Free Quality School Education

Ministry of Basic and Senior Secondary Education

Pupils' handbook for

JSS Mathematics

JSS 1

Term 2

STRICTLY NOT FOR SALE

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Please see final page for further information.

FOREWORD

The production of Teachers' Guides and Pupils' handbooks in respect of English and Mathematics for Junior Secondary Schools (JSSs) in Sierra Leone is an innovation. This would undoubtedly lead to improvement in the performance of pupils in the Basic Education Certificate Examination in these subjects. As Minister of Basic and Senior Secondary Education, I am pleased with this development in the educational sector.

The Teachers' Guides give teachers the support they need to utilize appropriate pedagogical skills to teach; and the Pupils' Handbooks are designed to support self-study by the pupils, and to give them additional opportunities to learn independently.

These Teachers' Guides and Pupils' Handbooks had been written by experienced Sierra Leonean and international educators. They have been reviewed by officials of my Ministry to ensure that they meet specific needs of the Sierra Leonean population.

I call on the teachers and pupils across the country to make the best use of these educational resources.

This is just the start of educational transformation in Sierra Leone as pronounced by His Excellency, the President of the Republic of Sierra Leone, Brigadier Rtd. Julius Maada Bio. I am committed to continue to strive for the changes that will make our country stronger and better.

I do thank the Department for International Development (DFID) for their continued support. Finally, I also thank the teachers of our country - for their hard work in securing our future.

Timbo

Mr. Alpha Osman Timbo
Minister of Basic and Senior Secondary Education

The Ministry of Basic and Senior Secondary Education, Sierra Leone, policy stipulates that every printed book should have a lifespan of 3 years.

To achieve this DO NOT WRITE IN THE BOOKS.

Table of contents

Introduction

to the Pupils' Handbook

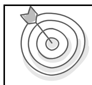
These practice activities are aligned to the lesson plans in the Teachers' Guide, and are based on the National Curriculum and the West Africa Examination Council syllabus guidelines. They meet the requirements established by the Ministry of Education, Science and Technology.

1. The practice activities will not take the whole term, so use any extra time to revise material or re-do activities where you made mistakes.
2. Use other textbooks or resources to help you learn better and practise what you have learned in the lessons.
3. Read the questions carefully before answering them. After completing the practice activities, check your answers using the answer key at the end of the book.
4. Make sure you understand the learning outcomes for the practice activities and check to see that you have achieved them. Each lesson plan shows these using the symbol to the right.

   Learning Outcomes

5. Organise yourself so that you have enough time to complete all of the practice activities. If there is time, quickly revise what you learned in the lesson before starting the practice activities. If it is taking you too long to complete the activities, you may need more practice on that particular topic.
6. Seek help from your teacher or your peers if you are having trouble completing the practice activities independently.
7. Make sure you write the answers in your exercise book in a clear and systematic way so that your teacher can check your work and you can refer back to it when you prepare for examinations.
8. Congratulate yourself when you get questions right! Do not worry if you do not get the right answer – ask for help and continue practising!

Lesson Title: Introduction to Ratio	Theme: Numbers and Numeration
Practice Activity: PHM-07-046	**Class:** JSS 1



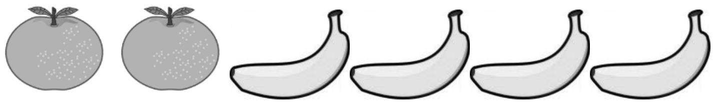
Learning Outcomes

By the end of the lesson, you will be able to:

1. Identify that ratio compares two quantities by division.
2. Use ratio language to compare quantities of things in their surroundings.

Overview

Ratio is a way of comparing two or more quantities. For example, consider the pictures below:



We can compare the oranges and bananas using the words "is to". In this case, we say "2 oranges is to 4 bananas".

The symbol for "is to" is a colon. In ratio form, we write:

2 oranges : 4 bananas

This is a ratio, and is read "2 oranges is to 4 bananas". We can also compare the oranges and bananas by saying "for every 2 oranges, we have 4 bananas".

Ratios are often just written with the numbers. For example, 2 : 4. The ratio of oranges to bananas is 2 : 4, and the ratio of bananas to oranges is 4 : 2.

Solved Examples

1. There are 25 girls and 26 boys in a classroom.
   a. What is the ratio of girls to boys?
   b. What is the ratio of boys to girls?

   Solutions

   a. The ratio of girls to boys is 25 : 26.
   b. The ratio of boys to girls is 26 : 25.

2. In a certain school, there are 35 JSS 1 pupils, 32 JSS 2 pupils, and 27 JSS 3 pupils. Write the following ratios:
   a. JSS 3 pupils to JSS 2 pupils.
   b. JSS 2 pupils to JSS 1 pupils.
   c. JSS 1 pupils to JSS 2 pupils.

   Solutions

   Note that order matters. The ratios are:

   a. 27 : 32
   b. 32 : 35
   c. 35 : 32

3. Write the following statements as ratios with only numbers:
   a. For every 3 teachers we have 30 pupils.
   b. 5 mangoes is to 2 pawpaws.
   c. For every 5 goats we have 3 sheep.
   d. 17 pens is to 20 exercise books.

   Solutions

   Remember that there are 2 ways to write ratios with words. For example, "For every 3 teachers, we have 30 pupils" is the same as "3 teachers is to 30 pupils" and both statements can be written 3 : 30.

   a. 3 : 30
   b. 5 : 2
   c. 5 : 3
   d. 17 : 20

Practice

1. Mrs. Bangura is a farmer. Today she harvested 45 eggplants, 23 cabbages, and 30 cassava. Write the following as ratios:
    a. Eggplants to cabbages.
    b. Cabbages to cassava.
    c. Cassava to eggplants.
    d. Eggplants to cassava.

2. Write the following statements as ratios with only numbers:
    a. For every 5 cups of rice, we have 2 cups of garri.
    b. Seven pupils is to 1 teacher.
    c. For every doctor, we have 35 sick people.
    d. Ten exercise books is to 3 pupils.

Lesson Title: Ratio of a Whole	**Theme:** Numbers and Numeration
Practice Activity: PHM-07-047	**Class:** JSS 1

Learning Outcomes

By the end of the lesson, you will be able to:

1. Identify that ratio can compare part of something to the whole.
2. Use ratio language to compare parts to the whole in their surroundings.

Overview

Remember that ratio is a way of comparing 2 or more quantities. We can also use ratios to compare a part of something to the whole. In the previous lesson, you wrote 'part-to-part' ratios. This lesson is on 'part-to-whole' ratios.

For example, consider a story: Ali is carrying a basket of fruits. He has 10 mangoes and 8 pineapples.

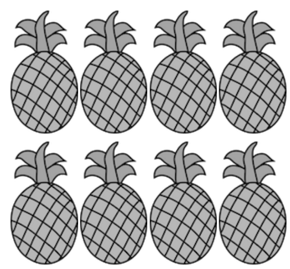
In the previous lesson, you wrote **part-to-part** ratios, such as the ratio of mangoes to pineapples (10 : 8) and the ratio of pineapples to mangoes (8 : 10).

We can also write **part-to-whole** ratios for Ali's fruit:

- The ratio of mangoes to all fruit is 10 : 18.
- The ratio of pineapples to all fruit is 8 : 18.

In part-to-whole ratios, the second number is the total number of items. This can be shown with a diagram. These are the fruit in Ali's basket:

10 : 18



8 : 18

Solved Examples

1. Fatu has 24 animals on her farm. She has 12 sheep, 5 goats and 7 chickens. Find the following:
    a. The ratio of sheep to all animals on the farm.
    b. The ratio of goats to all animals on the farm.
    c. The ratio of chickens to all animals on the farm.

Solutions

The total number of animals on the farm is 24. This will be the second number in each ratio. The ratios are:

a. 12 : 24
b. 5 : 24
c. 7 : 24

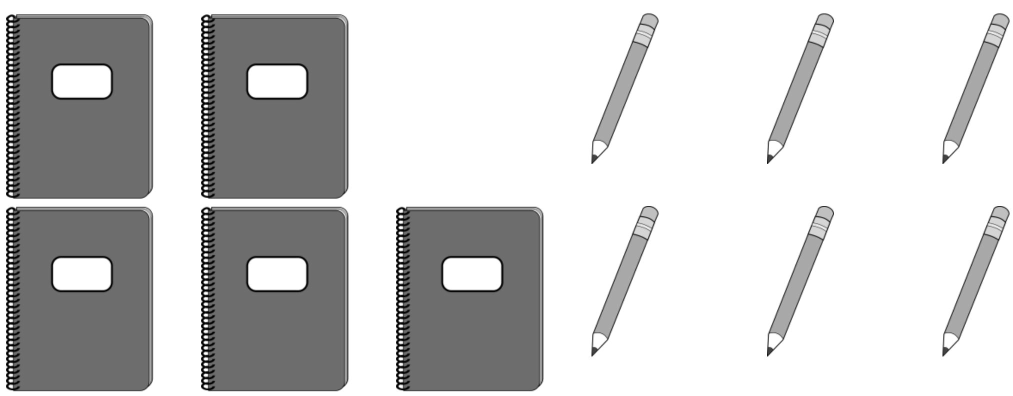
2. There are 45 pupils in a JSS 1 classroom. Twenty pupils are 12 years old, 18 pupils are 13 years old, and 7 pupils are 14 years old. Find the following:
   a. The ratio of 12-year-old pupils to all pupils.
   b. The ratio of 14-year-old pupils to all pupils.
   c. The ratio of 12-year-old pupils to 13-year-old pupils.
   d. The ratio of 12-year-old pupils to 14-year-old pupils.

Solutions

Note that parts a. and b. are part-to-whole ratios. Parts c. and d. are part-to-part ratios. The ratios are:

a. 20 : 45
b. 7 : 45
c. 20 : 18
d. 20 : 7

3. Abu returned from school and emptied his bag. He had these pencils and exercise books in his bag:



Write the following ratios:
   a. Exercise books to pencils.
   b. Pencils to exercise books.
   c. Exercise books to all items.
   d. Pencils to all items.

Solutions

First, identify how many of each item there are. There are 5 exercise books and 6 pencils. This means there are 5 + 6 = 11 items in total. The ratios are:

a. 5 : 6
b. 6 : 5
c. 5 : 11
d. 6 : 11

Practice

1. In a JSS 1 classroom, there are 18 girls and 19 boys. Write the following ratios:
   a. Girls to boys.
   b. Boys to girls.
   c. Girls to all pupils.
   d. Boys to all pupils.

2. Foday saw 26 animals in a pond. He saw 15 fish, 8 frogs, and 3 crocodiles. Find the following:
   a. The ratio of fish to frogs.
   b. The ratio of fish to all animals.
   c. The ratio of crocodiles to frogs.
   d. The ratio of frogs to all animals.

3. Fatu stopped at the market on the way home from school and bought fruit. She bought 14 bananas, 8 oranges, and 9 mangoes. Write the following ratios:
   a. Bananas to all fruit.
   b. Oranges to all fruit.
   c. Bananas to oranges.
   d. Bananas to mangoes.

Lesson Title: Ratios and Fractions	**Theme:** Numbers and Numeration
Practice Activity: PHM-07-048	**Class:** JSS 1

Learning Outcome
By the end of the lesson, you will be able to express ratios as fractions.

Overview

Any ratio can be expressed as a fraction. Say that you are in a JSS 2 class where there are 22 boys and 24 girls. This can be written as a part-to-part ratio in two ways: 22 : 24 or $\frac{22}{24}$. Note that the first number in the ratio becomes the numerator and the second number becomes the denominator.

A ratio can be simplified to its lowest terms, like any fraction. It is easiest to write the ratio in the fractional form before simplifying: $\frac{22}{24}=\frac{22\div 2}{24\div 2}=\frac{11}{12}$. The simplified form can be written $\frac{11}{12}$ or 11 : 12. This means that for every 11 boys in the class, there are 12 girls.

The order in which ratios are written is very important and must be maintained when solving a problem. A ratio written as 2 : 3 means $\frac{2}{3}$, while a ratio written as 3 : 2 means $\frac{3}{2}$. The fractions are different and give different answers.

When working with ratios, improper fractions are not changed to mixed numbers. For example, you would give the answer as $\frac{3}{2}$. You would **not** change it to $1\frac{1}{2}$.

We can only simplify ratios when the quantities are in the same units. If the quantities are not in the same unit, we must convert one to the other before we simplify.

Solved Examples

1. Express $24:14$ as a fraction in its lowest terms.

   Solution

   Write $24:14$ as a fraction and simplify it: $24:14=\frac{24}{14}=\frac{12}{7}$

2. Mr. Koroma has 20 animals on his farm. Eight are goats and 12 are cows. Write the following ratios as fractions in their simplest form:
   a. Goats to all animals on his farm.
   b. Cows to all animals on his farm.
   c. Goats to cows.
   d. Cows to goats.

   Solutions

   Write each ratio as you did in the previous lesson. Then, change it to a fraction. Remember that order matters, and the first number should be in the numerator. Simplify each fraction if possible.

   a. $8:20=\frac{8}{20}=\frac{2}{5}$
   b. $12:20=\frac{12}{20}=\frac{3}{5}$
   c. $8:12=\frac{8}{12}=\frac{2}{3}$
   d. $12:8=\frac{12}{8}=\frac{3}{2}$

3. What is 20 centimetres as a ratio of 80 centimetres? Give your answer as a fraction in its simplified form.

   Solution

   Quantities must be in the same unit before writing them as a ratio. Both units are in centimetres, so we have 20 : 80. Write this as a fraction and simplify it:

   $20:80=\frac{20}{80}=\frac{1}{4}$

4. Two sellers travelled different distances to the market. Samuel went 6 km and Alice went 8 km. What is the ratio of Samuel's travel to Alice's travel? Give your answer as a fraction in its lowest terms.

   Solution

   The ratio of Samuel's travel to Alice's travel is 6 : 8. We can write these as a ratio because they are in the same units.

   Write the ratio as a fraction and simplify it: $\frac{6}{8}=\frac{3}{4}$

5. Marie sold 250 kg of cassava. She has 750 kg remaining. What is the ratio of sold cassava to unsold cassava expressed as a fraction in the lowest terms?

   Solution

   The ratio of sold cassava to unsold cassava is 250 : 750. We can write these as a ratio because they are in the same units.

   Write the ratio as a fraction and simplify it: $\frac{250}{750} = \frac{1}{3}$

Practice

1. Express $20 : 35$ as a fraction in its lowest terms.
2. Express $12 : 48$ as a fraction in its lowest terms.
3. Express $200 : 800$ as a fraction in its lowest terms.
4. Express $150 : 450$ as a fraction in its lowest terms.
5. What is 10 centimetres as ratio of 100 centimetres? Give your answer as a fraction in its lowest terms.
6. A village chief counted the people living in the village. He counted 20 girls, 25 boys, 30 women, and 25 men.
   a. What is the total number of people living in the village?
   b. What is the ratio of girls to boys? Give your answer as a fraction in its lowest terms.
   c. What is the ratio of women to people in the village? Give your answer as a fraction in its lowest terms.
7. Mohamed has a shop near a school. He sold 50 exercise books this year, and 150 exercise books remain in his shop. What is the ratio of sold exercise books to unsold exercise books? Give your answer as a fraction in its lowest terms.

Lesson Title: Ratios and Percentages	Theme: Numbers and Numeration
Practice Activity: PHM-07-049	**Class:** JSS 1

Learning Outcomes

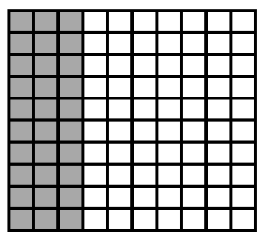
By the end of the lesson, you will be able to:

1. Identify that a percent is a ratio that compares a number to 100.
2. Express ratios as percentages.

Overview

In this lesson, you will express ratios as percent. This will help us, for instance, to compare our test marks to 100. Remember that percentage is parts out of 100.

Recall that the square below represents 30% or $\frac{30}{100}$. In ratio form, it is 30 : 100.



To write a ratio as a percentage, there are 2 steps:

1. Convert the ratio to a fraction.
2. Convert the fraction to percentage.

Remember that to express fractions as percentages, we multiply the fraction by 100, and simplify the result. For example, consider $\frac{3}{10}$. Multiply: $\frac{3}{10} \times \frac{100}{1} = \frac{300}{10} = 30\%$.

You may also be asked to convert percentage to ratio. Follow the above 2 steps in reverse:

1. Convert the percentage to a fraction.
2. Convert the fraction to a ratio.

To convert a percentage to a fraction, simply place the numbers over a denominator of 100 and simplify to its lowest term. For example, 50% is the same as $\frac{50}{100}$. We also need to simplify it: $50\% = \frac{50}{100} = \frac{1}{2} = 1 : 2$.

Solved Examples

1. Write the following percentages as ratios:
    a. 25%
    b. 40%
    c. 85%
    d. 90%

    Solutions

    Each percentage can be written as a part-to-whole ratio, where 100 is the whole.

    a. 25 : 100
    b. 40 : 100
    c. 85 : 100
    d. 90 : 100

2. Express the following ratios as percentages:
    a. 42 : 100
    b. 3 : 10
    c. 1 : 20
    d. 3 : 4

    Solutions

    Convert each ratio to a fraction before converting it to a percentage:

    a. Convert to a fraction: $42 : 100 = \frac{42}{100}$
       Convert to a percentage: $\frac{42}{100} = \frac{42}{100} \times \frac{100}{1} = \frac{42}{1} = 42\%$
    b. Convert to a fraction: $3 : 10 = \frac{3}{10}$
       Convert to a percentage: $\frac{3}{10} = \frac{3}{10} \times \frac{100}{1} = \frac{300}{10} = 30\%$
    c. Convert to a fraction: $1 : 20 = \frac{1}{20}$
       Convert to a percentage: $\frac{1}{20} = \frac{1}{20} \times \frac{100}{1} = \frac{100}{20} = 5\%$
    d. Convert to a fraction: $3 : 4 = \frac{3}{4}$
       Convert to a percentage: $\frac{3}{4} = \frac{3}{4} \times \frac{100}{1} = \frac{300}{4} = 75\%$

3. Hawa received 14 out of 20 marks on an exam. Express her score as:
    a. A ratio
    b. A fraction in its lowest term
    c. A percentage

Solutions

a. "14 out of 20" is a part-to-whole ratio, $14:20$.

b. Convert the ratio to a fraction: $14:20 = \frac{14}{20} = \frac{7}{10}$

c. Convert the fraction to a percentage: $\frac{7}{10} = \frac{7}{10} \times \frac{100}{1} = \frac{700}{10} = 70\%$

4. Mohamed received 71% mark on his biology exam. Express Mohamed's mark as a ratio.

Solution

To solve this, we must remember that when we talk about percent, we compare a number to 100. 71% is the same as $\frac{71}{100}$. We also know that $\frac{71}{100} = 71:100$.

Mohamed's mark as a ratio is $71:100$.

5. Fatu received an 85% mark on an exam. What ratio of correct answers did Fatu get? Write your answer as a fraction in its simplified form.

Solution

85% is the same as $\frac{85}{100}$. We also know that $\frac{85}{100} = 85:100$.

This is the ratio that we want to simplify: $\frac{85}{100} = \frac{17}{20}$.

Practice

1. Write the following percentages as ratios:
   a. 55% b. 60% c. 81% d. 8%
2. Express the following ratios as percentages:
   a. 31 : 100 b. 3 : 5 c. 1 : 2 d. 7 : 10
3. David received 32 out of 40 marks on an exam. Express his score as:
   a. A ratio
   b. A fraction in its lowest term
   c. A percentage
4. Sia scored 87% on a maths exam. Write her score as a ratio.
5. Foday received a 95% mark on an exam. What ratio of correct answers did Foday get? Write your answer as a fraction in its simplified form.

Lesson Title: Ratios and Decimals	**Theme:** Numbers and Numeration
Practice Activity: PHM-07-050	**Class:** JSS 1

Learning Outcome
By the end of the lesson, you will be able to express ratios as decimals.

Overview

In this lesson you will express ratios as decimals. To write a ratio as a decimal, there are 2 steps:

1. Convert the ratio to a fraction.
2. Convert the fraction to decimal.

Remember that to express fractions as decimals, we divide the numerator by the denominator.

You may also be asked to convert a decimal to a ratio. Follow the above 2 steps in reverse:

1. Convert the decimal to a fraction.
2. Convert the fraction to a ratio.

Remember that to express a decimal as a fraction, we write the decimal digits in the numerator of the fraction, and the correct power of 10 in the denominator.

Solved Examples

1. Express the following ratios as decimals:
   a. 35 : 100
   b. 9 : 10
   c. 3 : 5
   d. 1 : 4

   Solutions

   a. Convert to a fraction: $35 : 100 = \frac{35}{100}$
   Convert to a decimal: $\frac{35}{100} = 0.35$
   b. Convert to a fraction: $9 : 10 = \frac{9}{10}$
   Convert to a decimal: $\frac{9}{10} = 0.9$

c. Convert to a fraction: $3 : 5 = \frac{3}{5}$

Convert to a decimal: $\frac{3}{5} = 0.6$. Show the division →

$$\begin{array}{r|rr} & 0. & 6 \\ \cline{2-3} 5 & 3. & 0 \\ & -3. & 0 \\ \cline{2-3} & & 0 \end{array}$$

d. Convert to a fraction: $1 : 4 = \frac{1}{4}$

Convert to a decimal: $\frac{1}{4} = 0.25$. Show the division →

$$\begin{array}{r|rrr} & 0. & 2 & 5 \\ \cline{2-4} 4 & 1. & 0 & 0 \\ & - & 8 & \\ \cline{2-4} & & 2 & 0 \\ & - & 2 & 0 \\ \cline{2-4} & & & 0 \end{array}$$

2. Express the following decimals as ratios:

a. 0.85
b. 0.7
c. 0.08
d. 0.12

Solutions

a. Convert to a fraction: $0.85 = \frac{85}{100} = \frac{17}{20}$

Convert to a ratio: $\frac{17}{20} = 17 : 20$

b. Convert to a fraction: $0.7 = \frac{7}{10}$

Convert to a ratio: $\frac{7}{10} = 7 : 10$

c. Convert to a fraction: $0.08 = \frac{8}{100} = \frac{2}{25}$

Convert to a ratio: $\frac{2}{25} = 2 : 25$

d. Convert to a fraction: $0.12 = \frac{12}{100} = \frac{3}{25}$

Convert to a ratio: $\frac{3}{25} = 3 : 25$

3. Express 30 cm : 200 cm as a decimal.

Solution

Convert the ratio to a fraction: $30 : 200 = \frac{30}{200} = \frac{3}{20}$

Convert the fraction to a decimal: Note that $\frac{3}{20} = \frac{15}{100}$. It is easiest to convert to the equivalent fraction with a power of 10 in the denominator because this can be easily converted to a decimal: $\frac{15}{100} = 0.15$.

Or we can do the long division as shown on the right.
Note that $\frac{3}{20} = 3 \div 20$.

The answer is 0.15.

$$
\begin{array}{r|l}
 & 0.\ 15 \\
\hline
20 & 3.\ 00 \\
 & -2\ 0 \\
\cline{2-2}
 & \ \ 1\ 00 \\
 & -1\ 00 \\
\cline{2-2}
 & \qquad 0
\end{array}
$$

4. Express 4 m : 10 m as a fraction, decimal, and percentage.

Solution
First, convert the ratio to a fraction. Then, convert the fraction to both a decimal and a percentage.

Fraction: $4 \ :\ 10 = \frac{4}{10} = \frac{2}{5}$
Decimal: $4 \ :\ 10 = \frac{4}{10} = 0.4$
Percentage: $4 \ :\ 10 = 0.4 = 40\%$

These are all equivalent, so we have $4 \ :\ 10 = \frac{2}{5} = 0.4 = 40\%$.

5. Express 0.65 as a fraction, ratio, and percentage.

Solution
Fraction: $0.65 = \frac{65}{100} = \frac{13}{20}$
Ratio: $0.65 = \frac{13}{20} = 13 : 20$
Percentage: $0.65 = 65\%$

These are all equivalent, so we have $0.65 = \frac{13}{20} = 13 : 20 = 65\%$

Practice

1. Express the following ratios as decimals:
    a. 19 : 100
    b. 3 : 10
    c. 4 : 20
    d. 2 : 5
2. Express the following decimals as ratios:
    a. 0.8
    b. 0.03
    c. 0.5
    d. 0.79
3. Express 15 minutes: 30 minutes as a fraction, decimal and percentage.
4. Express 0.8 as a fraction, ratio and percentage.

Lesson Title: Simplification of Ratios	**Theme:** Numbers and Numeration
Practice Activity: PHM-07-051	**Class:** JSS 1

Learning Outcomes

By the end of the lesson, you will be able to:

1. Identify equivalent ratios.
2. Simplify a ratio to its lowest terms.

Overview

This lesson is on simplifying ratios. Simplifying ratios is similar to simplifying fractions. Remember that to simplify fractions, we divide the numerator and denominator by the highest common factor (HCF). For example, consider $\frac{12}{42}$. The HCF of 12 and 42 is 6, so divide by 6: $\frac{12}{42} = \frac{12 \div 6}{42 \div 6} = \frac{2}{7}$.

We follow the same process with ratios. To write a ratio to its simplest form, we must divide the numbers in the ratio by their HCF. For example, consider the ratio that is equivalent to $\frac{12}{42}$. It is $12 : 42$. Divide both parts of the ratio by the HCF of 12 and 42: $\frac{12}{6} : \frac{42}{6} = 2 : 7$.

We now have 2 **equivalent ratios**. We can write $12 : 42 = 2 : 7$. Any two ratios that are equal to one another are called equivalent ratios.

We can also get equivalent ratios when we multiply the numbers in the ratio by the same amount. Consider $2 : 7$. Let's find another equivalent ratio by multiplying both sides by 3: $2 \times 3 : 7 \times 3$. This gives $6 : 21$.

We now have 3 equivalent ratios, which we found by multiplying and dividing:

$$2 : 7 = 6 : 21 = 12 : 42$$

Solved Examples

1. Reduce the following ratios to their lowest terms:
   a. $2 : 10$ b. $15 : 30$ c. $8 : 12$ d. $18 : 24$

Solutions

Find the HCF for each ratio, then divide both sides of the ratio by it.

a. The HCF of 2 and 10 is 2.
   Divide: $2 : 10 = \frac{2}{2} : \frac{10}{2} = 1 : 5$
b. The HCF of 15 and 30 is 15.
   Divide: $15 : 30 = \frac{15}{15} : \frac{30}{15} = 1 : 2$
c. The HCF of 8 and 12 is 4.
   Divide: $8 : 12 = \frac{8}{4} : \frac{12}{4} = 2 : 3$
d. The HCF of 18 and 24 is 6.
   Divide: $18 : 24 = \frac{18}{6} : \frac{24}{6} = 3 : 4$

2. Find the missing number in the set of equivalent ratios: 14 : 24 = 7 : $\square$

Solution

There is a missing number. We can find it because we know that these two ratios are equivalent. Look at the first terms of the ratios, and notice that $14 \div 2 = 7$. Divide 24 by 2 to find the missing number: $24 \div 2 = 12$.

The missing number is 12, and the equivalent ratios are 14 : 24 = 7 : 12.

3. Find the missing number in each set of equivalent ratios:
   a. 2 : $\square$ = 16 : 40
   b. 6 : 8 = $\square$: 4
   c. $\square$: 10 = 60 : 50
   d. 24 : 36 = 8 : $\square$

   Solutions

   a. Note that $2 \times 8 = 16$, so we know that $\square \times 8 = 40$
      Divide to find the missing number: $40 \div 8 = 5$
   b. Note that $8 \div 2 = 4$, so we know that $6 \div 2 = \square$
      Divide to find the missing number: $6 \div 2 = 3$
   c. Note that $10 \times 5 = 50$, so we know that $\square \times 5 = 60$
      Divide to find the missing number: $60 \div 5 = 12$
   d. Note that $24 \div 3 = 8$, so we know that $36 \div 3 = \square$
      Divide to find the missing number: $36 \div 3 = 12$

Practice

1. Reduce the following ratios to their lowest terms:
   a. $5 : 35$ b. $20 : 50$ c. $18 : 4$ d. $28 : 14$
2. Find the missing number in each set of equivalent ratios:
   a. 3 : $\square$ = 18 : 54
   b. 9 : 18 = $\square$: 6
   c. $\square$: 32 = 60 : 64
   d. 90 : 36 = 30 : $\square$
   e. 30 : 60 = 90 : $\square$
   f. $\square$: 300 =25 : 100

Lesson Title: Ratio Problems with Two Terms	Theme: Numbers and Numeration
Practice Activity: PHM-07-052	**Class:** JSS 1

Learning Outcome
By the end of the lesson, you will be able to share a given quantity among a ratio with two terms (m : n).

Overview

In this lesson, you will learn how to share a quantity in a given ratio. For example, consider the problem: Divide 20 mangoes between Amie and Hawa in the ratio $2:3$.

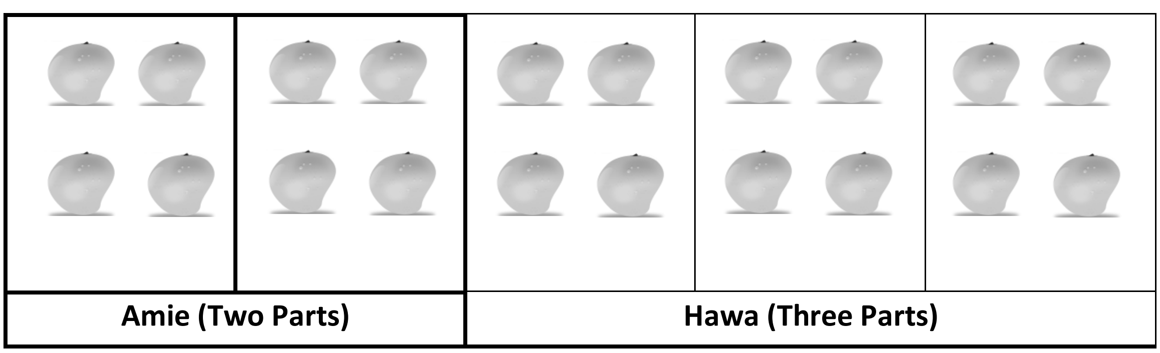
In this problem, the first number in the ratio is for Amie and the second number is for Hawa. We can write $A:H = 2:3$. This means that for every 2 mangoes that Amie gets, Hawa gets 3 mangoes. We are not sharing the mangoes equally. Hawa will get more mangoes than Amie.

These are the steps to divide a quantity into a given ratio:

Step	Example Problem
1. Find the total number of parts in the ratio. Add the 2 parts together to find the total parts.	In the example, there are 2 + 3 = 5 parts.
2. Write a fraction for each group you are sharing into.	Amie gets $\frac{2}{5}$ of the mangoes Hawa gets $\frac{3}{5}$ of the mangoes
3. Multiply each fraction by the number you are sharing.	Amie gets $\frac{2}{5} \times 20 = 8$ mangoes Hawa gets $\frac{3}{5} \times 20 = 12$ mangoes

You can check your answer by adding the results. They should equal the original amount you shared. In this example problem, $8 + 12 = 20$ mangoes.

This problem can be illustrated with a picture:



Solved Examples

1. Mustapha picks 48 mangoes from a tree. He wants to share them between his 2 friends in the ratio 3 : 1. How much does each friend get?

   Solution

   Step 1. Find the total number of parts: $3 + 1 = 4$ parts

   Step 2. Write a fraction for each friend: $\frac{3}{4}$ and $\frac{1}{4}$

   Step 3. Multiply the total quantity (48) by each fraction to find each friend's share:

   $$\frac{3}{4} \times 48 = 3 \times 12 = 36 \qquad \frac{1}{4} \times 48 = 1 \times 12 = 12$$

   Answer: His friends get 36 mangoes and 12 mangoes.

2. Share 96 ml in the ratio 5 : 3.

   Solution

   Step 1. Find the total number of parts: $5 + 3 = 8$ parts

   Step 2. Write a fraction for each group you are sharing into: $\frac{5}{8}$ and $\frac{3}{8}$

   Step 3. Multiply the total quantity (96) by each fraction to find the size of each group:

   $$\frac{5}{8} \times 96 = 5 \times 12 = 60 \qquad \frac{3}{8} \times 96 = 3 \times 12 = 36$$

   Answer: 60 ml and 36 ml.

3. Divide 90 in the ratio 2 : 7.

Solution

Step 1. Find the total number of parts: $2 + 7 = 9$ parts

Step 2. Write a fraction for each group: $\frac{2}{9}$ and $\frac{7}{9}$

Step 3. Multiply the total quantity (90) by each fraction to find the size of each group:

$\frac{2}{9} \times 90 = 2 \times 10 = 20$ $\qquad$ $\frac{7}{9} \times 90 = 7 \times 10 = 70$

Answer: 20, 70.

Practice

1. Sia has 108 bananas. She wants to share them between 2 of her friends in the ratio 4 : 5. How many bananas does each friend get?
2. Share 200 grammes in the ratio 1 : 3.
3. Share 150 exercise books in the ratio 2 : 3.
4. Foday has 24 pieces of candy. He wants to share them between his 2 children in the ratio 5 : 1. How many pieces of candy does each child get?
5. Share 120 in the ratio 1 : 2.
6. Share Le 80,000.00 between Mattu and Bondu in the ratio 3 : 5.

Lesson Title: Ratio Problems with Three or More Terms	**Theme:** Numbers and Numeration
Practice Activity: PHM-07-053	**Class:** JSS 1

Learning Outcome
By the end of the lesson, you will be able to share quantities among given ratios with three or more terms.

Overview

In the previous lesson, you shared a given quantity into 2 groups. In this lesson, you will share a quantity into 3 or more groups. This means that the ratio in the problem will have 3 or more terms.

For example, consider the problem: 3 sisters divided 30 pineapples between them in the ratio 3 : 1 : 2.

Note that this problem asks us to divide the 30 pineapples into 3 groups of different sizes. For every 3 pineapples that the first sister gets, the second sister gets 1 pineapple and the third sister gets 2 pineapples.

You will follow the same steps in the previous lesson to solve problems in this lesson. Instead of adding 2 terms to find the total parts, you will add up all of the terms in the ratio.

These are the steps to solve the problem above:

Step	Example Problem
1. Find the total number of parts in the ratio. Add the 3 parts together to find the total parts.	In the example, there are 3 + 1 + 2 = 6 parts.
2. Write a fraction for each group you are sharing into.	Sister 1 gets $\frac{3}{6}$ of the pineapples Sister 2 gets $\frac{1}{6}$ of the pineapples Sister 3 gets $\frac{2}{6}$ of the pineapples
3. Multiply each fraction by the number you are sharing.	Sister 1 gets $\frac{3}{6} \times 30 = 15$ pineapples Sister 2 gets $\frac{1}{6} \times 30 = 5$ pineapples Sister 3 gets $\frac{2}{6} \times 30 = 10$ pineapples

You can check your answer by adding the results. They should equal the original amount you shared. In this example problem, $15 + 5 + 10 = 30$ pineapples.

Solved Examples

1. Share $\mathrm{Le}\ 100{,}000.00$ among 4 people, Fatu, David, Alice and Mustapha in the ratio $3 : 4 : 2 : 1$.

   Solution
   Step 1. Find the total number of parts: $3 + 4 + 2 + 1 = 10$ parts

   Step 2. Write a fraction for each group: $\frac{3}{10}, \frac{4}{10}, \frac{2}{10}, \frac{1}{10}$

   Step 3. Multiply the total quantity (100,000) by each fraction to find the size of each person's share:

   Fatu's share: $\frac{3}{10} \times 100{,}000 = 3 \times 10{,}000 = \mathrm{Le}\ 30{,}000.00$

   David's share: $\frac{4}{10} \times 100{,}000 = 4 \times 10{,}000 = \mathrm{Le}\ 40{,}000.00$

   Alice's share: $\frac{2}{10} \times 100{,}000 = 2 \times 10{,}000 = \mathrm{Le}\ 20{,}000.00$

   Mustapha's share: $\frac{1}{10} \times 100{,}000 = 1 \times 10{,}000 = \mathrm{Le}\ 10{,}000.00$

2. Divide 72 plums among 3 friends in the ratio 4 : 2 : 3.

Solution

Step 1. Find the total number of parts: $4 + 2 + 3 = 9$ parts

Step 2. Write a fraction for each group: $\frac{4}{9}, \frac{2}{9}, \frac{3}{9}$

Step 3. Multiply the total quantity (72) by each fraction to find the size of each person's share:

Friend 1: $\frac{4}{9} \times 72 = 4 \times 8 = 32$ plums

Friend 2: $\frac{2}{9} \times 72 = 2 \times 8 = 16$ plums

Friend 3: $\frac{3}{9} \times 72 = 3 \times 8 = 24$ plums

Practice

1. Share Le 24,000.00 among 3 friends in the ratio 3 : 5 : 4.
2. Divide 45 kg of pepper among 3 people in the ratio 2 : 3 : 4.
3. Four friends, Ben, Juliet, Sia and John picked 48 mangoes from a tree. They want to divide them in the ratio 4 : 2 : 3 : 3. How many mangoes does each person get?
4. Share 60 bananas among 3 children in the ratio 3 : 2 : 5.

Lesson Title: Relating Ratios to Measurement	Theme: Numbers and Numeration
Practice Activity: PHM-07-054	**Class:** JSS 1

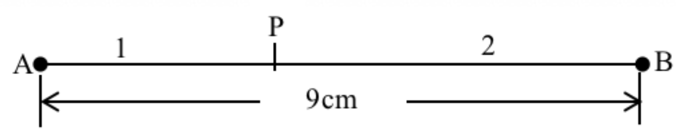
Learning Outcome
By the end of the lesson, you will be able to solve ratio problems involving measurement.

Overview

In this lesson, you will solve ratio problems involving measurement. This helps us to compare lengths or distances, in a ratio format. Instead of calculating ratios with real objects, you will calculate ratios with units of length or distance, such as metres, centimetres, or kilometres.

Solved Examples

1. Line AB is 9 cm long. If P divides the line AB in the ratio 1:2, calculate the lengths of AP and BP.



Solution
Follow the same process you used in previous lessons to find the sizes of the 2 parts. Instead of dividing objects, you are dividing 9 cm.

Step 1. Find the total number of parts: $1 + 2 = 3$ parts

Step 2. Write a fraction for each group you are sharing into: $\frac{1}{3}$ and $\frac{2}{3}$

Step 3. Multiply the total length (9) by each fraction to find the length of each part:

$AP = \frac{1}{3} \times 9 = 3$ cm

$BP = \frac{2}{3} \times 9 = 6$ cm

2. The length of a desk is 120 cm and the width of the desk is 50 cm. Find the ratio of the desk's length to its width. Give your answer in its simplest form.

Solution

The ratio of length to width is $120 : 50$. Simplify the ratio by dividing both terms by the GCF, 10.

$$120 : 50 = \frac{120}{10} : \frac{50}{10} = 12 : 5$$

3. Divide a distance of 40 km in the ratio 3 : 4 : 1.

   Solution

   Follow the same process as in the previous lesson on dividing a quantity into 3 or more parts.

   Step 1. Find the total number of parts: $3 + 4 + 1 = 8$ parts

   Step 2. Write a fraction for each group: $\frac{3}{8}, \frac{4}{8}, \frac{1}{8}$

   Step 3. Multiply the total distance (40 km) by each fraction to find the size of each part:

   Part 1: $\frac{3}{8} \times 40 = 3 \times 5 = 15$ km

   Part 2: $\frac{4}{8} \times 40 = 4 \times 5 = 20$ km

   Part 3: $\frac{1}{8} \times 40 = 1 \times 5 = 5$ km

Practice

1. A football field is 100 metres long. Divide the length of the field in the following ratios:
   a. 2 : 3
   b. 2 : 3 : 5
2. A rectangle is 50 cm long and 35 cm wide. Write the ratio of its length to its width. Give your answer in its lowest terms.
3. Divide a distance of 42 km in the ratio 5 : 1.
4. Three friends are painting a wall. The wall is 24 metres long, and they divide the work in the ratio 2 : 3 : 1. How much of the wall does each friend paint?
5. Divide a length of 18 metres in the ratio 1 : 2.

Lesson Title: Ratio Story Problems	**Theme:** Numbers and Numeration
Practice Activity: PHM-07-055	**Class:** JSS 1

Learning Outcome
By the end of the lesson, you will be able to solve story problems involving ratios.

Overview

In this lesson you will solve story problems involving ratios. You will use the information you learned from the previous 9 lessons on ratios.

Solved Examples

1. Abass has 3 children. Fatu is 3 years old, Sahr is 5 years old and Hawa is 8 years old. Abass will share 64 bananas among them in the ratio of their ages. How many bananas does each child get?

   Solution

   The ratio of their ages is 3 : 5 : 8.

   Step 1. Find the total number of parts: $3 + 5 + 8 = 16$ parts

   Step 2. Write a fraction for each group: $\frac{3}{16}, \frac{5}{16}, \frac{8}{16}$

   Step 3. Multiply the total quantity (64) by each fraction to find the size of each person's share:

   Fatu's share: $\frac{3}{16} \times 64 = 3 \times 4 = 12$ bananas

   Sahr's share: $\frac{5}{16} \times 64 = 5 \times 4 = 20$ bananas

   Hawa's share: $\frac{8}{16} \times 64 = 8 \times 4 = 32$ bananas

2. Abu and Mustapha started a business together. This month, the profit is Le 9,000,000.00. Abu and Mustapha invested in the business in the ratio 2 : 1. They want to share the profit in the same ratio. How much does each person receive?

Solution

Step 1. Find the total number of parts: $2+1=3$ parts

Step 2. Write a fraction for each person's share: $\frac{2}{3}, \frac{1}{3}$

Step 3. Multiply the total quantity (9,000,000) by each fraction to find the size of each person's share:

Abu's share: $\frac{2}{3} \times 9{,}000{,}000 = 2 \times 3{,}000{,}000 = \text{Le } 6{,}000{,}000.00$

Mustapha's share: $\frac{1}{3} \times 9{,}000{,}000 = 1 \times 3{,}000{,}000 = \text{Le } 3{,}000{,}000.00$

3. A rope is 18 m long. Alpha will divide it in the ratio 2 : 3 : 1. Find the length of each part.

Solution

Step 1. Find the total number of parts: $2+3+1 \;=\; 6$ parts

Step 2. Write a fraction for each group: $\frac{2}{6}, \frac{3}{6}, \frac{1}{6}$

Step 3. Multiply the total length (18 m) by each fraction to find the size of each part:

Part 1: $\frac{2}{6} \times 18 = 2 \times 3 = 6$ m

Part 2: $\frac{3}{6} \times 18 = 3 \times 3 = 9$ m

Part 3: $\frac{1}{6} \times 18 = 1 \times 3 = 3$ m

4. In a village, there are 21 dogs, 18 cats, 20 chickens and 24 ducks. Write the following ratios in their lowest terms:

   a. Dogs to cats
   b. Chickens to ducks
   c. Dogs to chickens
   d. Dogs to cats to ducks

Solution

Write each ratio, and divide the parts by their GCF.

a. $21 : 18 = \frac{21}{3} : \frac{18}{3} = 7 : 6$

b. $20 : 24 = \frac{20}{4} : \frac{24}{4} = 5 : 6$

c. $21 : 20$

d. $21 : 18 : 24 = \frac{21}{3} : \frac{18}{3} : \frac{24}{3} = 7 : 6 : 8$

Practice

1. A building is 20 feet tall. Mustapha wants to paint the bottom part green and the top part yellow, in the ratio 2 : 3. Find how many feet are green and how many feet are yellow.
2. In a bag of fruit, there are 18 bananas, 20 mangoes and 4 pineapples. Write the following ratios in their lowest terms:
    a. Bananas to mangoes
    b. Mangoes to pineapples
    c. Bananas to pineapples
    d. Bananas to mangoes to pineapples
3. Mary has Le 800,000.00 to share between her 4 children. She will share it such that Alice gets 2 parts, Foday gets 1 part, Sia gets 3 parts and Michael gets 4 parts. Find the share of each child.
4. Emmanuel scored 86% on his maths exam. Write his score as a ratio in its lowest term.
5. The length and width of a rectangular field are 36 m and 32 m, respectively. What is the ratio of its length to its width?

Lesson Title: Introduction to Integers	**Theme:** Numbers and Numeration
Practice Activity: PHM-07-056	**Class:** JSS 1

Learning Outcome
By the end of the lesson, you will be able to recognise and count positive and negative numbers, and zero.

Overview

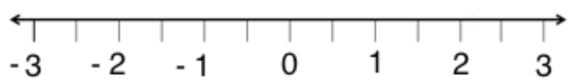
It is possible for numbers to be less than zero. We call these **negative** numbers, and they are the topic of this lesson. All numbers greater than zero are positive numbers and all numbers less than zero are negative numbers. Most of the numbers that you have been using in maths are positive numbers. **Integers** are all of the positive and negative numbers, and zero.

Positive integers are numbers that have a **plus** sign before the number. If a number has no sign, it is also a positive number. These are examples of positive numbers: 3, +5, 1, +10.

Negative integers have a **minus** sign before the number. These are examples of negative numbers: –3, –1, –12, –60.

Zero is neither positive nor negative.

Positive and negative numbers can be shown on a number line:



Remember that numbers to the right on a number line are bigger than numbers to the left. This number line shows that positive numbers are greater than zero and negative numbers are less than zero.

These are some real-life situations when we use negative numbers:

- If you have money, you have a positive amount. If you owe people money, you have a negative amount.
- When we talk about temperature, very cold temperatures are negative. The temperature of anything frozen, such as ice, is negative degrees Celsius. Hot temperatures are positive.
- When we talk about elevation, we measure height above or below sea level in metres. The highest point in Sierra Leone is Bintimani, which is 1,948 metres above

sea level. If we enter the ocean and touch the bottom, we are **below** sea level. Below sea level, we talk about elevation in negative numbers. The Atlantic Ocean next to Sierra Leone is 8,605 metres deep at its deepest point. That is an elevation of **negative** 8,605 metres.

Solved Examples

1. Identify the negative integers:

    a. –2   b. 0   c. 9   d. +10   e. –15

    Solutions

    The negative numbers are a. –2 and e. –15. These are the only numbers with minus signs.

    Remember that 0 is neither positive nor negative. The other integers, 9 and +10, are both positive numbers.

2. Determine whether each of the integers in the box below is positive or negative. Write each number in the correct box on the right.

21 –30 4 0
+12 9 –39
40 +91 –20 +7
–18 8 –1
16 87 –100
+101 –56 99

Positive Integers

Negative Integers

Solution

Any number with a minus sign is negative. Any integer with a plus sign or no sign is positive. The only exception is 0, which is not positive or negative. It cannot be written in either box.

Positive Integers
21, 4, 9, +12, 40, +91, +7, 8, 16, 87, +101, 99

Negative Integers
–30, –39, –20, –18, –1, –100, –56

Practice

1. Determine which of the numbers are negative:
   a. –5 b. + 5 c. –16 d. 16 e. –1

2. Determine which of the numbers are positive:
   a. 0 b. 1 c. +3 d. –4 e. 5

3. Circle the positive numbers, and draw a triangle around the negative numbers:

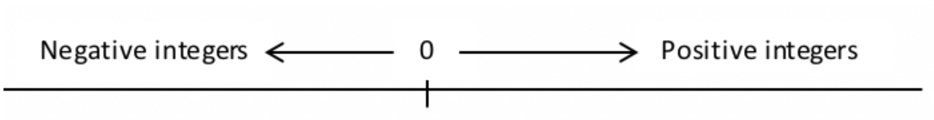
+10	–21	16	–6	–8	47	91	–80	1	–19
3	100	–24	+4	11	–70	+5	–99	–14	+75

Lesson Title: Positive and Negative Integers	Theme: Numbers and Numeration
Practice Activity: PHM-07-057	**Class:** JSS 1

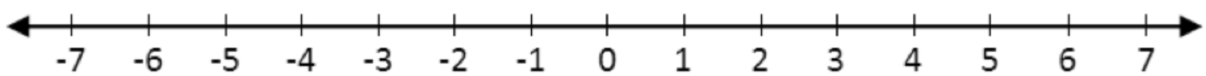
Learning Outcome
By the end of the lesson, you will be able to locate positive and negative integers on a number line.

Overview

In this lesson, you will locate positive and negative integers on a number line. Remember that negative integers are to the left of 0, and positive integers are to the right:



The number line below shows the integers from –7 to +7:

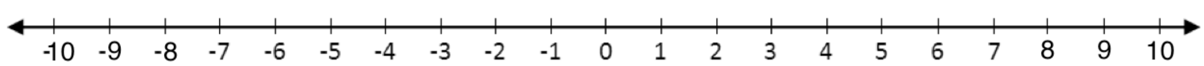


Notice that it looks like the numbers increase in both directions as you move away from zero. It appears that negative numbers increase to the left, but this is actually not the case. All numbers increase to the right. For example, -1 is actually greater than -2. You will learn to compare positive and negative numbers in the next lesson.

Solved Examples

1. Draw a number line from –10 to +10.

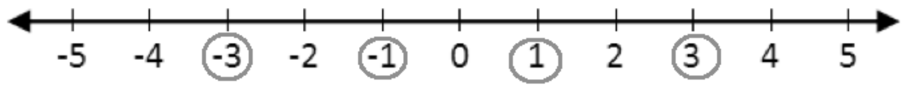
   Solution
   Remember that the marks on a number line should be an equal distance apart. Zero should be in the middle. The number line is:



2. Draw a number line from –5 to 5, and circle the following numbers: –3, –1, 1, 3.

Solution

The number line with circled numbers is:



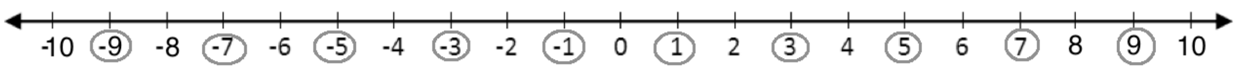
Notice that each negative number has a corresponding positive number. They are an equal distance from 0.

3. Draw a number line from –10 to 10 and circle all of the odd numbers.

Solution

Recall that positive odd numbers are 1, 3, 5, 7, 9. Negative odd numbers have the same digits. They are –1, –3, –5, –7, –9.

The number line with circled numbers is:



Practice

1. Draw a number line from -5 to 5 and circle the following integers: -5, -3, 0, 4
2. Draw a number line from -6 to 6 and circle all of the even numbers.
3. Draw a number line from -6 to 6 and do the following:
    a. Draw a star around the largest integer.
    b. Draw a square around 0.
    c. Draw a triangle around the largest odd number.
    d. Circle the negative odd numbers.

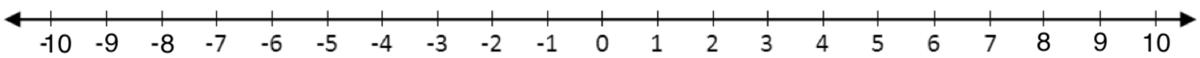
Lesson Title: Comparing Integers	**Theme:** Numbers and Numeration
Practice Activity: PHM-07-058	**Class:** JSS 1

Learning Outcome
By the end of the lesson, you will be able to compare positive and negative integers.

Overview

In this lesson, you will compare and order positive and negative integers. Recall the symbols for greater than (>) and less than (<) that are used to compare 2 numbers. The symbol opens to the larger number. You have compared positive numbers with these symbols. For example, 6 < 9.

You can use the number line to help compare 2 integers:



Remember that numbers to the right on a number line are bigger than numbers to the left.

The farther positive integers are to the right of zero, the greater their value. For example, 9 > 8 because 9 is to the right of 8.

The farther negative integers are to the left of zero, the lesser the value. For example, –9 < –8 because –9 is to the left of –8.

Also, positive numbers are always greater than negative numbers. That is, integers on the right of zero are always greater than those on the left. For example, 9 > –9 and 8 > –8.

You may be asked to compare numbers beyond what you can draw on the number line. For example, –21 and –40. You can imagine the number line. –21 is to the right of –40. It is closer to 0. This tells us that –21 > –40.

Solved Examples

1. Write the correct symbol in between each set of numbers, > or <.
   a. 5 –6   b. – 3 3   c. 0 –6

Solutions

a. 5 > –6 because positive numbers are always greater than negative numbers.
b. –3 < 3 because positive numbers are always greater than negative numbers.
c. 0 > –6. Because -6 is to the left of 0 on the number line.

2. Write the correct symbol in between each set of numbers, > or <.

a. 8 12 b. 15 14 c. 9 2

Solutions

a. 8 < 12 because 12 is to the right of 8 on the number line.
b. 15 > 14 because 15 is to the right of 14 on the number line.
c. 9 > 2 because 9 is to the right of 2 on the number line.

3. Write the correct symbol in between each set of numbers, > or <.

a. –2 –3 b. – 19 –8 c. –7 –2

Solutions

a. –2 > – 3 because –2 is to the right of –3 on the number line.
b. –19 < – 8 because –8 is to the right of –10 on the number line.
c. –7 < – 2 because –2 is to the right of –7 on the number line.

4. List these integers in order from least to greatest: –9, 15, –8, –12, 4, 7

Solution

Remember that negative numbers are always less than positive numbers. You can start ordering them by writing negative integers first: –9, –8, –12, 15, 4, 7

Now, compare the negative numbers to write them in order. Note that –12 < –9 and –9 < –8. From least to greatest, the negative numbers are –12, –9, –8.

Next, compare the positive numbers to write them in order. Note that 4 < 7 and 7 < 15. From least to greatest, the positive numbers are 4, 7, 15.

Write the entire list of numbers in order from least to greatest: $-12, -9, -8,\ 4,\ 7, 15$.

5. List these integers in order from greatest to least: –20, 30, 40, –50, –60, 70, –80, 90.

Solution

In this problem we want to start with the greater numbers. Start ordering them by writing the positive integers first: 30, 40, 70, 90, –20, –50, –60, –80.

Compare the positive numbers to write them in order. Note that 90 > 70, 70 > 40, and 40 > 30. From greatest to least, the positive numbers are 90, 70, 40, 30.

Compare the negative numbers to write them in order. Note that –20 > –50, –50 > –60, and –60 > –80. From greatest to least, the negative numbers are –20, –50, –60, –80.

Write the entire list of numbers from greatest to least: 90, 70, 40, 30, –20, –50, –60, –80

Practice

1. Write the correct symbol in between each set of numbers, > or <.
   a. –8 7 b. – 9 9 c. –12 0
2. Write the correct symbol in between each set of numbers, > or <.
   a. 21 12 b. 9 8 c. 17 21
3. Write the correct symbol in between each set of numbers, > or <.
   a. –12 –30 b. – 20 –80 c. –9 –7
4. List these integers in order from least to greatest: –14, 18, 17, –15, –19, 13, –12, –16
5. List these integers in order from greatest to least: –9, 18, –15, –7, 16, 5, –6, –16, 7

Lesson Title: Addition of Integers Using a Number Line	Theme: Numbers and Numeration
Practice Activity: PHM-07-059	**Class:** JSS 1

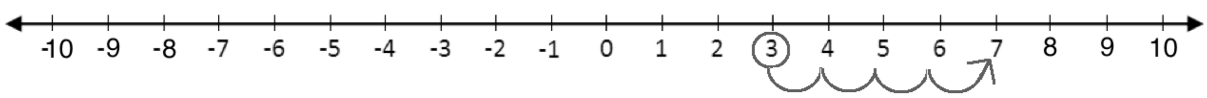
Learning Outcome
By the end of the lesson, you will be able to add any two positive and negative integers using a number line.

Overview

The number line can be used to add or subtract positive and negative numbers. In this lesson, you will be adding numbers.

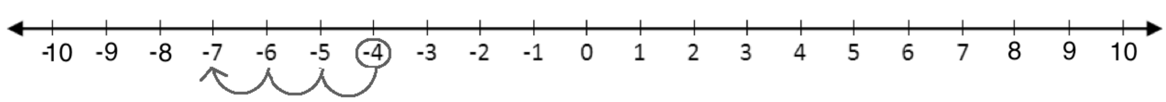
Recall how the number line is used for addition of positive numbers. The first integer in the problem tells us where to start and the second integer tells us the number of steps to move.

For example, 3 + 4 = 7 is shown on the number line:



To add **positive** numbers, we move to the **right** the given number of steps. However, to add **negative** numbers, we move to the **left** the given number of steps.

For example, consider the problem $-4 + (-3)$. This is read "negative four plus negative three". To solve this problem with a number line, start at –4 and move to the left 3 steps:



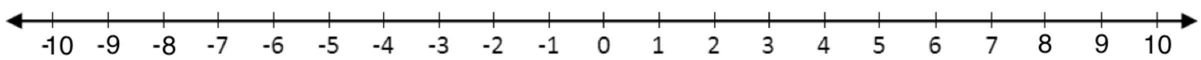
The answer is $-4 + (-3) = -7$.

Note that adding a negative number is the same as subtracting the number. We can rewrite this problem: $-4 + (-3) = -4 - 3 = -7$.

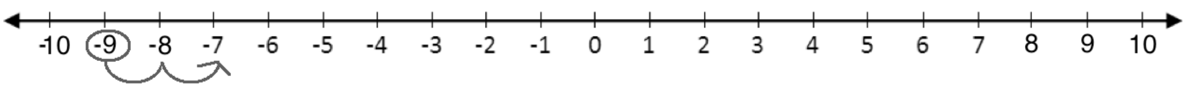
Note that any time you add a negative and positive integer, the answer will take the sign of the bigger number. A big negative and a small positive will give a negative answer. A big positive and a small negative will give a positive answer.

Solved Examples

1. Use the number line below to solve $-9 + 2$.

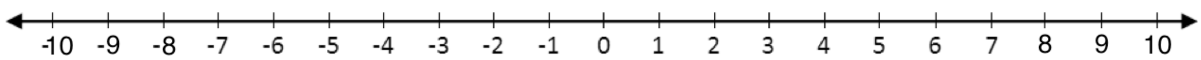


Solution
Start at -9 and move to the right 2 places.

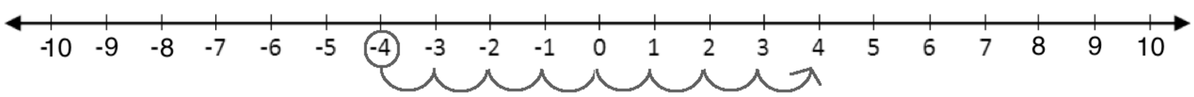


Answer: $-9 + 2 = -7$

2. Use the number line below to solve $-4 + 8$.

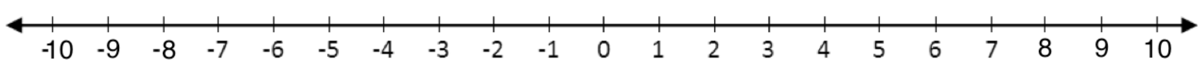


Solution
Start at -4 and move to the right 8 places.

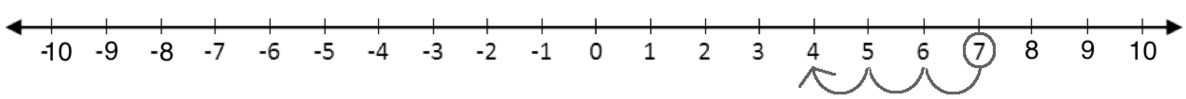


Answer: $-4 + 8 = 4$

3. Use the number line below to solve $7 + (-3)$.

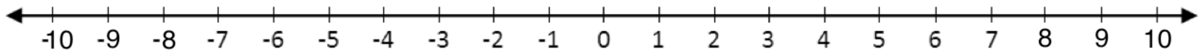


Solution
Start at 7 and move to the left 3 places.



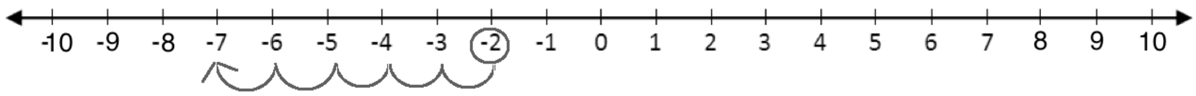
Answer: $7 + (-3) = 4$

4. Use the number line below to solve $-2+(-5)$.



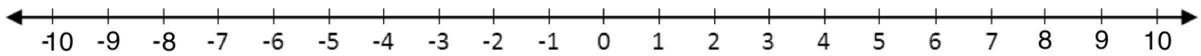
Solution

Start at -2 and move to the left 5 places.



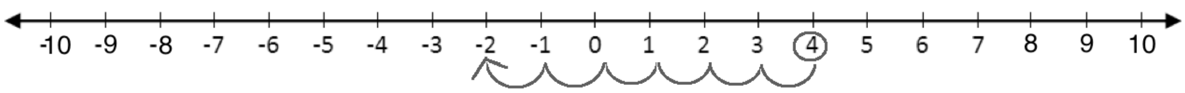
Answer: $-2+(-5)=-7$

5. Use the number line below to solve $4+(-6)$.



Solution

Start at 4 and move to the left 6 places.



Answer: $4+(-6)=-2$

Practice

Draw a number line and use it to solve each of the following problems:

1. $-8+2$
2. $10+(-5)$
3. $-4+(-3)$
4. $3+(-7)$
5. $-5+12$
6. $-1+(-7)$
7. $0+(-3)$

Lesson Title: Addition of Integers	**Theme:** Numbers and Numeration
Practice Activity: PHM-07-060	**Class:** JSS 1

Learning Outcome

By the end of the lesson, you will be able to apply rules for adding integers to 2 positive or negative integers.

Overview

In the previous lesson, you solved addition problems with a number line. In this lesson, you will apply rules for adding integers to 2 positive or negative integers. You will be able to do this without using a number line.

The first 2 rules are simple. Adding 2 positive numbers gives a positive answer. Adding 2 negative integers gives a negative answer.

- positive + positive = positive, $(+)+(+)=+$
- negative + negative = negative, $(-)+(-)=-$

To add integers with the same signs, add the numbers without regard to the signs and use the same sign for the answer. You are familiar with adding positive numbers. To add 2 negative numbers, add them as you would positive numbers, but make sure a minus symbol is on the answer.

Here are some examples: $3+5=8$ and $-3+(-5)=-8$.

Now let's look at adding positive and negative numbers together. For example, $3+(-8)$.

When we add integers with different signs, the answer can be positive or negative. The answer will take the sign of the bigger number. To add integers with different signs, **subtract** the smaller number from the bigger number and put the correct sign on the answer.

- positive + negative = sign of the bigger number, $(+)+(-)=-$ or $+$

Let's solve the example above. For $3+(-8)$, subtract $8-3=5$. The bigger number in this problem is 8, which has a negative sign. The answer is $3+(-8)=-5$.

Note that addition of a negative number can also be written as a subtraction problem. For example, $3 + (-8) = 3 - 8 = -5$. Subtraction is covered in the next lesson.

Solved Examples

1. Solve the addition problems:

    a. $12 + 7$
    b. $-5 + (-8)$
    c. $3 + 19$
    d. $-14 + (-6)$

    Solutions

    a. Add the positive numbers: $12 + 7 = 19$
    b. Add the negative numbers: $-5 + (-8) = -13$
    c. Add the positive numbers: $3 + 19 = 22$
    d. Add the negative numbers: $-14 + (-6) = -20$

2. Solve the addition problems:

    a. $-9 + 3$
    b. $-5 + 16$
    c. $9 + (-4)$
    d. $7 + (-18)$

    Solutions
    All of these problems are addition of a positive and negative number. Subtract the numbers and write the sign of the larger number on the answer.

    a. $-9 + 3 \rightarrow 9 - 3 = 6 \rightarrow -9 + 3 = -6$
    b. $-5 + 16 \rightarrow 16 - 5 = 11 \rightarrow -5 + 16 = 11$
    c. $9 + (-4) \rightarrow 9 - 4 = 5 \rightarrow 9 + (-4) = 5$
    d. $7 + (-18) \rightarrow 18 - 7 = 11 \rightarrow 7 + (-18) = -11$

Practice

Add the numbers:

1. $-5 + (-3)$
2. $-1 + 8$
3. $9 + 15$
4. $-2 + (-21)$
5. $2 + (-12)$
6. $15 + (-4)$
7. $-5 + (-5)$
8. $-10 + 10$
9. $-4 + 14$
10. $3 + (-18)$

Lesson Title: Subtraction of Integers	**Theme:** Numbers and Numeration
Practice Activity: PHM-07-061	**Class:** JSS 1

Learning Outcome
By the end of the lesson, you will be able to subtract any two positive or negative integers.

Overview

In this lesson, you will subtract positive and negative integers using rules.

Remember from the previous lesson that adding a negative is the same as subtraction. For example, $-3+(-4)=-3-4=-7$. A plus and minus sign together always make a minus sign. For example:

- $-2-(+6)=-2-6=-8$
- $+5-(+10)=5-10=-5$

When adding a negative and positive in the previous lesson, the answer took the sign of the bigger number. You will use that rule again in this lesson. Subtract the numbers, and use the sign of the bigger number. Consider the example above. The step-by-step subtraction is shown below:

$5-10$ → $10-5=5$ → $5-10=-5$

Subtracting a positive number from a positive number is normal subtraction. For example, $+4-(+6)=4-6=-2$.

Subtracting a negative number is the same as addition. For example, $-5-(-3)=-5+3=-2$.

It is important to remember that a minus sign always changes the sign on the number you are adding or subtracting. Here are the rules:

Subtracting a Negative → Addition	$-(-)$ → $+$
Subtracting a Positive → Subtraction	$-(+)$ → $-$

Solved Examples

1. Subtract the numbers:
    a. $6-(-7)$
    b. $-2-(-5)$
    c. $-10-(-3)$

    Solutions

    All of these problems involve subtracting a negative. Remember that this is the same as addition.

    a. $6-(-7)=6+7=13$

    This becomes a normal addition problem.

    b. $-2-(-5)=-2+5=3$

    Note that $-2+5$ is the same as $5-2$. Subtract and give the sign of the bigger number, 5, to the answer. The result is positive 3.

    c. $-10-(-3)=-10+3=-7$

    Note that to solve $-10+3$, we should subtract and give the sign of the bigger number, -10, to the answer. $-10+3$ → $10-3=7$ → $-10+3=-7$

2. Subtract the numbers:
    a. $5-(+6)$
    b. $-9-(+5)$
    c. $10-(+4)$

    Solutions

    All of these problems involve subtracting a positive. Remember that is the same as subtraction.

    a. $5-(+6)=5-6=-1$

    Subtract the numbers and give the sign of the bigger number, -6, to the answer:
    $5-6$ → $6-5=1$ → $5-6=-1$

    b. $-9-(+5)=-9-5=-14$

This is the same as adding 2 negative numbers. Add the numbers and give a negative sign to the answer.

c. $10 - (+4) = 10 - 4 = 6$

This is a normal subtraction problem.

Practice

Subtract the numbers:

1. $-4 - (+7)$
2. $5 - (+12)$
3. $-16 - (-7)$
4. $-5 - (-6)$
5. $17 - (-10)$
6. $-5 - (+13)$
7. $19 - (+2)$
8. $-2 - (-2)$
9. $0 - (+6)$
10. $0 - (-7)$

Lesson Title: Multiplication of Integers Using a Number Line	**Theme:** Numbers and Numeration
Practice Activity: PHM-07-062	**Class:** JSS 1

Learning Outcome
By the end of the lesson, you will be able to multiply positive and negative integers using number line.

Overview

Multiplication is a short way of writing repeated addition. We can solve multiplication problems on the number line in a similar way to addition.

To multiply a positive number by a positive number, we start from zero and move to the right along the number line. This is normal multiplication of positive numbers that you are familiar with. See Solved Example 1.

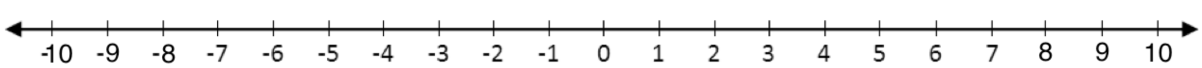
When we multiply with negative numbers, we may move to the right or left, depending on the signs of both of the numbers. The table below gives rules for multiplying negative numbers. If the result is positive, move in the positive direction on the number line (to the right). If the result is negative, move in the negative direction on the number line (to the left). Always start from zero.

There are some rules for multiplying positive and negative numbers.

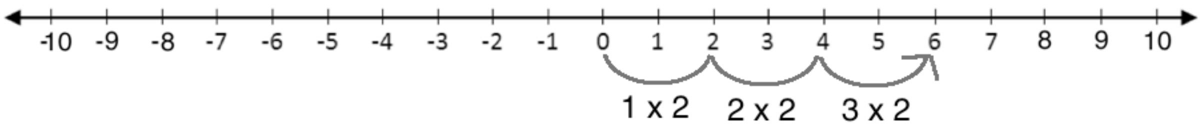
Positive × Positive = Positive	$(+) \times (+) = +$
Negative × Negative = Positive	$(-) \times (-) = +$
Positive × Negative = Negative	$(+) \times (-) = -$
Negative × Positive = Negative	$(-) \times (+) = -$

Solved Examples

1. Use the number line below to multiply 3×2.

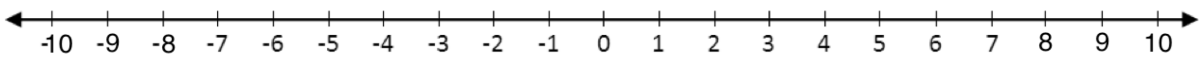


Solution
3×2 is the same as the repeated addition problem $2\ +\ 2\ +\ 2$. Three times 2 means we add three 2's. We can show this on the number line by counting by 2's three times.



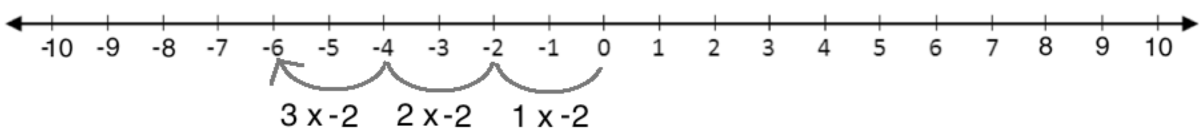
Answer: $3 \times 2 = 6$

2. Use the number line below to multiply $3 \times (-2)$.



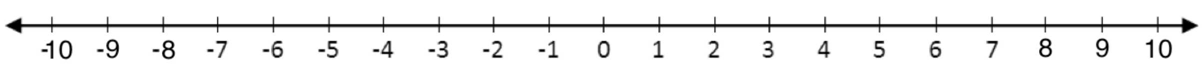
Solution
$3 \times (-2)$ is the same as the repeated addition problem $(-2) + (-2) + (-2)$. This means we subtract three 2's. We can show this on the number line by counting 2's in the negative direction.

Also note that from the table in the Overview, $(+) \times (-) \rightarrow\ -$. This rule tells us that we will move in the negative direction, to the left.

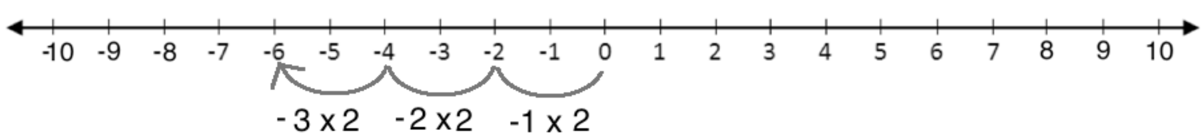


Answer: $3 \times (-2) = -6$

3. Use the number line below to multiply -3×2.

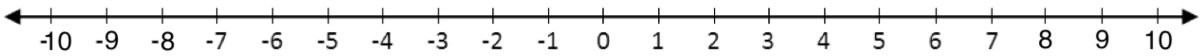


Solution
Note that from the table in the Overview, $(-) \times (+) \rightarrow\ -$. This rule tells us that we will move in the negative direction, to the left.



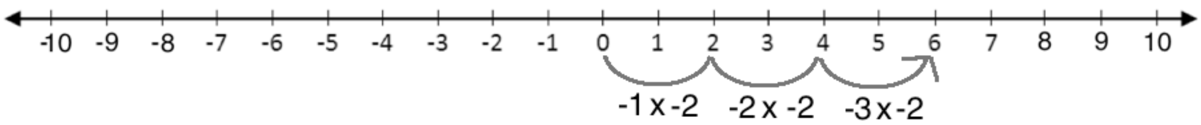
Answer: $-3 \times 2 = -6$

4. Use the number line below to multiply $(-3) \times (-2)$.



Solution

Note that from the table in the Overview, $(-) \times (-) \rightarrow +$. This rule tells us that we will move in the positive direction, to the right.



Answer: $(-3) \times (-2) = 6$

Practice

Draw a number line and use it to solve each of the following problems:

1. -3×3
2. $2 \times (-5)$
3. -1×4
4. $3 \times (-1)$
5. 2×3
6. $-4 \times (-2)$
7. -4×2
8. $-3 \times (0)$
9. $-5 \times (-2)$
10. $(+2) \times (-2)$

Lesson Title: Multiplication of Integers	**Theme:** Numbers and Numeration
Practice Activity: PHM-07-063	**Class:** JSS 1

Learning Outcome

By the end of the lesson, you will be able to apply rules for multiplying integers to multiply any two positive or negative integers.

Overview

In this lesson, you will solve multiplication problems without using a number line. Use the rules:

Positive × Positive = Positive	$(+) \times (+) = +$
Negative × Negative = Positive	$(-) \times (-) = +$
Positive × Negative = Negative	$(+) \times (-) = -$
Negative × Positive = Negative	$(-) \times (+) = -$

If the two integers have the same sign, their product will carry a positive sign. If the two integers have different signs, their product will carry a negative sign.

Multiply the numbers using normal multiplication facts, and make sure you give the correct sign to the answer. We can use the same multiplication table when multiplying negative numbers. We just need to make sure we use the correct sign on the product.

Try to use multiplication facts that you have memorised. If you need help, locate the product using the multiplication table:

×	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

Solved Examples

1. Multiply the following numbers:
    a. $(-3) \times (-7)$
    b. $(-10) \times (-4)$
    c. $(-5) \times (-9)$

    Solutions

    Each problem is multiplication of 2 negative integers. Use the rule $(-) \times (-) = +$. The answers will all be positive.

    To find the answers, multiply positive numbers using the multiplication table. For example, $3 \times 7 = 21$. This means $(-3) \times (-7) = 21$.

    a. $(-3) \times (-7) = 21$
    b. $(-10) \times (-4) = 40$
    c. $(-5) \times (-9) = 45$

2. Multiply the following numbers:
    a. $(-7) \times 8$
    b. $6 \times (-3)$
    c. $(-10) \times 9$
    d. $4 \times (-8)$

    Solutions

    Each problem is multiplication of a positive and negative integer. Use the rules $(+) \times (-) = -$ and $(-) \times (+) = -$. The answers will all be negative.

    To find the answers, multiply positive numbers using the multiplication table. Then, change the answer to a negative number. For example, $7 \times 8 = 56$. This means $(-7) \times 8 = -56$.

    a. $(-7) \times 8 = -56$
    b. $6 \times (-3) = -18$
    c. $(-10) \times 9 = -90$
    d. $4 \times (-8) = -32$

Practice

Multiply the numbers:

1. -4×9
2. $-3 \times (-5)$
3. $4 \times (-7)$
4. $-1 \times (-1)$
5. -1×9
6. -10×0
7. $4 \times (-10)$
8. $3 \times (-9)$
9. $-6 \times (-8)$
10. -5×7

Lesson Title: Division of Integers	**Theme:** Numbers and Numeration
Practice Activity: PHM-07-064	**Class:** JSS 1

Learning Outcome
By the end of the lesson, you will be able to divide any two positive or negative integers.

Overview
In this lesson, you will solve division problems with positive and negative numbers. The rules in dividing integers are the same as in multiplying integers.

Positive ÷ Positive = Positive	$(+) \div (+) = +$
Negative ÷ Negative = Positive	$(-) \div (-) = +$
Positive ÷ Negative = Negative	$(+) \div (-) = -$
Negative ÷ Positive = Negative	$(-) \div (+) = -$

When we divide integers with the same sign, we get a positive quotient. When we divide integers with different sign we get a negative quotient.

Try to solve division problems on your own using multiplication facts that you know. If you need help, use the multiplication table in the previous lesson to divide.

Solved Examples

1. Divide the following numbers:
   a. $(-21) \div (-7)$
   b. $\frac{-40}{-8}$
   c. $(-25) \div (-5)$

   Solutions

   Each problem is division of 2 negative integers. Use the rule $(-) \div (-) = +$. The answers will all be positive.

   Remember that fractions are the same as division, so $\frac{-40}{-8} = -40 \div -8$.

   a. $(-21) \div (-7) = 3$
   b. $\frac{-40}{-8} = 5$
   c. $(-25) \div (-5) = 5$

2. Divide the following numbers:
    a. $(-60) \div 3$
    b. $6 \div (-3)$
    c. $(-10) \div 10$
    d. $48 \div (-8)$

    Solutions

    Each problem is division of a positive and negative integer. Use the rules $(+) \div (-) = -$ and $(-) \div (+) = -$. The answers will all be negative.

    a. $(-60) \div 3 = -20$
    b. $6 \div (-3) = -2$
    c. $(-10) \div 10 = -1$
    d. $48 \div (-8) = -6$

Practice

Divide the numbers:

1. $-45 \div 9$
2. $-30 \div (-5)$
3. $32 \div (-4)$
4. $-14 \div (-7)$
5. $\frac{-1}{1}$
6. $-320 \div 10$
7. $\frac{400}{-20}$
8. $9 \div (-3)$
9. $-16 \div (-8)$
10. $-500 \div (-10)$

Lesson Title: Story Problems on Integers	**Theme:** Numbers and Numeration
Practice Activity: PHM-07-065	**Class:** JSS 1

Learning Outcomes

By the end of the lesson, you will be able to:

1. Solve story problems involving positive and negative integers.
2. Apply the correct order of operations (BODMAS) to problems involving positive and negative integers.

Overview

Story problems are maths problems that are expressed as stories. Take your time to read story problems. Find the numbers, and decide which operation to use.

Here are some key words to look for in story problems. These will help you decide which operation to use:

- **Addition** words: sum, total, more than, all together
- **Subtraction** words: difference, less than, left, owe
- **Multiplication** words: times, each, total, all together
- **Division** words: share, each

Make sure to write your answer with units. For example, in Solved Example 1, the unit is degrees Celsius (°C). This must be included with the answer.

Remember to carry out calculations using the correct order of operations (BODMAS). The letters BODMAS stand for: Bracket, Of, Division Multiplication, Addition and Subtraction. When working problems which have more than one operation (of, ×, +, –, ÷), we use BODMAS. The letters of the word (BODMAS) tell us the order in which we should work the operations in a math problem.

Lesson 56 describes real-life situations when we use negative numbers. These are when money is borrowed, negative temperatures, and negative elevations. The same examples are used in this lesson.

Solved Examples

1. If it is 34°C in Freetown and -12°C in New York City, what is the temperature difference between the two cities?

Solution
We are asked to find the difference between 2 temperatures, but one of them is negative. Remember that negative temperatures are very cold. Remember that "difference" is a subtraction word, so we want to write a subtraction problem for this story: $34℃ - (-12℃)$.

Solve the subtraction problem using the rule for subtracting negative numbers. Remember that subtracting a negative is the same as addition: $-(-) \rightarrow +$.

$34℃ - (-12℃) = 34℃ + 12℃ = 46℃$

The difference in temperature is $46℃$.

2. Hawa started the day without money, but she earned Le 100,000.00 selling goods. However, she owes her sister Le 150,000.00. What is the balance of Hawa's money?

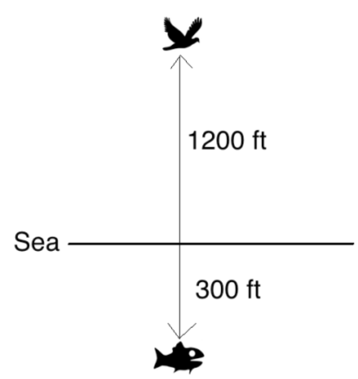
Solution
Remember that money owed is negative. Subtract the money she owes from the amount that Hawa has:

$\text{Le } 100{,}000.00 - \text{Le } 150{,}000.00 = -\text{Le } 50{,}000.00$

Hawa owes her sister Le 50,000.00 more than she has. Hawa is still in debt, and her balance is negative, $-\text{Le } 50{,}000.00$.

3. A bird is flying high above the sea, at a height of 1,200 feet above sea level. A fish is swimming directly under the bird, at a depth of 300 feet below sea level. What is their difference in elevation?

Solution
Remember that height above sea level is positive, and the distance below sea level is negative. You can draw a picture to help. →



The bird's elevation from sea level is +1,200 ft, and the fish's elevation from sea level is -300 ft.

Subtract to find their difference in elevation:

$1{,}200 - (-300) = 1{,}200 + 300 = 1{,}500$ ft.

4. Foday is making his budget for the week. He has Le 80,000.00. For each of the next 5 days, he will spend Le 15,000.00 on food and Le 5,000.00 on transportation. What will the balance of his money be?

   Solution

   To find the balance, we need to subtract Le 15,000.00 and Le 5,000.00 for each of the 5 days. In one day, he spends $(\text{Le } 15{,}000.00 + \text{Le } 5{,}000.00)$. Multiply this by 5 to find how much he spends in 5 days. Subtract the money he will spend from the money he has.

$$
\begin{aligned}
\text{Le } 80{,}000.00 - 5 \times (\text{Le } 15{,}000.00 + \text{Le } 5{,}000.00) &= \text{Le } 80{,}000.00 - 5 \times (\text{Le } 20{,}000.00) \\
&= \text{Le } 80{,}000.00 - \text{Le } 100{,}000.00 \\
&= -\text{Le } 20{,}000.00
\end{aligned}
$$

   His balance will be $-\text{Le } 20{,}000.00$. He doesn't have enough money for the week. He will need to earn or borrow money.

Practice

1. Abu has a cold storage where he sells fish and chicken. Inside the cold storage, the temperature is –8°C. The temperature outside is 30°C. What is the temperature difference?
2. Mustapha owed his sister Le 8,000.00. Today he earned Le 12,000.00. What is the balance of Mustapha's money?
3. Sia earned Le 80,000.00 today. However, she owes each of her 2 sisters Le 45,000.00. What is the balance of Sia's money?
4. A gold miner is underground, 120 feet below sea level. His brother is working on a mountain, 300 feet above sea level. What is their difference in elevation?

Lesson Title: Simple Proportion	Theme: Everyday Arithmetic
Practice Activity: PHM-07-066	**Class:** JSS 1

Learning Outcome
By the end of the lesson, you will be able to solve simple proportion problems.

Overview

A **proportion** is just two ratios that are equivalent or equal. A proportion can be written as 2 equivalent fractions. For example, these fractions are equivalent: $\frac{1}{2} = \frac{5}{10}$

Consider a story: Hawa reads 1 book every 2 days. She reads 5 books every 10 days.

We can write a proportion for this story. A proportion is a set of equivalent fractions with units. For this example, we have $\frac{1 \text{ book}}{2 \text{ days}} = \frac{5 \text{ books}}{10 \text{ days}}$. The units used on the numerators and denominators must be the same.

Proportions are good for figuring out something that you don't know from something you know. In this lesson, you will set up proportions for story problems. These problems will have an unknown amount, which you will solve for using cross-multiplication. In **cross-multiplication**, if we multiply the numbers diagonally across from each other, the two products will be equal.

For example, consider the question: How many books does Hawa read in 18 days?

We can write the proportion: $\frac{1 \text{ book}}{2 \text{ days}} = \frac{x \text{ books}}{18 \text{ days}}$. x is the unknown amount, which is the number of books she can read in 18 days.

To solve this problem and find how many books she reads in 18 days, cross-multiply. That is, multiply the diagonal numbers and set them equal: $1 \times 18 = 2 \times x$.

Follow these steps to solve for the unknown amount:

$$\begin{aligned} 1 \times 18 &= 2 \times x && \text{Cross-multiply} \\ 18 &= 2 \times x && \text{Multiply the left-hand side} \\ \tfrac{18}{2} &= x && \text{Divide both sides by 2} \\ 9 &= x \end{aligned}$$

Hawa can read 9 books in 18 days. Note that we now have 3 equivalent fractions for Hawa's story: $\frac{1 \text{ book}}{2 \text{ days}} = \frac{5 \text{ books}}{10 \text{ days}} = \frac{9 \text{ books}}{18 \text{ days}}$.

Solved Examples

1. Abu buys new exercise books for school. He buys 6 exercise books for Le 3,000.00. How much would it cost to buy 10 exercise books?

   Solution
   Let x be the cost of buying 10 exercise books.
   Write a proportion for the problem: $\frac{6 \text{ exercise books}}{\text{Le } 3{,}000.00} = \frac{10 \text{ exercise books}}{\text{Le } x}$

   Cross-multiply and solve for x:

$$\begin{aligned} 6 \times x &= 3{,}000 \times 10 && \text{Cross-multiply} \\ 6 \times x &= 30{,}000 && \text{Multiply the right-hand side} \\ x &= \tfrac{30{,}000}{6} && \text{Divide both sides by 6} \\ x &= 5{,}000 \end{aligned}$$

   It would cost Le 5,000.00 to buy 10 exercise books.

2. Fatu walks to school each day. She walks 4 kilometres in 36 minutes. How long would it take her to walk 3 kilometres?

   Solution
   Let x be the time Fatu takes to walk the 3 km.
   Write a proportion for the problem: $\frac{4 \text{ km}}{36 \text{ minutes}} = \frac{3 \text{ km}}{x \text{ minutes}}$

Cross-multiply and solve for x:

$$\begin{aligned} 4 \times x &= 36 \times 3 && \text{Cross-multiply} \\ 4 \times x &= 108 && \text{Multiply the right-hand side} \\ x &= \tfrac{108}{4} && \text{Divide both sides by 4} \\ x &= 27 \end{aligned}$$

It would take her 27 minutes to walk 3 km.

3. Sia sells goods in the market. She estimates that she earns Le 20,000.00 for every 6 hours that she works. She wants to save Le 80,000.00 for a new maths textbook. How many hours does she need to work to earn Le 80,000.00?

Solution
Let x be the number of hours she will work to earn Le 80,000.00.
Write a proportion for the problem: $\frac{\text{Le } 20{,}000.00}{6 \text{ hours}} = \frac{\text{Le } 80{,}000.00}{x \text{ hours}}$

Cross-multiply and solve for x:

$$\begin{aligned} 20{,}000 \times x &= 6 \times 80{,}000 && \text{Cross-multiply} \\ 20{,}000 \times x &= 480{,}000 && \text{Multiply the right-hand side} \\ x &= \tfrac{480{,}000}{20{,}000} && \text{Divide both sides by 20,000} \\ x &= 24 \end{aligned}$$

She needs to work 24 hours to earn Le 80,000.00.

4. A car travels 160 kilometres in 2 hours. How far will it travel in 7 hours?

Solution
Let x be the distance travelled in 7 hours.
Write a proportion for the problem: $\frac{160 \text{ km}}{2 \text{ hours}} = \frac{x \text{ km}}{7 \text{ hours}}$

Cross-multiply and solve for x:

$$\begin{aligned} 160 \times 7 &= 2 \times x && \text{Cross-multiply} \\ 1120 &= 2 \times x && \text{Multiply the left-hand side} \\ \tfrac{1120}{2} &= x && \text{Divide both sides by 2} \\ 560 &= x \end{aligned}$$

The car will travel 560 km in 7 hours.

Practice

1. A man can ride his bicycle 30 kilometres in 2 hours. How far can he ride in 5 hours?
2. Mustapha bought 8 oranges for Le 16,000.00. How much would he pay for 20 oranges?
3. Juliet earns Le 20,000.00 for 3 hours of work. She wants to save Le 120,000.00 to buy a new tire for her bicycle. How many hours does she need to work to earn Le 120,000.00?
4. Aminata is a doctor. In an 8-hour shift at the hospital, she treated 24 patients. If she works 40 hours in a week, how many patients does she treat each week?
5. A bus can carry 35 passengers in one trip. How many such buses will be needed to carry 210 pupils on an outing?
6. Five packets of candles cost Le 30,000.00. Kadie was given Le 90,000.00 to buy packets of candles. How many packets can she get with this amount?

Lesson Title: Simple Interest	**Theme:** Everyday Arithmetic
Practice Activity: PHM-07-067	**Class:** JSS 1

Learning Outcome
By the end of the lesson, you will be able to solve problems involving simple interest.

Overview

Interest is extra money that is charged or paid. For example, banks sometimes add interest to the amount of money in a savings account. They also add interest to the amount of money a person owes for a loan. Interest is paid regularly at a certain rate, given as a percentage.

The equation to calculate simple interest is $I = PRT$, where P is the principal, R is the rate, and T is time in years.

Principal is the original amount of money borrowed, lent, or invested at the given rate.

In simple interest problems, **time** should be in years. We often see 'per annum' in interest problems, which means yearly or annually.

The **rate** R is a percentage. It should be converted to a fraction before substituting it in the formula. So, if the rate is 5%, you should convert it to a fraction ($\frac{5}{100}$) before substituting it into the formula.

Solved Examples

1. Find the simple interest on Le 400,000.00 for 3 years at an interest rate of 5% per annum.

   Solution

   Substitute the given numbers into the formula $I = PRT$ and find the simple interest, I.

$$\begin{aligned} I &= PRT \\ &= 400{,}000 \times \frac{5}{100} \times 3 \\ &= 4000 \times 5 \times 3 \\ &= \text{Le } 60{,}000.00 \end{aligned}$$

The simple interest is Le 60,000.00

2. Find the amount of interest on Le 350,000.00 invested in a bank for a period of 4 years at a rate of 12%.

Solution

Substitute the given numbers into the formula $I = PRT$ and find the simple interest, I.

$$\begin{aligned} I &= PRT \\ &= 350{,}000 \times \frac{12}{100} \times 4 \\ &= 3500 \times 12 \times 4 \\ &= \text{Le } 168{,}000.00 \end{aligned}$$

The simple interest is Le 168,000.00

3. What is the interest paid on Le 45,000.00 borrowed for 3 years at a rate of 10% per annum?

Solution

Substitute the given numbers into the formula $I = PRT$ and find the simple interest, I.

$$\begin{aligned} I &= PRT \\ &= 45{,}000 \times \frac{10}{100} \times 3 \\ &= 450 \times 10 \times 3 \\ &= \text{Le } 13{,}500.00 \end{aligned}$$

The simple interest is Le 13,500.00

Practice

1. Find the interest on Le 500,000.00 for 9 years at 6% interest per annum.
2. Find the simple interest on Le 700,000.00 for 5 years at 4% per annum.
3. Find the simple interest on Le 120,000.00 at 2% for 5 years.
4. What is the interest paid on Le 50,000.00 borrowed for 2 years at a rate of 2% per annum?

Lesson Title: Discount	Theme: Everyday Arithmetic
Practice Activity: PHM-07-068	**Class:** JSS 1

Learning Outcome
By the end of the lesson, you will be able to calculate discount on a given sum of money.

Overview

In many stores, items have set prices. For example, in one store an exercise book may cost Le 2,000.00. This is the **marked price**. Marked price is the amount written on the price ticket for the item. It is also known as the original price, normal price, non-discounted price, or the price tag price.

The owner of the shop might sell an exercise book for less than Le 2,000.00. For example, the owner might meet a pupil with only Le 1,800.00. She can decide to sell the exercise book for that price. The price at which an item is sold is called the **selling price**.

Discount is an amount that is subtracted from the marked price. In this example, the discount is Le 200.00. We can find this by subtracting the selling price from the marked price:

Discount = Marked Price – Selling Price = Le 2,000.00 – Le 1,800.00 = Le 200.00

In most cases, we are not given the selling price of an item. We are often given the original price, and the discount as a percentage. We can use the percentage discount to find the amount of the discount in Leones, and the selling price of the item.

To find the discount, use the formula:

$$\text{Discount} = \text{Original price} \times \text{Rate}$$

Note that rate is given as a percentage, but should be written as a fraction in the formula. For example, $10\% = \frac{10}{100}$.

If you know the marked price and the discount of an item, you can subtract to find the selling price:

$$\text{Selling Price} = \text{Marked Price} - \text{Discount}$$

Solved Examples

1. In a shop, a bag of flour is marked "Get a 20% discount". If its marked price is Le 40,000.00, find the following:
   a. The discount
   b. The selling price

   Solutions

   a. Use the formula to find the discount:

$$\begin{aligned} \text{Discount} &= \text{Original price} \times \text{Rate} \\ &= 40{,}000 \times \tfrac{20}{100} \\ &= 400 \times 20 \\ &= 8{,}000 \end{aligned}$$

   The discount is Le 8,000.00.

   b. Use the formula to find the selling price:

$$\begin{aligned} \text{Selling Price} &= \text{Marked Price} - \text{Discount} \\ &= 40{,}000 - 8{,}000 \\ &= 32{,}000.00 \end{aligned}$$

   The selling price is Le 32,000.00.

2. During a clearance sale, a shop reduced the price of its goods by 10%. If the original price of a maths textbook was Le 70,000.00, what is the selling price?

   Solution
   We are not asked to find the amount of the discount in this problem. However, we will find the discount first and use it to find the selling price.

   Step 1. Find the discount:

$$\begin{aligned} \text{Discount} &= \text{Original price} \times \text{Rate} \\ &= 70{,}000 \times \tfrac{10}{100} \\ &= 700 \times 10 \\ &= 7{,}000 \end{aligned}$$

   The discount is Le 7,000.00.

Step 2. Find the selling price:

$$\begin{aligned}\text{Selling Price} &= \text{Marked Price} - \text{Discount}\\ &= 70{,}000 - 7{,}000\\ &= 63{,}000.00\end{aligned}$$

The selling price is Le 63,000.00.

3. Fatu is shopping for a new phone. She has Le 300,000.00 to spend. She found a phone marked 25% off. The original price of the phone is Le 350,000.00. Does Fatu have enough money to buy it?

Solution

We need to find the selling price of the phone. If it is Le 300,000.00 or less, then Fatu will have enough money to buy it.

Step 1. Find the discount:

$$\begin{aligned}\text{Discount} &= \text{Original price} \times \text{Rate}\\ &= 350{,}000 \times \frac{25}{100}\\ &= 3{,}500 \times 25\\ &= 87{,}500\end{aligned}$$

The discount is Le 87,500.00.

Step 2. Find the selling price:

$$\begin{aligned}\text{Selling Price} &= \text{Marked Price} - \text{Discount}\\ &= 350{,}000 - 87{,}500\\ &= 262{,}500.00\end{aligned}$$

The selling price is Le 262,500.00

Answer: Yes, Fatu has enough money because 262,500 < 300,000.

Practice

1. A shop owner is giving a 15% discount on a book that originally cost Le 50,000.00. Find the following:
    a. The discount
    b. The selling price
2. Mark wants to buy a new refrigerator for his restaurant. He found a refrigerator with a marked price of Le 5,000,000.00. It is marked 10% off. What is the selling price of the refrigerator?
3. Sia is shopping for a new bicycle. She has Le 800,000.00 to spend. She found a bicycle with a selling price of Le 900,000.00. The shop owner will give her a 10% discount. Does she have enough money?
4. Find the sale price for an item that has a price tag of Le 200,000.00 and a discount rate of 25%.

Lesson Title: Commission	**Theme:** Everyday Arithmetic
Practice Activity: PHM-07-069	**Class:** JSS 1

Learning Outcome
By the end of the lesson, you will be able to calculate commission on a given sum of money.

Overview

Commission is an amount of money that someone receives when they sell something. Commission is usually a percent of the sale that has been made.

For example, real estate agents are often paid commission. Real estate agents are people who sell other people's houses. They often earn a percentage of the sales price of each house. If the house they sell is more expensive, they earn more money.

To calculate commission, use the formula:

$$\text{Commission} = \text{Selling price} \times \text{Rate of commission}$$

Note that rate is given as a percentage, but should be written as a fraction in the formula. For example, $10\% = \frac{10}{100}$.

Solved Examples

1. Alice works as a real estate agent. She has just sold a house for Le 7,000,000.00. If Alice makes a 2% commission, how much money did she make when she sold the house?

   Solution
   Apply the commission formula:

$$\begin{aligned} \text{Commission} &= \text{Selling price} \times \\ &\quad \text{Rate of commission} \\ &= 7{,}000{,}000 \times \tfrac{2}{100} \\ &= 70{,}000 \times 2 \\ &= 140{,}000 \end{aligned}$$

   Alice's commission was Le 140,000.00

2. Fatu works in a mobile phone store. She earns a 5% commission on each phone she sells. Today she sold a phone for Le 650,000.00. What commission did she earn?

Solution

Apply the commission formula:

$$\begin{aligned}\text{Commission} &= \text{Selling price} \times \\ & \text{Rate of commission} \\ &= 650{,}000 \times \frac{5}{100} \\ &= 6{,}500 \times 5 \\ &= 32{,}500\end{aligned}$$

Fatu's commission was Le 32,500.00

3. Abass works as a real estate agent and earns a commission of 3%. Last month, he sold 3 houses for the following amounts. Find his commission for each house:
   a. Le 9,000,000.00
   b. Le 5,500,000.00
   c. Le 12,000,000.00

Solutions

Use the same rate, 3%, to find the commission for each house:

a. 3% commission on a Le 9,000,000.00 house:

$$\begin{aligned}\text{Commission} &= \text{Selling price} \times \\ & \text{Rate of commission} \\ &= 9{,}000{,}000 \times \frac{3}{100} \\ &= 90{,}000 \times 3 \\ &= \text{Le } 270{,}000.00\end{aligned}$$

b. 3% commission on a Le 5,500,000.00 house:

$$\begin{aligned}\text{Commission} &= \text{Selling price} \times \\ & \text{Rate of commission} \\ &= 5{,}500{,}000 \times \frac{3}{100} \\ &= 55{,}000 \times 3 \\ &= \text{Le } 165{,}000.00\end{aligned}$$

c. 3% commission on a Le 12,000,000.00 house:

$$\begin{aligned} \text{Commission} &= \text{Selling price} \times \\ &\quad \text{Rate of commission} \\ &= 12{,}000{,}000 \times \tfrac{3}{100} \\ &= 120{,}000 \times 3 \\ &= \text{Le } 360{,}000.00 \end{aligned}$$

Practice

1. Martin works in a furniture shop. He is paid 5% commission on his sales. One day he made the following 3 sales: a bed for Le 200,000.00, a table for Le 400,000.00 and a chair for Le 100,000.00. What was Martin's commission on his total sales?
2. Sia is a real estate agent. Today she sold a house for Le 9,800,000.00. If her rate of commission is 6%, how much money did she make?
3. Aminata works as a car salesperson and earns a commission of 4%. Last month, she sold 3 cars for the following amounts. Find her commission for each car:
   a. Le 25,000,000.00
   b. Le 10,000,000.00
   c. Le 8,500,000.00

Lesson Title: Tax	Theme: Everyday Arithmetic
Practice Activity: PHM-07-070	Class: JSS 1

Learning Outcome
By the end of the lesson, you will be able to calculate tax on a given sum of money.

Overview

Taxes are how a government raises money to cover public costs. For example, tax money pays for hospitals, roads and schools. Governments have different ways of raising money through taxes. Income tax is an amount that people pay from the money they earn working. Sales tax is an amount that people pay when they buy something from a store. It can also be called "goods and services tax" (GST).

In this lesson, you will calculate sales tax. We don't always pay sales tax at every small shop. However, more and more shops are adding sales tax to the cost of items. This money goes to the government to help Sierra Leone pay for things we need.

The formula for calculating sales tax on an item is:

$$\text{Sales tax} = \text{cost of the item} \times \text{tax rate}$$

As in the previous lessons, rate is given as a percentage, but should be written as a fraction in the formula. For example, $10\% = \frac{10}{100}$.

You can find the total amount you will pay for an item by adding sales tax to its cost.

Solved Examples

1. Ama is buying a new maths textbook. The textbook costs Le 30,000.00. If the sales tax rate is 6%, find:
   a. The sales tax
   b. The total amount she will pay

   Solutions

   a. Calculate the sales tax using the formula:

   $$\begin{aligned} \text{Sales tax} &= \text{cost of the item} \times \text{tax rate} \\ &= 30{,}000 \times \frac{6}{100} \\ &= 300 \times 6 \\ &= \text{Le } 1{,}800.00 \end{aligned}$$

   b. Add the sales tax to the price to find the total amount she will pay:
   $30{,}000 + 1{,}800 = \text{Le } 31{,}800.00$

2. A mobile phone costs Le 350,000.00. A 5% goods and services tax (GST) is now imposed. What is the new price of the mobile phone?

Solution

We are not asked to find the amount of the tax in this problem. However, we will find the tax first and use it to find the new price.

Step 1. Calculate the sales tax:

$$\begin{aligned}\text{Sales tax} &= \text{cost of the item} \times \text{tax rate}\\ &= 350{,}000 \times \frac{5}{100}\\ &= 3500 \times 5\\ &= \text{Le } 17{,}500.00\end{aligned}$$

Step 2. Calculate the total cost:

$$350{,}000 + 17{,}500 = \text{Le } 367{,}500.00$$

3. George buys a car with a selling price of Le 5,000,000.00. If the sales tax rate is 4%, how much will he pay for the car?

Solution

Step 1. Calculate the sales tax:

$$\begin{aligned}\text{Sales tax} &= \text{cost of the item} \times \text{tax rate}\\ &= 5{,}000{,}000 \times \frac{4}{100}\\ &= 50{,}000 \times 4\\ &= \text{Le } 200{,}000.00\end{aligned}$$

Step 2. Calculate the total cost:

$$5{,}000{,}000 + 200{,}000 = \text{Le } 5{,}200{,}000.00$$

Practice

1. Hawa is buying a new phone. The phone costs Le 450,000.00. If the sales tax rate is 5%, find:
    a. The sales tax
    b. The total amount she will pay
2. A fan costs Le 500,000.00. A 6% goods and services tax (GST) is now imposed. What is the new price of the fan?
3. Michael buys a new stove for Le 3,000,000.00. If the sales tax rate is 4%, how much will he pay for the stove?

Lesson Title: Units of Measurements	Theme: Measurement and Estimation
Practice Activity: PHM-07-071	**Class:** JSS 1

Learning Outcomes

By the end of the lesson, you will be able to:

1. Identify the units of measurement for length, mass and volume.
2. Compare mass and volume.

Overview

This is the first lesson on units of measurement. In this lesson, you will identify the units of measurement and understand the difference between mass and volume. In the following lessons, you will convert between units of measurement.

Length is how long or short something is, or the distance between 2 points. Length is used by tailors to take measurements and make clothes. It is used by carpenters to build furniture, and by builders to build houses.

Mass is the quantity of matter an object contains. It is related to the weight of the object. It is used to measure the amount of rice, or the size of our bodies.

Volume is the capacity or space a substance occupies. We measure liquids in volume. For example, petrol and oil are sold by volume.

We use the metric system of measurement. Common units and their abbreviations are:

- **Length**: millimetre (mm), centimetre (cm), metre (m), kilometre (km).
- **Mass**: milligramme (mg), gramme (g), kilogramme (kg), tonne (t).
- **Volume**: millilitre (ml), decilitre (dl), litre (l), kilolitre (kl).

To understand the difference between mass and volume, imagine that you have 2 bottles of the same size. Each holds 1 litre of liquid. One is full of water, the other one is full of palm oil. You place them on a scale to find their mass. One litre of water has a mass of 1,000 g. One litre of palm oil has a mass of 890 g. In other words, they have the same volume but different mass.

1 litre water
1,000 g water

1 litre palm oil
890 g palm oil

Solved Examples

1. From the list of items, choose the items that are measurements of mass, and the items that are measurements of volume. Write the letters that match each type of measurement in the table.

   a. A person's weight (kg)
   b. Palm oil (l)
   c. Gold (mg)
   d. A bottle of cola (ml)
   e. A bag of cassava (kg)
   f. A pile of pepper (g)
   g. Water in a lake (kl)
   h. A vaccine (ml)
   i. A bottle of water (l)

Mass	Volume

Solution

Remember that the following are units of mass: kg, mg, and g. Any item with these units belongs in the box for mass. The following are units of volume: l, ml, and kl. Any item with these units belongs in the box for volume.

The answers are:

Mass	Volume
a, c, e, f	b, d, g, h, i

2. List all of the units that can be used to measure the size of the following:

   a. The length of your classroom.
   b. The distance from Foday's house to school.
   c. The amount of juice in a bottle.
   d. The size of a newborn baby.

Solutions

a. Length can be measured in mm, cm, m, or km.
b. Distance and length are the same. Correct answers are mm, cm, m, or km.
c. The amount of a liquid is volume. Correct answers are ml, dl, l, kl.
d. The size of a newborn baby is its mass. Correct answers are mg, g, kg, and t.

Practice

1. From the list of items, choose the items that are measurements of mass, and the items that are measurements of volume. Write the letters that match each type of measurement in the table.

    a. A farmer's eggplant harvest (kg)
    b. Water in a cup (ml)
    c. A diamond (mg)
    d. Water in a river (dl)
    e. Petrol in a car (l)
    f. A bag of coal (kg)
    g. A bag of salt (g)
    h. Water in a coconut (ml)
    i. Size of a goat (kg)

Mass	Volume

2. List all of the units that can be used to measure the size of the following:

    a. The size of a chicken.
    b. The amount of water in a bucket.
    c. The distance from your house to the nearest health clinic.

Lesson Title: Conversion of Length	Theme: Measurement and Estimation
Practice Activity: PHM-07-072	**Class:** JSS 1

Learning Outcome
By the end of the lesson, you will be able to convert between units of length within the same system.

Overview

In this lesson, you will convert between different units of length. Recall that common units for length in the metric system are millimetre (mm), centimetre (cm), metre (m) and kilometre (km).

These are the relationships between these units:

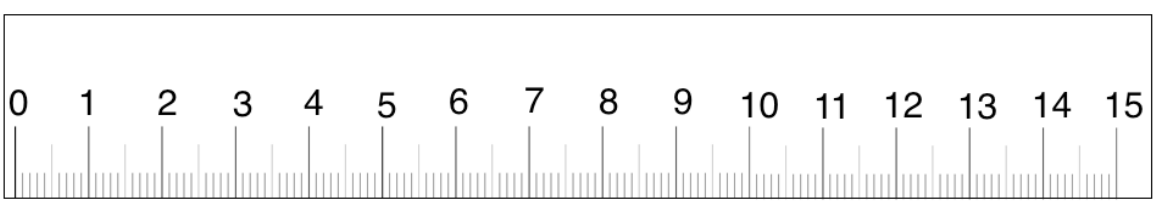
10 mm = 1 cm
100 cm = 1 m
1,000 mm = 1 m
1,000 m = 1 km

These relationships are what we use to convert units in the measurement of length.

Of these 4 units, **kilometre** is the longest. We use kilometre to measure distance, such as the difference between two villages.

Metre is the next longest. The distance between your finger tips and the centre of your body is about one metre. There are 1,000 metres in one kilometre. Metres are used to measure the length of a football field, or the length of a room.

Centimetre is the next longest. Look at your fingernails. They are approximately 1 cm wide. Centimetres are used to measure small items. For example, tailors use centimetres to measure pieces when they make clothing. Centimetres can be measured using rulers like the one shown below:



Millimetres are very small. The ruler above shows 15 cm. In each centimetre, there are 10 mm. These are shown with the small lines on the ruler. The ruler above has 150 millimetres! Millimetres are used to measure very small objects. For example, a cucumber seed could be measured in millimetres.

Any length of distance can be given in km, m, cm, or mm. We can convert between units of measurement. We use the relationship between the 2 units (for example, 1 km = 1,000 m) and we either multiply or divide.

- To change from larger unit to smaller unit: multiply by the power of 10. See Solved Example 1.
- To change from a smaller unit to a larger unit: divide by the power of 10. See Solved Example 2.

When converting between units in the metric system, you will be multiplying and dividing by powers of 10 (10, 100, 1,000). Remember the rules for multiplying and dividing by 10. These were covered in lessons 29 and 30.

Solved Examples

1. Convert 25 kilometres to metres.

   Solution

   Use the relationship between kilometres and metres, 1 km = 1,000 m. We are converting from a larger unit (km) to a smaller unit (m), so we will multiply.

   $$25\ \text{km} = (25 \times 1{,}000)\ \text{m} = 25{,}000\ \text{m}$$

   25 kilometres is the same as 25,000 metres.

2. Convert 350 metres to kilometres.

   Solution

   Use the relationship between kilometres and metres again, 1 km = 1,000 m. We are now converting from a smaller unit (m) to a larger unit (km), so we will divide.

$350 \text{ m} = (350 \div 1{,}000) \text{ km} = 0.35 \text{ km}$

350 metres is the same as 0.35 kilometres.

3. Convert the following measurements to metres:
   a. 850 cm   b. 75 cm   c. 4 km   d. 0.7 km

   Solutions

   Note that to convert from cm to m, we must divide. To convert from km to m, we must multiply.

   a. $850 \text{ cm} = (850 \div 100) \text{ m} = 8.5 \text{ m}$
   b. $75 \text{ cm} = (75 \div 100) \text{ m} = 0.75 \text{ m}$
   c. $4 \text{ km} = (4 \times 1{,}000) \text{ m} = 4{,}000 \text{ m}$
   d. $0.7 \text{ km} = (0.7 \times 1{,}000) \text{ m} = 700 \text{ m}$

4. Add the following measurements, and express your answer in kilometres: 500 m, 700 m, 400 m.

   Solution
   Step 1. Add the measurements: $500 + 700 + 400 = 1{,}600 \text{ m}$
   Step 2. Convert to kilometres: $1{,}600 \text{ m} = (1600 \div 1{,}000) \text{ km} = 1.6 \text{ km}$

Practice

1. Convert 25 mm to cm.
2. Convert 10 cm to mm.
3. Convert 220 mm to cm.
4. Convert 150 cm to m.
5. Convert 600 m to km.
6. Convert 55 m to km.
7. Convert 7 km to m.
8. Convert 0.075 km to m.
9. Add the following measurements, and give your answer in metres: 20 cm, 8 cm, 75 cm, 10 cm.

Lesson Title: Conversion of Mass	Theme: Measurement and Estimation
Practice Activity: PHM-07-073	**Class:** JSS 1

Learning Outcome
By the end of the lesson, you will be able to convert between units of mass within the same system.

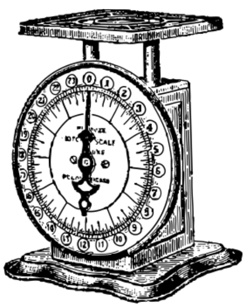
Overview

In this lesson, you will convert between different units of mass. Recall that common units for mass in the metric system are milligramme (mg), gramme (g), kilogramme (kg) and tonne (t).

These are the relationships between these units:

1,000 mg	=	1 g
1,000 g	=	1 kg
1,000 kg	=	1 t

These relationships are what we use to convert units in the measurement of mass. Mass can be measured with a scale. There are different types of scales. Here are some examples:



Tonne is the largest unit we will use for mass. We use tonnes to measure very large things, such as lorries or shipping containers.

Kilogramme is the next largest. Kilogrammes are commonly used to measure our bodies, or bags of rice. If you go to the hospital, you may measure your own mass with a kilogramme scale.

Grammes are smaller than kilogrammes and can measure smaller amounts of things. For example, a cup of sugar could be measured in grammes. Some scales measure in grammes. You may see these scales in some shops.

Milligrammes are smaller than grammes. Milligrammes are used when you have a very small quantity of something. For example, a medicine tablet or some gold.

Any mass can be given in mg, g, kg, or t. You will use the same process that you used to convert between lengths. We can convert between units of measurement. We use the relationship between the 2 units (for example, 1 kg = 1,000 g) and we either multiply or divide.

- To change from larger unit to smaller unit: multiply by the power of 10. See Solved Example 1.
- To change from a smaller unit to a larger unit: divide by the power of 10. See Solved Example 2.

When converting between units in the metric system, you will be multiplying and dividing by powers of 10 (10, 100, 1,000). Remember the rules for multiplying and dividing by 10. These were covered in lessons 29 and 30.

Solved Examples

1. Convert 2.5 kilogrammes to grammes.

   Solution

   Use the relationship between kilogrammes and grammes, 1 kg = 1,000 g. We are converting from a larger unit (kg) to a smaller unit (g), so we will multiply.

   $$2.5 \text{ kg} = (2.5 \times 1{,}000) \text{ g} = 2{,}500 \text{ g}$$

   2.5 kilogrammes is the same as 2,500 grammes.

2. Convert 800 grammes to kilogrammes.

   Solution

   Use the relationship between kilogrammes and grammes again, 1 kg = 1,000 g. We are now converting from a smaller unit (g) to a larger unit (kg), so we will divide.

$800 \text{ g} = (800 \div 1{,}000)\text{kg} = 0.8 \text{ kg}$

800 grammes is the same as 0.8 kilogrammes.

3. Convert the following measurements to kilogrammes:
   a. 2 t   b. 0.5 t   c. 4,000 g   d. 20.25 g

   Solutions

   Note that to convert from t to kg, we must multiply. To convert from g to kg, we must divide.

   a. $2 \text{ t} = (2 \times 1{,}000) \text{ kg} = 2{,}000 \text{ kg}$
   b. $0.5 \text{ t} = (0.5 \times 1{,}000)\text{kg} = 500 \text{ kg}$
   c. $4{,}000 \text{ g} = (4{,}000 \div 1{,}000) \text{ kg} = 4 \text{ kg}$
   d. $20.25 = (20.25 \div 1{,}000) \text{ kg} = 0.02025 \text{ kg}$

4. Add the following measurements, and express your answer in grammes: 300 mg, 250 mg, 180 mg, 700 mg.

   Solution
   Step 1. Add the measurements: $300 + 250 + 180 + 700 = 1{,}430 \text{ mg}$
   Step 2. Convert to grammes: $1{,}430 \text{ mg} = (1{,}430 \div 1{,}000) \text{ g} = 1.43 \text{ g}$

Practice

1. Convert 25 mg to g.
2. Convert 250 g to kg.
3. Convert 3,300 g to kg.
4. Convert 2,430 kg to t.
5. Convert 560 kg to t.
6. Convert 2,600 mg to g.
7. Convert 8.5 t to kg.
8. Convert 0.05 t to kg.
9. The following are the weights of 3 small cars in kilogrammes: 900 kg, 750 kg, 875 kg. Find the total weight of the 3 cars in tonnes.

Lesson Title: Conversion of Volume	Theme: Measurement and Estimation
Practice Activity: PHM-07-074	**Class:** JSS 1

Learning Outcome
By the end of the lesson, you will be able to convert between units of volume within the same system.

Overview

In this lesson, you will convert between different units of volume. Recall that common units for volume in the metric system are millilitre (ml), decilitre (dl), litre (l) and kilolitre (kl). Of these, millilitres and litres are most common in everyday life.

These are the relationships between these units:

1,000 ml	=	1 l
100 cl	=	1 l
10 dl	=	1 l
1 kl	=	1,000 l

These relationships are what we use to convert units in the measurement of volume.

Litre is the standard unit used to measure volume. It is very common, and liquids such as petrol and water are often sold by the litre.

Kilolitres are bigger than litres. The volume of very large things such as water tanks or petrol trucks can be measured in kilolitres.

Centilitres, **decilitres** and **millilitres** are smaller than litres. Centilitres and decilitres are not used often, because we normally measure small amounts of liquid in millilitres. For example, if you buy a small bottle of water it may say 500 ml on it. This is the same as 5 dl or 50 cl.

Any mass can be given in kl, l, cl, dl or ml. You will use the same process that you used to convert between lengths. We can convert between units of measurement. We use the relationship between the 2 units (for example, 1 l = 1,000 ml) and we either multiply or divide.

- To change from larger unit to smaller unit: multiply by the power of 10. See Solved Example 1.
- To change from a smaller unit to a larger unit: divide by the power of 10. See Solved Example 2.

When converting between units in the metric system, you will be multiplying and dividing by powers of 10 (10, 100, 1,000). Remember the rules for multiplying and dividing by 10. These were covered in lessons 29 and 30.

Solved Examples

1. Convert 1.5 litres to millilitres.

   Solution

   Use the relationship between litres and millilitres, 1 l = 1,000 ml. We are converting from a larger unit (l) to a smaller unit (ml), so we will multiply.

   $$1.5\ \mathrm{l} = (1.5 \times 1{,}000)\ \mathrm{ml} = 1{,}500\ \mathrm{ml}$$

   1.5 litres is the same as 1,500 millilitres.

2. Convert 3,200 ml to litres.

   Solution

   Use the relationship between litres and millilitres again, 1 l = 1,000 ml. We are now converting from a smaller unit (ml) to a larger unit (l), so we will divide.

   $$3{,}200\ \mathrm{ml} = (3{,}200 \div 1{,}000)\ \mathrm{l} = 3.2\ \mathrm{l}$$

   3,200 ml is the same as 3.2 litres.

3. Convert the following measurements to litres:

   a. 200 dl b. 20 cl c. 1.7 kl d. 450 ml

Solutions

Note that to convert from kl to l, we must multiply. To convert from dl, cl or ml, we must divide.

a. $200 \text{ dl} = (200 \div 10) \text{ l} = 20 \text{ l}$
b. $20 \text{ cl} = (20 \div 100) \text{ l} = 0.2 \text{ l}$
c. $1.7 \text{ kl} = (1.7 \times 1{,}000) \text{ l} = 1{,}700 \text{ l}$
d. $450 \text{ ml} = (450 \div 1{,}000) \text{ l} = 0.45 \text{ l}$

4. Abu drinks 3 cups of juice. If each cup is 400 ml, how much does he drink all together? Give your answer in litres.

Solution

Step 1. Multiply to find how much he drank: $3 \times 400 \text{ ml} = 1{,}200 \text{ ml}$
Step 2. Convert to litres: $1{,}200 \text{ ml} = (1{,}200 \div 1{,}000)\text{l} = 1.2 \text{ l}$

Practice

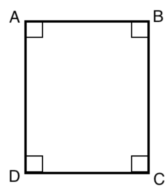
1. Convert 36 ml to l.
2. Convert 3 dl to l.
3. Convert 0.75 l to ml.
4. Convert 420 cl to l.
5. Convert 0.6 kl to l.
6. Convert 0.25 l to dl.
7. Convert 25 kl to l.
8. Convert 3,670 ml to l.
9. Convert 0.075 l to ml.
10. Fatu sells 3 bottles of palm oil in one day. The volumes of the bottles are 500 ml, 750 ml, and 800 ml. How much palm oil did she sell? Give your answer in litres.

Lesson Title: Review of Plane Shapes	Theme: Measurement and Estimation
Practice Activity: PHM-07-075	**Class:** JSS 1

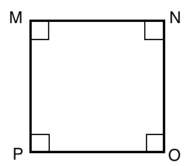
Learning Outcome
By the end of the lesson, you will be able to identify and label the parts of rectangles, squares and triangles.

Overview

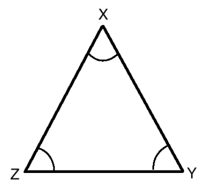
This lesson is on 3 shapes, rectangles, squares, and triangles. Examples of these shapes and their characteristics are given below. Small lines are used to show that sides are equal in length.



Rectangle
- 4 sides
- 4 angles
- Opposite sides equal in length
- Opposite sides parallel
- Angles are 90° (right angles)



Square
- 4 sides
- 4 angles
- All sides are equal in length
- Angles are 90° (right angles)

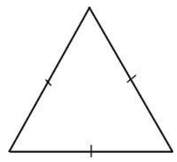


Triangle
- 3 sides
- 3 angles

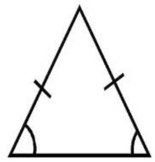
The point at which any two sides meet is an angle. Each angle is named by a letter. We name shapes according to the labels on their angles. Above, we have rectangle ABCD, Square MNOP, and triangle XYZ.

Squares and rectangles are called **quadrilaterals**. 'Quad' means four, and they're called quadrilaterals because they both have 4 sides.

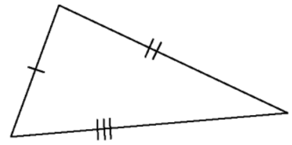
There are some special types of triangles that you should also know:



Equilateral Triangle
3 sides are equal
3 angles are equal



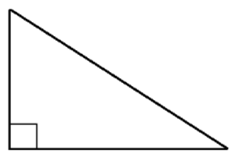
Isosceles Triangle
2 sides are equal
2 angles are equal



Scalene Triangle
All sides are different
All angles are different

Small marks on lines are used to show that lines are equal in length. If the lines have a different number of marks, it means they have different lengths. For example, the scalene triangle has sides with 1, 2 and 3 marks. This means the sides are 3 different lengths.

There is another type of triangle called a right-angled triangle. Right-angled triangles have 1 right angle, which is like the angle of a square or rectangle. Right angles are always shown with a small square. Right-angled triangles can be isosceles or scalene triangles.



Solved Examples

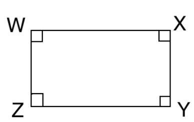
1. Draw the following shapes:
   a. Rectangle WXYZ
   b. Square DEFG
   c. Triangle RST

   Solutions

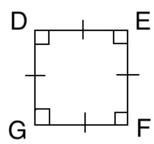
   Your shapes may have any size. Make sure they fit with the characteristics given in the Overview.

   These are examples:

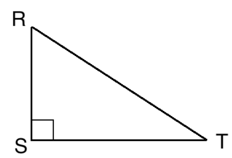
a.



b.



c.

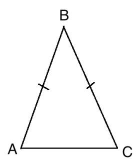


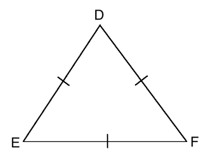
2. Draw the following shapes:

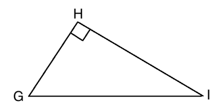
   a. An isosceles triangle ABC.
   b. An equilateral triangle DEF.
   c. A right-angled triangle GHI.
   d. A scalene triangle JKL.

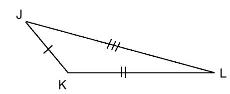
   Solutions

   Your shapes may have any size. Make sure they fit with the characteristics given in the Overview.

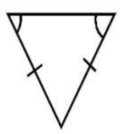
a.


b.


c.


d.


3. Identify each what type of triangle each triangle is. Give your reasons.

a.


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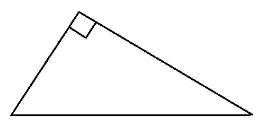
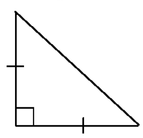


c.


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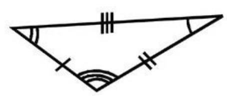
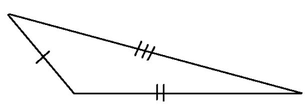
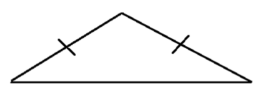


e.


f.


   Solutions

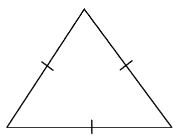
   Determine the type of triangle by looking at the marks on the sides and angles.

   a. Isosceles triangle. Two sides are equal.
   b. Right-angled triangle. It has a right angle. It also appears to be a scalene triangle.
   c. Right-angled triangle and isosceles triangle. It has a right angle and 2 equal sides.
   d. Scalene triangle. All 3 sides are different.
   e. Scalene triangle. All 3 sides are different.
   f. Isosceles triangle. Two sides are equal.

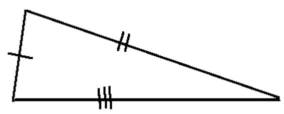
Practice

1. Draw the following shapes:
    a. Square GOAT
    b. Rectangle BIRD
    c. Triangle CAT
2. Draw the following triangles:
    a. Scalene triangle RAT
    b. Isosceles triangle DOG
    c. Equilateral triangle ANT
    d. Right-angled triangle COW
3. Identify what type of triangle each triangle is. Give your reasons.

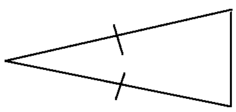
a.



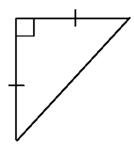
b.



c.



d.



Lesson Title: Perimeter	**Theme:** Measurement and Estimation
Practice Activity: PHM-07-076	**Class:** JSS 1

Learning Outcome
By the end of the lesson, you will be able to find the perimeter of rectangles, triangles and squares.

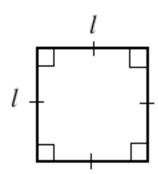
Overview

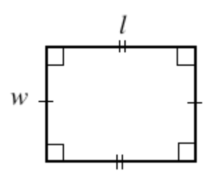
In maths, **perimeter** is the total length around a shape. To find the perimeter of a rectangle or square, add the lengths of all 4 sides together. We have some short cuts for square and rectangle, because they have sides of the same length.

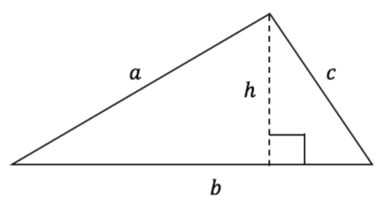
The formulae are:

Shape	Perimeter
Square	$P = l + l + l + l = 4l$
Rectangle	$P = l + l + w + w = 2l + 2w$
Triangle	$P = a + b + c$

For the shapes:



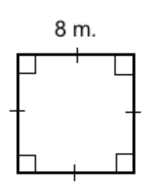




Always remember to write the unit of measurement with your answer. For example, m or cm.

Solved Examples

1. Find the perimeter of the square:



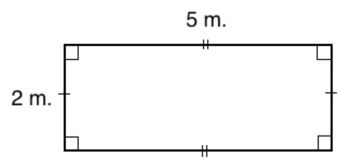
Solution

Find the perimeter by adding the length 4 times, or by multiplying the length of 1 side by 4. Both methods are shown:

$$\mathrm{P} = l + l + l + l = 8 + 8 + 8 + 8 = 32 \text{ m}$$

$$\mathrm{P} = 4l = 4 \times 8 \text{ m} = 32 \text{ m}$$

2. Find the perimeter of the rectangle:



Solution

Find the perimeter by adding lengths of the 4 sides. You can also find the perimeter by multiplying each the length and width by 2, and adding the results. Both methods are shown:

$$\mathrm{P} = l + l + w + w = 5 + 5 + 2 + 2 = 14 \text{ m}$$

$$\mathrm{P} = 2l + 2w = 2 \times 5 \text{ m} + 2 \times 2 \text{ m} = 10 \text{ m} + 4 \text{ m} = 14 \text{ m}$$

3. Find the perimeter of a square with sides of length 15 cm.

Solution

Substitute $l = 15$ cm into the formula:

$$\mathrm{P} = 4l = 4 \times 15 \text{ cm} = 60 \text{ cm}$$

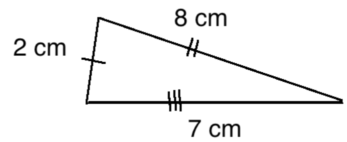
4. Find the perimeter of a rectangle of breadth 4 m and length 12 m.

Solution

"Breadth" is another way of saying width. The rectangle is 4 m by 12 m. Apply the formula:

$$\mathrm{P} = 2l + 2w = 2 \times 12 \text{ m} + 2 \times 4 \text{ m} = 24 \text{ m} + 8 \text{ m} = 32\text{m}$$

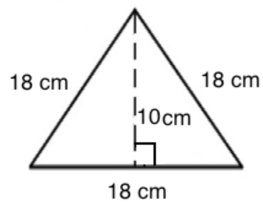
5. Find the perimeter of the triangle:



Solution

Add the lengths of the 3 sides: $P = a + b + c = 2 + 8 + 7 = 17 \text{ cm}$

6. The equilateral triangle shown below has sides of length 18 cm, and a height of 10 cm. Find the perimeter of the triangle.



Solution

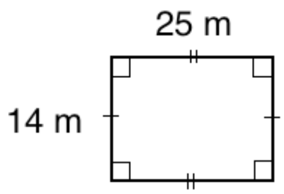
In this problem, you are given information that you do not need. You only need the lengths of the 3 sides. You will see later how the height is used to find the area of a triangle.

Add the lengths of the 3 sides: $P = a + b + c = 18 + 18 + 18 = 54 \text{ cm}$

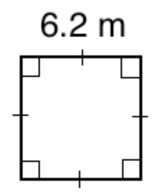
Practice

1. Find the perimeter of shapes a. and b.

a.

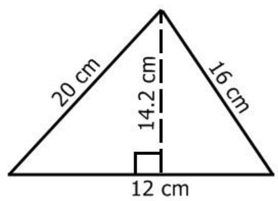


b.

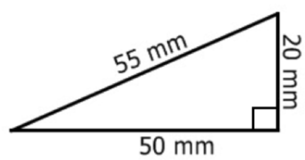


2. Find the perimeter of a square with sides of 14 cm.
3. Find the perimeter of a rectangle with length 5 metres and width 3 metres.
4. Find the perimeters of the triangles below:

a.



b.



Lesson Title: Area of Rectangles and Squares	**Theme:** Measurement and Estimation
Practice Activity: PHM-07-077	**Class:** JSS 1

Learning Outcome

By the end of the lesson, you will be able to calculate the area of rectangles and squares using the formulae (Square: $A = l^2$; Rectangle: $A = l \times w$).

Overview

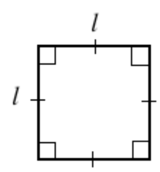
In maths, “area” is the size of the space inside of a shape. Imagine you want to cover the floor of your house with mats. Your floor is a certain area. If you measure the area of your floor, you will be able to buy the correct number of mats.

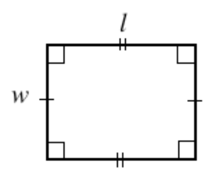
To find the **area** of a square or rectangle, multiply the measurements of the two sides, length and width. For a square, the sides are the same length, so the area will be length squared.

The formulae are:

Shape	Area
Square	$A = l \times l = l^2$ $Area = length \times length$
Rectangle	$A = l \times w$ $Area = length \times width$

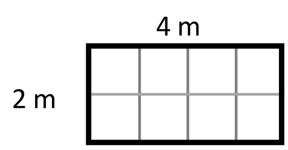
For the shapes:





Area is always given in units squared. Remember to write the unit squared with your answer. For example, 64 m^2 is read as “64 square metres” or “64 metres squared”.

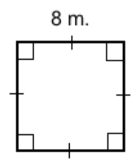
To see why the formulae work, consider a rectangle with length 4 metres and width 2 metres. We can draw a grid on the rectangle like this:



The area gives the number of squares inside of a shape. So if I ask you to find the area of this rectangle, it means to find the number of square metres that fit inside. You can count the squares inside this rectangle, and notice that there are 8. The size of the rectangle is 8 square metres. We can also find this with the formula: $A = l \times w = 4 \times 2 = 8 \text{ m}^2$.

Solved Examples

1. Find the area of the square:

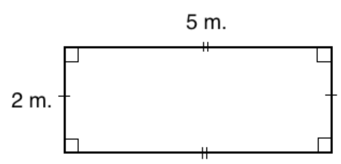


Solution

Substitute the given length in the formula:

$$\text{A} = l^2 = 8 \text{ m} \times 8 \text{ m} = 64 \text{ m}^2$$

2. Find the area of the rectangle:



Solution

Substitute the given length and width in the formula:

$$\text{A} = l \times w = 5 \text{ m} \times 2 \text{ m} = 10 \text{ m}^2$$

3. Find the area of a square with sides of length 15 cm.

Solution

Substitute $l = 15$ cm into the formula:

$$\text{A} = l^2 = 15 \text{ cm} \times 15 \text{ cm} = 225 \text{ cm}^2$$

4. Find the area of a rectangle with length 20 metres and width 3 metres.

Solution

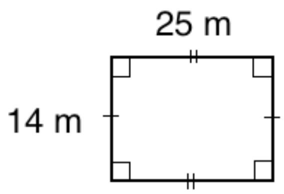
Substitute $l = 20$ m and $w = 3$ m into the formula:

$$\text{A} = l \times w = 20 \text{ m} \times 3 \text{ m} = 60 \text{ m}^2$$

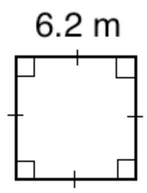
Practice

1. Find the area of the following shapes:

a.



b.



2. Find the area of a square with sides of 14 cm.
3. Find the area of a rectangle with a length of 35 cm and a width of 4 cm.
4. A square has sides of length 4 cm. A rectangle has a length of 5 cm and a width of 3 cm. Which shape has a greater area?

Lesson Title: Area of Triangles	**Theme:** Measurement and Estimation
Practice Activity: PHM-07-078	**Class:** JSS 1

Learning Outcome

By the end of the lesson, you will be able to calculate the area of triangles using the formula $\frac{1}{2}$ base $\times$ height.

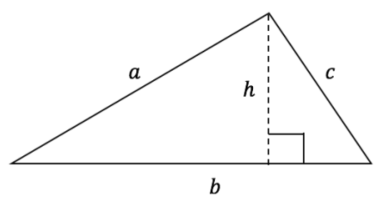
Overview

The area is found using a formula that is specifically for triangles. Multiply the base and height by $\frac{1}{2}$.

The formula is:

Shape	Area
Triangle	$A = \frac{1}{2} b \times h$ $Area = \frac{1}{2} \times base \times height$

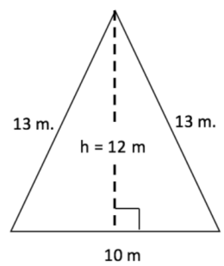
For the shape:



Base and height are always perpendicular to each other. You can take any side of the triangle as its base. Then you find the height of the triangle from that base. The height is a perpendicular line drawn from the base to the opposite angle of the triangle.

Solved Examples

1. Find the area of the triangle:



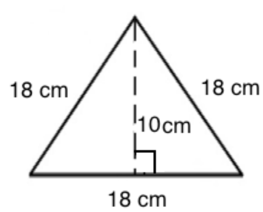
Solution

In this triangle, the base is 10 m and the height is 12 m. We know that they are perpendicular to each other because of the small box.

Apply the formula:

$$
\begin{aligned}
A &= \frac{1}{2}\ b \times h \\
&= \frac{1}{2}(10 \times 12) && \text{Substitute values} \\
&= \frac{1}{2}(120) && \text{Simplify} \\
&= 60\ \text{cm}^2
\end{aligned}
$$

2. The equilateral triangle shown below has sides of length 18 cm, and a height of 10 cm. Find the area of the triangle.

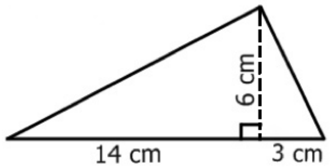


Solution

In this triangle, the base is 18 cm and the height is 10 cm. Substitute them into the formula and solve:

$$
\begin{aligned}
A &= \frac{1}{2}\ b \times h \\
&= \frac{1}{2} \times 18 \times 10 && \text{Substitute values} \\
&= \frac{1}{2}(180) && \text{Simplify} \\
&= 90\ \text{cm}^2
\end{aligned}
$$

3. Find the area of the triangle:



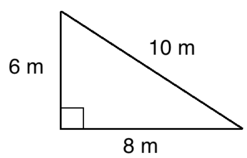
Solution

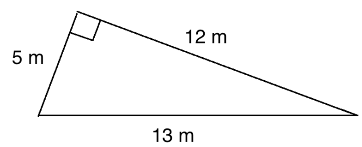
In this diagram, 2 parts of the base are labelled separately. We need to add to find the length of the base: $14 + 3 = 17$ cm.

Now substitute the base and height into the equation and solve:

$$
\begin{aligned}
A &= \frac{1}{2}\, b \times h \\
&= \frac{1}{2} \times 17 \times 6 && \text{Substitute values} \\
&= \frac{1}{2} \times (102) && \text{Simplify} \\
&= 51 \text{ cm}^2
\end{aligned}
$$

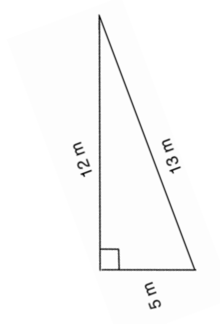
4. Find the area of the triangles:

a. 

b. 

Solutions

Remember that any side of the triangle can be taken as the base. It is important that the base and height are perpendicular.



The base and height of triangle a. are 6 m and 8 m. The base and height of triangle b are 5 m and 12 m. To see why, rotate the triangle. →

a. Substitute $b = 6$ m and $h = 8$ m, and solve:

$$
\begin{aligned}
A &= \frac{1}{2}\, b \times h \\
&= \frac{1}{2} \times 6 \times 8 && \text{Substitute values} \\
&= \frac{1}{2} \times 48 && \text{Simplify} \\
&= 24 \text{ m}^2
\end{aligned}
$$

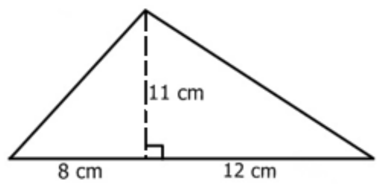
b. Substitute $b = 5$ m and $h = 12$ m, and solve:

$$
\begin{aligned}
A &= \tfrac{1}{2}\, b \times h \\
&= \tfrac{1}{2} \times 5 \times 12 && \text{Substitute values} \\
&= \tfrac{1}{2} \times 60 && \text{Simplify} \\
&= 30\ \mathrm{m}^2
\end{aligned}
$$

Practice

1. Find the area of the triangles below:

a.



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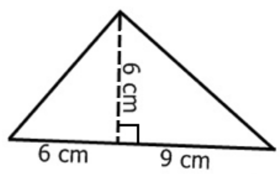
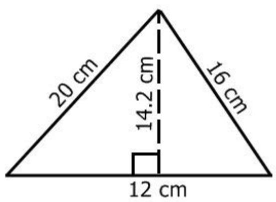
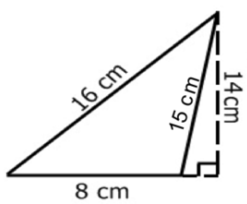


2. Calculate the area of the following triangle

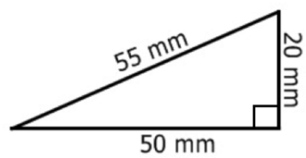
a.



b.



c.



Lesson Title: Perimeter Story Problems	**Theme:** Measurement and Estimation
Practice Activity: PHM-07-079	**Class:** JSS 1

Learning Outcome

By the end of the lesson, you will be able to solve story problems involving the perimeter of plane shapes.

Overview

In this lesson, you will solve perimeter story problems. Remember that perimeter is the distance around a shape. You will also apply perimeter formulae to find the distance around real-life objects. You will also use maths skills that you learned from other lessons to solve problems involving perimeter. For example, you may be asked to convert units or increase or decrease the size of a shape.

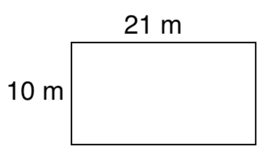
It can be helpful to draw a diagram before solving story problems. A diagram is given for each of the Solved Examples below.

Solved Examples

1. A rectangular field has width of 10 m and a length of 21 m. The owner wants to build a fence around the field. Find the perimeter of the field.

   Solution

   Draw a diagram:

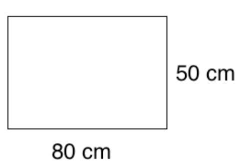


   Apply the perimeter formula: $P = 2 \times 10 + 2 \times 21 = 20 + 42 = 62 \text{ m}$

2. A rectangle is 80 cm long and 50 cm wide. What is the perimeter of the rectangle? Give your answer in metres.

   Solution

   Draw a diagram:



In this problem, the measurements are given in centimetres but we are asked to give the answer in metres. We can convert the cm to m either before or after doing the perimeter calculation. In this case, it is better to convert the units after. If you convert the units before, you will do calculations on decimal numbers.

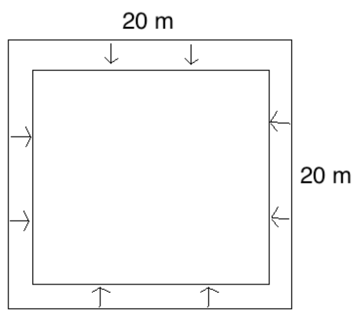
Apply the perimeter formula: $P = 2 \times 80 + 2 \times 50 = 160 + 100 = 260 \text{ cm}$

Convert the perimeter to metres. Remember that 1 m = 100 cm. Divide because you are converting from the smaller unit to the larger unit: $260 \text{ cm} = 260 \div 100 = 2.6 \text{ m}$

3. A square of side 20 m had its sides reduced by 10%. Calculate the perimeter of the new square.

Solution

Draw a diagram:



To solve this problem, use percentage decrease. The length of each side of the rectangle is reduced by 10%. Apply the formula for percentage decrease to find the new length:

$$\begin{aligned}
\text{New length} &= \frac{100-\text{percentage decrease}}{100} \times \frac{\text{the given length}}{1} \\
&= \frac{100-10}{100} \times \frac{20}{1} \\
&= \frac{90}{100} \times \frac{20}{1} \\
&= \frac{9}{1} \times \frac{2}{1} \\
&= 9 \times 2 \\
&= 18 \text{ m}
\end{aligned}$$

The new length of the sides of the square are 18 metres. Apply the formula to find the perimeter:

$P = 4l = 4 \times 18 \text{ m} = 72 \text{ m}$

Practice

1. A plot of land to be cultivated is in the shape of a triangle which has sides of length 24 m, 20 m and 18 m. Find the perimeter of the plot of land.
2. Sam built a small building to store his farm equipment. The building measures 3 m by 1.5 m. What is the perimeter of the building?
3. A farmer wants to build a fence around his field. The field is 30 metres long by 25 metres wide. What is the perimeter of the field?
4. A square of side 50 cm had its sides increased by 6%. What is the perimeter of the square now?
5. A square has sides of 90 cm. What is the perimeter of the square? Give your answer in metres.

Lesson Title: Area Story Problems	**Theme:** Measurement and Estimation
Practice Activity: PHM-07-080	**Class:** JSS 1

Learning Outcome
By the end of the lesson, you will be able to solve story problems involving the area of plane shapes.

Overview

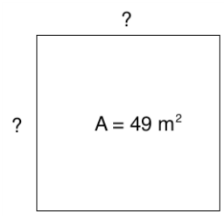
In this lesson, you will solve area story problems. Remember that area is the space inside a shape. You will also apply area formulae to find the space inside real-life objects, and use maths skills that you learned from other lessons to solve problems involving area. For example, you may be asked to convert units or use ratio.

It can be helpful to draw a diagram before solving story problems.

Solved Examples

1. The area of a square is 49 m^2. Find the length of its side.

   Solution
   Draw a diagram:

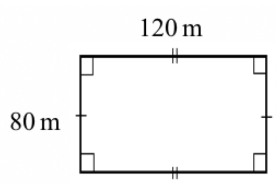
   Recall that the area formula for a square is $A = l \times l = l^2$. In this case, the area is 49 m^2, so we have $l^2 = 49$ m^2.

   You can use the multiplication table or recall perfect squares to identify that $7^2 = 49$. The length of each side of the square is 7 m.

2. Bright Secondary School has a football field that measures 120 metres on one side and 80 metres on the other side. A gardener is hired to plant carpet grass on the field, and he needs to know the area of the field. Calculate the area.

   Solution

   First, draw a diagram:

Calculate the area:

$$
\begin{aligned}
\text{A} &= l \times w \\
&= 120\text{ m} \times 80\text{ m} \\
&= 9{,}600\text{ m}^2
\end{aligned}
$$

3. If the area of a square is 25 cm^2, find its perimeter.

   Solution
   This problem involves 2 steps. First, use the area to find the length l of the square. Use l to calculate the perimeter.

   Step 1. Find l using the area. Identify that $5^2 = 25$, so each side of the square is 5 cm.

   Step 2. Find the perimeter: $\text{P} = 4l = 4 \times 5\text{ cm} = 20\text{ cm}$

4. The dimensions of the floor of a room are 20 ft by 15 ft. If rectangular tiles of dimensions 1.0 ft by 1.5 ft are used to tile the room, find the number of tiles required.

   Solution
   This problem involves multiple steps. Find the area of the room, then the area of each tile. Divide the area of the room by the area of 1 tile to find the total number of times required.

   Step 1. Area of the room: $A = l \times w = 20 \times 15 = 300\text{ ft}^2$

   Step 2. Area of 1 tile: $A = l \times w = 1.5 \times 1.0 = 1.5\text{ ft}^2$

   Step 3. Divide the area of the room by the area of 1 tile:

   $$300\text{ ft}^2 \div 1.5\text{ ft}^2 = 3{,}000 \div 15 = 200$$

   Answer: 200 tiles are needed.

5. How many tiles of 50 cm by 50 cm are needed to tile a floor of 3 m by 4 m? (1 m = 100 cm).

   Solution
   This problem is similar to problem 4, but the numbers are given with different units. All of the measurements should be in the same units. You may convert them all to cm, or all to m.

Step 1. Convert the sides of the tiles to metres: $50 \text{ cm} \; = \; 50 \div 100 = 0.5 \text{ m}$

Step 2. Area of the floor: $A = l \times w = 4 \times 3 = 12 \text{ m}^2$

Step 3. Area of 1 tile: $A = l^2 = 0.5 \times 0.5 = 0.25 \text{ m}^2$

Step 4. Divide the area of the room by the area of 1 tile:

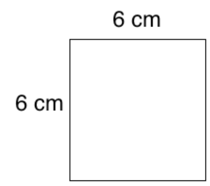
$$12 \text{ m}^2 \div 0.25 \text{ m}^2 = 1200 \div 25 = 48$$

Answer: 48 tiles are needed.

6. A square has a side of 6 cm. A triangle has a base of 8 cm and a height of 4 cm. What is the ratio of the area of the square to that of the triangle? Give your answer in its lowest term.

Solution

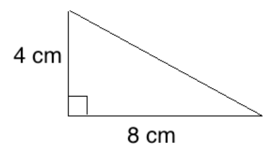
You may first draw the shapes, shown on the right.



Find the area of each shape, then write them in ratio form.

Step 1. Area of the square: $A = l^2 = 6^2 = 36 \text{ cm}^2$

Step 2. Area of the triangle: $A = \frac{1}{2}bh = \frac{1}{2} \times 8 \times 4 = 16 \text{ cm}^2$



Step 3. Ratio of the area of the square to that of the triangle: $36 : \; 16 = \frac{36}{4} : \; \frac{16}{4} = 9 : 4$

Practice

1. The area of a square is 36 m^2. Find the perimeter of the square.
2. The dimensions of the floor of a rectangular room are 10 metres by 12 metres.
    a. Find the area of the floor of the room.
    b. How many tiles would you need to cover the floor if the tiles are 20 cm by 20 cm?
3. A square has a side of 5 cm. A triangle has a base of 10 cm and a height of 3 cm. What is the ratio of the area of the square to that of the triangle? Give your answer in its lowest term.
4. A school principal has a rectangular office with a length 10 metres, and an area of 70 square metres.
    a. Draw a diagram of the principal's office.
    b. Find the width of her office.
    c. Find the perimeter of her office.

Lesson Title: Circles	**Theme:** Measurement and Estimation
Practice Activity: PHM-07-081	**Class:** JSS 1

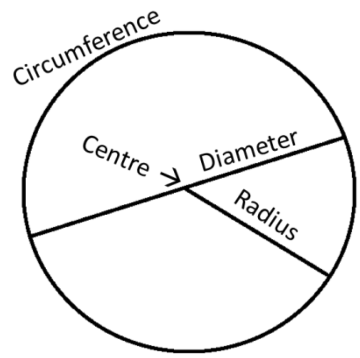
Learning Outcomes

By the end of the lesson, you will be able to:

1. Identify and label parts of a circle.
2. Identify that the diameter is twice the radius.

Overview

This lesson is a review lesson on identifying the parts of a circle. The parts of a circle are shown in the diagram.



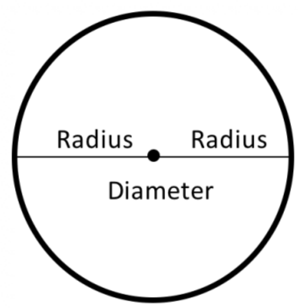
The **centre** of the circle is the point exactly in the middle of the circle.

The **circumference** is the distance around the circle.

The **radius** of the circle is the distance from the centre of the circle to any point on the circumference. The plural of radius is radii.

The **diameter** is the distance across the circle, passing through the centre.

The radius and diameter have an important relationship. The radius is half of the diameter, and the diameter is twice the radius. This is shown in the second diagram.



From this information, we can write two equations for diameter and radius:

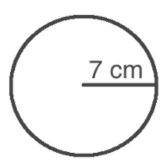
$d = 2r$ and $r = \frac{d}{2}$ where d is diameter and r is radius.

Solved Examples

1. The radius of a circle is 7 cm. What is the length of its diameter?

   Solution

   You may first draw a picture to help solve the problem:

   Apply the formula to find the diameter, $d = 2r$.

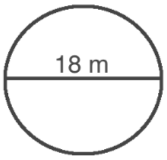
$$\begin{aligned} d &= 2r \\ &= 2(7) \\ &= 14 \text{ cm} \end{aligned}$$

Note that the radius and diameter should always be given in the same units. Remember to write the units with your answer.

2. Find the radius of a circle with diameter 18 metres.

Solution

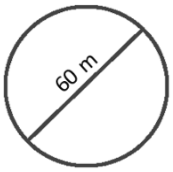
You may first draw a picture to help solve the problem:



Apply the formula to find the radius, $r = \frac{d}{2}$.

$$\begin{aligned} r &= \frac{d}{2} \\ &= \frac{18}{2} \\ &= 9 \text{ m} \end{aligned}$$

3. Find the radius of the circle:



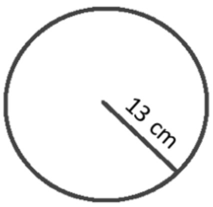
Solution

In the diagram, the diameter of the circle is 60 m.

Apply the formula to find the radius, $r = \frac{d}{2}$.

$$\begin{aligned} r &= \frac{d}{2} \\ &= \frac{60}{2} \\ &= 30 \text{ m} \end{aligned}$$

4. Find the diameter of the circle:



Solution

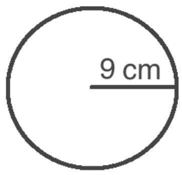
In the diagram, the radius of the circle is 13 cm.

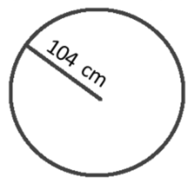
Apply the formula to find the diameter, $d = 2r$.

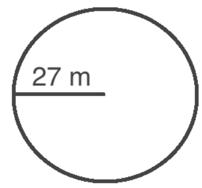
$$\begin{aligned} d &= 2r \\ &= 2(13) \\ &= 26 \text{ cm} \end{aligned}$$

Practice

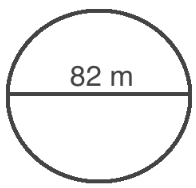
1. Sketch a circle with radius 19 m. What is the diameter?
2. Sketch a circle with diameter 3 m. What is the radius?
3. Find the diameter of each circle:

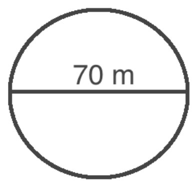
a. 

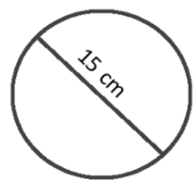
b. 

c. 

4. Find the radius of each circle:

a. 

b. 

c. 

Lesson Title: Circumference of Circles	**Theme:** Measurement and Estimation
Practice Activity: PHM-07-082	**Class:** JSS 1

Learning Outcome

By the end of the lesson, you will be able to calculate the circumference of a circle using the formula ($C = 2\pi r$).

Overview

The circumference is the distance around a circle, or its perimeter. The circumference of a circle is given by the formula $C = 2\pi r$ where r is the radius of the circle.

The π symbol in the formula is called 'pi' and it is a constant number that never changes. If we write pi as a decimal, it will be very long. We won't find the end of it. We have a fraction and a rounded off decimal that we use in this formula to give us an **approximate** value for pi.

Pi is approximately $\pi \approx \frac{22}{7} \approx 3.14$.

You will often be told which estimated value of pi to use to solve a problem, the fraction or the decimal.

Solved Examples

1. Find the circumference of a circle whose radius is 21 cm. Take $\pi = \frac{22}{7}$.

   Solution

   Substitute $r = 21$ cm and $\pi = \frac{22}{7}$ into the formula for circumference:

C	$=$	$2\pi r$	Formula
	$=$	$2 \times \frac{22}{7} \times 21$	Substitute values
	$=$	$2 \times 22 \times 3$	Cancel 7
	$=$	132 cm	

The circumference of the circle is 132 cm.

2. The radius of a circle is 7 cm. What is the length of its circumference? Take $\pi = \frac{22}{7}$.

Solution

$$\begin{array}{rcll} C & = & 2\pi r & \text{Formula} \\ & = & 2 \times \frac{22}{7} \times 7 & \text{Substitute values} \\ & = & 2 \times 22 & \text{Cancel 7} \\ & = & 44 \text{ cm} & \end{array}$$

3. Find the circumference of a circle with diameter 28 m. Take $\pi = \frac{22}{7}$.

Solution

This problem gives the diameter. We need to find the radius to use it in the circumference formula.

Step 1. Find the radius:

$$\begin{array}{rcl} r & = & \frac{d}{2} \\ & = & \frac{28}{2} \\ & = & 14 \text{ m} \end{array}$$

Step 2. Find the circumference:

$$\begin{array}{rcll} C & = & 2\pi r & \text{Formula} \\ & = & 2 \times \frac{22}{7} \times 14 & \text{Substitute values} \\ & = & 2 \times 22 \times 2 & \text{Cancel 7} \\ & = & 88 \text{ m} & \end{array}$$

The circumference is 88 m.

4. The diameter of a circle is 40 cm. Find its circumference using $\pi = 3.14$.

Solution

This problem gives the diameter. We need to find the radius to use it in the circumference formula.

Step 1. Find the radius:

$$r = \frac{d}{2}$$
$$= \frac{40}{2}$$
$$= 20 \text{ cm}$$

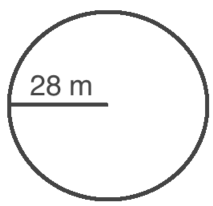
Step 2. Find the circumference:

C	$=$	$2\pi r$	Formula
	$=$	$2 \times 3.14 \times 20$	Substitute values
	$=$	6.28×20	Multiply
	$=$	125.6 cm	

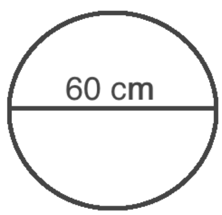
The circumference is 125.6 cm.

Practice

1. Find the circumference of a circle with diameter 14 cm, using $\pi = \frac{22}{7}$.
2. The radius of a circle is 100 cm. Find its circumference using $\pi = 3.14$.
3. Find the circumference of a circle whose radius is 35 cm. Take $\pi = \frac{22}{7}$.
4. Find the circumference of a circle whose diameter is 140 cm. Take $\pi = \frac{22}{7}$.
5. Find the circumference of the circle, using $\pi = \frac{22}{7}$:



6. Find the circumference of the circle, using $\pi = 3.14$:



Lesson Title: Area of Circles	**Theme:** Measurement and Estimation
Practice Activity: PHM-07-083	**Class:** JSS 1

Learning Outcome
By the end of the lesson, you will be able to calculate the area of circle using the formula ($A = \pi r^2$).

Overview

This lesson is on calculating the area of a circle. The area of a circle is the total space inside the circumference.

The area of a circle is given by the formula $A = \pi r^2$ where r is the radius of the circle. Remember that pi is approximately $\pi \approx \frac{22}{7} \approx 3.14$.

Remember that area is always given in units squared. For example, m^2 and cm^2.

Solved Examples

1. Find the area of a circle with radius 7 cm. Take $\pi = \frac{22}{7}$.

   Solution

   Apply the formula:

$$
\begin{aligned}
A &= \pi r^2 \\
&= \frac{22}{7} \times 7^2 && \text{Substitute values} \\
&= \frac{22}{1} \times 7 && \text{Cancel 7} \\
&= 22 \times 7 && \text{Multiply} \\
A &= 154 \text{ cm}^2
\end{aligned}
$$

   The area of the circle is 154 cm^2.

2. Find the area of a circle with diameter 28 m. Take $\pi = \frac{22}{7}$.

   Solution
   This problem gives the diameter. We need to find the radius to use it in the area formula.

Step 1. Find the radius:

$$
\begin{aligned}
r &= \frac{d}{2} \\
&= \frac{28}{2} \\
&= 14 \text{ m}
\end{aligned}
$$

Step 2. Find the area:

$$
\begin{aligned}
A &= \pi r^2 \\
&= \frac{22}{7} \times 14^2 && \text{Substitute values} \\
&= \frac{22}{7} \times 196 && \text{Square 14} \\
&= \frac{22}{1} \times 28 && \text{Cancel 7} \\
&= 22 \times 28 \\
A &= 616 \text{ m}^2
\end{aligned}
$$

The area of the circle is 616 m^2.

3. Find the area of a circle with radius 6 m. Give your answer to 1 decimal place. (Take $\pi = \frac{22}{7}$)

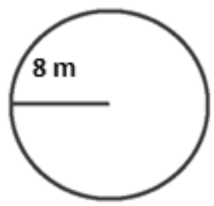
Solution

Note that the radius is 6 m. Apply the formula:

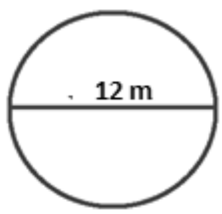
$$
\begin{aligned}
A &= \pi r^2 \\
&= \frac{22}{7} \times 6^2 \\
&= \frac{22}{7} \times 36 \\
&= \frac{792}{7} \\
&= 113.14 \\
A &= 113.1 \text{ m}^2
\end{aligned}
$$

Practice

1. Find the area of a circle with a radius of 10 cm. (Take $\pi = 3.14$)
2. Find the area of a circle with diameter of 10 m. Give your answer to 1 decimal place. (Take $\pi = \frac{22}{7}$)
3. Find the area of a circle with radius 3 cm. Give your answer to 1 decimal place. (Take $\pi = 3.14$)
4. Find the area of a circle with diameter 8 m. Give your answer to 2 decimal places. (Take $\pi = \frac{22}{7}$)
5. Find the area of the circle below. Give your answer to 1 decimal place. (Take $\pi = \frac{22}{7}$)



6. Find the area of the circle below. Give your answer to the nearest whole number. (Take $\pi = 3.14$)



Lesson Title: Problem Solving with Circles	**Theme:** Measurement and Estimation
Practice Activity: PHM-07-084	**Class:** JSS 1

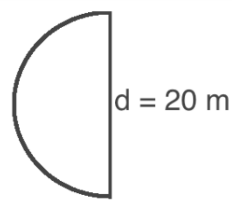
Learning Outcome
By the end of the lesson, you will be able to solve multi-step problems involving circle measurements, including radius, diameter, circumference, and area.

Overview

In this lesson, you will use the information from the previous three lessons to solve more challenging problems.

Solved Examples

1. The diagram below shows a semi-circle with a diameter of 20 m. Use $\pi = 3.14$ to calculate the following:
    a. The perimeter of the semicircle.
    b. The area of the semicircle.



Solutions

Note that a semi-circle is exactly half of a circle. Its straight edge is the diameter of the circle.

Also note that we need to calculate the radius: $r = \frac{d}{2} = \frac{20}{2} = 10 \text{ m}$.

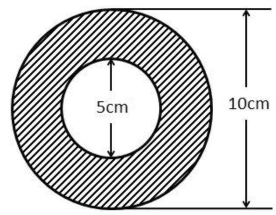
a. The curved line is exactly half of the circumference of the full circle. The straight line is exactly the diameter. To find the perimeter of this semi-circle, we need to add half of the circumference, plus the diameter.

$$\begin{array}{lcll} P & = & \frac{1}{2}C + d & \text{Formula} \\ & = & \frac{1}{2}2\pi r + d & \\ & = & \pi r + d & \text{Simplify the formula} \\ & = & 3.14 \times 10 + 20 & \text{Substitute values} \\ & = & 31.4 + 20 & \text{Multiply} \\ & = & 51.4 \text{ m} & \end{array}$$

b. The area inside this semi-circle is exactly half the area inside the full circle. To find the area of this semi-circle, we need to multiply the area by one half.

A	$=$	$\frac{1}{2}\pi r^2$	Formula
	$=$	$\frac{1}{2}(3.14)(10^2)$	Substitute values
	$=$	$\frac{1}{2}(3.14)(100)$	Square 10
	$=$	$\frac{1}{2}(314)$	Multiply
	$=$	$157\ \text{m}^2$	

2. Find the area of the shaded portion in the figure shown:



Solution

Note that the area of the shaded part is the area of the outer (large) circle, minus the area of the inner (small) circle.

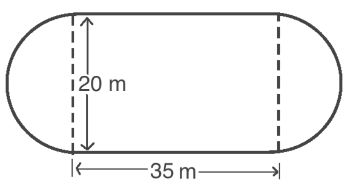
We need to find the radius of each circle. Let's call them R and r:

- Radius of the outer circle (R): $\frac{10}{2} = 5$ cm
- Radius of the inner circle (r): $\frac{5}{2} = 2.5$ cm

Calculate the area:

Area of the shaded portion	$=$	Area of the outer circle − area of the inner circle
Area of the shaded portion	$=$	$\pi R^2 - \pi r^2$
	$=$	$\frac{22}{7} \times 5^2 - \frac{22}{7} \times 2.5^2$
	$=$	$\frac{550}{7} - \frac{137.5}{7}$
	$=$	$78.57 - 19.64$
Area of the shaded portion	$=$	$58.93\ \text{cm}^2$

3. A farmer's field is in the shape of the figure below. Calculate the area of the field. Use $\pi = 3.14$.



Solution

Note that this shape is a rectangle with a semi-circle on each side. Together, 2 semi-circles make 1 full circle. This is because each semicircle is exactly half of the circle. To find the area of the shape, find the area of the circle and the area of the rectangle. Add them together to find the total area.

The diameter of this circle is 20 m, which means the radius is $r = \frac{20}{2} = 10$ m.

Area of circle:

$$\begin{aligned} A &= \pi r^2 \\ &= 3.14 \times 10^2 && \text{Substitute values} \\ &= 3.14 \times 100 && \text{Square 10} \\ A &= 314 \text{ m}^2 \end{aligned}$$

Area of the rectangle:

$$\begin{aligned} A &= l \times w \\ &= 35 \times 20 && \text{Substitute values} \\ &= 700 \text{ m}^2 && \text{Multiply} \end{aligned}$$

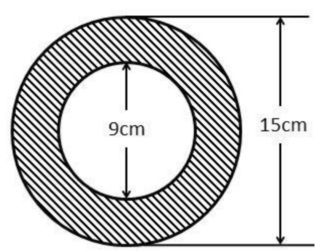
Total area:

$$\begin{aligned} A &= \text{Area of circle + Area of rectangle} \\ &= 314 + 700 \\ &= 1{,}014 \text{ m}^2 \end{aligned}$$

The area of the field is 1,014 m^2.

Practice

1. Find the perimeter and area of a semi-circle with radius 21 m. (Use $\pi = \frac{22}{7}$)
2. Find the perimeter and area of a semi-circle with diameter 10 cm. (Use $\pi = 3.14$)
3. Find the perimeter of the field in Solved Example 3. (Take $\pi = 3.14$)
4. Find the area of the shaded portion in the figure shown below. Give your answer to the nearest whole number. (Take $\pi = 3.14$)



Lesson Title: Circle Story Problems	**Theme:** Measurement and Estimation
Practice Activity: PHM-07-085	**Class:** JSS 1

Learning Outcome
By the end of the lesson, you will be able to solve story problems involving the circumference and area of circles.

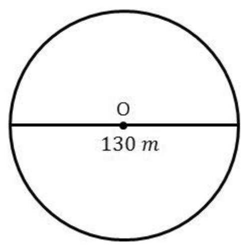
Overview

Area and circumference have many applications to everyday life. In this lesson, you will use the information from previous lessons to solve story problems. Remember that it is often helpful to draw a diagram before solving a story problem.

Solved Examples

1. A sports ground is circular in shape. If the diameter is 130 m, what is the distance around the field? (Take $\pi = \frac{22}{7}$)

   Solution
   Draw a diagram:



   The distance around the field is its circumference. We need to first find the radius, then use it in the circumference formula.

   Radius: $r = \frac{130 \text{ m}}{2} = 65 \text{ m}$

   Circumference:

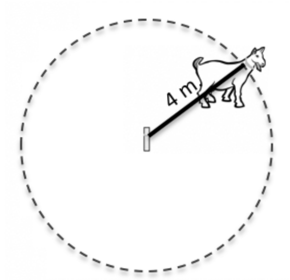
$$\begin{aligned} \text{C} &= 2 \times \frac{22}{7} \times 65 \text{ m} \\ &= \frac{2,860}{7} \text{m} \\ &= 408.57 \text{ m} \end{aligned}$$

   The distance around the field is 408.57 m.

2. A goat is tied to a peg in the ground. The rope is 4 m long. What area of grass can the goat eat? (use $\pi = 3.14$)

Solution

Draw a diagram →



We want to find the area of the grass the goat can eat. The area in which it can walk is in the shape of a circle.

Apply the area formula:

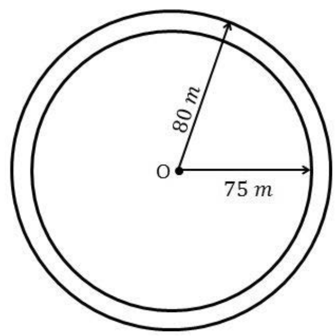
$$\begin{aligned} A &= \pi r^2 \\ &= 3.14 \times 4^2 && \text{Substitute values} \\ &= 3.14 \times 16 && \text{Square 4} \\ A &= 50.24 \text{ m}^2 \end{aligned}$$

The goat can eat grass in an area of 50.24 m^2.

3. There are five lanes in a circular running track. The radius of the edge of the track is 80 m; the radius of the first lane is 75 m. What is the difference in the distances run by two athletes if one runs around the edge of the track and the other runs around the first lane? (Take $\pi = \frac{22}{7}$)

Solution

Draw a diagram →



The distance that each runner travels is the circumference of a circle. Find the circumference of each circle, and **subtract** to find the **difference**.

The distance around the edge of the track is:

$$\begin{aligned} \text{C} &= 2 \times \frac{22}{7} \times 80 \text{ m} \\ &= \frac{3{,}520}{7} \\ &= 502.86 \text{ m} \end{aligned}$$

The distance around the first lane is:

$$\begin{aligned} \text{C} &= 2 \times \frac{22}{7} \times 75 \text{ m} \\ &= \frac{3{,}300}{7} \\ &= 471.43 \text{ m} \end{aligned}$$

The difference in the distances is:

$$\begin{aligned} D &= \text{Distance of the edge of track - Distance of the first lane} \\ &= 502.86 \text{ m} - 471.43 \text{ m} \\ &= 31.43 \text{ m} \end{aligned}$$

The difference in the distances run by the 2 athletes is 31.43 metres.

Practice

1. Mary's garden is circular in shape; its diameter is 7 m. If she plants rose trees 50 cm apart round the edge of the garden, how many rose trees does she need? (Use $\pi = \frac{22}{7}$)
2. A circular mat has a radius of 2.5 m. Calculate the area of the mat. Give your answer to 1 decimal place. (Use $\pi = 3.14$)
3. A farmer has a circular piece of land with radius 50 metres. He wants to build a fence around the land.
    a. How long will the fence be? (Use $\pi = 3.14$)
    b. If the fence costs Le 3,000.00 per metre, what will the total cost be?
4. Sam has a pet dog. He wants to tie his dog to a peg, and he has a rope that is 14 feet long. How much area will his dog have to walk about? (Use $\pi = \frac{22}{7}$)

Lesson Title: Volume of Solids	Theme: Measurement and Estimation
Practice Activity: PHM-07-086	**Class:** JSS 1

Learning Outcomes

By the end of the lesson, you will be able to:

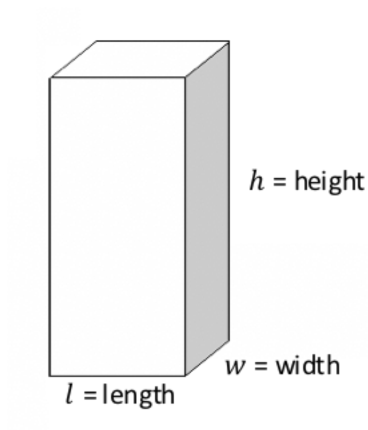
1. Identify the general formula for volume of prisms and cylinders as cross-section multiplied by height.
2. Identify and interpret measurements for volume (units cubed).

Overview

This is the first lesson on calculating the volume of a 3-dimensional shape. These are often called solids. Volume is the measurement of space taken up by a 3-dimensional solid.

Let us consider a box of chalk, a balloon and a tin of milk. In the tin of milk, the milk inside is taking up space. In a balloon, air or gas is taken up space. In the chalk box, the chalks are taking up space. This shows that solids, liquids and gases all take up space. This space is called volume.

Rectangular prisms are solids with 6 rectangular faces. They are also called cuboids. Boxes are rectangular prisms. A rectangular prism is shown on the right. It has a length, width and height.



Recall that a rectangle has a length and width. We found the area of rectangles with the formula $A = l \times w$.

To find the volume of the rectangular prism, we use the area of one of its faces. This is called the **cross-section**. The cross-section of the rectangular prism shown is $A = l \times w$.

To calculate the volume of the rectangular solid, multiply the cross-section by its height:

$$Volume = length \times width \times height$$
$$V = l \times w \times h$$

Since we multiply 3 lengths together, the units for volume are cubed. We use a power of 3 to show **cubic**. This is the same as the **cubed** we use for indices. For example, if the size of a rectangular prism is measured in centimetres, its volume is given in cm^3. This is read as “cubic centimetres” or “centimetres cubed”.

To find volume, it is important that the measurements of the solid are all given in the same units.

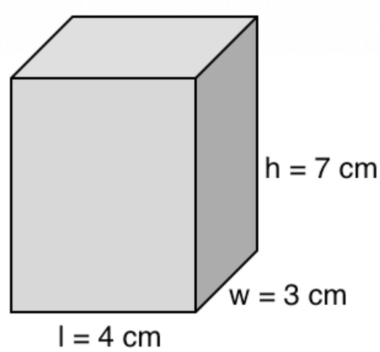
Solved Examples

1. Sketch a rectangular prism with height 7 cm, length 4 cm and width 3 cm. Your measurements do not need to be precise.

   Solution
   This problem asks you to “sketch” and states that the measurements do not need to be precise. You do not need to use a ruler to draw exactly 7 cm, 4 cm and 3 cm.

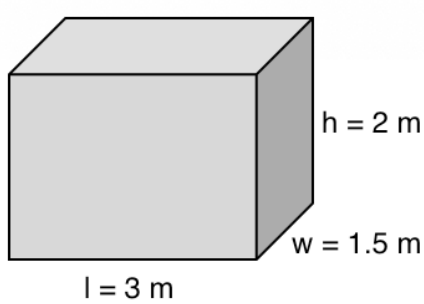
   It is important that sides with longer measures are drawn longer. It is also important that the shape is labelled correctly, so that the height is labelled as 7 cm, and so on.

   Sketch:



2. A water tank is in the shape of a rectangular prism. It is 2 metres tall, 3 metres long, and 1.5 metres wide. Sketch the water tank.

   Solution



3. The following statements describe rectangular prisms. Write down what units the volume will be measured in.
    a. A box of chalk is 10 cm tall, 8 cm long and 2 cm wide.
    b. A shipping container is 3 metres tall, 6 metres long and 2 metres wide.
    c. A box of biscuits is 30 cm tall, 10 cm long and 5 cm wide.
    d. A water tank is 6 feet tall, 5 feet long and 3 feet wide.

    Solutions

    Give each answer in units cubed.

    a. cm^3
    b. m^3
    c. cm^3
    d. ft^3

Practice

1. Write down the formula for volume of a rectangular prism.
2. Sketch a rectangular prism with a height of 5 cm, length of 7 cm, and width of 3 cm.
3. A dictionary is laying on a table. It has height of 5 cm, length of 20 cm, and width of 15 cm. Sketch the dictionary.
4. A building is in the shape of rectangular prism. It has height of 2 metres, length of 10 metres, and width of 5 metres.
    a. Sketch the building.
    b. In what units would you measure the building's volume?
5. The following statements describe rectangular prisms. Write down what units the volume will be measured in.
    a. A box of juice is 25 cm tall, 10 cm long and 6 cm wide.
    b. A water tank is 4 metres tall, 3 metres long and 1.75 metres wide.
    c. A container of petrol is 60 cm tall, 40 cm long and 35 cm wide.
    d. A building is 14 feet tall, 25 feet long and 10 feet wide.
6. Determine whether each of the following is a measurement of area or volume:
    a. cm^2
    b. m^3
    c. ft^3
    d. km^2
    e. m^2

Lesson Title: Volume of Cubes	**Theme:** Measurement and Estimation
Practice Activity: PHM-07-087	**Class:** JSS 1

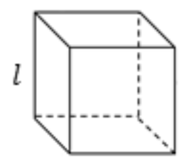
Learning Outcome
By the end of the lesson, you will be able to calculate the volume of a cube using the formula ($A = l^3$).

Overview

A cube is a special type of rectangular prism. The height, length and width are all equal.

Remember that we find volume of a rectangular prism by multiplying the length, width and height: $V = l \times w \times h = lwh$. Since all the sides are of the same length, then one number will represent all the sides.

Volume of a cube is $V = l \times l \times l = l^3$

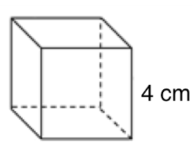


If you know only the length of one side of the cube then you will know its volume.

Solved Examples

1. Find the volume of a cube of side length $l = 4$ cm.

   Solution
   You may sketch the cube to help solve the problem. →

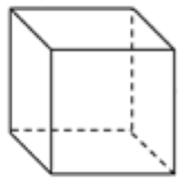
   

   Apply the formula for volume of a cube:

$$\begin{array}{rcll} V & = & l^3 & \text{Formula} \\ & = & 4^3 & \text{Substitute } l = 4 \\ & = & 4 \times 4 \times 4 & \text{Calculate } 4^3 \\ & = & 16 \times 4 & \\ & = & 64 \text{ cm}^3 & \end{array}$$

   Remember to give your answer in units cubed.

2. Find the volume of the cube shown:



Solution

Apply the formula for volume of a cube:

$$
\begin{aligned}
V &= l^3 && \text{Formula} \\
&= 2^3 && \text{Substitute } l = 2 \\
&= 2 \times 2 \times 2 && \text{Calculate } 2^3 \\
&= 4 \times 2 && \\
&= 8 \text{ m}^3 &&
\end{aligned}
$$

Remember to give your answer in units cubed.

3. The edges of a cube are 8.5 cm long. Find its volume. Give your answer to the nearest whole number.

Solution

Apply the formula for volume of a cube:

$$
\begin{aligned}
V &= l^3 && \text{Formula} \\
&= 8.5^3 && \text{Substitute } l = 8.5 \\
&= 8.5 \times 8.5 \times 8.5 && \text{Calculate } 8.5^3 \\
&= 72.25 \times 8.5 && \\
&= 614.125 \text{ cm}^3 && \\
&= 614 \text{ cm}^3 && \text{Round to the nearest whole number}
\end{aligned}
$$

Remember to give your answer in units cubed.

4. Determine the length of the side of a cube if its volume is 27 cm^3.

Solution

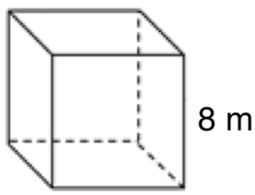
This problem asks us to do the opposite of the problems above. In this case, you are given the volume and asked to find the lengths of the sides of the cube. Since $V = l^3$, we have $27 = l^3$.

Think of a number that can be multiplied by itself 3 times to get 27. You may use a multiplication table to help you brainstorm. Note that $3^3 = 3 \times 3 \times 3 = 27$.

The sides of the cube must be 3 cm.

Practice

1. Find the volume of a cube of sides 7 cm.
2. Find the volume of a cube with sides 2.5 metres. Give your answer to 1 decimal place.
3. The edges of a cube are 9 cm long. Find its volume.
4. Find the volume of the cube shown:



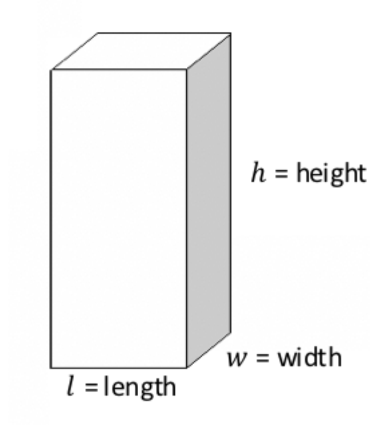
5. If the volume of a cube is 8 ft^3, what is the length of its sides?

Lesson Title: Volume of Cuboids	Theme: Measurement and Estimation
Practice Activity: PHM-07-088	**Class:** JSS 1

Learning Outcome
By the end of the lesson, you will be able to calculate the volume of a cuboid (rectangular prism) using the formula ($V = l \times w \times h$).

Overview

Recall that the formula for finding the volume of a cuboid or rectangular prism is $V = l \times w \times h$.

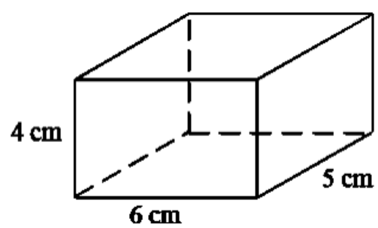


In this lesson, you will find the volume of cuboids. You are able to find the volume of any cuboid if you have its 3 measurements. Remember that volume is always given in units cubed.

You may sometimes see **width** called **breadth**. These 2 words have the same meaning.

Solved Examples

1. Find the volume of the given cuboid:



Solution
Note that the measurements of the cuboid are $l = 6$ cm, $w = 5$ cm, $h = 4$ cm.

Apply the volume formula:

V	$=$	lwh	Formula
	$=$	$6 \times 5 \times 4$	Substitute values
	$=$	120 cm^3	Multiply

2. Find the volume of a box with a length of 20 feet, width of 10 feet, and height of 4 feet.

Solution

Apply the volume formula:

V	$=$	lwh	Formula
	$=$	$20 \times 10 \times 4$	Substitute values
	$=$	800 ft^3	Multiply

3. A rectangular block measures 12 cm by 8 cm by 4 cm. What is its volume?

Solution

The 3 dimensions of the cuboid are given. We do not need to know which side is the length, base or height. A rectangular block can be turned in any direction, so that any of the measurements could be the height. Also, the order does not matter in multiplication.

Multiply the 3 measurements together to find the volume:

V	$=$	lwh	Formula
	$=$	$12 \times 8 \times 4$	Substitute values
	$=$	384 cm^3	Multiply

4. The length of a cuboid is 1 metre; its breadth is 15 cm and its height is 5 cm. Find its volume in cm^3.

Solution

Note that the measurements are given in m, cm, and cm. You are asked to find the answer in cm^3. This means that you must convert all units to cm first.

Convert 1 m to cm: 1 m = 100 cm

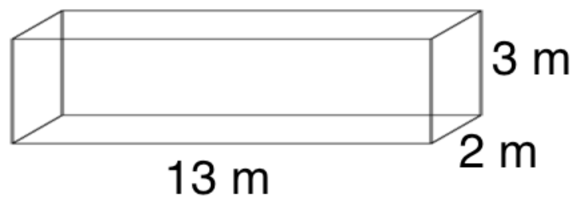
Apply the volume formula:

V	$=$	lwh	Formula
	$=$	$100 \times 15 \times 5$	Substitute values
	$=$	7,500 cm^3	Multiply

Practice

1. Find the volume of cuboids with these dimensions:
    a. Length 8 cm, width 5 cm and height 4 cm.
    b. Length 10 cm, breadth 7 cm and height 4 cm.

2. Find the volume of the cuboid shown:



3. The length of a cuboid is 2 m; its breadth is 30 cm and its height is 20 cm. Find its volume in cm^3.
4. The dimensions of a cuboid are 4 m, 3 m, and 50 cm. Find its volume in m^3.

Lesson Title: Problem Solving with Volume	Theme: Measurement and Estimation
Practice Activity: PHM-07-089	**Class:** JSS 1

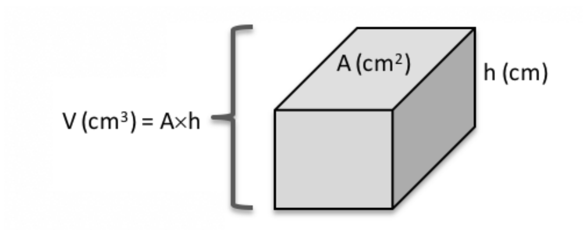
Learning Outcome
By the end of the lesson, you will be able to solve multi-step problems involving length, area and volume measurements.

Overview

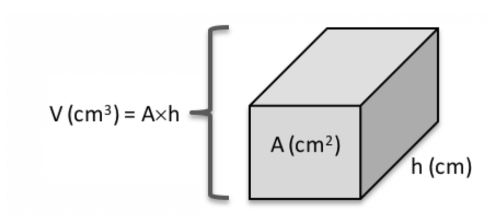
In this lesson, you will use information from previous lessons to solve problems on volume. These will be multi-step problems involving measurements of length, area and volume.

Remember from PHM-07-086 that the volume of a shape is the area of one of its cross-section times its height. The area of any of its faces can be taken as the cross-section. The cross-section of the rectangular prism shown is $A = l \times w$.

To calculate the volume of the rectangular solid, multiply the cross-section by its height: $V = l \times w \times h$. This can also be written as $V = A \times h$.



Any face can be taken as the cross-section, but take the correct height for that cross-section. For example, we can take the front face as the cross-section, and the height will be as shown:



From the formula $V = A \times h$, we can get 2 more formulae.

If the volume and the height are given, then we can find the area of the cross-section of the prism. It will be the volume divided by the height: $A = \frac{V}{h}$.

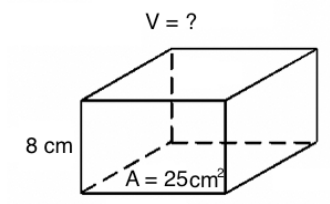
If the volume and area of a cross-section are given, we can find the height of the prism. It will be volume divided by area: $h = \frac{V}{A}$.

Solved Examples

1. A box has a base with an area of $25\ \mathrm{cm}^2$. Calculate the volume of the box if it is 8 cm deep.

Solution

Draw a diagram:



We are given the area of a cross-section and the height of the box. Use the formula $V = A \times h$ to find the volume:

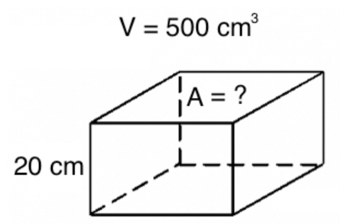
V	$=$	$A \times h$	Formula
	$=$	$25\ \mathrm{cm}^2 \times 8\ \mathrm{cm}$	Substitute values
	$=$	$200\ \mathrm{cm}^3$	Multiply

The volume of the box is $200\ \mathrm{cm}^3$.

2. The volume of a rectangular box is 500 cm^2. The height of the box is 20 cm. What is the area of the top of the box?

Solution

Draw a diagram:



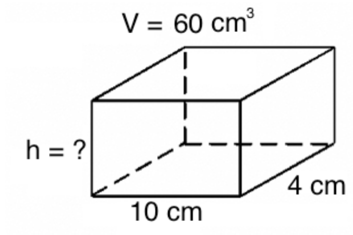
We are given volume and height. Use the formula $A = \frac{V}{h}$ to find area:

A	$=$	$\frac{V}{h}$	Formula
	$=$	$\frac{500\ \mathrm{cm}^3}{20\ \mathrm{cm}}$	Substitute values
	$=$	$25\ \mathrm{cm}^2$	Divide

Remember that area is always given in units squared. The answer is $25\ \mathrm{cm}^2$.

3. The volume of a cuboid is 60 cm^3. Its length is 10 cm and its breadth is 4 cm. Find its height.

Solution
Draw a diagram:



Remember that length and breadth give the area of one side. If we find the area of that side, we can use the formula $A = \frac{V}{h}$ to find the height.

Step 1. Find the area of the cross-section:

A	$=$	lw	Formula
	$=$	10×4	Substitute values
	$=$	40 cm^2	Multiply

Step 2. Find the height:

h	$=$	$\frac{V}{A}$	Formula
	$=$	$\frac{60\text{ cm}^3}{40\text{ cm}}$	Substitute values
	$=$	$\frac{3}{2}$ cm	Divide
	$=$	1.5 cm	

The height of the cuboid is 1.5 cm.

Practice

1. The top of a wooden box has an area of 35 cm^2. Calculate the volume of the box if it is 20 cm deep.
2. The volume of a rectangular box is 250 cm^3. The height of the box is 10 cm. What is the area of the top of the box?
3. The volume of a rectangular box is 420 cm^3. The height of the box is 80 cm. What is the area of the cross-section of the box? Give your answer to 1 decimal place.
4. The volume of a cuboid is 520 cm^3. If the area of its cross-section is 20 cm^2, what is the height of the cuboid?
5. The volume of a cuboid is 400 cm^3. Its length is 10 cm and its width is 8 cm. Find its height.

Lesson Title: Volume Story Problems	Theme: Measurement and Estimation
Practice Activity: PHM-07-090	**Class:** JSS 1

Learning Outcome
By the end of the lesson, you will be able to solve story problems involving the volumes of cubes and cuboids.

Overview

In this lesson, you will solve story problems involving the volume of cuboids. There are many applications of volume to everyday life.

Volume and capacity problems are solved using the same steps. Volume is the space that a substance takes up, and capacity refers to the amount of space inside a container. For example, a tank might have the capacity to hold 200 m^3, but be only half filled with water. There would be 100 m^3 of water inside.

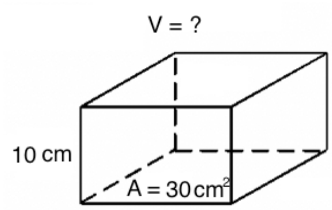
Some problems may ask you to find the number of litres in a certain volume of water. Use the fact that there are 1,000 cubic centimetres in 1 litre (1,000 cm^3 = 1 litre). Divide the volume by 1,000 cm^3 to find how many litres there are. See Solved Example 3 for an example.

Solved Examples

1. Fatu has a tin container of palm oil. The base has area 30 cm^2, and the height of the container is 10 cm.
    a. If the container is full, calculate the volume of oil.
    b. If Fatu used half of the oil, calculate the volume that remains.

Solutions

First, draw a diagram:



a. Calculate the volume of the container using the formula $V = A \times h$.

V	$=$	$A \times h$	Formula
	$=$	$30 \text{ cm}^2 \times 10 \text{ cm}$	Substitute values
	$=$	300 cm^3	Multiply

b. Find half of the volume by multiplying by $\frac{1}{2}$:

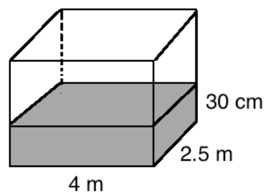
Oil remaining	$= \frac{1}{2}V$	Formula
		Substitute values
	$= \frac{1}{2}(300 \text{ cm}^3)$	Multiply
	$= 150 \text{ cm}^3$	

The volume of the remaining oil is 150 cm^3.

2. A water tank is 4 metres long and 2.5 metres wide. The depth of the water inside is 30 cm. What is the volume of water in the tank?

Solution

Draw a diagram:



All of the measurements should be in the same units before calculating volume. Convert 30 cm to metres: $30 \text{ cm} = 30 \div 100 = 0.3 \text{ m}$

Use the formula for volume:

V	$=$	$l \times w \times h$	Formula
	$=$	$4 \text{ m} \times 2.5 \text{ m} \times 0.3 \text{ m}$	Substitute values
	$=$	$10 \text{ m}^2 \times 0.3 \text{ m}$	Multiply
	$=$	3 m^3	

The volume of the water is 3 m^3.

3. A water tank is in the shape of a cuboid. Its base has area 400 cm^2, and its height is 1 metre.
   a. Find the capacity of the tank.
   b. What is the capacity in litres? (1,000 cm^3 = 1 litre)

Solutions

a. Remember that capacity is the total volume that the tank can hold. Find the volume of the tank. All measurements must be the same units. Use h = 100 cm.

V	$=$	$A \times h$	Formula
	$=$	$400 \text{ cm}^2 \times 100 \text{ cm}$	Substitute values
	$=$	$40{,}000 \text{ cm}^3$	Multiply

The tank has capacity of $40{,}000 \text{ cm}^3$.

b. Convert the capacity of the tank from cm^3 to litres. Use 1,000 cm^3 = 1 litre.

$$V = 40{,}000\ \text{cm}^3 = 40{,}000\ \div 1{,}000 = 40\ \text{litres}$$

The capacity of the tank is 40 litres.

4. A room measures 10 m by 6 m by 3 m. The volume of airspace needed for one person is 5 m^3. Find the maximum number of people who may use the room at the same time.

Solution
Find the volume of the room. Then, divide by the airspace needed for 1 person.

Step 1. Volume of the room:

$$\begin{aligned} V &= l \times w \times h && \text{Formula} \\ &= 10\ \text{m} \times 6\ \text{m} \times 3\ \text{m} && \text{Substitute values} \\ &= 180\ \text{m}^3 && \text{Multiply} \end{aligned}$$

Step 2. Divide total volume by the volume needed for each person:

$$\begin{aligned} \text{Number of people} &= \frac{180\ \text{m}^3}{5\ \text{m}^3} \\ &= 36\ \text{people} \end{aligned}$$

A maximum of 36 people can use the room at one time.

Practice

1. A rectangular tank has dimensions of 4.5 m by 6 m by 8 m. It is filled with water to the brim. If 100 m^3 of the water is used, how much water is left in the tank?
2. A rectangular tank has a length of 4 m, a width of 12 m and a height of 3.5 m. If the tank is half filled with water, calculate the volume of the water in the tank.
3. A water tank is in the shape of a cuboid. Its base has area 300 cm^2, and its height is 2 metres.
   a. Find the capacity of the tank in cm^3.
   b. What is the capacity in litres? (1,000 cm^3 = 1 litre)
4. A water container is 1 m long and 0.8 m wide. The depth of the water inside is 50 cm. What is the volume of water in the tank? Give your answer in m^3.

Lesson Title: Introduction to Angles	**Theme:** Geometry
Practice Activity: PHM-07-091	**Class:** JSS 1

Learning Outcomes

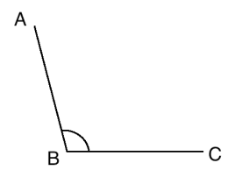
By the end of the lesson, you will be able to:

1. Identify and compare types of angles (acute, obtuse, right angle).
2. Identify degrees as angles measurement.

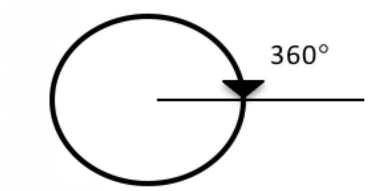
Overview

An angle is made up of 2 lines. The corner point of an angle is called **vertex** and the two straight lines are called **arms.** For example, $\angle ABC$ ('angle ABC') is shown below. It can be measured in degrees. Degrees are used to measure turn. There are 360 degrees in one full rotation (one complete circle). We use little circle (°) following the number to mean degrees.

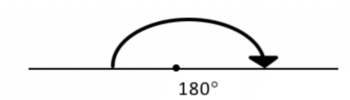
In this case, $\angle ABC = 105°$



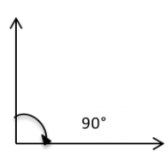
An angle measures the amount of turn. There are 360° in a full turn.



A half turn is 180°.

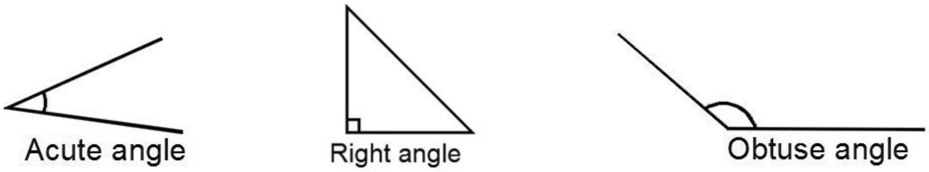


A quarter turn is 90°.



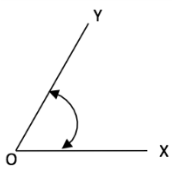
A 90° angle is also called a **right angle**. It is shown with a small square in the angle. There are 2 more types of angles that you will learn in this lesson. An **acute angle** is less than 90°, and an **obtuse angle** is more than 90° but less than 180°.

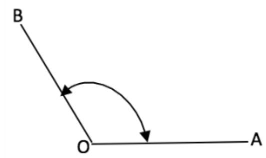
Examples of these 3 types of angles are shown below:

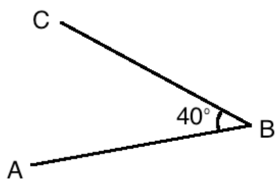


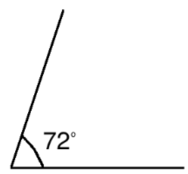
Solved Examples

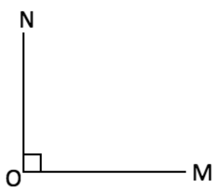
1. Identify whether each angle is obtuse, acute, or a right angle. Give your reasons.

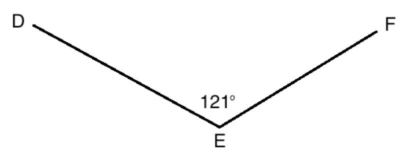
a.


b.


c.


d.


e.


f.


Solutions

a. Acute angle, because it is smaller than 90°
b. Obtuse angle, because it is larger than 90°
c. Acute angle, because it is smaller than 90°
d. Acute angle, because it is smaller than 90°
e. Right angle, because it is exactly 90°
f. Obtuse angle, because it is larger than 90°

2. Write the following angle measurements in words:
   a. 104°
   b. 16.3°

Solutions

a. One hundred and four degrees
b. Sixteen point three degrees

3. Write the following angular measurements in figures:
    a. Three hundred twenty point three degrees
    b. Ten and a half degrees

Solutions

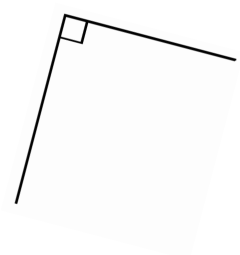
a. $320.3°$
b. $10.5°$

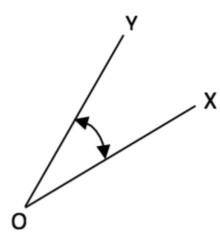
4. Classify the types of angles listed below as acute, obtuse, or right angles.
    a. $3°$ b. $179°$ c. $69°$ d. $170°$ e. $90°$

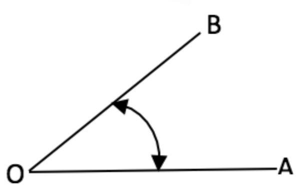
    Solutions
    a. Acute angle, b. Obtuse angle, c. Acute angle, d. Obtuse angle, e. Right angle.

Practice

1. Identify whether each angle is obtuse, acute, or a right angle. Give your reasons.

a.


b.


c.


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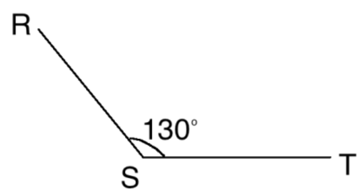


2. Write the following angular measurements in words:
    a. $280°$
    b. $80.39°$
    c. $16.5°$
3. Write the following angular measurements in figures:
    a. Fifty-five degrees
    b. Sixty-six and a half degrees.

c. Ninety point four degrees.

4. Classify the types of angles listed below as acute, obtuse, or right angles.

a. $33°$ b. $109°$ c. $90°$ d. $9°$ e. $91°$

Lesson Title: Right Angles	**Theme:** Geometry
Practice Activity: PHM-07-092	**Class:** JSS 1

Learning Outcomes

By the end of the lesson, you will be able to:

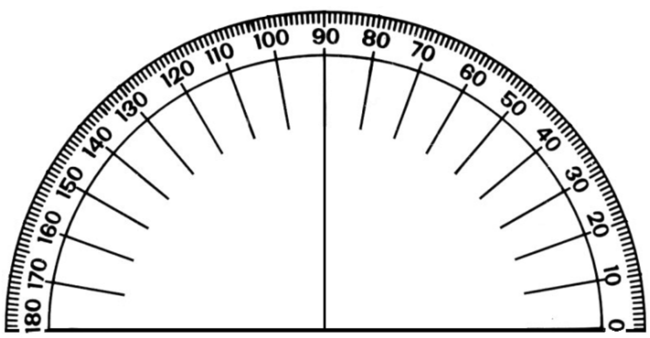
1. Identify that each angle of a rectangle or square measures 90°.
2. Measure 90° angles with a protractor.

Overview

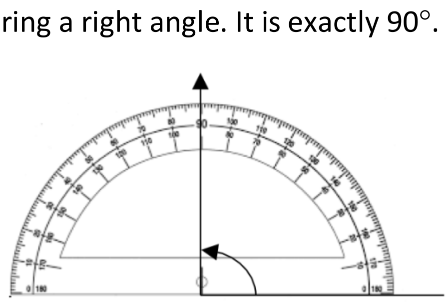
This lesson is on right angles. Recall from the previous lesson that right angles measure exactly 90°.

We use a tool called a protractor to measure and draw angles. Angles are measured in degrees, and this protractor can measure any angle less than 180 degrees. Look at the numbers on the protractor. They count by tens from 0° to 180°. This is like a ruler, but instead of measuring length, we use it to measure how much an angle opens.

You will be using a protractor in this lesson to measure right angles. If you do not have a protractor, you can make one with paper. Trace the protractor below with a pen onto another piece of paper.



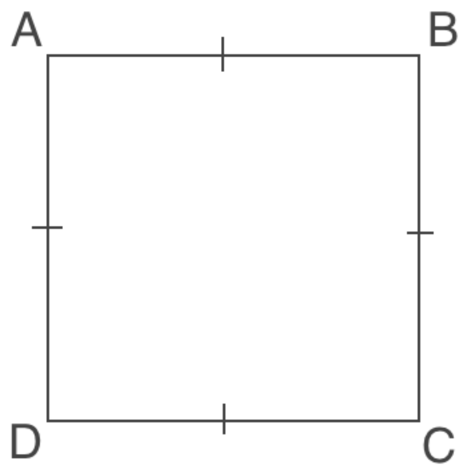
To measure an angle, place a protractor over one arm so that its centre is exactly over the vertex of the angle and the baseline is exactly along one line of the angle. In the diagram below, a protractor is measuring a right angle. It is exactly 90°.



Note that the angles of squares and rectangles are always 90°. This is one of the characteristics of both squares and rectangles.

Solved Examples

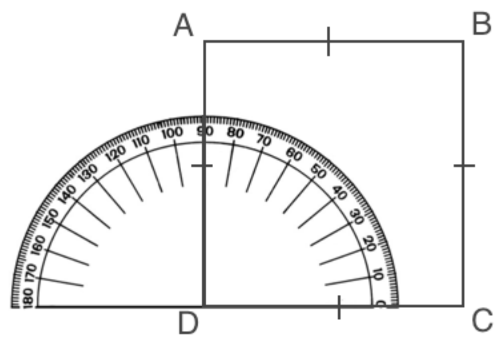
1. Use a protractor to measure each angle of the square below. Record the angle measures. The measure of angle A is given below.



$\angle A = 90°$ $\angle B =$ ______ $\angle C =$ ______ $\angle D =$ _______

Solutions

Use the protractor to measure each angle of the square as shown:



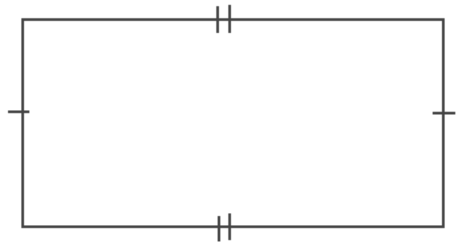
The answers are:

$\angle A = 90°$ $\angle B = 90°$ $\angle C = 90°$ $\angle D = 90°$

2. Draw a rectangle and use a protractor to measure its angles. Find the sum of the angles of the rectangle.

Solution

Your rectangle may have any shape and size. It is important that all of the angles are right angles. For example:

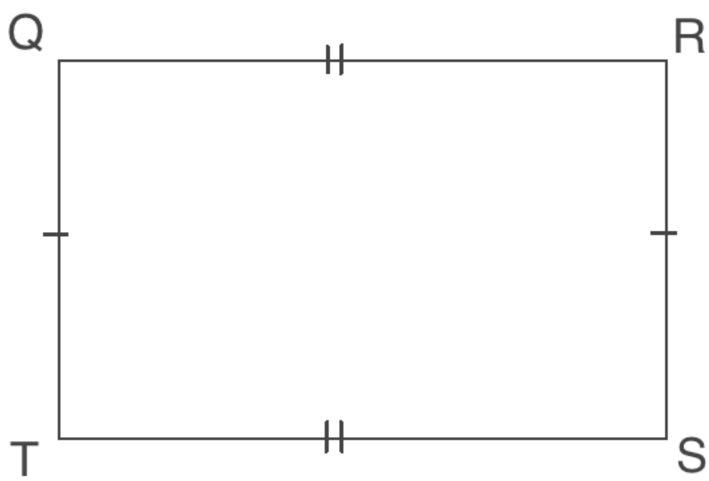


If you measure the angles of your rectangle, you should find that each angle is equal to 90°. Find the sum of the angles: $90° + 90° + 90° + 90° = 4 \times 90° = 360°$.

The sum of the angles of a rectangle is $360°$. Note that this is always true for any rectangle.

Practice

1. Use a protractor to measure each angle of the rectangle below. Record the angle measures. The measure of angle Q is given below.



$\angle Q = 90°$ $\angle R =$ ______ $\angle S =$ ______ $\angle T =$ _______

2. Draw a square and label it WXYZ. Find the measure of each of the angles using a protractor and give them below:

$\angle W =$ ______ $\angle X =$ ______ $\angle Y =$ ______ $\angle Z =$ _______

Lesson Title: Measurement of Angles	**Theme:** Geometry
Practice Activity: PHM-07-093	**Class:** JSS 1

Learning Outcomes

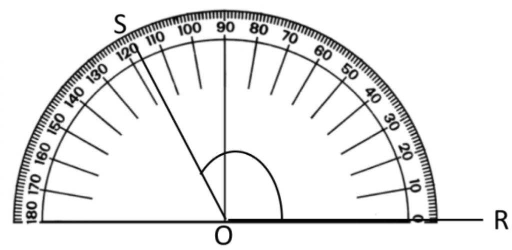
By the end of the lesson, you will be able to:

1. Estimate the measure of a given angle.
2. Measure given angles (acute, obtuse, right angle) using a protractor.

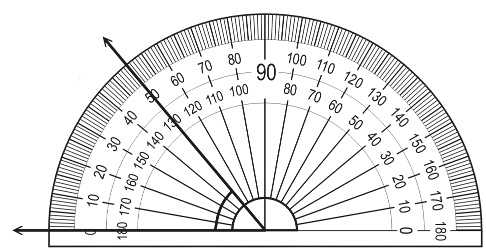
Overview

To measure an angle, place a protractor over the angle so that its centre is exactly over the vertex of the angle. The baseline of the protractor is exactly along one line of the angle. Count the degrees from the baseline to the other line of the angle.

The first diagram shows angle SOR. The baseline of the protractor is along the line OR. The line SO tells us the measure of the angle. It passes through 117°. The measure of angle SOR is 117°. This can be written $\angle SOR = 117°$.



You can measure an angle using either side of the protractor. The angle shown here opens on the left. If your protractor has 2 sets of numbers, count the degrees using the outside numbers, starting on the left, from the baseline to where the other ray of the angle is pointing. The angle shown here is 50°.



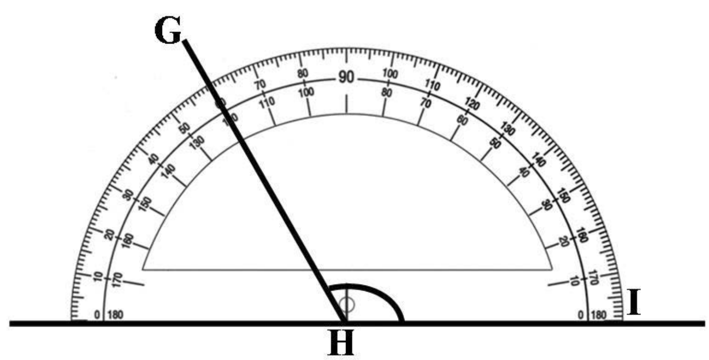
Solved Examples

1. Identify the measure of angle GHI.

   Solution

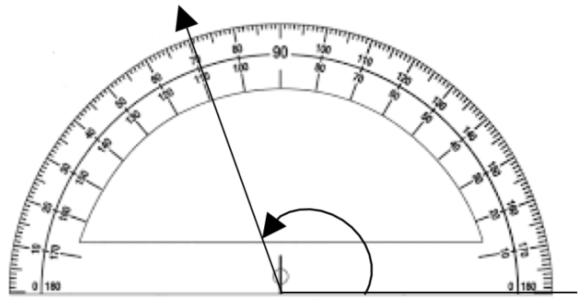
   The measure of the angle is given by the degree that GH passes through.

   $\angle GHI = 120°$

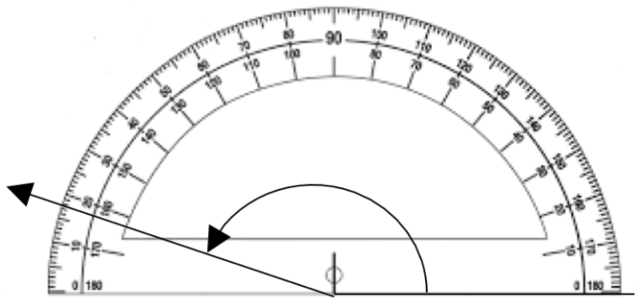


2. Identify the values of the angles illustrated below. Determine whether each angle is acute or obtuse.

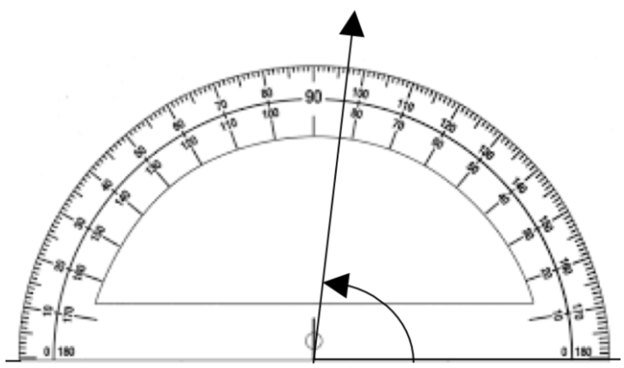
a.



b.



c.

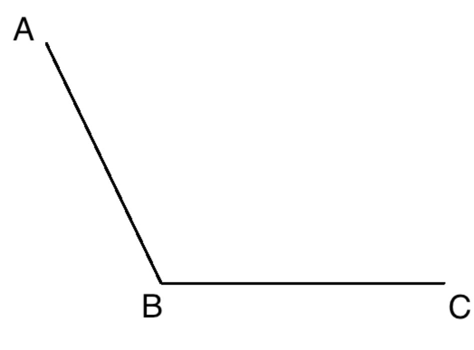


Solutions

Identify which degree the arm of each triangle passes through. The answers are:

a. 110°, obtuse
b. 163°, obtuse
c. 83°, acute

3. Use a protractor to measure angle ABC:



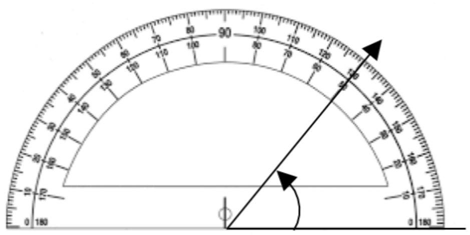
Solution

Place your protractor on the angle as shown in the diagrams above. You will find that its measure is $\angle ABC = 115°$

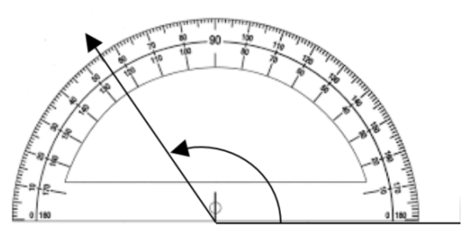
Practice

1. Give the values of the angles illustrated in the measurements below. Determine whether each angle is acute or obtuse.

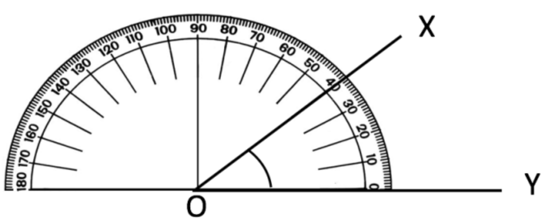
a.

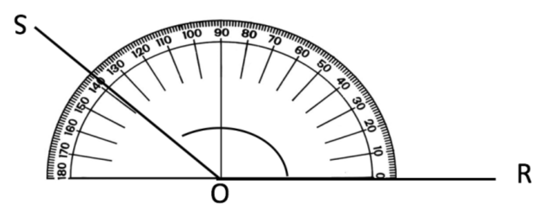


b.



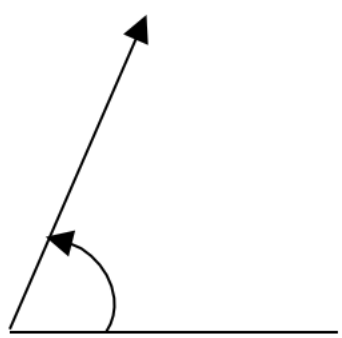
c.



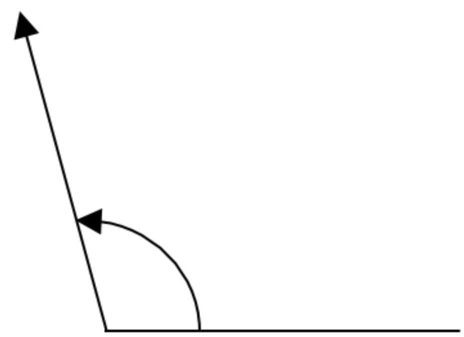


2. Use a protractor to measure the following angles:

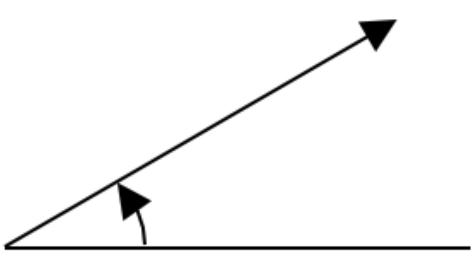
a.



b.



c.

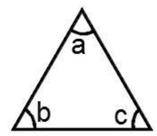


Lesson Title: Finding Unknown Angles in Triangles	**Theme:** Geometry
Practice Activity: PHM-07-094	**Class:** JSS 1

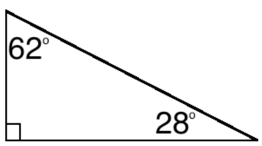
Learning Outcomes
By the end of the lesson, you will be able to:
1. Identify that the sum of the angles in a triangle is 180°.
2. Find unknown angles in triangle.

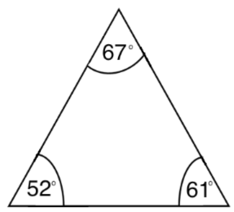
Overview

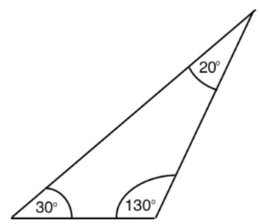
In the triangle on the right, the angles $a, b,$ and c are called **interior** angles. The sum of the interior angles of any triangle is equal to $180°$.



These are examples of triangles. You can try adding the angle measures – they always sum to $180°$.



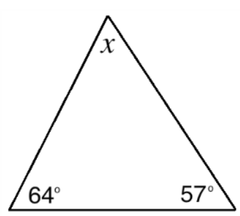




When the measure of an angle is unknown, you can find its measure by subtracting the known angles from $180°$.

Solved Examples

1. Find the measure of the angle marked x in the triangle below:



Solution

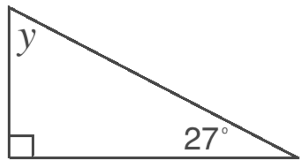
The sum of the interior angles of a triangle is $180°$.

$$\begin{aligned} x + 64° + 57° &= 180° \\ x + 121° &= 180° \\ x &= 180° - 121° \\ x &= 59° \end{aligned}$$

Check: You can always check your answer by adding the 3 angle measures together. They should sum to $180°$:

$$64° + 57° + 59° = 180°$$

2. Find the measure of angle y in the right-angled triangle below:

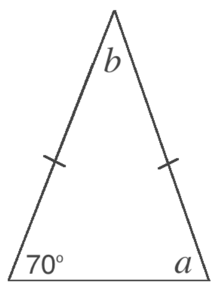


Solution

Remember that the right angle is 90°.

$$\begin{aligned} y + 90° + 27° &= 180° \\ y + 117° &= 180° \\ y &= 180° - 117° \\ y &= 63° \end{aligned}$$

3. Find the measure of angles a and b in the isosceles triangle below:



Solution

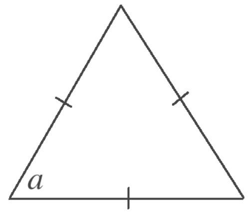
Remember that an isosceles triangle has 2 sides of equal length. These are marked in the diagram with small lines. An isosceles triangle also has 2 equal angles. The angles between each of the equal sides and the 3rd side are equal. In this triangle, $a = 70°$.

We know 2 of the angles because $a = 70°$. Use this to solve for b:

$$\begin{aligned} b + 70° + 70° &= 180° \\ b + 140° &= 180° \\ b &= 180° - 140° \\ b &= 40° \end{aligned}$$

The answer is $a = 70°$ and $b = 40°$.

4. Find the measure of angle a in the equilateral triangle below:



Solution
Remember that an equilateral triangle has all 3 sides of equal length. The 3 angle measures are also equal. Thus, we have $a + a + a = 180°$ or $3a = 180°$.

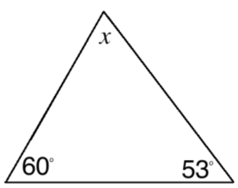
Solve for a:

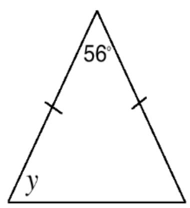
$$3a = 180°$$
$$a = \frac{180°}{3} \qquad \text{Divide both sides by 3}$$
$$a = 60°$$

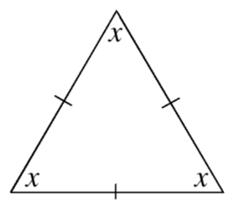
The angles of any equilateral triangle are all $60°$.

Practice

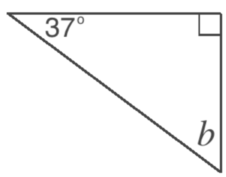
1. Find the value of the lettered angles in the diagrams below:

a. 

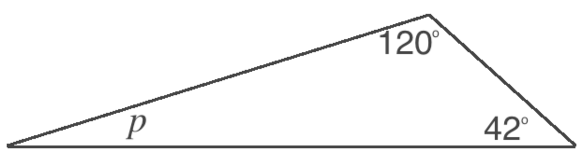
b. 

c. 

2. Find the measure of angle b in the triangle below:



3. Find the measure of angle p:



Lesson Title: Finding Unknown Angles in Composite Shapes	Theme: Geometry
Practice Activity: PHM-07-095	**Class:** JSS 1

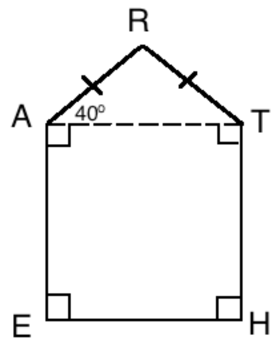
Learning Outcome
By the end of the lesson, you will be able to identify unknown angles in composite shapes involving rectangles, squares and triangles.

Overview

In the previous lessons, you learned to find the measures of angles in squares, rectangles and triangles. In this lesson, you will find the angles of **composite** shapes. Composite means made up of different parts. Composite shapes are made of other, smaller shapes. You already know all of the information you need to find angles in composite shapes. Today you will practise using your problem-solving skills.

Solved Examples

1. The 5-sided shape below is EARTH. Find the measure of each of its angles.



Solution
EARTH is a composite shape made up of rectangle EATH and triangle ART. Find the angles of the smaller shapes and use them to find the angles of EARTH.

First, identify that $E = H = 90°$. These are marked right angles.

The angle A is made up of smaller angles $90°$ and $40°$. Add them to find the measure of angle A: $90° + 40° = 130°$.

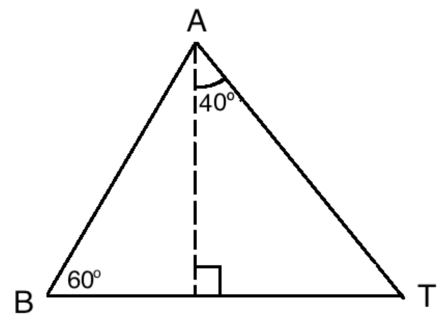
To find angles R and T, we need to first solve for the missing angles in the triangle. It is isosceles, so the angle of the triangle at A is equal to the angle of the triangle at T. The angle of the triangle at T is $40°$. This gives us $T = 90° + 40° = 130°$.

Find the measure of angle R using triangle ART: $R = 180° - 40° - 40° = 100°$

$$\begin{aligned} R + 40° + 40° &= 180° \\ R + 80° &= 180° \\ R &= 180° - 80° \\ R &= 100° \end{aligned}$$

We have now found all of the angles of EARTH: $E = 90°$, $A = 130°$, $R = 100°$, $T = 130°, H = 90°$.

2. Find the measure of angles A and T in triangle BAT.



Solution

Triangle BAT is a composite shape made up of 2 smaller right-angled triangles. We can solve for angle T using its right-angled triangle.

$$\begin{aligned} T + 90° + 40° &= 180° \\ T + 130° &= 180° \\ T &= 180° - 130° \\ T &= 50° \end{aligned}$$

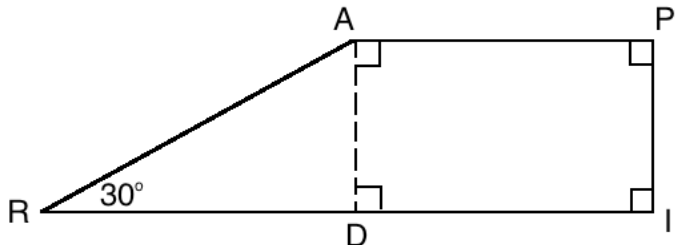
Now that we know the measures of B and T, we can solve for angle A in triangle BAT:

$$\begin{aligned} A + 60° + 50° &= 180° \\ A + 110° &= 180° \\ A &= 180° - 110° \\ A &= 70° \end{aligned}$$

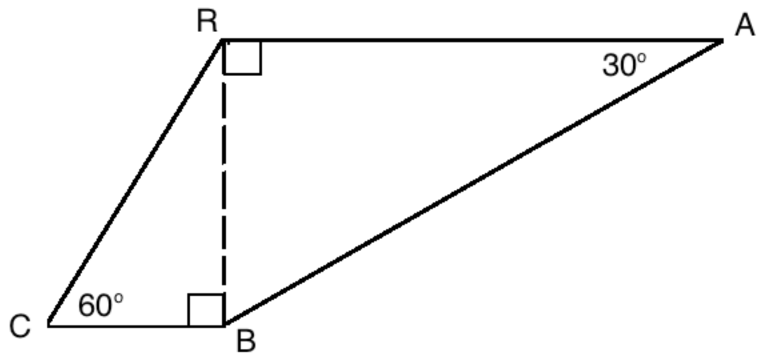
The answer is $T = 50°$ and $A = 70°$.

Practice

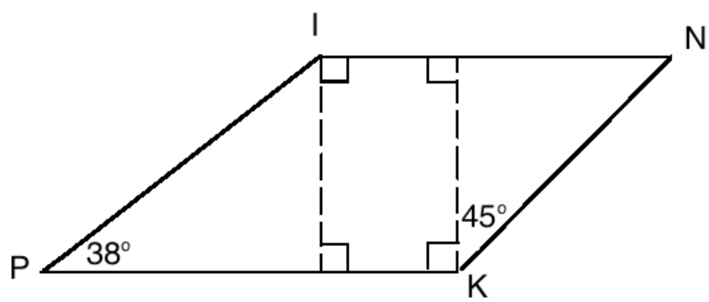
1. Find the measure of angles R, A, P, I and D in the figure below.



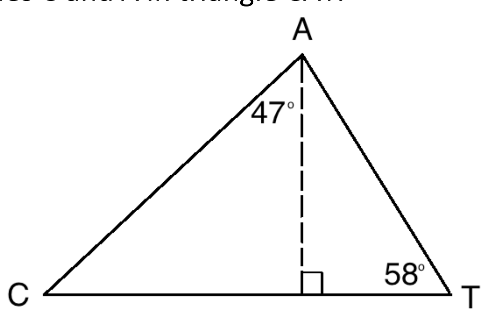
2. The quadrilateral shape below is CRAB. Find the measure of each of its angles.



3. The quadrilateral below is PINK. Find the measure of each of its angles.



4. Find the measures of angles C and A in triangle CAT.



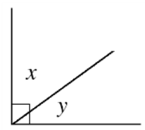
Lesson Title: Introduction to Complementary and Supplementary Angles	**Theme:** Geometry
Practice Activity: PHM-07-096	**Class:** JSS 1

Learning Outcome

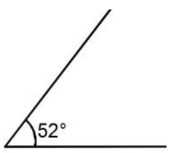
By the end of the lesson, you will be able to identify and compare complementary and supplementary angles.

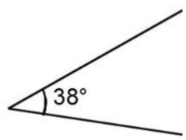
Overview

Two angles are **complementary** when they add up to $90°$. For example, in the diagram below, x and y are complementary angles:

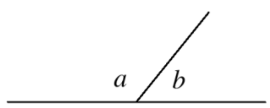


Complementary angles are not necessarily adjacent. For example, the following angles are complementary because their sum is $90°$:

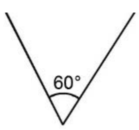


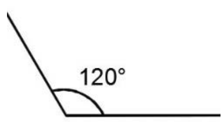


Two angles are **supplementary** when they add up to $180°$. For example, in the diagram below, a and b are supplementary.



Supplementary angles are not necessarily adjacent. For example, the following angles are supplementary because their sum is $180°$:





The easy way to remember complementary and supplementary angles are:

- **C** of complementary stands for "corner" ∟ (a right angle, 90°)
- **S** of Supplementary stands for "straight" — (a straight angle, 180°).

Solved Examples

1. Determine whether each of the following sets of angles are complementary or supplementary:

    a. $45° + 135°$
    b. $3° + 177°$
    c. $80° + 10°$
    d. $36° + 54°$
    e. $61° + 119°$
    f. $98° + 82°$

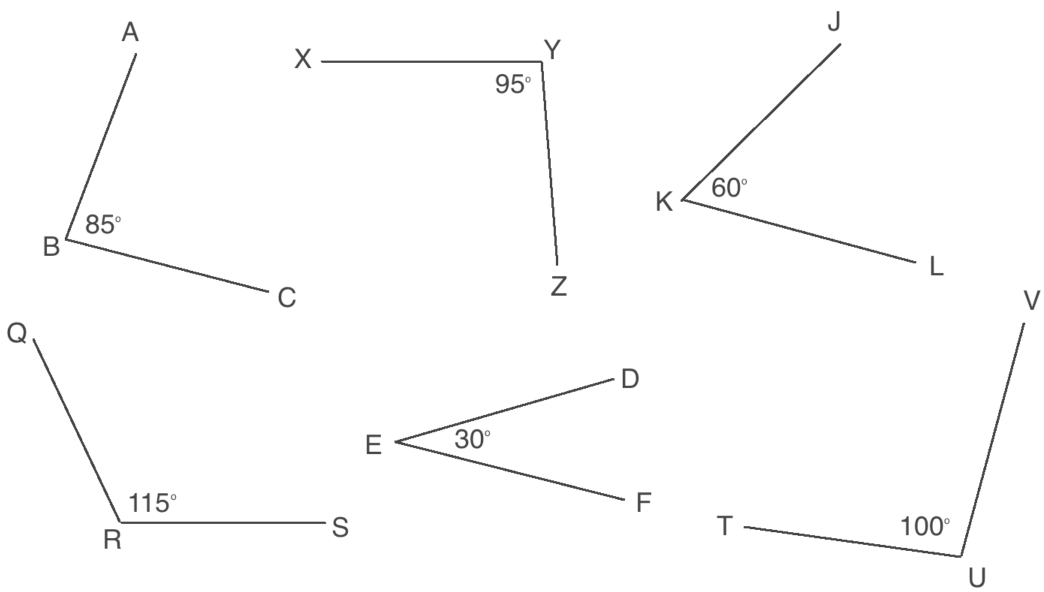
    Solutions

    If the angles sum to 90°, they are complementary. If they sum to 180°, they are supplementary. The answers are:

    a. Supplementary
    b. Supplementary
    c. Complementary
    d. Complementary
    e. Supplementary
    f. Supplementary

2. Determine which of the angles below are:

    a. Supplementary
    b. Complementary



Solutions

a. $\angle ABC$ and $\angle XYZ$ are supplementary, because they sum to $180°$:

$$85° + 95° = 180°$$

b. $\angle DEF$ and $\angle JKL$ are complementary, because they sum to $90°$:

$$30° + 60° = 90°$$

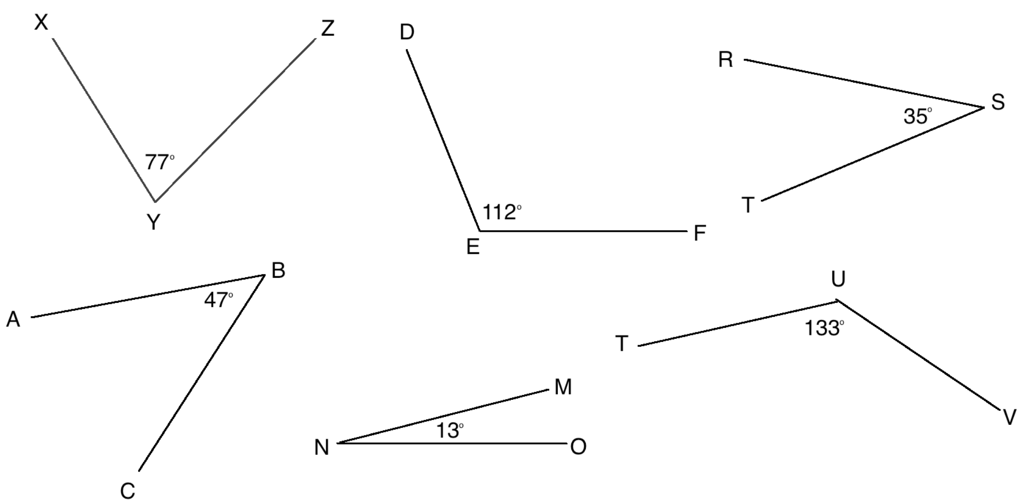
Practice

1. Determine whether each of the following sets of angles are complementary or supplementary:

   a. $2° + 88°$
   b. $140° + 40°$
   c. $72° + 18°$
   d. $102° + 78°$
   e. $61° + 29°$
   f. $15° + 75°$

2. Determine which of the angles below are:

   a. Supplementary
   b. Complementary



Lesson Title: Complementary Angles	Theme: Geometry
Practice Activity: PHM-07-097	**Class:** JSS 1

Learning Outcome
By the end of the lesson, you will be able to find an unknown angle given two complementary angles.

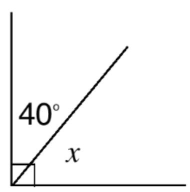
Overview

Remember that complementary angles add up to 90°, and supplementary angles add up to 180°. In this lesson, you will solve problems on complementary angles. For complementary angles, we can solve for an unknown angle by subtracting the known angle from 90 degrees.

If you are asked to find the **complement** of an angle, you want to find the angle that you can add to it to make 90°. For example, 60° is the complement of 30°. This is because $60° + 30° = 90°$.

Solved Examples

1. Find the value of x in the diagram below:



Solution

We know that $x + 40° = 90°$ because these are complementary angles. To find the unknown angle x, subtract the known angle from 90°.

$x = 90° - 40°$

$x = 50°$

You can always check your answers by adding: $50° + 40° = 90°$

2. Find the complement of each of the following angles:

   a. $42°$ b. 60° c. $75°$ d. $25°$

   Solutions

   Subtract from 90° to find the complement of each angle:

   a. $90 - 42 = 48°$
      $48°$ is the complement of $42°$
   b. $90 - 60 = 30°$
      $30°$ is the complement of $60°$
   c. $90 - 75 = 15°$
      $15°$ is the complement of $75°$
   d. $90 - 25 = 65°$
      $65°$ is the complement of $25°$

3. If p and $35°$ are complementary angles, find the value of p.

   Solution

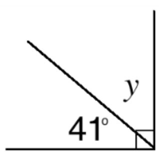
   Subtract $35°$ from $90°$ to find the value of complementary angle p:

   $p = 90 - 35 = 55°$

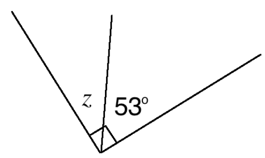
Practice

1. Find the values of the unknown angles in the diagrams below:

   a.



   b.



2. Give the complement of the following angles:
   a. $34°$ b. $37°$ c. $30°$ d. $70°$
3. If a and $82°$ are complementary angles, find the value of a.
4. If m and $59°$ are complementary angles, find the value of m.

Lesson Title: Supplementary Angles	**Theme:** Geometry
Practice Activity: PHM-07-098	**Class:** JSS 1

Learning Outcome
By the end of the lesson, you will be able to find an unknown angle given two supplementary angles.

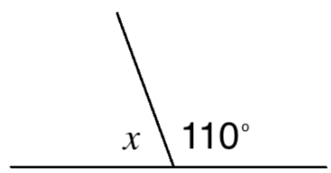
Overview

Remember that complementary angles add up to 90°, and supplementary angles add up to 180°. In this lesson, you will solve problems on supplementary angles. For supplementary angles, we can solve for an unknown angle by subtracting the known angle from 180 degrees.

If you are asked to find the **supplement** of an angle, you want to find the angle that you can add to it to make 180°. For example, 60° is the supplement of 120°. This is because $120° + 60° = 180°$.

Solved Examples

1. Find the value of x in the diagram below:



Solution

We know that $x + 110° = 180°$ because these are supplementary angles. To find the unknown angle x, subtract the known angle from $180°$.

$x = 180° - 110°$

$x = 70°$

2. Find the supplement of each of the following angles:
   a. $100°$ b. $90°$ c. $179°$ d. $120°$

Solution

Remember that supplementary angles are two angles that sum up to $180°$.
Therefore, subtract from $180°$ to find the supplement of each angle:

a. $180 - 100 = 80°$
80° is the supplement of $100°$
b. $180 - 90 = 90°$
90° is the supplement of $90°$
c. $180 - 179 = 1°$
$1°$ is supplement of $179°$
d. $180 - 120 = 60°$
$60°$ is the supplement of $120°$

3. If m and $150°$ are supplementary angles, find the value of m.

Solution

Subtract $150°$ from $180°$ to find the value of supplementary angle m:

$m = 180 - 150$
$m = 30°$

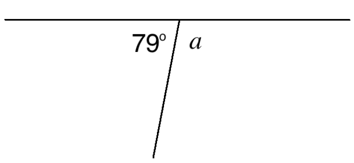
4. If p and 89° are supplementary angles, what is the value of p?

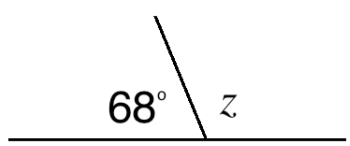
Solution

Supplementary angles add to 180°. Therefore, we have $p + 89° = 180°$. Subtract to find p: $p = 180° - 89° = 91°$

Practice

1. Find the values of the unknown angles in the diagrams below:

a. 

b. 

2. Find the supplement of the following angles:
a. $110°$ b. $130°$ c. $75°$ d. $95°$
3. If q and $17°$ are supplementary angles, find the value of q.
4. If p and $55°$ are supplementary angles, what is the value of p?

Lesson Title: Intersecting Lines	**Theme:** Geometry
Practice Activity: PHM-07-099	**Class:** JSS 1

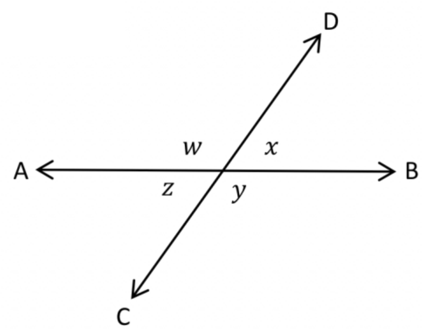
Learning Outcomes

By the end of the lesson, you will be able to:

1. Identify that intersecting lines make supplementary angles.
2. Find unknown angles formed by two intersecting lines.

Overview

In this lesson, you will solve problems that involve the angles formed by intersecting lines. For example, in the diagram below, AB and CD are intersecting lines. They form 4 angles: w, x, y and z.



The angles that are next to each other are called **adjacent** angles. Adjacent angles are supplementary to one another. In this diagram, we have the following sets of adjacent angles:

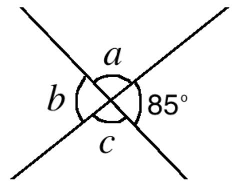
$$w + x = 180° \qquad z + y = 180°$$
$$x + y = 180° \qquad w + z = 180°$$

Angles that are opposite from each other are called **opposite** angles. Opposite angles are equal. In this diagram, we have the following sets of opposite angles:

$$w = y \qquad x = z$$

Solved Examples

1. Find the value of the angles marked with small letters in the diagram below. State your reasons.



Solution

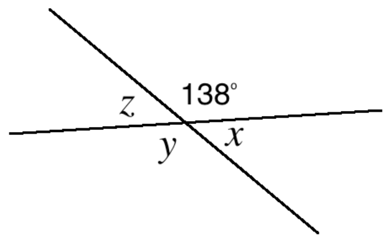
a and $85°$ are supplementary angles because they are adjacent:

$$\begin{aligned} a + 85° &= 180° \\ a &= 180° - 85° \\ a &= 95° \end{aligned}$$

$b = 85°$ because these are opposite angles

$c = a = 95°$ because these are opposite angles

2. Find the measures of angles x, y and z in the diagram below. Give your reasons.



Solution

x and $138°$ are supplementary angles because they are adjacent:

$$\begin{aligned} x + 138° &= 180° \\ x &= 180° - 138° \\ x &= 42° \end{aligned}$$

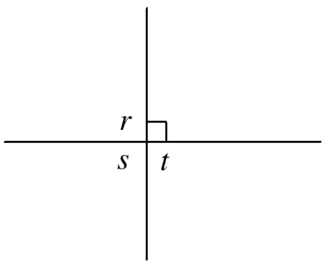
$y = 138°$ because these are opposite angles

$z = x = 42°$ because these are opposite angles

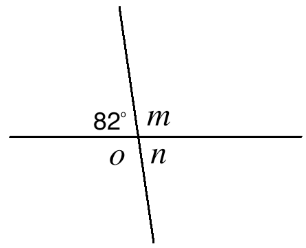
Practice

Find the values of the angles marked with small letters in the diagrams below:

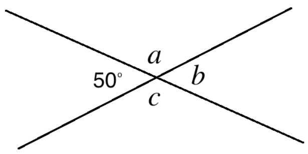
1.



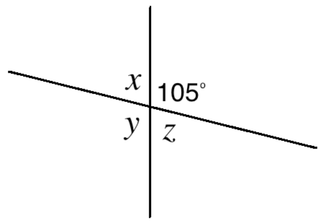
2.



3.



4.



Lesson Title: Transversal of Parallel Lines	**Theme:** Geometry
Practice Activity: PHM-07-100	**Class:** JSS 1

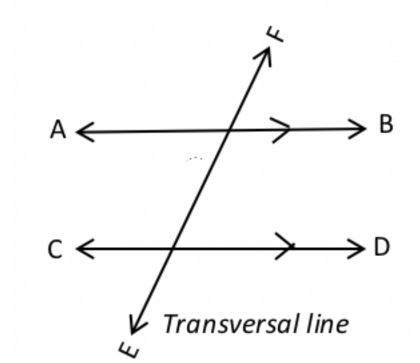
Learning Outcomes

By the end of the lesson, you will be able to:

1. Identify angles that are supplementary and angles that are equal when a transversal cuts two parallel lines.
2. Find unknown angles formed by parallel lines and a transversal line.

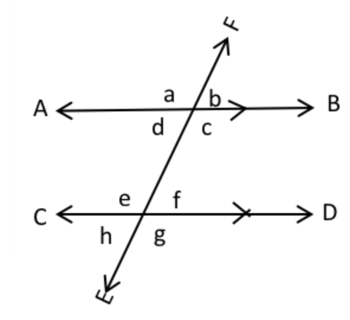
Overview

A **transversal line** is one that intersects two or more parallel lines. Parallel lines are two lines on a plane that never meet. They are always the same distance apart. In the diagram, AB and CD are parallel lines. EF is a transversal line.



If you know the measure of one angle in a set of parallel lines with a transversal, you can find all of the other angles.

Many sets of supplementary angles are formed by the transversal. All **adjacent angles** are supplementary. See the letters labelling the angles in the second diagram. The following sets of angles are adjacent:



$a + b = 180°$ $\quad b + c = 180°$ $\quad c + d = 180°$

$d + a = 180°$ $\quad e + f = 180°$ $\quad f + g = 180°$

$g + h = 180°$ $\quad h + e = 180°$

In the diagram, d and e are called **co-interior angles**, which means they are both inside the parallel lines on the same side of the transversal. They are supplementary angles even though they are not next to each other. Co-interior angles always sum up to 180°.

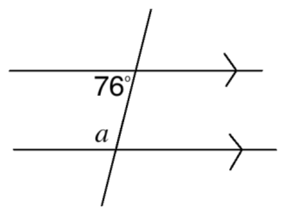
Co-interior angles: $d + e = 180°$ $\quad c + f = 180°$

Recall that **opposite angles** are across from each other in an intersection and are always equal. For example, in this diagram, $a = c$ and $b = d$.

Corresponding angles are in the same position on the two different intersections and are always equal. For example, $a = e$ and $d = h$.

Solved Examples

1. Find the value of a in the diagram below:



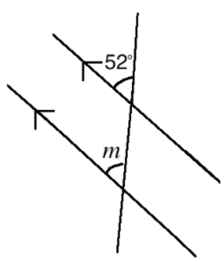
Solution

a and $76°$ are co-interior angles, so they are supplementary. Therefore:

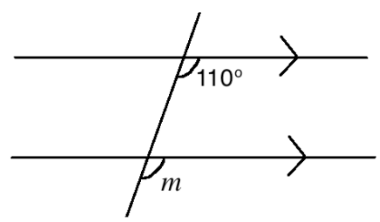
$$\begin{aligned} a + 76° &= 180° \\ a &= 180° - 76° \\ a &= 104° \end{aligned}$$

2. Find the value of m in the diagrams below. State your reasons.

a.



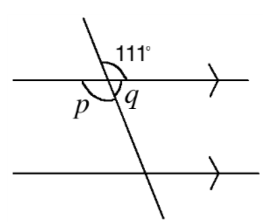
b.



Solution

a. $m = 52°$, because they are corresponding angles.
b. $m = 110°$, because they are corresponding angles.

3. Find the measures of p and q. State your reasons.



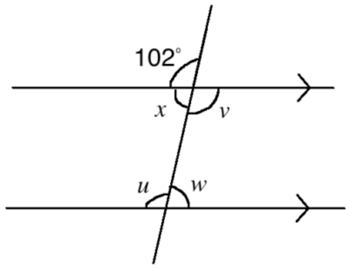
Solution

$p = 111°$ because they are opposite angles.

q and $111°$ are supplementary angles because they are adjacent:

$$\begin{aligned} q + 111° &= 180° \\ q &= 180° - 111° \\ q &= 69° \end{aligned}$$

4. Find the value of the angles marked with small letters in the diagram below. State your reasons.



Solution

$v = 102°$, because it is an opposite angle to $102°$.
$u = 102°$, because it is a corresponding angle to $102°$.

x and $102°$ are supplementary angles; use this to find x:

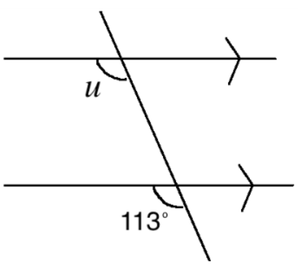
$$\begin{aligned} x + 102° &= 180° \\ x &= 180° - 102° \\ x &= 78° \end{aligned}$$

w is a supplementary angle with u, which is also $102°$. Therefore, $w = 78°$

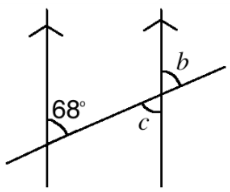
Practice

1. Find the value of the angles marked with small letters in the diagrams below.
   State your reasons.

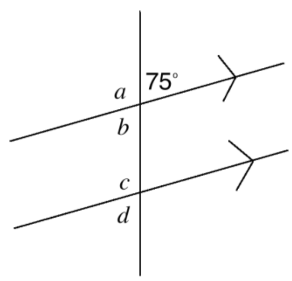
a.



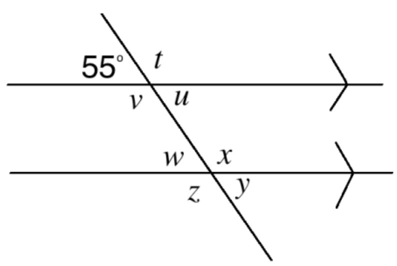
b.



c.



2. Find the measure of each angle in the diagram below:

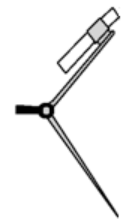


Lesson Title: Construction of Circles	**Theme:** Geometry
Practice Activity: PHM-07-101	**Class:** JSS 1

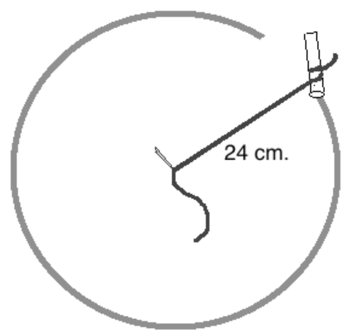
Learning Outcome
By the end of the lesson, you will be able to use a pair of compasses to construct a circle within a given radius.

Overview

The tool pictured is a pair of compasses. It makes circles, and can be used to construct many different angles and shapes in geometry. It will be used for the next 4 lessons. If you do not have a pair of compasses, you can make one using string or a strip of paper.

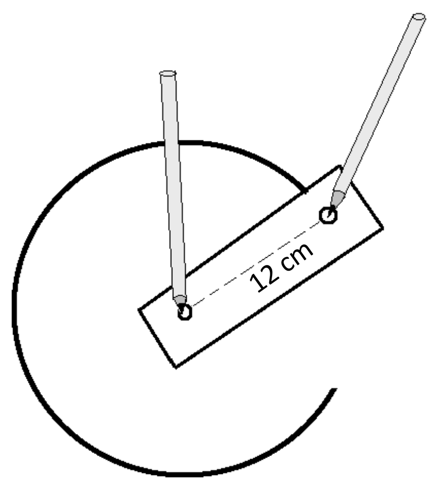


Using **string** to make a pair of compasses:



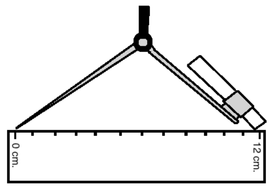
1. Cut a piece of string longer than the radius of the circle you will make.
2. Tie one end of the string to a pencil.
3. Hold the string to your paper. The distance between the place you hold and the pencil will be the **radius** of the circle. In the diagram, the radius of the circle is 24 cm.
4. Use one hand to hold the string to the same place on the paper. Use the other hand to move the pencil around and draw a circle.

Using **paper** to make a pair of compasses:

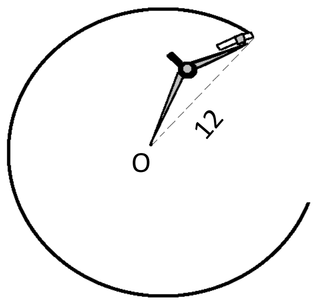


1. Cut or tear any piece of paper, longer than the radius of the circle you will make.
2. Make two small holes in the paper. The distance between the two holes will be the radius of your circle. In the diagram below, the radius of the circle is 12 cm.
3. Put something sharp (a pen or pencil will work) through one hole. Place this in your exercise book, at the centre of the circle you will draw.
4. Put your pen or pencil through the other hole, and move it in a circle on your paper.

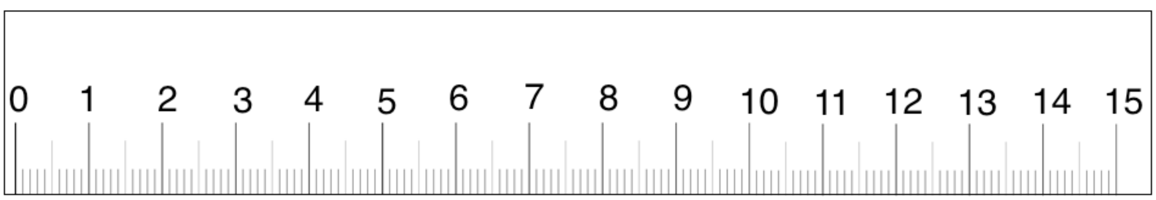
In this lesson, you will construct circles with a given radius. For example, you may be asked to construct a circle with radius of 12 cm. You would open your pair of compasses to 12 cm using a ruler, as shown:



Place the point of the compass on your paper. Keep the pair of compasses at a distance of 12 cm, and move it around to draw a circle.



If you have a ruler, use it in class and at home to do construction. If you do not have a real ruler, use the printed one below. This ruler is not exactly to scale (1 cm is not exactly 1 cm). However, it can be used for the purpose of learning construction.

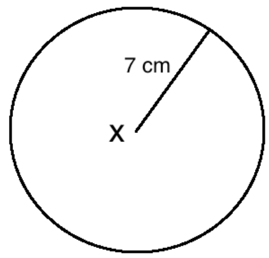


Solved Examples

1. Construct a circle with radius of 7 cm, with its centre at point x.

   Solution

   Follow these steps to construct the circle:

   Not to Scale

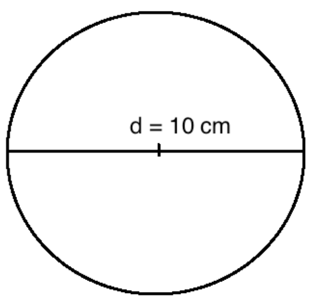
   1. Mark a point on your paper and label it x.
   2. Open your pair of compasses to 7 cm.
   3. Place the point of your compass at x.
   4. Keep the pair of compasses at a distance of 12 cm, and move it around to draw a circle.

2. Construct a circle with diameter 10 cm.

   Solution

   Note that this problem gives the diameter of the circle, 10 cm. To construct a circle, you must have the radius. The radius is $r = \frac{d}{2} = \frac{10}{2} = 5$ cm.

   Open your compass to 5 cm and construct the circle:

   Not to Scale

Practice

1. Construct a circle with radius 8 cm, and its centre at point A.
2. Construct a circle with diameter 8 cm, and its centre at point B.
3. Construct a circle with radius 3 cm.
4. Construct a circle with diameter 13 cm

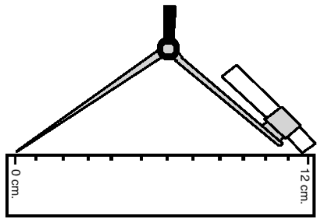
Lesson Title: Construction of Triangles	**Theme:** Geometry
Practice Activity: PHM-07-102	**Class:** JSS 1

Learning Outcome
By the end of the lesson, you will be able to use a pair of compasses to construct a triangle given the lengths of the 3 sides.

Overview

In this lesson, you will construct a triangle given the lengths of its 3 sides.

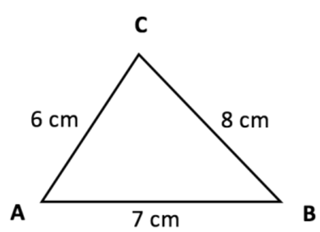
To construct a triangle given 3 sides, you will need to set the radius of your compass equal to the lengths of the sides of the triangle. This is done in the same way as the previous lesson. →



Consider an example problem: Construct a triangle ABC with sides 6 cm, 7 cm, and 8 cm.

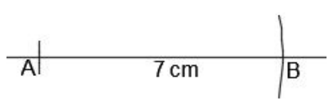
Before starting your construction, it is helpful to quickly sketch a picture of what the constructed triangle will look like. Don't worry about sketching it to scale, this sketch is just to guide your construction.

Sketch:

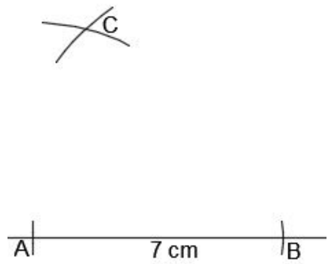


To construct ABC, follow these steps:

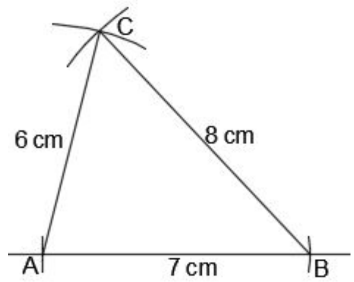
- Draw a line and label point A on one end.
- Open your compass to the length of 7 cm. Use it to mark point B, 7 cm from point A. This gives line segment $\overline{AB} = 7$ cm.



- Open your compass to the length of 6 cm. Use A as the centre, and draw an arc of 6 cm above $\overline{AB}$.
- Open your compass to the length of 8 cm. With the point B as the centre, draw an arc that intersects with the arc you drew from point A. Label the point of intersection C.



- Join $\overline{AC}$ and $\overline{BC}$. This is the required triangle ABC.
- Label the sides with the correct lengths:

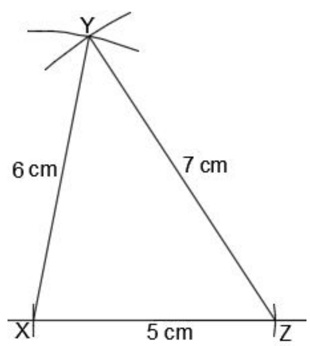


Note that the triangles in this lesson are not drawn to the correct length. They are in the correct shape. Use a ruler to draw your triangles with the correct length.

Solved Examples

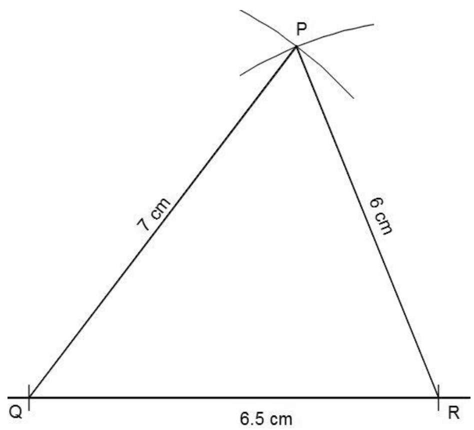
1. Construct triangle XYZ with sides of length 5 cm, 6 cm, and 7 cm.

   Solution
   Note that your triangles could be turned in any direction. For example, line XY or YZ could be the base.



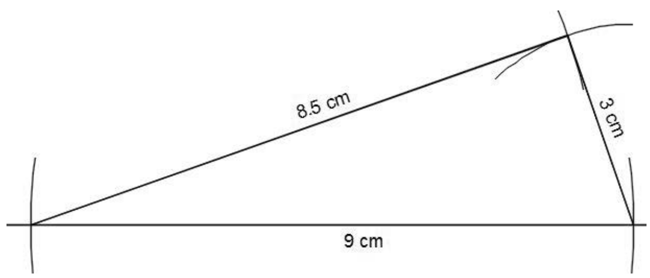
2. Using a ruler and a pair of compasses only, construct a triangle PQR with $|PR| = 6$ cm, $|PQ| = 7$ cm and $|QR| = 6.5$ cm.

Solution



3. Construct a triangle with sides of length 3 cm, 9 cm and 8.5 cm.

Solution



Practice

1. Construct a triangle ABC such that $|AB| = 5$ cm, $|BC| = 6$ cm and $|AC| = 4$ cm.
2. Construct triangle JKL where $|JK|$ is 9 cm, $|KL|$ is 7 cm and $|LJ|$ is 6 cm.
3. Using a ruler and a pair of compasses, construct a triangle ABC such that $|AB| = 6.0$ cm, $|BC| = 8.0$ cm and $|AC| = 7.0$ cm.

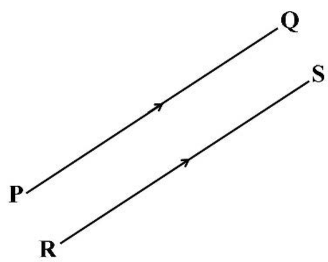
Lesson Title: Construction of Parallel Lines	**Theme:** Geometry
Practice Activity: PHM-07-103	**Class:** JSS 1

Learning Outcome
By the end of the lesson, you will be able to use a pair of compasses to construct parallel lines.

Overview

Parallel lines are two lines that are always the same distance apart and never touch.

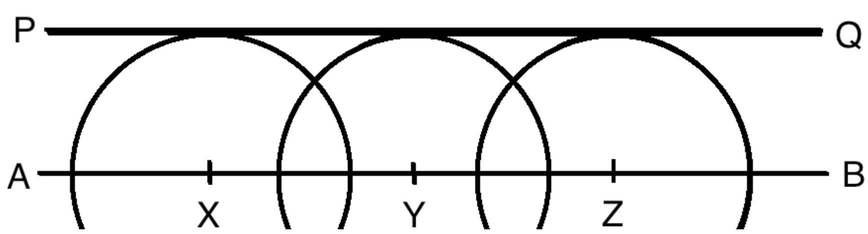
In order for two lines to be parallel, they must be drawn in the same plane, a perfectly flat surface like a wall or sheet of paper.



To show that lines are parallel, we draw small arrow marks on them.

The line segments above are parallel. In symbols, we write $\overline{PQ}||\overline{RS}$. The symbol || means 'is parallel to.' Recall that the horizontal bar over the letters indicates it is a line segment**.**

Parallel lines can be constructed using a pair of compasses and any straight edge. Step-by-step instructions for drawing this construction are below.



To construct parallel lines PQ and AB, follow these steps:

1. Draw a horizontal line and label the ends with A and B
2. Mark 3 points anywhere on the line. Let's call these X, Y and Z.
3. Place the point of the pair of compasses at X. Using any convenient radius, draw an arc above the line AB.
4. Do the same thing for points Y and Z. Use exactly the same radius for all 3 points.
5. Place a straight edge above the 3 arcs, touching each of them at the highest point.
6. Draw a line along the straight edge and label the ends of the line with P and Q.

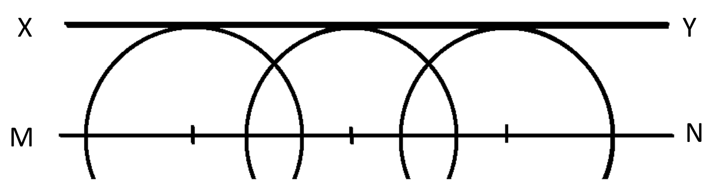
We now have $\overline{PQ}||\overline{AB}$.

Solved Examples

1. Construct two parallel lines, $\overline{MN}$ and $\overline{XY}$.

   Solution

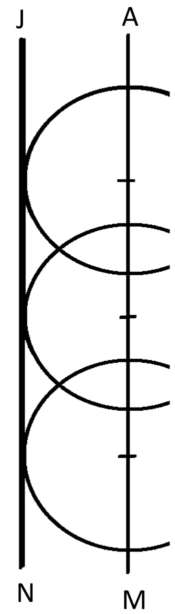
   Follow the steps given above, but label the ends of the lines MN and XY, as shown:



2. Draw a vertical line $\overline{AM}$. Construct line $\overline{JN}$ parallel to it.

Solution

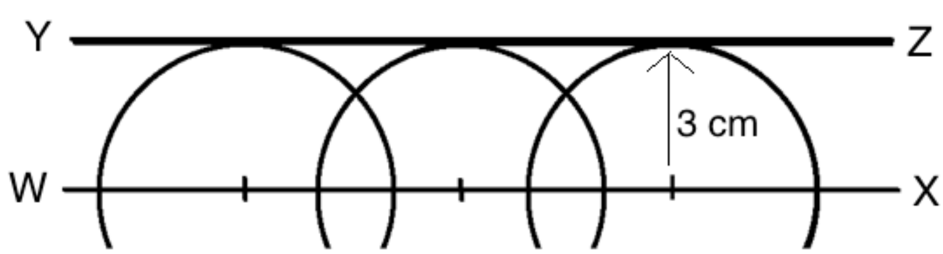
Follow the same steps as given above. The only difference is that you will start with a vertical line $\overline{AM}$, instead of starting with a horizontal line.



3. Draw a horizontal line $\overline{WX}$. Construct parallel line $\overline{YZ}$ at a distance of 3 cm from $\overline{WX}$.

Solution

Follow the same steps as in the above problems. The difference is that you will open your pair of compasses to 3 cm. Keep it open to exactly 3 cm to make each of the 3 arcs. This will give you a parallel line 3 cm away from the first line.



Practice

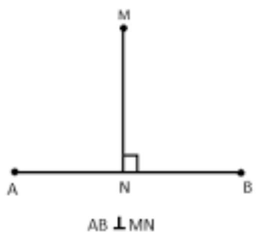
1. Draw a horizontal line $\overline{MB}$. Construct line $\overline{DZ}$ parallel to it.
2. Construct parallel lines $\overline{DE}$ and $\overline{FG}$.
3. Draw a vertical line $\overline{AB}$. Construct parallel line $\overline{CD}$ at a distance of 5 cm from $\overline{AB}$.

Lesson Title: Construction of Perpendicular Lines	**Theme:** Geometry
Practice Activity: PHM-07-104	**Class:** JSS 1

Learning Outcome
By the end of the lesson, you will be able to use a pair of compasses to construct perpendicular lines.

Overview

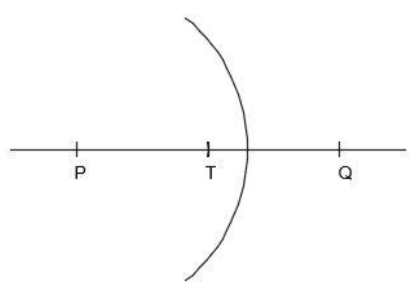
Perpendicular means that two lines meet at a right angle, 90°. A line is said to be perpendicular to another line if the two lines intersect at a right angle. In the diagram below, $\overline{MN}$ is perpendicular to $\overline{AB}$. In symbols, this is written $\overline{AB} \perp \overline{MN}$.



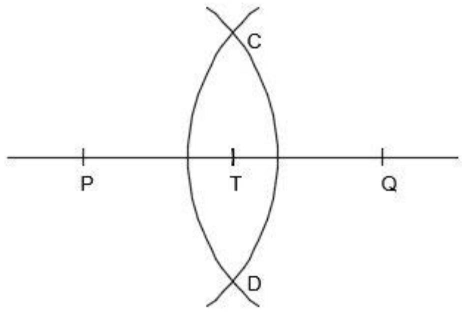
Perpendicular lines are often shown with a small square on the right angle. You can measure the angle with a protractor to check if the measure is 90°.

In the diagram below, $\overline{CD}$ is perpendicular to $\overline{PQ}$ at point T. To do this construction, follow these steps:

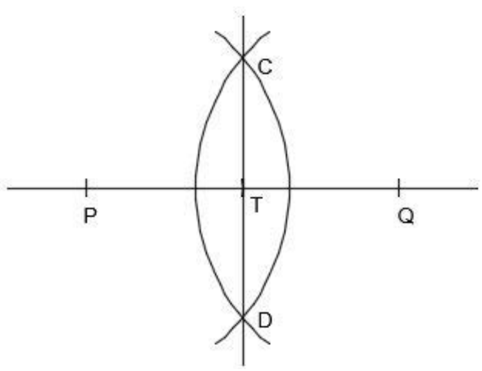
1. Draw a line, and mark point T around the middle.
2. Draw arcs to cut the line segment at 2 points the same distance from T, and label these points P and Q.
3. With point P as centre, open your compass more than half way to point Q. Then draw an arc that intersects $\overline{PQ}$.



4. Using the same radius and point Q as the centre, draw an arc that intersects the first arc. Label the points where the 2 arcs intersect as C and D.



5. Draw $\overline{CD}$.

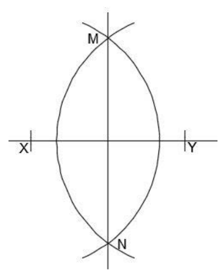


Solved Examples

1. Construct perpendicular lines XY and MN.

   Solution

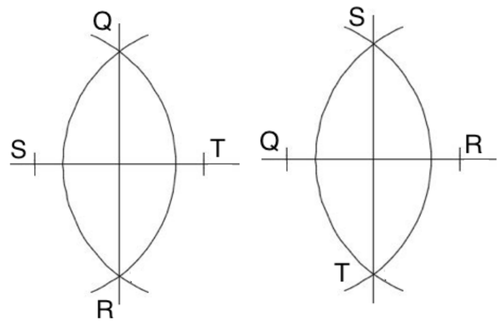
   Follow the steps described above. It is not necessary to label the point of intersection, because a letter is not given for that point.



2. Line ST and QR are perpendicular lines. Construct the 2 lines.

Solution

Either construction is correct:



Practice

1. Draw a line segment $\overline{PQ}$ with the centre at point O. Construct a perpendicular line at O.
2. Construct perpendicular lines BO and SL.
3. Draw a line segment PQ with the centre at point T. Construct a perpendicular line CD at T.
4. Construct perpendicular lines XY and MN.

Lesson Title: Construction Practice	Theme: Geometry
Practice Activity: PHM-07-105	Class: JSS 1

Learning Outcome
By the end of the lesson, you will be able to construct circles, triangles, parallel and perpendicular lines.

Overview

In this lesson, you will practise what you learned in the previous 4 lessons to draw various constructions.

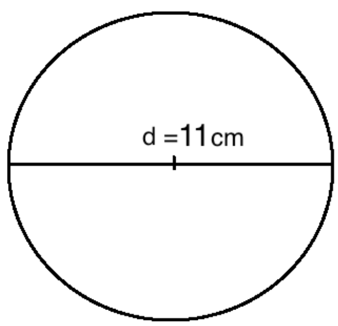
Solved Examples

1. Construct a circle with diameter 11 cm.

   Solution

   Note that this problem gives the diameter of the circle, 11 cm. To construct a circle, you must have the radius. The radius is $r = \frac{d}{2} = \frac{11}{2} = 5.5$ cm.

   Open your compass to 5.5 cm and construct the circle:



Not to Scale

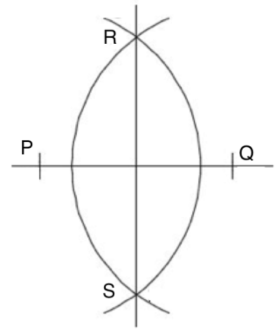
2. Construct perpendicular lines $\overline{PQ}$ and $\overline{RS}$.

   Solution

   Follow the steps:

   1. Draw a line and choose a point around the middle.
   2. Draw arcs to cut the line segment at 2 points on the line and label these points P and Q.

3. With point P as the centre, open your compass more than halfway to point Q. Then draw an arc that intersects $\overline{PQ}$.
4. Using the same radius and point Q as centre, draw an arc that intersects the first arc. Label the points where the 2 arcs intersect as R and S.
5. Draw $\overline{RS}$.

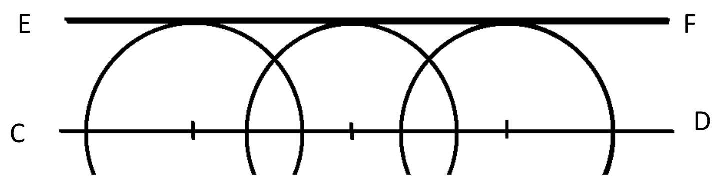


3. Construct two parallel lines, $\overline{CD}$ and $\overline{EF}$.

Solution

Follow these steps:

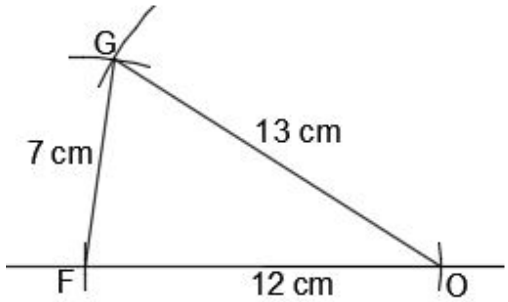
1. Draw a horizontal line and label the ends with C and D
2. Mark 3 points anywhere on the line.
3. Place the point of the pair of compasses at each point. Using any convenient radius, draw an arc above the line AB. Use exactly the same radius for all 3 points.
4. Place a straight edge above the 3 arcs, touching each of them at the highest point.
5. Draw a line along the straight edge and label the ends of the line with E and F.



4. Construct triangle FOG where $\overline{FO} = 12$ cm, $\overline{OG} = 13$ cm, and $\overline{FG} = 7$ cm.

Solution
Follow the steps:
1. Draw a line and label point F on one end.
2. Open your compass to the length of 12 cm. Use it to mark point O, 12 cm from point F. This gives line segment $\overline{FO} = 12$ cm.
3. Open your compass to the length of 13 cm. Use O as centre, and draw an arc of 13 cm above $\overline{FO}$.

4. Open your compass to the length of 7 cm. With the point F as the centre, draw an arc that intersects with the arc you drew from point O. Label the point of intersection G.
5. Join $\overline{FG}$ and $\overline{OG}$. This is the required triangle FOG.
6. Label the sides with the correct lengths.



Practice

1. Construct parallel lines $\overline{LI}$ and $\overline{ON}$.
2. Construct triangle FLY where $\overline{FL}$ is 10 cm, $\overline{LY}$ is 8 cm, and $\overline{YF}$ is 6 cm.
3. Construct perpendicular lines $\overline{DU}$ and $\overline{CK}$.
4. Construct a circle with radius 6 cm.
5. Construct a circle with diameter 7 cm.

Answer Key – JSS 1 Term 2

Lesson Title: Introduction to Ratio	
Practice Activity: PHM-07-046	

1. a. 45 : 23; b. 23 : 30; c. 30 : 45; d. 45 : 30
2. a. 5 : 2; b. 7 : 1; c. 1 : 35; d. 10 : 3

Lesson Title: Ratio of a Whole	
Practice Activity: PHM-07-047	

1. a. 18 : 19; b. 19 : 18; c. 18 : 37; d. 19 : 37
2. a. 15 : 8; b. 15 : 26; c. 3 : 8; d. 8 : 26
3. a. 14 : 31; b. 8 : 31; c. 14 : 8; d. 14 : 9

Lesson Title: Ratios and Fractions	
Practice Activity: PHM-07-048	

1. $\frac{4}{7}$
2. $\frac{1}{4}$
3. $\frac{1}{4}$
4. $\frac{1}{3}$
5. $\frac{1}{10}$
6. a. 100 people; b. $\frac{4}{5}$; c. $\frac{3}{10}$
7. $\frac{1}{3}$

Lesson Title: Ratios and Percentages	
Practice Activity: PHM-07-049	

1. a. 55 : 100; b. 60 : 100; c. 81 : 100; d. 8 : 100
2. a. 31%; b. 60%; c. 50%; d. 70%
3. a. 32 : 40; b. $\frac{4}{5}$; c. 80%
4. 87 : 100
5. $\frac{19}{20}$

Lesson Title: Ratios and Decimals
Practice Activity: PHM-07-050

1. a. 0.19; b. 0.3; c. 0.2; d. 0.4
2. a. 8 : 10 or 4 : 5; b. 3 : 100; c. 5 : 10 or 1 : 2; d. 79 : 100
3. $\frac{1}{2}$, 0.5, 50%
4. $\frac{4}{5}$, 4 : 5, 80%

Lesson Title: Simplification of Ratios
Practice Activity: PHM-07-051

1. a. 1 : 7; b. 2 : 5; c. 9 : 2; d. 2 : 1
2. a. 9; b. 3; c. 30; d. 12; e. 180; f. 75

Lesson Title: Ratio Problems with Two Terms
Practice Activity: PHM-07-052

1. 48 bananas and 60 bananas
2. 50 grammes and 150 grammes
3. 60 exercise books and 90 exercise books
4. 20 pieces of candy and 4 pieces of candy
5. 40 and 80
6. Mattu gets Le 30,000.00, Bondu gets Le 50,000.00.

Lesson Title: Ratio Problems with Three or More Terms
Practice Activity: PHM-07-053

1. Le 6,000.00, Le 10,000.00, and Le 8,000.00
2. 10 kg, 15 kg, and 20 kg
3. Ben's share: 16 mangoes; Juliet's share: 8 mangoes; Sia's share: 12 mangoes; John's share: 12 mangoes
4. 18 bananas, 12 bananas, 30 bananas

Lesson Title: Relating Ratios to Measurement
Practice Activity: PHM-07-054

1. a. 40 m and 60 m; b. 20 m, 30 m, and 50 m
2. 10 : 7
3. 35 km and 7 km
4. 8 m, 12 m, and 4 m
5. 6 m and 12 m

Lesson Title:	Ratio Story Problems
Practice Activity:	PHM-07-055

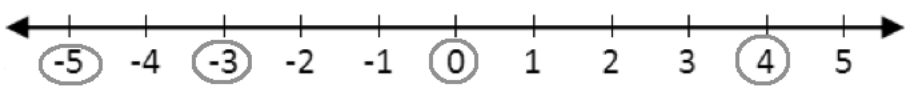
1. 8 feet green and 12 feet yellow
2. a. 9 : 10; b. 5 : 1; c. 9 : 2; d. 9 : 10 : 2
3. Alice gets Le 160,000.00, Foday gets Le 80,000.00, Sia gets Le 240,000.00, Michael gets Le 320,000.00
4. 43 : 50
5. 9 : 8

Lesson Title:	Introduction to Integers
Practice Activity:	PHM-07-056

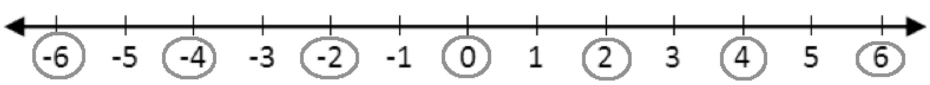
1. a -5, c. -16, e. -1
2. b. 1, c. +3, e. 5
3. Circled numbers: +10, 16, 47, 91, 1, 3, 100, +4, 11, 5, +75; Triangle numbers: -21, -6, -8,
   -80, -19, -24, -70, -99, -14

Lesson Title:	Positive and Negative Integers
Practice Activity:	PHM-07-057

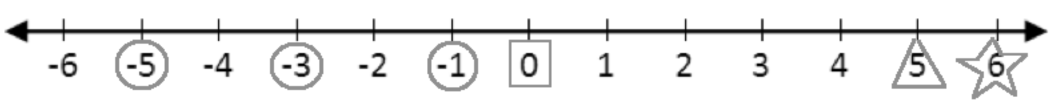
1.



2.



3.



Lesson Title:	Comparing Integers
Practice Activity:	PHM-07-058

1. a.–8 < 7; b. –9 < 9; c. –12 < 0
2. a. 21 > 12; b. 9 > 8; c. 17 < 21
3. a. –12 > –30; b. –20 > –80; c. –9 < –7
4. –19, –16, –15, –14, –12, 13, 17, 18
5. 18, 16, 7, 5, -6, -7, -9, -15, -16.

Lesson Title:	Addition of Integers Using a Number Line
Practice Activity:	PHM-07-059

1. -6
2. 5
3. -7
4. -4
5. 7
6. -8
7. -3

Lesson Title:	Addition of Integers
Practice Activity:	PHM-07-060

1. -8
2. 7
3. 24
4. -23
5. -10
6. 11
7. -10
8. 0
9. 10
10. -15

Lesson Title:	Subtraction of Integers
Practice Activity:	PHM-07-061

1. -11
2. -7
3. -9
4. 1
5. 27
6. -18
7. 17
8. 0
9. -6
10. 7

Lesson Title:	Multiplication of Integers Using a Number Line
Practice Activity:	PHM-07-062

1. -9
2. -10
3. -4
4. -3
5. 6
6. 8
7. -8
8. 0
9. 10
10. - 4

Lesson Title:	Multiplication of Integers
Practice Activity:	PHM-07-063

1. -36
2. 15
3. -28
4. 1
5. -9
6. 0
7. -40
8. -27
9. 48
10. -35

Lesson Title:	Division of Integers
Practice Activity:	PHM-07-064

1. -5
2. 6
3. -8
4. 2
5. -1
6. -32
7. -20
8. -3
9. 2
10. 50

Lesson Title:	Story Problems on Integers
Practice Activity:	PHM-07-065

1. 38°C
2. Le 4,000.00
3. – Le 10,000.00
4. 420 feet

Lesson Title: Simple Proportion
Practice Activity: PHM-07-066

1. 75 km
2. Le 40,000.00
3. 18 hours
4. 120 patients
5. 6 buses
6. 15 packets

Lesson Title: Simple Interest
Practice Activity: PHM-07-067

1. Le 270,000.00
2. Le 140,000.00
3. Le 12,000.00
4. Le 2,000.00

Lesson Title: Discount
Practice Activity: PHM-07-068

1. a. Le 7,500.00; b. Le 42,500.00
2. Le 4,500,000.00
3. No, she doesn't have enough money.
4. Le 150,000.00

Lesson Title: Commission
Practice Activity: PHM-07-069

1. Le 35,000.00
2. Le 588,000.00
3. a. Le 1,000,000.00; b. Le 400,000.00; c. Le 340,000.00

Lesson Title: Tax
Practice Activity: PHM-07-070

1. a. Le 22,500.00; b. Le 472,500.00
2. Le 530,000.00
3. Le 3,120,000.00

Lesson Title: Units of Measurements
Practice Activity: PHM-07-071

1. Answers are given in the table:

Mass	Volume
a, c, f, g, i	b, d, e, h

2. a. mg, g, kg, t; b. ml, dl, l, kl; c. mm, cm, m, km

Lesson Title: Conversion of Length
Practice Activity: PHM-07-072

1. 2.5 cm
2. 100 mm
3. 22 cm
4. 1.5 m
5. 0.6 km
6. 0.055 km
7. 7,000 m
8. 75 m
9. 1.13 m

Lesson Title: Conversion of Mass
Practice Activity: PHM-07-073

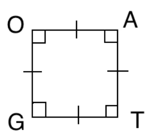
1. 0.025 g
2. 0.25 kg
3. 3.3 kg
4. 2.43 t
5. 0.56 t
6. 2.6 g
7. 8,500 kg
8. 50 kg
9. 2.525 t

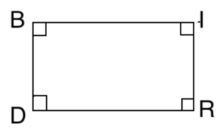
Lesson Title: Conversion of Volume	
Practice Activity: PHM-07-074	

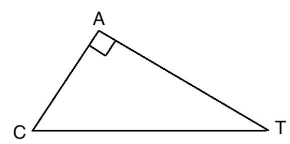
1. 0.036 l
2. 0.3 l
3. 750 ml
4. 4.2 l
5. 600 l
6. 2.5 dl
7. 25,000 l
8. 3.67 l
9. 75 ml
10. 2.05 l

Lesson Title: Review of Plane Shapes	
Practice Activity: PHM-07-075	

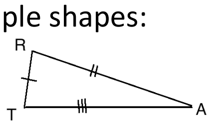
1. Example shapes:

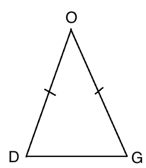
a. 

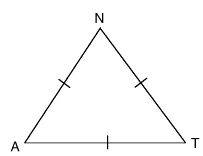
b. 

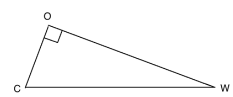
c. 

2. Example shapes:

a. 

b. 

c. 

d. 

3. a. equilateral. All sides are equal
   b. Scalene. No sides are equal.
   c. Isosceles. Two Sides are equal.

   d. Isosceles and right-angled triangle. Two sides are equal and there is a right angle.

Lesson Title: Perimeter	
Practice Activity: PHM-07-076	

1. a. 78 m; b. 24.8 m
2. 56 cm
3. 16 m
4. 48 cm
5. 125 mm

Lesson Title: Area of Rectangles and Squares
Practice Activity: PHM-07-077

1. a. 350 m^2; b. 38.44 m^2
2. 196 cm^2
3. 140 cm^2
4. The square has a greater area. The square has an area of 16 cm^2, and the rectangle has an area of 15 cm^2.

Lesson Title: Area of Triangles
Practice Activity: PHM-07-078

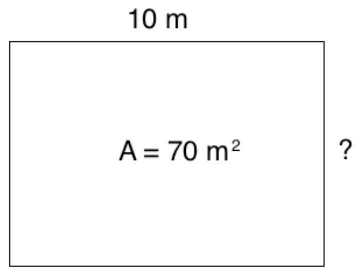
1. a. 110 cm^2; b. 45 cm^2
2. a. 85.2 cm^2; b. 56 cm^2; c. 500 mm^2

Lesson Title: Perimeter Story Problems
Practice Activity: PHM-07-079

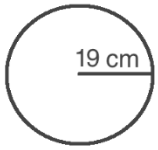
1. 62 m
2. 9 m
3. 110 m
4. 212 cm
5. 3.6 m

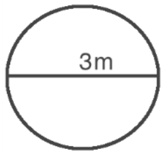
Lesson Title: Area Story Problems
Practice Activity: PHM-07-080

1. P = 24 m
2. a. 120 m^2; b. 3000 tiles
3. 5 : 3
4. a. See diagram below; b. w = 7 m; c. P = 34 m



Lesson Title:	Circles
Practice Activity:	PHM-07-081

1. $d = 38$ m; Sketch: 

2. $r = 1.5$ m; Sketch: 

3. a. 18 cm; b. 208 cm; c. 54 m
4. a. 41 m; b. 35 m; c. 7.5 cm

Lesson Title:	Circumference of Circles
Practice Activity:	PHM-07-082

1. 44 cm
2. 628 cm
3. 220 cm
4. 440 cm
5. 176 m
6. 188.4 cm

Lesson Title:	Area of Circles
Practice Activity:	PHM-07-083

1. 314 cm^2
2. 78.6 m^2
3. 28.3 cm^2
4. 50.29 m^2
5. 201.1 m^2
6. 113 m^2

Lesson Title:	Problem Solving with Circles
Practice Activity:	PHM-07-084

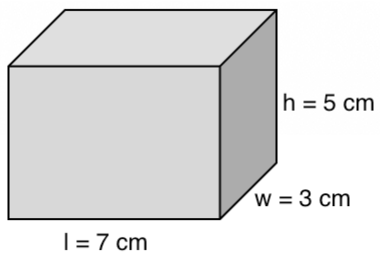
1. P = 87 m; A = 693 m^2
2. P = 25.7cm; A = 39.25 cm^2
3. 132.8 m
4. 113 cm^2

Lesson Title: Circle Story Problems
Practice Activity: PHM-07-085

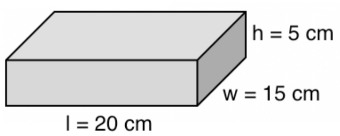
1. 44 rose trees
2. 19.6 m^2
3. a. 314 m; b. Le 942,000.00
4. 616 ft^2

Lesson Title: Volume of Solids
Practice Activity: PHM-07-086

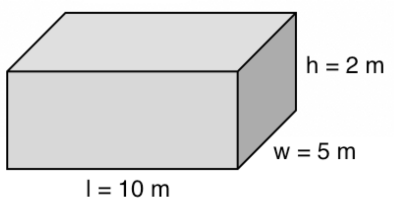
1. $V = l \times w \times h$ or $V = A \times h$
2. Rectangular solid:



3. Dictionary:



4. a. see below; b. m^3



5. a. cm^3; b. m^3; c. cm^3; d. ft^3
6. a. area; b. volume; c. volume; d. area; e. area

Lesson Title: Volume of Cubes
Practice Activity: PHM-07-087

1. 343 cm^3
2. 15.6 m^3
3. 729 cm^3
4. 512 m^3
5. 2 ft

Lesson Title: Volume of Cuboids
Practice Activity: PHM-07-088

1. 160 cm^3
2. 280 cm^3
3. 78 m^3
4. 120,000 cm^3
5. 6 m^3

Lesson Title: Problem Solving with Volume
Practice Activity: PHM-07-089

1. $V = 700\ cm^3$
2. $A = 25\ cm^2$
3. $A = 5.3\ cm^2$
4. $h = 26$ cm
5. $h = 5$ cm

Lesson Title: Volume Story Problems
Practice Activity: PHM-07-090

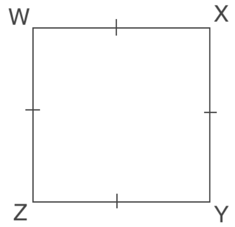
1. 116 m^3
2. 84 m^3
3. a. 60,000 cm^3; b. 60 litres
4. 0.4 m^3

Lesson Title: Introduction to Angles
Practice Activity: PHM-07-091

1. a. right; b. acute; c. acute; d. obtuse
2. a. Two hundred eighty degrees
   b. Eighty point three nine degrees
   c. Sixteen and a half degrees **or** Sixteen point five degrees
3. a. $55°$; b. $66.5°$; c. $90.4°$
4. a. acute; b. obtuse; c. right; d. acute; e. obtuse

Lesson Title: Right Angles
Practice Activity: PHM-07-092

1. $\angle Q = 90°$, $\angle R = 90°$, $\angle S = 90°$, $\angle T = 90°$
2. See square below; $\angle W = 90°$, $\angle X = 90°$, $\angle Y = 90°$, $\angle Z = 90°$



Lesson Title: Measurement of Angles
Practice Activity: PHM-07-093

1. a. $50°$, acute; b. $125°$, obtuse; c. $\angle XOY = 40°$, acute; d. $\angle SOR = 139°$, obtuse
2. a. $67°$; b. $105°$; c. $30°$

Lesson Title: Finding Unknown Angles in Triangles
Practice Activity: PHM-07-094

1. a. $x = 67°$; b. $x = 62°$; c. $x = 60°$
2. $b = 53°$
3. $p = 18°$

Lesson Title: Finding Unknown Angles in Composite Shapes
Practice Activity: PHM-07-095

1. $R = 30°$, $A = 150°$, $P = 90°$, $I = 90°, D = 180°$.
2. $C = 60°$, $R = 120°$, $A = 30°$, $B = 150°$
3. $P = 38°$, $I = 142°$, $N = 45°$, $K = 135°$
4. $C = 43°$, $A = 79°$

Lesson Title: Introduction to Complementary and Supplementary Angles
Practice Activity: PHM-07-096

1. a. Complementary; b. Supplementary; c. Complementary; d. Supplementary; e. Complementary; f. Complementary
2. a. $\angle ABC$ and $\angle TUV$; b. $\angle XYZ$ and $\angle MNO$

Lesson Title: Complementary Angles	
Practice Activity: PHM-07-097	

1. a. $y = 49°$; b. $z = 37°$
2. a. $56°$; b. $53°$; c. $60°$; d. $20°$
3. $a = 8°$
4. $m = 31°$

Lesson Title: Supplementary Angles	
Practice Activity: PHM-07-098	

1. a. $a = 101°$; b. $z = 112°$
2. a. $70°$; b. $50°$; c. $105°$; d. $85°$
3. $q = 163°$
4. $p = 125°$

Lesson Title: Intersecting Lines	
Practice Activity: PHM-07-099	

1. $r = 90°$, $s = 90°$, $t = 90°$
2. $m = 98°$, $n = 82°$, $o = 98°$
3. $a = 130°$, $b = 50°$, $c = 130°$
4. $x = 75°$, $y = 105°$, $z = 75°$

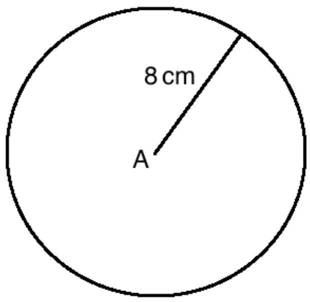
Lesson Title: Transversal of Parallel Lines	
Practice Activity: PHM-07-100	

1. a. $u = 113°$; b. $b = 68°$, $c = 68°$; c. $a = 105°$, $b = 75°$, $c = 105°$, $d = 75°$
2. $t = 125°$, $u = 55°$, $v = 125°$, $w = 55°$, $x = 125°$, $y = 55°$, $z = 125°$

Lesson Title:	Construction of Circles
Practice Activity:	PHM-07-101

Note that the diagrams below are not drawn to the correct length. Use a ruler to draw your constructions to the correct length.

1.



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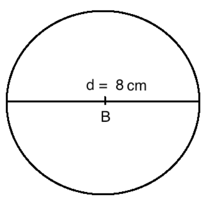

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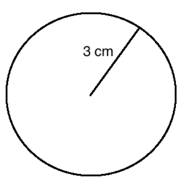

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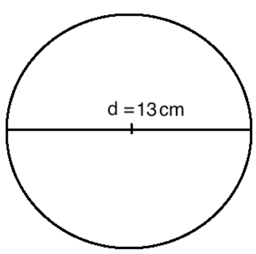
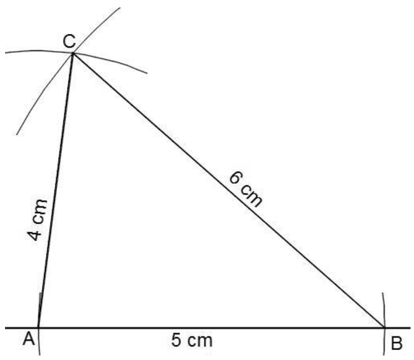


Lesson Title: Construction of Triangles	
Practice Activity: PHM-07-102	

Note that the diagrams below are not drawn to the correct length. Use a ruler to draw your constructions to the correct length.

1.



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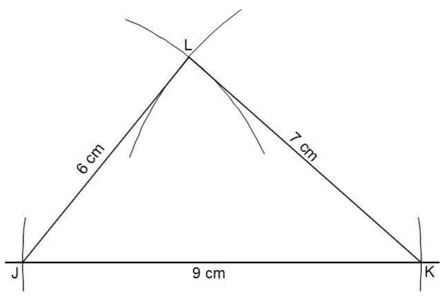

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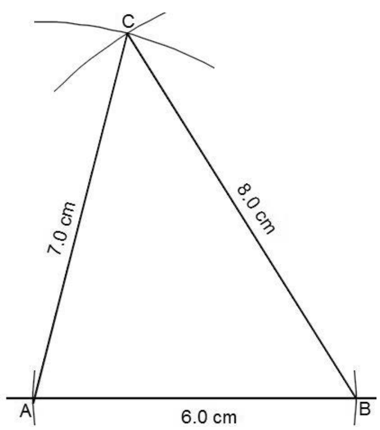
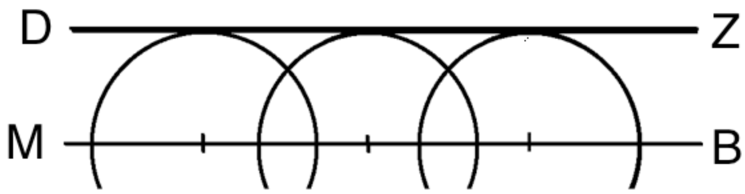


Lesson Title: Construction of Parallel Lines
Practice Activity: PHM-07-103

1.



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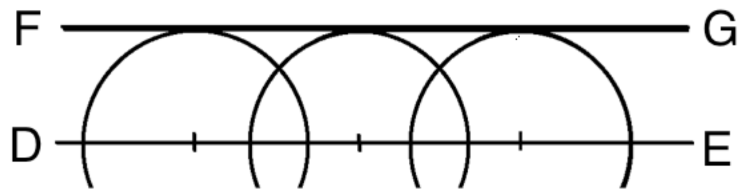
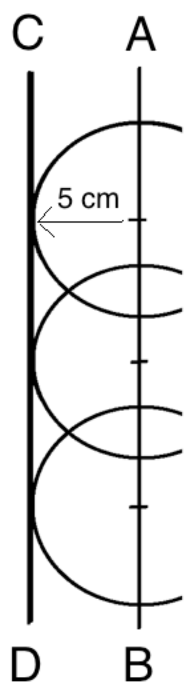
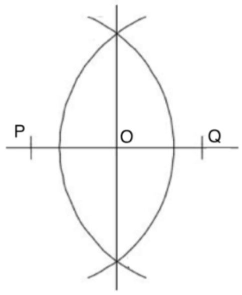


3. Note that the diagram below is not drawn the correct size. Use a ruler to draw your construction to the correct size.



Lesson Title: Construction of Perpendicular Lines	
Practice Activity: PHM-07-104	

1.



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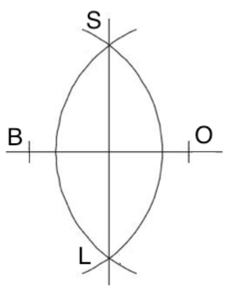

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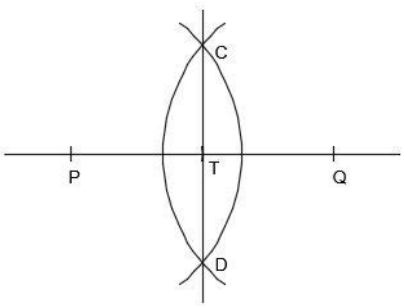

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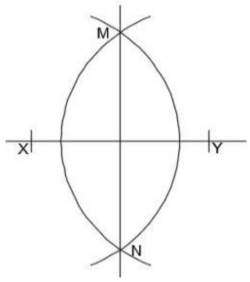
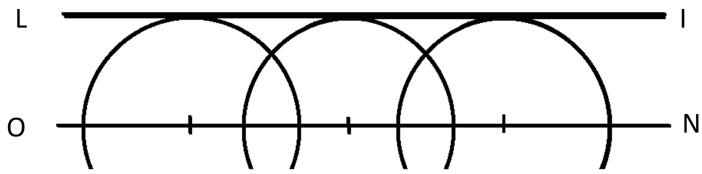


Lesson Title: Construction Practice	
Practice Activity: PHM-07-105	

1.



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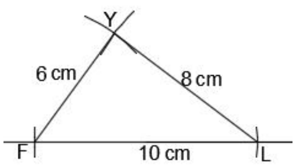

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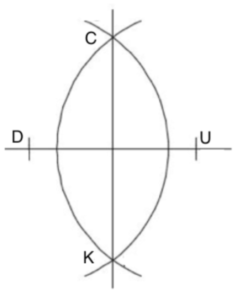

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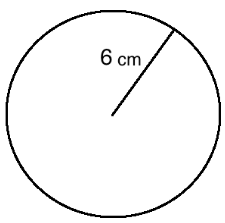

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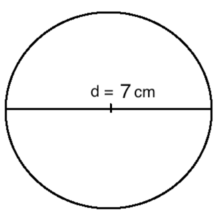

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