 2 sides and the included angle; since they are equal, the triangles are congruent.
20. **Say**: There is one last test that we carry out to find out if 2 triangles are congruent.
21. **Ask**: Look at the triangles in Question b. What do you notice? Raise your hand.
22. Guide a pupil to say: 2 angles and 1 side are the same in both triangles.
23. **Say:** If 2 angles and the length of one side are equal in 2 triangles, then the triangles are congruent. These angles are referred to as corresponding angles. They are marked as shown with the arcs to show they are equal. The 4 tests we looked at last lesson and today are used to test whether 2 (or more) triangles are congruent.

Guided Practice (*10 minutes*)

1. Ask pupils to work in pairs to answer Questions c., d. and e.
2. Walk around, if possible, to check answers and correct any misconceptions.
3. Have pupils from around the classroom volunteer to give their answers to the questions.
4. **Write** the correct answers on the board. Ask pupils to check their work. (Answers: Shown below the questions at the end of this lesson plan)

Independent Practice (*10 minutes*)

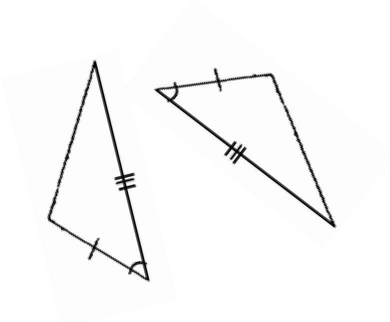
1. Ask the pupils to work independently to answer Question f.
2. Walk around, if possible, to check answers and clear up any misconceptions.
3. Have pupils from around the classroom volunteer to give their answers to the questions.
4. **Write** the correct answers on the board. Ask pupils to check their work. (Answers: Shown below the questions at the end of this lesson plan)

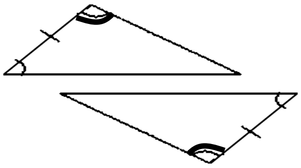
Closing (*2 minutes*)

1. **Say**: Look through your notes from last lesson and today.
2. **Ask**: What are the 4 tests we found out for triangles to be congruent?
3. Allow pupils to discuss and share ideas.
4. Have pupils from around the classroom volunteer to answer.
5. **Write** (or dictate) the tests on the board with appropriate diagrams:
    - If the lengths of all 3 sides of 2 triangles are equal, then the 2 triangles are congruent.
    - If the lengths of the hypotenuse and a corresponding side of 2 right-angled triangles are equal, then the triangles are congruent.
    - If the lengths of 2 sides and the angle between them in 2 triangles are equal, then the triangles are congruent.
    - If 2 angles and the length of one side are equal in 2 triangles, then the triangles are congruent.
6. Ask pupils to copy them into their exercise books for reference.
7. **Say**: These tests with some modifications can be used to test other shapes apart from triangles are congruent.

[QUESTIONS FOR INTRODUCTION TO THE NEW MATERIALS]

a. Show that the 2 triangles shown right are congruent.
b. Show that the 2 triangles shown below are congruent.

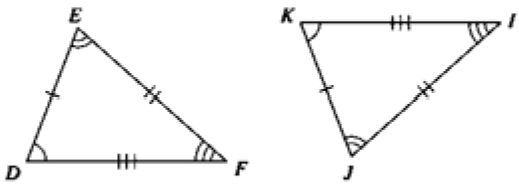




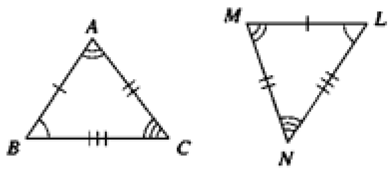
[QUESTIONS FOR GUIDED PRACTICE]

c. Show that the 2 triangles are congruent by naming the corresponding sides and angles

i.



ii.



(Answers: c. i. $ED = JK, EF = JI, DF = KI, \angle EDF = \angle JKI, \angle DEF = \angle KJI, \angle DFE = \angle KIJ$
ii. $AB = ML, AC = MN, BC = LN, \angle ABC = \angle MLN, \angle BAC = \angle LMN, \angle ACB = \angle MNL$)

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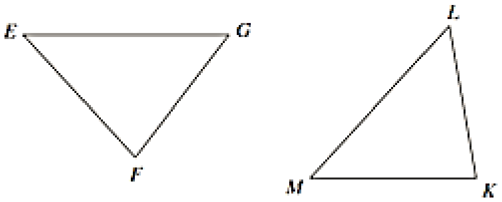
d. Mark the angles and sides of each pair of triangles to indicate that they are congruent.

i.



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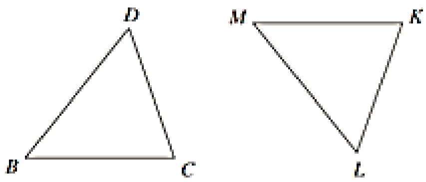
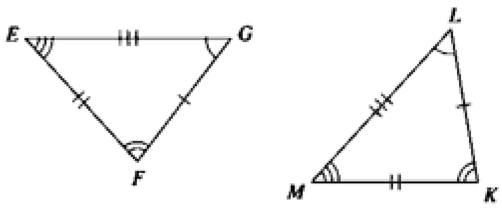


(Answers: d. i. and d. ii. Shown on triangles below)

d. i.



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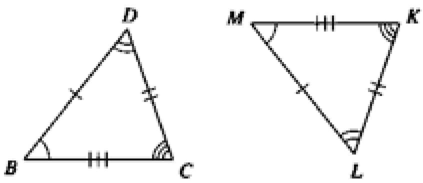
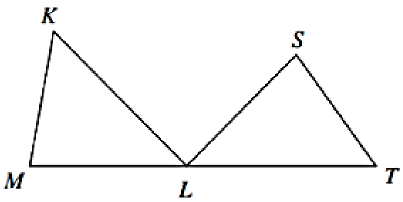


[QUESTIONS FOR INDEPENDENT PRACTICE]

f. Mark the angles and sides of each pair of triangles to indicate that they are congruent.

i.



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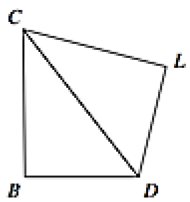
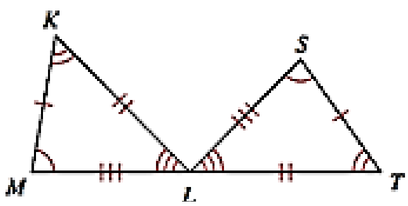


Answers
f. i.



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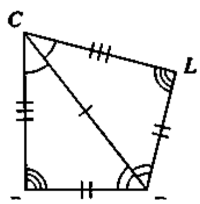


Lesson Title: Enlargement	**Theme**: Geometry	
Lesson Number: M-09-052	**Class/Level**: JSS 3	**Time**: 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to: 1. Identify that enlargement creates an object of the same shape, but a different size. 2. Recognise and perform enlargement.	None	1. Draw the diagrams for the different sections of the lesson found at the end of this lesson plan on the board. 2. Write the Vocabulary list on the board: centre of enlargement, scale factor.

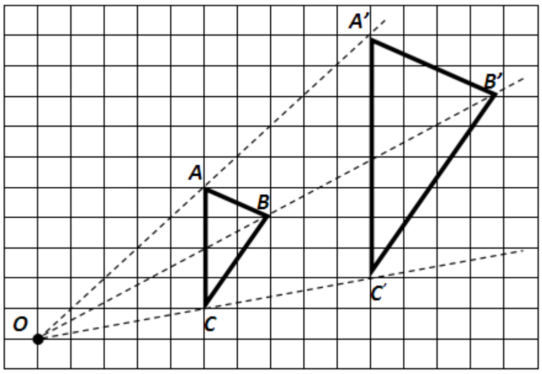
Opening (*3 minutes*)

1. **Ask:** Who can remind the class of the 3 transformations we have done so far? Raise your hand.
2. Have a pupil from the back of the classroom volunteer to answer. (Answer: translation, reflection and rotation)
3. **Ask**: What is common to the object undergoing any of the 3 transformations? Raise your hand.
4. Guide a pupil to say: The shape and size of the object does not change; the object changes its position; the objects before and after the transformation are congruent.
5. **Say**: Today we are going to identify that enlargement creates an object of the same shape, but a different size. We will also recognise and perform enlargement.

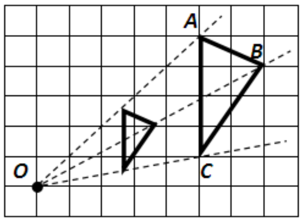
Introduction to the New Material (*10 minutes*)

1. **Ask:** What do you understand by the word 'enlarge'?
2. Have pupils from around the classroom volunteer to answer. (Example answers: To become bigger; the size of an object becomes larger)
3. **Say:** In everyday language when we say 'enlarge', we just mean to make something bigger. Both the shape and the size of the object may change. When we say we 'enlarge' an object in maths, we mean that we increase or decrease its size without changing its shape. We use the word 'enlargement' to describe the transformation. The size of the final object will depend on the scale factor that we use.
4. Ask a pupil to read Question a. on the board.
5. **Say**: Copy the diagram into your exercise books. We will enlarge the shape together.
6. The instructions to enlarge an object follow.
   - **Say:** We are given 2 pieces of information in order to enlarge our shape: the scale factor and the centre of rotation.
   - **Ask**: What is the scale factor for this enlargement? Raise your hand. (Answer: 2)
   - **Ask**: What do you think the scale factor is telling us to do? Raise your hand. (Example answers: To double the shape; To make the shape twice as big)
   - **Ask**: What is the centre of enlargement? Raise your hand. (Answer: Point O)
   - **Say**: The centre of enlargement is the point of reference from which we draw the enlarged shape.
   - **Say**: Draw a straight line from the centre of enlargement through one of the points, e.g. A.
   - **Say:** Measure the distance from point O to point A.

- Encourage the pupils to be accurate in their measurements in order to get an accurate result. They can check with each other that they have roughly the same measurement.
- **Say**: We will multiply the distance by the scale factor, in this case by 2.
- **Say**: Measure the distance from O on the line and mark the new distance on the point A prime (A').
- **Say**: This is the new point of the object after enlargement. Note that $OA' = 2 \times OA$.
- **Say**: Do the same for the other points.
- **Say**: Join the points to draw the shape at its new position (shown right).



7. Allow time for the pupils to complete the enlargement.
8. **Say**: What can you say about the lengths of the sides of the triangle after the enlargement?
9. Guide a pupil to say: The length of each side doubles after enlargement.
10. **Say:** We multiplied the distance from O to each of our original points by the scale factor of 2 so everything doubled.
11. **Ask:** What will happen when the scale factor is $\frac{1}{2}$? Raise your hand. (Answer: all the sides will be halved)
12. **Say:** Let us do the enlargement in part ii. of Question a. and see what happens.
13. Ask pupils to make a fresh copy of the triangle and mark the point O.
14. Go through the procedure again and ask the pupils to follow in their exercise books.
15. Allow pupils to work together to check their measurements.
16. Allow pupils time to finish their enlargements (shown right).
17. **Ask**: What has happened to the length of the sides of the triangle? Raise your hand. (Answer: each side has halved in size).
18. **Say**: The object will always increase or decrease by a multiple given by the scale factor. We know we have done the enlargement correctly when we measure the side and it has increased or decreased by the scale factor.
19. Ask pupils to measure 1 or 2 sides to verify this.



Guided Practice (*10 minutes*)

1. Ask pupils to continue to work in pairs to answer Question b.
2. Walk around, if possible, to check answers and correct any misconceptions.
3. Have pupils from around the classroom volunteer to share their answers on the board.
4. Correct any errors. (Answers: shown below the questions at the end of this lesson plan)

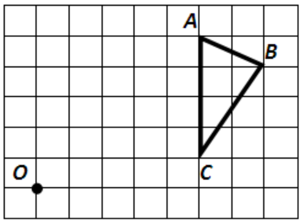
Independent Practice (*10 minutes*)

1. Ask the pupils to work independently to answer Question c.
2. Walk around, if possible, to check answers and clear misconceptions.
3. Have pupils from around the classroom volunteer to share their answers with the class.
4. Correct any errors in the solution on the board. Ask pupils to check their work. (Answers: shown below the questions at the end of this lesson plan).

Closing (*2 minutes*)

1. **Ask**: Write down everything you have found out during this lesson about enlargement that is similar or different to the other transformations?
2. Allow time for pupils to write down their answers.
3. Have pupils from around the classroom volunteer to share their answers with the class. (Example answers: enlargement makes an object larger or smaller depending on the scale factor, the other transformations only change the position; the shapes before and after an enlargement are not congruent; enlargements change both the position and the size of an object, but the object does not change its shape; none of the 4 transformations changes the shape of an object).
4. **Say**: Translation, reflection and rotation all result in congruent shapes. Enlargements result in a shape that is similar in shape to the original object but different in size. We will look at what we mean by 'similar objects' in more detail next lesson.

[QUESTIONS FOR INTRODUCTION TO THE NEW MATERIAL]

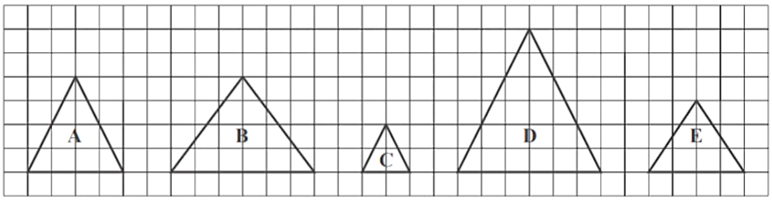


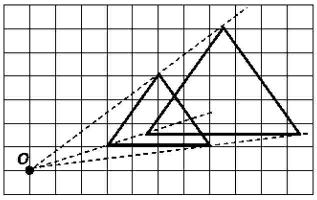
x. Draw enlargements to the triangle ABC with scale factors:

i. 2 ii. $\frac{1}{2}$

[QUESTIONS FOR GUIDED PRACTICE]

xi. i. Which of the triangles shown below are enlargements of shape A?
ii. State the scale factor of each enlargement.
iii. Draw an enlargement of shape E with scale factor $1\frac{1}{2}$.

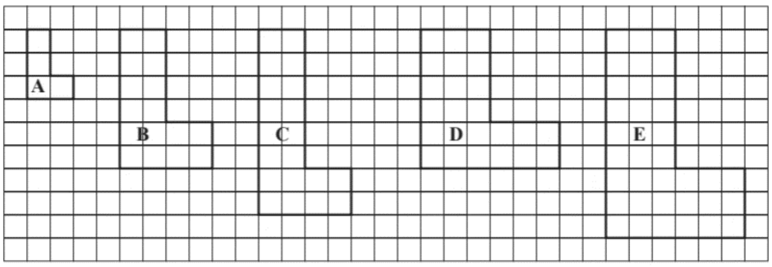


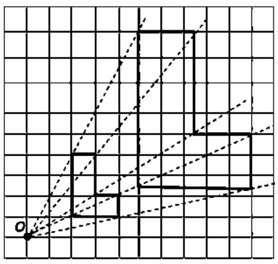


(Answers: i. enlargement; ii. (scale factor): C ($\frac{1}{2}$); D ($1\frac{1}{2}$); iii. enlargement shown below).

[QUESTIONS FOR INDEPENDENT PRACTICE]

xii. i. Which of the shapes shown below are <u>not</u> enlargements of shape A?
ii. Draw an enlargement of shape A with scale factor $2\frac{1}{2}$.





(Answers: i. B and C are not enlargements; ii. requested enlargement shown above).

Lesson Title: Similarity	Theme: Geometry	
Lesson Number: M-09-053	**Class/Level**: JSS 3	**Time**: 35 minutes

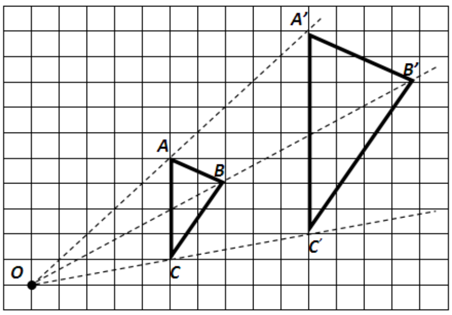
Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to identify that enlarged shapes are similar because angles are preserved but lengths are not.	None	1. Draw the diagrams for the different sections of the lesson found at the end of this lesson plan on the board. 2. Write the Vocabulary List on the board: similar.

Opening (*3 minutes*)

1. **Say:** Write down 2 things we discovered last lesson about shapes that have been enlarged.
2. Allow pupils time to write down their answer.
3. Have pupils from around the classroom volunteer to give their answers. (Example answers: Enlarged shape can increase or decrease in size depending on the scale factor; Enlarged shapes are not congruent because they change size; Enlarged shapes have the same shape as each other but have different sizes)
4. **Say**: Today we are going to identify that enlarged shapes are similar because angles are preserved but lengths are not.

Introduction to the New Material (*10 minutes*)

1. **Ask:** When you hear the word 'similar', what comes to mind?
2. Have pupils from around the classroom volunteer to answer. (Example answers: Things that look alike; Things that have something in common)
3. **Say:** We learnt last lesson that 'enlarge' has a special meaning in maths. 'Similar' also has a special meaning. When we say objects are 'similar' in Maths, we mean that they have the same shape but have different sizes.
4. **Ask**: What do we do to an object that causes it to have the same shape but a different size? Raise your hand.
5. Guide a pupil to say: Enlargement, because after the object has been enlarged it keeps its shape but changes its size.
6. **Say**: Similar objects are enlargements of each other. We can increase or decrease any one of them and we will get another one like it. Look back in your exercise books at the enlargements we did last lesson.
7. Refer to the diagram on the right from the last lesson.
8. **Say**: Notice how the enlarged shape A'B'C' have the same shape as ABC.
9. **Ask:** What do you notice about lengths of the sides in triangle A'B'C'? Raise your hand. (Answer: They all doubled in size)
10. **Ask:** What do you notice about the angles in both triangles? Raise your hand.
11. Guide a pupil to say: The angles have stayed the same.
12. **Say**: In enlargements, the lengths of the sides of the shape changes according to the scale factor. The angles always stay the same. If we know the lengths of the sides of one object and we also



know the scale factor by which it was enlarged, we can find the lengths of the enlarged object. The reverse also works. We can use the same method to find the lengths of the original shape.

13. Ask a pupil to read Question a.
14. **Ask**: Look at the lengths of sides AB and DE of triangle ABC. What do you notice?
15. Guide pupils to say: We can find the scale factor by comparing the lengths of corresponding sides DE and AB.
16. **Say**: We can do this by comparing the ratio of corresponding sides $\frac{\text{DE}}{\text{AB}} = \frac{4.5}{3} = 1.5$. This gives us $\text{DE} = 1.5 \times \text{AB}$.
17. **Ask**: Will all the ratio of the lengths of the sides of DEF always be 1.5 times the lengths of the sides of ABC? Raise your hand. (Answer: Yes)
18. Do a visual check that pupils recognise this fact.
19. **Say:** Use that fact to find the lengths of BC and DE.
20. Allow time for pupils to do this. Allow them to discuss and share ideas with each other.
21. Have 2 pupils volunteer to explain on the board how to calculate the answers.
22. Correct any errors in the solution on the board. Ask pupils to check their work. Example calculations are shown below (the simpler calculation has been done first).

$$\begin{aligned} \text{DF} &= 1.5 \times AC \\ &= 1.5 \times 5 \\ &= 7.5 \text{ cm} \end{aligned} \qquad \begin{aligned} \text{EF} &= 1.5 \times AC \\ 6 &= 1.5 \times AC \\ \text{AC} &= \frac{6}{1.5} \\ &= 4 \text{ cm} \end{aligned}$$

23. **Say**: All calculations to find the lengths of missing sides of similar triangles or any other shape are done in a similar way. We use the fact that the ratios of corresponding sides are equal.

Guided Practice (*10 minutes*)

1. Ask pupils to continue to work in pairs to answer Questions b. and c.
2. Walk around, if possible, to check answers and correct any misconceptions.
3. Have pupils from around the classroom volunteer to give their answers to the questions.
4. **Write** the correct answers on the board. Ask pupils to check their work. (Answers: Shown below the questions at the end of this lesson plan)

Independent Practice (*10 minutes*)

1. Ask the pupils to work independently to answer Question d.
2. Walk around, if possible, to check answers and clear up any misconceptions.
3. Have pupils from around the classroom volunteer to give their answers to the questions.
4. **Write** the correct answers on the board. Ask pupils to check their work. (Answers: Shown below the questions at the end of this lesson plan)

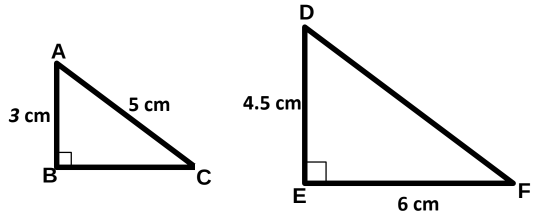
Closing (*2 minutes*)

5. **Say**: Add what you have learnt this lesson about similar shapes to your notes on enlargements from the beginning of the lesson.
6. Allow time for pupils to do this.

7. Have 2-3 pupils volunteer to share their notes with the class. (Example answers: All enlarged shapes are similar; Lengths of sides in similar shapes change, but the angles stay the same; The ratio of corresponding sides are equal)
8. **Say:** For any pair of similar shapes, corresponding **sides** are in the **same ratio** and corresponding **angles** are **equal**.

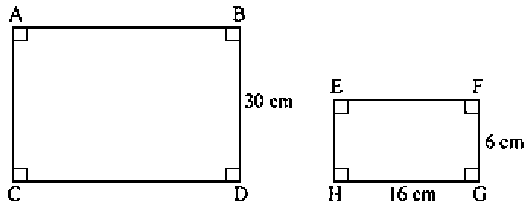
[QUESTIONS FOR INTRODUCTION TO THE NEW MATERIAL]

xiii. The diagram shown right shows 2 similar right-angled triangles.
Calculate the lengths of sides BC and DE.

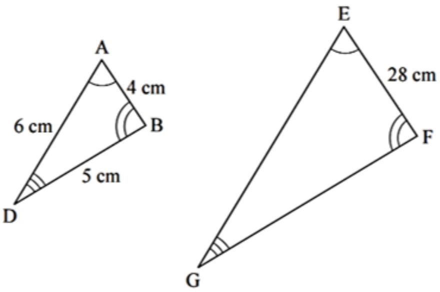


[QUESTIONS FOR GUIDED PRACTICE]

xiv. The diagram below shows 2 similar rectangles. Calculate the length of side CD.



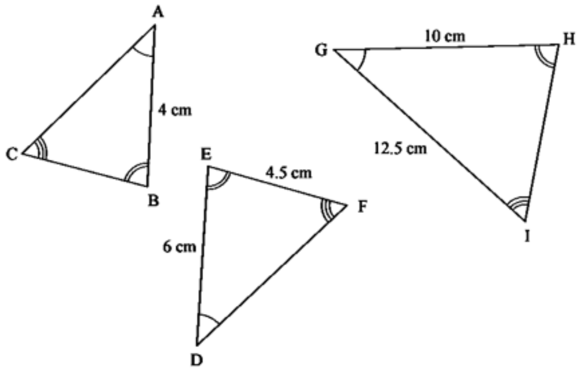
xv. Triangle ABD has been enlarged to give EFG.



i. Give reasons why the 2 triangles are similar.
ii. What is the scale factor for the enlargement?
iii. Calculate the lengths of sides GE and FG.

(Answers: b. $\frac{BD}{FG} = \frac{30}{6} = 5$; $CD = 5HG = 5 \times 16 = 80$ cm; c. i. the triangles are similar because:
$\angle ABD = \angle EFG$; $\angle BDA = \angle FGEG$; $\angle DAB = \angle GEF$;
ii. scale factor: $\frac{EF}{AB} = \frac{28}{4} = 7$; ii. $GE = 7DA = 7 \times 6 = 42$ cm, $FG = 7BD = 7 \times 5 = 35$ cm).

[QUESTIONS FOR INDEPENDENT PRACTICE]



xvi. Triangles ABC, DEF and GHI are shown right

i. State whether the triangles are similar. Give reasons for your answers.

ii. Calculate the lengths of sides HI, BC, AC and DF.

(Answers: d. i. The triangles are similar. $\angle ABC = \angle DEF = \angle GHI$; $\angle BCA = \angle EFD = \angle HIG$; $\angle CAB = \angle FDE = \angle IGH$

ii. Accept all reasonable methods. Example calculations are shown below.

$\frac{GH}{DE} = \frac{10}{6} = \frac{5}{3}$, $HI = \frac{5}{3}EF = \frac{5}{3} \times 4.5 = 7.5$ cm;

$\frac{DE}{AB} = \frac{6}{4} = 1.5, EF = 1.5BC, BC = \frac{4.5}{1.5} = 3$ cm;

$\frac{AC}{GI} = \frac{4}{10}, AC = \frac{4}{10} \times GI = 0.4 \times 12.5 = 5$ cm;

$\frac{DF}{GI} = \frac{6}{10}, DF = \frac{6}{10} \times 12.5 = 7.5$ cm)

Lesson Title: Comparing Congruent and Similar Shapes	Theme: Geometry	
Lesson Number: M-09-054	**Class/Level:** JSS 3	**Time:** 35 minutes

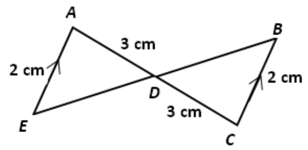
Learning Outcomes	Teaching Aids	Preparation
By the end of the lesson, pupils will be able to differentiate between congruency and similarity of shapes.	None	Draw the diagrams for the different sections of the lesson found at the end of this lesson plan on the board.

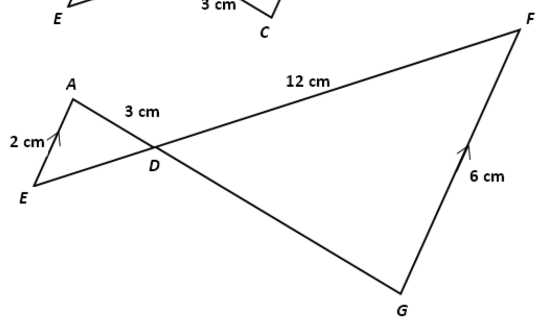
Opening (*3 minutes*)

1. **Ask**: Look at the shapes on the board. Which of the shapes shown are congruent and which are similar?
2. Allow time for pupils to inspect the shapes and write down their answers.
3. Have pupils from around the classroom volunteer to give their answers. (Answers: C, D and F are congruent to each other because they are the same shape and size, so are B and G; they are all similar to each other because corresponding sides are in the same ratio and corresponding angles are equal.)
4. Pupils might think that shapes of the same size cannot be similar, only congruent. Draw their attention to the everyday meaning of 'similar'.
5. **Say**: Similar means objects look alike. They can have the same or different size. Think of it as an enlargement with a scale factor of 1.
6. **Ask:** What fact did we use to find the lengths of sides in similar shapes? Raise your hand.
7. Guide a pupil to answer: The fact that the ratio of corresponding sides is equal.
8. **Ask:** What else is equal in similar shapes? Raise your hand. (Answer: Corresponding angles)
9. **Say**: Today we are going to differentiate between congruency and similarity of shapes.

Introduction to the New Material (*10 minutes*)

1. **Say:** We wish to compare congruent and similar triangles and see how they are alike or different.
2. Ask a pupil to read Question a.
3. **Say**: Since the 2 lines AE and BC are parallel, it means that $\angle \text{EAD} = \angle \text{BCD}$.
4. Mark these (z-angles) with single arcs.
5. **Ask:** Let us look back at our tests for congruency. Who can tell us which test can help us here? Raise your hand.
6. Guide a pupil to say: 2 sets of corresponding sides are equal and the included angle is also equal in both triangles.
7. **Ask:** What can we conclude about triangles ADE and CDB? Raise your hand.
8. Guide a pupil to say: They are congruent triangles.
9. **Say**: Let us now look at triangles ADE and GDF.
10. **Ask**: Who knows what type of angles $\angle \text{ADE and} \angle \text{GDF}$ are? Raise your hand.
11. Guide a pupil to say: They are equal and opposite angles.
12. Mark the 2 angles to show they are equal with double arcs.





13. **Say**: Since the 2 lines AE and FG are parallel, it means that $\angle \mathrm{EAD} = \angle \mathrm{FGD}$.
14. **Ask:** Which other angles are equal in the 2 triangles? Raise your hand.
15. Guide a pupil to say: $\angle \mathrm{DFG} = \angle \mathrm{DEA}$.
16. Mark these 2 angles with triple arcs.
17. **Say**: The 3 corresponding angles are equal in both triangles ADE and GDF.
18. **Ask**: What can we say about the 2 triangles? Raise your hand.
19. Guide a pupil to say: They are similar triangles. (Remember, similar triangles have corresponding angles equal).
20. **Say**: Triangles ADE and CDB are congruent and have 3 corresponding angles equal. Triangles ADE and GDF are similar and also have 3 corresponding angles equal.
21. **Ask**: What is different between the triangles? Raise your hand.
22. Allow pupils to discuss and share ideas.
23. Guide a pupil to say: The lengths of the sides are the same between ADE and CDB. They are different between triangles ADE and GDF.
24. **Say:** In congruent triangles, the lengths of all 3 sides are the same. In similar triangles, the lengths of the sides are in proportion according to the scale factor. This is also called the similarity ratio. We can find missing lengths using this ratio.
25. **Say:** Remember though that all congruent triangles are similar because they pass the test that the corresponding angles are equal. However, similar triangles are not congruent because they fail the test that the corresponding lengths are equal.

Guided Practice (*10 minutes*)

1. Ask pupils to work in pairs to answer Question c.
2. Walk around, if possible, to check answers and correct any misconceptions.
3. Have pupils from around the classroom volunteer to give their answers to the questions.
4. **Write** the correct answers on the board. Ask pupils to check their work. (Answers: Shown below the questions at the end of this lesson plan)

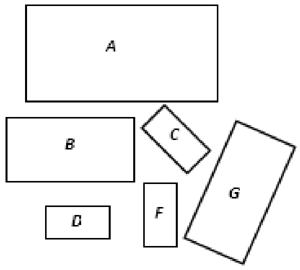
Independent Practice (*10 minutes*)

1. Ask the pupils to work independently to answer Question d.
2. Walk around, if possible, to check answers and clear up any misconceptions.
3. Have pupils from around the classroom volunteer to give their answers to the questions.
4. **Write** the correct answers on the board. Ask pupils to check their work. (Answers: Shown below the questions at the end of this lesson plan)

Closing (*2 minutes*)

1. Write down in your own words how you would tell the difference between congruent triangles and similar triangles.
2. Allow time for pupils to write down their answers.
3. Have pupils from around the classroom volunteer to answer. (Answer: In congruent triangles the lengths of all 3 sides are the same; In similar triangles the lengths of the sides are in proportion according to the scale factor or similarity ratio)

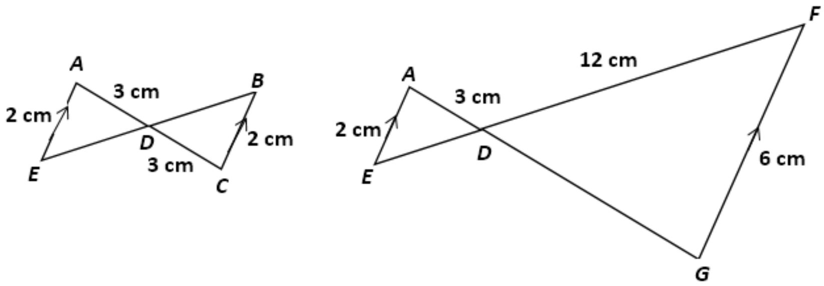
[QUESTIONS FOR OPENING]



xvii. Which of the shapes shown are congruent and which are similar? Give reasons for your answer.

[QUESTIONS FOR INTRODUCTION TO THE NEW MATERIAL]

xviii. In the diagram below, the sides AE, BC and FG are parallel.



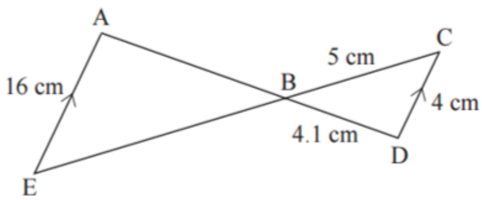
i. Explain why triangles ADE and CDB are congruent

ii. Explain why triangles ADE and GDF are similar

[QUESTIONS FOR GUIDED PRACTICE]

xix. In the diagram right, the lines AE and CD are parallel.

i. Copy and complete the following statements:
   - $\angle ABE = \angle$
   - $\angle BAE = \angle$
   - $\angle AEB = \angle$

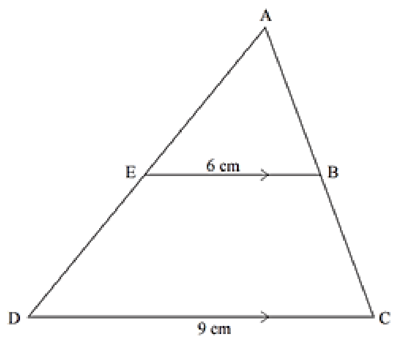


ii. State whether the triangles are similar or congruent. Give a reason for your answer.

iii. Calculate the lengths of AB and BE

(Answers: i. $\angle ABE = \angle DBC$, $\angle BAE = \angle BDC$, $\angle AEB = \angle BCD$
ii. the triangles are similar and not congruent because the corresponding lengths are not equal; iii. $\frac{AE}{CD} = \frac{16}{4} = 4, AB = 4BD = 4 \times 4.1 = 16.4$ cm; $BE = 4BC = 4 \times 5 = 20$ cm)

[QUESTIONS FOR INDEPENDENT PRACTICE]



xx. Lines BE and CD are parallel in the triangles shown right.

i. Explain why triangles ABE and ACD are similar

ii. If the length of AB is 4.4 cm, calculate the lengths of AC and BC.

iii. If the length of AD is 13.5 cm, determine the lengths of AE and DE.

(Answers: i. $\angle ADC = \angle AEB$, $\angle ACD = \angle ABE$
ii. $\frac{AC}{AB} = \frac{CD}{BE}, \frac{AC}{4.4} = \frac{9}{6} = \frac{3}{2}$, $AC = \frac{3}{2} \times 4.4 = 6.6$ cm, $BC = 6.6 - 4.4 = 2.2$ cm.
iii. $\frac{AD}{AE} = \frac{3}{2}, AE = \frac{2}{3}AD = \frac{2}{3} \times 13.5 = 9$ cm; $DE = 13.5 - 9 = 4.5$ cm)

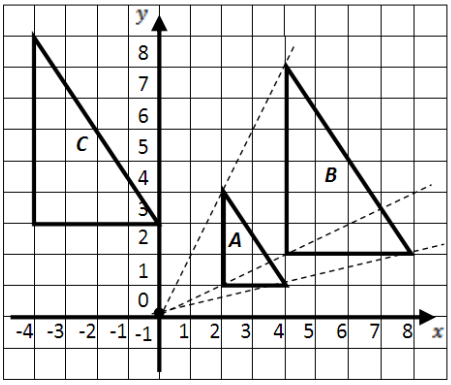
Lesson Title: Transformation Practice	**Theme:** Geometry	
Lesson Number: M-09-055	**Class/Level:** JSS 3	**Time:** 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to: 1. Carry out combinations of the 4 common transformations. 2. Identify shapes as either congruent or similar after carrying out a combination of transformations.	None	1. Draw the diagrams for the different sections of the lesson found at the end of this lesson plan on the board.

Opening (*3 minutes*)

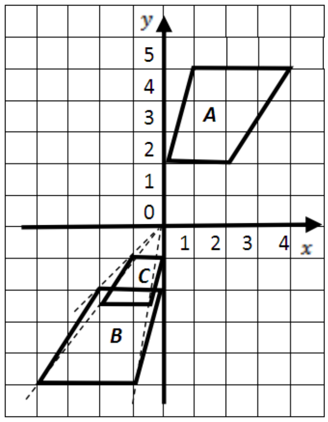
1. **Say**: We have done a lot of work on transformations, congruency and similarity. Let us see if we can predict the outcome of doing a combination of transformations before we actually start.
2. **Ask**: If we translate a shape and then enlarge the translated shape, do you think the final shape will be similar or congruent to the original shape? Raise your hand.
3. Allow pupils to discuss and share ideas.
4. Have a pupil volunteer to share their ideas with the class. (Answer: The final shape will be similar to the original shape.)
5. **Say**: We will see if our prediction is correct. Today we are going to carry out combinations of the 4 common transformations. We will identify shapes as either congruent or similar after carrying out a combination of transformations.

Introduction to the New Material (*10 minutes*)



1. **Say:** Ask a pupil to read Question a. on the board.
2. Allow time for pupils to copy the diagram in their exercise books.
3. **Ask:** Who can briefly remind the class how to do an enlargement with a scale factor of 2? Raise your hand. (Example answer: Draw lines from the centre of enlargement (O) through the points of the original shape. Measure the distances from O to the points and double it for the enlarged points.)
4. **Say:** Follow the instructions. Draw the enlarged shape.
5. Allow pupils time to do the enlargement.
6. Have a pupil volunteer to do the enlargement on the board.
7. Correct any errors in the solution on the board. Ask pupils to check their work.
8. Ask pupils to finish the instructions and complete the transformation by translating the triangle.
9. **Ask**: Look at our shapes, write down in your exercise books which shapes are congruent and which are similar.
10. Allow time for pupils to do this.

11. Have pupils volunteer to answer. (Answers: B and C are similar to A and congruent to each other).
12. **Say**: Let us look at another combination of transformations,
13. Ask a pupil to read Question b.
14. Ask pupils to work in pairs to complete the full transformation.
15. Select a pupil to show the transformation on the board (shown right).
16. Correct any errors in the solution on the board. Ask pupils to check their work.
17. **Ask**: In this combined transformation, which shapes are similar and which are congruent? Raise your hand. (Answer: A and B are congruent and they are both similar to C.)
18. **Ask:** What difference does it make expanding first before doing other transformations as opposed to doing the other transformations first and expanding last? Raise your hand.
19. Guide pupils to say: Expanding first makes the shapes all similar to the original and congruent with each other. Expanding last makes all the shapes, except the last one, congruent with each other; The last shape will be similar to all the other shapes including the original.
20. **Say:** We will see if this remains true when we do a few more combined transformations.



Guided Practice (*10 minutes*)

1. Ask pupils to continue to work in pairs to answer Question c.
2. Walk around, if possible, to check answers and correct any misconceptions.
3. Have pupils from around the classroom volunteer to give their answers on the board.
4. Correct any errors in the solution on the board. Ask pupils to check their work. (Answers: Shown below the questions at the end of this lesson plan)

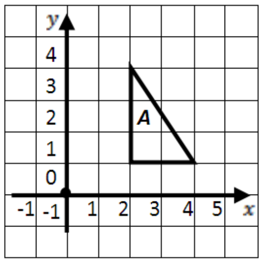
Independent Practice (*10 minutes*)

1. Ask the pupils to continue to work in pairs to discuss and share ideas.
2. Ask pupils to follow the instructions to draw and transform each shape.
   a. Draw x- and y-axes and mark them from -7 to $+7$
   b. i. Draw a 4-sided irregular shape (i.e. with unequal sides and angles).
      ii. Choose how to translate then enlarge the shape.
   c. i. Draw a 5-sided irregular shape
      ii. Choose how to enlarge then reflect or rotate the shape
3. Allow time for pupils to draw their shapes and perform the transformations.
4. Ask pupils to comment on the shape and size of their final objects.
5. Walk around, if possible, to check answers and clear up any misconceptions.
6. Have pairs from around the classroom volunteer to present their shapes on the board.
7. Ask the class whether they agree that the shapes are congruent or similar. Correct any errors in the solution on the board. Ask pupils to check their work. (Answers: Various shapes which should show congruency for translation, reflection and rotation, and similarity for enlargement)

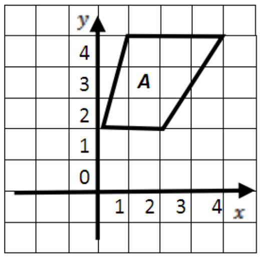
Closing (*2 minutes*)

1. **Ask:** What can we say about the shapes we get when we do a series of transformations? Raise your hand. (Example answers: Expanding first makes the shapes all similar to the original and congruent with each other; Expanding last makes all the shapes, except the last one, congruent with each other; the last shape will be similar to all the other shapes including the original)
2. **Say**: We can now say this with more confidence than before because we have shown it with all the transformations we did today.
3. **Ask**: Can we conclude that this will always be the case? Raise your hand.
4. Guide a pupil to say: Tes, because when we think of the results we get after each type of transformation, this will always be the case.

[QUESTIONS FOR INTRODUCTION TO THE NEW MATERIAL]

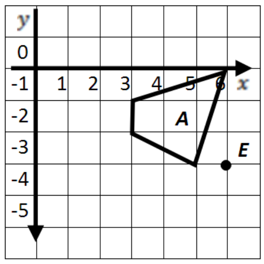


xxi. Perform the following combined transformation on the triangle (A) shown in the diagram below:
   i. Enlarge with a scale factor of 2 using the origin as the centre of enlargement (B).
   ii. Translate 8 units up and I unit left (C).
   iii. Are the final shapes congruent or similar?

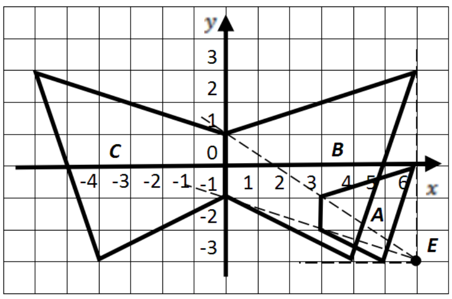


xxii. i. Rotate the shape (A) shown right 180 ^{o} about the origin to give B.
   ii. Enlarge the result by a scale factor of $\frac{1}{2}$ with centre of enlargement at the origin to give C.
   ii. Make a comment about the resulting shapes.
   (Answers: i. and ii. shown below; iii. A and B are congruent, C is similar to both A and B)

[QUESTIONS FOR GUIDED PRACTICE]



c. i. Enlarge the shape (A) shown right by a scale factor of 2, with centre of enlargement at E, to give B.
   ii. Reflect the result in the y-axis to give C.
   iii. Make a comment about the resulting shapes.
   (Answers: i. and ii. shown below; iii. B and C are similar to A, and congruent to each other)



Lesson Title: Introduction to Trigonometry	**Theme**: Geometry	
Lesson Number: M-09-056	**Class/Level**: JSS 3	**Time:** 35 minutes

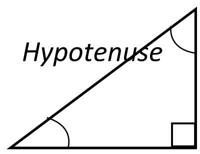
Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson pupils will be able to**:** 1. Identify the right and acute angles of a right-angled triangle. 2. Identify the relative sides of a right-angled triangle (adjacent, opposite, hypotenuse). 3. Identify SOHCAHTOA as a rule for remembering trigonometric ratios.	None	1. Draw the triangles for Guided and Independent Practice found at the end of this lesson plan on one side of the board.

Opening (*3 minutes*)

1. **Ask**: Who can remind the class of some of the properties of a right-angled triangle? You can look back on the notes you made during Pythagoras' Theorem.
2. Allow pupils to check their notes. Have pupils from around the classroom volunteer to answer. (Example answers: Right-angled triangles have one 90° angle; they have 2 acute angles which are also complementary angles as they add up to 90°; they have one long side called the hypotenuse and 2 shorter sides)
3. **Say**: Today we are going to identify the right and acute angles of a right-angled triangle. We will also identify the relative sides of a right-angled triangle and learn about SOHCAHTOA.

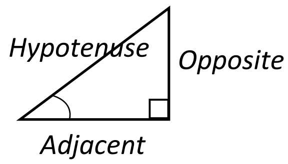
Introduction to the New Material (*10 minutes*)

1. Draw a right-angled triangle on the board with no labels or angles marked.
2. **Ask**: Who can identify the right angle and the hypotenuse on this triangle? Raise your hand.
3. Guide a pupil to come to the board to identify the 90° angle and the hypotenuse.
4. **Say**: The triangle also has 2 acute angles. Who can mark the acute angles on this triangle? Raise your hand.
5. Guide a pupil to mark the acute angles with arcs. (Answer: Complete answer shown at right)

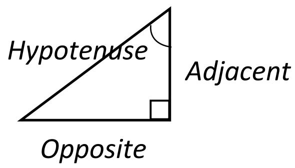


6. **Say:** When we talk about the sides of a right-angled triangle we like to be able to name each side. We already know the name of the longest side. It is called the hypotenuse.
7. **Ask**: Does anyone know what the other 2 sides of a right-angled triangle are called? Raise your hand.
8. Allow a few moments for pupils to think. Pupils may already know the terms 'opposite' and 'adjacent'. Select a pupil to answer. (Answer: Opposite and adjacent)
9. Pupils are often confused about the terms 'opposite' and 'adjacent' of a right-angled triangle. It is essential that they understand the terms are relative to a particular angle.
10. **Say**: The other 2 sides of the triangle are the 'opposite' and 'adjacent' sides. The 'opposite' and 'adjacent' sides of the triangle relate to a particular angle. The 'opposite' side is the side opposite the angle. The 'adjacent' side is the side next to the angle.

11. Draw the 2 right-angled triangles shown at right. Point to and label the angle and sides as you go through the following information.



12. **Say:** If we are interested in this angle, then this is the side opposite the angle, and this is the side adjacent, or next to the angle.
13. **Say:** If we are interested in this angle, then this is the side opposite the angle, and this is the side adjacent, or next to the angle.
    Allow a few moments for this difference to be understood.
    Repeat the information if necessary.

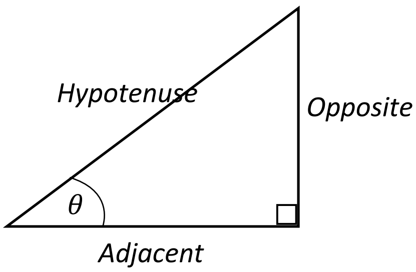


14. **Say**: We are now going to talk about trigonometric ratios which are very important in Maths, Science and Engineering. Trigonometric ratios are ratios of the sides of a right-angled triangle. We have a right-angled triangle labelled with its sides and one of its angle called θ (theta) as shown.
15. **Write** the trigonometric ratios and point to the relevant sides for each one.
16. **Say**: Here are the 3 basic trigonometric ratios:

$$\text{sine } \theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{O}{H}$$

$$\text{cosine } \theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{A}{H}$$

$$\text{tangent } \theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{O}{A}$$

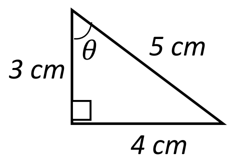


17. **Say**: We usually abbreviate sine, cosine and tangent to sin, cos and tan.
18. **Write** each abbreviation on the board next to its ratio. Note we pronounce sin in the same way as sine.
19. **Say**: We use the term SOHCAHTOA as a way of remembering the ratios.
    - SOH stands for sine equals opposite over hypotenuse
    - CAH stands for cosine equals adjacent over hypotenuse
    - TOA stands for tangent equals opposite over adjacent
20. Draw and label the triangle shown below on the board.
21. **Say:** Let us find the trigonometric ratios for θ in this right-angled triangle.
22. **Write** the trigonometric (trig) ratios. Explain each part of the formula and the sides to which they relate. Use SOHCAHTOA

$$\sin\theta = \frac{O}{H} = \frac{4}{5} = 0.8$$

$$\cos\theta = \frac{A}{H} = \frac{3}{5} = 0.6$$

$$\text{tang } \theta = \frac{O}{A} = \frac{4}{3}$$



23. **Say**: We will leave the tangent as a fraction because it is easier to work with like that.

Guided Practice (*10 minutes*)

1. Ask the pupils to work in pairs to answer Questions a. to d.
2. Walk around, if possible, to check their answers and clear up any misconceptions.
3. Have pupils from around the classroom volunteer to give their answers to the questions.

4. Correct any errors in the solution on the board. Ask pupils to check their work. (Answers: Shown next to the questions at the end of this lesson plan)

Independent Practice (*10 minutes*)

1. Ask the pupils to work independently to answer Questions e. and f.
2. Walk around, if possible, to check their answers and clear up any misconceptions.
3. Ask pupils to exchange their exercise books and check each other's work.
4. Have pupils from around the classroom volunteer to give their answers to the questions.
5. Correct any errors in the solution on the board. Ask pupils to check their answers. (Answers: Shown next to the questions at the end of this lesson plan)

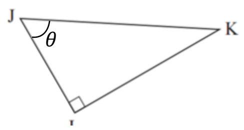
Closing (*2 minutes*)

1. **Say:** Please write down in pairs 2 different things you learned today.
2. Allow pupils 1 minute to discuss and share their ideas.
3. Have one pupil from the front, and one from the back of the classroom volunteer to answer. (Example answers: How to identify the hypotenuse, adjacent and opposite sides to angles in a triangle; How to calculate the trigonometric ratios; SOHCAHTOA)

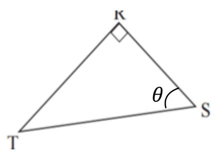
[QUESTIONS FOR GUIDED PRACTICE]

For each triangle, state which is the hypotenuse, the adjacent and the opposite side to the angle, θ.

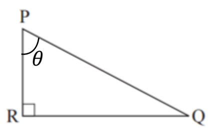
a.



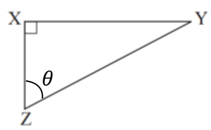
b.



c.



d.



Answers

a. Hypotenuse: JK
Adjacent: JL
Opposite: KL

b. Hypotenuse: ST
Adjacent: RS
Opposite: RT

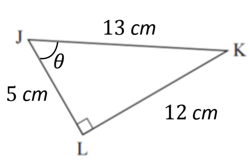
c. Hypotenuse: PQ
Adjacent: PR
Opposite: QR

d. Hypotenuse: YZ
Adjacent: XZ
Opposite: XY

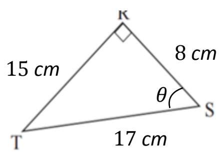
[QUESTIONS FOR INDEPENDENT PRACTICE]

For each triangle, write sin θ, cos θ and tan θ as fractions.

e.



f.



Answers:

e.
$\sin\theta = \frac{12}{13}$
$\cos\theta = \frac{5}{13}$
$\tan\theta = \frac{12}{5}$

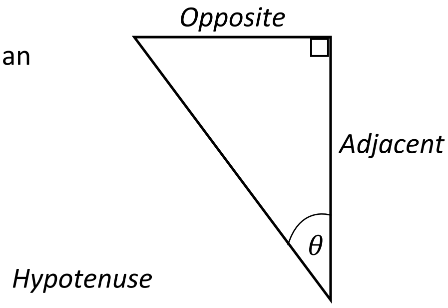
f.
$\sin\theta = \frac{15}{17}$
$\cos\theta = \frac{8}{17}$
$\tan\theta = \frac{15}{8}$

Lesson Title: Sine	**Theme**: Geometry	
Lesson Number: M-09-057	**Class/Level**: JSS 3	**Time:** 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to apply the sine ratio to solve for an unknown side.	None	1. Draw the triangles for the lesson found at the end of this lesson plan on one side of the board. 2. Write the table of sines and cosines of common angles given at the end on the board.

Opening (*3 minutes*)

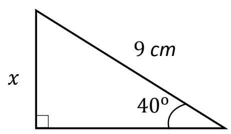
1. **Say**: Please turn to the back of your exercise books. Write the sine, cosine and tangent ratios for the triangle in Question a. and raise your hand when you finish.
2. Allow time for pupils to answer the question.
3. Have a pupil volunteer to come to the board and write the answers to the trig ratios.
4. Correct any errors in the solution on the board. Ask pupils to check their work. (Answers: Shown next to question at end of lesson plan)
5. **Say**: Today we are going to apply the sine ratio to solve for an unknown side.



Introduction to the New Material (*10 minutes*)

1. Draw the triangle shown on the right.
2. **Write** SOHCAHTOA on the board.
3. **Ask**: Who can remind the class what this stands for?
4. Have a pupil volunteer to give the answer shown below:
   - SOH stands for $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$
   - CAH stands for $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$
   - TOA stands for $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$
5. **Say**: If we look at any of the ratios, we see that each one has 3 terms in it. The angle and 2 of the sides of the triangle. If we know any 2 of the values, we can find the third value.
6. Ask a pupil to read Question b. on the board.
7. Show on the board how to work out the length of the side marked x. Use the value for $\sin 40^{\circ}$ found at the end of this lesson plan.

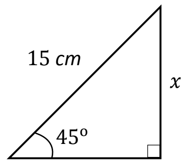
$$\sin\theta = \frac{\text{O}}{\text{H}}$$



$$\sin 40^{\circ} = \frac{\text{x}}{9}$$

$$9 \times \sin 40^{\circ} = \text{x}$$

$$9 \times 0.6428 = x$$
$$= 5.7852$$
$$x = 5.8 \text{ cm to 1 d.p.}$$



8. **Say**: We will do one more question together. You will then have the chance to practise with your neighbour.
9. Ask a pupil to read Question c. on the board.
10. **Ask**: Who would like to explain how to work out the length of the side marked x? Raise your hand.
11. Guide a pupil to do the calculations for Question c. Ask other pupils to observe carefully to see if they agree with the calculation.
12. Use the value for $\sin 45^{\circ}$ found at the end of this lesson plan.

$$\sin \theta = \frac{O}{H}$$
$$\sin 45^{\circ} = \frac{x}{15}$$
$$15 \times \sin 45^{\circ} = x$$
$$15 \times 0.7071 = x$$
$$= 10.6065$$
$$x = 10.6 \text{ cm to 1 d.p.}$$

Guided Practice (*10 minutes*)

1. Ask the pupils to work in pairs to answer Questions d. and e.
2. Walk around, if possible, to check their answers and clear up any misconceptions.
3. Have pupils from around the classroom volunteer to show the answer on the board.
4. Correct any errors in the solution on the board. Ask pupils to check their work. (Answers: Shown next to the questions at the end of this lesson plan)

Independent Practice (*10 minutes*)

1. Ask the pupils to work to answer Questions f. and g.
2. Walk around, if possible, to check their answers and clear up any misconceptions.
3. Ask pupils to exchange exercise books and check each other's work.
4. Have a pupil from around the classroom volunteer to show the answer to Question f. on the board.
5. Correct any errors in the solution on the board. Ask pupils to check their work. (Answers: Shown next to the questions at the end of this lesson plan) Do not do the answer for Questions g. Use it to check pupils' understanding of the work.

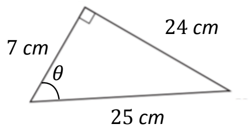
Closing (*2 minutes*)

1. **Say:** Please write your name on a piece of paper.

2. **Say**: Write your working out and answer for Question g. on the paper. Hand the paper in at the end of the lesson.
3. Check the work done by the pupils after the lesson. Use it as a guide to which pupils need additional assistance during the next lesson when pupils will be applying the cosine ratio to solve for an unknown side.

[QUESTIONS FOR THE OPENING ACTIVITY]

a. Write sin θ, cos θ and tan θ as fractions for the triangle shown

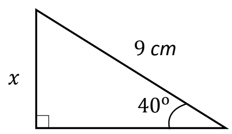


Answer:
$\sin\theta = \frac{24}{25}, \cos\theta = \frac{7}{25}, \tan\theta = \frac{24}{7}$

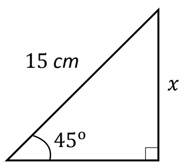
[QUESTIONS FOR INTRODUCTION TO THE NEW MATERIAL]

Use the formula for the sine to find the length of the side marked x in the triangle below.
Give your answers to 1 decimal place.

b.



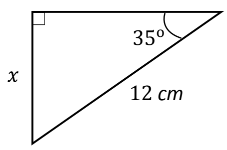
c.



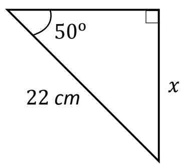
[QUESTIONS FOR GUIDED PRACTICE]

Use the formula for the sine to find the length of the side marked x in each of the triangles below.
Give your answers to 1 decimal place.

d.



e.

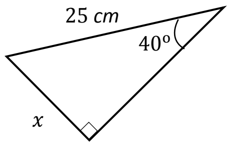


(Answers: d. $\sin 35^{o} = \frac{O}{H} = \frac{x}{12}$, $x = 12 \times \sin 35^{o} = 12 \times 0.5736 = 6.8832 = 6.9$ cm to 1 d.p.
e. $\sin 50^{o} = \frac{O}{H} = \frac{x}{22}$, $x = 22 \times \sin 50^{o} = 22 \times 0.7660 = 16.852 = 16.9$ cm to 1 d.p.)

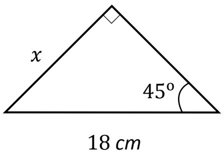
[QUESTIONS FOR INDEPENDENT PRACTICE]

Use the formula for the sine to find the length of the side marked x in each of the triangles below.
Give your answer to 1 decimal place.

f.



g.



(Answers: f. $\sin 40^o = \frac{O}{H} = \frac{x}{25}$, $x = 25 \times \sin 40^o = 25 \times 0.6428 = 16.07 = 16.1$ cm to 1 d.p.

g. $\sin 45^o = \frac{O}{H} = \frac{x}{18}$, $x = 18 \times \sin 45^o = 18 \times 0.7071 = 12.7278 = 12.7$ cm to 1 d.p.)

[SINES AND COSINES OF SOME COMMON ANGLES (TO 4 SIGNIFICANT FIGURES)]

Angle θ	sin θ	cos θ
30	0.5000	0.8660
35	0.5736	0.8192
40	0.6428	0.7660
45	0.7071	0.7071
50	0.7660	0.6428
55	0.8192	0.5736
60	0.8660	0.5000

Lesson Title: Cosine	**Theme**: Geometry	
Lesson Number: M-09-058	**Class/Level**: JSS 3	**Time:** 35 minutes

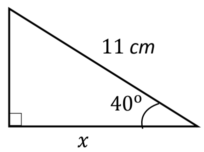
Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to apply the cosine ratio to solve for an unknown side.	None	1. Draw the triangles for the lesson found at the end of this lesson plan on one side of the board. 2. Write the table of sines and cosines of common angles given at the end.

Opening (*3 minutes*)

1. **Say**: In the last lesson, we applied the sine ratio to solve for an unknown side. Let us solve for x in the triangle in Question a. Please raise your hand when you finish.
2. Allow time for pupils to answer the question.
3. Have a pupil volunteer to write the solution on the board.
4. Correct any errors in the solution on the board. Ask pupils to check their work. (Answers: Shown next to the question at the end of the lesson plan).
5. **Say**: Today we are going to apply the cosine ratio to solve for an unknown side.

Introduction to the New Material (*10 minutes*)

1. Ask a pupil to read Question b. on the board.
2. Show on the board how to work out the length of the side marked x. Use the value for $\cos 40^o$ found at the end of this lesson plan.



$$\cos\theta = \frac{A}{H}$$

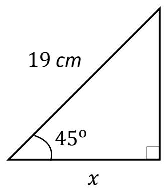
$$\cos 40^o = \frac{x}{11}$$

$$11 \times \cos 40^o = x$$

$$11 \times 0.7660 = x$$

$$= 8.426$$

$$x = 8.4 \text{ cm to 1 d.p.}$$



3. **Say**: We will do one more question together. You will then have the chance to practise with your neighbour.
4. Ask a pupil to read Question c. on the board.
5. **Ask**: Who would like to explain how to work out the length of the side marked x? Raise your hand.
6. Guide a pupil to do the calculations for Question c. Ask other pupils to observe carefully to see if they agree with the calculation.
7. Correct any errors in the solution on the board. Ask pupils to check their work. Use the value for $\cos 45^o$ found at the end of this lesson plan.

$$\cos\theta = \frac{A}{H}$$

$$\cos 45^o = \frac{x}{19}$$

$$19 \times \cos 45^o = x$$

$$19 \times 0.7071 = x$$

$$= 13.4349$$

$$x = 13.4 \text{ cm to 1 d.p.}$$

Guided Practice (*10 minutes*)

1. Ask pupils to work in pairs to answer Questions d. and e.
2. Walk around, if possible, to check their answers and clear up any misconceptions.
3. Have pupils from around the classroom volunteer to give their answers to the questions. Correct any errors in the solution on the board. Ask pupils to check their work. (Answers: Shown next to the questions at the end of this lesson plan)

Independent Practice (*10 minutes*)

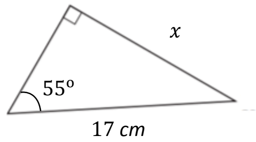
1. Ask the pupils to work independently to answer Questions f. and g.
2. Walk around, if possible, to check their answers and clear up any misconceptions.
3. Ask pupils to exchange exercise books and check each other's work.
4. Have a pupil from around the classroom volunteer to give their answers to Question f.
5. Correct any errors in the solution on the board. Ask pupils to check their work. (Answers: Shown next to the questions at the end of this lesson plan)
6. Do not do the answer for Questions g. Use it to check pupils' understanding of the work.

Closing (*2 minutes*)

1. **Say:** Please write your name on a piece of paper.
2. **Say**: Write your working out and answer for Questions g. on the paper. Hand the paper in at the end of the lesson.
3. Check the work done by the pupils after the lesson. Use it as a guide to which pupils need additional assistance during the next lesson when pupils will be applying the tangent ratio to solve for an unknown side.

[QUESTIONS FOR THE OPENING ACTIVITY]

b. Use the formula for the sine to find the length of the side marked x in the triangle below. Give your answers to 1 decimal place.



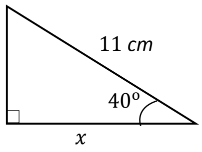
Answer:

$sin\,55^o = \frac{O}{H} = \frac{x}{17}$,
$x = 17 \times sin\,55^o = 17 \times 0.8192$
$= 13.9264 = 13.9\ cm$ to 1 d.p.

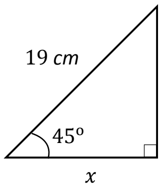
[QUESTIONS FOR INTRODUCTION TO THE NEW MATERIAL]

Use the formula for the cosine to find the length of the side marked x in the triangle below.
Give your answers to 1 decimal place.

e.



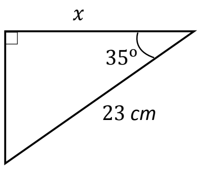
f.



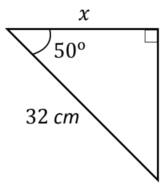
[QUESTIONS FOR GUIDED PRACTICE]

Use the formula for the cosine to find the length of the side marked x in each of the triangles below.
Give your answers to 1 decimal place.

i.



j

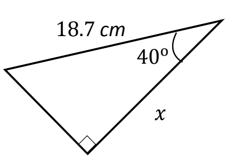


(Answers: d. $\cos 35^o = \frac{A}{H} = \frac{x}{23}$, $x = 23 \times \cos 35^o = 23 \times 0.8192 = 18.8416 = 18.8$ cm to 1 d.p.
e. $\cos 50^o = \frac{A}{H} = \frac{x}{32}$, $x = 32 \times \cos 50^o = 32 \times 0.6428 = 20.5696 = 20.6$ cm to 1 d.p.)

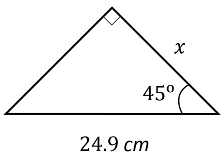
[QUESTIONS FOR INDEPENDENT PRACTICE]

Use the formula for the cosine to find the length of the side marked x in each of the triangles below.
Give your answer to 1 decimal place.

m.



n



(Answers: f. $\cos 40^o = \frac{A}{H} = \frac{x}{18.7}$, $x = 18.7 \times \cos 40^o = 18.7 \times 0.7660$
$= 14.3242 = 14.3$ cm to 1 d.p.
g. $\cos 45^o = \frac{A}{H} = \frac{x}{24.9}$, $x = 24.9 \times \cos 45^o = 24.9 \times 0.7071 = 17.60679 = 17.6$ cm to 1 d.p.)

[SINES AND COSINES OF SOME COMMON ANGLES (TO 4 SIGNIFICANT FIGURES)]

Angle θ	sin θ	cos θ
30	0.5000	0.8660
35	0.5736	0.8192
40	0.6428	0.7660
45	0.7071	0.7071
50	0.7660	0.6428
55	0.8192	0.5736
60	0.8660	0.5000

Lesson Title: Tangent	**Theme**: Geometry	
Lesson Number: M-09-059	**Class/Level**: JSS 3	**Time**: 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson pupils will be able to: 1. Apply the tangent ratio to solve for an unknown side. 2. Identify that tangent is a ratio of sine and cosine: $\tan\theta = \frac{\sin\theta}{\cos\theta}$	None	1. Draw on the triangles for the lesson found at the end of this lesson plan one side of the board. 2. Write the table of sines, cosines and tangents of common angles given at the end.

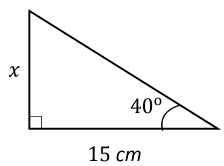
Opening (*3 minutes*)

1. **Say**: We have now looked at both the sine and cosine ratios to solve for an unknown side. Let us solve for x in the triangle in Question a. Please raise your hand when you finish.
2. Allow time for pupils to answer the question.
3. Have a pupil volunteer to write the solution on the board.
4. Correct any errors in the solution on the board. Ask pupils to check their work. (Answers: Shown next to the questions at the end of the lesson plan)
5. **Say**: Today we are going to apply the tangent ratio to solve for an unknown side. We will also identify that $\tan\theta = \frac{\sin\theta}{\cos\theta}$.

Introduction to the New Material (*10 minutes*)

1. Ask a pupil to read Question b. on the board.
2. Show on the board how to work out the length of the side marked x.
   Use the value for tan 40^{o} found at the end of this lesson plan.

$$\tan\theta = \frac{O}{A}$$



$$\tan 40^{o} = \frac{x}{15}$$

$$15 \times \tan 40^{o} = x$$

$$15 \times 0.8391 = x$$

$$12.5865 = x$$

$$x = \text{12.6 cm to 1 d.p.}$$

3. **Say**: Let us look at the ratio for tangent in a little more detail.
4. Ask a pupil to volunteer to remind the class of the 3 ratios for sine, cosine and tangent. (Answer: $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}, \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$ and $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$).
5. **Say**: Let us take the ratio of sin θ to cos θ.
6. Show the calculation below on the board. Explain each step as dividing by 2 fractions.

$$\frac{\sin\theta}{\cos\theta} = \frac{\frac{\text{opposite}}{\text{hypotenuse}}}{\frac{\text{adjacent}}{\text{hypotenuse}}} \quad \longleftarrow \text{ divide the 2 fractions}$$

$$= \frac{\text{opposite}}{\text{hypotenuse}} \times \frac{\text{hypotenuse}}{\text{adjacent}} \quad \longleftarrow \text{ multiply by the reciprocal of the denominator}$$

$$= \frac{opposite}{adjacent} \quad \longleftarrow \text{ simplify}$$

$$\frac{\sin\theta}{\cos\theta} = \tan\theta$$

7. **Say**: We use this form of the tangent ratio when we know the sine and cosine of the angle, but do not know the actual lengths of the sides.
8. Ask a pupil to read Question c. on the board.
9. **Ask**: Who would like to explain how to work out the length of the side marked x? Raise your hand.
10. Guide a pupil to do the calculations for Question c.
    Use the values for $sin\,50, cos\,50$ and $tan\,50^o$ found at the end of this lesson plan.

$$\tan\theta = \frac{O}{A}$$

$$tan\,50^o = \frac{x}{26}$$

$$26 \times tan\,50^o = x$$

$$26 \times 1.192 = x$$

$$30.992 = x$$

$$x = 31.0 \text{ cm to 1 d.p.}$$

Verify that $tan\,50 = \frac{sin\,50}{cos\,50}$

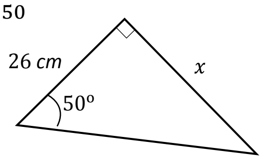
$$tan\,50 = \frac{sin\,50}{cos\,50}$$

$$1.192 = \frac{0.7660}{0.6428}$$

$$1.192 = 1.19166$$

$1.192 = 1.192$ to 4 significant figures

LHS = RHS



Guided Practice (*10 minutes*)

1. Ask the pupils to work in pairs to answer Questions d. and e.
2. Walk around, if possible, to check their answers and clear up any misconceptions.
3. Have pupils from around the classroom volunteer to give their answers to the questions.
4. Correct any errors in the solution on the board. Ask pupils to check their work. (Answers: Shown next to the questions at the end of this lesson plan)

Independent Practice (*10 minutes*)

1. Ask the pupils to work independently to answer Questions f. and g.
2. Walk around, if possible, to check their answers and clear up any misconceptions.
3. Ask pupils to exchange exercise books and check each other's work.
4. Do not do the answer for Questions g. Use it to check pupils' understanding of the work.
5. Have a pupil volunteer to give their answers to the question.
6. Correct any errors in the solution on the board. Ask pupils to check their work. (Answers: Shown next to the questions at the end of this lesson plan)

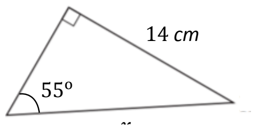
7. Do not do the answer for Questions g. Use it to check pupils' understanding of the work.

Closing (*2 minutes*)

1. **Say:** Please write your name on a piece of paper.
2. **Say**: Write your working out and answer for Question g. on the paper. Hand the paper in at the end of the lesson.
3. Check the work done by the pupils after the lesson. Use it as a guide to which pupils need additional assistance during the next lesson when pupils will be applying the trigonometric ratios to solve for an unknown side.

[QUESTION FOR THE OPENING ACTIVITY]

Find the length of the side marked x in the triangle below. Give your answer to 1 decimal place.



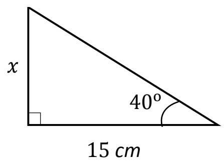
Answer:

$sin\,55^o = \frac{O}{H} = \frac{14}{x}, x = \frac{14}{sin\,55^o} = \frac{14}{0.8192}$
$= 17.0898 = 17.1$ cm to 1 d.p.

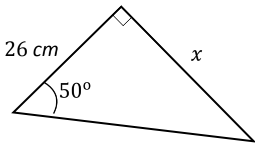
[QUESTIONS FOR INTRODUCTION TO THE NEW MATERIAL]

Use the formula for the tangent to find the length of the side marked x in the triangles below. Give your answers to 1 decimal place. For c., verify that $tan\,50 = \frac{sin\,50}{cos\,50}$.

h.



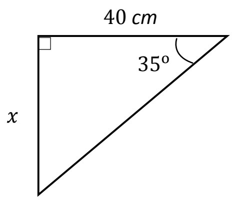
i.



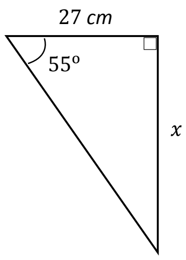
[QUESTIONS FOR GUIDED PRACTICE]

Use the formula for the tangent to find the length of the side marked x in each of the triangles below. Give your answers to 1 decimal place. For v., verify that $tan\,55 = \frac{sin\,55}{cos\,55}$

n.



o.



(Answers: d. $tan\ 35^o = \frac{O}{A} = \frac{x}{40}$, $x = 40 \times tan\ 35^o = 40 \times 0.7002 = 28.008 = 28.0$ cm to 1 d.p.

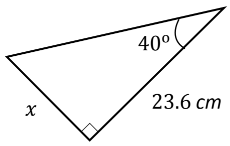
e. $cos\ 55^o = \frac{O}{A} = \frac{x}{27}$, $x = 27 \times tan\ 55^o = 27 \times 1.428 = 38.556 = 38.6$ cm to 1 d.p.;

$tan\ 55 = 1.428, \frac{sin\ 55}{cos\ 55} = \frac{0.8192}{0.5736} = 1.42817 = 1.428\ to\ 4\ s.f.$, LHS = RHS)

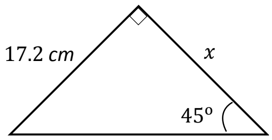
[QUESTIONS FOR INDEPENDENT PRACTICE]

Use the formula for the tangent to find the length of the side marked x in each of the triangles below. Give your answer to 1 decimal place.

t.



u.



(Answers: f. $tan\ 40^o = \frac{O}{A} = \frac{x}{23.6}$, $x = 23.6 \times tan\ 40^o = 23.6 \times 0.8391$
$= 19.80273 = 19.8$ cm to 1 d.p.

g. $tan\ 45^o = \frac{O}{A} = \frac{x}{17.2}$, $x = 17.2 \times tan\ 45^o = 17.2 \times 1.000 = 17.2$ cm to 1 d.p.)

[SINES, COSINES AND TANGENTS OF SOME COMMON ANGLES (TO 4 SIGNIFICANT FIGURES)]

Angle θ	sin θ	cos θ	tan θ	Angle θ	sin θ	cos θ	tan θ
30	0.5000	0.8660	0.5774	50	0.7660	0.6428	1.192
35	0.5736	0.8192	0.7002	55	0.8192	0.5736	1.428
40	0.6428	0.7660	0.8391	60	0.8660	0.5000	1.732
45	0.7071	0.7071	1.000	65	0.9063	0.4226	2.145

Lesson Title: Applying the Trigonometric Ratios	**Theme**: Geometry	
Lesson Number: M-09-060	**Class/Level**: JSS 3	**Time:** 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to find the lengths of the sides of a triangle using sine, cosine, and tangent of given angles.	None	1. Draw the triangles for the lesson found at the end of this lesson plan on one side of the board. 2. Write the table of sines, cosines and tangents of common angles given at the end.

Opening (*3 minutes*)

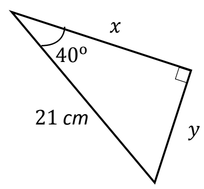
1. **Say**: We now know the basics of solving for the opposite and adjacent sides for a right–angled triangle. Let us see if we can apply what we have learned. Please work in pairs to match each triangle with the correct ratio to use to solve it in Question a. Raise your hand when you finish.
2. Allow time for pupils to answer the question.
3. Have pupils volunteer to give the answers to each triangle. (Answers: Shown next to question at the end of the lesson plan)
4. **Say**: Today we are going to find the lengths of the sides of a triangle using sine, cosine, and tangent of given angles.

Introduction to the New Material (*10 minutes*)

1. Ask a pupil to read Question b. on the board.
2. **Say**: In our triangle, we are asked to solve for x and y. Take a moment to look at the triangle and the information we have.
3. **Ask:** Which side should we solve for first? Raise your hand.
4. Allow pupils time to discuss and share their ideas.
5. Select a pupil who has raised their hand to answer. (Example answers: Either side can be found first. As we know the angle and the hypotenuse, we can use sine or cosine first)
6. **Say**: Please work in pairs to find the unknown sides. Decide who will solve for x and who will solve for y. When you finish give your book to your partner to check your solution.
7. Allow pupils time to solve for the unknown sides and to check each other's work.
8. **Say**: We need 2 volunteers to come to the board to solve for x and y. Raise your hand.
9. Select 2 pupils to come to the board one at a time and write down their solutions.
10. Correct any errors in the solution on the board. Ask pupils to check their work.
    Example calculations are shown below.

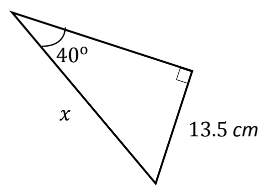
$$\begin{aligned} \cos\theta &= \frac{\mathrm{A}}{\mathrm{H}} \\ \cos 40^{\mathrm{o}} &= \frac{\mathrm{x}}{21} \\ 21\times\cos 40^{\mathrm{o}} &= \mathrm{x} \\ 21\times 0.7660 &= \mathrm{x} \\ &= 16.086 \end{aligned}$$

$$\begin{aligned} \sin\theta &= \frac{\mathrm{O}}{\mathrm{H}} \\ \sin 40^{\mathrm{o}} &= \frac{\mathrm{y}}{21} \\ 21\times\sin 40^{\mathrm{o}} &= \mathrm{y} \\ 21\times 0.6428 &= \mathrm{y} \\ &= 13.4988 \end{aligned}$$



x = 16.1 cm to 1 d.p. $\quad\quad$ y = 13.5 cm to 1 d.p.

11. **Say**: Suppose we are given the value of y and asked to find the hypotenuse instead in the triangle in Question b.?
12. **Ask**: What do you think we should do? Raise your hand.
13. Guide a pupil to say that since we know the angle and its opposite side, we can use the sine ratio.
14. Ask pupils to apply the sine ratio to find the hypotenuse for the new triangle on the right.



$$\sin\theta = \frac{O}{H}$$

$$sin\ 40^o = \frac{13.5}{x}$$

$$x = \frac{13.5}{sin\ 40^o}$$

$$x = \frac{13.5}{0.6428}$$

$$= 21.0018$$

$$x = 21.0 \text{ cm to 1 d.p.}$$

Guided Practice (*10 minutes*)

1. Ask the pupils to work in pairs to answer Questions c. and d.
2. Walk around, if possible, to check their answers and clear up any misconceptions.
3. Have pupils from around the classroom volunteer to give their answers to the questions.
4. Correct any errors in the solution on the board. Ask pupils to check their work. (Answers: Shown next to the questions at the end of this lesson plan)

Independent Practice (*10 minutes*)

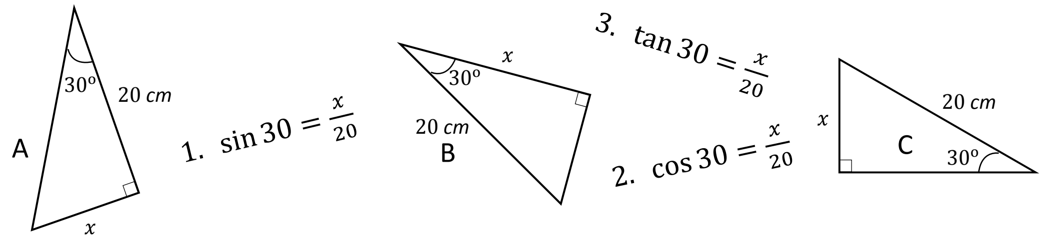
1. Ask the pupils to work independently to answer Questions e. and f.
2. Walk around, if possible, to check their answers and clear up any misconceptions.
3. Ask pupils to exchange exercise books and check each other's work.
4. Have pupils from around the classroom volunteer to give their answers to the questions.
5. Correct any errors in the solution on the board. Ask pupils to check their work. (Answers: Shown next to the questions at the end of this lesson plan)

Closing (*2 minutes*)

1. **Say:** Please write down one thing you learned today.
2. Allow pupils 1 minute to write down their thoughts.
3. Have one pupil from the front, and one from the back of the classroom volunteer to answer. (Example answers: If we know one angle and the hypotenuse we can find the lengths of the other 2 sides; How to find for more than one unknown side in a triangle. Accept all reasonable answers.)

[QUESTION FOR THE OPENING ACTIVITY]

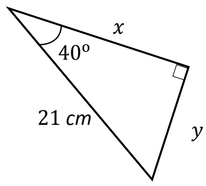
Match the triangle with the correct ratio to solve for the unknown side.



(Answer: A and 3, B and 2, C and 1)

[QUESTIONS FOR INTRODUCTION TO THE NEW MATERIAL]

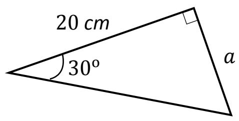
k. Find the lengths of the sides marked x and y in the triangle shown.
Give your answers to 1 decimal place.



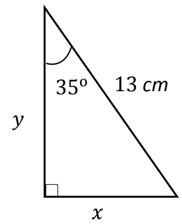
[QUESTIONS FOR GUIDED PRACTICE]

Find the length of each of the unknown sides marked with a letter in the triangles below.
Give your answers to 1 decimal place.

r.



s.

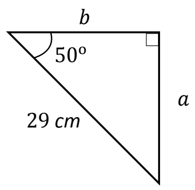


(Answers: c. $tan\ 30^o = \frac{O}{A} = \frac{a}{20}$, $a = 20 \times tan\ 30^o\ = 20 \times 0.5774 = 11.54 = 11.5$ cm to 1 d.p.;
d. $sin\ 35^o\ = \frac{O}{H} = \frac{x}{13}$, $x = 13 \times sin\ 35^o\ = 13 \times 0.5736 = 7.4568 = 7.5$ cm to 1 d.p.;
$cos\ 35^o = \frac{A}{H} = \frac{y}{13}$, $y = 13 \times cos\ 35^o\ = 13 \times 0.8192 = 10.6496 = 10.6$ cm to 1 d.p.)

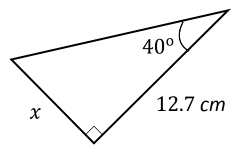
[QUESTIONS FOR INDEPENDENT PRACTICE]

Find the length of each of the unknown sides marked with a letter in the triangles below.
Give your answers to 1 decimal place.

z.



aa.



(Answers: e. $sin\ 50^o = \frac{O}{H} = \frac{a}{29}$, $a = 29 \times sin\ 50^o\ = 13 \times 0.7660 = 9.192 = 9.2$ cm to 1 d.p.;
$cos\ 50^o = \frac{A}{H} = \frac{yb}{29}$, $b = 29 \times cos\ 50^o\ = 29 \times 0.6428 = 18.6412 = 18.6$ cm to 1 d.p.;
f. $tan\ 40^o = \frac{O}{A} = \frac{x}{12.7}$, $x = 12.7 \times tan\ 40^o = 12.7 \times 0.8391 = 10.65657 = 10.7$ cm to 1 d.p.)

[SINES, COSINES AND TANGENTS OF SOME COMMON ANGLES]

Angle θ	sin θ	cos θ	tan θ	Angle θ	sin θ	cos θ	tan θ
30	0.5000	0.8660	0.5774	50	0.7660	0.6428	1.192
35	0.5736	0.8192	0.7002	55	0.8192	0.5736	1.428
40	0.6428	0.7660	0.8391	60	0.8660	0.5000	1.732
45	0.7071	0.7071	1.000	65	0.9063	0.4226	2.145

Lesson Title: Trigonometric Tables for Sine	**Theme**: Geometry	
Lesson Number: M-09-061	**Class/Level**: JSS 3	**Time:** 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to use trigonometric tables to find the sine of an angle.	Copies of Mathematical and Statistical Tables and Formulae (log books)	1. Bring the log books to class. This lesson plan will refer to the tables by name. It will not specify a table number or the page where the table is found. 2. Write the questions for each section of the lesson on the board (see end of lesson plan).

Opening *(3 minutes)*

1. **Say**: So far we have been finding lengths of unknown sides using sines, cosines and tangents of angles already written on the board. These are the trig ratios for some of the more common angles. In everyday life, we will come across many different angles. We need to know how to find the trig ratios of these angles. `Today we are going to use trigonometric tables to find the sine of an angle.

Introduction to the New Material *(10 minutes)*

1. **Say:** Please open your maths tables to the table of sines of angles.
2. **Say:** Look at the table. First notice that all the angles are measured in degrees. This will become important in future lessons when angles are measured in other units.
3. **Say**: Now look at the first column. What is the smallest angle and what is the largest angle in that column?
4. Guide a pupil to say: Smallest 0, largest 89.
5. **Say**: Notice at the top and bottom of the page we have the numbers .0, .1, .2, up to .9. So we can actually find sines of angles up to 90 ^{o}.
6. **Say**: Remember our right-angled triangle has one angle of 90 ^{o} and 2 other angles which are smaller than 90 ^{o}.
7. **Say:** The way the table is written means we can read the sines of whole number angles or angles with 1 decimal place directly from the table.
8. **Say**: Let us find the sine of a whole number angle.
9. Ask a pupil to read Question a. on the board.
10. **Say**: Look down the page until we get to 8. As it is a whole number, we will read the value at 8.0.
11. **Ask:** What is the value of the sine of 8.0 ^{o}? Raise your hand. (Answer: 0.1392)
12. **Say:** Let us try the next angle on the board: 32^o
13. **Say:** Look down the page until we get to 32. Again, as it is a whole number, we will read the value at 32.0 ^{o}.
14. **Ask:** What is the value of sine 32 ^{o}? Raise your hand. (Answer: 0.5299)
15. **Say:** Now read the value of the sine of the next angle on the board: 68 ^{o}. Please raise your hand when you have found the value.
16. Allow the pupils a few moments to read the angle.
17. Select a pupil to give the answer. (Answer: 0.9272)

18. **Say**: To find the sine of an angle with 1 decimal place, we must first find the whole number part of the angle and simply move across the page until we get to the correct decimal number.
19. **Say**: Let us find the sine of 21.8 ^{o}.
20. Allow pupils a few moments to look up this angle.
21. **Ask**: Do you notice anything when you get to .8? Raise your hand.
22. Guide a pupil to say what they observe. (Example answer: Only the numbers in the decimal places are given, there is no 0. in front of the number.)
23. **Say**: When we find 21 and read across to .8, the number there says 3714. We have to remember that the actual sine of 21.8 ^{o} is 0.3714.
24. **Say**: We have to do this for all angles which are not whole numbers. This is to save space in the table. However, we must not forget to put the 0 and the decimal point in front of the sine of the angle.
25. **Say**: Please work with your neighbour to look up the values of 67.5 ^{o} and 71.2 ^{o}. The numbers at the top and bottom of the page help you keep track of the decimal number you are looking for.
26. Allow time for the pupils to read the values from the table.
27. Have pupils volunteer to give the values found. (Answers: sin 67.5 ^{o} = 0.9239, sin 71.2 ^{o} = 0.9466)
28. **Say:** You now know how to read the sine tables for whole number angles and those with 1 decimal place.
29. **Say:** You will notice at the right hand side of the tables a number of columns called 'Add differences'. We use these to calculate sines of angles up to 2 decimal places.
30. Show on the board how to calculate the sine for angle 16.23 ^{o}.

0.2790	⟵	Value of sine at <u>16.2</u>
+ 0.0005	⟵	Value in the 'Add differences' column under <u>3</u> written as a decimal number
0.2795	⟵	Add the 2 values to give sine 16.23 ^{o}

31. **Say**: Calculate the sines for the angles in Questions h. and i.: 33.79 ^{o} and 46.28 ^{o}
32. Allow time for pupils to do this.
33. **Say**: Please compare your answers with your neighbour and discuss any differences you have.
34. Allow time for pupils to do this.
35. **Say:** We need 2 volunteers to explain their calculations on the board.
36. Select 2 pupils to come one at a time to the board to explain their calculations.
37. Correct any errors in the solution on the board. Ask pupils to check their work.
    The calculations are shown below.

sin 33.79 ^{o}	sin 46.28 ^{o}
0.5548	0.7218
+ 0.0013	+ 0.0010
0.5561	0.7228

Guided Practice *(10 minutes)*

1. Ask pupils to work in pairs to answer the Questions for Guided Practice.
2. Walk around, if possible, to check their answers and clear up any misconceptions.
3. Have pupils from around the classroom volunteer to give their answers for the required sines.
4. **Write** the correct answers on the board. Ask pupils to check their work.

Independent Practice *(10 minutes)*

1. Ask the pupils to work independently to answer the Questions for Independent Practice.
2. Walk around, if possible, to check their answers and clear up any misconceptions.
3. Ask pupils to exchange exercise books and check each other's work.
4. Have pupils from around the classroom volunteer to give their answers for the required sines.
5. **Write** the correct answers on the board. Ask pupils to check their work.

Closing *(2 minutes)*

1. **Say:** Please look at the table of sines.
2. **Ask:** What do you notice about the values of the sines as the angles increase from 0^o to 90^o? Raise your hand.
3. Allow a few moments for pupils to look at the sine table and to discuss and share their ideas.
4. Guide a pupil to say that as the angles increase from 0^o to 90^o, the sines increase from *0* to *1.*
5. Ask pupils to copy this in their exercise books.
6. **Say:** In tomorrow's lesson, we will be looking at the trig tables to find cosines of angles.

[QUESTIONS FOR INTRODUCTION TO THE NEW MATERIAL]

Use the trigonometric tables to find the sines of the following angles:

a. 8^o	b. 32^o	c. 68^o
d. 21.8^o	e. 67.5^o	f. 71.2^o
g. 16.23^o	h. 33.79^o	i. 46.28^o

[QUESTIONS FOR GUIDED PRACTICE]

Use the trigonometric tables to find the sines of the following angles:

a. 6^o	b. 55^o	c. 77^o
d. 2.9^o	e. 47.2^o	f. 64.8^o
g. 27.24^o	h. 48.17^o	i. 59.61^o

Answers:

a. $\sin 6^o = 0.1045$	b. $\sin 55^o = 0.8192$	c. $\sin 77^o = 0.9744$
d. $\sin 12.9^o = 0.2233$	e. $\sin 47.2^o = 0.7337$	f. $\sin 64.8^o = 0.9048$
g. $\sin 27.24^o = 0.4577$	h. $\sin 48.17^o = 0.7451$	i. $\sin 59.61^o = 0.8626$

[QUESTIONS FOR INDEPENDENT PRACTICE]

Use the trigonometric tables to find the sines of the following angles:

a. 9^o b. 46^o c. 89^o
d. 15.4^o e. 51.3^o f. 87.2^o
g. 18.74^o h. 24.87^o i. 67.24^o

Answers:

a. $\sin 9^o$	= 0.1564	b. $\sin 46^o$	= 0.7193	c. $\sin 89^o$	= 0.9998
d. $\sin 15.4^o$	= 0.2656	e. $\sin 51.3^o$	= 0.7804	f. $\sin 87.2^o$	= 0.9988
g. $\sin 18.74^o$	= 0.3213	h. $\sin 24.87^o$	= 0.4206	i. $\sin 67.24^o$	= 0.9222

Lesson Title: Trigonometric Tables for Cosine	**Theme**: Geometry	
Lesson Number: M-09-062	**Class/Level**: JSS 3	**Time**: 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to use trigonometric tables to find the cosine of an angle.	Copies of Mathematical and Statistical Tables and Formulae (log books)	1. Bring the log books to class. This lesson plan will refer to the tables by name. It will not specify a table number or the page where the table is found. 2. Write the questions for each section of the lesson on the board (see end of lesson plan).

Opening (*3 minutes*)

1. **Ask**: Who can remind the class of what we did in the last lesson? Raise your hand. (Answer: Used the trigonometric tables to find sines of angles.)
2. **Say**: Find the sine of the angle 24.89°. Please check your answer with your neighbour.
3. Allow time for pupils to find the required sine.
4. Have a pupil volunteer to give the answer. (Answer: 0.4195 + 0.0014 = 0.4209)
5. **Say**: Remember that we add the differences to the rightmost digits of the main sine as a decimal number. Today we are going to use trigonometric tables to find the cosine of an angle.

Introduction to the New Material (*10 minutes*)

1. **Say:** Please open your maths tables to the table of cosines of angles.
2. **Say:** The procedure to find the cosines of angles is similar to how we learned in the last lesson to find the sines.
3. **Say:** Everyone will have the chance to practise how to use the tables some more. As we did with the sines in the last lesson, we can read the cosines of whole number angles or angles with 1 decimal place directly from the table.
4. **Say**: We will find the cosines of some of the angles from the last lesson. But, we will concentrate on cosines of numbers with 1 and 2 decimal places.
5. Have a pupil volunteer to read Question a. on the board.
6. **Say**: Look down the page until we get to 18. As it is a whole number, we will read the value at 18.0.
7. **Ask:** What is the value of the cosine of 18.0°? (Answer: 0.9511)
8. **Say:** Now we need someone to explain to the class how to find the cosine of 32.3°.
9. **Say:** Please raise your hand if you think you can do so.
10. Select a pupil who has raised their hand to explain. (Example answer: First find the whole number part of the angle and move across the page until we get to the correct decimal number)
11. **Ask**: What do we get? Raise your hand. (Answer: 0.8453)
12. Some pupils may have had problems in the previous lesson finding sines of angles with 2 decimal places.
13. **Say:** We will now find the cosine of angles with 2 decimal places. We will do an example together on the board for the angle 55.29°.

14. Show each step on the board as you ask questions and guide the pupils to answer. The completed calculation is shown below.
15. **Ask:** What is the first step? Raise your hand.
16. Guide a pupil to say we first find the value of cosine 55.2 °. (Answer: 0.5707)
17. **Ask**: What do we do next? Raise your hand.
18. Guide a pupil to say we look in the 'Subtract differences' column to find the value for 9.
19. **Ask:** What is the answer? (Answer: 13)
20. **Ask**: Who can tell the difference from our last lesson, when we were finding the sines of angles? Raise your hand.
21. Guide a pupil to say that with sine we added the differences. With cosine, we are subtracting the differences.

0. 5707	←	Value of cosine at 55.2
– 0.0013	←	Value in the 'Subtract differences' column under 9 written as a decimal number
0.5694	←	Subtract the 2 values to give cosine 55.29 °

22. **Say**: Please calculate the cosines of the angles in Questions d. and e.: 33.79 ° and 46.28 °
23. Allow time for pupils to do this.
24. **Say**: Compare your answer with your neighbour and discuss any differences you have.
25. Allow time for pupils to do this.
26. **Say:** We need 2 volunteers to explain their calculations on the board.
27. Select 2 pupils to come one at a time to the board to explain their calculations.
28. Correct any errors in the solution on the board. Ask pupils to check their work. The calculations are shown below.

cos 33.79 °	cos 46.28 °
0.8320	0.6921
– 0.0009	– 0.0010
0.8311	0.6911

Guided Practice (*10 minutes*)

1. Ask the pupils to work in pairs to match the angle to the correct cosine in the questions for Guided Practice.
2. Walk around, if possible, to check their answers and clear up any misconceptions.
3. Have pupils from around the classroom volunteer to give their answers for the required cosines.
4. **Write** the correct answers on the board. Ask pupils to check their work. (Answers: Shown next to the questions at the end of this lesson plan)

Independent Practice (*10 minutes*)

1. Ask the pupils to work independently to answer the Questions for Independent Practice.

2. Walk around, if possible, to check their answers and clear up any misconceptions.
3. Ask pupils to exchange exercise books and check each other's work.
4. Have pupils from around the classroom volunteer to give their answers for the required cosines.
5. **Write** the correct answers on the board. Ask pupils to check their work. (Answers: Shown next to the questions at the end of this lesson plan)

Closing (*2 minutes*)

1. **Say:** Please look at the table of cosines.
2. **Ask:** What do you notice about the values of the cosines as the angles increase from 0 ° to 90 °?
3. Allow a few moments for pupils to look at the cosine table and to discuss and share ideas.
4. Guide a pupil to say that as the angles increase from 0 ° to 90 °, the cosines decrease from 1 to 0.
5. Ask pupils to copy this in their exercise books.
6. **Say:** In tomorrow's lesson, we will be looking at the trig tables to find tangents of angles.

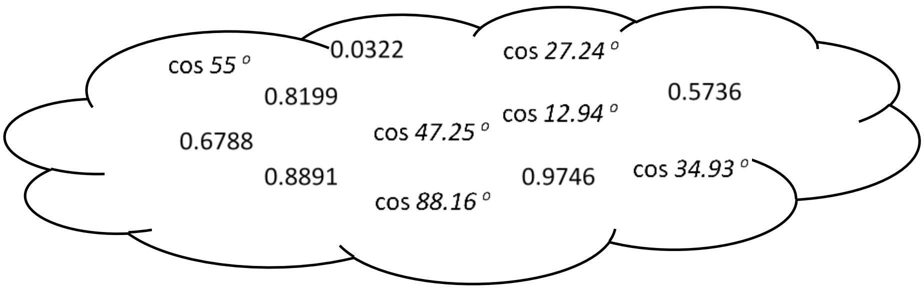
[QUESTIONS FOR INTRODUCTION TO THE NEW MATERIAL]

Use the trigonometric tables to find the cosines of the following angles:

j. 18 ° k. 32.3 ° l. 55.29 °
m. 33.79 ° n. 46.28 °

[QUESTIONS FOR GUIDED PRACTICE]

Match the angle to the correct cosine:



Answers:

cos 55 °	= 0.5736	cos 27.24 °	= 0.8891	cos 12.94 °	= 0.9746
cos 88.16 °	= 0.0322	cos 47.25 °	= 0.6788	cos 34.93 °	= 0.8199

[QUESTIONS FOR INDEPENDENT PRACTICE]

Use the trigonometric tables to find the cosines of the following angles:

xix. 15.42 ° xx. 51.3 ° xxi. 87 °
xxii. 18.74 ° xxiii. 34.87 ° xxiv. 67.24 °

Answers:

xix.	$\cos 15.42^{o}$	= 0.9640	xx.	$\cos 51.3^{o}$	= 0.6252	xxi.	$\cos 87^{o}$	= 0.0523
xxii.	$\cos 18.74^{o}$	= 0.9470	xxiii.	$\cos 34.87^{o}$	= 0.8204	xxiv.	$\cos 67.24^{o}$	= 0.3869

Lesson Title: Trigonometric Tables for Tangent	**Theme**: Geometry	
Lesson Number: M-09-063	**Class/Level**: JSS 3	**Time**: 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to use trigonometric tables to find the tangent of an angle.	Copies of Mathematical and Statistical Tables and Formulae (log books)	1. Bring the log books to class. This lesson plan will refer to the tables by name. It will not specify a table number or the page where the table is found. 2. Write the questions for each section of the lesson on the board (see end of lesson plan).

Opening (*3 minutes*)

1. **Ask**: Who can remind the class of the main difference between finding the sine and cosine of angles which have 2 decimal places? Raise your hand.
2. Guide a pupil to say that we add the difference for sines and subtract the difference for cosine.
3. **Say**: Find the sine and cosine of the angle 54.37°. Please check your answer with your neighbour.
4. Allow time for pupils to find the required ratios.
5. Have a pupil volunteer to give the answer. (Answer: $\sin 54.37^{\circ} = 0.8128$, $\cos 54.37^{\circ} = 0.5825$)
6. **Say**: Today we are going to use trigonometric tables to find the tangent of an angle.

Introduction to the New Material (*10 minutes*)

1. **Say:** Please open your maths tables to the table of tangents of angles.
2. **Say:** The procedure to find the tangents of angles is the same as we learned with sines.
3. **Say:** Find the tangents of the angles on the board.
4. Allow pupils time to calculate the tangents.
5. **Say**: Compare your answer with your neighbour and see if you agree.
6. Allow a few moments for pupils to compare answers.
7. **Say**: Who would like to share their answers with the class? Raise your hand.
8. Have pupils from around the classroom volunteer to share their answers. (Answers: Shown next to the questions at the end of the lesson plan)
9. **Say**: Look at the values of the tangents you have just calculated.
10. **Ask**: Do you notice anything about them that is different from sines and cosines? Raise your hand.
11. Allow pupils time to discuss and share their ideas.
12. Guide a pupil to say sines and cosines only have values between 0 and 1 for the angles in the tables. Tangents have values that start at 0 but then increase very quickly. Accept all reasonable answers.
13. **Say:** Work with your neighbour to find the tangents of angles 87.9° and 89.9°.
14. Allow time for pupils to read the tangents from the tables.
15. Have pupils volunteer to give their answers. (Answers; $\tan 87.9^{\circ} = 27.27$, $\tan 89.9^{\circ} = 573.0$)
16. **Say:** This is an interesting feature of tangents of angles that they can get so large compared with the sines and cosines of the angles. You will learn more about this in senior secondary school.

Guided Practice (*10 minutes*)

1. Ask the pupils to work in pairs to match the angle to the correct tangent in the questions for Guided Practice.
2. Walk around, if possible, to check their answers and clear up any misconceptions.
3. Have pupils from around the classroom volunteer to give their answers.
4. **Write** the correct answers and steps on the board. Ask pupils to check their work. (Answers: Shown at the end of this lesson plan)

Independent Practice (*10 minutes*)

1. Ask the pupils to work independently to answer the questions for Independent Practice.
2. Walk around, if possible, to check their answers and clear up any misconceptions.
3. Ask pupils to exchange exercise books and check each other's work.
4. Have pupils from around the classroom volunteer to give their answers.
5. **Write** the correct answers and steps on the board. Ask pupils to check their work. (Answers: shown at the end of this lesson plan)

Closing (*2 minutes*)

1. **Say:** Please write down one thing you learned about tangents in this lesson that you did not know before.
2. Allow pupils time to write in their exercise books.
3. Have pupils from around the classroom volunteer to give their answers. (Example answer: Tangents can get very large compared to sines and cosines)

[QUESTIONS FOR INTRODUCTION TO THE NEW MATERIAL]

Use the trigonometric tables to find the tangents of the following angles:

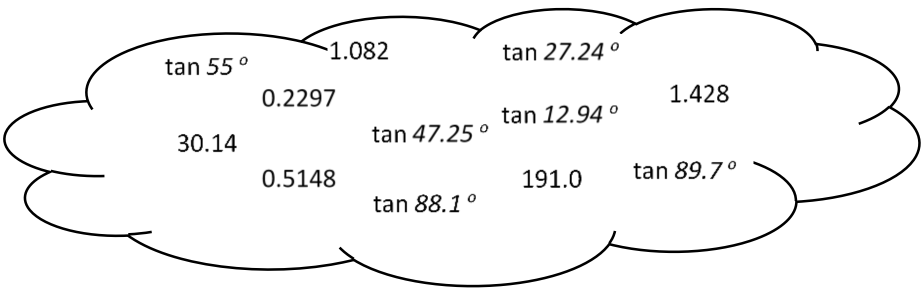
o. 13.8° p. 46.28° q. 89.5°

Answers:

a. $\tan 13.8^{\circ} = 0.2456$ b. $\tan 46.28^{\circ} = 1.046$ c. $\tan 89.5^{\circ} = 114.6$

[QUESTIONS FOR GUIDED PRACTICE]

Match the angle to the correct tangent:



Answers:

tan 55 ^{o}	= 1.428	tan 27.24 ^{o}	= 0.5148	tan 12.94 ^{o}	= 0.2297
tan 88.1 ^{o}	= 30.14	tan 47.25 ^{o}	= 1.082	tan 89.7 ^{o}	= 191.0

[QUESTIONS FOR INDEPENDENT PRACTICE]

Use the trigonometric tables to find the tangents of the following angles:

a. 15.42 ^{o}	b. 51.3 ^{o}	c. 67 ^{o}
d. 71.74 ^{o}	e. 34.87 ^{o}	f. 87.24 ^{o}

Answers:

a. tan 15.42 ^{o}	= 0.2758	b. tan 51.3 ^{o}	= 1.248	c. tan 67 ^{o}	= 2.356
d. tan 71.74 ^{o}	= 3.031	e. tan 34.87 ^{o}	= 0.6968	f. tan 87.2 ^{o}	= 20.45

Lesson Title: Trigonometric Practice	**Theme**: Geometry	
Lesson Number: M-09-064	**Class/Level**: JSS 3	**Time:** 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to: 1. Determine which trigonometric function should be applied to a given problem. 2. Apply the trigonometric functions.	Copies of Mathematical and Statistical Tables and Formulae (log books)	1. Bring the log books to class. 2. Write the questions for each section of the lesson on the board (see end of lesson plan).

Opening *(3 minutes)*

1. **Say**: Please find the sine, cosine and tangent of angle 52.56°. Check your answer with your neighbour.
2. Allow time for pupils to find the required ratios.
3. Have pupils from around the classroom volunteer to give the answers. (Answers: $\sin 52.56^{\circ} = 0.7940$, $\cos 52.56^{\circ} = 0.6080$, $\tan 52.56 = 1.306$)
4. **Say**: Today we are going to determine and apply the correct trigonometric function to a given problem.

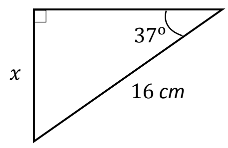
Introduction to the New Material *(10 minutes)*

Note: Wait a few moments after each question for pupils to think before selecting a pupil to answer.

1. **Say:** Now that we know how to read the trig tables, we can answer a wide variety of problems involving sine, cosine and tangent.
2. **Say:** In the previous exercises that we did, we were told which ratio to use. Now we have to decide for ourselves.
3. **Say**: Let us remind ourselves of the 3 trig ratios we are going to use in this lesson.
4. **Write** SOHCAHTOA on the board and ask pupils to write the expanded form in their exercise books.
5. Have a pupil volunteer to read Question a. on the board.
6. **Ask**: The question asks us to find x. What information do we have for this triangle? Raise your hand.
7. Guide a pupil to answer. Write the solution on the board. (Answer: shown below)

Hypotenuse = 16 cm

Opposite = x



8. **Ask**: Please look at the 3 ratios. Which one should we choose? Raise your hand. (Answer: sine)
9. **Ask:** Why do we choose the sine ratio? Raise your hand. (Example answers: It is the one which involves hypotenuse and opposite; because we know S and H in SOH and we want to find O)
10. **Say:** Now that we have determined which ratio to use, we can continue as we have done before.

11. **Ask:** Who would like to show us the calculation on the board? Raise your hand.

12. Select a volunteer to show the calculation on the board. Ask other pupils to solve it in their exercise books.
13. Correct any errors in the solution on the board. Ask pupils to check their work. (Example answer: See below)

$$\sin\theta = \frac{\mathrm{O}}{\mathrm{H}}$$

$$\sin 37^{\mathrm{o}} = \frac{x}{16}$$

$$16 \times \sin 37^{\mathrm{o}} = x$$

$$16 \times 0.6018 = x$$ using $\sin 37^{\mathrm{o}} = 0.6018$ from the sine tables

$$= 9.6288$$

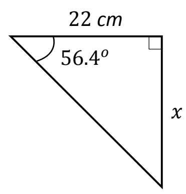
x = 9.6 *cm to 1 d.p.*

14. Ask a pupil to read Question b. on the board.

15. **Ask**: The question again asks us to find x. What information do we have for this triangle? Raise your hand.
16. Guide a pupil to answer. (Answer: shown below; write on the board)

Opposite = x

Adjacent = 22 cm



17. **Ask**: Please look at the 3 ratios. Which one should we choose? Raise your hand.
(Answer: tangent)
18. **Ask:** Why do we choose the tangent ratio? Raise your hand.
(Example answers: It is the one which involves opposite and adjacent; because we know T and A in TOA and we want to find O)
19. **Say:** Now that we have determined which ratio to use, we can continue as we have done before.
20. **Say**: Work with your neighbour to find the length of side x.

21. Allow time for pupils to calculate for x.

22. **Ask:** Who would like to show us their calculation on the board? Raise your hand.

23. Select a volunteer to show the calculation on the board. Ask other pupils to observe carefully to see if they agree with the calculation.
24. Correct any errors in the solution on the board. Ask pupils to check their work. (Example answer: see below)

$$\tan\theta = \frac{\mathrm{O}}{\mathrm{A}}$$

$$\tan 56.4^{\mathrm{o}} = \frac{x}{22}$$

$22 \times \tan 56.4 = x$

$22 \times 1.505 = x$ using $\tan 56.4^{o} = 1.505$ from the tangent tables

$= 33.11$

$x = 33.1$ cm *to 1 d.p.*

Guided Practice *(10 minutes)*

1. Ask the pupils to continue to work in pairs to answer the Questions for Guided Practice.
2. Walk around, if possible, to check their answers and clear up any misconceptions.
3. Have pupils from around the classroom volunteer to give their answers.
4. **Write** the correct answers on the board. Ask pupils to check their work. (Answers: Shown at the end of this lesson plan)

Independent Practice *(10 minutes)*

1. Ask the pupils to work independently to answer the Questions for Independent Practice.
2. Walk around, if possible, to check their answers and clear up any misconceptions.
3. Have pupils from around the classroom volunteer to give their answers.
4. **Write** the correct answers on the board. Ask pupils to check their work. (Answers: Shown at the end of this lesson plan)

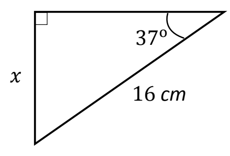
Closing *(2 minutes)*

1. **Say:** Please write on a piece of paper one thing you still have a problem with on finding the sides of triangles using the trig ratios. Do not write your name, just what you do not understand.
2. Allow pupils time to write one problem they have on a piece of paper.
3. Collect the papers. Review the problems. Keep the issues in mind so you can assist pupils during the next lesson when they are solving word problems using the trig ratios.

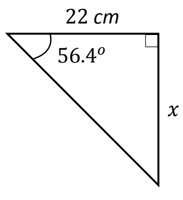
[QUESTIONS FOR INTRODUCTION TO THE NEW MATERIAL]

Find the length of the side marked x in each of the triangles below.
Give your answers to 1 decimal place.

a.



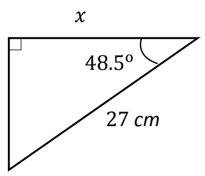
b.



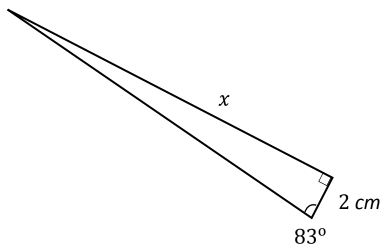
[QUESTIONS FOR GUIDED PRACTICE]

Find the length of the side marked x in each of the triangles below.
Give your answers to 1 decimal place.

c.



d.



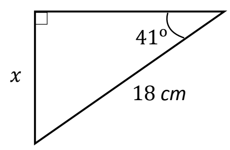
(Answers: c. $\cos 48.5^{o} = \frac{A}{H} = \frac{x}{27}$, $x = 27 \times \cos 48.5^{o} = 27 \times 0.6626 = 17.8902 = 17.9$ *cm* to 1 d.p.

d. $\tan 83^{o} = \frac{O}{A} = \frac{x}{2}$, $x = 2 \times \tan 83^{o} = 2 \times 8.144 = 16.288 = 16.3$ *cm* to 1 d.p.)

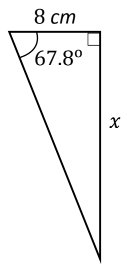
[QUESTIONS FOR INDEPENDENT PRACTICE]

Find the length of the side marked x in each of the triangles below.
Give your answers to 1 decimal place.

e.



f.



(Answers: e. $\sin 41^{o} = \frac{O}{H} = \frac{x}{18}$, $x = 18 \times \sin 41^{o} = 18 \times 0.6561 = 11.8098 = 11.8$ *cm* to 1 d.p.

f. $\tan 67.8^{o} = \frac{O}{A} = \frac{x}{8}$, $x = 8 \times \tan 67.8^{o} = 8 \times 2.450 = 19.6$ *cm* to 1 d.p.)

Lesson Title: Trigonometric Word Problems	**Theme**: Geometry	
Lesson Number: M-09-065	**Class/Level**: JSS 3	**Time:** 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to solve trigonometry word problems with and without diagrams.	Copies of Mathematical and Statistical Tables and Formulae (log books).	1. Bring the log books to class. This lesson plan will refer to the tables by name. 2. Write the questions for each section of the lesson on the board (see end of lesson plan).

Opening *(3 minutes)*

1. **Say**: We are now familiar with the trig ratios. We are able to solve for unknown sides when we know one side and the angle. We are also familiar with the trig tables and can read or work out the ratios from the tables.
2. **Say**: Today we are going to solve trigonometry word problems with and without diagrams.

Introduction to the New Material *(10 minutes)*

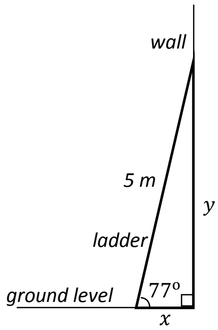
Note: Wait a few moments after each question for pupils to think before selecting a pupil to answer.

1. Ask a pupil to read Question a. on the board.
2. **Say**: Let us label the unknown sides x and y
3. **Ask**: What information do we have for the triangle in this situation? Raise your hand.
4. Guide a pupil to answer. **Write** the answer on the board. (Answer: Shown below)

Hypotenuse = 5 m

Adjacent = x

Opposite = y



5. **Ask**: What did we find out in the last lesson about which side to solve for first if we know the hypotenuse? Raise your hand.
   (Answer: We can solve for either x or y first)
6. **Say**: Please work with your neighbour to find the lengths of sides x and y.
7. Allow time for pupils to calculate for x and y.
8. **Ask:** Who would like to show us the calculation for x on the board?
9. Select a volunteer to show the calculation.
11. Select another volunteer to show the calculation to find y.
12. Correct any errors in the solutions on the board. Ask pupils to check their work. (Example answer: see below)

$$\cos\theta = \frac{A}{\mathrm{H}} \qquad \sin\theta = \frac{\mathrm{O}}{\mathrm{H}}$$

$$\cos 77^{\circ} = \frac{x}{5} \qquad \sin 77^{\circ} = \frac{y}{21}$$

$$5 \times \cos 77^{\circ} = x \qquad 5 \times \sin 77^{\circ} = y$$

$$5 \times 0.2250 = x \qquad 5 \times 0.9744 = y$$

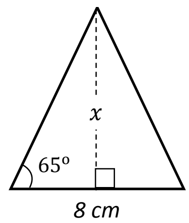
$$= 1.125 \qquad\qquad = 4.872$$

$$x = 1.1 \textit{ m to 1 d.p.} \qquad\qquad y = 4.9 \textit{ m to 1 d.p.}$$

13. Ask a pupil to read Question b. on the board.
14. **Say:** We do not have a diagram for this problem.
    Who would like to explain what our first step should be? Raise your hand.
15. Guide a pupil to say we should draw a diagram.
16. If a pupil gives the first step as a calculation, **say**: There is a step before that one which will help us decide whether we are solving for the opposite or adjacent side.
17. Show how to interpret the question as a diagram (shown below on the right).
18. **Ask**: What information do we have for the triangle in this situation? Raise your hand.
19. Guide a pupil to answer. **Write** the answer on the board. (Answer: shown below)

Opposite = x

Adjacent = $\frac{1}{2}$ of the base = $\frac{1}{2} \times 8 = 4$ cm



20. **Ask**: Which of the 3 ratios should we choose? Raise your hand.
    (Answer: tangent)
21. **Say**: Please work with your neighbour to find the length of side x.
22. Allow time for pupils to calculate for x.
23. **Ask:** Who would like to show us their calculation on the board? Raise your hand.
24. Select a volunteer to show the calculation. Ask other pupils to observe carefully to see if they agree with the calculation.
25. Correct any errors in the solution on the board. Ask pupils to check their work. (Example answer: See below)

$$\tan\theta = \frac{\text{O}}{\text{A}}$$

$$\tan 65^{\circ} = \frac{x}{4}$$

$$4 \times \tan 65 = x$$

$$4 \times 2.145 = x$$

$$= 8.58$$

$$x = 8.6 \textit{ cm to 1 d.p.}$$

Guided Practice *(10 minutes)*

1. Ask pupils to continue to work in pairs to answer the Questions for Guided Practice.
2. Walk around, if possible, to check their answers and clear up any misconceptions.
3. Have pupils from around the classroom volunteer to give their answers.
4. **Write** the correct answers on the board. Ask pupils to check their work. (Answers: Shown at the end of this lesson plan)

Independent Practice *(10 minutes)*

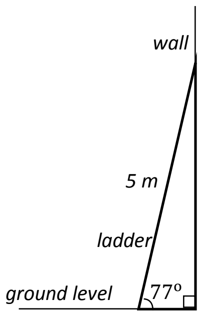
1. Ask the pupils to work independently to answer the Questions for Independent Practice.
2. Walk around, if possible, to check answers and clear misconceptions.
3. Have pupils from around the classroom volunteer to give their answers.
4. **Write** the correct answers on the board. Ask pupils to check their work. (Answers: Shown at the end of this lesson plan)

Closing *(2 minutes)*

1. **Say**: Trigonometry together with Pythagoras' Theorem are used to solve for unknown sides in right-angled triangles. You will learn in senior secondary how trigonometry is also used to solve for unknown angles.

[QUESTIONS FOR INTRODUCTION TO THE NEW MATERIAL]

a. A ladder of length 5 m leans against a wall as shown on the right.
   a) How far is the bottom of the ladder from the wall?
   b) How far is the top of the ladder from ground level?
   Give your answers to 1 decimal place.

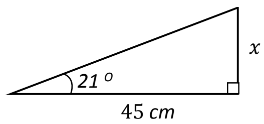


b. An isosceles triangle has a base of 8 cm. Its equal sides make an angle of 65 ° with the base. What is its perpendicular height?

[QUESTIONS FOR GUIDED PRACTICE]

Give all answers correct to 1 decimal place.

c. Calculate the area of this triangle.



d. A diagonal of a square is 30 cm long. How long is each side?

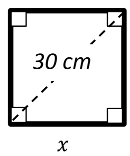
(Answers:

c. $\tan 21^{\circ} = \frac{O}{A} = \frac{x}{45}$, $x = 45 \times \tan 21^{\circ} = 45 \times 0.3839 = 17.2755 = 17.3$ *cm* to 1 d.p.

area of triangle $= \frac{1}{2}bh = \frac{1}{2} \times 45 \times 17.3 = 389.25 = 389.3$ *cm*^{2}*to* 1 d.p.)

d. Draw a diagram as shown right, diagonals bisect the right angles so each angle in the triangle is 45 °. we can use either sine 45 ° or cosine 45 ° – they are equal to each other.

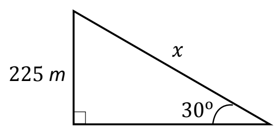
$\sin 45^{\circ} = \frac{O}{H} = \frac{x}{30}$, $x = 30 \times \sin 45^{\circ} = 30 \times 0.7071 = 21.213 = 21.2$ *m* to 1 d.p.)



[QUESTIONS FOR INDEPENDENT PRACTICE]

Give all answers correct to 1 decimal place.

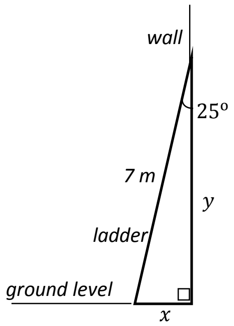
e. The diagram shows the path of a stone as it rolls down a hill 225 m high. Find how far it rolled down the hill.



f. A ladder of length 7 m leans against a wall so that it makes an angle of 25 ° with the wall.

i. How far is the bottom of the ladder from the wall?

ii. How far is the top of the ladder from ground level?



(Answers: e. $\sin 30^{o} = \frac{O}{H} = \frac{225}{x}$, $x = \frac{225}{\sin 30^{o}} = \frac{225}{0.5} = 450$ *m.*

f. Draw a diagram as shown right.

i. $\sin 25^{o} = \frac{O}{H} = \frac{x}{7}$, $x = 7 \times \sin 25^{o} = 7 \times 0.4226$
$= 2.9582 = 3.0$ *cm* to 1 d.p.

ii. $\cos 25^{o} = \frac{A}{H} = \frac{y}{7}$, $y = 7 \times \cos 25^{o} = 7 \times 0.9063$
$= 6.3441 = 6.3$ *cm* to 1 d.p.)

Lesson Title: Changing the Subject of a Formula	**Theme**: Algebra	
Lesson Number: M-09-066	**Class/Level**: JSS 3	**Time:** 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to balance an equation using addition, subtraction, multiplication, and division.	None	1. Write on the board: $C = 2\pi r$ 2. Write the vocabulary list on the board: Vocabulary List - formula, formulas , formulae 3. Write the questions from the Guided Practice section on the board. 4. Write the questions from the Independent Practice section on the board.

Opening (*3 minutes*)

1. **Say**: Please raise your hand if you recognise the formula on the board.
2. Select a pupil who raised their hand to describe the formula. (Answer: Circumference of a circle = 2 × pi × radius)
3. **Say**: A formula is an equation where we use letters to represent quantities. It always has at least 2 variables and gives the relationship between the variables.
4. **Say:** The plural of formula is formulae. Some people say formulas. For this lesson plan, we will use formulas.
5. **Ask**: We use formulas a lot in maths, science and other subjects. Who can tell the class some other formulas they know? Raise your hand.
6. Guide 2-4 pupils to give examples of other formulas. They should explain what the formulas represent. (Example answers: $A = \pi r^2$, (area of a circle = pi × radius squared); A = ½ b h, (area of a triangle = ½ × base × height). Accept other correct formulas.)
7. **Say**: Today we are going to learn how to change the subject of a formula. We will use addition, subtraction, multiplication, and division in order to balance the equation.

Introduction to the New Material (*10 minutes*)

1. **Say**: $C = 2\pi r$ relates the circumference of a circle to its radius. This means that if we know the radius of a circle, we can find its circumference.
2. **Say**: C is called the subject of the formula. It stands by itself on one side of the equal sign. It can be calculated directly by substituting known values of the radius r.
3. **Say**: If we know the circumference of a circle and want to find its radius, we change the subject of the formula from C to r. To do this, we rearrange the letters so that r stands by itself on one side of the equal sign.
4. Show pupils how to change the subject of the formula for the circumference of a circle from C to r:

$$C = 2\pi r$$

$$\frac{C}{2\pi} = \frac{2\pi r}{2\pi} \quad \longleftarrow \text{Divide both sides by } 2\pi$$

$$\frac{C}{2\pi} = r \quad \longleftarrow \text{Cancel } 2\pi \text{ from right-hand side}$$

$$r = \frac{C}{2\pi} \quad \longleftarrow \quad \text{Formula with r as the subject}$$

5. **Say**: In maths, the subject of a formula or equation is usually written on the left-hand side of the equal sign.
6. **Write** on the board: $F = 1.8C + 32$
7. **Ask:** Who knows what this formula represents? Raise your hand. (Answer: The relationship between degrees in Fahrenheit and degrees in Celsius both used to measure temperature. Accept other reasonable answers.)
8. **Ask**: What is the subject of the formula? Raise your hand. (Answer: F)
9. Who can explain to the class how to make C the subject of the formula?
10. Guide a pupil to show how to change the subject of the formula from F to C.
11. **Say:** We only need to write down the result from the balance method for each step.

$$F = 1.8C + 32$$

$$F - 32 = 1.8C \quad \longleftarrow \quad \text{Subtract 32 from both sides}$$

$$\frac{F-32}{1.8} = C \quad \longleftarrow \quad \text{Divide both sides by 1.8}$$

$$C = \frac{F-32}{1.8} \quad \longleftarrow \quad \text{Formula with C as the subject}$$

12. **Say**: Note the main points to remember when changing the subject of a formula.
13. **Write** on the board:
    - Do the same thing to both sides of the equation
    - Use the inverse of the sign or operation
        - adding and subtracting
        - multiplying and dividing
        - square and square root
        - powers and related roots (e.g. $x^3$and $\sqrt[3]{x}$)
14. **Say**: Let us look at the formula for the area of a circle, πr^2. Suppose we know the area and we want to find the radius of a circle.
15. **Ask**: Who can explain to the class how to make r the subject of the formula? Raise your hand.
16. Guide a pupil to show how to change the subject of the circle area formula from A to r on the board. Ask other pupils to observe carefully to see if they agree with the calculation.
17. The pupil may become stuck after dividing both sides by π. Remind them about using the inverse operation. So square root is inverse of the square.
18. Correct any errors in the solution on the board. (Answer: see below)

$$A = \pi r^2$$

$$\frac{A}{\pi} = r^2 \quad \longleftarrow \quad \text{Divide both sides by } \pi$$

$$\sqrt{\frac{A}{\pi}} = r \quad \longleftarrow \quad \text{Take the square root of both sides}$$

$$r = \sqrt{\frac{A}{\pi}} \quad \longleftarrow \quad \text{Formula with r as the subject}$$

Guided Practice (*10 minutes*)

1. Ask pupils to work in pairs.
2. Point to the questions on the board:

Make the letter or letters in brackets the subject of the formula.

a. $V = IR$, (I)
b. $v = u + at$, (u, t)
c. $y = 2x + 3$, (x)
d. $s = \frac{d}{t}$, (d, t)

3. Walk around, if possible, to check the answers and clear up any misconceptions.
4. Have pupils from around the classroom volunteer to give their answers.
5. **Write** the correct answers on the board. Ask pupils to check their work.

(Answers: a. $I = \frac{V}{R}$, (divide both sides by R); b. $u = v - at$ (subtract a t from both sides), $t = \frac{\mathrm{v} - u}{a}$ (subtract u from both sides, then divide both sides by a);
c. $x = \frac{y-3}{2}$ (subtract 3 from both sides, then divide both sides by 2);
d. $d = st$ (multiply both sides by t), $t = \frac{d}{s}$ (multiply both sides by t, then divide both sides by s)

Independent Practice (*10 minutes*)

1. Ask the pupils to work independently to answer the questions.
2. Point to the questions on the board:

Make the letter or letters in brackets the subject of the formula.

a. $\frac{y}{x} = 2$, (y, x)
b. $s = \sqrt{\frac{d}{g}}$, (d, g)
c. $I = \frac{PRT}{100}$, (P)
d. $V = \frac{1}{3}\pi r^2\mathrm{h}$, (h)

3. Walk around, if possible, to check the answers and clear up any misconceptions.
4. Ask pupils to exchange their exercise books and check each other's work.
5. Have pupils from around the classroom volunteer to give their answers.
6. **Write** the correct answers on the board. Ask pupils to check their work.

(Answers: a. $y = 2x$ (multiply both sides by x), $x = \frac{y}{2}$ (multiply both sides by x, then divide both sides by 2; b. $d = gs^2$ (square both sides, then multiply both sides by g), $g = \frac{d}{s^2}$ (square both sides, multiply both sides by g, then divide both sides by s^2); c. $P = \frac{100I}{RT}$ (multiply both sides by 100, then divide by RT; d. $h = \frac{3V}{\pi r^2}$ (multiply both sides by 3, divide both sides by π r^2)

Closing (*2 minutes*)

1. **Say:** Please write your name on a piece of paper. You will do the next question on it and hand it in.
2. **Say**: Make r the subject in Question d.
3. Collect the work from pupils at the end of the lesson. Use it to check how much pupils understood from today's lesson.
4. Start the next lesson with review work from this lesson depending on the issues raised.

(Answer: $r = \sqrt{\frac{3V}{\pi h}}$ (multiply both sides by 3, divide both sides by π h, take the square root of both sides)).

Lesson Title: Combining Like Terms	**Theme**: Algebra	
Lesson Number: M-09-067	**Class/Level**: JSS 3	**Time**: 35 minutes

Learning Outcomes
By the end of the lesson pupils will be able to identify and combine like terms.

Teaching Aids
None

Preparation
1. Write on the board:
   1. a. Make t the subject of the formula $r = \sqrt{\frac{3t}{2}}$

1. b. Simplify the following:
   i. $3 + 5$ ii. $a + b$
   iii. $a + a$ iv. $a + b + a$

1. c. Which are like? Which are unlike?

2x, x, 0x, -x	17x, 17z
15, -2, $\frac{2}{5}$, 0.6	a, ab, b
$3y^2$, y^2, $-y^2$, $26y^2$	$15y$, $19y^2$, $31y^5$

2. Write on the board: Vocabulary list – terms, like terms
3. Write the questions from the Guided Practice section on the board.
4. Write the questions from the Independent Practice section on the board.

Opening (*3 minutes*)

1. **Say**: In the last lesson, we were looking at changing the subject of a formula.
2. Point to Question 1. a. Make t the subject of the formula $r = \sqrt{\frac{3t}{2}}$
3. **Say**: Please answer the question on the board. You have 1 minute.
4. Have a pupil from the back of the classroom volunteer to explain their answer. (Answer: $t = \frac{2r^2}{3}$).
5. **Say**: Today we are going to look at identifying and combining like terms.

Introduction to the New Material (10 minutes)

1. Point to Question 2.b.
2. **Say:** Please raise your hand if you know how to answer Question 1.b.i.
3. Select a pupil who has raised their hand (which should be the whole class) to answer. (Answer: 8)
4. **Say:** Look at Questions ii. and iii. on the board. Work in pairs to answer them.
5. Allow 1 minute for pupils to look at the questions and discuss the answers with each other.
6. **Say**: Please raise your right hand if you know how to answer Question ii.
7. **Say:** Please raise your left hand if you know how to answer Question iii.
8. Repeat the instructions so pupils understand what to do.
9. Have pupils volunteer to come to the board to write their answers.
10. Look at the answers on the board. You may have answers as shown below:

ii. $a + b = 2a,\ 2b,\ a + b$ etc.
iii. $a + a = 2a,\ a + a$ etc.

11. Ask pupils if they agree with the answers on the board. Ask them for their reasons for agreeing or disagreeing with the answers.
12. Stop the discussion after 2 minutes and put the correct answers on the board. (Answers: ii. $a + b = a + b$; iii. $a + a = 2a$)
13. **Say**: We can simplify Questions i. and iii. because they are 'like terms'.
14. **Say**: 'Terms' is the word we use to describe the parts of an expression or equation. For example, 5 is a term, so is a, b and 2a are also terms. The terms are separated by + or – signs.
15. **Say**: There are 2 terms in Question i. They are both numbers, 5 and 3, and we can add them together.
16. **Say**: The terms in Question iii. are exactly the same letter, a, and we can also add them together.
17. **Say**: We cannot simplify Question ii. because they are not 'like terms'. a is different from b and we cannot add them together.
18. **Say:** Please simplify Question iv. What answer do you get?
19. Give pupils a few moments to think. Have a pupil volunteer to answer. (Answer: $2a + b$)
20. Point to the table in 1.c.
21. **Say**: Look at the expressions in the table. Which column gives like terms, right or left?
22. Give pupils a few moments to think. Have a pupil volunteer to answer. (Answer: Left)
23. **Say**: For like terms, every part of the expression is the same. If one part of the expression is different, it is not a like term.
24. **Say**: The numbers can be different, but the rest of the term must be exactly the same.
25. **Write** on the board: Simplify $5x + 2y - 3x - y$.
26. **Say:** In this expression, we have terms in x and terms in y. Put a box around the terms in x and circle the terms in y.
27. **Say**: Collect all the x terms together and all the y terms together. The number and any sign in front of the term is part of the term.
28. Ask pupils to exchange books for their neighbour to check.
29. Show on the board how to simplify the expression by collecting like terms. Ask pupils to check the answers.

$$\boxed{5x}\,(+2y)\,\boxed{-3x}\,(-y) = 5x - 3x + 2y - y$$
$$= 2x + y$$

Guided Practice (*10 minutes*)

1. Ask pupils to work in pairs.
2. Point to the questions on the board:

Simplify by collecting like terms:
(Hint: Put a box around one group of like terms.
Put a circle around another group of like terms. Any number or sign is part of the term.)

e. $a + b + 2b + 2a$
f. $4s + 11 - 2s$
g. $12x + 4 - 6x - 7$
h. $p - 2q - 2p + 3q$
i. $3x - 2y + 6x + y$
j. $x^2 + y^2 + 2y^2 + x^2$

3. Walk around, if possible, to check the answers and clear up any misconceptions.

4. Have pupils from around the classroom volunteer to give their answers to the questions.
5. **Write** the correct answers on the board. Ask pupils to check their work.

(Answers: a. $\boxed{a}$ $\textcircled{+b}$ $\textcircled{+2b}$ $\boxed{+2a}$ $= a + 2a + b + 2b = 3a + 3b$;

b. $\boxed{4s} + 11 \boxed{-2s} = 4s - 2s + 11 = 2s + 11$

Continue putting boxes and circles around groups of like terms.

c. $12x + 4 - 6x - 7 = 12x - 6x + 4 - 7 = 6x - 3$;
d. $p - 2q - 2p + 3q = p - 2p - 2q + 3q = -p + q$;
e. $3x - 2y + 6x + y = 3x + 6x - 2y + y = 9x - y$;
f. $x^2 + y^2 + 2y^{`2} + x^2 = x^2 + x^2 + y^2 + 2y^2 = 2x^2 + 3y^2$).

Independent Practice (*10 minutes*)

1. Ask the pupils to work independently to answer the questions.
2. Point to the questions on the board:

Simplify by collecting like terms:

a. $a + b + c + 2b + 2a - 3c$
b. $4r + s - 2r + 2t + s$
c. $a + b + 2b + 2c + b - 2a - c$
d. $4x - 3y + 2x + 2z - y - z$
e. $x^2 + y^2 + 2y^2 - x^2$
f. $x^2y + xy^2 + 2yx^2 - 3y^2x$

3. Walk around, if possible, to check the answers and clear up any misconceptions.
4. Ask pupils to exchange their exercise books and check each other's work.
5. Have pupils from around the classroom volunteer to give their answers to the questions.
6. **Write** the correct answers on the board. Ask pupils to check their work.
(Answers: a. $a + b + c + 2b + 2a - 3c = a + 2a + b + 2b + c - 3c = 3a + 3b - 3c$;
b. $4r + s - 2r + 2t + s = 4r - 2r + s + s + 2t = 2r + 2s + 2t$;
c. $a + b + 2b + 2c + b - 2a - c = a - 2a + b + 2b + b + 2c - c = -a + 4b - c$;
d. $4x - 3y + 2x + 2z - y - z = 4x + 2x - 3y - y + 2z - z = 6x - 4y + z$;
e. $x^2 + y^2 + 2y^2 - x^2 = x^2 - x^2 + y^2 + 2y^2 = 3y^2$;
f. $x^2y + xy^2 + 2yx^2 - 3y^2x = x^2y + 2yx^2 + xy^2 - 3y^2x = 3x^2y - 2xy^2$,
since $xy^2 = y^2x$ and $x^2y = yx^2$).

Closing (*2 minutes*)

1. **Say:** Who would like to tell the class one new thing they learned today that they did not know before? Raise your hand.
2. Select 1-2 pupils who raised their hands. (Example answers: Learned about collecting like terms; that the number and sign are part of the term; that $xy^2 = y^2x$)

Lesson Title: Solving Linear Equations	**Theme**: Algebra	
Lesson Number: M-09-068	**Class/Level**: JSS 3	**Time:** 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to solve linear equations in one variable by balancing the equation and combining like terms.	None	1. Write on the board: 1.a. Simplify the following expression: $3x + 7 - x - 11$ Solve for x in the equations: 1.b. $3x + 7 = x + 11$ 1.c. $5(x + 1) = 20$ 2. Write on the board: Vocabulary list – variable. 3. Write the questions from the Guided Practice section on the board. 4. Write the questions from the Independent Practice section on the board.

Opening (*3 minutes*)

1. **Say**: Please simplify the expression 1.a. $3x + 7 - x - 11$, written on the board. Raise your hand when you finish.
2. Allow 1 minute for pupils to answer the question.
3. Select a pupil who has raised their hand to answer. (Answer: $3x + 7 - x - 11 = 2x - 4$)
4. **Say**: Today we are going to solve linear equations in one variable by balancing the equation and combining like terms.

Introduction to the New Material (*10 minutes*)

1. **Ask**: Who can explain to the class what a variable is? Raise your hand.
2. Guide a pupil who has raised their hand to answer. (Example answer: A variable is a letter like x or y; a letter for a number we do not know.)
3. **Say:** A variable is a quantity that changes or 'varies'. We use letters to represent it such as x or y. Sometimes, we choose a letter that reminds us of the quantity it represents, such as t for time, or d for distance. Look at Questions 1.b and 1.c. What is similar between the 2 statements? Raise your hand.
4. Select a pupil with a raised hand to answer. (Example answer: Both contain the variable x; both have the numbers 3, 7 and 11.)
5. If no one volunteers, **say:** The variable x is present in both statements.
6. **Say:** Look at Questions 1.a and 1.c.
7. **Ask:** What is different between the 2 statements? Raise your hand.
8. Select a pupil who has raised their hand to answer. (Example answers: The statement in Question 1.a is an expression, while the one in Question 1.c. is an equation; the difference is that Question 1.c. is an equation because it has an equal sign.)
9. **Say:** The statement in Question 1.a. is an expression, while the one in Questions 1.b. and 1.c. is an equation because they have an equal sign.
10. **Say:** We want to solve for x using the balance method. Who can explain to the class how to do this? Raise your hand.

11. Guide a pupil to show step-by-step how the balance method works on the board. Ask other pupils to observe carefully to see if they agree with the calculation. (Answer: See below)

$$3x + 7 = x + 11$$

$$3x - x + 7 = x - x + 11 \quad \leftarrow \text{Subtract x from both sides of the equation}$$

$$2x + 7 = 11 \quad \leftarrow \text{Collect the terms in x}$$

$$2x + 7 - 7 = 11 - 7 \quad \leftarrow \text{Subtract 7 from both sides}$$

$$2x = 4$$

$$\frac{2x}{2} = \frac{4}{2} \quad \leftarrow \text{Divide both sides by 2}$$

$$x = 2 \quad \leftarrow \text{Solution for x}$$

Check when $x = 2$

$$\text{LHS} = (3 \times 2) + 7$$

$$= 6 + 7$$

$$= 13$$

$$\text{RHS} = 2 + 11 \qquad \text{RHS means right-hand side}$$

$$= 13$$

$$= \text{LHS} \qquad \text{LHS means left-hand side}$$

12. **Say**: It is good practice to check the answer by substituting the solution of x back into the equation. You will know when you have made a mistake if you do not get RHS = LHS.
13. **Say:** Let us now look at Question 1.c. where we solve for x in an equation with brackets.
14. Show pupils step-by-step how to solve for x on the board. Explain that we must always clear brackets first before balancing the equation.

$$5(x + 1) = 20$$

$$5x + 5 = 20 \quad \leftarrow \text{Remove brackets (multiply by 5)}$$

$$5x + 5 - 5 = 20 - 5 \quad \leftarrow \text{Subtract 5 from both sides}$$

$$5x = 15$$

$$\frac{5x}{5} = \frac{15}{5} \quad \leftarrow \text{Divide both sides by 5}$$

$$x = 3 \quad \leftarrow \text{Solution for x}$$

Check when $x = 3$

$$\text{LHS} = 5(3 + 1) = 5 \times 4 = 20 = \text{RHS}$$

Guided Practice (*10 minutes*)

1. Ask pupils to work in pairs.
2. Point to the questions on the board:

Solve for x in the following equations. Check the solution to the equation.

g. $2x + 3 = 5$ h. $12 = 7 - x$
i. $4x - 3 = x + 6$ j. $x + 4 = 2x + 7$
k. $4 - 2x = x - 5$ l. $4x - 1 = x + 5$

3. Walk around, if possible, to check the answers and clear up any misconceptions.
4. Have pupils from around the classroom volunteer to give their answers.
5. **Write** the correct answers on the board. Ask pupils to check their work.
   Note: It is very important for pupils to check their answers to problems of this type. It will tell them immediately if their answers are correct. This will give them the opportunity to self-correct any errors.
   (Answers: a. $x = 1$ (subtract 3 from both sides, then divide by 2),
   Check when $x = 1$, $\text{LHS} = (2 \times 1) + 3 = 2 + 3 = \text{RHS}$;
   b. $x = -5$ (subtract 5 from both sides, multiply both sides by -1 to make x positive),
   Check when $x = -5$, $\text{RHS} = 7 - (-5) = 7 + 5 = 12 = \text{LHS}$;
   c. $x = 3$ (add 3 to both sides, subtract 6 from both sides, collect the x terms, divide by 3),
   Check when $x = 3$, $\text{LHS} = (4 \times 3) - 3 = 12 - 3 = 9, \text{RHS} = 3 + 6 = 9 = \text{LHS}$;
   d. $x = -3$ (subtract 4 from both sides, subtract 7 from both sides, collect x terms),
   Check when $x = -3$, $\text{LHS} = -3 + 4 = 1, \text{RHS} = (2 \times (-3) + 7 = -6 + 7 = 1 = \text{LHS}$;
   e. $x = 3$ (subtract 4 from both sides, add 5 to both sides, collect x terms, divide by 3),
   Check when $x = 3$, $\text{LHS} = 4 - (2 \times 3) = 4 - 6 = -2, \text{RHS} = 3 - 5 = -2 = \text{LHS}$;
   f. $x = 2$ (add 1 to both sides, subtract 5 from both sides, collect x terms, divide by 3)
   Check when $x = 2$, $\text{LHS} = (4 \times 2) - 1 = 8 - 1 = 7, \text{RHS} = 2 + 5 = 7 = \text{LHS}$).

Independent Practice (*10 minutes*)

1. Ask the pupils to work independently to answer the questions.
2. Point to the questions on the board:

Solve for the variable in the following equations. Check the solution to the equation.

a. $3 - 5y = 8$ b. $15 = 9 - 2t$
c. $6p + 15 = 8 - p$ d. $s + 23 = 3s + 45$
e. $3(n + 5) = 18$ f. $2(m + 1) - 8 = 0$

3. Walk around, if possible, to check the answers and clear up any misconceptions.
4. Have pupils from around the classroom volunteer to give their answers.
5. **Write** the correct answers on the board. Ask pupils to check their work.
   (Answers: a. $y = -1$ (subtract 3 from both sides, then divide by -5),
   Check when y= -1, $\text{LHS} = 3 - \left(5 \times (-1)\right) = 3 — 5 = 3 + 5 = 8 = \text{RHS}$;
   b. $t = -3$ (subtract 9 from both sides, then divide by -2),
   Check when $t = -3$, $\text{RHS} = 9 - \left(2 \times (-3)\right) = 9 — 6 = 9 + 6 = 15 = \text{LHS}$;
   c. $p = -1$ (subtract 15 from both sides, subtract 8 from both sides, collect the x terms, divide by 7), Check when $p = -1$, $\text{LHS} = \left(6 \times (-1)\right) + 15 = -6 + 15 = 9$,
   $\text{RHS} = 8 - (-1) - 8 + 1 = 9 = \text{LHS}$;
   d. $s = -11$ (subtract 23 from both sides, subtract 45 from both sides, collect x terms, divide by

-2), Check when $x = -11$, $LHS = -11 + 23 = 12, R$
$HS = (3 \times (-11) + 45 = -33 + 45 = 12 = LHS$;
e. $n = 1$ (remove brackets – multiply both n and 5 by 3, subtract 15 from both sides, divide by 3),
Check when $n = 1$, $LHS = 3 \times (1 + 5) = 3 \times 6 = 18 = RHS$;
f. $m = 3$ $(2(m + 1) - 8 = 0, 2m + 2 - 8 = 0, 2m - 6 = 0, m - 3 = 0, m = 3)$
Check when $m = 3$, $LHS = 2 \times (3 + 1) = 2 \times 4 = 8 = RHS$).

Closing (*2 minutes*)

1. **Ask:** Who would like to tell the class one important thing they can do to correct errors when they solve a linear equation? Raise your hand.
2. Select a pupil who has raised their hand. (Example answer: Check the solution by putting the answer in the equation. If LHS=RHS then we know we have solved the equation correctly)
3. **Say:** Well done class! We will be looking at solving more linear equations later this year.

Lesson Title: Substituting	**Theme**: Algebra	
Lesson Number: M-09-069	**Class/Level**: JSS 3	**Time**: 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to find the value of an algebraic expression by substituting.	None	1. Write on the board: 1. Substitute $a = 5, b = 3, c = 6$ into the expressions below. Evaluate and simplify your answers: a. $a + b$ b. $a - b$ c. $2a + b - c$ d. $\frac{c}{b}$ e. $\frac{b}{c}$ 2. Write on the board: Vocabulary List- substitute, evaluate, simplify 3. Write the questions from the Guided Practice section on the board. 4. Write the questions from the Independent Practice section on the board.

Opening (*3 minutes*)

Note: After each question, wait for pupils to think and raise their hands to answer. Select pupils from around the classroom.

1. **Ask**: Who can remind the class what we did last lesson? Raise your hand. (Answer: Solving linear equations, solve for x.)
2. **Ask:** What do we do after we solve the equation and find x? Raise your hand. (Answer: Check if our solution is right; Put the value of x back in our equation. Accept all reasonable answers.)
3. **Say:** When we put the value of x back in the equation, we say we are substituting for x. We can substitute values into expressions as well as equations. Today we are going to find the value of an algebraic expression by substituting.

Introduction to the New Material (*10 minutes*)

1. **Say:** Write the vocabulary list on the board in your exercise books. We will be using the words throughout the lesson.
2. **Say:** We will substitute the given values in the algebraic expressions on the board and evaluate the answers.
3. Show how to substitute and calculate the value of the expression in Questions 1.a and 1.b.

a. $a + b = 5 + 3$ ← Substitute for a and b

$= 8$ ← Evaluate (find the value)

b. $a - b = 5 - 3 = 2$

4. Ask pupils to work in pairs for 1 minute to discuss and share ideas for Questions 1.c.

5. **Say**: Use BODMAS to help you remember in what order to carry out the operations.
6. **Write** the expanded form of BODMAS on the board (see box on the right)
7. **Say**: Please raise your hand when you finish.
4. Select a pupil who has raised their hand to explain on the board how to evaluate the expression.
5. Ask other pupils to observe carefully to see if they agree with the calculation
6. Correct any errors in the solution on the board.
   (Answer: c. $2a + b - c = (2 \times 5) + 3 - 6 = 10 + 3 - 6 = 7$)
8. **Say:** We do calculations involving division in the same way.
9. Show how to substitute and calculate the value of the expression in Questions d. and e.

Reminder
Brackets
Off
Division
Multiplication
Addition
Subtraction

d. $\frac{c}{b} = \frac{6}{3}$ ⟵ Substitute for b and c

$= 2$ ⟵ Evaluate

e. $\frac{b}{c} = \frac{3}{6} = \frac{1}{2}$

Guided Practice (*10 minutes*)

1. Ask pupils to continue to work in pairs.
2. Point to the questions on the board:

Substitute $p = 4, q = 2, r = 5$ into the expressions below. Evaluate and simplify your answers:

a. $p + q$
b. $p + r$
c. $2p + r$
d. $p + q + r$
e. $3q - 4$
f. $2p - 4q - r$

3. Walk around, if possible, to check the answers and clear up any misconceptions.
4. Have pupils from around the classroom volunteer to give their answers.
5. **Write** the correct answers on the board. Ask pupils to check their work.
   (Answers: a. $p + q = 4 + 2 = 6$; b. $p + r = 4 + 5 = 9$; c. $2p + r = (2 \times 4) + 5 = 8 + 5 = 13$; d. $p + q + r = 4 + 2 + 5 = 11$; e. $3q - 4 = (3 \times 2) - 4 = 6 - 4 = 2$; f. $2p - 4q - r = (2 \times 4) - (4 \times 2) + 5 = 8 - 8 - 5 = -5$).

Independent Practice (*10 minutes*)

1. Ask the pupils to work independently to answer the questions.
2. Point to the questions on the board:

Substitute $u = 5, v = 10, w = 15$ into the expressions below. Evaluate and simplify your answers:

a. uv
b. $3uw + v$
c. uvw
d. $\frac{v}{u}$
e. $\frac{uw}{3}$
f. $\frac{uv+w}{u}$

g. u^2v h. $u^2 + v^2$

3. Walk around, if possible, to check the answers and clear up any misconceptions.
4. Ask pupils to exchange their exercise books and check each other's work.
5. Have pupils from around the classroom volunteer to give their answers to the questions. Do not do the answers for Questions b. and f. Use them to check pupils' understanding of the work.
6. **Write** the correct answers on the board (except for b. and f.). Ask pupils to check their work. (Answers: a. $uv = 5 \times 10 = 50$; b. $3uw + v = (3 \times 5) + 15 = 15 + 15 = 30$; c. $uvw = 5 \times 10 \times 15 = 750$; d. $\frac{v}{u} = \frac{10}{5} = 2$; e. $\frac{uw}{3} = \frac{5\times15}{3} = \frac{75}{3} = 25$; f. $\frac{uv+w}{u} = \frac{(5\times10)+15}{5} = \frac{50+15}{5} = \frac{65}{5} = 13$; g. $u^2v = 5^2 \times 10 = 25 \times 10 = 250$; h. $u^2 + v^2 = 5^2 + 10^2 = 25 + 100 = 125$).

Closing (*2 minutes*)

1. **Say:** Please write your name on a piece of paper.
2. **Say**: Now write your working out and answer for Questions b. and f. on the paper. Hand the paper in at the end of the lesson.
3. Check the work done by the pupils after the lesson. Use it as a guide for which pupils need additional assistance during the next lesson when pupils will be practising solving problems with algebraic expressions.

Lesson Title: Practice Solving Algebraic Expressions	**Theme**: Algebra	
Lesson Number: M-09-070	**Class/Level**: JSS 3	**Time:** 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to solve algebraic expressions using various techniques.	None	1. Write on the board: 1. Collect like terms and simplify: a. $3x - z + 2y - 4x - 3y + 4z$ b. $5ab - a + b - 2ab + 3a - 2ab + 2b$ 2. Substitute the given values and evaluate: $p = 2, q = 4, r = 3$ a. $3p + q + r$ b. $3pq + 2qr$ 2. Write the questions from the Guided Practice section on the board. 3. Write the questions from the Independent Practice section on the board.

Opening (*3 minutes*)

1. **Say**: We have looked at solving different types of algebraic expressions in the last few lessons.
2. **Ask:** Who can remind the class of what they are? Raise your hand.
3. Allow 1 minute for pupils to look back in their exercise books and discuss the types of algebraic expressions with each other.
4. Have pupils from around the classroom volunteer to answer. (Answers: Collecting like terms, substitution, simplifying expressions)
5. **Say**: Today we are going to practise solving algebraic expressions using various techniques.

Introduction to the New Material (*10 minutes*)

Note: This lesson is to practise collecting like terms and substitution into algebraic expressions.

1. **Say:** We are going to use all the skills and techniques we have learned so far to solve for algebraic expressions.
2. **Say:** You have 4 minutes to work on your own to solve the Questions 1a., 1b. 2a., and 2b. on the board.
3. Allow 4 minutes for pupils to work independently.
4. If pupils finish early, ask them to write algebraic expressions of their own to solve.
5. Ask the pupils to pair up with a neighbour and discuss how they solved the problems for 1 minute.
6. Allow another minute for paired discussion.
7. **Ask**: Who would like to share their ideas with the class? Raise your hand.
8. Select different pupils to explain their answers to the 4 questions.
9. Ask whether the class agrees after each answer. Discuss any differences in the answer.
10. **Write** the correct answers and steps on the board. Ask pupils to check their work. (Answer: a. $3x - z + 2y - 4x - 3y + 4z = 3x - 4x + 2y - 3y - z + 4z = -x - y + 3z$; b. $5ab - a + b - 2ab + 3a - 2ab + 2b = -a + 3a + b + 2b + 5ab - 2ab - 2ab =$

$2a + 3b + ab$; c. $3p + q + r = (3 \times 2) + 4 + 3 = 6 + 4 + 3 = 13$;
d. $3pq + 2qr = (3 \times 2 \times 4) + (2 \times 4 \times 3) = 24 + 24 = 48$).

Guided Practice (*10 minutes*)

1. Ask pupils to continue to work in pairs.
2. Point to the questions on the board:

   Simplify by collecting like terms:

   a. $rp + pr$
   b. $pqr + 2pr + rpq + prq - 3rp$
   c. $p^2qr + pqr + 2rp - rpq + rp^2q$

   Substitute $r = -2, s = 5, t = -3$ into the expressions below. Evaluate and simplify your answers:

   d. $2r + s + t$
   e. rs
   f. rt

3. Walk around, if possible, to check answers and correct any misconceptions.
4. Have pupils from around the classroom volunteer to give their answers.
5. **Write** the correct answers on the board. Ask pupils to check their work. (Answers: a. $rp + pr = 2pr$; b. $pqr + 2pr + rpq + prq - 3rp = pqr + rpq + prq + 2pr - 3rp = 3pqr - pr$;
   c. $p^2qr + pqr + 2pr - rpq + rp^2q = p^2qr + rp^2q + pqr - rpq + 2pr = 2p^2qr + 2pr$;
   d. $2r + s + t = \left(2 \times (-2)\right) + 5 + (-3) = -4 + 5 - 3 = -2$; e. $rs = (-2) \times 5 = -10$
   f. $rt = (-2) \times (-3) = 6$).

Independent Practice (*10 minutes*)

1. Ask the pupils to work independently to answer the questions.
2. Write the following questions on the board:

   Simplify by collecting like terms:

   a. $a^2 + ab - 2a^2 + 3ba$
   b. $3cd + e + cde + 4dc - e$
   c. $p^3qr + pqr + qrp^3 - 2rpq + rp^3q$

   Substitute $u = 8, v = 12, w = -3$ into the expressions below. Evaluate and simplify your answers:

   d. $\frac{v}{w}$
   e. $uv - \frac{v}{4}$
   f. $\frac{u}{v} - \frac{v}{w}$

3. Walk around, if possible, to check the answers and clear up any misconceptions.
4. Ask pupils to exchange their exercise books and check each other's work.
5. Have pupils from around the classroom volunteer to give their answers.
6. **Write** the correct answers on the board. Ask pupils to check their answers. (Answers: a. $a^2 + ab - 2a^2 + 3ba = a^2 - 2a^2 + ab + 3ba = -a^2 + 4ab$;
   b. $3cd + e + cde + 4dc - e = 3cd + 4dc + e - e + cde = 7cd + cde$;
   c. $p^3qr + pqr + qrp^3 - 2rpq + rp^3q = p^3qr + qrp^3 + rp^3q + pqr - 2rpq$

$= 3p^3qr - pqr$; d. $\frac{v}{w} = \frac{12}{-3} = -4$; e. $uv - \frac{v}{4} = 8 \times 12 - \frac{12}{4} = 96 - 3 = 93$;
f. $\frac{8}{12} - \frac{12}{-2} = \frac{2}{3} + 6 = 6\frac{2}{3}$).

Closing (*2 minutes*)

1. Ask pupils to make an expression using 2 variables for their partners to substitute values they give to them. They should only use positive and negative integer values less than 10.
2. Have 2-3 pupils volunteer to share their expressions and solutions with the class. (Example answer: Evaluate the answer when you substitute a=2, b=3 into the expression 2a+b)

Lesson Title: Multiplying an Algebraic Expression by an Integer	**Theme**: Algebra	
Lesson Number: M-09-071	**Class/Level**: JSS 3	**Time:** 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to expand an algebraic expression by multiplying an expression by an integer.	None	1. Write on the board: 1. Expand: c. $2(x+3)$ d. $4(2x-1)$ e. $-2(3-2x)$

Opening (*3 minutes*)

1. **Say**: Please look at Question 1a. How do we solve this type of expression? Raise your hand.
2. Select a pupil to answer. (Example answers: By expanding brackets; removing brackets)
3. **Ask**: Where have you seen this type of expression before? Raise your hand. (Example answer: Solving linear equations, changing the subject of a formula)
4. **Say**: Today we are going to expand algebraic expressions by multiplying the expression by an integer.

Introduction to the New Material (*10 minutes*)

1. **Say:** We encounter expanding algebraic expressions all the time in Maths. In the next few lessons, we are going to review all the methods we use step-by-step. This review will help us when we come to working with quadratic equations.
2. Show on the board how to solve Question 1a. Multiply each term inside the bracket by 2:
   - **Say**: $(2 \times x) = 2x$
   - **Say:** $+(2 \times 3) = +6$

$$2(x+3) = (2 \times x) + (2 \times 3)$$ ← Multiply each term inside the bracket by 2

$$= 2x + 6$$

3. Repeat this so it is clear what happens when we multiply an expression by an integer.
4. **Say:** You now have 1 minute to solve Question 1b.
5. Allow pupils to think and write down ideas for 1 minute.
6. After 1 minute, ask them to pair up with their neighbour and discuss how to solve the problem for another minute.
7. **Ask**: Who would like to share their ideas with the class? Raise your hand.
8. Select a pupil who has raised their hand to explain their answer step by step on the board. Ask other pupils to observe carefully to see if they agree with the calculation.
9. Correct any errors in the solution on the board. (Answer: see below)

$$4(2x-1) = (4 \times 2x) + (4 \times (-1))$$ Multiply each term inside the bracket by 4

$$= 8x - 4$$

10. Draw pupils' attention to the fact that a + is always used between the terms.
11. **Say:** We always use a + sign between the terms. In our example, the negative sign in front of the 1 is part of the number. So we multiply 4 by (–1). This gives us (–4).
12. **Say**: Let us look at a question were we have to multiply by a negative integer.
13. Show on the board how to solve Question 1c.
14. As you multiply each term in the solution below, remind pupils about the rules of multiplying positive and negative integers together:
    - **Say**: $((-2) \times 3) = -6$ because (negative × positive) is negative
    - **Say:** $((-2) \times (-2x)) = +4x$ because (negative × negative) is positive

$-2(3 - 2x) = ((-2) \times 3) + ((-2) \times (-2x))$ ⟵ Multiply each term inside the bracket by –2

$= -6 + 4x$

15. **Say**: We will now practice multiplying with both positive and negative integers.

Guided Practice (*10 minutes*)

1. Ask pupils to continue to work in pairs.
2. Point to the questions on the board:

   Expand the following expressions:

a. $3(x + 4)$	b. $4(x - 1)$
c. $-5(x + 1)$	d. $3(2x - 4)$
e. $4(2x + 5)$	f. $-3(2 - 3y)$

3. **Say:** Use brackets to help you with your calculations as shown in the examples.
4. Walk around, if possible, to check the answers and clear up any misconceptions.
5. Have pupils from around the classroom volunteer to give their answers.
   Write the correct answers on the board. (Answers: a. $3(x + 4) = (3 \times x) + (3 \times 4) = 3x + 12$;
   b. $4(x - 1) = (4 \times x) + (4 \times (-1)) = 4x - 4$;
   c. $-5(x + 1) = (-5 \times x) + (-5 \times 1) = -5x - 5$;
   d. $3(2x - 4) = (3 \times 2x) + (3 \times (-4)) = 6x - 12$;
   e. $4(2x + 5) = (4 \times 2x) + (4 \times 5) = 8x + 20$;
   f. $-3(2 - 3y) = ((-3) \times 2) + ((-3) \times (-3y)) = -6 + 9y$.

Independent Practice (*10 minutes*)

1. Ask the pupils to work independently to answer the questions.
2. Point to the questions on the board:

   Expand the following expressions:

a. $4(3x - 1)$	b. $4(1 - 3x)$
c. $-5(x + 1)$	d. $-3(3x - 5)$
e. $-7(2 - 3x)$	f. $-5(-2 - 4y)$

3. Walk around, if possible, to check the answers and clear up any misconceptions.
4. Ask pupils to exchange their exercise books and check each other's work.
5. Have pupils from around the classroom volunteer to give their answers. Do not do the answers for Question d. Use it to check pupils' understanding of the work.

6. **Write** the correct answers on the board (except for Question d.) Explain Questions e. and f. in more detail if necessary. Pupils can multiply each part separately before solving the problem.
   (Answers: a. $4(3x-1) = (4 \times 3x) + (4 \times (-1)) = 12x - 4$;
   b. $4(1-3x) = (4 \times 1) + (4 \times (-3x)) = 4 - 12x$;
   c. $-5(x+1) = (-5 \times x) + (-5 \times 1) = -5x - 5$;
   d. $-3(3x-5) = (-3 \times 3x) + ((-3) \times (-5)) = -9x + 15$;
   e. $-7(2-3x) = (-7 \times 2) + ((-7) \times (-3x)) = -14 + 21x$;
   f. $-5(-2-4y) = ((-5) \times (-2)) + (-5) \times (-4y) = 10 + 20y$).

Closing (*2 minutes*)

1. **Say:** Please write your name on a piece of paper.
2. **Say**: Now write your working-out and answer for Questions d. on the paper. Hand the paper in at the end of the lesson.
3. Check the work done by pupils after the lesson. Use it as a guide for which pupils need additional assistance during the next lesson when pupils will be practicing solving problems with algebraic expressions.

Lesson Title: Multiplying Variables	Theme: Algebra	
Lesson Number: M-09-072	**Class/Level**: JSS 3	**Time**: 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to multiply 2 monomials with variables, applying the rules of indices.	None	1. Write on the board: 1. Use the laws of indices to answer the following: a. $x \times x$ b. $(x^2)^3$ c. $(3x)^2$ d. $xy \times xy$ e. $x \times 4x^2$ f. $3x^2 \times 6x^4$ 2. Write on the board: Examples of Monomials - x, y, 3, 7, x^2, y^2, 3xy, $6x^2y$ 2. Write on the board: Vocabulary List- monomial, co-efficient 3. Write the questions from the Guided Practice section on the board. 4. Write the questions from the Independent Practice section on the board.

Opening (*3 minutes*)

1. **Say:** Show how you use the laws of indices to answer Questions 1a. to 1c. on the board.
2. Allow 1 minute for pupils to answer the questions.
3. Have pupils volunteer to give their answers. (Answers: 1. a. $x \times x = x^{1+1} = x^2$; b. $(x^2)^3 = x^{2\times3} = x^6$; c. $(3x)^2 = 3^2 \times x^2 = 9x^2$).
4. **Say**: Today we are going to multiply 2 monomials with variables. We will apply the rules of indices to multiply them.

Introduction to the New Material (*10 minutes*)

1. **Say:** A monomial is an expression that consists of only one term. 'Mono' means one. Please look at the examples of monomials on the board. Monomials can be numbers, variables, or a mixture of both.
2. **Say:** Give me an example of a monomial with only numbers on the board. Raise your hand. (Example answers: 3, 7)
3. **Say:** Give me an example of a monomial with only variables. Raise your hand. (Example answers: x, y, x^2, y^2)
4. **Say:** 3xy and $6x^2y$ are examples of monomials which are a mixture of numbers and variables. The numbers 3 and 6 are the 'co-efficients' of the monomials. Monomials cannot have negative or fractional indices or powers. They have no operations like addition or subtraction in them.
5. **Say:** We are going to multiply 2 monomials with variables using the laws of indices.
6. Use a process which encourages pupils to work together called 'Think-Pair-Share'. It can be used with any topic to get pupils to share ideas.
7. **Say:** We will remind ourselves how to use the laws of indices to multiply.
8. **Say:** Look at the expressions in Questions d. and e. Spend 1 minute to 'think' about them. Then use the laws of indices to answer them.

9. After 1 minute ask pupils to 'pair' up with a neighbour to discuss their ideas and answers for another minute.
10. **Ask**: Who would like to 'share' their ideas with the class? Raise your hand.
11. Have pupils volunteer to share their ideas with the class. They should explain how they answered the questions.
12. **Write** the correct answers and steps on the board. Ask pupils to check their work. (Answers: iv. $xy \times xy = x^{1+1}y^{1+1} = x^2y^2$; v. $x \times 4x^2 = 4x^{1+2} = 4x^3$)
13. Show on the board how to calculate $3x^2 \times 6x^4$:

f. $3x^2 \times 6x^4 = 3 \times 6 \times x^{2+4}$ ← Multiply the co-efficients (3 and 6), use 1st law of indices to multiply x^2 and x^4

$= 18x^6$ ← Simplify

14. **Ask**: What do you notice about the answers to Questions d. to f.? Raise your hand.
15. Guide a pupil to say what they notice about the answers. (Example answers: The answers are themselves monomials; the answers are the same type of expression as the questions – just one term)

Guided Practice (*10 minutes*)

1. Ask pupils to continue to work in pairs.
2. Point to the questions on the board:

Multiply the monomials below. Simplify your answers.

a. $x^3 \times x^5$
b. $3y^4 \times y^6$
c. $4p^2 \times 7p^3$
d. $2x \times 8y$
e. $6x^2 \times 3xy$
f. $(x^4)^3$

3. Walk around, if possible, to check the answers and clear up any misconceptions.
4. Have pupils from around the classroom volunteer to give their answers.
Write the correct answers on the board. Ask pupils to check their work. (Answers: a. $x^3 \times x^5 = x^{3+5} = x^8$; b. $3y^4 \times y^6 = 3y^{4+6} = 3y^{10}$;
c. $4p^2 \times 7p^3 = 4 \times 7 \times p^{2+3} = 28p^5$; d. $2x \times 8y = 2 \times 8 \times x \times y = 16xy$;
e. $6x^2 \times 3xy = 6 \times 3 \times x^2 \times xy = 18x^3y$; f. $(x^4)^3 = x^{4\times3} = x^{12}$).

Independent Practice (*10 minutes*)

1. Ask the pupils to work independently to answer the questions.
2. Point to the questions on the board:

Multiply the monomials below. Simplify your answers.

a. $(rs \times rs)^2$
b. $(xy)^3$
c. $(p^3q^4)^2$
d. $(-5a^3)^2$
e. $(-3x^3y)^3$
f. $4(xy)^2$

3. Walk around, if possible, to check the answers and clear up any misconceptions.
4. Ask pupils to exchange their exercise books and check each other's work.
5. Have pupils from around the classroom volunteer to give their answers.

6. **Write** the correct answers on the board. Ask pupils to check the answers. (Answers: a. $(\text{rs} \times \text{rs})^2 = (\text{r}^2\text{s}^2)^2 = \text{r}^{2\times2}\text{s}^{2\times2} = \text{r}^4\text{s}^4$; b. $(\text{xy})^3 = \text{x}^3\text{y}^3$; c. $(\text{p}^3\text{q}^4)^2 = \text{p}^{3\times2} \times \text{q}^{4\times2} = \text{p}^6\text{q}^8$; d. $(-5\text{a}^3)^2 = (-5)^2\text{a}^{3\times2} = 25\text{a}^6$; e. $(-3\text{x}^3\text{y})^3 = (-3)^3\text{x}^{3\times3}\text{y}^3 = -27\text{x}^9\text{y}^3$; f $4(\text{xy})^2 = 4\text{x}^2\text{y}^2$.)

Closing (*2 minutes*)

1. **Ask:** What is a monomial? Raise your hand.
2. Select a pupil who has raised their hand. (Example answer: An expression that consists of only one term)
3. **Ask:** What type of expression do we get when we multiply monomials? Raise your hand.
4. Select a pupil who has raised their hand. (Example answer: We get another monomial)

Lesson Title: Multiplying an Algebraic Expression by a Variable	**Theme**: Algebra	
Lesson Number: M-09-073	**Class/Level**: JSS 3	**Time**: 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to expand an algebraic expression by multiplying an expression by a variable.	None	1. Write on the board: 1. Expand: g. $x \times x$ h. $x(x+3)$ i. $x(x-2)$ j. $-x(3-2x)$ 2. Write the questions from the Guided Practice section on the board. 3. Write the questions from the Independent Practice section on the board.

Opening (*3 minutes*)

1. **Say**: You should all be familiar by now how to solve $x \times x$. What is the answer? Raise your hand. (Answer: $x \times x = x^2$).
2. **Say:** We will use this fact in today's lesson when we expand algebraic expressions by multiplying the expression by a variable.

Introduction to the New Material (*10 minutes*)

1. Point to Question 1b. on the board.
2. **Say:** We want to expand $x(x+3)$. We use the same method as we did before when we multiplied by an integer. The only difference is that here, we are multiplying $x \times x$.
3. Show on the board how to solve Question 1b.
4. Multiply each term inside the bracket by x:
   - **Say**: $(x \times x) = x^2$
   - **Say:** $+(x \times 3) = +3x$. We put the number before the variable when we multiply a variable and a number.

$$x(x+3) = (x \times x) + (x \times 3) \quad \longleftarrow \text{Multiply each term inside the bracket by x}$$

$$= x^2 + 3x$$

5. Repeat this so it is clear what happens when we multiply an expression by a variable.
6. **Say:** You now have 1 minute to solve Question 1c.
7. Give pupils time to think and write down their ideas for 1 minute.
8. After 1 minute, ask them to pair up with their neighbour and discuss how to solve the problem for another minute.
9. **Ask**: Who would like to share their ideas with the class? Raise your hand.
10. Select a pupil who raised their hand to explain their answer step-by-step on the board. Ask other pupils to observe carefully to see if they agree with the calculation.
11. Correct any errors in the solution on the board. Ask pupils to check their work

$x(x-2) = (x \times x) + (x \times (-2))$ ← Multiply each term inside the bracket by x

$= x^2 - 2x$

12. Remind pupils about the rules of multiplying positive and negative variables and integers together:
    - **Say:** $(x \times x) = x^2$. We already know this.
    - **Say**: $\big((x) \times (-2)\big) = -2x$ because (negative × positive) is negative. We also put the number before the variable.
13. **Say**: Let us look at a question where we have to multiply by a variable with a negative sign.
14. Show on the board how to solve Question 1d.
15. Ask pupils to give you the answer when we multiply each term below by $-x$.
    Ask pupils about the rules of multiplying positive and negative variables and integers together.
    Give pupils some time to think after each question. Ask them to raise their hand to answer.
    - **Ask**: What is $\big((-x) \times 3\big)$? (Answer: $-3x$)
    - **Ask:** Who can explain why? (Example answers: Because (negative × positive) is negative; We put the number before the letter.)
    - **Ask:** What is $(-x) \times (-2x)$? (Answer: $= +2x^2$)
    - **Ask:** Who can explain why? (Example answers: Because (negative × negative) is positive; We already know that $x \times x = x^2$; We put the number before the variable.)

$-x(3-2x) = ((-x) \times 3) + ((-x) \times (-2x))$ ← Multiply each term inside the bracket by $-x$

$= -3x + 2x^2$

16. **Say**: We will now practice multiplying with both positive and negative integers.

Guided Practice (*10 minutes*)

1. Ask pupils to continue to work in pairs.
2. Point to the questions on the board:

   Expand the following expressions:

   g. $x(x+4)$
   h. $x(x-1)$
   i. $-x(x+1)$
   j. $x(2x-3)$
   k. $x(3x+5)$
   l. $-x(2-3x)$

3. **Say:** Please use brackets to help you with your calculations as shown in the examples. You can also multiply each part separately as rough work before solving the problem.
4. Walk around, if possible, to check the answers and clear up any misconceptions.
5. Have pupils from around the classroom volunteer to give their answers.
6. **Write** the correct answers on the board. Ask pupils to check their work. Explain Questions e. to f. in more detail if necessary.

(Answers: a. $x(x+4) = (x \times x) + (x \times 4) = x^2 + 4x$;
b. $x(x-1) = (x \times x) + (x \times (-1)) = x^2 - x$;
c. $-x(x+1) = (-x \times x) + (-x \times 1) = -x^2 - x$;
d. $x(2x-3) = (x \times 2x) + (x \times (-3)) = 2x^2 - 3x)$;
e. $x(3x+5) = (x \times 3x) + (x \times 5) = 3x^2 + 5x$;
f. $-x(2-3x) = ((-x) \times 2) + ((-x) \times (-3x)) = -2x + 3x^2$).

Independent Practice (*10 minutes*)

1. Ask the pupils to work independently to answer the questions.
2. Point to the questions on the board:

Expand the following expressions:

g. $x(3x-1)$	h. $x(1-3x)$
i. $-x(x+1)$	j. $-x(3x-5)$
k. $-x(2-3x)$	l. $-x(-2-4x)$
m. $2x(x-3) - 4(x-2)$	n. $3x(x+2) - x(6-x)$

3. Walk around, if possible, to check the answers and clear up any misconceptions.
4. Ask pupils to exchange their exercise books and check each other's work.
5. Have pupils from around the classroom volunteer to give their answers.
6. **Write** the correct answers on the board. Ask pupils to check their work. Explain Questions e. to h. in more detail if necessary.
   (Answers: a. $x(3x-1) = (x \times 3x) + \left(x \times (-1)\right) = 3x^2 - x$;
   b. $x(1-3x) = (x \times 1) + \left(x \times (-3x)\right) = x - 3x^2$;
   c. $-x(x+1) = (-x \times x) + (-x \times 1) = -x^2 - x$;
   d. $-x(3x-5) = (-x \times 3x) + \left((-x) \times (-5)\right) = -3x^2 + 5x$;
   e. $-x(2-3x) = (-x \times 2) + \left((-x) \times (-3x)\right) = -2x + 3x^2$;
   f. $-x(-2-4x) = \left((-x) \times (-2)\right) + (-x) \times (-4x) = 2x + 4x^2$).
   g. $2x(x-3) - 4(x-2) = (2x \times x) + (2x \times (-3)) - ((4 \times x) + (4 \times (-2))$
   $= 2x^2 - 6x - (4x - 8) = 2x^2 - 6x - 4x + 8 = 2x^2 - 6x - 4x + 8 = 2x^2 - 10x + 8$;
   h. $3x(x+2) - x(6-x) = (3x \times x) + (3x \times 2) - ((x \times 6) + \left(x \times (-x)\right)$
   $= 3x^2 + 6x - (6x - x^2) = 3x^2 + 6x - 6x + x^2 = 4x^2$).

Closing (*2 minutes*)

1. **Say:** Please remember, it can become very confusing when multiplying positive and negative integers and variables. We can do the question in parts. We can do each part of the multiplication as rough work and then put the answer into our expression.
2. **Write** on the board: $2 - x(6-x)$
3. **Ask**: How can we answer this question? Raise your hand.
4. Guide a pupil to say we can do the question in parts. (Answer: We can calculate $x(6-x)$ separately to get $6x - x^2$)
5. Show how this works on the board.

$$2 - x(6-x) = 2 - (6x - x^2)$$
$$= 2 - 6x + x^2$$

6. **Say**: Good work class! In the next lesson, we will put all this together when we solve story problems.

Lesson Title: Algebraic Expressions Story Problems	**Theme**: Algebra	
Lesson Number: M-09-074	**Class/Level**: JSS 3	**Time:** 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to write and simplify algebraic expressions for situations in story problems.	None	1. Write on the board: 1. Simplify: a. $2(x+1)+x(3+x)$ b. $(2p^2q^3)^4$ 2. Write an expression for: a. The perimeter of a triangle with sides x, $x-3$ and $x+3$ b. The area of a square with side length $2x$ c. The combined ages of 4 members of a family aged x, x^2, x^2-5 and $x+7$ 2. Write the questions from the Guided Practice section on the board. 3. Write the questions from the Independent Practice section on the board.

Opening (*3 minutes*)

1. **Say**: Please simplify the expressions (Questions 1a. and 1b.) on the board. Raise your hand when you finish. You have 2 minutes.
2. Select a pupil who has raised their hand to give the answers to the problems. (Answers:
   a. $2(x+1)+x(3+x)=2x+2+3x+x^2=x^2+2x+6x+2=x^2+5x+2$.
   b. $(2p^2q^3)^4=2^4p^{2\times4}q^{3\times4}=16p^8q^{12}$)
3. **Say**: Today we are going to write and simplify algebraic expressions for situations in story problems.

Introduction to the New Material (*10 minutes*)

1. **Say:** Question 2a. is asking us to find an expression for the perimeter of a triangle with sides x, $x-3$ and $x+3$.
2. **Ask:** How do we find the perimeter of a triangle? Raise your hand. (Example answer: Add the 3 sides.)
3. Show how to find the expression for the perimeter on the board.
   Note: Omit the equal sign so pupils do not confuse the expression for an equation.

$x+(x-3)+(x+3)$	⟵	Add the sides of the triangle
$x+x+x-3+3$	⟵	Collect like terms
$3x$	⟵	Perimeter of triangle with sides x, $x-3$ and $x+3$.

4. **Say:** Question 2b. is asking us to find an expression for area of a square. Take a few moments to think about the formula for the area of a square. Raise your hand if you know the answer when the sides are 2x.

5. Select a pupil with a raised hand to explain their answer step by step on the board. Ask other pupils to observe carefully to see if they agree with the calculation.
6. Correct any errors in the solution on the board. An example is shown below.

$2x \times 2x$	←	Multiply the sides of the square
$2 \times 2 \times x^{1+1}$	←	Use 1st law of indices
$4x^2$	←	Area of a square of side length 2x

7. **Say**: Work in pairs to answer Question 1c. You have 2 minutes.
8. **Ask**: Who would like to show us how to solve Question 1c. on the board? Raise your hand.
9. Select a pupil who raised their hand to explain their answer step-by-step on the board. Ask other pupils to observe carefully to see if they agree with the calculation.
10. Correct any errors in the solution on the board. Ask pupils to check their work. An example is shown below:

$x + x^2 + x^2 - 5 + x + 7$	←	Add the ages
$x + x + x^2 + x^2 - 5 + 7$	←	Collect like terms
$2x^2 + 2x + 2$	←	Combined ages

Guided Practice (*10 minutes*)

1. Ask pupils to work in pairs.
2. Point to the questions on the board:
   Write an expression for:
   a. the area of a rectangle with sides $2x$ and $x - 3$
   b. the width of a rectangle with area $8x^2$ and length $4x$
   c. the area of a circle whose radius is $2xy$

3. Walk around, if possible, to check the answers and clear up any misconceptions.
4. Have pupils from around the classroom volunteer to give their answers to the questions.
5. **Write** the correct answers on the board. Ask pupils to check their work. (Answers: a. (Area of a rectangle = length × width), $2x \times (x - 3) = 2x^{1+1} - 6x = 2x^2 - 6x$;
   b. (Area of a rectangle = length × width), so width = $\frac{\text{Area}}{\text{length}}, \frac{8x^2}{4x} = 2x$;
   c. (Area of circle = πr^2), $\pi \times (2xy)^2 = \pi \times 2xy \times 2xy = 2 \times 2 \times \pi \times x^{1+1} \times y^{1+1} = 4\pi x^2 y^2$)

Independent Practice (*10 minutes*)

1. Ask the pupils to work independently to answer the questions.
2. Point to the questions on the board:
   Write an expression for:
   a. The volume of a cuboid with length = x, width = 2x and height= $x + 5$
   b. The area of a rectangle with sides x and $x + 2$
   c. The area of a rectangle with sides $x + 1$ and $x + 2$. Use your answer to part b.

3. Walk around, if possible, to check answers and clear misconceptions.

4. Have pupils from around the classroom volunteer to give their answers to the questions.
5. **Write** the correct answers on the board. Ask pupils to check their work. (Answers: a. (Volume of cuboid = length × width × height), $x \times 2x \times (x + 5) = 2 \times x^{1+1+1} + 2 \times 5 \times x^{1+1} = 2x^3 + 10x^2$; b. (Area of a rectangle = length × width), $x \times (x + 2) = x^{1+1} + 2x = x^2 + 2x$;
   c. Expression for area of the rectangle is $(x + 1) \times (x + 2) = x \times (x + 2) + 1 \times (x + 2) = x^2 + 2x + (x + 2)$, using the answer from part b. This gives the expression $x^2 + 3x + 2$ for the area of the rectangle.

Closing (*2 minutes*)

1. **Say:** We can write the algebraic expressions for many situations in story problems. Please write an expression for the volume of a cube with sides 3x. Raise your hand when you finish.
2. Select a pupil who has raised their hand. (Answer: Volume of cube = $(\text{length})^3$, $(3x)^3 = 3^3 \times x^3 = 27x^3$)

Lesson Title: Introduction to Quadratic Equations	**Theme**: Algebra	
Lesson Number: M-09-075	**Class/Level**: JSS 3	**Time**: 35 minutes

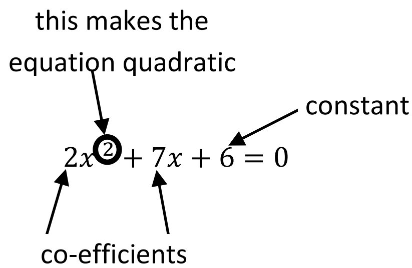
Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to identify a quadratic equation as one of the form $ax^2 + bx + c = 0$.	None	1. Write on the board: 1. Simplify: a. $x(2x + 5) + 2(x + 3)$ 2. Quadratic or Not Quadratic? a. $3x^2 + 11x + 6 = 0$ b. $x^2 - 4x = 0$ c. $7x + 4 = 0$ d. $x^2 = 4x + 2$ 2. Write on the board: Vocabulary List- quadratic, variable, co-efficient, constant 3. Write the questions from the Guided Practice section on the board. 4. Write the questions from the Independent Practice section on the board.

Opening (*3 minutes*)

1. **Say**: Please simplify Question 1a. on the board.
2. Allow 2 minutes for pupils to answer the question.
3. Have a pupil from the front of the classroom volunteer to explain their answer on the board. Ask other pupils to observe carefully to see if they agree with the calculation.
4. Correct any errors in the solution on the board. Ask pupils to check their work. (Answer: $x(2x + 5) + 2(x + 3) = 2x^2 + 5x + 2x + 6 = 2x^2 + 7x + 6$)
5. **Say**: Today we are going to identify a quadratic equation as one of the form $ax^2 + bx + c = 0$.

Introduction to the New Material (*10 minutes*)

1. Point to the result of Question 1a on the board.
2. **Ask**: How can we make this expression into an equation? Raise your hand.
3. Give pupils a few moments to think. Select a pupil from the back of the classroom to answer. (Answer: By adding an equal sign; by making it equal to 0. Accept all reasonable answers.)
4. **Say:** We can make this expression into an equation by making it equal to 0. When we make this expression into an equation we get a special type of equation called a ‘quadratic equation’.
5. Point to the features of a quadratic equation as you say the following statements.
6. **Say**: The quadratic equation is identified by these main features:
   - A variable, usually x
   - The ‘highest’ power of x in the equation is 2. This x^2 term is what makes it a quadratic equation. Without this term it will not be a quadratic equation.
   - It usually has 3 terms, though that is not always the case, it can also have 2 or just 1.
   - The numbers 2 and 7 are examples of co-efficients of each term. They are used to multiply the variables.
   - The 6 is the constant term. It does not change when x changes.



7. **Say:** The equation on the board is an example of a quadratic equation. The general or standard form of all quadratic equations look like this.
8. **Write:**

$$ax^2 + bx + c = 0$$

9. **Say:** You will always know the values of a, b and c. 'a' cannot be zero as there will be no x^2 term if it is zero.
10. **Say:** Look at the examples of the equations on the board, Questions 2a to 2d.
11. **Say:** Work in pairs. Please discuss your ideas and decide between you which of them are quadratic and which are not quadratic. If they are quadratic, write down the values of a, b and c for each one. Raise your hand when you finish.
12. Have pupils from around the classroom volunteer to give their answers.
13. **Write** the correct answers on the board. Ask pupils to check their work. (Answers: a. $3\mathrm{x}^2 - 11\mathrm{x} + 6 = 0$, quadratic, a = 3, b = –11, c = 6; b. $\mathrm{x}^2 - 4\mathrm{x} = 0$, quadratic, a = 1, b = –4, c = 0; c. $7\mathrm{x} + 4 = 0$; not quadratic, no x^2 term (it is a linear equation); d. $\mathrm{x}^2 = 4\mathrm{x} + 2$, quadratic, change to general form – subtract $(4\mathrm{x} + 2)$ from both sides of the equation, $\mathrm{x}^2 - 4\mathrm{x} - 2 = 0$, a = 1, b = –4, c = –2)

Guided Practice (*10 minutes*)

1. Ask pupils to continue to work in pairs.
2. Point to the questions on the board:

State which of these equations are quadratic and which are not quadratic.
If it is quadratic, write down the values of a, b and c.

m. $4\mathrm{x}^2 + 3\mathrm{x} + 5 = 0$	n. $\mathrm{y}^2 - \mathrm{y} + 6 = 0$
o. $5\mathrm{x} - 3 = 0$	p. $\mathrm{x}^2 + 11\mathrm{x} = 0$
q. $\mathrm{z}^3 + \mathrm{z}^2 + 6 = 0$	r. $\mathrm{w}^2 = 7$

3. Walk around, if possible, to check the answers and clear up any misconceptions.
4. Have pupils from around the classroom volunteer to give their answers.
5. **Write** the correct answers on the board. Ask pupils to check their work. (Answers: a. $4\mathrm{x}^2 + 3\mathrm{x} + 5 = 0$), quadratic, a = 4, b = 3, c = 5; b. $\mathrm{y}^2 - \mathrm{y} + 6 = 0$, quadratic, a = 1, b = –1, c = 6; c. not quadratic, no x^2 term (linear equation); d. $\mathrm{x}^2 + 11\mathrm{x} = 0$, quadratic, a = 1, b = 11, c = 0; e. $\mathrm{z}^3 + \mathrm{z}^2 + 6 = 0$, not quadratic (cubic equation – highest power of z is 3); f. $\mathrm{w}^2 = 7$, quadratic, change to general form – subtract 7 from both sides, $\mathrm{w}^2 - 7 = 0$, a = 1, b = 0, c = –7).

Independent Practice (*10 minutes*)

1. Ask the pupils to work independently to answer the questions.
2. Point to the questions on the board:

State which of these equations are quadratic and which are not quadratic.
If it is quadratic, write down the values of a, b and c.

a. $5\mathrm{x}^2 + 6\mathrm{x} - 1 = 0$	b. $\mathrm{x}^2 = \mathrm{x} + 4$
c. $\mathrm{x}^2 + \mathrm{x}^3 - 2 = 0$	d. $3\mathrm{x} - 4 = \mathrm{x} + 1$

e. $2(w^2 - 2w) = 5$ f. $p(p - 1) = 3$

3. Walk around, if possible, to check the answers and clear up any misconceptions.
4. Have pupils from around the classroom volunteer to give their answers.
5. **Write** the correct answers on the board. Ask pupils to check their work. (Answers: a. $5x^2 + 6x - 1 = 0$, quadratic, a = 5, b = 6, c = –1; b. $x^2 = x + 4$, quadratic, change to genera term – subtract $(x + 4)$ from both sides, $x^2 - x - 4 = 0$, a = 1, b = –1, c = –4;
   c. $x^2 + x^3 - 2 = 0$, not quadratic (cubic); d. $3x - 4 = x + 1$, not quadratic, no x^2 term, (linear equation), e. $2(w^2 - 2w) = 5$, quadratic, change to general form – expand brackets, subtract 5 from both sides, $2w^2 - 4w - 5 = 0$, a = 2, b = –4, c = –5; f. $p(p - 1) = 3$, quadratic, change to general form – expand brackets, subtract 3 from both sides; $p^2 - p - 3 = 0$, a = 1, b = –1, c= –3)

Closing (*2 minutes*)

1. **Ask:** What is the highest power of the variable in a quadratic equation? Raise your hand.
2. Select a pupil who has raised their hand. (Example answers: 2, x^2, y^2, w^2)
3. **Ask:** What is the general form of a quadratic equation? Raise your hand.
4. Select a pupil who has raised their hand. (Answer: $ax^2 + bx + c = 0$)
5. **Ask:** What can we do to change a quadratic equation that is not in the general form? Raise your hand.
6. **Say:** Select a pupil who has raised their hand. (Example answers: Expand brackets, perform inverse operations to both sides of the equation)

Lesson Title: Multiplying 2 Binomials	**Theme:** Algebra	
Lesson Number: M-09-076	**Class/Level**: JSS 3	**Time:** 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to: 1. Identify the FOIL (First Outside Inside Last) method as a rule for multiplying (expanding) 2 binomials. 2. Multiply 2 binomials.	None	1. Write on the board: <u>Examples of Binomials</u> $3x + 4$, $2x + 5$, $y - 2$, $5xy - x^2$ 2. Write on the board: <u>Vocabulary List</u>: binomial 3. Write the questions from the Guided Practice section on the board. 4. Write the questions from the Independent Practice section on the board.

Opening (*3 minutes*)

1. **Say**: We looked at monomials a little while ago. Please write down one fact and give one example of a monomial.
2. Allow 1 minute for pupils to answer the question.
3. Have pupils volunteer to give their answers including examples of monomial. (Example answers: An expression with one term; monomials are numbers, variables, or a mixture of both, they have no operations like addition or subtraction in them; examples of monomials are 4, x, $3x^3$, a^3b, etc.)
4. **Say**: Today we are going to identify the FOIL (First Outside Inside Last) method as a rule for multiplying (expanding) 2 binomials. We will also multiply 2 binomials using FOIL.

Introduction to the New Material (*10 minutes*)

1. **Say:** Please look at the examples of binomials on the board. What do you notice about them? Are there any similarities or difference with monomials? Share your ideas with your neighbour.
2. Allow 1 minute for pupils to discuss and share ideas.
3. Have 2-3 pupils volunteer to share their ideas with the class. (Example answers: Similarities – they are numbers, variables or a mixture of both; differences – binomials have 2 terms; they have operations like addition and subtraction in them)
4. **Say:** A binomial is an expression containing 2 terms. This is similar to binary numbers which are numbers which contain only 2 numerals, 0 and 1. The terms are either added to, or subtracted from each other. The terms can be numbers, variables or a mixture of both.
5. **Say:** Let us quickly remind ourselves how to multiply 2 monomials. Calculate $3x \times 2x$.
6. Allow a few moments for pupils to calculate the answer. They should by now be able to do these types of calculations in their heads.
7. Have a pupil volunteer to give the answer. (Answer: $3x \times 2x = 6x^2$)
8. **Say:** Just as with monomials, we can multiply 2 binomials together.
9. **Write** on the board: $(3x + 4) \times (2x + 5)$
10. **Ask:** How many terms are there altogether in the 2 binomials? Raise your hand.
11. Select a pupil to give the answer. (Answer: 4)
12. **Say:** We have to make sure we multiply every term in each bracket by every one of the other terms. We use a method called FOIL to help us do this.

FOIL

F irst
Outer
I nner
L ast

13. **Write** FOIL on the board as shown on the right.
14. **Say:** FOIL gives the order in which we are to multipy the terms together. F is for First, O for Outer, I for Inner and L for Last.
15. **Say:** The 2 binomial expressions have 4 terms. There are also 4 multiplications to do when we multiply them together, one for each of the letters in FOIL.
16. Show on the board how to use FOIL to multiply 2 binomials. Wait for pupils to provide the answer for each calculation before you write it on the board.
17. Step-by-step procedure for FOIL:

$$(3x+4)\times(2x+5) \quad = \quad (3x+4)(2x+5)$$

• **Say:** Multiply the <u>F</u>irst two terms $3x \times 2x$	$(3x+4)(2x+5)$	=	F $6x^2$
• **Say:** Multiply the <u>O</u>uter terms $3x \times 5$	$(3x+4)(2x+5)$	=	F O $6x^2+15x$
• **Say:** Multiply the <u>I</u>nner terms $4 \times 2x$	$(3x+4)(2x+5)$	=	F O I $6x^2+15x+8x$
• **Say:** Multiply the <u>L</u>ast terms 4×5	$(3x+4)(2x+5)$	=	F O I L $6x^2+15x+8x+20$
• **Say**: Collect the x terms	$(3x+4)(2x+5)$	=	$6x^2+23x+20$

18. **Say:** We must always remember to collect any like terms once we have finished multiplying.

Guided Practice (*10 minutes*)

1. Ask pupils to work in pairs and follow the procedure on the board to multiply 2 binomials.
2. Point to the questions on the board:

   Multiply these binomials:
   a. $(x+2)(x+3)$ b. $(x+5)(x-2)$
   c. $(x+5)(2-x)$ d. $(x-5)(x-3)$
   e. $(5-x)(2-x)$ f. $(x^2+4)(x^2+3)$

3. **Say:** Take note that in Question f, we have x^2 instead of x.
4. Walk around, if possible, to check the answers and clear up any misconceptions.
5. **Ask**: Who would like to explain to the class how to multiply the binomials? Raise your hand.
6. Select pupils who raised their hand to explain their answer step-by-step on the board for each question.
7. Discuss any problems that pupils may have had in applying FOIL to multiply the 2 binomials.
8. Correct any errors in the solution on the board. Ask pupils to check their work.
   (Answers: a. $(x+2)(x+3) = x^2+3x+2x+6 = x^2+5x+6$;
   b. $(x+5)(x-2) = x^2-2x+5x-10 = x^2+3x-10$;
   c. $(x+5)(2-x) = 2x-x^2+10-5x = -x^2-3x+10$;
   d. $(x-5)(x-3) = x^2-3x-5x+15 = x^2-8x+15$;
   e. $(5-x)(2-x) = 10-5x-2x+x^2 = x^2-7x+10$;
   f. $(x^2+4)(x^2+3) = x^4+3x^2+4x^2+12 = x^4+7x^2+12$).

Independent Practice (*10 minutes*)

1. Ask the pupils to work independently to answer the questions.
2. Point to the questions on the board:

Multiply these binomials:

a. $(x + 4)(x + 5)$ b. $(x - 5)(3 - x)$

c. $(2x - 1)(x + 2)$ d. $(3x - 1)(2x + 2)$

e. $(2x^2 + 4)(x - 3)$ f. $(3x^2 - 5)(x^2 - 2)$

3. Walk around, if possible, to check the answers and clear up any misconceptions.
4. Ask pupils to exchange their exercise books and check each other's work.
5. Have pupils from around the classroom volunteer to give their answers. Do not do the answer for Question d. Use it to check pupils' understanding of the work.
6. **Write** the correct answers on the board (except for Question d.). Ask pupils to check the answers. (Answers: a. $(x + 4)(x + 5) = x^2 + 5x + 4x + 20 = x^2 + 9x + 20$;
   b. $(x - 5)(3 - x) = 3x - x^2 - 15 + 5x = -x^2 + 8x - 15$;
   c. $(2x - 1)(x + 2) = 2x^2 + 4x - x - 2 = 2x^2 + 3x - 2$;
   d. $(3x - 1)(2x + 2) = 6x^2 + 6x - 2x - 2 = 6x^2 + 4x - 2$;
   e. $(2x^2 + 4)(x - 3) = 2x^3 - 6x^2 + 4x - 12$;
   f. $(3x^2 - 5)(x^2 - 2) = 3x^4 - 6x^2 - 5x^2 + 10 = 3x^4 - 11x^2 + 10$).

Closing (*2 minutes*)

1. **Say:** Please write your name on a piece of paper.
2. **Say**: Write your working out and answer for Question d. on the paper. Hand the paper in at the end of the lesson.
3. Check the work done by the pupils after the lesson. Use it as a guide for which pupils need additional assistance during the next lesson when pupils will be multiplying (expanding) 2 binomials to form a quadratic equation.

[STEP-BY-STEP PROCEDURE FOR FOIL]

$(3x + 4) \times (2x + 5) = (3x + 4)(2x + 5)$

- **Say:** Multiply the First two terms $3x \times 2x$ $(3x + 4)(2x + 5) = 6x^2$ (F)
- **Say:** Multiply the Outer terms $3x \times 5$ $(3x + 4)(2x + 5) = 6x^2 + 15x$ (F O)
- **Say:** Multiply the Inner terms $4 \times 2x$ $(3x + 4)(2x + 5) = 6x^2 + 15x + 8x$ (F O I)
- **Say:** Multiply the Last terms 4×5 $(3x + 4)(2x + 5) = 6x^2 + 15x + 8x + 20$ (F O I L)
- **Say**: Collect the x terms $(3x + 4)(2x + 5) = 6x^2 + 23x + 20$

Lesson Title: Practice with Multiplying 2 Binomials	**Theme**: Algebra	
Lesson Number: M-09-077	**Class/Level**: JSS 3	**Time:** 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to multiply (expand) 2 binomials to form a quadratic equation.	None	1. Write on the board: 1. Multiply: a. $(2x + 5)(x + 3)$ 2. Expand and simplify: a. $(6x - 5)(3x - 7)$ b. $(x + 1)^2$ 2. Write on the board: Vocabulary List: expand 3. Write the questions from the Guided Practice section on the board. 4. Write the questions from the Independent Practice section on the board.

Opening (*3 minutes*)

1. **Say**: Please write down the answers to Questions 1a. on the board.
2. Allow 1 minute for pupils to answer the question.
3. Have a pupil volunteer to give their answer.
   (Answers: $(2x + 5)(x + 3) = 2x^2 + 6x + 5x + 15 = 2x^2 + 11x + 15$).
4. **Ask**: What method did you use to multiply the binomials? Raise your hand. (Example answer: FOIL. Accept all reasonable answers.)
5. **Ask**: What type of equation do we have as our answer? Raise your hand. (Answer: Quadratic equation)
6. **Say**: Today we are going to multiply (expand) 2 binomials to form a quadratic equation.

Introduction to the New Material (*10 minutes*)

1. **Say:** Last lesson, we looked at multiplying 2 binomial expressions. Most of the solutions we got were in the form of a quadratic equation.
2. **Say:** When we multiply 2 expressions as we have just done, we call that ‘expanding’. When we say to "expand" an expression or equation, we mean to multiply out the brackets.
3. **Say**: Let us look at a few more examples of expanding 2 binomials to form a quadratic equation.
4. **Ask**: Who would like to come to the board to explain how to expand Question 2a.?
5. Select a pupil who raised their hand to explain their answer step-by-step on the board. Ask other pupils to observe carefully to see if they agree with the calculation.
6. Encourage the pupil to use the FOIL method. Guide them if they are unclear. However, there are other methods which are valid and should be accepted.
7. Correct any errors in the solution on the board. The solution using FOIL is shown below.

$$
\begin{aligned}
(6x - 5)(3x - 7) &= 18x^2 - 42x - 15x + 35 \\
&= 18x^2 - 57x + 35
\end{aligned}
$$

8. **Say**: Please take a look at Question 2b. Work with your neighbour to discuss and share ideas. Raise your hand when you finish.

9. Allow pupils 1 minute to discuss. Have a pupil volunteer to come to the board and show their solution. Ask other pupils to observe carefully to see if they agree with the calculation.
10. Correct any errors in the solution on the board. Ask pupils to check their work.

$$\begin{aligned}(x+1)^2 &= (x+1)(x+1)\\ &= x^2+x+x+1\\ &= x^2+2x+1\end{aligned}$$

11. **Say**: Just as x^2 means $x \times x$ so $(x+1)^2$ means $(x+1) \times (x+1)$. The same applies to higher powers of any expression.

Guided Practice (*10 minutes*)

1. Ask pupils to continue to work in pairs.
2. Point to the questions on the board:

   Expand and simplify:
   - a. $(4x+2)(x+3)$
   - b. $(x+2)^2$
   - c. $(x+2)(x+4)+(x+1)(x+2)$
   - d. $(x+3)(x+7)+(x-1)(x+5)$

3. **Say:** Use BODMAS to help you with any of the calculations. You can add additional brackets if you wish.
4. Walk around, if possible, to check the answers and clear up any misconceptions.
5. Have pupils from around the classroom volunteer to give their answers.
6. **Write** the correct answers on the board. Ask pupils to check their work. (Answers: a. $(4x+2)(x+3)=4x^2+12x+2x+6=4x^2+14x+6$;
   b. $(x+2)^2=(x+2)(x+2)=x^2+2x+2x+4=x^2+4x+4$;
   c. $(x+2)(x+4)+(x+1)(x+2)=(x^2+4x+2x+8)+(x^2+2x+x+2)$ $=(x^2+6x+8)+(x^2+3x+2)=x^2+x^2+6x+3x+8+2=2x^2+9x+10$;
   d. $(x+3)(x+7)+(x-1)(x+5)=(x^2+7x+3x+21)+(x^2+5x-x-5)$ $=(x^2+10x+21)+(x^2+4x-5)=x^2+x^2+10x+4x+21-5=2x^2+14x+16$).

Independent Practice (*10 minutes*)

1. Ask the pupils to work independently to answer the questions.
2. Point to the questions on the board:

   Expand and simplify:
   - a. $(x+2)(x-2)$
   - b. $(4x+5)^2$
   - c. $(3x+2)(5x+9)+(4x-2)(3x-5)$
   - d. $(4x+6)(5x+1)-(2x+3)(3x+1)$

3. Walk around, if possible, to check the answers and clear up any misconceptions.
4. Ask pupils to exchange exercise books and check each other's work.
5. Have pupils from around the classroom volunteer to give their answers.
6. **Write** the correct answers on the board. Ask pupils to check their work. (Answers: a. $(x+2)(x-2)=x^2-2x+2x-4=x^2-4$;
   b. $(4x+5)^2=(4x+5)(4x+5)=16x^2+20x+20x+25=16x^2+40x+25$;
   c. $(3x+2)(5x+9)+(4x-2)(3x-5)=(15x^2+27x+10x+18)+(12x^2-20x-6x+10)=(15x^2+37x+18)+(12x^2-26x+10)=15x^2+12x^2+37x-26x+18+10$ $=27x^2+11x+28$;

d. $(4x + 6)(5x + 1) - (2x + 3)(3x + 1) = (20x^2 + 4x + 30x + 6) - (6x^2 + 2x + 9x + 3)$
$= (20x^2 + 34x + 6) - (6x^2 + 11x + 3) = 20x^2 - 6x^2 + 34x - 11x + 6 - 9 =$
$= 14x^2 + 23x - 3$).

Closing (*2 minutes*)

1. **Say:** Write down in your pairs 2 different things you learned today.
2. Allow pupils 1 minute to discuss and share their ideas.
3. Have one pupil from the front, and one from the back of the classroom volunteer to answer. (Example answers: How to square a binomial; how to expand and simplify several binomials)

Lesson Title: Review of Factorisation: Integers	Theme: Algebra	
Lesson Number: M-09-079	**Class/Level**: JSS 3	**Time**: 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to: 1. Identify that factorisation involves using division to break an expression into parts. 2. Identify and factor integers that are common factors in an algebraic expression.	None	1. Write on the board: 1. Find the highest common factor (HCF) of: a. 2 and 8 b. 10 and 25 c. Expand $2(x + 3)$ 2. Factorise: a. $6x + 9$ b. $8y + 12$ 2. Write on the board: <u>Vocabulary List</u>: factorisation 3. Write the questions from the Guided Practice section on the board. 4. Write the questions from the Independent Practice section on the board.

Opening (*3 minutes*)

1. **Say**: Please write down the answers to Questions 1a. and 1b. on the board.
2. Allow 1 minute for pupils to answer the question.
3. Have pupils volunteer to give their answers. (Answers: 1a. Factors of 2 are: { 1, 2 }, factors of 8 are
{ 1, 2 ,4, 8 }, HCF of 2 and 8 is 2; 1b. Factors of 10: { 1, 2, 5, 10 }, factors of 25: { 1, 5, 25 }, HCF of 10 and 25 is 5.)
4. **Say**: Today we are going to identify that factorisation involves using division to break an expression into parts. We will identify and factor integers that are common factors in an algebraic expression.

Introduction to the New Material (*10 minutes*)

1. **Say:** Who can remind the class how to expand brackets in the expression $2(x + 3)$ shown in Question 1c.? Raise your hand.
2. Select a pupil who raised their hand to give the answer. (Answer: $2(x + 3) = 2 \times x + 2 \times 3 = 2x + 6$)
3. **Say:** The process we are going to do today is the opposite of expanding a bracket. We multiply to expand a bracket. Today, we will take an expression like $2x + 6$ and use division to find all its factors.
4. **Say:** This is called factorisation and it breaks an expression into its smallest parts.
5. **Ask:** What is the HCF in $2(x + 3)$? Raise your hand. (Answer: 2)
6. **Say:** The HCF, 2, breaks up the expression into 2 parts: 2 and $(x + 3)$. We cannot break it any smaller.
7. **Say**: The HCF is written outside the brackets and the other part is written inside the brackets: $2(x + 3)$. We will find HCFs for other similar expressions. You will sometimes have to do rough work to find the HCF before factorising the expression.
8. **Say:** Let us factorise $6x + 9$. We will do it in stages so you remember the step-by-step method.
9. Show on the board how to factorise the expression $6x + 9$.
10. **Ask**: What is the HCF of 6 and 9? Raise your hand.
11. Allow pupils time to find the HCF, then have a pupil volunteer to answer. (Answer: 3)

$6x + 9$	$= 3(\qquad)$	←	Write the empty bracket to remind pupils to find the common factor first
	$3(2x + 3)$	←	Divide each term in $6x + 9$ by 3

Check

$3(2x + 3)$	$= (3 \times 2x) + (3 \times 3)$	←	Multiply each term in the brackets by 3
	$= 6x + 9$	←	Check takes us back to the original expression

12. **Say:** Let us factorise $8y + 12$. Who would like to come to the board and explain to the class how to do Question 2b.?
13. Select a pupil who has raised their hand to explain Question 2b. on the board. Tell the pupil they must check their answer by expanding the brackets.
14. Ask other pupils to observe carefully to see if they agree with the calculation.
15. Correct any errors in the solution on the board.

$8y - 12$	$= 4(\qquad)$	←	HCF = 4. Write outside the brackets
	$4(2y - 3)$	←	Divide each term in $8y - 12$ by 4

Check

$4(2y - 3)$	$= (4 \times 2y) - (4 \times 3)$	←	Multiply each term in the brackets by 4
	$= 8y - 12$	←	Check takes us back to the original expression

16. **Say:** If our check does not take us back to the original expression, we know we have made a mistake and need to look at our work again.

Guided Practice (*10 minutes*)

1. Ask pupils to work in pairs.
2. Point to the questions on the board:

Factorise the expressions below. Please check all answers.

a. $10x + 5$
b. $7t + 21$
c. $5 - 15p$
d. $4x + 4$
e. $12u - 3$
f. $8s + 2t$

3. Walk around, if possible, to check the answers and clear up any misconceptions.
4. Have pupils from around the classroom volunteer to give their answers.
Write the correct answers on the board. Ask pupils to check their work. (Answers: a. $10x + 5 = 5(2x + 1)$, Check: $5(2x + 1) = (5 \times 2x) + (5 \times 1) = 10x + 5$;
b. $7t + 21 = 7(t + 3)$, Check: $7(t + 3) = (7 \times t) + (7 \times 3) = 7t + 21$;
c. $5 - 15p = 5(1 - 3p)$, Check: $5(1 - 3p) = (5 \times 1) + (5 \times (-3p)) = 5 - 15p$;
d. $4x + 4 = 4(x + 1)$, Check: $4(x + 1) = (4 \times x) + (4 \times 1) = 4x + 4$;

e. $12u - 3 = 3(4u - 1)$, Check: $3(4u - 1) = (3 \times 4u) + (3 \times (-1)) = 12u - 3$;
f. $8s + 2t = 2(4s + t)$, Check: $2(4s + t) = (2 \times 4s) + (2 \times t) = 8s + 2t$).

Independent Practice (*10 minutes*)

1. Ask the pupils to work independently to answer the questions.
2. Point to the questions on the board:

Factorise the expressions below. Please check all answers.

a. $12x + 6$ b. $5p - 10q$
c. $3ab + 3de$ d. $4x^2 + 2$
e. $15ut - 3$ f. $3a^2 - 6b + 3$

3. Walk around, if possible, to check the answers and clear up any misconceptions.
4. Ask pupils to exchange their exercise books and check each other's work.
5. Have pupils from around the classroom volunteer to give their answers.
6. **Write** the correct answers on the board. Ask pupils to check the answers. (Answers: a. $12x + 6 = 6(2x + 1)$, Check: $6(2x + 1) = (6 \times 2x) + (6 \times 1) = 12x + 6$;
b. $5p - 10q = 5(p - 2q)$, Check: $5(p - 2q) = (5 \times p) + \left(5 \times (-2q)\right) = 5p - 10q$;
c. $3ab + 3de = 3(ab + de)$, Check: $3(ab + de) = (3 \times ab) + (3 \times de) = 3ab + 3de$;
d. $4x^2 + 2 = 2(2x^2 + 1)$, Check: $2(2x^2 + 1) = (2 \times 2x^2) + (2 \times 1) = 4x^2 + 2$;
e. $15ut - 3 = 3(5ut - 1)$, Check: $3(5ut - 1) = (3 \times 5ut) + \left(3 \times (-1)\right) = 15ut - 3$;
f. $3a^2 - 6b + 3 = 3(a^2 - 2b + 1)$, Check: $3(a^2 - 2b + 1)$
$= (3 \times a^2) + (3 \times (-2b) + (3 \times 1) = 3a^2 - 6b + 3$).

Closing (*2 minutes*)

1. **Ask:** Who would like to tell the class one important thing they can do to correct errors when they factorise an expression? Raise your hand.
2. Select a pupil who has raised their hand. (Example answer: Check the solution by expanding the bracket. If our check does not take us back to the original expression, we know we have made a mistake and need to look at our work again)
3. **Say:** Good job, class! In the next lesson, we will be looking at factorising variables that are common factors in an algebraic expression.

Lesson Title: Review of Factorisation: Variables	**Theme**: Algebra	
Lesson Number: M-09-079	**Class/Level**: JSS 3	**Time:** 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to identify and factor variables that are common factors in an algebraic expression.	None	1. Write on the board: 1. Factorise: a. $5 - 25\text{x}$ b. $4\text{a}^2 - 4$ c. $\text{x}^2 + 4\text{x}$ d. $8\text{xy}^2 - 4\text{x}^2\text{y}$

Opening (*3 minutes*)

1. **Say**: Please write down the answers to Questions 1a. and 1b. on the board. Make sure you check your answers.
2. Allow 1 minute for pupils to answer the question.
3. Have pupils volunteer to give their answers. (Answers: a. $5 - 25\text{x} = 5(1 - 5\text{x})$; b. $4\text{a}^2 - 4 = 4(\text{a}^2 - 1)$)
4. **Say**: Today we are going to identify and factorise variables that are common factors in an algebraic expression.

Introduction to the New Material (*10 minutes*)

1. **Say:** We factorise variables from expressions in the same way we do for integers.
2. **Say:** Let us factorise $\text{x}^2 + 4\text{x}$. We will do it in stages as we did before.
3. Show on the board how to factorise the expression $\text{x}^2 + 4\text{x}$
4. **Ask**: What factor is common to both x^2 and 4x?
5. Allow pupils to think about this for a moment. Have a pupil volunteer to answer. (Answer: x)

$\text{x}^2 + 4\text{x} = \text{x}(\quad)$ ⟵ Write the empty bracket to remind pupils to find the common factor first

$\text{x}(\text{x} + 4)$ ⟵ Divide each term in $\text{x}^2 + 4\text{x}$ by x

<u>Check</u>

$\text{x}(\text{x} + 4) = (\text{x} \times \text{x}) + (\text{x} \times 4)$ ⟵ Multiply each term in the brackets by x

$= \text{x}^2 + 4\text{x}$ ⟵ Check takes us back to the original expression

6. **Say:** Let us factorise $8\text{xy}^2 - 4\text{xy}^2$. Who would like to come to the board and explain to the class how to do Question d.?
7. Select a pupil who raised their hand to explain Question d. on the board. Ask other pupils to observe carefully to see if they agree with the calculation.
8. Ask the pupil to explain how they found the HCF of $8\text{xy}^2$and $4\text{x}^2\text{y}$.
9. Tell the pupil they must check their answer by expanding the brackets.

Find the HCF of $8xy^2$and $4x^2y$.

$8xy^2 = 2 \times 4 \times x \times y \times y$

$4x^2y = 4 \times x \times x \times y$

$HCF = 4xy$

10. Correct any errors in the solution on the board.
11. If there are no volunteers, guide a pupil to answer the question on the board.

$8xy^2 - 4x^2y$ = $4xy(\quad)$ ← HCF= $4xy$. Write outside the brackets

$4xy(2y - x)$ ← Divide each term in $8xy^2 - 4xy^2$ by $4xy$

<u>Check</u>

$4xy(2y - x)$ = $(4xy \times 2y) - (4xy \times x)$ ← Multiply each term in the brackets by $4xy$

= $8xy^2 - 4x^2y$ ← Check takes us back to the original expression

12. **Say:** If our check does not take us back to the original expression, we know we have made a mistake and need to look at our work again.

Guided Practice (*10 minutes*)

1. Ask pupils to work in pairs.
2. Point to the questions on the board:

Factorise the expressions below. Please check all answers.

a. $ab + ac$ b. $pq - p$
c. $5p^2 - 15p$ d. $4ax^2 + 2ax$
e. $s^2 + s^3$ f. $6x^4 + 3x^2 - 9x$

3. Walk around, if possible, to check the answers and clear up any misconceptions.
4. Have pupils from around the classroom volunteer to give their answers.
5. **Write** the correct answers on the board. Ask pupils to check their work. (Answers:
   a. $ab + ac = a(b + c)$ Check: $a(b + c) = (a \times b) + (a \times c) = ab + ac$;
   b. $pq - p = p(q - 1)$ Check: $p(q - 1) = (p \times q) + (p \times (-1) = pq - p$;
   c. $5p^2 - 15p = 5p(p - 3)$ Check: $5p(p - 3) = (5p \times p) + \big(5p \times (-3)\big)$ $= 5p^2 - 15p$;
   d. $4ax^2 + 2ax = 2ax(2x + 1)$ Check: $2ax(2x + 1) = (2ax \times 2x) + (2ax \times 1)$ $= 4ax^2 + 2ax$;
   e. $s^2 + s^3 = s^2(1 + s)$ Check: $s^2(1 + s) = (s^2 \times 1) + (s^2 \times s) = s^2 + s^3$;
   f. $6x^4 + 3x^2 - 9x = 3x(2x^3 + x - 3)$ Check: $3x(2x^3 + x - 3) = (3x \times 2x^3) + (3x \times x) + (3x \times (-3) = 6x^4 + 3x^2 - 9x)$.

Independent Practice (*10 minutes*)

1. Ask the pupils to work independently to answer the questions.
2. Point to the questions on the board:

Factorise the expressions below. Please check all answers.

a. $abc + bcd$ b. $3g + g^2$
c. $y^4 - y^2$ d. $2b^2c^3 + 4b^3c$
e. $2ab^2c^3 + 3a^2b^3c$ f. $(3y)^2 - 3y$

3. Walk around, if possible, to check the answers and clear up any misconceptions.
4. Ask pupils to exchange their exercise books and check each other's work.

5. Have pupils from around the classroom volunteer to give their answers to the questions. Do not do the answers for Question f. Use it to check the pupils' understanding of the work.
6. **Write** the correct answers on the board. Ask pupils to check the answers. (Answers:
    a. $abc + bcd = bc(a + d)$ Check: $bc(a + d) = (bc \times a) + (bc \times d) = abc + bcd$;
    b. $3g + g^2 = g(3 + g)$ Check: $g(3 + g) = (g \times 3) + (g \times g) = 3g + g^2$;
    c. $y^4 - y^2 = y^2(y^2 - 1)$ Check: $y^2(y^2 - 1) = (y^2 \times y^2) + \left(y^2 \times (-1)\right) = y^4 - y^2$;
    d. $2b^2c^3 + 4b^3c = 2b^2c(c^2 + 2b)$ Check: $2b^2c(c^2 + 2b) = (2b^2c \times c^2) + (2b^2c \times 2b) = 2b^2c^3 + 4b^3c$;
    e. $2abc^3 + 3a^2b^3c^3 = abc^3(2 + 3ab^2)$ Check: $abc^3(2 + 3ab^{2)} = (abc^3 \times 2) + (abc^3 \times 3ab^2) = 2abc^3 + 3a^2b^3c^3$;
    f. $(3y)^2 - 3y = 3y(3y - 1)$ Check: $3y(3y - 1) = (3y \times 3y) + \left(3y \times (-1)\right) = (3y)^2 - 3y)$).

Closing (*2 minutes*)

1. **Say:** Please write your name on a piece of paper.
2. **Say**: Write your working out and answer for Question f. on the paper. Hand the paper in at the end of the lesson.
3. Check the work done by the pupils after the lesson. Use it as a guide for which pupils need additional assistance during the next lesson which will be factorisation of quadratic equations.

Lesson Title: Factorisation of Quadratic Equations	**Theme:** Algebra	
Lesson Number: M-09-080	**Class/Level:** JSS 3	**Time:** 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson pupils will be able to identify the factorisation method of factoring a quadratic equation into 2 binomials.	Guide shown in 'Guided Practice' on how to factorise a quadratic when a = 1.	1. Make guide shown in 'Guided Practice' on how to factorise a quadratic when a = 1. 2. Write on the board: 1. Expand: a. $(x+3)(x+5)$ 2. Factorise: a. x^2+6x+8 b. x^2-5x+6 3. Write the questions from the Guided Practice section on the board. 4. Write the questions from the Independent Practice section on the board.

Opening (*3 minutes*)

1. **Say**: Expand $(x+3)(x+5)$. Raise your hands when you finish.
2. Allow 1 minute for pupils to answer the question.
3. Select a pupil to explain their answer on the board. They can use FOIL or any other appropriate method. (Answer: $(x+3)(x+5) = x^2+8x+15$).
4. **Say**: Today we are going to identify the factorisation method of factorising a quadratic equation into 2 binomials.

Introduction to the New Material (*10 minutes*)

1. **Say:** It is very important that we know how to expand 2 binomials to give a quadratic equation. It is also very important to know how to do the reverse process, which is to factorise a quadratic equation into 2 binomials. This means if we have the quadratic $x^2+8x+15$, we need to be able to find its factors.
2. **Ask:** What is the general form of a quadratic? Raise your hand. (Answer: ax^2+bx+c).
3. **Ask:** What are the values for a, b and c in this quadratic? Raise your hand. (Answer: a = 1, b = 8 and c = 15)
4. **Say**: We already know from our expansion that $x^2+8x+15=(x+3)(x+5)$.
5. **Say**: Note that this is the same as $x^2+8x+15=(x+5)(x+3)$ since we can multiply in any order.
6. **Say:** Look at both forms of the quadratic on the left-hand side (LHS) and the right-hand side (RHS). Work in pairs to discuss for 1 minute anything you notice about the 2 expressions.
7. **Say:** Think about what we did to expand the quadratic.
8. Allow a minute for pupils to discuss and share ideas.
9. **Say**: Who would like to share their ideas with the class? Raise your hand.
10. Guide a pupil to say what they notice about the 2 forms of the quadratic. (Answers: The x^2 term is from $x \times x$; the b term (8) is found by adding 3 and 5; the c term (15) is found by multiplying 3 and 5. Accept all reasonable answers.)

11. **Say:** When a = 1, it is relatively easy to find the 2 numbers which combine to make the terms.
12. **Say:** If we add the 2 numbers together, we get the b term. This is called the sum.
13. **Say:** If we multiply the 2 numbers together, we get the c term. This is called the product.
14. **Say**: Let us see how this works with another example.
15. Have a pupil volunteer to read Question 1a.: Factorise $x^2 + 6x + 8$.
16. **Say**: We want to find 2 numbers, m and n. This will factorise the quadratic to give an expression such as $(x + m)(x + n)$.
17. **Ask**: What does $(x + m)(x + n)$ expand to? Raise your hand. (Answer: $x^2 + (m + n)x + mn$).
18. **Say**: So in our example, we want to find m and n so that $m + n = b = 6$ and $m \times n = c = 8$.
19. **Say:** Sometimes we can see the values for m and n straight away. However, it will not always be so easy, so let us solve the quadratic step by step.
20. **Say:** We always find the product $m \times n$, which we know is the same as c, first.
21. Show how to solve the equation as shown below:

$x^2 + 6x + 8$

$a = 1, b = 6, c = 8$ ⟵ Identify the values for a, b and c in the given quadratic. Make sure a = 1

$x^2 + 6x + 8 = (x + m)(x + n)$ ⟵ Write the required equation with x, m and n filled in. This gives the x^2 term right away from $x \times x$

1	8
2	**4**

⟵ Find all the factor pairs so that $m \times n = c = 8$
Choose the factor pair so that $m + n = b = 6$.

We get: $m = 2, n = 4$.

$x^2 + 6x + 8 = (x + 2)(x + 4)$ ⟵ Factors of $x^2 + 6x + 8$ are $(x + 2)$ and $(x + 4)$

Check

$x^2 + 6x + 8 = (x + 2)(x + 4)$

$= x^2 + 4x + 2x + 8$ ⟵ Use FOIL or any other appropriate method to expand the RHS.

$= x^2 + 6x + 8$ ⟵ Expansion equals LHS, so factors are correct.

22. **Say**: Let us look at another example. Select a pupil to read Question 2b.

    2b. Factorise $x^2 - 5x + 6$

    $a = 1, b = -5, c = 6$

23. Write the 2 brackets with x, m and n filled in. $x^2 - 5x + 6 = (x + m)(x + n)$
24. **Ask:** What are the factor pairs for +6?

+6	+1
+3	+2

25. Guide pupils to give all the factor pairs so that $m \times n = c = +6$.
    - Include negative factors since $b = -5$
    - Choose the factor pair so that $m + n = b = -5$
      We get: $m = -3, n = -2$.

−6	−1
−3	**−2**

Factors of $x^2 - 5x + 6$ are $(x - 3)$ *and* $(x - 2)$

26. **Say**: Check that the factorisation is correct:
    - Use FOIL or any other appropriate method to expand the RHS.
    - Expansion equals LHS, so factors are correct.

$x^2 - 5x + 6 = (x - 3)(x - 2)$
$= x^2 - 2x - 3x + 6$
$= x^2 - 5x + 6$

Guided Practice (*10 minutes*)

1. Ask pupils to copy into their exercise books the steps shown on the board to factorise a quadratic when a = 1.
2. Ask pupils to work in pairs.
3. Point to the questions on the board

Factorise:
a. $x^2 + 5x + 6$ b. $x^2 + 7x + 6$

How to Factorise a Quadratic when *a* = 1

1. Write down the general form of the quadratic: $ax^2 + bx + c$.
2. Identify a, b and c for the given quadratic.
3. If *a = 1,* write the quadratic in the form $(x + m)(x + n)$
4. Write down all the factor pairs of *c,* then find the pair that gives *b*.
5. The 2 numbers, m and n, for the quadratic satisfy: $m \times n = c$, $m + n = b$.
6. Always check your answer using expansion, once you find the factors.

4. Walk around, if possible, to check answers and correct any misconceptions.
5. Have pupils from the front and the back of the classroom volunteer to give their answers to the questions.
6. **Write** the correct answers and steps on the board. Ask pupils to check their work.
7. (Answers:
   a. $x^2 + 5x + 6 = (x + 3)(x + 2)$
   Check: $LHS = x^2 + 5x + 6; RHS = x^2 + 2x + 3x + 6 = x^2 + 5x + 6; LHS = RHS$
   Note: LHS = left-hand side; RHS = right-hand side.
   b. $x^2 + 7x + 6 = (x + 6)(x + 1)$
   Check: $LHS = x^2 + 7x + 6; RHS = x^2 + x + 6x + 6 = x^2 + 7x + 6; LHS = RHS$)

Independent Practice (*10 minutes*)

1. Ask the pupils to work independently to answer the questions.
2. Point to the questions on the board:

Factorise:
a. $x^2 + 8x$ b. $x^2 + 8x + 12$
c. $x^2 + 7x + 10$ d. $x^2 + 7x + 12$
e. $x^2 + 2x - 8$ f. $x^2 - 6x + 8$

3. Walk around, if possible, to check answers and clear misconceptions.

4. Encourage pupils to check their answers.
5. Have pupils from around the classroom volunteer to give their answers for Questions a. to d. Do not do the answers for Questions e. and f. Use them to check pupils' understanding of the work.
6. **Write** the correct answers on the board. Ask pupils to check their work.
7. (Answers:
   a. $x^2 + 8x = x(x + 8)$, simple factorisation with x as the common factor.
   Check: $\text{LHS} = x^2 + 8x; \text{RHS} = x^2 + 8x; \text{LHS} = \text{RHS}$
   b. $x^2 + 8x + 12 = (x + 6)(x + 2)$
   Check: $\text{LHS} = x^2 + 8x + 12; \text{RHS} = x^2 + 2x + 6x + 12 = x^2 + 8x + 12; \text{LHS} = \text{RHS}$
   c. $x^2 + 7x + 10 = (x + 5)(x + 2)$
   Check: $\text{LHS} = x^2 + 7x + 10; \text{RHS} = x^2 + 2x + 5x + 10 = x^2 + 7x + 10; \text{LHS} = \text{RHS}$
   d. $x^2 + 7x + 12 = (x + 4)(x + 3)$
   Check: $\text{LHS} = x^2 + 7x + 12; \text{RHS} = x^2 + 3x + 4x + 12 = x^2 + 7x + 12; \text{LHS} = \text{RHS}$
   e. $x^2 + 2x - 8 = (x + 4)(x - 2)$
   Check: $\text{LHS} = x^2 + 2x - 8; \text{RHS} = x^2 - 2x + 4x - 8 = x^2 + 2x - 8; \text{LHS} = \text{RHS}$
   f. $x^2 - 6x + 8 = (x - 4)(x - 2)$
   Check: $\text{LHS} = x^2 - 6x + 8; \text{RHS} = x^2 - 2x - 4x + 8 = x^2 - 6x + 8; \text{LHS} = \text{RHS}$)

Closing (*2 minutes*)

1. **Say:** Write your name on a piece of paper.
2. **Say**: Write your working-out and answer for Questions e. and f. on the paper. Hand the paper in at the end of the lesson.
3. Check the work done by the pupils after the lesson. Use it as a guide to which pupils need additional assistance during the next lesson when pupils will be practising factorisation of quadratic equations.

Lesson Title: Practice with Factorisation of Quadratic Equations	Theme: Algebra	
Lesson Number: M-09-081	**Class/Level:** JSS 3	**Time:** 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson pupils will be able to identify the factorisation method to factor a quadratic equation into 2 binomials.	None	1. Write on the board: (see below) 2. Write the questions from the Independent Practice section on the board.

1. Factorise:	
a. $x^2 + x - 2$	b. $x^2 - 6x - 16$
b. Expand $(2x + 3)(x + 4)$	
c. $3x^2 + 7x + 2$	d. $3x^2 + 8x - 3$

Opening (*3 minutes*)

1. **Say**: Last lesson we looked at how to factorise a quadratic for when a = 1.
2. **Say**: We want to write the quadratic in Question 1a. in the form: $(x + m)(x + n)$.
3. **Ask**: What do we know about m and n for this quadratic? (Answer: $m \times n = -2, m + n = 1$).
4. **Say**: Follow the procedure from our last lesson to factorise the quadratic. You have 2 minutes.
5. Allow 2 minutes for pupils to answer the question.
6. Have a pupil volunteer to give their answer.
   (Answer: $x^2 + x - 2 = (x + 2)(x - 1);\ \text{Check RHS} = x^2 - x + 2x - 2 = x^2 + x - 2 = \text{LHS}$).
7. **Say**: Today we are going to apply the factorisation method to factor a quadratic equation into 2 binomials.

Introduction to the New Material (*10 minutes*)

1. **Say:** Let us look at one more quadratic where a = 1.
2. Ask a pupil to read Question 1b.: Factorise $x^2 - 6x - 16$.
3. **Ask**: What do we know about m and n for this quadratic? Raise your hand. (Answer: $m \times n = -16, m + n = -6$).
4. **Say:** Work in pairs. Follow the procedure to factorise the quadratic.
5. Have a pupil volunteer to give the values of m and n. (Answer: $m = 2, n = -8$)
6. Have another pupil volunteer to say what the factorisation gives as 2 binomials.
   (Answer: $(x + 2)(x - 8)$)
7. Have a third pupil volunteer to check the factorisation on the board.
   (Answer: Check RHS $= x^2 - 8x + 2x - 16 = x^2 - 6x - 16 =$ LHS).
8. **Say**: Let us now look at a quadratic where the co-efficient of x is not 1.
9. Have a pupil volunteer to read Question 1c.: Expand $(2x + 3)(x + 4)$.
10. **Say:** Let us expand $(2x + 3)(x + 4)$.
11. Allow pupils 1 minute to do the expansion.
12. Have a pupil volunteer to answer. (Answer: $2x^2 + 8x + 3x + 12 = 2x^2 + 11x + 12$).

13. **Say**: We now have a co-efficient that is not 1 in the first binomial. It multiplies both of the terms in the second binomial.
14. **Say**: This gives $2x \times x = 2x^2$ *and* $2x \times 4 = 8x$. It now makes the terms 2 times as big.
15. **Say**: Let us see if we can work backwards to get the required factors from the expanded quadratic.
16. **Say**: We will need to modify our procedure a little.

Factorise $2x^2 + 11x + 12$

$ax^2 + bx + c$: $a = 2, b = 11, c = 12$

17. **Write** the 2 brackets, this time with $2x, m$ *and* n filled. $2x^2 + 11x + 12 = (2x + m)(x + n)$
18. **Ask**: What is the expansion for $(2x + m)(x + n)$? Raise your hand.
(Answer: $2x^2 + (2n + m)x + mn$).
19. **Say**: c is still equal to $m \times n = 12$. $(2x + m)(x + n)$
20. **Ask:** What are the factor pairs for 12? Raise your hand.
21. Guide pupils to give all the factor pairs so that $m \times n = c = 12$.
    - Note no negative factors are required, but we have to write factor pairs for both 12×1 *and* 1×12 since we do not know which is the right way round.
    - Choose the factor pair so that $2n + m = b$ =11
    We get: $m = 3, n = 4$.

12	1
1	12
6	2
2	6
4	3
3	**4**

the n values are multiplied by $2x$

Factors of $2x^2 + 11x + 12$ *are* $(2x + 3)$ *and* $(x + 4)$

22. **Say**: Check that the factorisation is correct
    - Use FOIL or any other appropriate method to expand the RHS.
    - Expansion equals LHS, so factors are correct.

$2x^2 + 11x + 12 = (2x + 3)(x + 4)$
$= 2x^2 + 8x + 3x + 12$
$= 2x^2 + 11x + 12$

Guided Practice (*10 minutes*)

1. Ask pupils to work in pairs. Allow them a few moments to discuss each question before selecting a pupil to answer.
2. Have a pupil to volunteer r Question 1e: Factorise $3x^2 + 7x + 2$.
3. **Ask:** What are the values for a, b and c? Raise your hand. (Answer: a = 3, b = 7, c= 2)

4. **Ask:** What will the factorised expression be in terms of $m \text{ and } n$? Raise your hand. (Answer: $(3x+m)(x+n)$)
5. **Ask:** What is the expansion for $(3x+m)(x+n)$? Raise your hand. (Answer: $3x^2+(3n+m)x+mn$)
6. **Say**: We know that $mn = c = 2 \text{ and } 3n + m = b = 7.$
7. **Say**: Remember to find all the factor pairs for c, then choose the pair that matches b.
8. Ask pupils to continue working in pairs to answer Questions d. and e.
9. Walk around, if possible, to check answers and correct any misconceptions.
10. Have pupils from around the classroom volunteer to give their answers.
11. **Write** the correct answers on the board. Ask pupils to check their work. (Answers: Note that some steps can be skipped when pupils are confident of the method.

<u>Question d.</u>

$3x^2+7x+2$; a = 3, b = 7, c= 2

$(3x+m)(x+n) = 3x^2+(3n+m)x+mn$

$mn = 2$; $(3n+m) = 7$ gives $m = 1, n = 2$

$3x^2+7x+2 = (3x+1(x+2)$

Check: RHS = $(3x+1)(x+2) = 3x^2+6x$ $+x+2 = 3x^2+7x+2$ = LHS

<u>Question e.</u>

$3x^2+8x-3$; a = 3, b = 8, c= –3

$(3x+m)(x+n) = 3x^2+(3n+m)x+mn$

$mn = -3$; $(3n+m) = 8$ gives $m = -1, n = 3$

$3x^2+8x-3 = (3x-1(x+3)$

Check: RHS = $(3x-1)(x+3) = 3x^2+9x$ $-x+2 = 3x^2+8x-3$ = LHS).

Independent Practice (*10 minutes*)

1. Ask the pupils to work independently to answer the questions.
2. Point to the questions on the board:

Complete each of the following factorisations:

a. $2x^2+5x+3 = (2x+?)(x+1)$

b. $3x^2+11x+6 = (3x+?)(x+3)$

c. $2x^2+13x-7 = (2x-1)(x+?)$

d. $4x^2-5x-6 = (4x+?)(x-?)$

Factorise:

e. $6x^2-8x+2$

f. $4x^2-11x-3$

3. Walk around, if possible, to check answers and clear misconceptions.
4. Encourage pupils to check their answers.
5. Have pupils from around the classroom volunteer to give their answers. Do not do the answers for Questions e. and f. Use them to check pupils' understanding of the work.
6. **Write** the correct answers on the board. Ask pupils to check their work. (Answers:

a.	? = 3	Check:	LHS = $2x^2+5x+3$, RHS = $(2x+3)(x+1) = 2x^2+2x+3x+3 = 3x^2+5x+3$	RHS=LHS
b.	? = 2	Check:	LHS = $3x^2+11x+6$ RHS = $(3x+2)(x+3) = 3x^2+9x+2x+6 = 3x^2+11x+6$	RHS=LHS

c.	? = 7	Check:	LHS = $2x^2 + 13x - 7$ RHS = $(2x - 1)(x + 7) = 2x^2 + 14x - x - 7 = 2x^2 + 13x - 7$	RHS=LHS
d.	$4x^2 - 5x - 6 = (4x + m)(x + n), mn = -6, 4n + \mathrm{m} = -5, m = 3, n = -2,$ $Check: RHS = (4x + 3)(x - 2) = 4x^2 - 8x + 3x - 6 = 4x^2 - 5x - 6$			RHS=LHS
e.	$6x^2 - 8x + 2 = (6x + m)(x + n), mn = 2, 6n + m = -8, m = -2, n = -1,$ Check: RHS = $(6x - 2)(x - 1) = 6x^2 - 6x - 2x + 2 = 6x^2 - 8x + 2$			RHS=LHS
f.	$4x^2 - 11x - 3 = (4x + m)(x + n), mn = -3, 4n + m = -11, m = 1, n = -3,$ Check: RHS = $(4x + 1)(x - 3) = 4x^2 - 12x + x - 3 = 4x^2 - 11x - 3$			RHS=LHS

Closing (*2 minutes*)

1. **Say:** Write your name on a piece of paper.
2. **Say**: Write your working out and answer for Questions e. and f. on the paper. Hand the paper in at the end of the lesson.
3. Check the work done by the pupils after the lesson. Use it as a guide to which pupils need additional assistance during the next lesson when pupils will continue factorisation of quadratics.

Lesson Title: Factorisation by Completing the Squares Method	**Theme**: Algebra	
Lesson Number: M-09-082	**Class/Level**: JSS 3	**Time:** 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson pupils will be able to identify the 'completing the squares' method of factoring a quadratic equation into 2 binomials.	None	1. Write on the board: Expand: a. $(x+2)^2$ b. $(x-3)^2$ c. $(x+p)^2$ Factorise: d. x^2+4x+2 e. $x^2+8x+11$ 2. Write the questions from the Guided Practice section on the board. 3. Write the questions from the Independent Practice section on the board.

Opening (*3 minutes*)

1. **Say**: Expand the expressions on the board.
2. Allow 2 minutes for pupils to answer the question.
3. Have pupils from around the classroom give their answers.
   (Answers: a. $(x+2)^2 = x^2+4x+4$; b. $(x-3)^2 = x^2-6x+9$;
   c. $(x+p)^2 = x^2+2px+p^2$))
4. **Say**: Today we are going to identify the 'completing the squares' method of factoring a quadratic equation.

Introduction to the New Material (*10 minutes*)

1. **Say:** Let us factorise the quadratic x^2+4x+2. Work in pairs to find m and n to make $mn = 2$ and $m+n = 4$.
2. Stop the pupils after one minute.
3. **Say**: Who would like to tell the class what they found? Raise your hand.
4. If there is a volunteer, acknowledge any answers given. **Say**: You made a very good effort to try to find the factors of the quadratic.
5. Whether or not there is a volunteer, **say:** In fact, we cannot use the method we learnt to factorise x^2+4x+2.
6. **Say**: We have to use a process called 'completing the square' to solve this problem.
7. **Say:** We started the lesson expanding expressions such as $(x+2)^2$. These expressions are called perfect squares because they are squares of a binomial. They are easy to expand and easy to factorise back into their factors. We use them to help to solve quadratic expressions which cannot be factorised by the method we learnt.
8. **Say:** Let us see how it works. We will compare our quadratic x^2+4x+2 with the perfect square $(x+2)^2 = x^2+4x+4$.
9. **Ask:** Can anyone tell the class why we chose $(x+2)^2$ and not any other perfect square? Raise your hand.
10. Guide a pupil to say they have the same x term, that is, $4x$.
11. Show the 2 quadratics side by side on the board so pupils can see how the quadratic is factorised.

$x^2 + 4x + 4$	$x^2 + 4x + 2$
$x^2 + 4x + 4$	$x^2 + 4x + 4 - 2$
$(x + 2)^2$	$(x + 2)^2 - 2$

12. **Say**: We rewrite the given quadratic as a sum of a perfect square and a number. We work out the number as shown on the board. Since $4 - 2 = 2$, our quadratic remains unchanged.
13. **Say**: This will be very useful when we solve quadratic equations later. We are able to rewrite our quadratic in terms of a perfect square we already know. What do we do if we do not know the perfect square to use to rewrite the quadratic? There is a procedure we use called completing the square. This changes the quadratic to the sum of a perfect square and a number.
14. **Say**: Let us use $(x + p)^2$ to represent the perfect square. Let us use q to represent the number.
15. Show the general procedure how to find the perfect square and number to rewrite the quadratic.
16. **Write** the procedure on the board as you go through the explanation.
17. **Say**: We found from our example earlier that: $(x + p)^2 = x^2 + 2px + p^2$
18. **Say**: We can use this fact to write any quadratic in the form we want.
19. Have a pupil volunteer to read Question e.: Factorise $x^2 + 8x + 11$.
20. **Ask:** What are the values of $a, b \ and \ c$ in this quadratic? Raise your hand. (Answer: $a = 1, b = 8 \ and \ c = 11$)
21. **Say:** We can write our quadratic in the form we want: as a perfect square and a number.

    $x^2 + 8x + 11 = (x + p)^2 + q$

    $x^2 + 8x + 11 = x^2 + 2px + p^2 + q$
22. **Say**: We find the value of p from $2p = b, p = \frac{b}{2}$. What answer will we get? Raise your hand. (Answer; $p = \frac{8}{2} = 4$).

    $x^2 + 8x + 11 = x^2 + 8x + 16 + q$
23. **Say**: We find the value of q from $p^2 + q = c, q = c - p^2$. What answer will we get? (Answer: $q = 11 - 16 = -5$)

    $= x^2 + 8x + 16 - 5$
24. **Say:** We have now changed our quadratic into a perfect square and a number. $x^2 + 8x + 11 = (x + 4)^2 - 5$
25. **Say:** A check will prove that we have the right solution.

Guided Practice (*10 minutes*)

1. Ask pupils to work in pairs.
2. Point to the questions on the board:

Complete the square by writing the quadratic expression $x^2 + bx + c$ in the form $(x + p)^2 + q$. Use the equations $p = \frac{b}{2}$ and $q = c - p^2$. Check your answers.

a. $x^2 + 6x + 1$
b. $x^2 + 2x + 5$
c. $x^2 + 8x$
d. $x^2 + 10x + 3$

3. Walk around, if possible, to check answers and correct any misconceptions.
4. Encourage pupils to check their answers.
5. Have pupils from around the classroom volunteer to give their answers.
6. **Write** the correct answers and steps on the board. Ask pupils to check their work. (Answers:

a. $x^2 + 6x + 1$: $p = \frac{b}{2} = \frac{6}{2} = 3$, $q = c - p^2 = 1 - 9 = -8$, $x^2 + 6x + 1 = (x + 3)^2 - 8$,
Check: LHS = $x^2 + 6x + 1$; RHS = $(x + 3)^2 - 8 = x^2 + 6x + 9 - 8 = x^2 + 6x + 1, LHS = RHS$
b. $x^2 + 2x + 5$: $p = \frac{b}{2} = \frac{2}{2} = 1$, $q = c - p^2 = 5 - 1 = 4$, $x^2 + 2x + 5 = (x + 1)^2 + 4$,
Check: LHS = $x^2 + 2x + 5$; RHS = $(x + 1)^2 + 4 = x^2 + 2x + 1 + 4 = x^2 + 2x + 5, LHS = RHS$
c. $x^2 + 8x$: $p = \frac{b}{2} = \frac{8}{2} = 4$, $q = c - p^2 = 0 - 16 = -16$, $x^2 + 8x = (x + 4)^2 - 16$,
Check: LHS = $x^2 + 8x$; RHS = $(x + 4)^2 - 16 = x^2 + 8x + 16 - 16 = x^2 + 8x, LHS = RHS$
d. $x^2 + 10x + 3$: $p = \frac{\text{b}}{2} = \frac{10}{2} = 5, = c - p^2 = 3 - 25 = -22$, $x^2 + 10x + 3 = (x + 5)^2 - 22$,
Check: LHS = $x^2 + 10x + 3$; RHS = $(x + 5)^2 - 22 = x^2 + 10x + 25 - 22 = x^2 + 10x + 3, LHS = RHS$).

Independent Practice (*10 minutes*)

1. Ask the pupils to work independently to answer the questions.
2. Point to the questions on the board:

Complete the square by writing the quadratic expression $x^2 + bx + c$ in the form $(x + p)^2 + q$. Use the equations $p = \frac{b}{2}$ and $q = c - p^2$. Check your answers.

i. $x^2 + 6x - 1$
ii. $x^2 - 4x + 7$
iii. $x^2 - 8x - 6$
iv. $x^2 - 10x - 9$

3. Walk around, if possible, to check answers and clear misconceptions.
4. Encourage pupils to check their answers.
5. Have pupils from around the classroom volunteer to give their answers. Do not do the answers for Question d. Use them to check pupils' understanding of the work.
6. **Write** the correct answers and steps on the board. Ask pupils to check their work. (Answers:

a. $x^2 + 6x - 1$: $p = \frac{b}{2} = \frac{6}{2} = 3$, $q = c - p^2 = -1 - 9 = -10$, $x^2 + 6x - 1 = (x + 3)^2 - 10$
Check: LHS = $x^2 + 6x - 1$; RHS = $(x + 3)^2 - 10 = \text{x}^2 + 6x + 9 - 10 = x^2 + 6x - 1$, LHS=RHS
b. $x^2 - 4x + 7$: $p = \frac{b}{2} = \frac{-4}{2} = -2$, $q = c - p^2 = 7 - 4 = 3$, $x^2 - 4x + 7 = (x - 2)^2 + 3$
Check: LHS = $x^2 - 4x + 7$; RHS = $(x - 2)^2 + 3 = x^2 - 4x + 4 + 3 = x^2 - 4x + 7, LHS = RHS$
c. $x^2 - 8x - 6$: $p = \frac{b}{2} = \frac{-8}{2} = -4$, $q = c - p^2 = -6 - 16 = -22$, $x^2 - 8x - 6 = (x - 4)^2 - 22$, Check: LHS = $x^2 - 8x - 6$; RHS = $(x - 4)^2 - 22 = x^2 - 8x + 16 - 22 = x^2 - 8x - 6$
d. $x^2 - 10x - 9$: $p = \frac{b}{2} = \frac{-10}{2} = -5$, $q = c - p^2 = -9 - 25 = -34$, $x^2 - 10x - 9 =$

$(x-5)^2-34$
Check: LHS = $x^2-10x-9$; RHS = $(x-5)^2-34=x^2-10x+25-34==\ x^2-10x-9$)).

Closing (*2 minutes*)

1. **Say:** Write your name on a piece of paper.
2. **Say**: Write your working out and answer for Questions d. on the paper. Hand the paper in at the end of the lesson.
3. Check the work done by the pupils after the lesson. Use it as a guide to which pupils need additional assistance during the next lesson when pupils will be practising solving problems with completing the squares.

Lesson Title: Practice with Completing the Squares Method	**Theme:** Algebra	
Lesson Number: M-09-083	**Class/Level:** JSS 3	**Time:** 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson pupils will be able to identify the 'completing the squares' method of factoring a quadratic equation into 2 binomials.	None	1. Write on the board: Complete the square for: $x^2+12x+5$ 2. Write the questions from the Guided Practice section on the board. 3. Write the questions from the Independent Practice section on the board.

Opening (*3 minutes*)

1. **Say**: Last lesson we looked at how to complete the square of a quadratic for when a = 1.
2. **Ask**: What 2 parts do we use to rewrite the quadratic? Raise your hand. (Answer: A perfect square and a number)
3. **Ask**: How can we be sure that we have the correct square and number? Raise your hand. (Answer: By checking the answer; expanding the RHS and checking it is the same as the LHS)
4. **Say**: Today we are going to apply the 'completing the squares' method of factoring a quadratic equation.

Introduction to the New Material (*10 minutes*)

1. **Say:** Let us go through the method again so everyone is clear how it works.
2. Have a pupil volunteer to read the question on the board: Complete the square for $x^2+12x+5$.
3. **Ask:** What are the values of a, b and c in this quadratic? Raise your hand. (Answer: $a=1, b=12$ and $c=5$).
4. **Say:** We write our quadratic in the form we want, as a perfect square and a number:

$$x^2+12x+5 = (x+p)^2+q$$

$$x^2+12x+5 = x^2+2px+p^2+q$$

5. **Say**: We find the value of p from $2p=b, p=\frac{b}{2}$.
   What answer will we get? (Answer: $p=\frac{12}{2}=6$)

$$x^2+12x+5 = x^2+12x+36+q$$

6. **Say**: We find the value of q from $p^2+q=c, q=c-p^2$
   What answer will we get? (Answer: $q=5-36=-31$)

$$x^2+12x+5 = x^2+12x+36-31$$

$$x^2+12x+5 = (x+6)^2-31$$

7. **Say:** We have now changed our quadratic into a perfect square and a number. We used $p = \frac{b}{2}$ to work out the perfect square and to find the number. This is called 'completing the square' and is used with all quadratics.
8. **Say:** It is always a good idea to check our answer to make sure we have got it right. Who would like to do that for us on the board? Raise your hand.
9. Have a pupil volunteer to check the answer. Ask other pupils to observe carefully to see if they agree with the calculation. (Answer: Check- LHS = $x^2 + 12x + 5$; RHS = $(x + 6)^2 - 31 = x^2 + 12x + 36 - 31 = x^2 + 12x + 5, \text{LHS} = \text{RHS}$).
10. Correct any errors in the solution on the board.

Guided Practice (*10 minutes*)

1. Ask pupils to work in pairs.
2. Point to the questions on the board:

'Match the 2 expressions which give the same quadratic:

$x^2 - 8x + 12$	$(x - 6)^2 - 20$
$x^2 - 6x + 9$	$(x + 3)^2 - 8$
$x^2 - 6x + 7$	$(x - 6)^2 - 15$
$x^2 + 6x + 1$	$(x - 3)^2 - 2$
$x^2 - 12x + 16$	$(x - 4)^2 - 4$
$x^2 - 12x + 21$	$(x - 3)^2$ '

3. Walk around, if possible, to check answers and correct any misconceptions.
4. Encourage pupils to check their answers.
5. Have pupils from around the classroom volunteer to give their answers.
6. **Write** the correct answers on the board. Ask pupils to check their work.

(Answers:

		Check
$x^2 - 8x + 12$	$(x - 4)^2 - 4$	$x^2 - 8x + 16 - 4 = x^2 - 8x + 12$
$x^2 - 6x + 9$	$(x - 3)^2$	$x^2 - 6x + 9$
$x^2 - 6x + 7$	$(x - 3)^2 - 2$	$x^2 - 6x + 9 - 2 = x^2 - 6x + 7$
$x^2 + 6x + 1$	$(x + 3)^2 - 8$	$x^2 - 6x + 9 - 8 - x^2 + 6x + 1$
$x^2 - 12x + 16$	$(x - 6)^2 - 20$	$x^2 - 12x + 36 - 20 = x^2 - 12x + 16$
$x^2 - 12x + 21$	$(x - 6)^2 - 15$	$x^2 - 12x + 36 - 15 = x^2 - 12x + 21$)

Independent Practice (*10 minutes*)

1. Ask the pupils to work independently to answer the questions.
2. Point to the questions on the board:

'Complete the square by writing the quadratic expression $x^2 + bx + c$ in the form $(x + p)^2 + q$. Check your answers.

k. $x^2 + 10x$
l. $x^2 + 4x + 5$
m. $x^2 + 8x + 17$
n. $x^2 - 6x + 11$
o. $x^2 - x + 1$
p. $x^2 - 5x - 5$ ‘

3. Walk around, if possible, to check answers and clear misconceptions.
4. Encourage pupils to check their answers.
5. Have pupils from around the classroom volunteer to give their answers. Do not do the answers for Question d. Use them to check pupils' understanding of the work.
6. **Write** the correct answers on the board. Ask pupils to check their work.
(Answers:

a.	$x^2 + 10x$	$p = \frac{b}{2} = \frac{10}{2} = 5, q = c - p^2 = 0 - 25 = -25$	$(x + 5)^2 - 25$
	Check:	$(x + 5)^2 - 25 = x^2 + 10x + 25 - 25 = x^2 + 10x$	LHS = RHS
b.	$x^2 + 4x + 5$	$p = \frac{b}{2} = \frac{4}{2} = 2, q = c - p^2 = 5 - 4 = 1$	$(x + 2)^2 + 1$
	Check:	$(x + 2)^2 + 1 = x^2 + 4x + 4 + 1 = x^2 + 4x + 5$	LHS = RHS
c.	$x^2 + 8x + 17$	$p = \frac{b}{2} = \frac{8}{2} = 4, q = c - p^2 = 17 - 16 = 1$	$(x + 4)^2 + 1$
	Check:	$(x + 4)^2 + 1 = x^2 + 8x + 16 + 1 = x^2 + 8x + 17$	LHS = RHS
d.	$x^2 - 6x + 11$	$p = \frac{b}{2} = \frac{-6}{2} = -3, q = c - p^2 = 11 - 9 = 2$	$(x - 3)^2 + 2$
	Check:	$(x - 3)^2 + 2 = x^2 - 6x + 9 + 2 = x^2 - 6x + 11$	LHS = RHS
e.	$x^2 - x + 1$	$p = \frac{b}{2} = \frac{-1}{2} = -0.5, q = c - p^2 = 1 - 0.25 = 0.75$	$(x - 0.5)^2 + 0.75$
	Check:	$x^2 - 1x + 0.25 + 0.75 = x^2 - x + 1$	LHS = RHS
f.	$x^2 - 5x - 5$	$p = \frac{b}{2} = \frac{-5}{2} = -2.5, q = c - p^2 = -5 - 6.25 = -11.25$	$(x - 2.5)^2 + 11.25$
	Check:	$(x - 2.5)^2 - 11.25 = x^2 - 5x + 6.25 - 11.25$	LHS = RHS

Closing (*2 minutes*)

1. **Say:** Write down in your exercise books 2 different things you learnt today.
2. Allow pupils 1 minute to discuss and share ideas.
3. Have one pupil from the front and one from the back of the classroom volunteer answer. (Example answers: How to complete the square using decimal numbers; how to match cards with the same expressions)

Lesson Title: Practice with Factorisation	**Theme**: Algebra	
Lesson Number: M-09-084	**Class/Level**: JSS 3	**Time**: 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson pupils will be able to identify and apply the best method to factor a given algebraic expression, including quadratic expressions.	None	1. Write on the board: Factorise: a. $3x - 12$ b. $8xy - 12x$ c. $x^2 + 2x$ d. $x^2 - 8x + 12$ 2. Write the questions from the Guided Practice section on the board.

Opening (*3 minutes*)

1. **Say**: We have looked at a variety of algebraic expressions over the last 2 weeks. We have expanded and factorised expressions using different methods. Today we are going to identify and apply the best method to factor a given algebraic expression, including quadratic expressions.

Introduction to the New Material (*10 minutes*)

Note: Wait a few moments after each question before selecting a pupil to answer.

1. **Say**: We have learnt several methods to factorise expressions. In a real problem situation we must choose which one to use. It will depend on the type of algebraic expression.
2. Select a pupil to read Question a.: Factorise: $3x - 12$.
3. **Ask:** How do we factorise this expression? Raise your hand. (Answer: $3x - 12 = 3(x - 4)$)
4. **Say**: We factorise $3x - 12$ by taking out the common factor, 3.
5. Follow the same procedure as before.
6. Have different pupils volunteer to read and answer Questions b. and c. on the board. (Answer: b. $8xy - 12x = 4x(2y - 3)$; c. $x^2 + 2x = x(x + 2)$)
7. **Say:** We take out the common factor for Questions b. and c.
8. **Ask**: Is there any other way to factorise Question c., $x^2 + 2x$? Raise your hand.
9. Guide a pupil to say we can complete the square.
10. **Ask**: What answer do we get if we complete the square? Raise your hand.
11. Allow time for pupils to work this out. (Answer: $(x + 1)^2 - 1$)
12. **Say:** It is always a good idea to check our answer to make sure we have got it right. Who would like to do that for us on the board? Raise your hand.
13. Select a pupil to check the answer. Ask other pupils to observe carefully to see if they agree with the calculation.
14. Correct any errors in the solution on the board. (Answer: Check - LHS = $x^2 + 2x$; RHS = $(x + 1)^2 - 11 = x^2 + 2x + 1 - 1 = x^2 + 2x$= LHS).
15. **Say:** Which of the 2 methods is a better method to use to factorise $x^2 + 2x$? Raise your hand.
16. Guide a pupil to say taking out the common factor.
17. **Say**: Very often in Maths there is more than one method to solve a problem. We learn them all so we can decide which is the best method to use when we have a problem to solve.
18. **Say**: Use all the methods you know to solve Question d., $x^2 - 8x + 12$.
19. Allow pupils to work in pairs to discuss and share ideas.

20. Have a pupil volunteer to show one method on the board. Ask the pupil to check the answer.
21. Have another pupil volunteer to show a different method.
22. Ask other pupils to observe carefully to see if they agree with the calculation.
23. Correct any errors in the solution on the board. Ask pupils to check their work. The 2 methods taught are shown below.

Method 1 – Two binomials $(x+m)(x+n)$	Method 2 – Completing the Square $(x+p)^2+q$
$x^2-8x+12$	$x^2-8x+12$
Find 2 numbers so that $mn=12,\ m+n=-8$ List all the factors of +12 Include negative factors since $b=-8$ Select the one which gives $m+n=-8$. This gives $m=-6, n=-2$	Find $p=\frac{b}{2}=\frac{-8}{2}=-4$, $q=c-p^2=12-16=-4$ $x^2-8x+12=(x-4)^2-4$
$x^2-8x+12=(x-6)(x-2)$	
<u>Check</u>	<u>Check</u>
$\text{LHS}=x^2-8x+12 = (x-6)(x-2)$ $=x^2-2x-6x+12$ $=x^2-8x+12$ $=\text{RHS}$	$\text{LHS}=x^2-8x+12 = (x-4)^2-4$ $=x^2-8x+16-4$ $=x^2-8x+12$ $=\text{RHS}$

24. **Say**: We appear to get 2 different results. However, our checks show that they are just different forms of the same equation. We have just factorised them differently.
25. **Say**: Raise your hand if you think Method 1 is the better method to factorise the expression.
26. **Say**: Raise your hand if you think Method 2 is the better method to factorise the expression.
27. **Say**: With practise, many people can do Method 1 very quickly. They work out the factors of c in their heads to find m and n so that $mn=c,\ m+n=b$.
28. **Say:** Method 2 is used in cases where the quadratic does not factorise to give 2 binomials. All quadratics can be made into the form given by Method 2. But not all can be made into that of Method 1.

Guided Practice (*10 minutes*)

1. Ask pupils to work in pairs.
2. Point to the questions on the board:

‘Use the most suitable method to factorise the following algebraic expressions. Be sure to check your answers.

a. $9-3x$ b. $3a^2+4a$

c. $4x^2y - 2xy$ d. $x^2 - 10x + 25$
e. $x^2 - 10x + 23$ f. $x^2 - 10x + 24$ ‘

3. Walk around, if possible, to check answers and correct any misconceptions.
4. Encourage pupils to check their answers.
5. Have pupils from around the classroom volunteer to give their answers to the questions. **Write** the correct answers and steps on the board. Ask pupils to check their work. (Answers: See below. Pupils should be able to skip some steps in calculations and checks. More than one method has been given where possible.)

a. $9 - 3x$ $9 - 3x = 3(3 - x)$

Check: $3(3 - x) = (3 \times 3) - (3 \times x) = 9 - 3x$ LHS = RHS

b. $3a^2 + 4a$ $3a^2 + 4a = a(3a + 4)$

Check: $a(3a + 4) = a \times 3a + a \times 4 = 3a^2 + 4a$ LHS = RHS

c. $4x^2y - 2xy$ $4x^2y - 2xy = 2xy(2x - 1)$,

Check: $2xy(2x - 1) = 2xy \times 2x - 2xy \times 1 = 4x^2y - 2xy$ LHS = RHS

d. $\mathrm{x^2 - 10x + 25}$
EITHER:
$\mathrm{p = \frac{b}{2} = \frac{-10}{2} = -5, q = c - p^2 = 25 - 25 = 0}$
OR:
$\mathrm{mn = 25, m + n = -10, m = n = -5}$

EITHER:
$\mathrm{(x - 5)^2 + 0}$
OR:
$\mathrm{(x - 5)(x - 5)}$
$\mathrm{(x - 5)^2}$

Check: $\mathrm{(x - 5)^2 = x^2 - 5x - 5x + 25 = x^2 - 10x + 25}$ LHS = RHS

e. $\mathrm{x^2 - 10x + 23}$: $\mathrm{p = \frac{b}{2} = \frac{-10}{2} = -5, q = c - p^2 = 23 - 25 = -2}$ $\mathrm{(x - 5)^2 - 2}$

Check: $\mathrm{(x - 5)^2 - 2 = x^2 - 10x + 25 - 2 = x^2 - 10x + 23}$ LHS = RHS

f. $\mathrm{x^2 - 10x + 24}$
EITHER:
$\mathrm{p = \frac{b}{2} = \frac{-10}{2} = -5, q = c - p^2 = 24 - 25 = -1}$
OR:
$\mathrm{mn = 24, m + n = -10, m = -6, n = -4}$

EITHER:
$\mathrm{(x - 5)^2 - 1}$
OR:
$\mathrm{(x - 6)(x - 4)}$

Check (1): $\mathrm{(x - 5)^2 - 1 = x^2 - 10x + 25 - 1 = x^2 - 10x + 24}$ LHS = RHS

Check (2): $\mathrm{(x - 6)(x - 4) = x^2 - 4x - 6x + 24}$
$\mathrm{= x^2 - 10x + 24}$ LHS = RHS

Independent Practice (*10 minutes*)

1. Ask pupils to continue to work in pairs.
2. **Say:** You are going to create 2 algebraic expressions for your partner to factorise. One of them should be a simple expression and the other a quadratic. You must solve and check them first before giving them to your partner to solve.
3. Guide pupils to create simple expressions. The challenge here is for them to come up with their own expressions.
4. Walk around, if possible, to check answers and correct any misconceptions.
5. Have pupils from around the classroom volunteer to write their expressions on the board.
6. Correct any errors on the board.

Closing (*2 minutes*)

1. **Say:** Work in your pairs and write down 2 different things you learnt today.
2. Allow pupils 1 minute to discuss and share ideas.
3. Have one pupil from the front and one from the back of the classroom volunteer to answer. (Example answers: How to create our own algebraic expression; that it is possible for some quadratic expressions to be factorised in different ways. Accept all reasonable answers.)

Lesson Title: Story Problems with Quadratic Expressions	Theme: Algebra	
Lesson Number: M-09-085	**Class/Level:** JSS 3	**Time:** 35 minutes

Learning Outcomes	Teaching Aids	Preparation
By the end of the lesson pupils will be able to write quadratic expressions for situations in story problems.	None	1. Write on the board: a. Factorise $(x+1)^2 - 4$ b. 2 consecutive integers are denoted by n and $n+1$. Find an expression for their product. c. Find the combined age of 3 children in a family aged x, x^2, and $x + 8$. 2. Write the questions from the Independent Practice section on the board.

Opening (*3 minutes*)

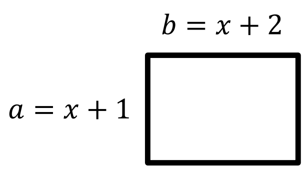
1. **Say**: Look at the quadratic expression in Question a. Factorise $(x+1)^2 - 4$. It is written in one of the forms we learnt about. We now want to find out if it can be factorised into 2 binomials.
2. Ask the pupils to spend 2 minutes to see if the expression can be factorised into 2 binomials.
3. Remind pupils to look for 2 numbers which will multiply to give c and add to give b in the general quadratic expression ($ax^2 + bx + c$).
4. Have a pupil volunteer to explain on the board how to factorise the expression. Ask other pupils to observe carefully to see if they agree with the calculation.
5. Correct any errors in the solution on the board. Ask pupils to check their work.
   (Answer: $(x+1)^2 - 4 = x^2 + 2x + 1 - 4 = x^2 + 2x - 3 = (x-1)(x+3)$
   Check: $(x-1)(x+3) = x^2 + 3x - x - 3 = x^2 + 2x - 3$, LHS = RHS)
6. **Say**: We are all now able to factorise quadratic expressions into different forms. We can even create our own for other people to factorise. Today we are going to write quadratic expressions for situations in story problems.

Introduction to the New Material (*10 minutes*)

Note: Wait a few moments after each question before selecting a pupil to answer.

1. Draw a rectangle with sides marked a and b on the board.
2. **Ask**: What formula do we use to calculate the area of a rectangle? Raise your hand. (Answer: Length × width)
3. **Ask:** What will be the area for the rectangle on the board? Raise your hand. (Answer: $a \times b = ab$)
4. **Say:** Supposing $a = x + 1$ and $b = x + 2$, how can we write the area of the rectangle? Raise your hand.
5. Guide a pupil to say they will substitute the values of a and b in the formula for the area. This gives:

$$(x+1)(x+2)$$



6. **Ask:** What type of expression have we created for the area of a rectangle? Raise your hand. (Answer: A quadratic expression)
7. **Say:** What will we get if we expand this expression? Raise your hand. (Answer: $x^2 + 3x + 2$)

8. **Say:** Let us look at another example. What expression will we get if we want to find the product of 2 consecutive integers as in Question b.?
9. Guide a pupil to say that the expression will be $n(n+1)$.
10. **Say**: Expand this expression in your exercise books. Raise your hand when you finish.
11. Select a pupil with raised hand to give the expansion. (Answer: $n^2 + n$)
12. **Say**: In this example $c = 0$. It is still a quadratic as long as we have a term where the highest power of the variable is 2.
13. Ask a pupil to read Question c. on the board: 'Find the combined age of 3 children in a family aged x, x^2, and $x + 8$.'
14. Ask pupils to work in pairs to discuss and share ideas.
15. Have a pupil volunteer to explain their answer on the board. (Answer: $x + x^2 + x + 8 = x^2 + 2x + 8$)
16. **Say**: We know how to convert between one form of a quadratic expression and another. Look at the answer for Question c. Discuss with your partner if this expression can be factorised into 2 binomials.
17. Allow 1 minute for pupils to discuss and share ideas.
18. Have a pupil volunteer to give the answer. (Answer: It cannot be factorised into 2 binomials, there are no 2 numbers which multiply to give 8 and add to give 2.)

Guided Practice (*10 minutes*)

1. **Say**: Even though we could not factorise our result into 2 binomials, we can still write it in the form $(x+p)^2 + q$.
2. **Write**: $(x+p)^2 + q$.
3. **Ask**: What do we call the procedure when we write a quadratic expression in this form? Raise your hand. (Answer: Completing the square)
4. **Say**: Continue to work in your pairs to complete the square for $x^2 + 2x + 8$.
5. **Write**: $x^2 + 2x + 8$
6. While the pupils are working on the problem, **write** on the board:

   'The 2 short sides of a right-angled triangle are given by x and $x + 1$. Find an expression for the hypotenuse of the triangle using Pythagoras' Theorem.'

7. Ask them to still work in pairs to solve the new problem on the board.
8. Walk around, if possible, to check answers and correct any misconceptions.
9. Encourage pupils to check their answers.
10. Have pupils from around the classroom volunteer to give their answers to the questions.
11. **Write** the correct answers and steps on the board. Ask pupils to check their work. (Answers:
    $x^2 + 2x + 8$: $p = \frac{b}{2} = \frac{2}{2} = 1, q = c - p^2 = 8 - 1 = 7$: $x^2 + 2x + 8 = (x+1)^2 + 7$
    Check: $x^2 + 2x + 8 = (x+1)^2 + 7 = x^2 + 2x + 1 + 7 = x^2 + 2x + 8$, $\text{LHS} = \text{RHS}$

    Pythagoras' Theorem states that the square of the hypotenuse is the sum of the squares of the 2 short sides.
    Expression for hypotenuse: $x^2 + (x+1)^2 = x^2 + x^2 + 2x + 1 = 2x^2 + 2x + 1$.
    No check necessary.)

Independent Practice (*10 minutes*)

1. Ask the pupils to work independently to answer the questions.
2. Point to the questions on the board:
   a. The area of a rectangle is given by the quadratic expression $x^2 + 6x + 8$. Find the length of the 2 sides in terms of x. (Hint: Factorise into 2 binomials)
   b. Find a quadratic expression for the product of 2 numbers denoted by $x + 5$ and $x + 7$.
3. Walk around, if possible, to check answers and clear misconceptions.
4. Encourage pupils to check their answers.
5. Have pupils from around the classroom volunteer to give their answers to the questions.
6. **Write** the correct answers and steps on the board. Ask pupils to check their work. (Answers: a. $x^2 + 6x + 8 = (x + 4)(x + 2)$; the 2 sides are $(x + 4)$ and $(x + 2)$.
   Check: $(x + 4)(x + 2) = x^2 + 2x + 4x + 8 = x^2 + 6x + 8$, LHS = RHS);
   b. Expression for the product of the 2 numbers: $(x + 5)(x + 7) = x^2 + 7x + 5x + 35 = x^2 + 12x + 35$. No check necessary.)

Closing (*2 minutes*)

1. **Say:** Write down one new thing you learnt today.
2. Allow pupils 1 minute to discuss and share ideas.
3. Have one pupil from the front and one from the back of the classroom volunteer to answer. (Example answer: How to write quadratic expressions for situations in story problems)

Lesson Title: Introduction to Linear Equations in 2 Variables	**Theme**: Algebra	
Lesson Number: M-09-086	**Class/Level**: JSS 3	**Time**: 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson pupils will be able to identify a simple linear equation in 2 variables and the form its solutions take: (x, y).	None	1. Write on the board: Solve for x: $2(3x - 1) = 4(x + 2)$ 2. Write on the board: <u>Vocabulary</u>: ordered pair 3. Write the questions from the Guided Practice section on the board. 4. Write the questions from the Independent Practice section on the board.

Opening (*3 minutes*)

1. **Ask**: Look at the equation on the board - how many variables does it have? Raise your hand. (Answer: One variable, x)
2. **Say**: We already know how to solve this type of linear equation in one variable. You have 1 minute to find its solution. Check your answer when you finish.
3. Allow 1 minute for pupils to answer the question.
4. **Say**: Who would like to come to the board to explain the answer?
5. Select a pupil with raised hand to explain their answer on the board. You may also want to select another pupil to check the solution on the board. Ask other pupils to observe carefully to see if they agree with the calculation.
6. Correct any errors in the solution on the board. Ask pupils to check their work.
   (Answer: $2(3x - 1) = 4(x + 2)$; $6x - 2 = 4x + 8; 2x = 10; x = 5$
   Check: $2(3 \times 5 - 1) = 4(5 + 2); 2 \times 14 = 4 \times 7; 28 = 28$).
7. **Say**: Today we are going to identify a simple linear equation in 2 variables and the form its solutions take: (x, y).

Introduction to the New Material (*10 minutes*)

1. **Say:** We just solved a linear equation which has one variable. We now want to look at linear equations which have 2 variables. Situations which give rise to these types of equations occur throughout our everyday life.
2. **Ask**: How can we write an equation to say that the number of girls and boys in a JSS 3 class is 82? Raise your hand.
3. Guide a pupil to say we can write it as: $x + y = 82$, where x is the number of girls and y is the number of boys.
4. **Say**: The equation we just described is an example of a linear equation in 2 variables, where

   $ax + by + c = 0$ a and b cannot both be zero at the same time,
   c is a constant term which can be zero.

5. **Say**: This is because we can rewrite our equation as: $x + y - 82 = 0$.
6. **Say**: There are many solutions to our equation. Give me any 2 whole numbers which add up to 82. Raise your hand.

7. Obtain a number of values for x and y, which add up to 82. **Write** a few on the board. (Example answer: $x = 45, y = 82 - 45 = 37$)
8. **Say**: One solution to our equation is written as: $(45, 37)$.
9. **Say**: This is called an ordered pair. It is always written in the format: (x, y). We always write the x value first, and the y value second.
10. **Ask:** What is the highest power of the variables x and y in the equation? Raise your hand. (Answer: 1)
11. **Say:** There are times when the linear equation is given in different formats. It is still a linear equation as long as the highest power of $x \text{ and } y$ is 1.The equation written in the format $ax + by + c = 0$ is not always easy to work with. We can make either x or y the subject of a linear equation depending on which value we are calculating.
12. **Say:** In most cases, we are interested in y, so we make it the subject of the equation. Let us look at the equation $2x - y + 1 = 0$.
13. **Write:** $2x - y + 1 = 0$
14. **Say**: We want to solve y for various values of x. To make y the subject of this equation, we simply add y to both sides of the equation.

$$\begin{aligned} 2x - y + 1 &= 0 \\ 2x - y + y + 1 &= 0 + y \\ 2x + 1 &= y \\ y &= 2x + 1 \end{aligned}$$

15. **Say:** When we make y the subject of the equation, the general equation changes to the format:

$$y = mx + c \qquad (m \text{ and } c \text{ are constants.})$$

16. **Say**: This makes the linear equation very easy to work with. You will find that when we come to draw graphs, this format of the linear equation allows us to find $m \text{ and } c$ very easily. Let us go back to our equation, $y = 2x + 1$. We can now solve *y* for various values of x. We write our solution as ordered pairs.
17. Ask the pupils to calculate the value of y for $x = 0, 1, 2 \text{ and } 3$.
18. Have pupils volunteer to give the answers as ordered pairs. (Answers:

$$\begin{array}{lllllll} & y & = & 2x + 1 & & & \\ \text{when } x = 0 & y & = & 2 \times 0 + 1 & = & 0 + 1 = 1 & (0, 1) \\ \text{when } x = 1 & y & = & 2 \times 1 + 1 & = & 2 + 1 = 3 & (1, 3) \\ \text{when } x = 2 & y & = & 2 \times 2 + 1 & = & 4 + 1 = 5 & (2, 5) \\ \text{when } x = 3 & y & = & 2 \times 3 + 1 & = & 6 + 1 = 7 & (2, 7)\ ) \end{array}$$

Guided Practice (*10 minutes*)

1. Ask pupils to work in pairs.

2. Point to the questions on the board:

   'Find the value of y for $x = 0$, $x = 1, x = 2$ and $x = 3$ in the following equations.
   Write your answers as ordered pairs, (x, y).

   q. $y = x$ r. $y = 2x$
   s. $y = 3x$ t. $y = x + 2$ '

3. Walk around, if possible, to check answers and correct any misconceptions.
4. Have pupils from around the classroom volunteer to give their answers.
5. **Write** the correct answers on the board. Ask pupils to check their work. (Answers: a. $y = x$: $(0, 0)$, $(1, 1)$, $(2, 2)$, $(3, 3)$; b. $y = 2x$: $(0, 0)$, $(1, 2)$, $(2, 4)$, $(3, 6)$; c. $y = 3x$: $(0, 0)$, $(1, 3)$, $(2, 6)$, $(3, 9)$; d. $y = x + 2$: $(0, 2)$, $(1, 3)$, $(2, 4)$, $(3, 5)$)

Independent Practice (*10 minutes*)

1. Ask the pupils to work independently to answer the questions.
2. Point to the questions on the board:

   'Find the value of y for $x = 0, 1, 2$ and 3 in the following equations.
   Write your answer as ordered pairs, (x, y).

   a. $y = 4x$ b. $y = 2x + 5$
   c. $y = 5 - x$ d. $y = 2(x + 1)$ '

3. Walk around, if possible, to check answers and clear misconceptions.
4. Have pupils from around the classroom volunteer to give their answers.
5. **Write** the correct answers on the board. Ask pupils to check their work. (Answers: a. $y = 4x$: $(0, 0)$, $(1, 4)$, $(2, 8)$, $(3, 12)$; b. $y = 2x + 5$: $(0, 5)$, $(1, 7)$, $(2, 9)$, $(3, 11)$; c. $y = 5 - x$: $(0, 5)$, $(1, 4)$, $(2, 3)$, $(3, 2)$; d. $y = 2(x + 1)$: $(0, 2)$, $(1, 4)$, $(2, 6)$, $(3, 8)$)

Closing (*2 minutes*)

1. **Say:** Work in your pairs and write down 2 different things you learnt today.
2. Allow pupils 1 minute to discuss and share ideas.
3. Have one pupil from the front and one from the back of the classroom volunteer to answer. (Example answers: Solutions to linear equations are written as ordered pairs, (x, y); linear equations can be written in more than one format; we can change the subject of the equation to make our calculations easier)

Lesson Title: Introduction to Linear Equations in 2 Variables	**Theme**: Algebra	
Lesson Number: M-09-087	**Class/Level**: JSS 3	**Time:** 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson pupils will be able to verify solutions to equations in 2 variables by substitution.	None	1. Write on the board: a. Find the value of y for $x = 1$ and $x = 2$ in the equation $y = 2x + 7$. Write your answer as an ordered pair. Verify if the ordered pair (given first) is a solution to the linear equation: b. (2, 11), $y = 3x + 5$. c. (3, 20), $y = 3(x + 3)$ d. (–2, 8), $x + y = 6$. 2. Write the questions from the Guided Practice section on the board. 3. Write the questions from the Independent Practice section on the board.

Opening (*3 minutes*)

1. **Say**: Last lesson we solved for y for values of x in various linear equations. Solve for y in the equation on the board, in Question a. You have 1 minute to find its solution.
2. Allow 1 minute for pupils to answer the question.
3. **Say**: Who would like to give the answers? Raise your hand.
4. Select a pupil to give their answer. (Answer: (1, 9) and (2, 11))
5. **Say**: Today we are going to verify solutions to equations in 2 variables by substitution.

Introduction to the New Material (*10 minutes*)

1. **Say:** What are the 2 formats of the linear equation we looked at last lesson? Raise your hand. (Answer: $y = ax + by + c = 0$ and $y = mx + c$)
2. **Say:** We will use the format $y = mx + c$ most of the time, as it is easier to work with. There are times when we need to determine whether an ordered pair actually is a solution to an equation.
3. **Say**: This is very useful when we solve simultaneous linear equations, as we have to verify that our ordered pair is a solution to 2 different linear equations. We know the linear equation is given in different formats. It is still a linear equation as long as the highest power of both x and y is 1.
4. **Say**: We verify a solution by substituting the values of the ordered pair into the equation. This is basically a check that our answer is correct. Let us show how this works for the simple linear equation in Question b.
5. Solve on the board:

$$\begin{array}{lrcl} & y & = & 3x + 5 \\ (2, 11) & 11 & = & 3 \times 2 + 5 \\ & & = & 6 + 5 \\ & 11 & = & 11 \end{array}$$

$$\text{LHS} = \text{RHS}$$

6. **Say**: This tells us that our solution is correct and the ordered pair is a solution to the equation. When the ordered pair is a solution to an equation, we say it 'satisfies' the equation. Let us see whether the ordered pair in Question c. satisfies the given linear equation.

$$\begin{aligned} & y &= &\ 3(x+3) \\ (3, 20) \quad & 20 &= &\ 3(3+3) \\ & &= &\ 3 \times 6 \\ & 20 &\neq &\ 18 \\ & \text{LHS} &\neq &\ \text{RHS} \end{aligned}$$

7. **Say**: This tells us that our solution is not correct and we need to take another look at our calculations to find where we have made an error. Who would like to explain to the class how to check if the ordered pair (–2, 8) satisfies the linear equation $x + y = 6$? Raise your hand.
8. Select a pupil with raised hand to come to the board. Ask other pupils to observe carefully to see if they agree with the calculation.
9. Correct any errors in the solution on the board. An example solution is shown below:

$$\begin{aligned} & x+y &= &\ 6 \\ (-2, 8) \quad & -2+8 &= &\ 6 \\ & 6 &= &\ 6 \\ & \text{LHS} &= &\ \text{RHS} \end{aligned}$$

10. **Say:** Therefore the ordered pair (–2, 8) satisfies the linear equation $x + y = 6$.
11. **Ask:** How many solutions do you think we have to the linear equation $x + y = 6$? Raise your hand.
12. Select 3-4 pupils to make suggestions to possible solutions. **Write** their suggestions on the board. They will probably only think of whole numbers for x and y.
13. Put a few additional examples of solutions on the board:

$$5 + \frac{1}{2} \qquad 4\frac{1}{2} + 1\frac{1}{2} \qquad 0.25 + 5.75$$

$$3\frac{1}{3} + 2\frac{2}{3} \qquad 7\frac{1}{2} + \left(-1\frac{1}{2}\right) \qquad 15.375 + (-9.375)$$

14. **Say:** These numbers are also solutions to $x + y = 6$. Check by adding to see if you get 6.
15. **Say**: In fact, there are an infinite number of solutions for every linear equation in 2 variables. You can pick any value for x and there will be a value for y which makes 6 when added together.
16. **Say:** Try it and see.
17. Allow pupils some time (about 1 minute) to verify that the above is true.

Guided Practice (*10 minutes*)

1. Ask pupils to work in pairs.
2. Point to the questions on the board:

Verify if the ordered pair (given first) satisfies the given linear equation(s):

a. $(3, 13)$ $y = 4x$ b. $(1, 7)$ $y - 2x = 5$

c. $(-2, 7)$ $y = 5 - x$ d. $(9, 21)$ $y = 2(x + 1)$

e. $(1, 2)$ $y = 3 - x$ and $y = x + 1$

3. Walk around, if possible, to check answers and correct any misconceptions.
4. Have pupils from around the classroom volunteer to give their answers.
5. **Write** the correct answers on the board. Ask pupils to check their work. (Answers:

a. $(3, 13)$ $y = 4x$ $\text{LHS} = 13; \text{RHS} = 4 \times 3 = 12$ $\text{LHS} \neq \text{RHS}$: not a solution

b. $(1, 7)$ $y - 2x = 5$ $\text{LHS} = 7 - 2 \times 1 = 5; \text{RHS} = 7$ $\text{LHS} = \text{RHS}$: solution

c. $(-2, 7)$ $y = 5 - x$ $\text{LHS} = 7;\ \text{RHS} = 5 - (-2)$ $\text{LHS} = \text{RHS}$: solution

$= 5 + 2 = 7$

d. $(9, 21)$ $y = 2(x + 1)$ $\text{LHS} = 21; \text{RHS} = 2 \times 10 = 20$ $\text{LHS} \neq \text{RHS}$: not a solution

e. $(1, 2)$ $y = 3 - x$ $\text{LHS} = 2; \text{RHS} = 3 - 1 = 2$ $\text{LHS} = \text{RHS}$: solution

$y = x + 1$ $\text{LHS} = 2; \text{RHS} = 1 + 1 = 2$ $\text{LHS} = \text{RHS}$: solution

$(1, 2)$ satisfies both equations simultaneously (at the same time).

Independent Practice (*10 minutes*)

1. Ask the pupils to work independently to answer the questions.
2. Point to the questions on the board:

'Verify if the ordered pair (given first) satisfies the given linear equation(s):

a. $(0, -7)$ $y = 3x - 7$ b. $(1, -3)$ $y = 2x - 5$

c. $(-4, 11)$ $x + y = 5$ d. $(4, 30)$ $y = 3(2x + 1)$

e. $(-3, 6)$ $2x + y = 0$ and $x + 2y = 3$ '

3. Walk around, if possible, to check answers and clear misconceptions.
4. Have pupils from around the classroom volunteer to give their answers.
5. **Write** the correct answers on the board. Ask pupils to check their work. (Answers:

a. $(0, -7)$ $y = 3x - 7$ $\text{LHS} = 7; \text{RHS} = 3 \times 0 - 7 = -7$ $\text{LHS} = \text{RHS}$: solution

b. $(1, -3)$ $y = 2x - 5$ $\text{LHS} = -3; \text{RHS} = 2 \times (1) - 5 = -3$ $\text{LHS} = \text{RHS}$: solution

c. $(-4, 11)$ $x + y = 5$ $\text{LHS} = -4 + 11 = 7;\ \text{RHS} = 5$ $\text{LHS} \neq \text{RHS}$: not a solution

d. $(4, 30)$ $y = 3(2x + 1)$ $\text{LHS} = 30; \text{RHS} = 3(2 \times 4 + 1)$ $\text{LHS} \neq \text{RHS}$: not a solution

$= 3 \times 9 = 27$

e. $(-3, 6)$ $2x + y = 0$ $\text{LHS} = 2 \times (-3) + 6 = 0; \text{RHS} = 0$ $\text{LHS} = \text{RHS}$: solution

$x + 2y = 3$ $\text{LHS} = -3 + 2 \times 6 = 9; \text{RHS} = 3$ $\text{LHS} \neq \text{RHS}$: not a solution

$(-3, 6)$ does not satisfy both equations simultaneously (at the same time).

Closing (*2 minutes*)

1. **Ask**: How do we verify that an ordered pair is a solution to a linear equation? Raise your hand. (Example answer: By substituting the values of the ordered pair into the equation)
2. **Ask:** How do we refer to an ordered pair which is a solution to a linear equation? Raise your hand. (Example answer: We say it satisfies the equation)
3. **Ask:** How many solutions are there to a linear equation in 2 variables? Raise your hand. (Answer: An infinite number).

Lesson Title: Finding Solutions to Linear Equations I	**Theme**: Algebra	
Lesson Number: M-09-088	**Class/Level**: JSS 3	**Time:** 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson pupils will be able to find solutions to equations in 2 variables by substituting a value for one variable and solving for the other.	None	1. Write on the board: Solve $y = 2x - 5$, when: a. $x = 1$ b. $y = 1$ Solve $y = 2(x - 5)$, when: c. $x = 6$ d. $y = -2$ 2. Write the questions from the Guided Practice section on the board. 3. Write the questions from the Independent Practice section on the board.

Opening (*3 minutes*)

1. **Say**: So far we have solved for y in various linear equations. In those cases, we were given values of x and asked to find y.
2. Point to Question a.i. on the board. **Say:** Show how to solve for y in the equation $y = 2x - 5$ when $x = 1$.
3. Allow 1 minute for pupils to answer the question.
4. **Say**: Who would like to give the answer? Raise your hand.
5. Select a pupil to give their answer. (Answer: a. i. $x = 1; y = 2x - 5 = 2 \times 1 - 5 = -3$, (1, -3))
6. **Say:** It is possible to know or be given values for y and be asked to solve for x. Today we are going to find solutions to equations in 2 variables by substituting a value for one variable and solving for the other.

Introduction to the New Material (10 minutes)

1. Point to Question a.ii. on the board. **Say:** Let us solve for x when $y = 1$ in our equation.
2. Show this on the board:

$$
\begin{array}{lrcll}
 & y & = & 2x - 5 & \\
\text{ii. When } y = 1 & 1 & = & 2x - 5 & \\
 & 6 & = & 2x & \leftarrow \text{Add 5 to both sides} \\
 & 3 & = & x & \leftarrow \text{Divide both sides by 2} \\
 & x & = & 3 & \leftarrow \text{Gives ordered pair: (3, 1)} \\
\underline{\text{Check}} & & & & \\
(3, 1) & 1 & = & 2 \times 3 - 5 & \\
 & & = & 6 - 5 & \\
 & 1 & = & 1 & \\
 & \text{LHS} & = & \text{RHS} &
\end{array}
$$

3. **Say**: If we have a lot of x values to find, it is easier to change the subject of the equation to x. We will then be able to substitute directly into the equation to find x.

$$y = 2x - 5$$

$$y + 5 = 2x \quad \longleftarrow \text{Add 5 to both sides}$$

$$\frac{y+5}{2} = x \quad \longleftarrow \text{Divide both sides by 2}$$

$$x = \frac{y+5}{2}$$

4. Point to Question b. on the board. **Say**: Let us look at the second equation on the board: $y = 2(x - 5)$. Work in pairs to answer the question.
5. Check your answers when you finish.
6. Allow pupils 3 minutes to work in pairs to discuss and share ideas.
7. **Say**: Who would like to explain the solution to Question b.i.? Raise your hand.
8. Select a pupil to come to the board and explain their solution to the class. Ask other pupils to observe carefully to see if they agree with the calculation.
9. Correct any errors in the solution on the board. Ask pupils to check their work. (Answer: See below)
10. **Ask**: Did anyone change the subject of the equation for Question b.ii. to x? Raise your hand.
11. Guide a pupil to show how to change the subject to x. (Answer: See the box below)
12. Have a second pupil volunteer to come to the board and explain their solution to Question b.ii. Ask other pupils to observe carefully to see if they agree with the calculation.
13. Correct any errors in the solution on the board. Ask pupils to check their work.
    Example calculations are shown below:

$$y = 2(x - 5)$$

b.i. When $x = 6$

$$y = 2 \times (6 - 5)$$
$$= 2 \times 1$$
$$= 2 \qquad (6, 2)$$

<u>Check (always)</u>

$(6, 2)$

$$2 = 2(6 - 5)$$
$$= 2 \times (1)$$
$$2 = 2$$

LHS = RHS

b.ii. When $y = -2$

$$-2 = 2(x-5)$$
$$= 2x - 10$$
$$8 = 2x$$
$$\frac{8}{2} = x$$
$$x = 4 \qquad (4, -2)$$

$y = 2(x-5)$
$= 2x - 10$
$y + 10 = 2x$
$\frac{y+10}{2} = x$
$x = \frac{y+10}{2}$

<u>Check</u>

$(4, -2)$
$$-2 = 2(4-5)$$
$$= 2 \times (-1)$$
$$-2 = -2$$
LHS = RHS

Guided Practice (*10 minutes*)

1. Ask pupils to continue to work in pairs.
2. Point to the questions on the board:
   Solve $y = 3x + 4$, when:
   a. $x = 2$ *b.* $x = -1$ *c.* $x = 4$
   d. $y = 0$ *e.* $y = -2$ *f.* $y = -5$
3. Encourage pupils to change the subject of the equation where appropriate. Checks can be done using the inverse equations.
4. Walk around, if possible, to check answers and correct any misconceptions.
5. Have pupils from around the classroom volunteer to give their answers to the questions, including checking the questions.
6. **Write** the correct answers and steps on the board. Ask pupils to check their work.

(Answers: $y = 3x + 4$, $x = \frac{y-4}{3}$

a. $x = 2$ $y = 3 \times 2 + 4 = 10$ $(2, 10)$ Check: $x = \frac{10-4}{3} = \frac{6}{3} = 2$

b. $x = -1$ $y = 3 \times (-1) + 4 = 1$ $(-2, 1)$ Check: $x = \frac{1-4}{3} = -\frac{-3}{3} = -1$

c. $x = 4$ $y = 3 \times 4 + 4 = 16$ $(4, 16)$ Check: $x = \frac{16-4}{3} = \frac{12}{3} = 4$

d. $y = 0$ $x = \frac{0-4}{3} = -\frac{4}{3}$ $(-\frac{4}{3}, 0)$ Check: $y = 3 \times \left(-\frac{4}{3}\right) + 4 = 0$

e. $y = -2$ $x = \frac{-2-4}{3} = \frac{-6}{3} = -2$ $(-2, -2)$ Check: $y = 3 \times (-2) + 4 = -2$

f. $y = -5$ $x = \frac{-5-4}{3} = \frac{-9}{3} = -3$ $(-3, -5)$ Check: $y = 3 \times (-3) + 4 = -5$).

Independent Practice (*10 minutes*)

1. Ask the pupils to work independently to answer the questions.
2. Point to the questions on the board:

   Solve $y = 4(x - 2)$, when:

a. $x = 0$	*b.* $x = 6$	*c.* $x = -1$
d. $y = 0$	*e.* $y = 2$	*f.* $y = -2$

3. Encourage pupils to change the subject of the equation where appropriate. The inverse equations can be used for checking.
4. Walk around, if possible, to check answers and clear misconceptions.
5. Have pupils from around the classroom volunteer to give their answers.
6. **Write** the correct answers on the board. Ask pupils to check their work.

   (Answers: $y = 4(x - 2)$, $x = \dfrac{y+8}{4}$

   a. $x = 0$ $y = 4 \times (0 - 2) = -8$ $(0, -8)$ Check: $x = \frac{-8+8}{4} = 0$

   b. $x = 6$ $y = 4 \times (6 - 2) = 16$ $(6, 16)$ Check: $x = \frac{16+8}{4} = -\frac{24}{4} = 6$

   c. $x = -1$ $y = 4 \times (-1 - 2) = -12$ $(-1, -12)$ Check: $x = \frac{-12+8}{4} = \frac{-4}{4} = -1$

   d. $y = 0$ $x = \dfrac{0+8}{4} = -\dfrac{8}{4} = 2$ $(2, 0)$ Check: $y = 4 \times (2 - 2) = 0$

   e. $y = 2$ $x = \dfrac{2+8}{4} = \dfrac{10}{4} = 2.5$ $(2.5, 2)$ Check: $y = 4 \times (2.5 - 2) = 2$

   f. $y = -2$ $x = \dfrac{-2+8}{4} = \dfrac{6}{4} = 1.5$ $(-1.5, -2)$ Check: $y = 4 \times (1.5 - 2) = -2$

Closing (*2 minutes*)

1. Ask pupils to write down in 1 minute one thing they learnt in this lesson they did not know before.
2. Have pupils from around the classroom volunteer to give their answers. (Example answers: Change the subject of the equation to x if there are a lot of x values to find; we can use the equation with x as the subject to check the solution)

Lesson Title: Finding Solutions to Linear Equations II	**Theme**: Algebra	
Lesson Number: M-09-089	**Class/Level**: JSS 3	**Time**: 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson pupils will be able to solve linear equations where the variable appears on both sides of the equation by balancing the equation and combining like terms.	None	1. Write on the board: Solve the following equations: a. $\frac{x+6}{2} = 5$ b. $5x + 2 = 4x + 5$ c. $2(x - 3) = 8 - 3x$ 2. Write the questions from the Guided Practice section on the board. 3. Write the questions from the Independent Practice section on the board.

Opening (*3 minutes*)

1. **Say**: Solve for x in Question a. on the board: $\frac{x+6}{2} = 5$. Raise your hand when you finish.
2. Allow 1 minute for pupils to answer the question.
3. Select a pupil with raised hand to explain their answer on the board.
4. Correct any errors in the solution on the board. Ask pupils to check their work.
   (Answer: $\frac{x+6}{2} = 5, x + 6 = 10, x - 10 - 6 = 4$)
5. **Say**: Today we are going to solve linear equations where the variable appears on both sides of the equation by balancing the equation and combining like terms.

Introduction to the New Material (*10 minutes*)

1. **Say:** We sometimes have linear equations where the variable is on both sides of the equation. An example of this type of equation is shown in Question b.
2. Ask a pupil to read Question b.: Solve $5x + 2 = 4x + 5$.
3. **Ask**: What format of the linear equation does each side have? Raise your hand. (Answer: $y = mx + c$).
4. **Say**: When we make 2 linear equations equal to each other, we end up with a variable on both sides of the equal sign. We use the balance method, which we are familiar with, to solve for the variable. We will go through each step for this question.
5. Show how to balance Question b. Ask questions at each step. Ask pupils to raise their hand to answer.
   Note that the order of the steps may be different. Follow the steps suggested by pupils as long as they are correct (e.g. subtract 2, then subtract 4x).
   An example calculation is shown below:

$$5x + 2 = 4x + 5$$

6. **Ask:** What is the first step?
   (Answer: Subtract 4x from both sides.)

$$5x + 2 - 4x = 4x - 4x + 5$$

7. **Ask**: What do we get?
   Answer: Shown as given by selected pupil)

$$x + 2 = 5$$

8. **Ask**: What is the next step?
(Answer: Subtract 2 from both sides.)

$$x + 2 - 2 = 5 - 2$$

9. **Ask:** What answer do we get? (Answer: 3)

$$x = 3$$

10. **Say:** We check our answer as we always do.
11. Have a pupil volunteer to come to the board to check.

<u>Check</u>

$$5 \times 3 + 2 = 4 \times 3 + 5$$
$$17 = 17$$
$$\text{LHS} = \text{RHS}$$

12. Do a visual check of the classroom to ensure pupils have understood the procedure. You may need to go through it one more time.
13. **Say:** From now on we will balance the equation in a more efficient manner. We will write the result from the inverse operation straight away.
14. **Say:** Let us do that with the next question.
15. Ask a pupil to read Question c.: Solve $2(x - 3) = 8 - 3x$.
16. Guide a volunteer to show how to balance the equation in a more efficient manner. Ask other pupils to observe carefully to see if they agree with the calculation. Ask the pupil to write each answer straight away without the intermediate steps.
The procedure is shown below:

$$2(x - 3) = 8 - 3x$$
$$4x - 6 = 8 - 3x$$ ← Remove the brackets (expand the equation)
$$7x - 6 = 8$$ ← Add 3x to both sides of the equation
$$7x = 14$$ ← Add 6 to both sides
$$x = 2$$ ← Divide both sides by 7

<u>Check</u>

$$4 \times 2 - 6 = 8 - 3 \times 2$$
$$2 = 2$$
$$\text{LHS} = \text{RHS}$$

Guided Practice (*10 minutes*)

1. Ask pupils to work in pairs.
2. Point to the questions on the board:
Solve $y = 3x + 7$ when:
a. $y = 5x + 3$
b. $y = 5x - 3$
c. $y = x - 1$
d. $y = -x + 3$

Check the solution to each pair of equations.

3. Walk around, if possible, to check answers and correct any misconceptions.
4. Have pupils from around the classroom volunteer to give their answers.
5. **Write** the correct answers on the board. Ask pupils to check their work.

(Answers:

	Equation	Solution	Check	
a.	$3x + 7 = 5x + 3$	$4 = 2x; x = 2$	$3 \times 2 + 7 = 5 \times 2 + 3; 13 = 13$	LHS=RHS
b.	$3x + 7 = 5x - 3$	$10 = 2x; x = 5$	$3 \times 5 + 7 = 5 \times 5 - 3; 22 = 22$	LHS=RHS
c.	$3x + 7 = x - 1$	$2x = -8; x = -4$	$3 \times (-4) + 7 = -4 - 1; -5 = -5$	LHS=RHS
d.	$3x + 7 = -x + 3$	$4x = -4; x = -1$	$3 \times (-1) + 7 = -(-1) + 3; 4 = 4$	LHS=RHS)

Independent Practice (*10 minutes*)

1. Ask the pupils to work independently to answer the questions.
2. Point to the questions on the board:

   Solve for x in the following equations. Check the solution to the equation.

   a. $3x + 2 = 5x - 8$ b. $2x - 2 = 3x - 7$
   c. $2 - x = x - 4$ d. $3 - 5x = -(x + 5)$
   e. $3(x + 1) = 7(x - 1)$ f. $2(5x + 2) = 3(x - 1)$

3. Walk around, if possible, to check answers and clear misconceptions.
4. Ask pupils to exchange exercise books and check each other's work.
5. Have pupils from around the classroom volunteer to give their answers.
6. **Write** the correct answers on the board. Ask pupils to check the answers.

(Answers:

	Equation	Solution	Check	
a.	$3x + 2 = 5x - 8$	$10 = 2x; x = 5$	$3 \times 5 + 2 = 5 \times 5 - 8; 17 = 17$	LHS=RHS
b.	$2x - 3 = 3x - 9$	$x = 6$	$2 \times 6 - 3 = 3 \times 6 - 9; 9 = 9$	LHS=RHS
c.	$2 - x = x - 4$	$6 = 2x; x = 3$	$2 - 3 = 3 - 4; -1 = -1$	LHS=RHS
d.	$3 - 5x = -(x + 5)$	$3 - 5x = -x - 5;$ $8 = 4x; x = 2$	$3 - 5 \times 2 = -(2 + 5); -7 = -7$	LHS=RHS
e.	$3(x + 1)$ $= 7(x - 1)$	$3x + 3 = 7x - 7;$ $10 = 4x; x - 2.5$	$3 \times 2.5 + 3 = 7 \times 2.5 - 7;$ $10.5 = 10.5$	LHS=RHS
f.	$2(5x + 2)$ $= 3(x - 1)$	$10x + 4 = 3x - 3;$ $7x = -7; x = -1$	$10 \times (-1) + 4 = 3 \times (-1) - 3;$ $-6 = -6$	LHS=RHS

Closing (*2 minutes*)

1. **Say:** Write down on a piece of paper one or 2 questions you did not get right this lesson. Do not write your names, just the questions.
2. Check the questions written by the pupils after the lesson. Use it as a guide to focus on questions for the next lesson when pupils will be answering general questions on solving linear equations.

Lesson Title: Practice Solving Linear Equations	Theme: Algebra	
Lesson Number: M-09-090	**Class/Level**: JSS 3	**Time**: 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson pupils will be able to solve any linear equation in 2 variables.	None	1. Write on the board: Question 1: Which of these are linear equations? a. $y = 4x + 3$ b. $p = 3q + 35$ c. $u + 2w = 15$ d. Solve $y = 4x + 3$ when: i. $x = 4$ ii. $y = 11$ iii. $y = x - 6$ e. Verify if (4, 20) is a solution to the equation. 2. Write the questions from the Guided Practice section on the board. 3. Write the questions from the Independent Practice section on the board.

Opening (*3 minutes*)

1. Point to Question 1 on the board. **Ask:** Which of the equations on the board is a linear equation? Raise your hand.
2. Allow the pupils one minute to think and discuss ideas.
3. Select a pupil to answer the question. (Answer: All of them)
4. **Ask**: How can you tell they are all linear equations? Raise your hand. (Answer: The highest power of both variables is 1.)
5. **Say**: Today we are going to solve any linear equation in 2 variables.

Introduction to the New Material (*10 minutes*)

1. **Say:** We will substitute different values for y in linear equations. We will also verify that an ordered pair is a solution to our equation.
2. **Say:** We will work through all the different types of problems involving linear equations in Question d. together. We will write the answers as ordered pairs.

d. Solve $y = 4x + 3$ when:

i. $x = 4$

$$y = 4 \times 4 + 3$$
$$= 19 \qquad (4, 19)$$

$$\underline{\text{Check}}\ (4,19) \qquad 19 = 4 \times 4 + 3$$

$$19 = 19$$

$$\text{LHS} = \text{RHS}$$

ii. $y = 11$

$$11 = 4x + 3$$

$$8 = 4x$$

$$x = 2 \qquad (2,11)$$

$$\underline{\text{Check}}\ (2,11) \qquad 11 = 4 \times 2 + 3$$

$$11 = 11$$

$$\text{LHS} = \text{RHS}$$

iii. $y = x - 6$

$$x - 6 = 4x + 3:$$

$$-9 = 3x$$

$$x = -3$$

$y = x - 6$

$$y = -3 - 6$$

$$= -9 \qquad (-3,-9)$$

$$\underline{\text{Check}}\ (-3,-9) \qquad -3 - 6 = 4 \times (-3) + 3$$

$$-9 = -12 + 3$$

$$-9 = -9$$

$$\text{LHS} = \text{RHS}$$

iv. <u>Verify</u> $(4,20)$

$$20 = 4 \times 4 + 3$$

$$20 \neq 19$$

$$\text{LHS} \neq \text{RHS}$$

$(4,20)$ is not a solution to the linear equation.

3. **Say:** These are the main types of problems that are involved in solving linear equations. It is very important that we check after we have solved for x, y or both. This helps us to catch any errors we make in our calculations.

Guided Practice (*10 minutes*)

1. Ask pupils to work in pairs.

2. Point to the questions on the board:
   Solve $y = 2(x + 5)$ when:
   a. $x = 6$ b. $y = 18$ c. $y = 2(1 - x)$
   d. Verify if (2, 16) is a solution to the equation.

3. Walk around, if possible, to check answers and correct any misconceptions.
4. Have pupils from around the classroom volunteer to give their answers.
5. **Write** the correct answers on the board. Ask pupils to check their work.

(Answers:

Equation	Solution	Check		
$y = 2(x + 5)$				
a. $x = 6$	$y = 2(6 + 5) = 2 \times 11 = 22$	$(6, 22)$	$22 = 2(6 + 5) = 2 \times 11 = 22$	LHS=RHS
b. $y = 18$	$18 = 2(x + 5), 9 = x + 5, x = 4$	$(4, 18)$	$18 = 2(4 + 5) = 2 \times 9 = 18$	LHS=RHS
c. $y = 2(1 - x)$	$2(1 - x) = 2(x + 5)$, $1 - x = x + 5, 2x = -4, x = -2$, $y = 2 \times (-2 + 5) = 2 \times 3 = 6$	$(-2, 6)$	$2(1 - (-2)) = 2(-2 + 5)$, $2 \times 3 = 2 \times 3 \times 6 = 6$	LHS=RHS
d. Verify $(2, 16)$	$16 = 2(2 + 5) = 2 \times 7 = 14$	$(2, 16)$	is not a solution	LHS≠RHS

Independent Practice (*10 minutes*)

1. Ask the pupils to work independently to answer the questions.
2. Point to the questions on the board:
   Solve $y = 3(x - 2)$ when:
   a. $x = 4$ b. $y = 12$ c. $y = x - 4$
   d. Verify if (8, 18) is a solution to the equation.
3. Walk around, if possible, to check answers and clear misconceptions.
4. Have pupils from around the classroom volunteer to give their answers.
5. **Write** the correct answers on the board. Ask pupils to check their work.

(Answers:

Equation	Solution	Check		
$y = 3(x - 2)$				
a. $x = 4$	$y = 3(4 - 2) = 3 \times 2 = 6$	$(4, 6)$	$6 = 3(4 - 2) = 3 \times 2 = 6$	LHS=RHS
b. $y = 12$	$12 = 3(x - 2), 4 = x - 2, x = 6$	$(6, 12)$	$12 = 3(6 - 2) = 3 \times 4 = 12$	LHS=RHS
c. $y = x - 4$	$x - 4 = 3(x - 2)$, $x - 4 = 3x - 6, 2x = 2, x = 1$ $y = 1 - 4 = -3$	$(1, -3)$	$1 - 4 = 3(1 - 2)$, $-3 = 3 \times (-1) = -3$	LHS=RHS
d. Verify $(8, 18)$	$18 = 3(8 - 2) = 3 \times 6 = 18$	$(8, 18)$	is a solution	LHS=RHS

Closing (*2 minutes*)

1. **Say:** Write down on a piece of paper one or 2 questions you did not get right this lesson. Do not write your names, just the questions.
2. Check the questions written by the pupils after the lesson. Use it as a guide to focus on questions for the next lesson when pupils will be answering story problems on linear equations.

Lesson Title: Solving Linear Equation Story Problems I	**Theme**: Algebra	
Lesson Number: M-09-091	**Class/Level**: JSS 3	**Time**: 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson pupils will be able to solve simple story problems by creating and solving linear equations in 2 variables.	None	Write the questions at the end of this lesson plan on the board.

Opening (*3 minutes*)

1. **Say**: Solve for x in Question a. on the board (Question a. is provided with the other questions at the end of this lesson plan). Raise your hand when you finish.
2. Allow 1 minute for pupils to answer the question.
3. Select a pupil with raised hand to explain their answer on the board. (Answer: $y = 4x + 5 = 4 \times 3 + 5 = 17$)
4. **Say**: Today we are going to solve simple story problems by creating and solving linear equations in 2 variables.

Introduction to the New Material (*10 minutes*)

1. Ask a pupil to read Question b. (at the end of this lesson plan) on the board .
2. **Ask:** Who would like to explain on the board what to do?
3. Select a pupil with hand raised. (Answer: Equation shown right)
4. **Say:** We know that Amina is 15 years old. Show that in the equation.
5. **Say**: Solve for x.
6. Ask a pupil to read Question c. on the board.
7. **Ask:** What are we being asked to find? Raise your hand.
8. Allow pupils to discuss and share ideas for a few moments (less than 1 minute).
9. Have a pupil from the back of the classroom volunteer to answer. (Answer: Equation, or formula, for perimeter of a rectangle.)
10. Guide pupils to show how to set up and solve for the perimeter of a rectangle. Ask pupils to raise their hand to answer.
11. **Ask:** What is the general formula for the perimeter of a rectangle? (Answer: 2 x (length + width)
12. **Ask**: How do we write that as an equation involving both P and x? (Example answer: Shown; next equation may be given straightaway.)
13. **Ask**: What is the next step? (Answer: Collect like terms and simplify.)

<u>Question b.</u>

$$y = x + 3$$
$$15 = x + 3$$
$$15 - 3 = x$$
Marie's age: $x = 12$

<u>Question c.</u>

$$P = 2 \times (x + x + 5)$$
$$P = 2(2x + 5)$$

14. **Ask:** What variables do we have in this equation? (Answer: P and x)
15. **Say:** The value of P will change depending on the value of x. How do we find the value of P when $\mathrm{x} = 8$ cm? (Answer: Substitute 8 into the equation.)
16. Have a pupil volunteer to come to the board to carry out the calculation. Ask other pupils to solve it in their exercise books.
17. Correct any errors in the solution on the board. Ask pupils to check their work.

$$\begin{aligned} \mathrm{P} &= 2(2 \times 8 + 5) \\ &= 2 \times 21 \\ &= 42 \text{ cm} \end{aligned}$$

18. **Say:** We can also find x if we know the value of P.

Suppose $\mathrm{P} = 50$ cm

$$\begin{aligned} 50 &= 2(2\mathrm{x} + 5) \\ 25 &= 2\mathrm{x} + 5 \\ 20 &= 2\mathrm{x} \\ \mathrm{x} &= 10 \text{ cm} \end{aligned}$$

<u>Check</u>

$$\begin{aligned} 50 &= 2(2 \times 10 + 5) \\ &= 2 \times 25 \\ 50 &= 50 \\ \text{LHS} &= \text{RHS} \end{aligned}$$

Guided Practice (*10 minutes*)

1. Ask pupils to work in pairs to answer Questions d. and e. on the board.
2. Walk around, if possible, to check answers and correct any misconceptions.
3. Have pupils from around the classroom volunteer to give their answers.
4. **Write** the correct answers on the board. Ask pupils to check their work. (Answers: shown at the end of this lesson plan)

Independent Practice (*10 minutes*)

1. Ask the pupils to work independently to answer Questions f. and g.
2. Walk around, if possible, to check answers and clear misconceptions.
3. Ask pupils to exchange exercise books and check each other's work.
4. Have pupils from around the classroom volunteer to give their answers.
5. **Write** the correct answers on the board. Ask pupils to check their work. (Answers: shown at the end of this lesson plan)

Closing (*2 minutes*)

1. **Say:** Write down one thing you learnt today.
2. Allow pupils 1 minute to discuss and share ideas.
3. Have one pupil from the front and one from the back of the classroom volunteer to answer. (Example answers: How to set up and solve a linear equation in 2 variables)

[QUESTIONS FOR USE DURING THE LESSON]

a. Solve $y = 4x + 5$ when $x = 3$.
b. Amina is 3 years older than her sister Marie. If Amina's age is y and Marie's age is x, write the relationship between the ages in an equation. If Amina is 15 years old, how old is her sister Marie?
c. The width of a rectangle is x. The length is $x + 5$.
   i. Write an equation for its perimeter, P, in terms of x.
   ii. Find the perimeter when $x = 8$ cm.
   iii. Find x when the perimeter is 50 cm.
d. Abi thinks of a number. She multiplies the number by 5 and then subtracts 2. She writes this answer as y in an equation. Can you write the equation Abi wrote down? Abi calculates that y is equal to 23. What number did she think of?
e. 3 angles of a shape are given by x, $x + 20$ and $x + 40$. Write an equation for y, the sum of the 3 angles. If $y = 180$, find the value of x and the other 2 angles.
f. Ahmed thinks of a number, x. He multiplies it by 3, then adds 3. The answer, y, is the same if he adds 10 to the number. Find the number.
g. 2 consecutive integers are denoted by n and $n + 1$. Find an equation for their sum, S.
   If $S = 69$, find the 2 integers.

(Answers:

a. See Opening
b. See Introduction to the New Material
c. See Introduction to the New Material
d. $y = 5x - 2$
   $y = 23 = 5x - 2, 25 = 5x$ $\underline{x = 5}$
   Check: $23 = 5 \times 5 - 2$ $= 23 - 2 = 23$ LHS = RHS
e. $y = x + (x + 20) + (x + 40) = x + x + x + 20 + 40$
   $= 3x + 60$
   $180 = 3x + 60, 120 = 3x,$ $\underline{x = 40}$
   The other angles are: $x + 20 = 40 + 20 = 60$
   and $x + 40 = 40 + 40 = 80$
   Check: $180 = 3 \times 40 + 60$ $= 120 + 60 = 180$ LHS = RHS
   Check: $180 = 40 + 60 + 80, 180 = 180$ LHS = RHS
f. $y = 3x + 3 = x + 10, 2x = 7, x = \frac{7}{2} = 3.5$
   Check: $3 \times 3.5 + 3 = 3.5 + 10,$ $13.5 = 13.5$ LHS = RHS
g. $S = n + n + 1 = 2n + 1$
   $69 = 2n + 1, 68 = 2n, n = 34, n + 1 = 35$
   Check: $69 = 34 + 35$ $= 69$ LHS = RHS

Lesson Title: Solving Linear Equation Story Problems II	**Theme**: Algebra	
Lesson Number: M-09-092	**Class/Level**: JSS 3	**Time**: 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson pupils will be able to solve more difficult story problems by creating and solving linear equations in 2 variables.	None	Write the questions at the end of this lesson plan on the board.

Opening (*3 minutes*)

1. **Say**: Solve for x in Question a. on the board (all questions are written at the end of this lesson plan). Raise your hand when you finish.
2. Allow 1 minute for pupils to answer the question.
3. Select a pupil with raised hand to explain their answer on the board. Ask other pupils to observe carefully to see if they agree with the calculation.
4. Correct any errors in the solution on the board. Ask pupils to check their work.
   (Answer: $y = 5 = \frac{x+6}{2}, 10 = x + 6, 10 - 6 = x, \ x = 4$)
5. **Say**: Today we are going to solve more difficult story problems by creating and solving linear equations in 2 variables.

Introduction to the New Material (*10 minutes*)

1. Have a pupil volunteer to read Question b. on the board.
2. **Say:** We will solve the equation together. You will answer each question as we go along in your exercise books. Raise your hand to answer my questions.
3. **Ask:** Who would like to explain the first step?
4. Select a pupil with hand raised.
   (Example answer: Add the 3 numbers and make them equal to y.)
5. Ask pupils to write down the answer for the first step in their exercise books. (Answer: Equation shown at right)
6. **Ask**: What is the next step?
   (Answer: Collect like terms and simplify.)
7. Allow a few moments for pupils to do this.
8. Have a pupil volunteer to complete the equation on the board.
9. **Say:** We know that $y = 120$. Show that in the equation.
   Solve for x.
10. Allow some more time for pupils to solve the equation in their exercise books.
11. Have another pupil volunteer to show this on the board.
    (Answer: Solution shown at right)
12. Ask pupils to find the other 2 numbers. (Answers: 22.5 and 78)

<u>Question b.</u>

$$y = x + (x + 3) + 4x$$
$$y = x + x + 4x + 3$$
$$y = 6x + 3$$
$$120 = 6x + 3$$
$$117 = 6x$$
$$x = 19.5$$

13. Ask pupils to check their numbers are correct.

$$120 = 19.5 + 22.5 + 78$$
$$120 = 120$$
$$\text{LHS} = \text{RHS}$$

14. Ask a pupil to read Question c. on the board.
15. **Ask:** What equation are we being asked to find?
16. Allow pupils to discuss and share ideas for a few moments (less than 1 minute).
17. Have a pupil from the back of the classroom volunteer to answer.
    (Answer: Shown at right)
18. **Ask**: What is the next step? (Answer: Write the second equation and make the 2 equations equal.)
19. Show how to do this on the board.
20. Ask the pupils to copy it into their exercise books.

<u>Question c.</u>

$$y = 3x + 7$$

$$4x + 3 = 3x + 7$$

21. **Say:** Work in pairs to solve for x and y.
    Raise your hand when you finish.
22. Allow 3 minutes for pupils to do this in their exercise books. They should also check their solution.
23. Select a pupil with hand raised to explain the solution on the board. Ask other pupils to observe carefully to see if they agree with the calculation.
24. Correct any errors in the solution on the board. Ask pupils to check their work. (Answer: see below)

$$4x + 3 = 3x + 7$$
$$x = 4$$

$$y = 3x + 7$$
$$= 3 \times 4 + 7$$
$$y = 19$$

$$4x + 3 = 3x + 7$$
$$4 \times 4 + 3 = 3 \times 4 + 7$$
$$19 = 19$$
$$\text{LHS} = \text{RHS}$$

Guided Practice (*10 minutes*)

1. Ask pupils to work in pairs to answer Questions d. and e. on the board.

2. Walk around, if possible, to check answers and correct any misconceptions.
3. Have pupils from around the classroom volunteer to give their answers.
4. **Write** the correct answers on the board. Ask pupils to check their work. (Answers: shown at the end of this lesson plan.

Independent Practice (*10 minutes*)

1. Ask the pupils to work independently to answer Questions f. and g.
2. Walk around, if possible, to check answers and clear misconceptions.
3. Ask pupils to exchange exercise books and check each other's work.
4. Have pupils from around the classroom volunteer to give their answers.
5. **Write** the correct answers on the board. Ask pupils to check their work. (Answers: shown at the end of this lesson plan.

Closing (*2 minutes*)

1. **Say:** Write down one thing you learnt today.
2. Allow pupils 1 minute to discuss and share ideas.
3. Have one pupil from the front and one from the back of the classroom volunteer to answer. (Example answers: How to create and solve more difficult linear equations in 2 variables. Accept all reasonable answers.)

[QUESTIONS FOR USE DURING THE LESSON]

a. Solve $y = \frac{x+6}{2}$ when $y = 5$.

b. 3 numbers are denoted x, $x + 3$ and $4x$. Their sum is denoted by y. Write an equation showing this relationship between x and y. If $y = 120$, what are the 3 numbers?

c. The value of a quantity, y, is given by 3 times x plus 7.
   i. Write an equation connecting y and x.
   ii. If this same quantity, y, is equal to 4 times x minus 3, write the equation connecting the 2 relationships. Hence, find x and y.

d. 2 consecutive odd numbers are denoted by n and $n + 2$. Their sum is denoted by S. Write the equation to show this. If $S = 52$, find the numbers.

e. 2 rectangles have the same area, A. The sides of the first rectangle are 2 cm and $(x + 8)$ cm. The sides of the second rectangle are 3 cm and $(x + 4)$ cm. Find the value of x.

f. 2 friends are the same age, A. One friend says 'You will know my age if you multiply a number by 4 and then add 3.
   The second friend says 'You will know my age if you multiply the same number as my friend by 6 and then subtract 3. What age are the friends?

g. The length of each side of an equilateral triangle is $x + 5$.
   i. Write an equation for its perimeter, P, in terms of x.
   ii. Find the perimeter when $x = 6$ cm.
   iii. Find x when the perimeter is 45 cm.

(Answers:

a. See Opening
b. See Introduction to the New Material
c. See Introduction to the New Material

d.	$S = n + n + 2 = 2n + 2, S = 52 = 2n + 2, 50 = 2n,$ $n = 25$. The second number is 27.	Check: $52 = 2 \times 25 + 2$ $= 52$	LHS = RHS
e.	$A = 2(x + 8) = 3(x + 4), 2x + 16 = 3x + 12, x = 4$	Check: $2(4 + 8) = 3(4 + 4),$ $2 \times 12 = 3 \times 8, 24$ $= 24$	LHS = RHS
f.	$\text{Age} = 4x + 3 = 6x - 3, 2x = 6, x = 3,$ $\text{Age} = 4 \times 3 + 3 = 15$	Check: $4 \times 3 + 3 = 6 \times 3 - 3$ $15 = 15$	LHS = RHS
g.	i. $P = (x + 5) + (x + 5) + (x + 5)$ $= 3 \times (x + 5) = 3(x + 5)$ ii. $P = 3(6 + 5) = 3 \times 11 = 33$ cm iii. $45 = 3(x + 5), 15 = x + 5, x = 10$ cm)	Check: c) $45 = 3(10 + 5)$ $= 3 \times 15 = 45$).	LHS = RHS

Lesson Title: Table of Values	**Theme**: Algebra	
Lesson Number: M-09-093	**Class/Level**: JSS 3	**Time**: 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson pupils will be able to create a table of values for a simple linear equation in 2 variables.	None	1. Write on the board: Write these linear equations in the form $y = mx + c$: a. $x + y - 2 = 0$ b. $x - y = 2$ c. $2x - y + 1 = 0$ 2. Write on the board: <u>Vocabulary list:</u> dependent and independent variables 3. Copy the linear equations and tables at the end of the lesson plan for Guided Practice. 4. Write the questions from the Independent Practice section on the board.

Opening (*3 minutes*)

1. **Say**: Change the linear equations on the board to the form $y = mx + c$.
2. Allow 1-2 minutes for pupils to answer the question.
3. Have pupils volunteer to give their answers. (Answers: a. $y = 2 - x$, b. $y = x + 2$, c. $y = 2x + 1$)
4. **Say**: Today we are going to create a table of values for simple linear equations in 2 variables.

Introduction to the New Material (*10 minutes*)

1. The tables in this lesson plan will be used in future lessons. Ask pupils to copy the tables in their exercise books.
2. **Say:** The linear equation $y = mx + c$ has 2 variables, x and y. The value of y will change depending on the value of x. y is called the dependent variable, and x is called the independent variable. Let us create a table of values for the linear equation $y = 2 - x$.
3. Draw a table with values of x from -3 to $+3$.
4. Have pupils volunteer one at a time to give the y value for each x value.
5. Complete the table with the given values. Correct any errors.

x	-3	-2	-1	0	1	2	3
y	5	4	3	2	1	0	-1

6. **Say:** The table will be used in a future lesson to draw a straight line graph.
7. Select a pupil to read the amended form of Equation b., $y = x + 2$.
8. Ask pupils to copy and complete the table for $y = x + 2$ in their exercise books. They should use x values from -3 to $+3$.
9. Have a pupil volunteer to come to the board and complete the table with help from the class.
10. Correct any errors in the solution on the board.

x	-3	-2	-1	0	1	2	3
y	-1	0	1	2	3	4	5

11. Ask pupils to work in pairs to complete the table for the amended form of Equation c., $y = 2x + 1$.
12. Have a pupil volunteer to come to the board and complete the table with help from the class.
13. Correct any errors in the solution on the board.

x	-3	-2	-1	0	1	2	3
y	-5	-3	-1	1	3	5	7

14. **Ask:** Can anyone see a pattern in the y values in the tables?
15. Allow the pupils to discuss the tables to find any patterns.
16. Have pupils from around the room volunteer to give the pattern to the y values in each table. (Answer: $y = 2 - x$: the y values decrease in steps of one; $y = x + 2$: increase in steps of 1; $y = 2x + 1$: increase in steps of 2.)
17. **Say**: You can usually see a pattern when you start to complete a table. This helps you check you are not making a mistake.

Guided Practice (*10 minutes*)

1. Ask pupils to continue to work in pairs.
2. Ask them to match the linear equation with the correct table of values in the linear equations and corresponding table at the end of the lesson plan.
3. Walk around, if possible, to check answers and correct any misconceptions.
4. Have pupils from around the classroom volunteer to give their answers.
5. **Write** the correct answers on the board. Ask pupils to check their work. (Answers: shown at the end of this lesson plan)

Independent Practice (*10 minutes*)

1. Ask the pupils to work independently to answer the questions.
2. Point to the questions on the board:

   Draw a table with values of x from -3 to $+3$ for the linear equations below:

   a. $y = 3x$ b. $y = 2x + 3$
   c. $y = 3x + 5$ d. $3x - y = 2$
   e. $y = 6 - 4x$ f. $y = 5x - 4$

3. Walk around, if possible, to check answers and clear misconceptions.
4. Have pupils from around the classroom volunteer to give their answers.
5. **Write** the correct answers on the board. Ask pupils to check their work. (Answers: shown at the end of this lesson plan)

Closing (*2 minutes*)

1. **Say:** Work in your pairs and write down 2 different things you learnt today.
2. Allow pupils 1 minute to discuss and share ideas.

3. Have one pupil from the front and one from the back of the classroom volunteer to answer. (Example answers: How to create a table of values from a linear equation; how to recognise a pattern in a table of values)

[TABLE OF VALUES FOR GUIDED PRACTICE]

Match the linear equation with the correct table of values.

a. $y = x + 3$

b. $y = 3x + 2$

c. $y = x$

d. $y = 2x - 1$

e. $2x + y = 5$

f. $y = 3x + 1$

a.

x	−3	−2	−1	0	1	2	3
y	−3	−2	−1	0	1	2	3

b.

x	−3	−2	−1	0	1	2	3
y	11	9	7	5	3	1	−1

c.

x	−3	−2	−1	0	1	2	3
y	0	1	2	3	4	5	6

d.

x	−3	−2	−1	0	1	2	3
y	-8	−5	−2	1	4	7	10

e.

x	−3	−2	−1	0	1	2	3
y	−7	−4	−1	2	5	8	111

f.

x	−3	−2	−1	0	1	2	3
y	−7	−5	−3	−1	1	3	5

(Answers: Match linear equation and table: a. and c.; b. and e.; c. and a.; d. and f.; e. and b.; f. and d.)

[ANSWERS: TABLE OF VALUES FOR INDEPENDENT PRACTICE]

a. $y = 3x$

x	−3	−2	−1	0	1	2	3
y	−9	−6	−3	0	3	6	9

b. $y = 2x + 3$

x	−3	−2	−1	0	1	2	3
y	−3	−1	1	3	5	7	9

c. $y = 3x + 5$

x	−3	−2	−1	0	1	2	3
y	−4	−1	2	5	8	11	14

d. $3x - y = 2$

x	−3	−2	−1	0	1	2	3
y	−11	−8	−5	−2	1	4	7

e. $y = 6 - 4x$

x	−3	−2	−1	0	1	2	3
y	18	14	10	6	2	−2	−6

f. $y = 5x - 4$

x	−3	−2	−1	0	1	2	3
y	−19	−14	−9	−4	1	6	11

Lesson Title: Table of Values	**Theme**: Algebra	
Lesson Number: M-09-094	**Class/Level**: JSS 3	**Time**: 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson pupils will be able to create a table of values for a more complicated linear equation in 2 variables.	None	1. Write on the board: Write these linear equations in the form $y = mx + c$: a. $y - \frac{1}{2}x - 2 = 0$ b. $x - 2y = 2$ c. $2(x + 3y) = 5$ 2. Write the questions from the Guided Practice section on the board.

Opening (*3 minutes*)

1. **Say**: Change the linear equations on the board to the form $y = mx + c$.
2. Allow 1-2 minutes for pupils to answer the question.
3. Have pupils volunteer to give their answers.
4. **Write** the correct answers on the board. Ask pupils to check their work. (Answers: a. $y = \frac{1}{2}x + 2$; b. $y = \frac{x-2}{2}$; c. $y = \frac{5-2x}{2}$)
5. **Say**: Today we are going to create a table of values for more complicated linear equations in 2 variables.

Introduction to the New Material (*10 minutes*)

1. The tables in this lesson plan will be used in future lessons. Ask pupils to copy the tables in their exercise books.
2. **Say:** We will continue our work from the last lesson. Let us create a table of values for the linear equation $y = \frac{1}{2}x + 2$.
3. Draw a table with values of x from -3 to $+3$.
4. Have pupils volunteer one at a time to give the y value for each x value.
5. Complete the table with the given values. Answers can be given in fractions or decimals. Correct any errors.

	x	-3	-2	-1	0	1	2	3
EITHER	y	0.5	1	1.5	2	2.5	3	3.5
OR	y	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3	$3\frac{1}{2}$

6. Have a pupil volunteer to read the amended form of Equation b., $y = \frac{x-2}{2}$.
7. Ask pupils to copy and complete the table for $y = \frac{x-2}{2}$ in their exercise books.
8. **Say**: Use the pattern in the y values to help you check you are not making a mistake.
9. Have a pupil volunteer to come to the board and complete the table with help from the class. Answers can be given in fractions or decimals.

10. Correct any errors in the solution on the board.

	x	-3	-2	-1	0	1	2	3
EITHER	y	-2.5	-2	-1.5	-1	-0.5	0	0.5
OR	y	$-2\frac{1}{2}$	-2	$-1\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$

11. Ask pupils to work in pairs to complete the table for the amended form of Equation c., $y = \frac{5-2x}{2}$.
12. Have a pupil volunteer to come to the board and complete the table with help from the class. Answers can be given in fractions or decimals.
13. Correct any errors in the solution on the board.

	x	-3	-2	-1	0	1	2	3
EITHER	y	5.5	4.5	3.5	2.5	1.5	0.5	-0.5
OR	y	$5\frac{1}{2}$	$4\frac{1}{2}$	$3\frac{1}{2}$	$2\frac{1}{2}$	$1\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$

Guided Practice (*10 minutes*)

1. Ask pupils to continue to work in pairs.
2. Point to the questions on the board:

   Create a table with values of x from -3 to $+3$. Give your answers as fractions.

   a. $y = \frac{2}{5}x + 1$
   b. $y = \frac{1}{3}x + 1$
   c. $y = 2 - \frac{1}{2}x$
   d. $2x + 3y = 1$

3. Walk around, if possible, to check answers and clear misconceptions.
4. Have pupils from around the classroom volunteer to give their answers.
5. **Write** the correct answers on the board. Ask pupils to check their work. (Answers shown at the end of this lesson plan)

Independent Practice (10 minutes)

1. Ask pupils to continue to work in pairs.
2. Ask pupils to create **3** linear equations and a related table of values of x from -3 to $+3$.
3. Ask them to give the linear equations and the tables to their partner.
4. Their partner will then try to match the linear equation with the correct table of values.
5. Guide pupils to create simple equations (e.g. $y = x + 7, y = \frac{1}{2}x$). The challenge here is for them to come up with their own equations and related tables.
6. Walk around, if possible, to check answers and correct any misconceptions. For example, the equations are not linear (the power of x and/or y is higher than 1).
7. Have pupils from around the classroom volunteer to write their linear equations and related tables on the board.

8. Correct any errors on the board. (Answers: Various. Use errors or misconceptions as teaching points.)

Closing (*2 minutes*)

1. **Say:** Work in your pairs and write down 2 different things you learnt today.
2. Allow pupils 1 minute to discuss and share ideas.
3. Have one pupil from the front and one from the back of the classroom volunteer to answer. (Example answers: How to create a table of values with fractions or decimals; create own linear equation and table of values)

[ANSWERS: TABLE OF VALUES FOR GUIDED PRACTICE]

i. $y = \frac{2}{5}x + 1$

x	-3	-2	-1	0	1	2	3
y	$-\frac{1}{5}$	$\frac{1}{5}$	$\frac{3}{5}$	1	$1\frac{2}{5}$	$1\frac{4}{5}$	$2\frac{1}{5}$

ii. $y = \frac{1}{3}x + 1$

x	-3	-2	-1	0	1	2	3
y	0	$\frac{1}{3}$	$\frac{2}{3}$	1	$1\frac{1}{3}$	$1\frac{2}{3}$	2

iii. $y = 2 - \frac{1}{2}x$

x	-3	-2	-1	0	1	2	3
y	$3\frac{1}{2}$	3	$2\frac{1}{2}$	2	$1\frac{1}{2}$	1	$\frac{1}{2}$

iv. $2x + 3y = 1$

x	-3	-2	-1	0	1	2	3
y	$2\frac{1}{3}$	$1\frac{2}{3}$	1	$\frac{1}{3}$	$-\frac{1}{3}$	-1	$-1\frac{2}{3}$

Lesson Title: Review of the Cartesian Plane	**Theme**: Algebra	
Lesson Number: M-09-095	**Class/Level**: JSS 3	**Time**: 35 minutes

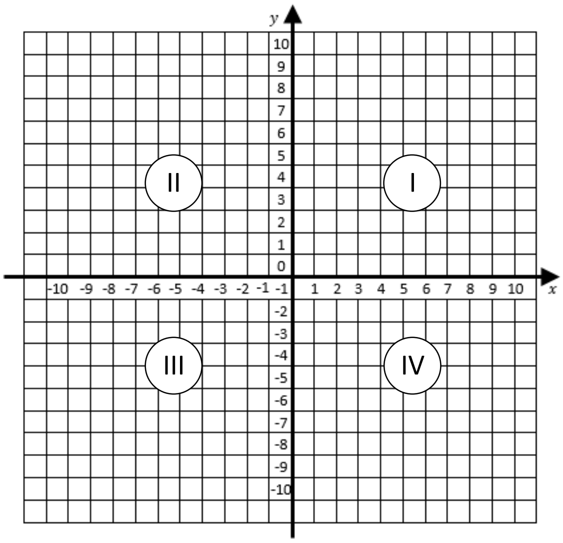
Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson pupils will be able to: 1. Draw a Cartesian plane. 2. Identify the x- and y-axes and label them with positive and negative values. 3. Identify points in each quadrant of a Cartesian plane and write them in the form (x, y).	None	1. If your board does not have a Cartesian plane (grid), you will need to prepare one on half of the board. You will need grids for all the lessons on graphs of linear equations. 2. Mark on a Cartesian plane the points from –10 to +10 for both axes as shown in the Introduction. 3. Write on the board: <u>Vocabulary List</u>: Cartesian plane, axis, axes (plural), quadrant, co-ordinates, point

Opening (*3 minutes*)

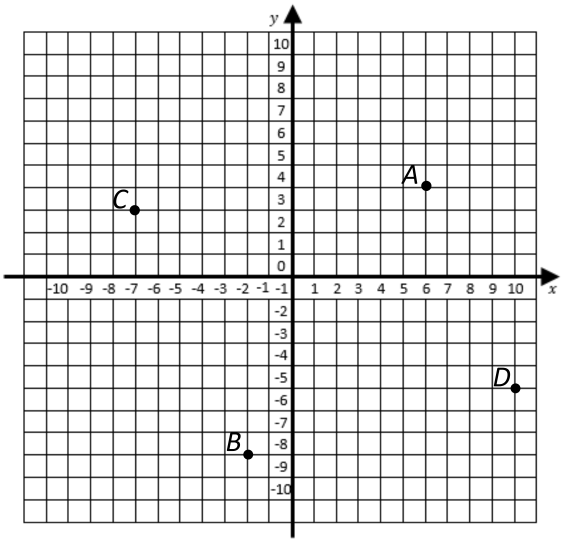
1. **Ask**: Who can remind the class what an ordered pair is? Raise your hand. (Answer: A solution to a linear equation written as (x, y))
2. **Say**: Today we are going to draw a Cartesian plane, identify the x- and y-axes in the plane. We will also be plotting points in each quadrant of a Cartesian plane and write them in the form (x, y).

Introduction to the New Material (*10 minutes*)

1. Mark the points from –10 to +10 for both axes as shown.
2. Note that the arrow points in the direction of increasing value (**not** both directions as often shown in textbooks). Also note the completed grid is shown here.
3. Pupils will initially see a blank grid with only the axes showing. Complete the grid as you go through the information.
4. **Say:** The grid system is called a 'Cartesian plane'. A plane is any flat 2-dimensional surface.
5. Point to the axes one at a time.
6. **Say:** We draw the 2 axes on the Cartesian plane. The x-axis goes from left to right and increases in value as shown by the arrow. Only a small part of the axis is shown from –10 to +10. It goes to infinity in both directions.
7. **Say:** The y-axis goes from the bottom of the board to the top. It also increases in value in the direction of the arrow. We have shown only the part from –10 to +10. It too goes to infinity in both directions.
8. **Say:** The 2 axes divide the Cartesian plane into 4 quadrants, numbered as shown with Roman numerals.
9. Number the quadrants at this point.



10. Mark the points A, B, C, and D on the plane as shown. Or any 4 points (one in each quadrant).
11. **Say**: Who can tell the class the coordinates of point A? Raise your hand.
12. Select a pupil who has raised their hand to give the answer. (Answer: $\mathrm{A}\ (6, 4)$)
13. Repeat the question for the other 3 points. (Answers: $\mathrm{B}\ (-2, -8), \mathrm{C}\ (-7, 3), \mathrm{D}\ (10, -5)$)



Guided Practice (*10 minutes*)

1. Mark any 8 points, 2 in each quadrant of the plane on the board. (Or use the example question for Guided Practice at the end of the lesson plan)
2. Ask pupils to work in pairs to answer the question on the board.
3. Ask pupils to write the coordinates as ordered pairs.
4. Walk around, if possible, to check their answers and clear up any misconceptions. For example, make sure pupils write the ordered pairs with the x co-ordinate first, then the y co-ordinate.
5. Have pupils from around the classroom volunteer to give their answers to the questions.
6. **Write** the correct answers on the board. Ask pupils to check their work. (Answers: Various answers possible. Answer to the example questions given at the end of this lesson plan.)

Independent Practice (*10 minutes*)

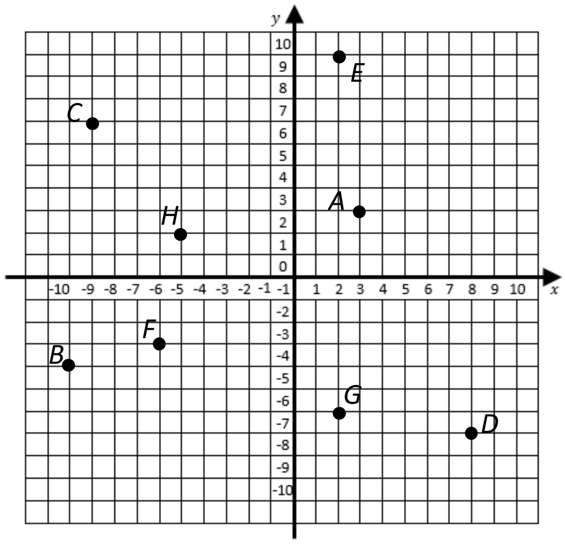
1. Mark any 8 points, 2 in each quadrant of the plane on the board. (Or use the example question for Independent Practice at the end of the lesson plan)
2. Ask the pupils to work independently to answer the questions.
3. Ask pupils to write the coordinates as ordered pairs.
4. Walk around, if possible, to check their answers and clear up any misconceptions. For example, make sure pupils write the ordered pairs with the x co-ordinate first, then the y co-ordinate.
5. Have pupils from around the classroom volunteer to give their answers to the questions.
6. **Write** the correct answers on the board. Ask pupils to check their work. (Answers: Various answers possible. Answer to the example questions given at the end of this lesson plan.)

Closing (*2 minutes*)

1. **Say:** Please write down one new thing you learned today.
2. Allow pupils 1 minute answer the question.
3. Have pupils from around the classroom volunteer to answer. (Example answers: That the Cartesian plane is a 2-dimensional surface; that the quadrants are identified by Roman numerals; that you always write the x co-ordinate first, then the y co-ordinate)

[EXAMPLE QUESTION FOR GUIDED PRACTICE]

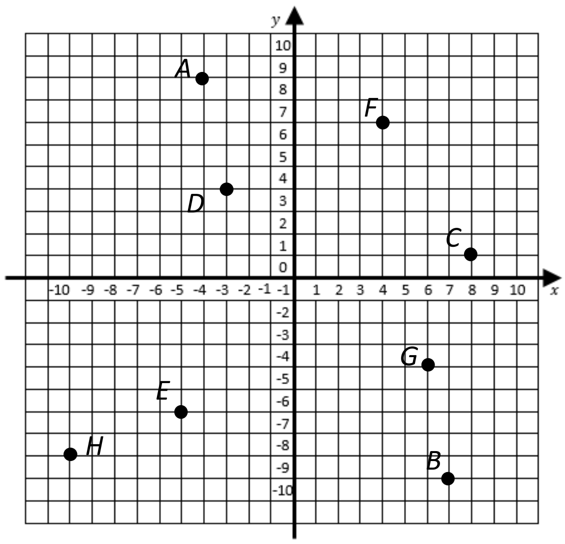
Write down the coordinates of the points marked on the Cartesian plane below.



(Answers: $\mathrm{A}(3,3), \mathrm{B}(-10,-4), \mathrm{C}(-9,7), \mathrm{D}(8,-7), \mathrm{E}(2,10), \mathrm{F}(-6,-3), \mathrm{G}(2,-6), \mathrm{H}(-6,2)$)

[EXAMPLE QUESTION FOR INDEPENDENT PRACTICE]

Write down the coordinates of the points marked on the Cartesian plane below.



(Answers: $\mathrm{A}(-4,9), \mathrm{B}(7,-9), \mathrm{C}(8,1), \mathrm{D}(-3,4), \mathrm{E}(-5,-6), \mathrm{F}(4,7), \mathrm{G}(6,-4), \mathrm{H}(-10,-8)$)

Lesson Title: Plotting Points in the Cartesian Plane	**Theme**: Algebra	
Lesson Number: M-09-096	**Class/Level**: JSS 3	**Time**: 35 minutes

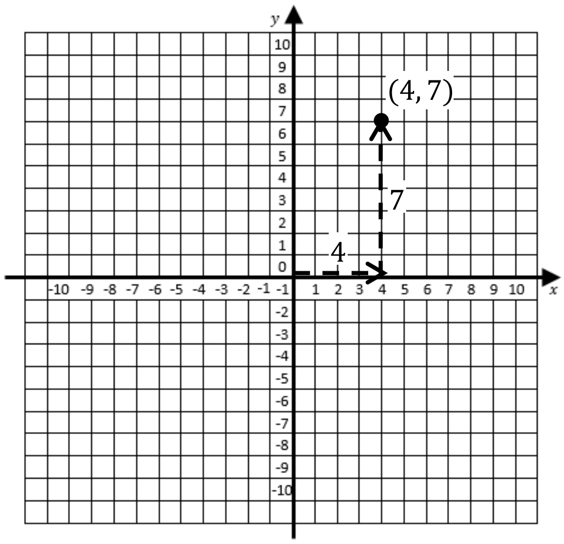
Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to plot given points in any quadrant of the Cartesian plane.	None	1. If your board does not have a Cartesian plane (grid), you will need to prepare one on half of the board. You will need grids for all the lessons on graphs of linear equations. 2. Draw a triangle on the Cartesian plane – see example question for the Opening Activity at the end of the lesson plan. 3. Write on the board: <u>Vocabulary List</u>: origin 4. Write the questions from the Guided Practice section on the board. 5. Write the questions from the Independent Practice section on the board.

Opening (*3 minutes*)

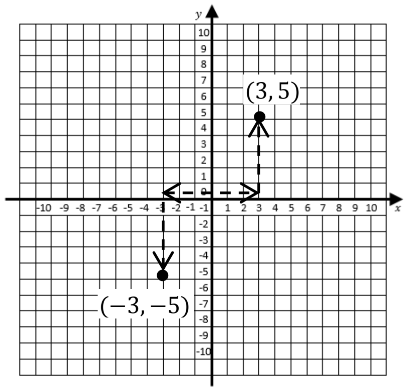
1. Ask pupils to write the co-ordinates of the 3 corners of the triangle.
2. Have one pupil from the front and one pupil from the back of the classroom volunteer to give the answers.
3. **Write** the correct answers on the board. Ask pupils to check their work. (Answers: various, see answer for example question at end of this lesson plan)
4. **Say**: Today we are going to plot given points in any quadrant of the Cartesian plane.

Introduction to the New Material (*10 minutes*)

1. **Ask**: Who would like to remind the class how to plot point $(4, 7)$on the plane on the board?
2. Guide a pupil who raised their hand to come to the board to plot the point. Ask the pupil to explain what they did. (Answer: shown right)
3. **Ask**: In which quadrant is the point $(4, 7)$? Raise your hand. (Answer: Quadrant I)
4. **Say**: Here is a little guide that will help us plot any point (x, y). We can use the procedure at any time.
5. **Say**: Draw the plane in your exercise books.



6. As the pupils do this, write the following on the board. Ask the pupils to copy it in their exercise books.
7. To plot the point (x, y):
   - Start at the origin $(0, 0)$;
   - Move along the x-axis x units from the origin, stop;
   - Move y units parallel to the y-axis to the required point;
   - Mark the point and write it as the ordered pair, (x, y).



8. Ask them to use this guide to plot the points $(3, 5)$ and $(-3, -5)$.
9. Guide a pupil to plot both points on the plane on the board.
   (Answer: shown right)
10. **Ask**: What do you notice about the 2 points? Raise your hand. Guide a pupil to comment about the direction we move when we plot positive and negative co-ordinates. (Answer: Move from the origin along the x-axis. For a positive x co-ordinate move right the required number of units, for a negative x co-ordinate move left. Then move up for positive y co-ordinate and down for negative.)
11. The procedure to plot points is very important for pupils to understand and do. It is required in many mathematical concepts. Spend time to ensure that all pupils can do this procedure.

Guided Practice (*10 minutes*)

1. Ask pupils to work in pairs. They should each do their own diagrams but they can discuss and share ideas. They should use the plane they drew earlier in their exercise books for these questions.
2. Point to the questions on the board:
   a. A shape has corners at the points with coordinates $(-2, 3), (7, 3), (-4, -4)$ and $(9, -4)$.
      i. Draw the shape.
      ii. What is the name of the shape?
   b. The coordinates of 3 corners of a parallelogram are $(-6, 3), (7, 3)$ and $(4, -7)$. What is the coordinate of the fourth corner?
3. Walk around, if possible, to check their answers and clear up any misconceptions.
4. Have pupils from around the classroom volunteer to give their answers.
5. **Write** the correct answers on the board. Ask pupils to check their work. (Answers: shown at the end of this lesson plan)

Independent Practice (*10 minutes*)

1. Ask the pupils to work independently to answer the questions.
2. Point to the questions on the board:
   a. Plot the points with co-ordinates $(5, 6), (-7, 1), (5, -4)$,
      i. Join the points to form a triangle.
      ii. What type of triangle have you drawn?
   b. A shape has corners at the points with coordinates $(-5, 8), (-9, 1), (5, 8)$, $(9, 1), (5, -6)$ and $(-5, -6)$.
      i. Draw the shape.

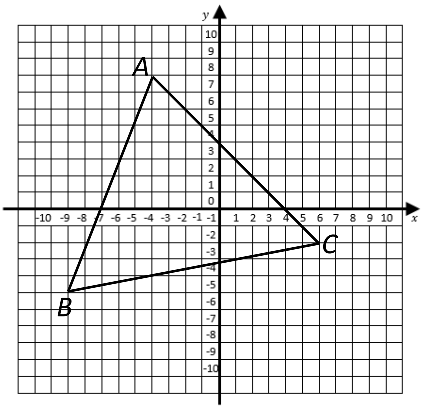
ii. What is the name of the shape?

3. Walk around, if possible, to check their answers and clear up any misconceptions.
4. Ask pupils to exchange exercise books and check each other’s work.
5. Have pupils from around the classroom volunteer to give their answers.
6. **Write** the correct answers on the board. Ask pupils to check the answers. (Answers: shown at the end of this lesson plan)

Closing (*2 minutes*)

1. **Say:** Please write your name on a piece of paper. Also write your answers to the question on the board. Your work will be collected at the end of the lesson.
2. **Write** on the board:
   Plot and join the points with coordinates $(-10, -6)$ and $(5, 8)$. What have you drawn?
3. Collect and check the work after the lesson is finished to see how much pupils have understood so far.
4. Use this to assist pupils in the next lesson when they will be plotting points from a table of values. (Answer: points join to make a straight line)

[EXAMPLE QUESTION FOR OPENING ACTIVITY]

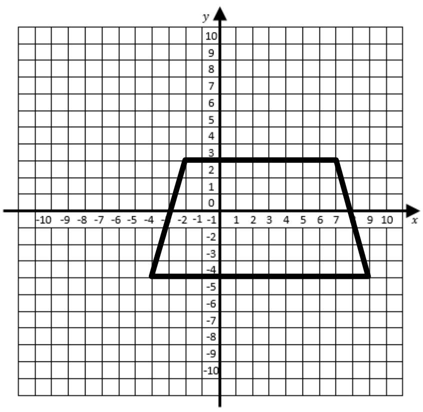


(Answers: $\mathrm{A}(-4, 8), \mathrm{B}(-9, -5), \mathrm{C}(6, -2)$)

[ANSWERS FOR QUESTIONS IN GUIDED PRACTICE]

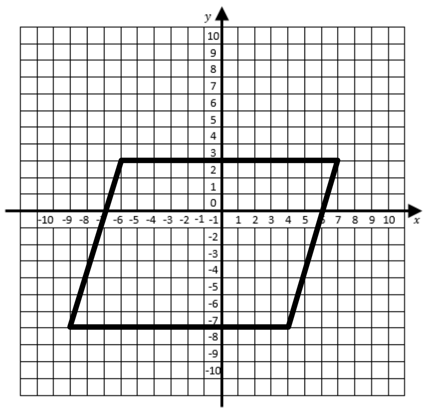
a.

i.



b.

i.



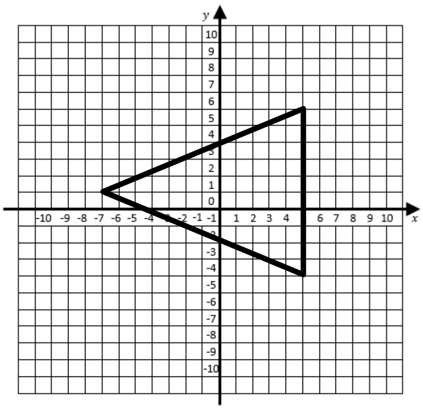
ii. Trapezium

ii. $(-9, -7)$

[ANSWERS FOR QUESTIONS IN INDEPENDENT PRACTICE]

a.

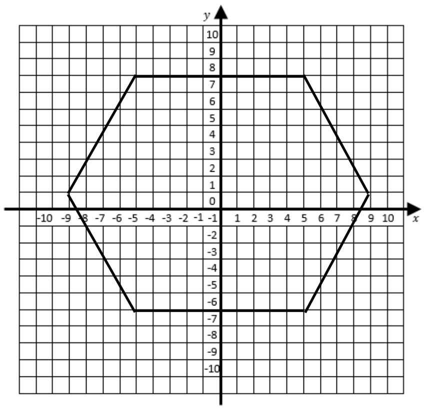
i.



ii. Isosceles triangle

b.

i.



ii. Hexagon

Lesson Title: Plotting Points from a Table of Values	**Theme**: Algebra	
Lesson Number: M-09-097	**Class/Level**: JSS 3	**Time:** 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to plot points from a given table of values on the Cartesian plane.	None	1. If your board does not have a Cartesian plane (grid), you will need to prepare one on half of the board. You will need grids for all the lessons on graphs of linear equations. 2. Draw the tables required for this lesson on the board. They can be found at the end of this lesson plan.

Opening (*3 minutes*)

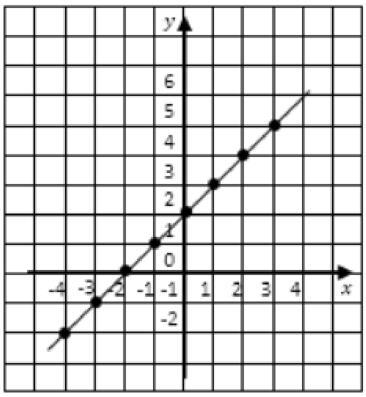
1. **Say**: In the last lesson we looked at plotting points on a Cartesian plane. Please turn to a blank page in your exercise books. Write down how you would explain to someone who was absent from the lesson how to plot a point on a plane.
2. Allow pupils 2 minutes to write down their explanations.
3. **Ask**: Who would like to explain to the class how to plot a point on the plane? Raise your hand.
4. Select a pupil who has raised their hand to answer. (Example answer: Start at the origin, move along the x-axis. For a positive x co-ordinate, move to the right the required number of units, for a negative x co-ordinate, move to the left. Then move up for a positive y co-ordinate and down for negative)
5. **Say**: Today we are going to plot points from a given table of values on the Cartesian plane.

Introduction to the New Material (*10 minutes*)

1. **Say**: We want to plot points for the equation $y = x + 2$.
   We already have the table we created from a previous lesson.
2. **Say**: Copy the table again from Question a. on the board.
3. Allow time for pupils to copy the table into their exercise books.
4. Draw the x- and y-axes on the plane.
5. You may have to do this freehand if there is no grid on the board.
6. Pupils can share rulers for this exercise.
7. **Say**: Remember x is the independent variable. We choose its values.
8. **Say**: The scale we use to plot the points depend on the x and y values we have in the table.
9. **Ask**: Can someone give the smallest and largest values of x in the table for $y = x + 2$? Raise your hand.
   (Answer: smallest –3, largest +3)
10. We usually give some space to the values on the table. We will mark the x-axis every 2 cm from –4 to +4 on. Ask the pupils to do the same on their x-axis.

g. $y = x + 2$

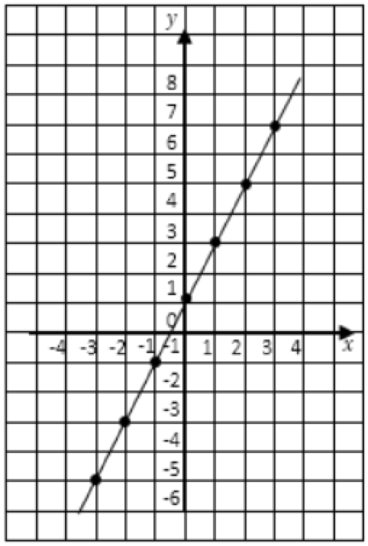
x	-3	-2	-1	0	1	2	3
y	-1	0	1	2	3	4	5



11. **Ask**: Can someone give the smallest and largest values of y in the table? Raise your hand. (Answer: –1 and 5)
12. **Say:** We will mark the y- axis every 2 cm from –2 to +6.
13. Mark the scale every 2 cm from –5 to +5 on the y-axis. Ask pupils to do the same on their y-axis.
14. Plot the points on the plane using the values from the table. Use a light chalk mark to move along the x-axis from the origin the required number of units. Do the same for the y- axis.
15. **Say:** We draw a straight line through all the points once we have plotted them all.
16. The table and completed graph is shown above. Ask the pupils to copy this into their exercise books.
17. Ask a pupil to read Question b.
18. **Ask**: What scale do we use on the x-axis? Raise your hand.
19. Select a pupil to give the scale for the x-axis. (Answer: Every 2 cm from –4 to +4)
20. **Ask**: What scale do we use on the y-axis? Raise your hand.
21. Select a pupil to give the scale for the y-axis. (Answer: Every 2 cm from –6 to +8)
22. Have another pupil volunteer to come to the board to draw the axes using the scales.
23. Have pupils from around the classroom volunteer to plot 1 or 2 points each on the plane.
24. The completed graph is shown on the right.
25. **Say:** Please look at the graphs we have just drawn. I want to see how many points I really need to be able to draw a straight line.
26. **Say**: Even if I rub out nearly all the points and just leave 2 behind, I will still be able to draw the straight line graph.
27. Erase the points on the graph. Leave the 2 points where the graph goes through the axes. Show pupils that even with only those 2 points, the straight line can still be drawn accurately.
28. **Say**: We only need 2 points to draw a straight line. We usually plot a third point to make sure we have not made a mistake. This will help you if have a point and it is not on the line. You will then be able to check your calculations and correct your error.

h. $y = 2x + 1$

X	-3	-2	-1	0	1	2	3
Y	-5	-3	-1	1	3	5	7



Guided Practice (*10 minutes*)

1. Ask pupils to work in pairs. They should each draw their own individual graphs, then check their partners' work.
2. Ask pupils to draw graphs from the tables in Questions c. and d. on the board. They should use suitable scales for the axes.
3. Walk around, if possible, to check their answers and clear up any misconceptions.
4. Have pupils from around the classroom volunteer to give their answers.
5. **Write** the correct answers on the board. Ask pupils to check their work. (Answers: shown at the end of this lesson plan)

Independent Practice (*10 minutes*)

1. Ask the pupils to work independently to answer Questions e. and f. on the board.

2. Walk around, if possible, to check their answers and clear up any misconceptions.
3. Have pupils from around the classroom volunteer to give their answers.
4. **Write** the correct answers on the board. Ask pupils to check their work. (Answers: shown at the end of this lesson plan)

Closing (*2 minutes*)

1. **Ask**: How many points do we need to draw a straight line graph? Raise your hand. (Answer: 2 points)
2. **Ask:** How many points should we draw to make sure we have not made a mistake? Raise your hand. (Answer: 3 points)
3. **Say:** In the next lesson, we will complete a table of values from a linear equation. We will plot the straight line graph from the table.

[QUESTIONS FOR INTRODUCTION TO THE NEW MATERIAL]

Draw the graphs for the values of the linear equations given in the tables below:

a. $y = x + 2$

x	-3	-2	-1	0	1	2	3
y	-1	0	1	2	3	4	5

b. $y = 2x + 1$

x	-3	-2	-1	0	1	2	3
y	-5	-3	-1	1	3	5	7

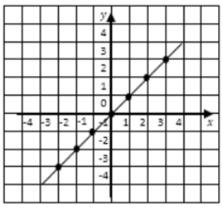
[QUESTIONS FOR GUIDED PRACTICE]

Draw the graphs for the values of the linear equations given in the tables below:

c. $y = x$

x	-3	-2	-1	0	1	2	3
y	-3	-2	-1	0	1	2	3

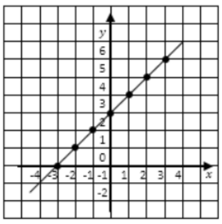
Answer:



d. $y = x + 3$

x	-3	-2	-1	0	1	2	3
y	0	1	2	3	4	5	6

Answer:



[QUESTIONS FOR INDEPENDENT PRACTICE]

Draw the graphs for the values of the linear equations given in the tables below:

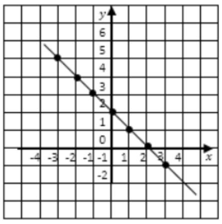
e. $y = 2 - x$

x	-3	-2	-1	0	1	2	3
y	5	4	3	2	1	0	-1

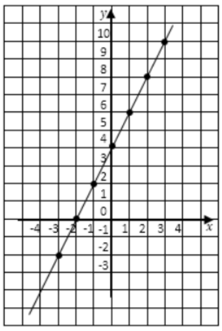
f. $y = 2x + 4$

x	-3	-2	-1	0	1	2	3
y	-2	0	2	4	6	8	10

Answer:



Answer:



Lesson Title: Graphing a Line I	**Theme**: Algebra	
Lesson Number: M-09-098	**Class/Level**: JSS 3	**Time**: 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to create a table of values for a given linear equation in 2 variables and graph it on the Cartesian plane.	None	1. 1. Write the questions for this lesson found at the end of this lesson plan on the board. 2. Write on the board: Vocabulary List: steep, steepness, slope, gradient

Opening (*3 minutes*)

1. **Say**: Can someone remind the class what we did in the last lesson? Raise your hand.
2. Select a pupil who has raised their hand to answer. (Answer: Plotted graphs of linear equations from a given table of values, plotted straight line graphs. Accept all reasonable answers.)
3. **Say**: Today we are going to create a table of values for a given linear equation in 2 variables and graph it on the Cartesian plane.

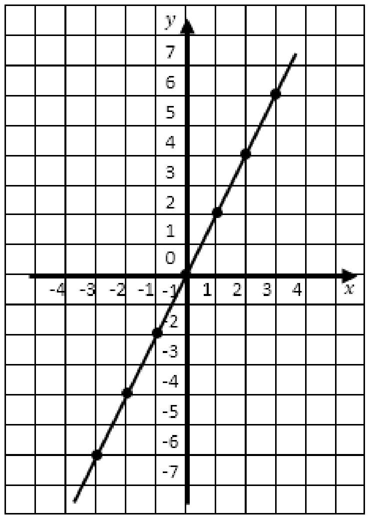
Introduction to the New Material (*10 minutes*)

1. **Say**: Before we can plot a graph of a linear equation we need a table of values.
   In the last lesson we were given the table of values from which we plotted the graphs.
2. **Say**: Today we are going to create a table of values ourselves from the linear equation. We will then plot the graph from the table.
3. Ask a pupil to read Question a. (at the end of this plan)
4. Draw a table of values of x from -3 to +3.
5. Ask pupils to copy the table as it is being completed, in their exercise books.
6. Have pupils from around the classroom volunteer to give the y-values for each x.
7. Complete the table of values. (Answers: shown on the right)
8. **Say**: We will now draw the graph for this table of values.
9. **Ask**: What scale do we use on the x-axis? Raise your hand.
10. Select a pupil to give the scale for the x-axis. (Answer: every 2 cm from –4 to +4)
11. **Ask**: What scale do we use on the y-axis? Raise your hand
12. Select a pupil to give the scale for the y-axis. (Answer: every 2 cm from –7 to +7)
13. Have another pupil volunteer to come to the board to draw the axes using the scales.
14. **Ask**: Why do we draw the axes with a scale higher than the values on the table? Raise your hand
15. Allow pupils to discuss and share ideas. (Answer: to give a balanced view of the graph)
16. Have pupils from around the classroom volunteer to plot 1 or 2 points each on the plane.
17. The completed table and graph is shown on the right.

i. $y = 2x$

x	-3	-2	-1	0	1	2	3
y	-6	-4	-2	0	2	4	6

j.



18. Ask pupils to copy the graph in their exercise books.

Guided Practice (*10 minutes*)

1. Ask pupils to work in pairs.
2. They should create the table from the linear equation in Question c. written on the board. They should then draw the graph using a suitable scale in Question d. on the board.
3. They should each draw their own individual graph then check their partner's work.
4. Walk around, if possible, to check their answers and clear up any misconceptions.
5. Ask pupils to exchange exercise books and check each other's work.
6. Have a pupil volunteer to complete the required table on the board.
7. Have another pupil volunteer to draw the graph.
8. Correct any errors in the solution on the board. Ask pupils to check their work. (Answers: shown at the end of this lesson plan)

Independent Practice (*10 minutes*)

1. Ask pupils to work independently to answer Questions e. and f. written on the board.
2. Walk around, if possible, to check their answers and clear up any misconceptions.
3. Have a pupil volunteer to complete the required table on the board.
4. Have another pupil volunteer to draw the graph.
5. Correct any errors in the solution on the board. Ask pupils to check their work. (Answers: shown at the end of this lesson plan)

Closing (*2 minutes*)

1. **Ask**: What can you say about the graphs of $y = 2x$ and $y = 2x + 3$?
2. Allow pupils to discuss and share their ideas.
3. Have pupils from around the classroom volunteer to answer. (Example answers: They are at the same angle; they seem to be moving in the same direction; they look the same; $y = 2x$ goes through 0, $y = 2x + 3$ goes through 3)
4. **Say**: The co-efficient of x gives very important information about how steep a graph is. All the graphs with the same co-efficient have the same steepness. This steepness is called the 'slope' or gradient of the graph. It is the angle the line makes with the x-axis.
5. **Say:** Remember what we discovered about these 2 graphs. We will talk more about this in a future lesson. In the next lesson, we will graph more complicated linear equations.

[QUESTIONS FOR OPENING ACTIVITY]

a. Complete a table of values for the linear equation: $y = 2x$
b. Draw the graphs of the line with equation: $y = 2x$

[QUESTIONS FOR GUIDED PRACTICE]

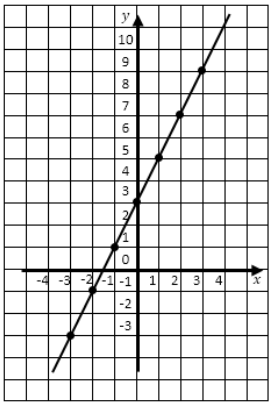
c. Complete a table of values for the linear equation: $y = 2x + 3$
d. Draw the graph of the line with equation: $y = 2x + 3$
   Answers:

$$y = 2x + 3$$

c.

x	−3	−2	−1	0	1	2	3
y	−3	−1	−1	3	5	7	9

d.



[QUESTIONS FOR INDEPENDENT PRACTICE]

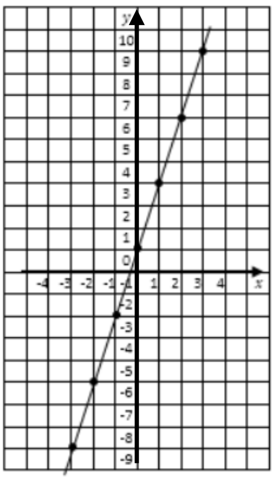
e. Complete a table of values for the linear equation: $y = 3x + 1$
f. Draw the graph of the line with equation: $y = 3x + 1$
Answers:

$y = 3x + 1$

e.

x	−3	−2	−1	0	1	2	3
y	−8	−5	−2	1	4	7	10

f.



Lesson Title: Graphing a Line II	**Theme**: Algebra	
Lesson Number: M-09-099	**Class/Level**: JSS 3	**Time**: 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to graph more complicated linear equations.	None	Write the questions for this lesson found at the end of this lesson plan on the board.

Opening (*3 minutes*)

1. **Say**: Can someone remind the class what we did in the last lesson? Raise your hand.
2. Select a pupil who raised their hand to answer. (Answer: Created our own table of values; plotted graphs of linear equations from our own table of values, plotted straight line graphs)
3. **Say**: Today we are going to graph more complicated linear equations.

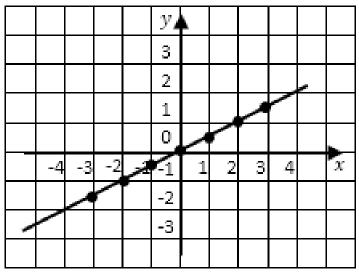
Introduction to the New Material (*10 minutes*)

1. **Say**: Today we are going to create a table of values from more complicated linear equations. We will then plot the graph from the table.
2. Ask a pupil to read Question a.
3. Draw a table of values of x from -3 to +3.
4. Ask volunteers to give the y-values for each x.
   Complete the table of values. (shown on the right)
5. **Say**: We will now draw the graph for this table of values as shown in b. We have been using the same x values so we know the scale for the x-axis.
6. **Ask**: What is the scale for the x-axis? Raise your hand.
7. Select a pupil to give the scale for the x-axis. (Answer: every 2 cm from –4 to +4)

k. $y = \frac{1}{2}x$

x	-3	-2	-1	0	1	2	3
y	-1.5	-1	-0.5	0	0.5	1	1.5

l.



8. **Ask**: What scale do we use on the y-axis? Raise your hand.
9. Select a pupil to give the scale for the y-axis. (Answer: every 2 cm from –3 to +3)
10. **Say**: We have whole numbers and decimals for our y-values. This makes plotting the points a little more complicated. We have to be careful to plot the points accurately to get a straight line.
11. Have a different pupil volunteer to come to the board to draw the axes using the scales.
12. Have pupils from around the classroom volunteer to plot 1 or 2 points each on the plane.
13. Correct any errors. The completed table and graph is shown on the right.

Guided Practice (*10 minutes*)

1. Ask pupils to work in pairs.
2. Ask pupils to work together to create the table from the linear equation in Question c. on the board.
3. Then they should each draw their own individual graph in Question d. then check their partner's work.
4. Walk around, if possible, to check their answers and clear up any misconceptions.

5. Ask pupils to exchange exercise books and check each other's work.
6. Have a pupil volunteer to draw the required table on the board.
7. Have another pupil volunteer to draw the graph.
8. Correct any errors in the solution on the board. Ask pupils to check their work. (Answers: see the end of this lesson plan)

Independent Practice (*10 minutes*)

1. Ask the pupils to work independently to answer Questions e. and f. on the board.
2. Walk around, if possible, to check their answers and clear up misconceptions.
3. Have a pupil volunteer to draw the required table on the board.
4. Have another pupil volunteer to draw the graph.
5. Correct any errors in the solution on the board. Ask pupils to check their work. (Answers: see the end of this lesson plan)

Closing (*2 minutes*)

Note: Allow pupils to think and discuss their ideas for a few moments. Have pupils from around the classroom volunteer to answer the question. Give some guidance if required.

1. **Ask**: What do you notice about the direction of the line in the graph of $y = \frac{1}{2}x$? Raise your hand. (Example answer: It is moving upward as x increases from left to right; it shows that as x increases, y also increases; it is moving in the positive direction at an angle)
2. **Ask**: How is it different from the directions of lines $y = -2x - 1$ and $y = -\frac{1}{2}(x + 1)$? Raise your hand. (Example answer: The 2 lines are moving downwards as x increases from left to right; the y-values are getting smaller; it is moving in a negative direction)
3. **Say:** There are clues to how a line will look like from whether there is a + sign or a – sign in front of the co-efficient of x. We will talk more about this in a future lesson.
4. **Say:** In the next lesson, we will practice graphing a line.

[QUESTIONS FOR OPENING ACTIVITY]

a. Complete a table of values for the linear equation: $y = \frac{1}{2}x$
b. Draw the graphs of the line with the equation: $y = \frac{1}{2}x$

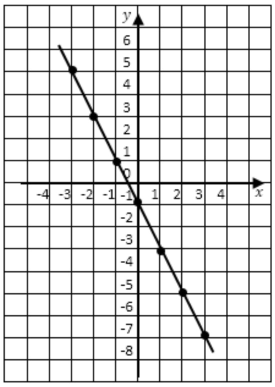
[QUESTIONS FOR GUIDED PRACTICE]

c. Complete a table of values for the linear equation: $y = -2x - 1$
d. Draw the graph of the line with the equation: $y = -2x - 1$
Answers:

$$y = -2x - 1$$

c.

x	−3	−2	−1	0	1	2	3
y	5	3	1	−1	−3	−5	−7

d. 

Questions for Independent Practice

e. Complete a table of values for the linear equation: $y = -\frac{1}{2}(x + 1)$

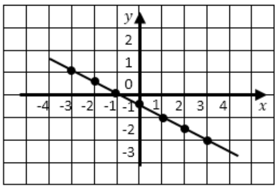
f. Draw the graph of the line with the equation: $y = -\frac{1}{2}(x + 1)$

Answers:

$$y = -\frac{1}{2}(x + 1)$$

e.

x	−3	−2	−1	0	1	2	3
y	1	0.5	0	−0.5	−1	−1.5	−2

f. 

Lesson Title: Graphing a Line III	**Theme**: Algebra	
Lesson Number: M-09-100	**Class/Level**: JSS 3	**Time**: 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to practice graphing a line.	None	Write the questions for this lesson found at the end of this lesson plan on the board.

Opening (*3 minutes*)

1. **Say:** In the last 2 lessons we noticed some important points about the co-efficient of x.
2. **Ask**: Who can remind the class what they were? Raise your hand.
3. Guide pupils to give the required answers. (Example answers: Graphs with the same co-efficient have the same steepness or slope. If there is a + sign in front of the co-efficient, the line moves upwards, if there is a minus sign, the line moves downwards)
4. **Say**: Today we are going to practice graphing a line.

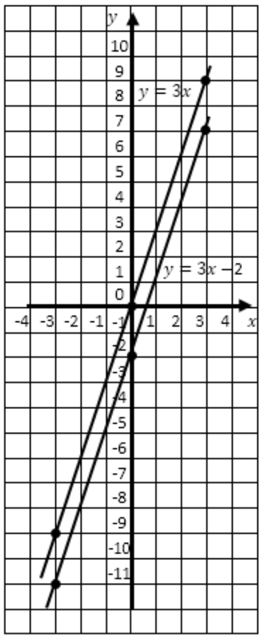
Introduction to the New Material (*10 minutes*)

1. **Say**: We are going to draw graphs from the linear equations on the board. We will take note of how lines with the same co-efficient of x look like. We will also take note of what the graph looks like when it has a + or – sign in front of x.
2. Ask a pupil to read Question a.
3. **Ask:** Who can remind the class how many points we need to plot to draw a straight line? Raise your hand.
4. Select a pupil who has raised their hand to answer. (Answer: 2 points required, but at least 3 are advisable to prevent mistakes.)
5. **Say**: For the graphs in this lesson, will find the y-values for x= -3, 0 and +3.
6. Draw the table shown on the right on the board. Complete only the y-values for 0.
7. **Say**: Please copy and complete the table on the board.
8. Allow the pupils time to copy and complete the table of values for the 2 equations in Question a.
9. Ask the pupils to draw the 2 graphs on the same axes (as shown on the right).
10. Allow time for pupils to draw the graphs.
11. **Ask**: What do you notice about the 2 graphs? (Example answer: They have the same steepness (or slope/gradient); they are parallel to each other)

a.

x	-3	0	3
$y = 3x$	-9	0	9
$y = 3x - 2$	-11	-2	7

b.



Guided Practice (*10 minutes*)

1. Ask pupils to work in pairs.

2. Ask pupils to create the table from the linear equation in Question c. on the board.
3. They should each draw their own individual graphs then check their partners' work.
4. Walk around, if possible, to check their answers and clear up any misconceptions.
5. Ask pupils to exchange exercise books and check each other's work.
6. Have 2 pupils volunteer to complete the tables and draw the required graphs on the board.
7. Correct any errors in the solution on the board. Ask pupils to check their work. (Answers: see the end of this lesson plan).

Independent Practice (*10 minutes*)

1. Ask the pupils to work independently to answer Questions e. and f. on the board.
2. Walk around, if possible, to check their answers and clear up any misconceptions.
3. Have 2 pupils volunteer to complete the tables and draw the required graphs on the board.
4. Correct any errors in the solution on the board. Ask pupils to check their work. (Answers: see the end of this lesson plan).

Closing (*2 minutes*)

1. **Say:** Write in your exercise books 2 things you have learned about graphs in this lesson.
2. Have pupils from around the classroom volunteer to share one answer. (Example answers: They can be drawn on the same axes; parallel lines have the same value and sign for the co-efficient of x)
3. **Say:** The topic for our next lesson will be an introduction to slopes of straight line graphs.

[QUESTIONS FOR OPENING ACTIVITY]

a. On the same axes, draw the graphs for the linear equations: $y = 3x$ and $y = 3x - 2$
b. What do you notice about the 2 graphs?

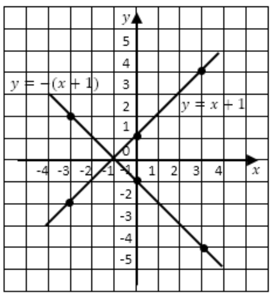
[QUESTIONS FOR GUIDED PRACTICE]

c. On the same axes, draw the graphs for the linear equations: $y = x + 1$ and $y = -x - 1$.
d. What do you notice about the 2 graphs?
Answers:

c.

X	-3	0	3
$y = x + 1$	-2	1	4
$y = -(x + 1)$	2	-1	-4

d.



The 2 lines are at right angles (perpendicular) to each other.

[QUESTIONS FOR INDEPENDENT PRACTICE]

e. On the same axes, draw the graphs for the linear equations: $y = -2x$ and $y = 4 - 2x$

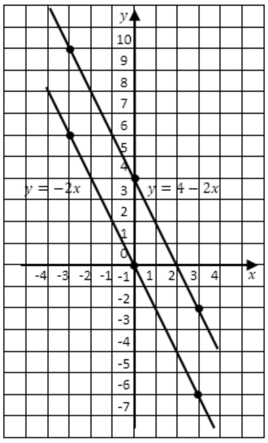
f. What do you notice about the 2 graphs?

Answers:

e.

x	-3	0	3
$y = -2x$	6	0	-6
$y = 4 - 2x$	10	4	-2

f.



The 2 lines are parallel to each other.

Lesson Title: Introduction to Slope	**Theme**: Algebra	
Lesson Number: M-09-101	**Class/Level**: JSS 3	**Time**: 35 minutes

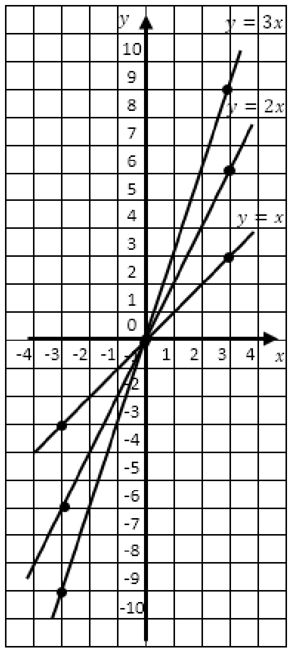
Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to: 1. Identify that the slope of a line describes its steepness, and is described by the fraction $\frac{\text{rise}}{\text{run}}$. 2. Identify the direction of positive and negative slope.	None	1. Draw the graphs for $\mathrm{y} = \mathrm{x}$, $\mathrm{y} = 2\mathrm{x}$ and $\mathrm{y} = 3\mathrm{x}$ in the centre of the board (see Introduction to the New Material of this lesson plan). 2. Draw the diagram from the Guided Practice on the board. 3. Draw the graph shown in the Guided Practice on one side of the board. 4. Write on the board: <u>Vocabulary List</u>: rise, run

Opening *(3 minutes)*

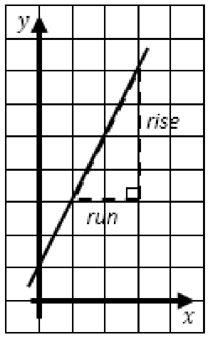
1. **Say**: We have already learned that graphs with the same value and sign for the co-efficient of x have the same steepness or slope.
2. **Ask:** What else did we learn about the direction of the lines?
3. Have a pupil volunteer to answer. Guide the pupil to give the answer connecting the sign to the direction of the line. (Example answer: Positive co-efficients give a line that moves upwards as x increases, negative co-efficients give a line that moves downwards).
4. **Say**: Today we are going to identify that the slope of a line describes its steepness. We will identify how to calculate the slope and the direction of positive and negative slopes.

Introduction to the New Material *(10 minutes)*

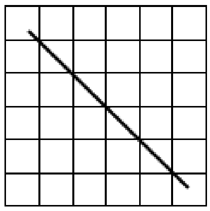
1. **Say**: We have drawn on the same axes, the graphs for $y = x$, $y = 2x$ and $y = 3x$ (shown on the right).
2. **Ask**: Look at the line for the x-axis. How would you compare the line for $y = x$ with the x-axis?
3. Have a pupil volunteer to answer. Guide the pupil to comment on the difference between the 2 lines. (Example answer: The x-axis is horizontal, the line for $y = x$ is at an angle which shows how steep it is compared to the x-axis.)
4. **Ask** the following questions and have pupils volunteer to answer:
   a. Is the line for the equation $y = 2x$ more steep or less steep than the x-axis? (Answer: More)
   b. What about the line for equation $y = 3x$? Is it more or less steep than the x-axis? (Answer: More)
5. **Say**: When we look at the 3 graphs, we can see which is more steep than the x-axis. We can also see how steep they are compared to each other. This steepness is called the 'gradient' or 'slope' of the line.
6. **Ask**: What do you notice about the co-efficient of x in the 3 equations?



7. Have a pupil volunteer to answer. Guide the pupil to comment on the value of the co-efficient of x. (Example answer: The bigger the co-efficient of x the steeper the graphs. Accept all reasonable answers.)
8. **Say**: The co-efficient of x tells us how steep a graph is. It is the same as the slope of the graph. We can calculate its value from the graph.
9. Point to the diagram on the right on the board. We will consider parts of lines in Quadrant I only.



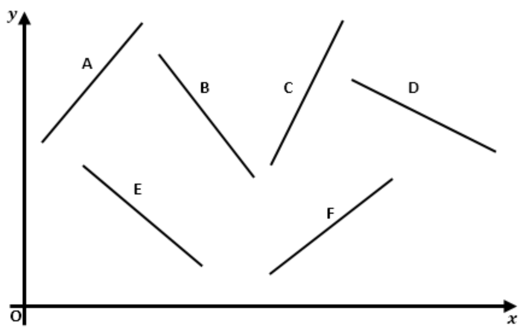
10. **Say:** We can draw a triangle as shown and use it to calculate the slope of the graph. We can find the value of the fraction: $\frac{\text{rise}}{\text{run}}$. This will give us the slope of the line. You can see the 'rise' is the same as the height of the triangle. The 'run' is the same as the base.
11. **Ask**: What is the rise of this triangle? (Answer: 4)
12. Let pupils raise their hands to answer.
13. **Ask**: What is the run of this triangle? (Answer: 2)
14. Allow pupils to raise their hands to answer.
15. **Say**: Calculate the fraction of $\frac{\text{rise}}{\text{run}}$ in your exercise books. Raise your hand when you finish.
16. Give pupils 1-2 minutes to do the calculation. Select a pupil to give the answer. (Answer: $\frac{\text{rise}}{\text{run}} = \frac{4}{2} = 2$)
17. **Say**: The slope or gradient of this line is equal to 2. A value of +2 tells us that the slope is positive and the direction of the line is as shown. It goes upwards from left to right on the axes. Remember the graphs we have drawn already.
18. **Ask:** Who can come to the board and draw a line with a negative slope?
19. Have a pupil volunteer to come to the board.
20. **Say**: We do not need a graph, just the direction of a line with a negative slope.
21. Guide the pupil to draw a line with a negative slope. (Answer: Shown on the right.) Correct any errors made by the pupil.



22. **Say**: A negative slope goes downwards from left to right on the axes. We will learn how to calculate a negative slope in the next lesson.

Guided Practice *(10 minutes)*

1. Ask pupils to work in pairs. Point to the graph on the right from this section.
2. **Say**: Draw this graph showing these lines in your exercise books. Discuss which of them have a positive slope and which ones have a negative slope.



3. Allow pupils to discuss and share their ideas. They should each write their answers in their exercise books.
4. Walk around, if possible, to check their answers and clear up any misconceptions.
5. Have pupils volunteer to give their answers to the questions.
6. Correct any errors and ask pupils t check their work.
   (Answers: positive slopes: A, C and F; negative slopes: B, D and E)

Independent Practice *(10 minutes)*

1. Ask the pupils to work independently to answer the questions in this section.
2. **Write** on the board:
    a. Which of these linear equations are for lines with a positive slope and which are for lines with a negative slope 1?

i.	$y = x + 5$	ii.	$y = 2x + 1$
iii.	$y = 6 - 2x$	iv.	$y = 4 - 3x$
v.	$y = 2x$	vi.	$y = 5 - x$
vii.	$y = -(3x + 2)$	viii.	$y = -2x$
ix.	$y = 2x + 8$	x.	$y = x + 9$

    b. Which of them are parallel to each other?
3. Walk around, if possible, to check their answers and clear up any misconceptions.
4. Have pupils from around the classroom volunteer to give their answers to the questions.
5. Correct any errors on the board. Have pupils check their work.
(Answers: a. positive slopes: i., ii., v., ix., and x.; negative slopes: iii., iv., vi., vii., and viii.;
b. parallel lines are: i. and x.; ii., v. and ix.; iii.) and viii.; iv. and vii.

Closing *(2 minutes)*

1. **Say:** Please write down one new thing you learned during this lesson.
2. Allow pupils time to write down one new thing they learned.
3. Have 2-3 pupils volunteer to share their thoughts with the class. (Example answer: How to find a gradient using $\frac{\text{rise}}{\text{run}}$; the rise is the same as the height of the triangle, the run is the same as the base; positive slopes go upward from left to right, negative slope goes downward from left to right.) Accept all reasonable answers.
4. **Say:** In the next lesson, we will continue to find the slopes of lines using: $\frac{\text{rise}}{\text{run}}$.

Lesson Title: Finding the Slope of a Line	**Theme**: Algebra	
Lesson Number: M-09-102	**Class/Level**: JSS 3	**Time**: 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to find the slope of a line by counting and dividing its rise and run.	None	Write the questions for this lesson, from the end of this lesson plan, on the board.

Opening *(3 minutes)*

1. **Say**: Raise your hand if you can tell the class the fraction we use to calculate the slope of a graph.
2. Select a pupil who raised their hand to give the answer. (Answer: $\frac{\text{rise}}{\text{run}}$)
3. **Ask**: Who can remind the class of what part of the linear equation we can find this fraction? (Answer: The co-efficient of x)
4. **Say**: Today we are going to find the slope of a line by counting and dividing its rise and run.

Introduction to the New Material *(10 minutes)*

Allow time after each question for pupils to think. Have pupils volunteer to give their answers to the questions, or to come to the board.

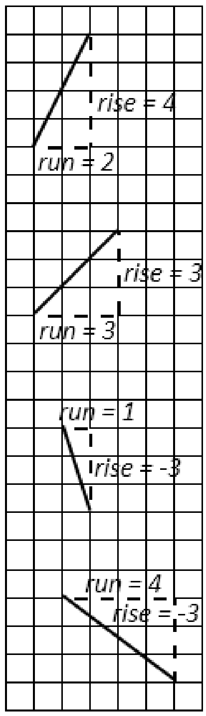
Note: The completed answers are shown on the right below. However, the pupils will not be shown these on the board. They will be answering questions and doing the calculations themselves on the board or in their exercise books.

a) $Slope = \frac{\text{rise}}{\text{run}} = \frac{4}{2} = 2$

b) $Slope = \frac{\text{rise}}{\text{run}} = \frac{3}{3} = 1$

c) $Slope = \frac{\text{rise}}{\text{run}} = \frac{-3}{1} = -3$

d) $Slope = \frac{\text{rise}}{\text{run}} = \frac{-3}{4} = -\frac{3}{4}$



1. **Say**: Please look at the lines shown in Question a. We can find the slope of each line by counting and dividing its rise and run. We draw a triangle on 2 points of the line as shown.
   The directions for counting are similar to when we are plotting points:
   a. We start at one point of the line. Going across to the right is positive. Going across to the left is negative.
   b. Going up is positive; down is negative.
2. Show how to work out the fraction $\frac{\text{rise}}{\text{run}}$ for the slope for line *a)*.
3. Write the answer on the board next to line *a)* as shown.
4. Ask the pupils to copy the line and calculation in their exercise books.
5. **Say**: It does not matter which side of the line we draw the triangle. What is important is that we remember that moving right on the x-axis is positive and moving up on the y-axis is negative. Work with your partner or the pupil sitting beside you to find the slope for the line in *b).* Please raise your hand when you finish.
6. Have a pupil volunteer to do the calculation on the board. (Answer: Shown on the right)
7. **Say**: We want to find the slope for the line in *c).* Remember which directions are positive.
8. **Ask:** What is the rise for the line in *c)*? (Answer: -3)

9. **Ask:** What is its run? (Answer: 1)
10. **Ask:** What do we get for the fraction $\frac{\text{rise}}{\text{run}}$?
11. Let any pupil volunteer to answer, and guide that pupil to show the calculation for line *c)* on the board. (Answer: Shown above)
12. Ask pupils to copy the line and calculation in their exercise books.
13. Ask pupils to do the fraction $\frac{\text{rise}}{\text{run}}$ for line *d)*.
14. Have a pupil volunteer to show the calculation for line *d)* on the board. (Answer: Shown above). Ask pupils to observe carefully or do the calculation in their exercise books.
15. Correct any errors made by the pupil on the board. Ask pupils to check their work.

Guided Practice *(10 minutes)*

1. Ask pupils to continue to work in pairs. They should each do their own diagrams but they can discuss and share their ideas.
2. **Say**: We have been drawing triangles and calculating the fraction $\frac{\text{rise}}{\text{run}}$ for each of the lines. The fraction should really be described as a ratio. The rise is directly proportional to the run. If you draw a bigger or smaller triangle, the values of the rise and run will change but the ratio will remain unchanged. Find the slope of the lines in Question b. by counting and dividing the rise and run.
   For *c),* one of you draw the triangle on top of the line. The other one draw the triangle underneath the line. See if there is any difference in the slope calcualted.
3. The triangles for *c)* are shown on the diagram for the teacher to guide pupils.
   They should not be drawn on the board.
4. Walk around, if possible, to check their answers and clear up any misconceptions.
5. Have pupils from around the classroom volunteer to give their answers to the questions.
6. Correct any errors made by the pupils on the board. Ask all pupils to check their work.
7. **Say**: We can see that the size of the triangle you draw or on which side you draw it do not matter. You will always get a constant answer which is the slope for the line.
   (Answers: See the end of this lesson plan)

Independent Practice *(10 minutes)*

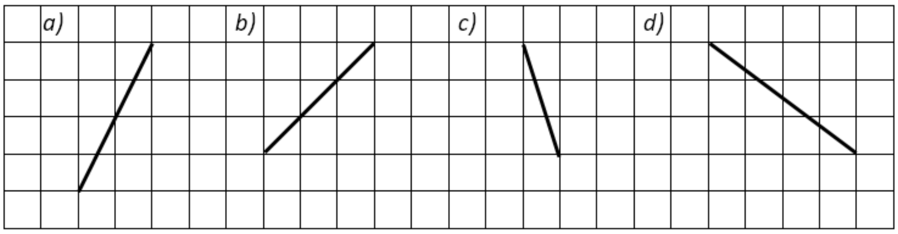
1. Ask the pupils to work independently to answer Questions 1 and 2 for this section.
2. Walk around, if possible, to check their answers and clear up any misconceptions.
3. Have pupils from around the classroom volunteer to give their answers to the questions.
4. Correct any errors on the board and ask pupils to check their work.
   (Answers: See the end of this lesson plan.)

Closing *(2 minutes)*

1. **Say:** Please write down 2 new things you have learned during this lesson.
2. Allow pupils time to write down their answers.
3. Select 2-3 pupils to share their thoughts with the class. (Example answers: It does not matter what size of triangle is drawn on the line, the slope is always the same; The triangle can be drawn on any side of the line, the slope does not change. Accept all reasonable answers.)

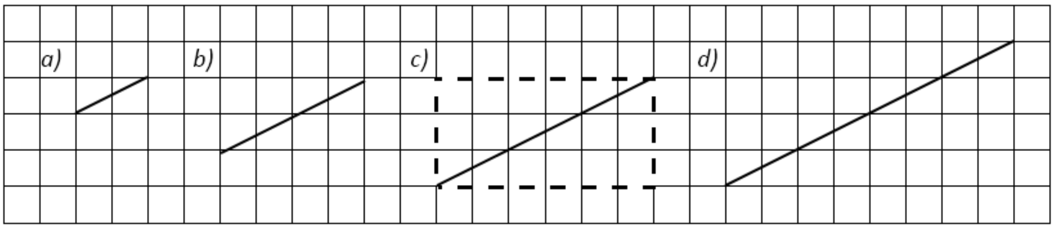
[QUESTIONS FOR INTRODUCTION TO THE NEW MATERIAL]

1. Determine the slope (gradient) of each of the lines shown below:



[QUESTIONS FOR GUIDED PRACTICE]

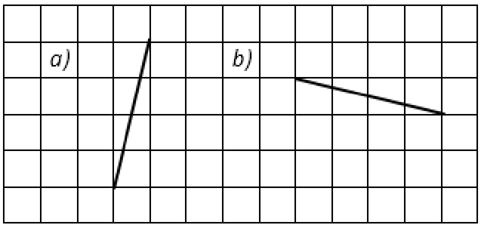
1. Determine the slope (gradient) of each of the lines shown below:



(Answers: *a)* $\frac{\text{rise}}{\text{run}} = \frac{1}{2}$; *b)* $\frac{\text{rise}}{\text{run}} = \frac{2}{4} = \frac{1}{2}$; *c)* $\frac{\text{rise}}{\text{run}} = \frac{3}{6} = \frac{1}{2}$ *or* $\frac{\text{rise}}{\text{run}} = \frac{-3}{-6} = \frac{1}{2}$; *d)* $\frac{\text{rise}}{\text{run}} = \frac{4}{8} = \frac{1}{2}$).

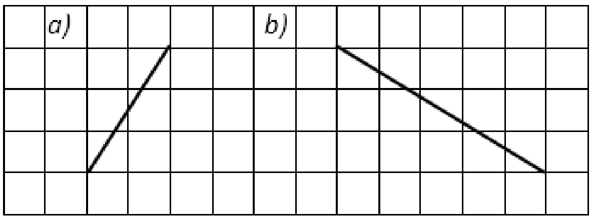
[QUESTIONS FOR INDEPENDENT PRACTICE]

1. Determine the slope (gradient) of each of the lines shown below:



(Answers: *a)* $\frac{\text{rise}}{\text{run}} = \frac{4}{1} = 4$; *b)* $\frac{\text{rise}}{\text{run}} = -\frac{1}{4}$)

2. Draw lines with these gradients: *a)* $\frac{3}{2}$; *b)* $-\frac{3}{5}$



(Answers: *a)* and *b)* *are as shown in the diagram above.*)

Lesson Title: Slope Formula	**Theme**: Algebra	
Lesson Number: M-09-103	**Class/Level**: JSS 3	**Time**: 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to find the slope of a line using 2 points (x_1, y_1) and (x_2, y_2) on the line, and the formula $m = \frac{y_2-y_1}{x_2-x_1}$.	None	1. Write the questions for this lesson on the board (see end of lesson). 2. Draw the graph with the axes from Introduction to the New Material on the board. Do not plot the points if the pupils have answered those questions.

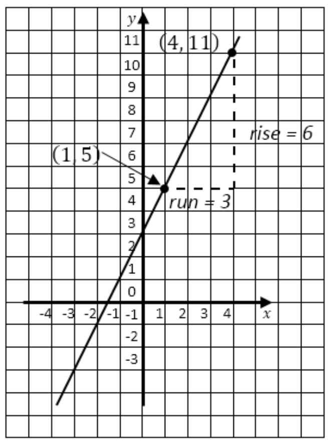
Opening *(3 minutes)*

1. **Say**: We have been using linear equations to draw straight line graphs. Who can remind the class the standard form of the linear equations we have been using? (Answer: $y = mx + c$)
2. Have a pupil volunteer to answer.
3. **Say**: We have also been plotting points and drawing straight lines using their linear equations. How many points do we need to draw a straight line graph? (Answer: 2 points, 3 to make sure we have not made a mistake.) We have learned how to find a ratio by counting and dividing rise and run on a straight line.
4. **Ask:** What does this ratio calculate in the line? (Answer: the slope (or gradient) of the line)
5. Have a pupil volunteer to answer.
6. **Say**: Today we are going to find the slope of a line using 2 points (x_1, y_1) and (x_2, y_2) on the line, and the formula: $m = \frac{y_2-y_1}{x_2-x_1}$

Introduction to the New Material *(10 minutes)*

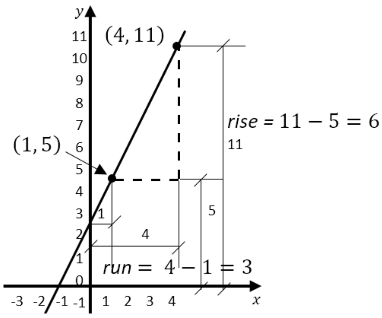
Give pupils some time to think after each question. Have pupils from around the classroom volunteer to give their answers or to come to the board.

The completed solution is shown below. However, the pupils will work through the procedure step-by-step to arrive at the final solution.



1. **Say**: Remember we just said we only need 2 points to draw a straight line graph? Well these 2 points can be used to find the gradient of a line. Suppose we know the co-ordinates of 2 points on a straight line are $(1, 5)$ and $(4, 11)$. We can plot these 2 points and join them with a straight line. We can extend the line beyond the points so we can see how the formula works.
2. Point to the axes shown on the right. Ask a volunteer pupil to plot the 2 points. (Answer: Shown on the right) Ask other pupils to observe carefully to see if they agree with the answer.
3. Draw a straight line through the 2 points. Extend the line so it is a good length as shown.
4. **Say**: Earlier, we found the gradient of a straight line using the ratio $\frac{\text{rise}}{\text{run}}$. Let us see how these 2 points fit into the formula.
5. Draw the triangle shown on the line.

6. **Say**: Look at the triangle. Who can tell the class the rise? (Answer: 6)
7. **Say:** We get the same answer for the rise if we subtract the 2 y co-ordinates.
8. On the graph, show how the distance for the rise $= (11 - 5)$. The little guide on the right is to help with the explanation. You do not need to draw this on the graph.



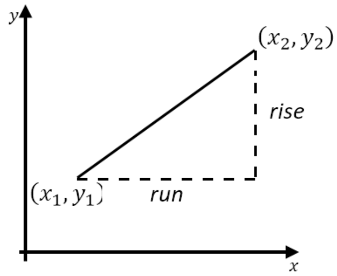
9. **Ask:** What is its run? (Answer: 3)
10. **Say:** We get the same answer for the run if we subtract the 2 x co-ordinates.
11. Show on the graph how the distance for the run $= (4 - 1)$.
12. **Say**: Let us put these values in our formula:
13. **Write** on the board:

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

$$= \frac{11-5}{4-1} = \frac{\text{difference in } y \text{ co} - \text{ordinates}}{\text{difference in } x \text{ co} - \text{ordinates}}$$

$$= \frac{6}{3}$$

$$= 2$$



14. **Say**: We can write a general formula for the slope. If we have 2 points on a line with co-ordinates (x_1, y_1) and (x_2, y_2), then the slope, m, of the line is given by:

$$m = \frac{\text{rise}}{\text{run}}$$

$$= \frac{\text{difference in } y \text{ co} - \text{ordinates}}{\text{difference in } x \text{ co} - \text{ordinates}}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

15. **Say**: The formula $m = \frac{y_2-y_1}{x_2-x_1}$ gives the value of the co-efficient of x in the equation $y = mx + c$.
16. Ask pupils to copy the graph and general formula in their exercise books.

Guided Practice *(10 minutes)*

1. Ask pupils to write the instructions below to find the gradient of a line using the formula.
   To find the gradient of a straight line:
   - Plot the 2 points on a Cartesian plane
   - Join the points with a straight line
   - Write the co-ordinates of the points on the line
   - Draw a triangle on the line
   - Let (x_1, y_1) be equal to one point and (x_2, y_2) be equal to the second point
   - Use the formula $m = \frac{y_2-y_1}{x_2-x_1}$ to find the gradient

2. **Say**: When calculating the slope from the co-ordinates of 2 points, we subtract the x-values in the same order that we subtract the y-values.
3. Ask pupils to continue to work in pairs to do Question 1 for Guided Practice. They should each do their own diagrams but they can discuss and share their ideas.
4. Walk around, if possible, to check their answers and clear up any misconceptions.
5. Have pupils from around the classroom volunteer to give their answers to the questions.
6. Correct any errors. (Answers: See the end of this lesson plan)

Independent Practice *(10 minutes)*

1. Ask the pupils to work independently to answer Question 2 written on the board.
2. Walk around, if possible, to check their answers and clear up any misconceptions.
3. Select pupils from around the classroom to give their answers to the questions.
4. Correct any errors and write the answers on the board. Ask pupils to check their work. (Answers: See the end of this lesson plan)

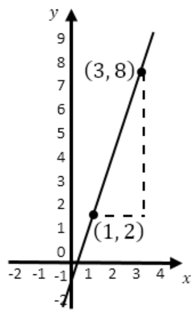
Closing *(2 minutes)*

1. **Say**: The formula we have been using to find the gradient means that if we know the co-ordinates of 2 points on a line, we can calculate the gradient without drawing the graph first.
2. Use the formula to find the gradient of the line joining $(5, 7)$ and $(-2, 4)$.
3. Allow time for the pupils to do the calculation.
4. Have a pupil volunteer to show the calculation on the board. (Answer: $m = \frac{y_2-y_1}{x_2-x_1} = \frac{4-7}{-2-5} = \frac{-3}{-7} = \frac{3}{7}$)

[QUESTIONS FOR GUIDED PRACTICE]

1. Find the gradient of the line joining the points $(1, 2)$ and $(3, 8)$.
   Answer: Let $(x_1, y_1) = (1, 2)$ and $(x_2, y_2) = (3, 8)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{8-2}{3-1}$$
$$= \frac{6}{2}$$
$$m = 3$$



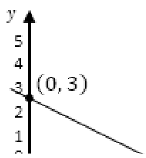
[QUESTIONS FOR INDEPENDENT PRACTICE]

2. Find the gradient of the line joining the points $(6, 0)$ and $(0, 3)$.
   Answer: Let $(x_1, y_1) = (6, 0)$ and $(x_2, y_2) = (0, 3)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{3-0}{0-6}$$

$$= \frac{3}{-6}$$

$$m = -\frac{1}{2}$$



Lesson Title: Slope-intercept Form of Linear Equations	**Theme**: Algebra	
Lesson Number: M-09-104	**Class/Level**: JSS3	**Time**: 35 minutes

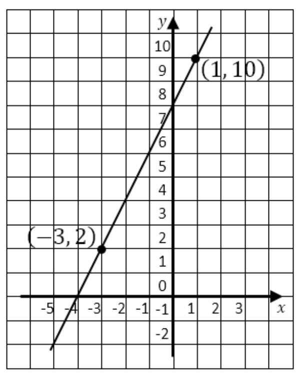
Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to identify the slope (m) and y-intercept (c) of a linear equation in slope-intercept form: $y = mx + c$	None	1. Write the questions for this lesson, found at the end of this lesson plan, on the board. 2. Write on the board: <u>Vocabulary List</u>: intercept

Opening *(3 minutes)*

1. **Ask**: Who can remind the class of what we did in the last lesson?
2. Have a pupil volunteer to answer. (Answer: How to find the gradient of a straight line using 2 known points on the line. Accept all reasonable answers.)
3. **Ask:** What is the gradient of the line joining the points with co-ordinates $(-1, -3)$ and $(4, -18)$?
4. Allow time for pupils to calculate the gradient.
5. Have a pupil volunteer to answer. (Answer: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-18-(-3)}{4-(-1)} = \frac{-15}{5} = \frac{-3}{1} = -3$)
6. **Say**: Today we are going to identify the slope (m) and y-intercept (c) of a linear equation in slope-intercept form: $y = mx + c$

Introduction to the New Material *(10 minutes)*

1. **Say**: We know that the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ gives the value of the coefficient of x in the equation of a straight line, $y = mx + c$. This is the coefficient of x and it gives the slope or gradient of the line. The constant term c is known as the intercept. It is where the line crosses the x-axis. We know how to find m when we know 2 points on the line. We now need to work out how to find the value of c in the equation. We will use Question 1. to show this.
2. Ask a pupil to read Question 1.
3. **Say**: We want to answer the questions for the points with coordinates $(-3, 2)$ and $(1, 10)$.
4. **Say:** Answer parts a, b and c in your exercise books. You can check with the pupil beside you when you finish.
5. Allow time for pupils to answer the questions in parts a, b and c.
6. Select a pupil to draw the graph for part a on the board. (Answer: a) shown on the right).
7. **Say**: We can find the gradient either by calculating the ratio $\frac{\text{rise}}{\text{run}}$ on the graph or by using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$.
8. **Ask**: We will use both methods on the board. We need 2 volunteers. Who wants to try to find the gradient?
9. Select 2 volunteer pupils to show their calculations on the board. Both pupils should show complete solutions including drawing the triangle for the ratio.
   (Answer using the formula: b) $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{10-2}{1-(-3)} = \frac{8}{4} = 2$)
10. **Ask:** Who can tell the class the value of y where the line crosses the y-axis? (Answer: $y = 8$)



11. **Say:** The value of y where the line crosses the y-axis is called the intercept. This is the value where $x = 0$ in the equation. It is given the special symbol c.
12. **Ask**: What is the value of c for this straight line? (Answer: c) $c = 8$)
13. **Say**: We can now write the equation of the straight line. We substitute the values of m and c we have just found. **Ask:** What is the equation?
14. Allow a few moments for the pupils to discuss and share their ideas.
15. Have a pupil volunteer to answer. (Answer: d) $y = 2x + 8 = 2(x + 4)$)
16. **Say**: We can check that the equation is correct by substituting the x- and y-values into the equation. Use the expanded form of your equation as it is possible to make a mistake factorising.
17. Allow a few moments for pupils to do this and confirm that the equation is correct. (Answer: Check: $(-3, 2), 2 = 2 \times (-3) + 8, = -6 + 8 = 2$, LHS = RHS*;* $(1, 10), 10 = 2 \times 1 + 8, = 2 + 8 = 10$, LHS = RHS)
18. **Say**: Our check confirms we have the correct equation. The equation of a straight line gives us all the information we need to know about the gradient and intercept of the line. For example, the straight line with equation:

    $y = 4x - 3$ has gradient $m = 4$ and intercept $c = -3$

    As you can see, if we put $x = 0$ in the equation $y = -3$. This is the intercept of the line.
19. **Ask**: What is the gradient (m) and intercept (c) of the straight line $y = -5x - 2$?
20. Allow a few moments for the pupils work in pairs to discuss and share their ideas.
21. Have a pair volunteer to share their answer. (Answer: gradient $m = -5$, intercept $c = -2$)

Guided Practice *(10 minutes)*

1. Ask pupils to continue to work in pairs to do Question 2.
2. Walk around, if possible, to check their answers and clear up any misconceptions.
3. Have pupils from around the classroom volunteer to give their answers to the questions. Correct any errors and write them on the board. Ask pupils to check their work. (Answers: See the end of this lesson plan)

Independent Practice *(10 minutes)*

1. Ask the pupils to work independently to answer Questions 3.
2. Walk around, if possible, to check their answers and clear up any misconceptions.
3. Have pupils from around the classroom volunteer to give their answers to the questions.
4. Correct any errors and write the answers on the board. Ask pupils to check their work. (Answers: See the end of this lesson plan)

Closing *(2 minutes)*

1. Write your name on a piece of paper.
2. **Say**: Write on the paper the equation of the line with gradient –1 and intercept 4. Hand the paper in at the end of the lesson.
3. Check the work done by pupils after the lesson. Use it as a guide to which pupils need additional assistance during the next lesson when pupils will be drawing straight line graphs and verifying the gradient and intercept of straight line equations.

[QUESTIONS FOR INTRODUCTION TO THE NEW MATERIAL]

1. The points with co-ordinates $(-3, 2)$ and $(1, 10)$ lie on a straight line.

a. Plot the points and draw the line joining them.
b. What is the gradient of the line?
c. What is the intercept of the line?
d. Write down the equation of the line.

[QUESTIONS FOR GUIDED PRACTICE]

2. Copy and complete the table below:

Equation	*Gradient*	*Intercept*
$y = 5x + 4$		
	1	3
$y = 4 - 3x$		
	-5	1
	-4	-5
	-1	-7

Answers:

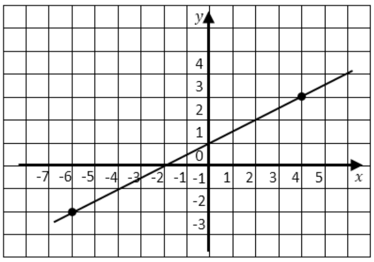
Equation	*Gradient*	*Intercept*
	5	4
$y = x + 3$		
	-3	4
$y = 1 - 5x$		
$y = -4x - 5$		
$y = -x - 7$		

[QUESTIONS FOR INDEPENDENT PRACTICE]

3. The points with co-ordinates $(-6, -2)$ and $(4, 3)$ lie on a straight line.
   a. Plot the points and draw the line joining them.
   b. What is the gradient (m) of the line?
   c. What is the intercept (c) of the line?
   d. Write down the equation of the line.

Answers:

a.



b. $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-(-2)}{4-(-6)} = \frac{5}{10} = \frac{1}{2}$

c. $c = 1$

d. $y = \frac{1}{2}x + 1$

Lesson Title: Graphing Lines in Slope-intercept Form	**Theme**: Algebra	
Lesson Number: M-09-105	**Class/Level**: JSS 3	**Time**: 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to graph a linear equation in slope-intercept form using a table of values, and verify its slope and y-intercept.	None	1. Write the questions for this lesson (found at the end of this lesson plan) on the board. 2. Draw the table from Introduction to the New Material on the board.

Opening *(3 minutes)*

1. **Say**: Write down the gradient and intercept of the line $y = 5 - x$. Please raise your hand when you finish.
2. Allow a few moments for the pupils to respond.
3. Select a pupil who raised their hand to answer. (Answer: gradient $= -1$, intercept $= 5$)
4. **Say**: Today we are going to graph a linear equation in slope-intercept form using a table of values, and verify its slope and y-intercept.

Introduction to the New Material *(10 minutes)*

Allow time for pupils to complete each part of the question in their exercise books before calling on volunteers to explain the answer on the board.

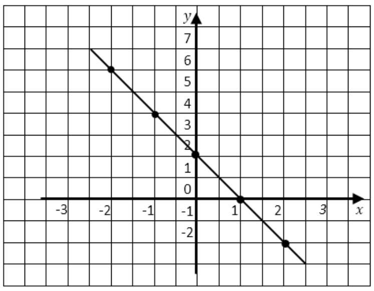
1. **Say**: We have looked at a lot of linear equations over the last few weeks. Here is a list of equations on the board. We will go through the procedure together for one equation. You will then have the chance to do the others with a partner or by yourself.
2. Ask a pupil to read Question 1 a.
3. Point to the table for x-values from -2 to $+2$.
4. **Say**: Copy and complete the table of values in your exercise books.
5. Have pupils from around the classroom volunteer to give the y-value for each x-value. Complete the table of values on the board. (shown on the right)
6. **Say**: We need a volunteer to come to the board to draw the axes for the graph.
7. Let the pupil draw the axes. They should mark the axes using suitable scales for each axis.
8. Ask pupils to copy and complete plotting the graph in their exercise books.
9. Have pupils volunteer to plot 1 or 2 points on the graph on the board. (The completed graph is shown on the right.)
10. **Say**: We need to confirm that the gradient of the graph $m = -2$.
11. Ask pupils to choose any 2 points on the graph and find the gradient of the line.

1. $y = 2(1 - x)$

a.

x	-2	-1	0	1	2
y	6	4	2	0	-2

b.



c. Using the points $(0, 2)$ and $(1, 0)$:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 2}{1 - 0} = \frac{-2}{1} = -2$$

d. From the graph the line crosses the *y-axis at* $y = 2$.
So the intercept $c = 2$

12. **Say**: Choose 2 points which make the calculations easy to carry out. If possible, find points where x or y is equal to zero.
13. Have 2 pupils volunteer to show their calculations on the board. Make sure they have chosen different points. An example calculation is shown for the points with co-ordinates $(0, 2)$and $(1, 0)$.
14. **Say**: We have verified that the gradient $m = -2$.
15. Have a pupil volunteer to give the value of y where the line crosses the y –axis. (Answer: *at* $y = 2$)
16. **Say**: We have confirmed that the intercept $c = 2$. The questions we have just answered are standard questions on linear equations. You will now get some practice answering them for yourselves.

Guided Practice *(10 minutes)*

1. Ask pupils to work in pairs.
2. They should create the table from the linear equation in Question 2 given on the board. They should then draw the graph using a suitable scale. They should each draw their own individual graph then check their partner's work.
3. **Say**: Choose different points for part c. Check with each other that you both have the same gradient.
4. Walk around, if possible, to check their answers and clear up any misconceptions.
5. Ask pupils to exchange exercise books and check each other's work.
6. Have some pupils volunteer to complete the answers on the board.
7. Correct any errors on the board and ask all pupils to check their work. (Answers: *S*ee the end of this lesson plan)

Independent Practice *(10 minutes)*

1. Ask the pupils to work independently to answer Question 3. from the board.
2. Walk around, if possible, to check their answers and clear up any misconceptions.
3. Have some pupils volunteer to complete the answers on the board.
4. Correct any errors on the board and ask all pupils to check their work. (Answers: See the end of this lesson plan)

Closing *(2 minutes)*

1. **Say:** In your exercise book, write how you would explain to a friend who has been absent for the last 3 weeks, how to find the gradient of a line using the co-ordinates of 2 points on the line.
2. Allow pupils time to think and write down their explanations.
3. Have 2-3 pupils volunteer to explain how to do the calculation. (Example answers: Subtract the y-values, subtract the x-values, make sure the values are subtracted in the same order, divide the 2 answers; Change in y co-ordinates, divided by change in x co-ordinates. Accept all reasonable answers.)

[QUESTIONS FOR INTRODUCTION OF THE NEW MATERIAL]

1. $y = 2(1 - x)$
   a. Complete a table of values for the linear equation.
   b. Draw the graph of the line.
   c. Verify its slope $\mathrm{m} = -2$ using any 2 points on the graph.
   d. Confirm that the intercept $\mathrm{c} = 2$.

[QUESTIONS FOR GUIDED PRACTICE]

2. $y = 1 - \frac{1}{2}x$
   a. Complete a table of values for the linear equation.
   b. Draw the graph of the line.
   c. Verify its slope $\mathrm{m} = -\frac{1}{2}$ using any 2 points on the graph.
   d. Confirm that the intercept $\mathrm{c} = 1$.

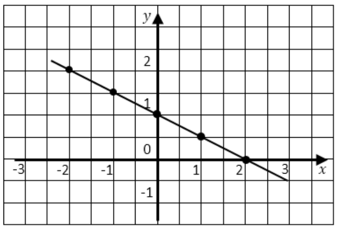
Answers:

2. $$y = 1 - \frac{1}{2}x$$

a.

x	-2	-1	0	1	2
y	2	1.5	1	0.5	0

b.



c. Using the points $(0,1)$and $(2,0)$:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-1}{2-0} = \frac{-1}{2} = -\frac{1}{2}$$

d. From the graph the line crosses the y-axis at $y = 1$. So the intercept $c = 1$

[QUESTIONS FOR INDEPENDENT PRACTICE]

3. $y = \frac{3}{5}x + 2$
   a. Complete a table of values for the linear equation.
   b. Draw the graph of the line.
   c. Verify its slope $\mathrm{m} = \frac{3}{5}$ using any 2 points on the graph.
   d. Confirm that the intercept $c = 2$.

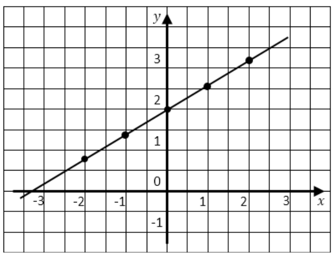
Answers:

3. $$y = \frac{3}{5}x + 2$$

a.

x	-2	-1	0	1	2
y	0.8	1.4	2	2.6	3.2

b.



c. Using the points $(0,2)$and $(2,3.2)$:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3.2-2}{2-0} = \frac{1.2}{2} = \frac{12}{20} = \frac{3}{5}$$

d. From the graph the line crosses the y-axis *at* $y = 2$. So the intercept $c = 2$

Appendix I: Squares of Numbers 10-100

Squares of Numbers

$x \rightarrow x^2$

x	0	1	2	3	4	5	6	7	8	9	Differences 1	2	3	4	5	6	7	8	9
10	1000	1020	1040	1061	1082	1103	1124	1145	1166	1188	2	4	6	8	10	13	15	17	19
11	1210	1232	1254	1277	1300	1323	1346	1369	1392	1416	2	5	7	9	11	14	16	18	21
12	1440	1464	1488	1513	1538	1563	1588	1613	1638	1664	2	5	7	10	12	15	17	20	22
13	1690	1716	1742	1769	1796	1823	1850	1877	1904	1932	3	5	8	11	13	16	19	22	24
14	1960	1988	2016	2045	2074	2103	2132	2161	2190	2220	3	6	9	12	14	17	20	23	26
15	2250	2280	2310	2341	2372	2403	2434	2465	2496	2528	3	6	9	12	15	19	22	25	28
16	2560	2592	2624	2657	2690	2723	2756	2789	2822	2856	3	7	10	13	16	20	23	26	30
17	2890	2924	2958	2993	3028	3063	3098	3133	3168	3204	3	7	10	14	17	21	24	28	31
18	3240	3276	3312	3349	3386	3423	3460	3497	3534	3572	4	7	11	15	18	22	26	30	33
19	3610	3648	3686	3725	3764	3803	3842	3881	3920	3960	4	8	12	16	19	23	27	31	35
20	4000	4040	4080	4121	4162	4203	4244	4285	4326	4368	4	8	12	16	20	25	29	33	37
21	4410	4452	4494	4537	4580	4623	4666	4709	4752	4796	4	9	13	17	21	26	30	34	39
22	4840	4884	4928	4973	5018	5063	5108	5153	5198	5244	4	9	13	18	22	27	31	36	40
23	5290	5336	5382	5429	5476	5523	5570	5617	5664	5712	5	9	14	19	23	28	33	38	42
24	5760	5808	5856	5905	5954	6003	6052	6101	6150	6200	5	10	15	20	24	29	34	39	44
25	6250	6300	6350	6401	6452	6503	6554	6605	6656	6708	5	10	15	20	25	31	36	41	46
26	6760	6812	6864	6917	6970	7023	7076	7129	7182	7236	5	11	16	21	26	32	37	42	48
27	7290	7344	7398	7453	7508	7563	7618	7673	7728	7784	5	11	16	22	27	33	38	44	49
28	7840	7896	7952	8009	8066	8123	8180	8237	8294	8352	6	11	17	23	28	34	40	46	51
29	8410	8486	8526	8585	8644	8703	8762	8821	8880	8940	6	12	18	24	29	35	41	47	53
30	9000	9060	9120	9181	9242	9303	9364	9425	9486	9548	6	12	18	24	30	37	43	49	55
31	9610	9672	9734	9797	9860	9923	9986	–	–	–	6	13	19	25	31	38	44	50	57
31	–	–	–	–	–	–	–	1005	1011	1018	1	1	2	3	3	4	5	5	6
32	1024	1030	1037	1043	1050	1056	1063	1069	1076	1082	1	1	2	3	3	4	5	5	6
33	1089	1096	1102	1109	1116	1122	1129	1136	1142	1149	1	1	2	3	3	4	5	5	6
34	1156	1163	1170	1176	1183	1190	1197	1204	1211	1218	1	1	2	3	3	4	5	6	6
35	1225	1232	1239	1246	1253	1260	1267	1274	1282	1289	1	1	2	3	4	4	5	6	6
36	1296	1303	1310	1318	1325	1332	1340	1347	1354	1362	1	1	2	3	4	4	5	6	7
37	1369	1376	1384	1391	1399	1406	1414	1421	1429	1436	1	2	2	3	4	5	5	6	7
38	1444	1452	1459	1467	1475	1482	1490	1498	1505	1513	1	2	2	3	4	5	5	6	7
39	1521	1529	1537	1544	1552	1560	1568	1576	1584	1592	1	2	2	3	4	5	6	6	7
40	1600	1608	1616	1624	1632	1640	1648	1656	1665	1673	1	2	2	3	4	5	6	6	7
41	1681	1689	1697	1706	1714	1722	1731	1739	1747	1756	1	2	2	3	4	5	6	7	7
42	1764	1772	1781	1789	1798	1806	1815	1823	1832	1840	1	2	3	3	4	5	6	7	8
43	1849	1858	1866	1875	1884	1892	1901	1910	1918	1927	1	2	3	3	4	5	6	7	8
44	1936	1945	1954	1962	1971	1980	1989	1998	2007	2016	1	2	3	4	5	5	6	7	8
45	2025	2034	2043	2052	2061	2070	2079	2088	2098	2107	1	2	3	4	5	5	6	7	8
46	2116	2125	2134	2144	2153	2162	2172	2181	2190	2200	1	2	3	4	5	6	7	7	8
47	2209	2218	2228	2237	2247	2256	2266	2275	2285	2294	1	2	3	4	5	6	7	8	9
48	2304	2314	2323	2333	2343	2352	2362	2372	2381	2391	1	2	3	4	5	6	7	8	9
49	2401	2411	2421	2430	2440	2450	2460	2470	2480	2490	1	2	3	4	5	6	7	8	9
50	2500	2510	2520	2530	2540	2550	2560	2570	2581	2591	1	2	3	4	5	6	7	8	9
51	2601	2611	2621	2632	2642	2652	2663	2673	2683	2694	1	2	3	4	5	6	7	8	9
52	2704	2714	2725	2735	2746	2756	2767	2777	2788	2798	1	2	3	4	5	6	7	8	9
53	2809	2820	2830	2841	2852	2862	2873	2884	2894	2905	1	2	3	4	5	6	7	9	10
54	2916	2927	2938	2948	2959	2970	2981	2992	3003	3014	1	2	3	4	5	7	8	9	10
x	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9

x	0	1	2	3	4	5	6	7	8	9	Differences 1	2	3	4	5	6	7	8	9
55	3025	3036	3047	3058	3069	3080	3091	3102	3114	3125	1	2	3	4	6	7	8	9	10
56	3136	3147	3158	3170	3181	3192	3204	3215	3226	3238	1	2	3	5	6	7	8	9	10
57	3249	3260	3272	3283	3295	3306	3318	3329	3341	3352	1	2	3	5	6	7	8	9	10
58	3364	3376	3387	3399	3411	3422	3434	3446	3457	3469	1	2	4	5	6	7	8	9	10
59	3481	3493	3505	3516	3528	3540	3552	3564	3576	3588	1	2	4	5	6	7	8	10	11
60	3600	3612	3624	3636	3648	3660	3672	3684	3697	3709	1	2	4	5	6	7	8	10	11
61	3721	3733	3745	3758	3770	3782	3795	3807	3819	3832	1	2	4	5	6	7	9	10	11
62	3844	3856	3869	3881	3894	3906	3919	3931	3944	3956	1	2	4	5	6	7	9	10	11
63	3969	3982	3994	4007	4020	4032	4054	4058	4070	4083	1	3	4	5	6	8	9	10	11
64	4096	4109	4122	4134	4147	4160	4173	4186	4199	4212	1	3	4	5	6	8	9	10	12
65	4225	4238	4251	4264	4277	4290	4303	4316	4330	4343	1	3	4	5	7	8	9	10	12
66	4356	4369	4382	4396	4409	4422	4436	4449	4462	4476	1	3	4	5	7	8	9	11	12
67	4489	4502	4516	4529	4543	4556	4570	4583	4597	4610	1	3	4	5	7	8	9	11	12
68	4624	4638	4651	4665	4679	4692	4706	4720	4733	4747	1	3	4	5	7	8	10	11	12
69	4761	4775	4789	4802	4816	4830	4844	4858	4872	4886	1	3	4	6	7	8	10	11	13
70	4900	4914	4928	4942	4956	4970	4984	4998	5013	5027	1	3	4	6	7	8	10	11	13
71	5041	5055	5069	5084	5098	5112	5127	5141	5155	5170	1	3	4	6	7	9	10	11	13
72	5184	5198	5213	5227	5242	5256	5271	5285	5300	5314	1	3	4	6	7	9	10	11	13
73	5329	5344	5358	5373	5388	5402	5417	5432	5446	5461	1	3	4	6	7	9	10	12	13
74	5476	5491	5506	5520	5535	5550	5565	5580	5595	5610	1	3	4	6	7	9	10	12	13
75	5625	5640	5655	5670	5685	5700	5715	5730	5746	5761	2	3	5	6	8	9	11	12	14
76	5776	5791	5806	5822	5837	5852	5868	5883	5898	5914	2	3	5	6	8	9	11	12	14
77	5929	5944	5960	5975	5991	6006	6022	6037	6053	6068	2	3	5	6	8	9	11	12	14
78	6084	6100	6115	6131	6147	6162	6178	6194	6209	6225	2	3	5	6	8	9	11	13	14
79	6241	6257	6273	6288	6304	6320	6336	6352	6368	6384	2	3	5	6	8	10	11	13	14
80	6400	6416	6432	6448	6464	6480	6496	6512	6529	6545	2	3	5	6	8	10	11	13	14
81	6561	6577	6593	6610	6626	6642	6659	6675	6691	6708	2	3	5	7	8	10	11	13	15
82	6724	6740	6757	6773	6790	6806	6823	6839	6856	6872	2	3	5	7	8	10	12	13	15
83	6889	6906	6922	6939	6956	6972	6989	7006	7022	7039	2	3	5	7	8	10	12	13	15
84	7056	7073	7090	7106	7123	7140	7157	7174	7191	7208	2	3	5	7	8	10	12	14	15
85	7225	7242	7259	7276	7293	7310	7327	7344	7362	7379	2	3	5	7	9	10	12	14	15
86	7396	7413	7430	7448	7465	7482	7500	7517	7534	7552	2	3	5	7	9	10	12	14	16
87	7569	7586	7604	7621	7639	7656	7674	7691	7709	7726	2	3	5	7	9	10	12	14	16
88	7744	7762	7779	7797	7815	7832	7850	7868	7885	7903	2	4	5	7	9	11	12	14	16
89	7921	7939	7957	7974	7992	8010	8028	8046	8064	8082	2	4	5	7	9	11	13	14	16
90	8100	8118	8136	8154	8172	8190	8208	8226	8245	8263	2	4	5	7	9	11	13	14	16
91	8281	8299	8317	8336	8354	8372	8391	8409	8427	8446	2	4	5	7	9	11	13	15	16
92	8464	8482	8501	8519	8538	8556	8575	8593	8612	8630	2	4	6	7	9	11	13	15	17
93	8649	8668	8686	8705	8724	8742	8761	8780	8798	8817	2	4	6	7	9	11	13	15	17
94	8836	8855	8874	8892	8911	8930	8949	8968	8987	9006	2	4	6	8	9	11	13	15	17
95	9025	9044	9063	9082	9101	9120	9139	9158	9178	9197	2	4	6	8	10	11	13	15	17
96	9216	9235	9254	9274	9293	9312	9332	9351	9370	9390	2	4	6	8	10	12	14	15	17
97	9409	9428	9448	9467	9487	9506	9526	9545	9565	9584	2	4	6	8	10	12	14	16	18
98	9604	9624	9643	9663	9683	9702	9722	9742	9761	9781	2	4	6	8	10	12	14	16	18
99	9801	9821	9841	9860	9880	9900	9920	9940	9960	9980	2	4	6	8	10	12	14	16	18
x	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9

Sines of Angles (*x* in degrees)

x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	ADD Differences								
											1	2	3	4	5	6	7	8	9
0	0·0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	2	3	5	7	9	10	12	14	16
1	0·0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	2	3	5	7	9	10	12	14	16
2	0·0349	0366	0384	0401	0419	0436	0454	0471	0488	0506	2	3	5	7	9	10	12	14	16
3	0·0523	0541	0558	0576	0593	0610	0628	0645	0663	0680	2	3	5	7	9	10	12	14	16
4	0·0698	0715	0732	0750	0767	0785	0802	0819	0837	0854	2	3	5	7	9	10	12	14	16
5	0·0872	0889	0906	0924	0941	0958	0976	0993	1011	1028	2	3	5	7	9	10	12	14	16
6	0·1045	1063	1080	1097	1115	1132	1149	1167	1184	1201	2	3	5	7	9	10	12	14	16
7	0·1219	1236	1253	1271	1288	1305	1323	1340	1357	1374	2	3	5	7	9	10	12	14	16
8	0·1392	1409	1426	1444	1461	1478	1495	1513	1530	1547	2	3	5	7	9	10	12	14	16
9	0·1564	1582	1599	1616	1633	1650	1668	1685	1702	1719	2	3	5	7	9	10	12	14	15
10	0·1736	1754	1771	1788	1805	1822	1840	1857	1874	1891	2	3	5	7	9	10	12	14	15
11	0·1908	1925	1942	1959	1977	1994	2011	2028	2045	2062	2	3	5	7	9	10	12	14	15
12	0·2079	2096	2113	2130	2147	2164	2181	2198	2215	2233	2	3	5	7	9	10	12	14	15
13	0·2250	2267	2284	2300	2317	2334	2351	2368	2385	2402	2	3	5	7	8	10	12	14	15
14	0·2419	2436	2453	2470	2487	2504	2521	2538	2554	2571	2	3	5	7	8	10	12	14	15
15	0·2588	2605	2622	2639	2656	2672	2689	2706	2723	2740	2	3	5	7	8	10	12	13	15
16	0·2756	2773	2790	2807	2823	2840	2857	2874	2890	2907	2	3	5	7	8	10	12	13	15
17	0·2924	2940	2957	2974	2990	3007	3024	3040	3057	3074	2	3	5	7	8	10	12	13	15
18	0·3090	3107	3123	3140	3156	3173	3190	3206	3223	3239	2	3	5	7	8	10	12	13	15
19	0·3256	3272	3289	3305	3322	3338	3355	3371	3387	3404	2	3	5	7	8	10	12	13	15
20	0·3420	3437	3453	3469	3486	3502	3518	3535	3551	3567	2	3	5	7	8	10	11	13	15
21	0·3584	3600	3616	3633	3649	3665	3681	3697	3714	3730	2	3	5	6	8	10	11	13	15
22	0·3746	3762	3778	3795	3811	3827	3843	3859	3875	3891	2	3	5	6	8	10	11	13	15
23	0·3907	3923	3939	3955	3971	3987	4003	4019	4035	4051	2	3	5	6	8	10	11	13	14
24	0·4067	4083	4099	4115	4131	4147	4163	4179	4195	4210	2	3	5	6	8	10	11	13	14
25	0·4226	4242	4258	4274	4289	4305	4321	4337	4352	4368	2	3	5	6	8	9	11	13	14
26	0·4384	4399	4415	4431	4446	4462	4478	4493	4509	4524	2	3	5	5	8	9	11	12	14
27	0·4540	4555	4571	4586	4602	4617	4633	4648	4664	4679	2	3	5	6	8	9	11	12	14
28	0·4695	4710	4726	4741	4756	4772	4787	4802	4818	4833	2	3	5	6	8	9	11	12	14
29	0·4848	4863	4879	4894	4909	4924	4939	4955	4970	4985	2	3	5	6	8	9	11	12	14
30	0·5000	5015	5030	5045	5060	5075	5090	5105	5120	5135	2	3	5	6	8	9	11	12	14
31	0·5150	5165	5180	5195	5210	5225	5240	5255	5270	5284	1	3	4	6	7	9	10	12	13
32	0·5299	5314	5329	5344	5358	5373	5388	5402	5417	5432	1	3	4	6	7	9	10	12	13
33	0·5446	5461	5476	5490	5505	5519	5534	5548	5563	5577	1	3	4	6	7	9	10	12	13
34	0·5592	5606	5621	5635	5650	5664	5678	5693	5707	5721	1	3	4	6	7	9	10	12	13
35	0·5736	5750	5764	5779	5793	5807	5821	5835	5850	5864	1	3	4	6	7	9	10	11	13
36	0·5878	5892	5906	5920	5934	5948	5962	5976	5990	6004	1	3	4	6	7	8	10	11	13
37	0·6018	6032	6046	6060	6074	6088	6101	6115	6129	6143	1	3	4	6	7	8	10	11	12
38	0·6157	6170	6184	6198	6211	6225	6239	6252	6266	6280	1	3	4	5	7	8	10	11	12
39	0·6293	6307	6320	6334	6347	6361	6374	6388	6401	6414	1	3	4	5	7	8	9	11	12
40	0·6428	6441	6455	6468	6481	6494	6508	6521	6534	6547	1	3	4	5	7	8	9	11	12
41	0·6561	6574	6587	6600	6613	6626	6639	6652	6665	6678	1	3	4	5	7	8	9	10	12
42	0·6691	6704	6717	6730	6743	6756	6769	6782	6794	6807	1	3	4	5	6	8	9	10	12
43	0·6820	6833	6845	6858	6871	6884	6896	6909	6921	6934	1	3	4	5	6	8	9	10	11
44	0·6947	6959	6972	6984	6997	7009	7022	7034	7046	7059	1	2	4	5	6	7	9	10	11
x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9

x → sin *x*

x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	ADD Differences								
											1	2	3	4	5	6	7	8	9
45	0·7071	7083	7096	7108	7120	7133	7145	7157	7169	7181	1	2	4	5	6	7	9	10	11
46	0·7193	7206	7218	7230	7242	7254	7266	7278	7290	7302	1	2	4	5	6	7	8	10	11
47	0·7314	7325	7337	7349	7361	7373	7385	7396	7408	7420	1	2	4	5	6	7	8	9	11
48	0·7431	7443	7455	7466	7478	7490	7501	7513	7524	7536	1	2	3	5	6	7	8	9	10
49	0·7547	7559	7570	7581	7593	7604	7615	7627	7638	7649	1	2	3	5	6	7	8	9	10
50	0·7660	7672	7683	7694	7705	7716	7727	7738	7749	7760	1	2	3	4	6	7	8	9	10
51	0·7771	7782	7793	7804	7815	7826	7837	7848	7859	7869	1	2	3	4	5	7	8	9	10
52	0·7880	7891	7902	7912	7923	7934	7944	7955	7965	7976	1	2	3	4	5	6	7	8	10
53	0·7986	7997	8007	8018	8028	8039	8049	8059	8070	8080	1	2	3	4	5	6	7	8	9
54	0·8090	8100	8111	8121	8131	8141	8151	8161	8171	8181	1	2	3	4	5	6	7	8	9
55	0·8192	8202	8211	8221	8231	8241	8251	8261	8271	8281	1	2	3	4	5	6	7	8	9
56	0·8290	8300	8310	8320	8329	8339	8348	8358	8368	8377	1	2	3	4	5	6	7	8	9
57	0·8387	8396	8406	8415	8425	8434	8443	8453	8462	8471	1	2	3	4	5	6	7	8	8
58	0·8480	8490	8499	8508	8517	8526	8536	8545	8554	8563	1	2	3	4	5	5	6	7	8
59	0·8572	8581	8590	8599	8607	8616	8625	8634	8643	8652	1	2	3	4	4	5	6	7	8
60	0·8660	8669	8678	8686	8695	8704	8712	8721	8729	8738	1	2	3	3	4	5	6	7	8
61	0·8746	8755	8763	8771	8780	8788	8796	8805	8813	8821	1	2	2	3	4	5	6	7	7
62	0·8829	8838	8846	8854	8862	8870	8878	8886	8894	8902	1	2	2	3	4	5	6	6	7
63	0·8910	8918	8926	8934	8942	8949	8957	8965	8973	8980	1	2	2	3	4	5	5	6	7
64	0·8988	8996	9003	9011	9018	9026	9033	9041	9048	9056	1	2	2	3	4	5	5	6	7
65	0·9063	9070	9078	9085	9092	9100	9107	9114	9121	9128	1	1	2	3	4	4	5	6	7
66	0·9135	9143	9150	9157	9164	9171	9178	9184	9191	9198	1	1	2	3	3	4	5	6	6
67	0·9205	9212	9219	9225	9232	9239	9245	9252	9259	9265	1	1	2	3	3	4	5	5	6
68	0·9272	9278	9285	9291	9298	9304	9311	9317	9323	9330	1	1	2	3	3	4	4	5	6
69	0·9336	9342	9348	9354	9361	9367	9373	9379	9385	9391	1	1	2	2	3	4	4	5	6
70	0·9397	9403	9409	9415	9421	9426	9432	9438	9444	9449	1	1	2	2	3	3	4	5	5
71	0·9455	9461	9466	9472	9478	9483	9489	9494	9500	9505	1	1	2	2	3	3	4	4	5
72	0·9511	9516	9521	9527	9532	9537	9542	9548	9553	9558	1	1	2	2	3	3	4	4	5
73	0·9563	9568	9573	9578	9583	9588	9593	9598	9603	9608	0	1	1	2	2	3	3	4	4
74	0·9613	9617	9622	9627	9632	9636	9641	9646	9650	9655	0	1	1	2	2	3	3	4	4
75	0·9659	9664	9668	9673	9677	9681	9686	9690	9694	9699	0	1	1	2	2	3	3	3	4
76	0·9703	9707	9711	9715	9720	9724	9728	9732	9736	9740	0	1	1	2	2	2	3	3	4
77	0·9744	9748	9751	9755	9759	9763	9767	9770	9774	9778	0	1	1	2	2	2	3	3	3
78	0·9781	9785	9789	9792	9796	9799	9803	9806	9810	9813	0	1	1	1	2	2	2	3	3
79	0·9816	9820	9823	9826	9829	9833	9836	9839	9842	9845	0	1	1	1	2	2	2	3	3
80	0·9848	9851	9854	9857	9860	9863	9866	9869	9871	9874	0	1	1	1	1	2	2	2	3
81	0·9877	9880	9882	9885	9888	9890	9893	9895	9898	9900	0	1	1	1	1	2	2	2	2
82	0·9903	9905	9907	9910	9912	9914	9917	9919	9921	9923	0	0	1	1	1	1	2	2	2
83	0·9925	9928	9930	9932	9934	9936	9938	9940	9942	9943	0	0	1	1	1	1	1	2	2
84	0·9945	9947	9949	9951	9952	9954	9956	9957	9959	9960	0	0	1	1	1	1	1	1	2
85	0·9962	9963	9965	9966	9968	9969	9971	9972	9973	9974	0	0	0	1	1	1	1	1	1
86	0·9976	9977	9978	9979	9980	9981	9982	9983	9984	9985	0	0	0	0	1	1	1	1	1
87	0·9986	9987	9988	9989	9990	9990	9991	9992	9993	9993	0	0	0	0	0	0	1	1	1
88	0·9994	9995	9995	9996	9996	9997	9997	9997	9998	9998									
89	0·9998	9999	9999	9999	9999	1·0000	1·0000	1·0000	1·0000	1·0000									
x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9

Appendix III: Cosines of Angles

Cosines of Angles (*x* in degrees)

$x \to \cos x$

x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	SUBTRACT Differences 1	2	3	4	5	6	7	8	9
0	1·0000	1·000	1·000	1·000	1·000	1·000	·9999	·9999	·9999	·9999									
1	0·9998	·9998	·9998	·9997	·9997	·9997	·9996	·9996	·9995	·9995									
2	0·9994	·9993	·9993	·9992	·9991	·9990	·9990	·9989	·9988	·9987	0	0	0	0	0	0	1	1	1
3	0·9986	·9985	·9984	·9983	·9982	·9981	·9980	·9979	·9978	·9977	0	0	0	0	1	1	1	1	1
4	0·9976	·9974	·9973	·9972	·9971	·9969	·9968	·9966	·9965	·9963	0	0	0	1	1	1	1	1	1
5	0·9962	·9960	·9959	·9957	·9956	·9954	·9952	·9951	·9949	·9947	0	0	1	1	1	1	1	1	2
6	0·9945	·9943	·9942	·9940	·9938	·9936	·9934	·9932	·9930	·9928	0	0	1	1	1	1	1	2	2
7	0·9925	·9923	·9921	·9919	·9917	·9914	·9912	·9910	·9907	·9905	0	0	1	1	1	1	2	2	2
8	0·9903	·9900	·9898	·9895	·9893	·9890	·9888	·9885	·9882	·9880	0	1	1	1	1	2	2	2	2
9	0·9877	·9874	·9871	·9869	·9866	·9863	·9860	·9857	·9854	·9851	0	1	1	1	1	2	2	2	3
10	0·9848	·9845	·9842	·9839	·9836	·9833	·9829	·9826	·9823	·9820	0	1	1	1	2	2	2	3	3
11	0·9816	·9813	·9810	·9806	·9803	·9799	·9796	·9792	·9789	·9785	0	1	1	1	2	2	2	3	3
12	0·9781	·9778	·9774	·9770	·9767	·9763	·9759	·9755	·9751	·9748	0	1	1	2	2	2	3	3	3
13	0·9744	·9740	·9736	·9732	·9728	·9724	·9720	·9715	·9711	·9707	0	1	1	2	2	2	3	3	4
14	0·9703	·9699	·9694	·9690	·9686	·9681	·9677	·9673	·9668	·9664	0	1	1	2	2	3	3	3	4
15	0·9659	·9655	·9650	·9646	·9641	·9636	·9632	·9627	·9622	·9617	0	1	1	2	2	3	3	4	4
16	0·9613	·9608	·9603	·9598	·9593	·9588	·9583	·9578	·9573	·9568	0	1	1	2	2	3	3	4	4
17	0·9563	·9558	·9553	·9548	·9542	·9537	·9532	·9527	·9521	·9516	1	1	2	2	3	3	4	4	5
18	0·9511	·9505	·9500	·9494	·9489	·9483	·9478	·9472	·9466	·9461	1	1	2	2	3	3	4	4	5
19	0·9455	·9449	·9444	·9438	·9432	·9426	·9421	·9415	·9409	·9403	1	1	2	2	3	3	4	5	5
20	0·9397	·9391	·9385	·9379	·9373	·9367	·9361	·9354	·9348	·9342	1	1	2	2	3	4	4	5	6
21	0·9336	·9330	·9323	·9317	·9311	·9304	·9298	·9291	·9285	·9278	1	1	2	3	3	4	4	5	6
22	0·9272	·9265	·9259	·9252	·9245	·9239	·9232	·9225	·9219	·9212	1	1	2	3	3	4	5	5	6
23	0·9205	·9198	·9191	·9184	·9178	·9171	·9164	·9157	·9150	·9143	1	1	2	3	3	4	5	6	6
24	0·9135	·9128	·9121	·9114	·9107	·9100	·9092	·9085	·9078	·9070	1	1	2	3	4	4	5	6	7
25	0·9063	·9056	·9048	·9041	·9033	·9026	·9018	·9011	·9003	·8996	1	2	2	3	4	5	5	6	7
26	0·8988	·8980	·8973	·8965	·8957	·8949	·8942	·8934	·8926	·8918	1	2	2	3	4	5	5	6	7
27	0·8910	·8902	·8894	·8886	·8878	·8870	·8862	·8854	·8846	·8838	1	2	2	3	4	5	6	6	7
28	0·8829	·8821	·8813	·8805	·8796	·8788	·8780	·8771	·8763	·8755	1	2	2	3	4	5	6	7	7
29	0·8746	·8738	·8729	·8721	·8712	·8704	·8695	·8686	·8678	·8669	1	2	3	3	4	5	6	7	8
30	0·8660	·8652	·8643	·8634	·8625	·8616	·8607	·8599	·8590	·8581	1	2	3	4	4	5	6	7	8
31	0·8572	·8563	·8554	·8545	·8536	·8526	·8517	·8508	·8499	·8490	1	2	3	4	5	5	6	7	8
32	0·8480	·8471	·8462	·8453	·8443	·8434	·8425	·8415	·8406	·8396	1	2	3	4	5	6	7	8	8
33	0·8387	·8377	·8368	·8358	·8348	·8339	·8329	·8320	·8310	·8300	1	2	3	4	5	6	7	8	9
34	0·8290	·8281	·8271	·8261	·8251	·8241	·8231	·8221	·8211	·8202	1	2	3	4	5	6	7	8	9
35	0·8192	·8181	·8171	·8161	·8151	·8141	·8131	·8121	·8111	·8100	1	2	3	4	5	6	7	8	9
36	0·8090	·8080	·8070	·8059	·8049	·8039	·8028	·8018	·8007	·7997	1	2	3	4	5	6	7	8	9
37	0·7986	·7976	·7965	·7955	·7944	·7934	·7923	·7912	·7902	·7891	1	2	3	4	5	6	7	8	10
38	0·7880	·7869	·7859	·7848	·7837	·7826	·7815	·7804	·7793	·7782	1	2	3	4	5	6	7	9	10
39	0·7771	·7760	·7749	·7738	·7727	·7716	·7705	·7694	·7683	·7672	1	2	3	4	6	7	8	9	10
40	0·7660	·7649	·7638	·7627	·7615	·7604	·7593	·7581	·7570	·7559	1	2	3	5	6	7	8	9	10
41	0·7547	·7536	·7524	·7513	·7501	·7490	·7478	·7466	·7455	·7443	1	2	3	5	6	7	8	9	10
42	0·7431	·7420	·7408	·7396	·7385	·7373	·7361	·7349	·7337	·7325	1	2	4	5	6	7	8	9	11
43	0·7314	·7302	·7290	·7278	·7266	·7254	·7242	·7230	·7218	·7206	1	2	4	5	6	7	8	10	11
44	0·7193	·7181	·7169	·7157	·7145	·7133	·7120	·7108	·7096	·7083	1	2	4	5	6	7	9	10	11
x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9

x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	SUBTRACT Differences 1	2	3	4	5	6	7	8	9
45	0·7071	·7059	·7046	·7034	·7022	·7009	·6997	·6984	·6972	·6959	1	2	4	5	6	7	9	10	11
46	0·6947	·6934	·6921	·6909	·6896	·6884	·6871	·6858	·6845	·6833	1	3	4	5	6	8	9	10	11
47	0·6820	·6807	·6794	·6782	·6769	·6756	·6743	·6730	·6717	·6704	1	3	4	5	6	8	9	10	12
48	0·6691	·6678	·6665	·6652	·6639	·6626	·6613	·6600	·6587	·6574	1	3	4	5	7	8	9	10	12
49	0·6561	·6547	·6534	·6521	·6508	·6494	·6481	·6468	·6455	·6441	1	3	4	5	7	8	9	11	12
50	0·6428	·6414	·6401	·6388	·6374	·6361	·6347	·6334	·6320	·6307	1	3	4	5	7	8	9	11	12
51	0·6293	·6280	·6266	·6252	·6239	·6225	·6211	·6198	·6184	·6170	1	3	4	5	7	8	10	11	12
52	0·6157	·6143	·6129	·6115	·6101	·6088	·6074	·6060	·6046	·6032	1	3	4	5	7	8	10	11	12
53	0·6018	·6004	·5990	·5976	·5962	·5948	·5934	·5920	·5906	·5892	1	3	4	6	7	8	10	11	13
54	0·5878	·5864	·5850	·5835	·5821	·5807	·5793	·5779	·5764	·5750	1	3	4	6	7	9	10	11	13
55	0·5736	·5721	·5707	·5693	·5678	·5664	·5650	·5635	·5621	·5606	1	3	4	6	7	9	10	12	13
56	0·5592	·5577	·5563	·5548	·5534	·5519	·5505	·5490	·5476	·5461	1	3	4	6	7	9	10	12	13
57	0·5446	·5432	·5417	·5402	·5388	·5373	·5358	·5344	·5329	·5314	1	3	4	6	7	9	10	12	13
58	0·5299	·5284	·5270	·5255	·5240	·5225	·5210	·5195	·5180	·5165	1	3	4	6	7	9	10	12	13
59	0·5150	·5135	·5120	·5105	·5090	·5075	·5060	·5045	·5030	·5015	2	3	5	6	8	9	11	12	14
60	0·5000	·4985	·4970	·4955	·4939	·4924	·4909	·4894	·4879	·4863	2	3	5	6	8	9	11	12	14
61	0·4848	·4833	·4818	·4802	·4787	·4772	·4756	·4741	·4726	·4710	2	3	5	6	8	9	11	12	14
62	0·4695	·4679	·4664	·4648	·4633	·4617	·4602	·4586	·4571	·4555	2	3	5	6	8	9	11	12	14
63	0·4540	·4524	·4509	·4493	·4478	·4462	·4446	·4431	·4415	·4399	2	3	5	6	8	9	11	12	14
64	0·4384	·4368	·4352	·4337	·4321	·4305	·4289	·4274	·4258	·4242	2	3	5	6	8	9	11	13	14
65	0·4226	·4210	·4195	·4179	·4163	·4147	·4131	·4115	·4099	·4083	2	3	5	6	8	10	11	13	14
66	0·4067	·4051	·4035	·4019	·4003	·3987	·3971	·3955	·3939	·3923	2	3	5	6	8	10	11	13	14
67	0·3907	·3891	·3875	·3859	·3843	·3827	·3811	·3795	·3778	·3762	2	3	5	6	8	10	11	13	15
68	0·3746	·3730	·3714	·3697	·3681	·3665	·3649	·3633	·3616	·3600	2	3	5	6	8	10	11	13	15
69	0·3584	·3567	·3551	·3535	·3518	·3502	·3486	·3469	·3453	·3437	2	3	5	6	8	10	11	13	15
70	0·3420	·3404	·3387	·3371	·3355	·3338	·3322	·3305	·3289	·3272	2	3	5	7	8	10	12	13	15
71	0·3256	·3239	·3223	·3206	·3190	·3173	·3156	·3140	·3123	·3107	2	3	5	7	8	10	12	13	15
72	0·3090	·3074	·3057	·3040	·3024	·3007	·2990	·2974	·2957	·2940	2	3	5	7	8	10	12	13	15
73	0·2924	·2907	·2890	·2874	·2857	·2840	·2823	·2807	·2790	·2773	2	3	5	7	8	10	12	13	15
74	0·2756	·2740	·2723	·2706	·2689	·2672	·2656	·2639	·2622	·2605	2	3	5	7	8	10	12	13	15
75	0·2588	·2571	·2554	·2538	·2521	·2504	·2487	·2470	·2453	·2436	2	3	5	7	8	10	12	14	15
76	0·2419	·2402	·2385	·2368	·2351	·2334	·2317	·2300	·2284	·2267	2	3	5	7	8	10	12	14	15
77	0·2250	·2233	·2215	·2198	·2181	·2164	·2147	·2130	·2113	·2096	2	3	5	7	9	10	12	13	15
78	0·2079	·2062	·2045	·2028	·2011	·1994	·1977	·1959	·1942	·1925	2	3	5	7	9	10	12	14	15
79	0·1908	·1891	·1874	·1857	·1840	·1822	·1805	·1788	·1771	·1754	2	3	5	7	9	10	12	14	15
80	0·1736	·1719	·1702	·1685	·1668	·1650	·1633	·1616	·1599	·1582	2	3	5	7	9	10	12	14	15
81	0·1564	·1547	·1530	·1513	·1495	·1478	·1461	·1444	·1426	·1409	2	3	5	7	9	10	12	14	16
82	0·1392	·1374	·1357	·1340	·1323	·1305	·1288	·1271	·1253	·1236	2	3	5	7	9	10	12	14	16
83	0·1219	·1201	·1184	·1167	·1149	·1132	·1115	·1097	·1080	·1063	2	3	5	7	9	10	12	14	16
84	0·1045	·1028	·1011	·0993	·0976	·0958	·0941	·0924	·0906	·0889	2	3	5	7	9	10	12	14	16
85	0·0872	·0854	·0837	·0819	·0802	·0785	·0767	·0750	·0732	·0715	2	3	5	7	9	10	12	14	16
86	0·0698	·0680	·0663	·0645	·0628	·0610	·0593	·0576	·0558	·0541	2	3	5	7	9	10	12	14	16
87	0·0523	·0506	·0488	·0471	·0454	·0436	·0419	·0401	·0384	·0366	2	3	5	7	9	10	12	14	16
88	0·0349	·0332	·0314	·0297	·0279	·0262	·0244	·0227	·0209	·0192	2	3	5	7	9	10	12	14	16
89	0·0175	·0157	·0140	·0122	·0105	·0087	·0070	·0052	·0035	·0017	2	3	5	7	9	10	12	14	16
x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9

Appendix IV: Tangents of Angles

Tangents of Angles (*x* in degrees)

$x \to \tan x$

x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9
											ADD Differences								
0	0·0000	·0017	·0035	·0052	·0070	·0087	·0105	·0122	·0140	·0157	2	3	5	7	9	10	12	14	16
1	0·0175	·0192	·0209	·0227	·0244	·0262	·0279	·0297	·0314	·0332	2	3	5	7	9	10	12	14	16
2	0·0349	·0367	·0384	·0402	·0419	·0437	·0454	·0472	·0489	·0507	2	3	5	7	9	10	12	14	16
3	0·0524	·0542	·0559	·0577	·0594	·0612	·0629	·0647	·0664	·0682	2	4	5	7	9	11	12	14	16
4	0·0699	·0717	·0734	·0752	·0769	·0787	·0805	·0822	·0840	·0857	2	4	5	7	9	11	12	14	16
5	0·0875	·0892	·0910	·0928	·0945	·0963	·0981	·0998	·1016	·1033	2	4	5	7	9	11	12	14	16
6	0·1051	·1069	·1086	·1104	·1122	·1139	·1157	·1175	·1192	·1210	2	4	5	7	9	11	12	14	16
7	0·1228	·1246	·1263	·1281	·1299	·1317	·1334	·1352	·1370	·1388	2	4	5	7	9	11	12	14	16
8	0·1405	·1423	·1441	·1459	·1477	·1495	·1512	·1530	·1548	·1566	2	4	5	7	9	11	12	14	16
9	0·1584	·1602	·1620	·1638	·1655	·1673	·1691	·1709	·1727	·1745	2	4	5	7	9	11	13	14	16
10	0·1763	·1781	·1799	·1817	·1835	·1853	·1871	·1890	·1908	·1926	2	4	5	7	9	11	13	14	16
11	0·1944	·1962	·1980	·1998	·2016	·2035	·2053	·2071	·2089	·2107	2	4	5	7	9	11	13	15	16
12	0·2126	·2144	·2162	·2180	·2199	·2217	·2235	·2254	·2272	·2290	2	4	5	7	9	11	13	15	16
13	0·2309	·2327	·2345	·2364	·2382	·2401	·2419	·2438	·2456	·2475	2	4	6	7	9	11	13	15	17
14	0·2493	·2512	·2530	·2549	·2568	·2586	·2605	·2623	·2642	·2661	2	4	6	7	9	11	13	15	17
15	0·2679	·2698	·2717	·2736	·2754	·2773	·2792	·2811	·2830	·2849	2	4	6	8	9	11	13	15	17
16	0·2867	·2886	·2905	·2924	·2943	·2962	·2981	·3000	·3019	·3038	2	4	6	8	9	11	13	15	17
17	0·3057	·3076	·3096	·3115	·3134	·3153	·3172	·3191	·3211	·3230	2	4	6	8	10	12	13	15	17
18	0·3249	·3269	·3288	·3307	·3327	·3346	·3365	·3385	·3404	·3424	2	4	6	8	10	12	14	16	17
19	0·3443	·3463	·3482	·3502	·3522	·3541	·3561	·3581	·3600	·3620	2	4	6	8	10	12	14	16	18
20	0·3640	·3659	·3679	·3699	·3719	·3739	·3759	·3779	·3799	·3819	2	4	6	8	10	12	14	16	18
21	0·3839	·3859	·3879	·3899	·3919	·3939	·3959	·3979	·4000	·4020	2	4	6	8	10	12	14	16	18
22	0·4040	·4061	·4061	·4101	·4122	·4142	·4163	·4183	·4204	·4224	2	4	6	8	10	12	14	16	18
23	0·4245	·4265	·4286	·4307	·4327	·4348	·4369	·4390	·4411	·4431	2	4	6	8	10	12	15	17	19
24	0·4452	·4473	·4494	·4515	·4536	·4557	·4578	·4599	·4621	·4642	2	4	6	8	11	13	15	17	19
25	0·4663	·4684	·4706	·4727	·4748	·4770	·4791	·4813	·4834	·4856	2	4	6	9	11	13	15	17	19
26	0·4877	·4899	·4921	·4942	·4964	·4986	·5008	·5029	·5051	·5073	2	4	7	9	11	13	15	17	20
27	0·5095	·5117	·5139	·5161	·5184	·5206	·5228	·5250	·5272	·5295	2	4	7	9	11	13	16	18	20
28	0·5317	·5340	·5362	·5384	·5407	·5430	·5452	·5475	·5498	·5520	2	5	7	9	11	14	16	18	20
29	0·5543	·5566	·5589	·5612	·5635	·5658	·5681	·5704	·5727	·5750	2	5	7	9	12	14	16	18	21
30	0·5774	·5797	·5820	·5844	·5867	·5890	·5914	·5938	·5961	·5985	2	5	7	9	12	14	16	19	21
31	0·6009	·6032	·6056	·6080	·6104	·6128	·6152	·6176	·6200	·6224	2	5	7	10	12	14	17	19	22
32	0·6249	·6273	·6297	·6322	·6346	·6371	·6395	·6420	·6445	·6469	2	5	7	10	12	15	17	20	22
33	0·6494	·6519	·6544	·6569	·6594	·6619	·6644	·6669	·6694	·6720	3	5	8	10	13	15	18	20	23
34	0·6745	·6771	·6796	·6822	·6847	·6873	·6899	·6924	·6950	·6976	3	5	8	10	13	15	18	21	23
35	0·7002	·7028	·7054	·7080	·7107	·7133	·7159	·7186	·7212	·7239	3	5	8	11	13	16	18	21	24
36	0·7265	·7292	·7319	·7346	·7373	·7400	·7427	·7454	·7481	·7508	3	5	8	11	14	16	19	22	24
37	0·7536	·7563	·7590	·7618	·7646	·7673	·7701	·7729	·7757	·7785	3	6	8	11	14	17	19	22	25
38	0·7813	·7841	·7869	·7898	·7926	·7954	·7983	·8012	·8040	·8069	3	6	9	11	14	17	20	23	26
39	0·8098	·8127	·8156	·8185	·8214	·8243	·8273	·8302	·8332	·8361	3	6	9	12	15	18	21	23	26
40	0·8391	·8421	·8451	·8481	·8511	·8541	·8571	·8601	·8632	·8662	3	6	9	12	15	18	21	24	27
41	0·8693	·8724	·8754	·8785	·8816	·8847	·8878	·8910	·8941	·8972	3	6	9	12	16	19	22	25	28
42	0·9004	·9036	·9067	·9099	·9131	·9163	·9195	·9228	·9260	·9293	3	6	10	13	16	19	22	26	29
43	0·9325	·9358	·9391	·9424	·9457	·9490	·9523	·9556	·9590	·9623	3	7	10	13	17	20	23	27	30
44	0·9657	·9091	·9725	·9759	·9793	·9827	·9861	·9896	·9930	·9965	3	7	10	14	17	21	24	27	31
x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9

x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9
											ADD Differences								
45	1·000	1·003	1·007	1·011	1·014	1·018	1·021	1·025	1·028	1·032	0	1	1	1	2	2	2	3	3
46	1·036	1·039	1·043	1·046	1·050	1·054	1·057	1·061	1·065	1·069	0	1	1	1	2	2	3	3	3
47	1·072	1·076	1·080	1·084	1·087	1·091	1·095	1·099	1·103	1·107	0	1	1	2	2	2	3	3	3
48	1·111	1·115	1·118	1·122	1·126	1·130	1·134	1·138	1·142	1·146	0	1	1	2	2	2	3	3	4
49	1·150	1·154	1·159	1·163	1·167	1·171	1·175	1·179	1·183	1·188	0	1	1	2	2	2	3	3	4
50	1·192	1·196	1·200	1·205	1·209	1·213	1·217	1·222	1·226	1·230	0	1	1	2	2	3	3	4	4
51	1·235	1·239	1·244	1·248	1·253	1·257	1·262	1·266	1·271	1·275	0	1	1	2	2	3	3	4	4
52	1·280	1·285	1·289	1·294	1·299	1·303	1·308	1·313	1·317	1·322	0	1	1	2	2	3	3	4	4
53	1·327	1·332	1·337	1·342	1·347	1·351	1·356	1·361	1·366	1·371	0	1	1	2	2	3	3	4	4
54	1·376	1·381	1·387	1·392	1·397	1·402	1·407	1·412	1·418	1·423	1	1	2	2	3	3	4	4	5
55	1·428	1·433	1·439	1·444	1·450	1·455	1·460	1·466	1·471	1·477	1	1	2	2	3	3	4	4	5
56	1·483	1·488	1·494	1·499	1·505	1·511	1·517	1·522	1·528	1·534	1	1	2	2	3	3	4	5	5
57	1·540	1·546	1·552	1·558	1·564	1·570	1·576	1·582	1·588	1·594	1	1	2	2	3	4	4	5	5
58	1·600	1·607	1·613	1·619	1·525	1·632	1·638	1·645	1·651	1·658	1	1	2	3	3	4	4	5	6
59	1·664	1·671	1·678	1·684	1·691	1·698	1·704	1·711	1·718	1·725	1	1	2	3	3	4	5	5	6
60	1·732	1·739	1·746	1·753	1·760	1·767	1·775	1·782	1·789	1·797	1	1	2	3	4	4	5	6	6
61	1·804	1·811	1·819	1·827	1·834	1·842	1·849	1·857	1·865	1·873	1	2	2	3	4	5	5	6	7
62	1·881	1·889	1·897	1·905	1·913	1·921	1·929	1·937	1·946	1·954	1	2	2	3	4	5	6	7	7
63	1·963	1·971	1·980	1·988	1·997	2·006	2·014	2·023	2·032	2·041	1	2	3	4	4	5	6	7	8
64	2·050	2·059	2·069	2·078	2·087	2·097	2·106	2·116	2·125	2·135	1	2	3	4	5	6	7	8	8
65	2·145	2·154	2·164	2·174	2·184	2·194	2·204	2·215	2·225	2·236	1	2	3	4	5	6	7	8	9
66	2·246	2·257	2·267	2·278	2·289	2·300	2·311	2·322	2·333	2·344	1	2	3	4	5	7	8	9	10
67	2·356	2·367	2·379	2·391	2·402	2·414	2·426	2·438	2·450	2·463	1	2	4	5	6	7	8	10	11
68	2·475	2·488	2·500	2·513	2·526	2·539	2·552	2·565	2·578	2·592	1	3	4	5	7	8	9	10	12
69	2·605	2·619	2·633	2·646	2·660	2·675	2·689	2·703	2·718	2·733	1	3	4	6	7	9	10	11	13
70	2·747	2·762	2·778	2·793	2·808	2·824	2·840	2·856	2·872	2·888	2	3	5	6	8	9	11	13	14
71	2·904	2·921	2·937	2·954	2·971	2·989	3·006	3·024	3·042	3·060	2	3	5	7	9	10	12	14	16
72	3·078	3·096	3·115	3·133	3·152	3·172	3·191	3·211	3·230	3·251	2	4	6	8	10	12	14	15	17
73	3·271	3·291	3·312	3·333	3·354	3·376	3·398	3·420	3·442	3·465	2	4	6	9	11	13	15	17	19
74	3·487	3·511	3·534	3·558	3·582	3·606	3·630	3·655	3·681	3·706	2	5	7	10	12	15	17	20	22
75	3·732	3·758	3·785	3·812	3·839	3·867	3·895	3·923	3·952	3·981	3	6	8	11	14	17	20	22	25
76	4·011	4·041	4·071	4·102	4·134	4·165	4·198	4·230	4·264	4·297	3	6	10	13	16	19	22	26	29
77	4·331	4·366	4·402	4·437	4·474	4·511	4·548	4·586	4·625	4·665									
78	4·705	4·745	4·787	4·829	4·872	4·915	4·959	5·005	5·050	5·097									
79	5·145	5·193	5·242	5·292	5·343	5·396	5·449	5·503	5·558	5·614									
80	5·671	5·730	5·789	5·850	5·912	5·976	6·041	6·107	6·174	6·243	differences unreliable:								
81	6·314	6·386	6·460	6·535	6·612	6·691	6·772	6·855	6·940	7·026									
82	7·115	7·207	7·300	7·396	7·495	7·596	7·700	7·806	7·916	8·028									
83	8·144	8·264	8·386	8·513	8·643	8·777	8·915	9·058	9·205	9·357									
84	9·514	9·677	9·845	10·02	10·20	10·39	10·58	10·78	10·99	11·20									
85	11·43	11·66	11·91	12·16	12·43	12·71	13·00	13·30	13·62	13·95									
86	14·30	14·67	15·06	15·46	15·89	16·35	16·83	17·34	17·89	18·46									
87	19·08	19·74	20·45	21·20	22·02	22·90	23·86	24·90	26·03	27·27									
88	28·64	30·14	31·82	33·69	35·80	38·19	40·92	44·07	47·74	52·08									
89	57·29	63·66	71·62	81·85	95·46	114·6	143·2	191·0	286·5	573·0									
x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9

Appendix V: Square Roots of Numbers, 1-10

Square Roots of Numbers, 1-10

$x \rightarrow \sqrt{x}$

x	0	1	2	3	4	5	6	7	8	9	Differences								
											1	2	3	4	5	6	7	8	9
1.0	1.000	1.005	1.010	1.015	1.020	1.025	1.030	1.034	1.039	1.044	0	1	1	2	2	3	3	4	4
1.1	1.049	1.054	1.058	1.063	1.068	1.072	1.077	1.082	1.086	1.091	0	1	1	2	2	3	3	4	4
1.2	1.095	1.100	1.105	1.109	1.114	1.118	1.123	1.127	1.131	1.136	0	1	1	2	2	3	3	4	4
1.3	1.140	1.145	1.149	1.153	1.158	1.162	1.166	1.171	1.175	1.179	0	1	1	2	2	3	3	3	4
1.4	1.183	1.187	1.192	1.196	1.200	1.204	1.208	1.212	1.217	1.221	0	1	1	2	2	3	3	3	4
1.5	1.225	1.229	1.233	1.237	1.241	1.245	1.249	1.253	1.257	1.261	0	1	1	2	2	2	3	3	4
1.6	1.265	1.269	1.273	1.277	1.281	1.285	1.288	1.292	1.296	1.300	0	1	1	2	2	2	3	3	3
1.7	1.304	1.308	1.312	1.315	1.319	1.323	1.327	1.330	1.334	1.338	0	1	1	2	2	2	3	3	3
1.8	1.342	1.345	1.349	1.353	1.357	1.360	1.364	1.368	1.371	1.375	0	1	1	1	2	2	3	3	3
1.9	1.378	1.382	1.386	1.389	1.393	1.396	1.400	1.404	1.407	1.411	0	1	1	1	2	2	3	3	3
2.0	1.414	1.418	1.421	1.425	1.428	1.432	1.435	1.439	1.442	1.446	0	1	1	1	2	2	2	3	3
2.1	1.449	1.453	1.456	1.460	1.463	1.466	1.470	1.473	1.477	1.480	0	1	1	1	2	2	2	3	3
2.2	1.483	1.487	1.490	1.493	1.497	1.500	1.503	1.507	1.510	1.513	0	1	1	1	2	2	2	3	3
2.3	1.517	1.520	1.523	1.526	1.530	1.533	1.536	1.539	1.543	1.546	0	1	1	1	2	2	2	3	3
2.4	1.549	1.552	1.556	1.559	1.562	1.565	1.568	1.572	1.575	1.578	0	1	1	1	2	2	2	3	3
2.5	1.581	1.584	1.587	1.591	1.594	1.597	1.600	1.603	1.606	1.609	0	1	1	1	2	2	2	3	3
2.6	1.612	1.616	1.619	1.622	1.625	1.628	1.631	1.634	1.637	1.640	0	1	1	1	2	2	2	2	3
2.7	1.643	1.646	1.649	1.652	1.655	1.658	1.661	1.664	1.667	1.670	0	1	1	1	2	2	2	2	3
2.8	1.673	1.676	1.679	1.682	1.685	1.688	1.691	1.694	1.697	1.700	0	1	1	1	1	2	2	2	3
2.9	1.703	1.706	1.709	1.712	1.715	1.718	1.720	1.723	1.726	1.729	0	1	1	1	1	2	2	2	3
3.0	1.732	1.735	1.738	1.741	1.744	1.746	1.749	1.752	1.755	1.758	0	1	1	1	1	2	2	2	3
3.1	1.761	1.764	1.766	1.769	1.772	1.775	1.778	1.780	1.783	1.786	0	1	1	1	1	2	2	2	3
3.2	1.789	1.792	1.794	1.797	1.800	1.803	1.806	1.808	1.811	1.814	0	1	1	1	1	2	2	2	2
3.3	1.817	1.819	1.822	1.825	1.828	1.830	1.833	1.836	1.839	1.841	0	1	1	1	1	2	2	2	2
3.4	1.844	1.847	1.849	1.852	1.855	1.857	1.860	1.863	1.866	1.868	0	1	1	1	1	2	2	2	2
3.5	1.871	1.874	1.876	1.879	1.882	1.884	1.887	1.889	1.892	1.895	0	1	1	1	1	2	2	2	2
3.6	1.897	1.900	1.903	1.905	1.908	1.911	1.913	1.916	1.918	1.921	0	1	1	1	1	2	2	2	2
3.7	1.924	1.926	1.929	1.931	1.934	1.937	1.939	1.942	1.944	1.947	0	1	1	1	1	2	2	2	2
3.8	1.949	1.952	1.955	1.957	1.960	1.962	1.965	1.967	1.970	1.972	0	1	1	1	1	2	2	2	2
3.9	1.975	1.977	1.980	1.982	1.985	1.988	1.990	1.993	1.995	1.998	0	1	1	1	1	2	2	2	2
4.0	2.000	2.003	2.005	2.008	2.010	2.013	2.015	2.017	2.020	2.022	0	0	1	1	1	1	2	2	2
4.1	2.025	2.027	2.030	2.032	2.035	2.037	2.040	2.042	2.045	2.047	0	0	1	1	1	1	2	2	2
4.2	2.049	2.052	2.054	2.057	2.059	2.062	2.064	2.066	2.069	2.071	0	0	1	1	1	1	2	2	2
4.3	2.074	2.076	2.078	2.081	2.083	2.086	2.088	2.091	2.093	2.095	0	0	1	1	1	1	2	2	2
4.4	2.098	2.100	2.102	2.105	2.107	2.110	2.112	2.114	2.117	2.119	0	0	1	1	1	1	2	2	2
4.5	2.121	2.124	2.126	2.128	2.131	2.133	2.135	2.138	2.140	2.142	0	0	1	1	1	1	2	2	2
4.6	2.145	2.147	2.149	2.152	2.154	2.156	2.159	2.161	2.163	2.166	0	0	1	1	1	1	2	2	2
4.7	2.168	2.170	2.173	2.175	2.177	2.179	2.182	2.184	2.186	2.189	0	0	1	1	1	1	2	2	2
4.8	2.191	2.193	2.195	2.198	2.200	2.202	2.205	2.207	2.209	2.211	0	0	1	1	1	1	2	2	2
4.9	2.214	2.216	2.218	2.220	2.223	2.225	2.227	2.229	2.232	2.234	0	0	1	1	1	1	2	2	2
5.0	2.236	2.238	2.241	2.243	2.245	2.247	2.249	2.252	2.254	2.256	0	0	1	1	1	1	2	2	2
5.1	2.258	2.261	2.263	2.265	2.267	2.269	2.272	2.274	2.276	2.278	0	0	1	1	1	1	2	2	2
5.2	2.280	2.283	2.285	2.287	2.289	2.291	2.294	2.296	2.298	2.300	0	0	1	1	1	1	2	2	2
5.3	2.302	2.304	2.307	2.309	2.311	2.313	2.315	2.317	2.320	2.322	0	0	1	1	1	1	2	2	2
5.4	2.324	2.326	2.328	2.330	2.332	2.335	2.337	2.339	2.341	2.343	0	0	1	1	1	1	1	2	2
x	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9

x	0	1	2	3	4	5	6	7	8	9	Differences								
											1	2	3	4	5	6	7	8	9
5.5	2.345	2.347	2.350	2.352	2.354	2.356	2.358	2.360	2.362	2.364	0	0	1	1	1	1	1	2	2
5.6	2.366	2.369	2.371	2.373	2.375	2.377	2.379	2.381	2.383	2.385	0	0	1	1	1	1	1	2	2
5.7	2.388	2.390	2.392	2.394	2.396	2.398	2.400	2.402	2.404	2.406	0	0	1	1	1	1	1	2	2
5.8	2.408	2.410	2.412	2.415	2.417	2.419	2.421	2.423	2.425	2.427	0	0	1	1	1	1	1	2	2
5.9	2.429	2.431	2.433	2.435	2.437	2.439	2.441	2.443	2.445	2.447	0	0	1	1	1	1	1	2	2
6.0	2.450	2.452	2.454	2.456	2.458	2.460	2.462	2.464	2.466	2.468	0	0	1	1	1	1	1	2	2
6.1	2.470	2.472	2.474	2.476	2.478	2.480	2.482	2.484	2.486	2.488	0	0	1	1	1	1	1	2	2
6.2	2.490	2.492	2.494	2.496	2.498	2.500	2.502	2.504	2.506	2.508	0	0	1	1	1	1	1	2	2
6.3	2.510	2.512	2.514	2.516	2.518	2.520	2.522	2.524	2.526	2.528	0	0	1	1	1	1	1	2	2
6.4	2.530	2.532	2.534	2.536	2.538	2.540	2.542	2.544	2.546	2.548	0	0	1	1	1	1	1	2	2
6.5	2.550	2.551	2.553	2.555	2.557	2.559	2.561	2.563	2.565	2.567	0	0	1	1	1	1	1	2	2
6.6	2.569	2.571	2.573	2.575	2.577	2.579	2.581	2.583	2.585	2.587	0	0	1	1	1	1	1	2	2
6.7	2.588	2.590	2.592	2.594	2.596	2.598	2.600	2.602	2.604	2.606	0	0	1	1	1	1	1	2	2
6.8	2.608	2.610	2.612	2.613	2.615	2.617	2.619	2.621	2.623	2.625	0	0	1	1	1	1	1	2	2
6.9	2.627	2.629	2.631	2.632	2.634	2.636	2.638	2.640	2.642	2.644	0	0	1	1	1	1	1	2	2
7.0	2.646	2.648	2.650	2.651	2.653	2.655	2.657	2.659	2.661	2.663	0	0	1	1	1	1	1	2	2
7.1	2.665	2.667	2.668	2.670	2.672	2.674	2.676	2.678	2.680	2.681	0	0	1	1	1	1	1	1	2
7.2	2.683	2.685	2.687	2.689	2.691	2.693	2.694	2.696	2.698	2.700	0	0	1	1	1	1	1	1	2
7.3	2.702	2.704	2.706	2.707	2.709	2.711	2.713	2.715	2.717	2.719	0	0	1	1	1	1	1	1	2
7.4	2.720	2.722	2.724	2.726	2.728	2.729	2.731	2.733	2.735	2.737	0	0	1	1	1	1	1	1	2
7.5	2.739	2.740	2.742	2.744	2.746	2.748	2.750	2.751	2.753	2.755	0	0	1	1	1	1	1	1	2
7.6	2.757	2.759	2.760	2.762	2.764	2.766	2.768	2.769	2.771	2.773	0	0	1	1	1	1	1	1	2
7.7	2.775	2.777	2.778	2.780	2.782	2.784	2.786	2.787	2.789	2.791	0	0	1	1	1	1	1	1	2
7.8	2.793	2.795	2.796	2.798	2.800	2.802	2.804	2.805	2.807	2.809	0	0	1	1	1	1	1	1	2
7.9	2.811	2.812	2.814	2.816	2.818	2.820	2.821	2.823	2.825	2.827	0	0	1	1	1	1	1	1	2
8.0	2.828	2.830	2.832	2.834	2.835	2.837	2.839	2.841	2.843	2.844	0	0	1	1	1	1	1	1	2
8.1	2.846	2.848	2.850	2.851	2.853	2.855	2.857	2.858	2.860	2.862	0	0	1	1	1	1	1	1	2
8.2	2.864	2.865	2.867	2.869	2.871	2.872	2.874	2.876	2.877	2.879	0	0	1	1	1	1	1	1	2
8.3	2.881	2.883	2.884	2.886	2.888	2.890	2.891	2.893	2.895	2.897	0	0	1	1	1	1	1	1	2
8.4	2.898	2.900	2.902	2.903	2.905	2.907	2.909	2.910	2.912	2.914	0	0	1	1	1	1	1	1	2
8.5	2.915	2.917	2.919	2.921	2.922	2.924	2.926	2.927	2.929	2.931	0	0	1	1	1	1	1	1	2
8.6	2.933	2.934	2.936	2.938	2.939	2.941	2.943	2.944	2.946	2.948	0	0	1	1	1	1	1	1	2
8.7	2.950	2.951	2.953	2.955	2.956	2.958	2.960	2.961	2.963	2.965	0	0	1	1	1	1	1	1	2
8.8	2.966	2.968	2.970	2.972	2.973	2.975	2.977	2.978	2.980	2.982	0	0	1	1	1	1	1	1	2
8.9	2.983	2.985	2.987	2.988	2.990	2.992	2.993	2.995	2.997	2.998	0	0	1	1	1	1	1	1	2
9.0	3.000	3.002	3.003	3.005	3.007	3.008	3.010	3.012	3.013	3.015	0	0	0	1	1	1	1	1	1
9.1	3.017	3.018	3.020	3.022	3.023	3.025	3.027	3.028	3.030	3.032	0	0	0	1	1	1	1	1	1
9.2	3.033	3.035	3.036	3.038	3.040	3.041	3.043	3.045	3.046	3.048	0	0	0	1	1	1	1	1	1
9.3	3.050	3.051	3.053	3.055	3.056	3.058	3.059	3.061	3.063	3.064	0	0	0	1	1	1	1	1	1
9.4	3.066	3.068	3.069	3.071	3.072	3.074	3.076	3.077	3.079	3.081	0	0	0	1	1	1	1	1	1
9.5	3.082	3.084	3.085	3.087	3.089	3.090	3.092	3.094	3.095	3.097	0	0	0	1	1	1	1	1	1
9.6	3.098	3.100	3.102	3.103	3.105	3.106	3.108	3.110	3.111	3.113	0	0	0	1	1	1	1	1	1
9.7	3.115	3.116	3.118	3.119	3.121	3.123	3.124	3.126	3.127	3.129	0	0	0	1	1	1	1	1	1
9.8	3.131	3.132	3.134	3.135	3.137	3.139	3.140	3.142	3.143	2.145	0	0	0	1	1	1	1	1	1
9.9	3.146	3.148	3.150	3.151	3.153	3.154	3.156	3.158	3.159	3.161	0	0	0	1	1	1	1	1	1
x	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9

Appendix VI: Square Roots of Numbers, 10-100

Square Roots of Numbers 10-100

$x \rightarrow \sqrt{x}$

											Differences								
x	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	3·162	3·178	3·194	3·209	3·225	3·240	3·256	3·271	3·286	3·302	2	3	5	6	8	9	11	12	14
11	3·317	3·332	3·347	3·362	3·376	3·391	3·406	3·421	3·435	3·450	1	3	4	6	7	9	10	12	13
12	3·464	3·479	3·493	3·507	3·521	3·536	3·550	3·564	3·578	3·592	1	3	4	6	7	8	10	11	13
13	3·606	3·619	3·633	3·647	3·661	3·674	3·688	3·701	3·715	3·728	1	3	4	5	7	8	10	11	12
14	3·742	3·755	3·768	3·782	3·795	3·808	3·821	3·834	3·847	3·860	1	3	4	5	7	8	9	11	12
15	3·873	3·886	3·899	3·912	3·924	3·937	3·950	3·962	3·975	3·988	1	3	4	5	6	8	9	10	11
16	4·000	4·012	4·025	4·037	4·050	4·062	4·074	4·087	4·099	4·111	1	2	4	5	6	7	9	10	11
17	4·123	4·135	4·147	4·159	4·171	4·183	4·195	4·207	4·219	4·231	1	2	4	5	6	7	8	10	11
18	4·243	4·254	4·266	4·278	4·290	4·301	4·313	4·324	4·336	4·347	1	2	3	5	6	7	8	9	10
19	4·359	4·370	4·382	4·393	4·405	4·416	4·427	4·438	4·450	4·461	1	2	3	5	6	7	8	9	10
20	4·472	4·483	4·494	4·506	4·517	4·528	4·539	4·550	4·561	4·572	1	2	3	4	6	7	8	9	10
21	4·583	4·594	4·604	4·615	4·626	4·637	4·648	4·658	4·669	4·680	1	2	3	4	5	6	8	9	10
22	4·690	4·701	4·712	4·722	4·733	4·743	4·754	4·765	4·775	4·785	1	2	3	4	5	6	7	8	9
23	4·796	4·806	4·817	4·827	4·837	4·848	4·858	4·868	4·879	4·889	1	2	3	4	5	6	7	8	9
24	4·899	4·909	4·919	4·930	4·940	4·950	4·960	4·970	4·980	4·990	1	2	3	4	5	6	7	8	9
25	5·000	5·010	5·020	5·030	5·040	5·050	5·060	5·070	5·079	5·089	1	2	3	4	5	6	7	8	9
26	5·099	5·109	5·119	5·128	5·138	5·148	5·158	5·167	5·177	5·187	1	2	3	4	5	6	7	8	9
27	5·196	5·206	5·215	5·225	5·235	5·244	5·254	5·263	5·273	5·282	1	2	3	4	5	6	7	8	9
28	5·292	5·301	5·310	5·320	5·329	5·339	5·348	5·357	5·367	5·376	1	2	3	4	5	6	7	7	8
29	5·385	5·394	5·404	5·413	5·422	5·431	5·441	5·450	5·459	5·468	1	2	3	4	5	5	6	7	8
30	5·477	5·486	5·495	5·505	5·514	5·523	5·532	5·541	5·550	5·559	1	2	3	4	4	5	6	7	8
31	5·568	5·577	5·586	5·596	5·604	5·612	5·621	5·630	5·639	5·648	1	2	3	3	4	5	6	7	8
32	5·657	5·666	5·675	5·683	5·692	5·701	5·710	5·718	5·727	5·736	1	2	3	3	4	5	6	7	8
33	5·745	5·753	5·762	5·771	5·779	5·788	5·797	5·805	5·814	5·822	1	2	3	3	4	5	6	7	8
34	5·831	5·840	5·848	5·857	5·865	5·874	5·882	5·891	5·899	5·908	1	2	3	3	4	5	6	7	8
35	5·916	5·925	5·933	5·941	5·950	5·958	5·967	5·975	5·983	5·992	1	2	2	3	4	5	6	7	8
36	6·000	6·008	6·017	6·025	6·033	6·042	6·050	6·058	6·066	6·075	1	2	2	3	4	5	6	7	7
37	6·083	6·091	6·099	6·107	6·116	6·124	6·132	6·140	6·148	6·156	1	2	2	3	4	5	6	7	7
38	6·164	6·173	6·181	6·189	6·197	6·205	6·213	6·221	6·229	6·237	1	2	2	3	4	5	6	6	7
39	6·245	6·253	6·261	6·269	6·277	6·285	6·293	6·301	6·309	6·317	1	2	2	3	4	5	6	6	7
40	6·325	6·332	6·340	6·348	6·356	6·364	6·372	6·380	6·387	6·395	1	2	2	3	4	5	6	6	7
41	6·403	6·411	6·419	6·427	6·434	6·442	6·450	6·458	6·465	6·473	1	2	2	3	4	5	5	6	7
42	6·481	6·488	6·496	6·504	6·512	6·519	6·527	6·535	6·542	6·550	1	2	2	3	4	5	5	6	7
43	6·557	6·565	6·573	6·580	6·588	6·595	6·603	6·611	6·618	6·626	1	2	2	3	4	5	5	6	7
44	6·633	6·641	6·648	6·656	6·663	6·671	6·678	6·686	6·693	6·701	1	2	2	3	4	5	5	6	7
45	6·708	6·716	6·723	6·731	6·738	6·745	6·753	6·760	6·768	6·775	1	1	2	3	4	4	5	6	7
46	6·782	6·790	6·797	6·804	6·812	6·819	6·826	6·834	6·841	6·848	1	1	2	3	4	4	5	6	7
47	6·856	6·863	6·870	6·878	6·885	6·892	6·899	6·907	6·914	6·921	1	1	2	3	4	4	5	6	7
48	6·928	6·935	6·943	6·950	6·957	6·964	6·971	6·979	6·986	6·993	1	1	2	3	4	4	5	6	6
49	7·000	7·007	7·014	7·021	7·029	7·036	7·043	7·050	7·057	7·064	1	1	2	3	4	4	5	6	6
50	7·071	7·078	7·085	7·092	7·099	7·106	7·113	7·120	7·127	7·134	1	1	2	3	4	4	5	6	6
51	7·141	7·148	7·155	7·162	7·169	7·176	7·183	7·190	7·197	7·204	1	1	2	3	4	4	5	6	6
52	7·211	7·218	7·225	7·232	7·239	7·246	7·253	7·259	7·266	7·273	1	1	2	3	3	4	5	6	6
53	7·280	7·287	7·294	7·301	7·308	7·314	7·321	7·328	7·335	7·342	1	1	2	3	3	4	5	5	6
54	7·349	7·355	7·362	7·369	7·376	7·382	7·389	7·396	7·403	7·410	1	1	2	3	3	4	5	5	6
55	7·416	7·423	7·430	7·436	7·443	7·450	7·457	7·463	7·470	7·477	1	1	2	3	3	4	5	5	6
56	7·483	7·490	7·497	7·503	7·510	7·517	7·523	7·530	7·537	7·543	1	1	2	3	3	4	5	5	6
57	7·550	7·556	7·563	7·570	7·576	7·583	7·589	7·596	7·603	7·609	1	1	2	3	3	4	5	5	6
58	7·616	7·622	7·629	7·635	7·642	7·649	7·655	7·662	7·668	7·675	1	1	2	3	3	4	5	5	6
59	7·681	7·688	7·694	7·701	7·707	7·714	7·720	7·727	7·733	7·740	1	1	2	3	3	4	4	5	6
60	7·746	7·752	7·759	7·765	7·772	7·778	7·785	7·791	7·797	7·804	1	1	2	3	3	4	4	5	6
61	7·810	7·817	7·823	7·829	7·836	7·842	7·849	7·855	7·861	7·868	1	1	2	3	3	4	4	5	6
62	7·874	7·880	7·887	7·893	7·899	7·906	7·912	7·918	7·925	7·931	1	1	2	3	3	4	4	5	6
63	7·937	7·944	7·950	7·956	7·962	7·969	7·975	7·981	7·987	7·994	1	1	2	3	3	4	4	5	6
64	8·000	8·006	8·012	8·019	8·025	8·031	8·037	8·044	8·050	8·056	1	1	2	2	3	4	4	5	6
65	8·062	8·068	8·075	8·081	8·087	8·093	8·099	8·106	8·112	8·118	1	1	2	2	3	4	4	5	6
66	8·124	8·130	8·136	8·142	8·149	8·155	8·161	8·167	8·173	8·179	1	1	2	2	3	4	4	5	5
67	8·185	8·191	8·198	8·204	8·210	8·216	8·222	8·228	8·234	8·240	1	1	2	2	3	4	4	5	5
68	8·246	8·252	8·258	8·264	8·270	8·276	8·283	8·289	8·295	8·301	1	1	2	2	3	4	4	5	5
69	8·307	8·313	8·319	8·325	8·331	8·337	8·343	8·349	8·355	8·361	1	1	2	2	3	4	4	5	5
70	8·367	8·373	8·379	8·385	8·390	8·396	8·402	8·408	8·414	8·420	1	1	2	2	3	4	4	5	5
71	8·426	8·432	8·438	8·444	8·450	8·456	8·462	8·468	8·473	8·479	1	1	2	2	3	4	4	5	5
72	8·485	8·491	8·497	8·503	8·509	8·515	8·521	8·526	8·532	8·538	1	1	2	2	3	3	4	5	5
73	8·544	8·550	8·556	8·562	8·567	8·573	8·579	8·585	8·591	8·597	1	1	2	2	3	3	4	5	5
74	8·602	8·608	8·614	8·620	8·626	8·631	8·637	8·643	8·649	8·654	1	1	2	2	3	3	4	5	5
75	8·660	8·666	8·672	8·678	8·683	8·689	8·695	8·701	8·706	8·712	1	1	2	2	3	3	4	5	5
76	8·718	8·724	8·729	8·735	8·741	8·746	8·752	8·758	8·764	8·769	1	1	2	2	3	3	4	5	5
77	8·775	8·781	8·786	8·792	8·798	8·803	8·809	8·815	8·820	8·826	1	1	2	2	3	3	4	4	5
78	8·832	8·837	8·843	8·849	8·854	8·860	8·866	8·871	8·877	8·883	1	1	2	2	3	3	4	4	5
79	8·888	8·894	8·899	8·905	8·911	8·916	8·922	8·927	8·933	8·939	1	1	2	2	3	3	4	4	5
80	8·944	8·950	8·955	8·961	8·967	8·972	8·978	8·983	8·989	8·994	1	1	2	2	3	3	4	4	5
81	9·000	9·006	9·011	9·017	9·022	9·028	9·033	9·039	9·044	9·050	1	1	2	2	3	3	4	4	5
82	9·055	9·061	9·066	9·072	9·077	9·083	9·088	9·094	9·099	9·105	1	1	2	2	3	3	4	4	5
83	9·110	9·116	9·121	9·127	9·132	9·138	9·143	9·149	9·154	9·160	1	1	2	2	3	3	4	4	5
84	9·165	9·171	9·176	9·182	9·187	9·192	9·198	9·203	9·209	9·214	1	1	2	2	3	3	4	4	5
85	9·220	9·225	9·230	9·236	9·241	9·247	9·252	9·257	9·263	9·268	1	1	2	2	3	3	4	4	5
86	9·274	9·279	9·284	9·290	9·295	9·301	9·306	9·311	9·317	9·322	1	1	2	2	3	3	4	4	5
87	9·237	9·333	9·338	9·343	9·349	9·354	9·359	9·365	9·370	9·375	1	1	2	2	3	3	4	4	5
88	9·381	9·386	9·391	9·397	9·402	9·407	9·413	9·418	9·423	9·429	1	1	2	2	3	3	4	4	5
89	9·434	9·439	9·445	9·450	9·455	9·460	9·466	9·471	9·476	9·482	1	1	2	2	3	3	4	4	5
90	9·487	9·492	9·497	9·503	9·508	9·513	9·518	9·524	9·529	9·534	1	1	2	2	3	3	4	4	5
91	9·539	9·545	9·550	9·555	9·560	9·566	9·571	9·576	9·581	9·586	1	1	2	2	3	3	4	4	5
92	9·592	9·597	9·602	9·607	9·613	9·618	9·623	9·628	9·633	9·638	1	1	2	2	3	3	4	4	5
93	9·644	9·649	9·654	9·659	9·664	9·670	9·675	9·680	9·685	9·690	1	1	2	2	3	3	4	4	5
94	9·695	9·701	9·706	9·711	9·716	9·721	9·726	9·731	9·737	9·742	1	1	2	2	3	3	4	4	5
95	9·747	9·752	9·757	9·762	9·767	9·772	9·778	9·783	9·788	9·793	1	1	2	2	3	3	4	4	5
96	9·798	9·803	9·808	9·813	9·818	9·823	9·829	9·834	9·839	9·844	1	1	2	2	3	3	4	4	5
97	9·849	9·854	9·859	9·864	9·869	9·874	9·879	9·884	9·889	9·894	1	1	2	2	3	3	4	4	5
98	9·900	9·905	9·910	9·915	9·920	9·925	9·930	9·935	9·940	9·945	0	1	1	2	2	3	3	4	4
99	9·950	9·955	9·960	9·965	9·970	9·975	9·980	9·985	9·990	9·996	0	1	1	2	2	3	3	4	4
x	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9

Appendix VII: Reciprocals of Numbers

Reciprocals of Numbers

$x \rightarrow 1/x$

x	0	1	2	3	4	5	6	7	8	9	SUBTRACT Differences 1	2	3	4	5	6	7	8	9
10	1000	9901	9804	9709	9615	9524	9434	9346	9259	9174	9	18	28	37	46	55	64	74	83
11	9091	9009	8929	8850	8772	8696	8621	8547	8475	8403	8	15	23	31	38	46	53	61	69
12	8333	8264	8197	8130	8065	8000	7937	7874	7813	7752	7	13	20	26	33	39	46	52	59
13	7692	7634	7576	7519	7463	7407	7353	7299	7246	7194	6	11	17	22	28	33	39	44	50
14	7143	7092	7042	6993	6944	6897	6849	6803	6757	6711	5	10	14	19	24	29	33	38	43
15	6667	6623	6579	6536	6494	6452	6410	6369	6329	6289	4	8	13	17	21	25	29	33	38
16	6250	6211	6173	6135	6098	6061	6024	5988	5952	5917	4	7	11	15	18	22	26	29	33
17	5882	5848	5814	5780	5747	5714	5682	5650	5618	5587	3	7	10	13	16	20	23	26	29
18	5556	5525	5495	5464	5435	5405	5376	5348	5319	5291	3	6	9	12	15	17	20	23	26
19	5263	5236	5208	5181	5155	5128	5102	5076	5051	5025	3	5	8	11	13	16	18	21	24
20	5000	4975	4950	4926	4902	4878	4854	4831	4808	4785	2	5	7	10	12	14	17	19	21
21	4762	4739	4717	4695	4673	4651	4630	4608	4587	4566	2	4	7	9	11	13	15	17	19
22	4545	4525	4505	4484	4464	4444	4425	4405	4386	4367	2	4	6	8	10	12	14	16	18
23	4348	4329	4310	4292	4274	4255	4237	4219	4202	4184	2	4	5	7	9	11	13	14	16
24	4167	4149	4132	4115	4098	4082	4065	4049	4032	4016	2	3	5	7	8	10	12	13	15
25	4000	3981	3968	3953	3937	3922	3906	3891	3876	3861	2	3	5	6	8	9	11	12	14
26	3846	3831	3817	3802	3788	3774	3759	3745	3731	3717	1	3	4	6	7	9	10	11	13
27	3704	3690	3676	3663	3650	3636	3623	3610	3597	3584	1	3	4	5	7	8	9	11	12
28	3571	3559	3546	3534	3521	3509	3497	3484	3472	3460	1	2	4	5	6	7	9	10	11
29	3448	3436	3425	3413	3401	3390	3378	3367	3356	3344	1	2	3	5	6	7	8	9	10
30	3333	3322	3311	3300	3289	3279	3268	3257	3247	3236	1	2	3	4	5	6	8	9	10
31	3226	3215	3205	3195	3185	3175	3165	3155	3145	3135	1	2	3	4	5	6	7	8	9
32	3125	3115	3106	3096	3086	3077	3067	3058	3049	3040	1	2	3	4	5	6	7	8	9
33	3030	3021	3012	3003	2994	2985	2976	2967	2959	2950	1	2	3	4	4	5	6	7	8
34	2941	2933	2924	2915	2907	2899	2890	2882	2874	2865	1	2	3	3	4	5	6	7	8
35	2857	2849	2841	2833	2825	2817	2809	2801	2793	2786	1	2	2	3	4	5	6	6	7
36	2778	2770	2762	2755	2747	2740	2732	2725	2717	2710	1	2	2	3	4	5	5	6	7
37	2703	2695	2688	2681	2674	2667	2660	2653	2646	2639	1	1	2	3	4	4	5	6	6
38	2632	2625	2618	2611	2604	2597	2591	2584	2577	2571	1	1	2	3	3	4	5	5	6
39	2564	2558	2551	2545	2538	2532	2525	2519	2513	2506	1	1	2	3	3	4	4	5	6
40	2500	2494	2488	2481	2475	2469	2463	2457	2451	2445	1	1	2	2	3	4	4	5	5
41	2439	2433	2427	2421	2415	2410	2404	2398	2392	2387	1	1	2	2	3	3	4	5	5
42	2381	2375	2370	2364	2358	2353	2347	2342	2336	2331	1	1	2	2	3	3	4	4	5
43	2326	2320	2315	2309	2304	2299	2294	2288	2283	2278	1	1	2	2	3	3	4	4	5
44	2273	2268	2262	2257	2252	2247	2242	2237	2232	2227	1	1	2	2	3	3	4	4	5
45	2222	2217	2212	2208	2203	2198	2193	2188	2183	2179	0	1	1	2	2	3	3	4	4
46	2174	2169	2165	2160	2155	2151	2146	2141	2137	2132	0	1	1	2	2	3	3	4	4
47	2128	2123	2119	2114	2110	2105	2101	2096	2092	2088	0	1	1	2	2	3	3	4	4
48	2083	2079	2075	2070	2066	2062	2058	2053	2049	2045	0	1	1	2	2	3	3	3	4
49	2041	2037	2033	2028	2024	2020	2016	2012	2008	2004	0	1	1	2	2	2	3	3	4
50	2000	1996	1992	1988	1984	1980	1976	1972	1969	1965	0	1	1	2	2	2	3	3	4
51	1961	1957	1953	1949	1946	1941	1938	1934	1931	1927	0	1	1	2	2	2	3	3	3
52	1923	1919	1916	1912	1908	1905	1901	1898	1894	1890	0	1	1	1	2	2	3	3	3
53	1887	1883	1880	1876	1873	1869	1866	1862	1859	1855	0	1	1	1	2	2	2	3	3
54	1852	1848	1845	1842	1838	1835	1832	1828	1825	1821	0	1	1	1	2	2	2	3	3
x	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9

x	0	1	2	3	4	5	6	7	8	9	SUBTRACT Differences 1	2	3	4	5	6	7	8	9
55	1818	1815	1812	1808	1805	1802	1799	1795	1792	1789	0	1	1	1	2	2	2	3	3
56	1786	1783	1779	1776	1773	1770	1767	1764	1761	1757	0	1	1	1	2	2	2	3	3
57	1754	1751	1748	1745	1742	1739	1736	1733	1730	1727	0	1	1	1	1	2	2	2	3
58	1724	1721	1718	1715	1712	1709	1706	1704	1701	1696	0	1	1	1	1	2	2	2	3
59	1695	1692	1689	1686	1684	1681	1678	1675	1672	1669	0	1	1	1	1	2	2	2	3
60	1667	1664	1661	1658	1656	1653	1650	1647	1645	1642	0	1	1	1	1	2	2	2	3
61	1639	1637	1634	1631	1629	1626	1623	1621	1618	1616	0	1	1	1	1	2	2	2	2
62	1613	1610	1608	1605	1603	1600	1597	1595	1592	1590	0	1	1	1	1	2	2	2	2
63	1587	1585	1582	1580	1577	1575	1572	1570	1567	1565	0	0	1	1	1	1	2	2	2
64	1562	1560	1558	1555	1553	1550	1548	1546	1543	1541	0	0	1	1	1	1	2	2	2
65	1538	1536	1534	1531	1529	1527	1524	1522	1520	1517	0	0	1	1	1	1	2	2	2
66	1515	1513	1511	1508	1506	1504	1502	1499	1497	1495	0	0	1	1	1	1	2	2	2
67	0·1493	1490	1488	1486	1484	1481	1479	1477	1475	1473	0	0	1	1	1	1	2	2	2
68	1471	1468	1466	1464	1462	1460	1458	1456	1453	1451	0	0	1	1	1	1	2	2	2
69	1449	1447	1445	1443	1441	1439	1437	1435	1433	1431	0	0	1	1	1	1	2	2	2
70	1429	1427	1425	1422	1420	1418	1416	1414	1412	1410	0	0	1	1	1	1	1	2	2
71	1408	1406	1404	1403	1401	1399	1397	1395	1393	1391	0	0	1	1	1	1	1	2	2
72	1389	1387	1385	1383	1381	1379	1377	1376	1374	1372	0	0	1	1	1	1	1	2	2
73	1370	1368	1366	1364	1362	1361	1359	1357	1355	1353	0	0	1	1	1	1	1	2	2
74	1351	1350	1348	1346	1344	1342	1340	1339	1337	1335	0	0	1	1	1	1	1	1	2
75	1333	1332	1330	1328	1326	1325	1323	1321	1319	1318	0	0	1	1	1	1	1	1	2
76	1316	1314	1312	1311	1309	1307	1305	1304	1302	1300	0	0	1	1	1	1	1	1	2
77	1299	1297	1295	1294	1292	1290	1289	1287	1285	1284	0	0	0	1	1	1	1	1	1
78	1282	1280	1279	1277	1276	1274	1272	1271	1269	1267	0	0	0	1	1	1	1	1	1
79	1266	1264	1263	1261	1259	1258	1256	1255	1253	1252	0	0	0	1	1	1	1	1	1
80	1250	1248	1247	1245	1244	1242	1241	1239	1238	1236	0	0	0	1	1	1	1	1	1
81	1235	1233	1232	1230	1229	1227	1225	1224	1222	1221	0	0	0	1	1	1	1	1	1
82	1220	1218	1217	1215	1214	1212	1211	1209	1208	1206	0	0	0	1	1	1	1	1	1
83	1205	1203	1202	1200	1199	1198	1196	1195	1193	1192	0	0	0	1	1	1	1	1	1
84	1190	1189	1188	1186	1185	1183	1182	1181	1179	1178	0	0	0	1	1	1	1	1	1
85	1176	1175	1174	1172	1171	1170	1168	1167	1166	1164	0	0	0	1	1	1	1	1	1
86	1163	1161	1160	1159	1157	1156	1155	1153	1152	1151	0	0	0	1	1	1	1	1	1
87	1149	1148	1147	1145	1144	1143	1142	1140	1139	1138	0	0	0	1	1	1	1	1	1
88	1136	1135	1134	1133	1131	1130	1129	1127	1126	1125	0	0	0	1	1	1	1	1	1
89	1124	1122	1121	1120	1119	1117	1116	1115	1114	1112	0	0	0	1	1	1	1	1	1
90	1111	1110	1109	1107	1106	1105	1104	1103	1101	1100	0	0	0	1	1	1	1	1	1
91	1099	1098	1096	1095	1094	1093	1092	1090	1089	1088	0	0	0	0	1	1	1	1	1
92	1087	1086	1085	1083	1082	1081	1080	1079	1078	1076	0	0	0	0	1	1	1	1	1
93	1075	1074	1073	1072	1071	1070	1068	1067	1066	1065	0	0	0	0	1	1	1	1	1
94	1064	1063	1062	1060	1059	1058	1057	1056	1055	1054	0	0	0	0	1	1	1	1	1
95	1053	1052	1050	1049	1048	1047	1046	1045	1044	1043	0	0	0	0	1	1	1	1	1
96	1042	1041	1039	1038	1037	1036	1035	1034	1033	1032	0	0	0	0	1	1	1	1	1
97	1031	1030	1029	1028	1027	1026	1025	1024	1022	1021	0	0	0	0	1	1	1	1	1
98	1020	1019	1018	1017	1016	1015	1014	1013	1012	1011	0	0	0	0	1	1	1	1	1
99	1010	1009	1008	1007	1006	1005	1004	1003	1002	1001	0	0	0	0	0	1	1	1	1
x	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9

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