Free Quality School Education

Ministry of Basic and Senior Secondary Education

Lesson Plans for

Senior Secondary *Mathematics*

SSS II

Term I

STRICTLY NOT FOR SALE

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Please see final page for further information.

Foreword

These Lesson Plans and the accompanying Pupils' Handbooks are essential educational resources for the promotion of quality education in senior secondary schools in Sierra Leone. As Minister of Basic and Senior Secondary Education, I am pleased with the professional competencies demonstrated by the writers of these educational materials in English Language and Mathematics.

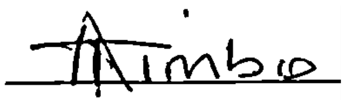
The Lesson Plans give teachers the support they need to cover each element of the national curriculum, as well as prepare pupils for the West African Examinations Council's (WAEC) examinations. The practice activities in the Pupils' Handbooks are designed to support self-study by pupils, and to give them additional opportunities to learn independently. In total, we have produced 516 lesson plans and 516 practice activities – one for each lesson, in each term, in each year, for each class. The production of these materials in a matter of months is a remarkable achievement.

These plans have been written by experienced Sierra Leoneans together with international educators. They have been reviewed by officials of my Ministry to ensure that they meet the specific needs of the Sierra Leonean population. They provide step-by-step guidance for each learning outcome, using a range of recognized techniques to deliver the best teaching.

I call on all teachers and heads of schools across the country to make the best use of these materials. We are supporting our teachers through a detailed training programme designed specifically for these new lesson plans. It is really important that the Lesson Plans and Pupils' Handbooks are used, together with any other materials they may have.

This is just the start of educational transformation in Sierra Leone as pronounced by His Excellency, the President of the Republic of Sierra Leone, Brigadier Rtd Julius Maada Bio. I am committed to continue to strive for the changes that will make our country stronger and better.

I do thank our partners for their continued support. Finally, I also thank the teachers of our country for their hard work in securing our future.



Mr. Alpha Osman Timbo

Minister of Basic and Senior Secondary Education

The policy of the Ministry of Basic and Senior Secondary Education, Sierra Leone, on textbooks stipulates that every printed book should have a lifespan of three years.

To achieve thus, <u>DO NOT WRITE IN THE BOOKS</u>.

Table of Contents

Introduction

to the Lesson Plans

These lesson plans are based on the National Curriculum and the West Africa Examination Council syllabus guidelines, and meet the requirements established by the Ministry of Basic and Senior Secondary Education.

1. The lesson plans will not take the whole term, so use spare time to review material or prepare for examinations.
2. Teachers can use other textbooks alongside or instead of these lesson plans.
3. Read the lesson plan before you start the lesson. Look ahead to the next lesson, and see if you need to tell pupils to bring materials for next time.
4. Make sure you understand the learning outcomes, and have teaching aids and other preparation ready – each lesson plan shows these using the symbols on the right.
5. If there is time, quickly review what you taught last time before starting each lesson.
6. Follow the suggested time allocations for each part of the lesson. If time permits, extend practice with additional work.
7. Lesson plans have a mix of activities for the whole class and for individuals or in pairs.
8. Use the board and other visual aids as you teach.
9. Interact with all pupils in the class – including the quiet ones.
10. Congratulate pupils when they get questions right! Offer solutions when they don't, and thank them for trying.

Learning outcomes

Preparation

KEY TAKEAWAYS FROM SIERRA LEONE'S PERFORMANCE IN WEST AFRICAN SENIOR SCHOOL CERTIFICATE EXAMINATION – GENERAL MATHEMATICS[1]

This section, seeks to outline key takeaways from assessing Sierra Leonean pupils' responses on the West African Senior School Certificate Examination. The common errors pupils make are highlighted below with the intention of giving teachers an insight into areas to focus on, to improve pupil performance on the examination. Suggestions are provided for addressing these issues.

Common errors

1. Errors in applying principles of BODMAS
2. Mistakes in simplifying fractions
3. Errors in application of Maths learned in class to real-life situations, and vis-a-versa.
4. Errors in solving geometric constructions.
5. Mistakes in solving problems on circle theorems.
6. Proofs are often left out from solutions, derivations are often missing from quadratic equations.

Suggested solutions

1. Practice answering questions to the detail requested
2. Practice re-reading questions to make sure all the components are answered.
3. If possible, procure as many geometry sets to practice geometry construction.
4. Check that depth and level of the lesson taught is appropriate for the grade level.

[1] This information is derived from an evaluation of WAEC Examiners' Reports, as well as input from their examiners and Sierra Leonean teachers.

FACILITATION STRATEGIES

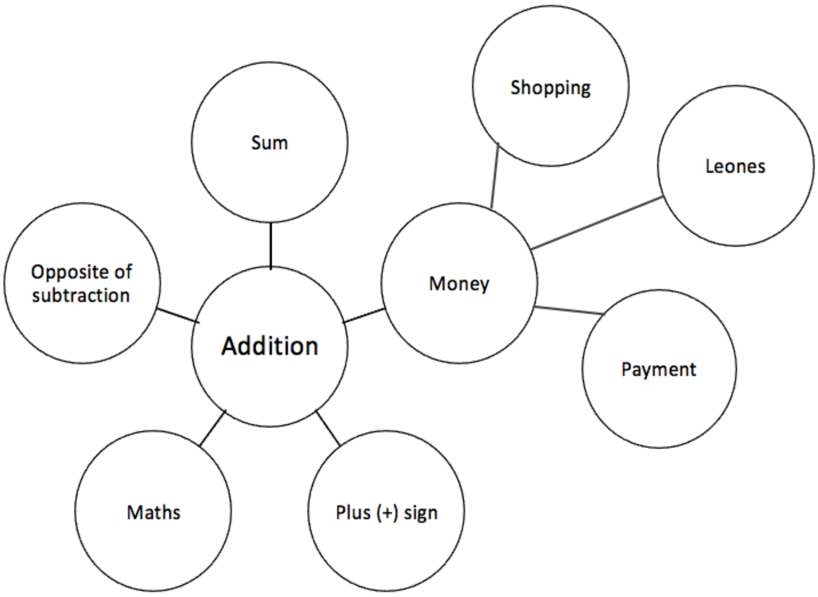
This section includes a list of suggested strategies for facilitating specific classroom and evaluation activities. These strategies were developed with input from national experts and international consultants during the materials development process for the Lesson Plans and Pupils' Handbooks for Senior Secondary Schools in Sierra Leone.

Strategies for introducing a new concept

- **Unpack prior knowledge:** Find out what pupils know about the topic before introducing new concepts, through questions and discussion. This will activate the relevant information in pupils' minds and give the teacher a good starting point for teaching, based on pupils' knowledge of the topic.
- **Relate to real-life experiences:** Ask questions or discuss real-life situations where the topic of the lesson can be applied. This will make the lesson relevant for pupils.
- **K-W-L:** Briefly tell pupils about the topic of the lesson, and ask them to discuss 'What I know' and 'What I want to know' about the topic. At the end of the lesson have pupils share 'What I learned' about the topic. This strategy activates prior knowledge, gives the teacher a sense of what pupils already know and gets pupils to think about how the lesson is relevant to what they want to learn.
- **Use teaching aids from the environment:** Use everyday objects available in the classroom or home as examples or tools to explain a concept. Being able to relate concepts to tangible examples will aid pupils' understanding and retention.
- **Brainstorming:** Freestyle brainstorming, where the teacher writes the topic on the board and pupils call out words or phrases related that topic, can be used to activate prior knowledge and engage pupils in the content which is going to be taught in the lesson.

Strategies for reviewing a concept in 3-5 minutes

- **Mind-mapping:** Write the name of the topic on the board. Ask pupils to identify words or phrases related to the topic. Draw lines from the topic to other related words. This will create a 'mind-map', showing pupils how the topic of the lesson can be mapped out to relate to other themes. Example below:



- **Ask questions:** Ask short questions to review key concepts. Questions that ask pupils to summarise the main idea or recall what was taught is an effective way to review a concept quickly. Remember to pick volunteers from all parts of the classroom to answer the questions.
- **Brainstorming:** Freestyle brainstorming, where the teacher writes the topic on the board and pupils call out words or phrases related that topic, is an effective way to review concepts as a whole group.
- **Matching:** Write the main concepts in one column and a word or a phrase related to each concept in the second column, in a jumbled order. Ask pupils to match the concept in the first column with the words or phrases that relate to in the second column.

Strategies for assessing learning without writing

- **Raise your hand:** Ask a question with multiple-choice answers. Give pupils time to think about the answer and then go through the multiple-choice options one by one, asking pupils to raise their hand if they agree with the option being presented. Then give the correct answer and explain why the other answers are incorrect.
- **Ask questions:** Ask short questions about the core concepts. Questions which require pupils to recall concepts and key information from the lesson are an effective way to assess understanding. Remember to pick volunteers from all parts of the classroom to answer the questions.
- **Think-pair-share:** Give pupils a question or topic and ask them to turn to seatmates to discuss it. Then, have pupils volunteer to share their ideas with the rest of the class.
- **Oral evaluation:** Invite volunteers to share their answers with the class to assess their work.

Strategies for assessing learning with writing

- **Exit ticket:** At the end of the lesson, assign a short 2-3 minute task to assess how much pupils have understood from the lesson. Pupils must hand in their answers on a sheet of paper before the end of the lesson.
- **Answer on the board:** Ask pupils to volunteer to come up to the board and answer a question. In order to keep all pupils engaged, the rest of the class can also answer the question in their exercise books. Check the answers together. If needed, correct the answer on the board and ask pupils to correct their own work.
- **Continuous assessment of written work:** Collect a set number of exercise books per day/per week to review pupils' written work in order to get a sense of their level of understanding. This is a useful way to review all the exercise books in a class which may have a large number of pupils.
- **Write and share:** Have pupils answer a question in their exercise books and then invite volunteers to read their answers aloud. Answer the question on the board at the end for the benefit of all pupils.
- **Paired check:** After pupils have completed a given activity, ask them to exchange their exercise books with someone sitting near them. Provide the answers, and ask pupils to check their partner's work.
- **Move around:** If there is enough space, move around the classroom and check pupils' work as they are working on a given task or after they have completed a given task and are working on a different activity.

Strategies for engaging different kinds of learners

- For pupils who progress faster than others:
  - Plan extension activities in the lesson.
  - Plan a small writing project which they can work on independently.
  - Plan more challenging tasks than the ones assigned to the rest of the class.
  - Pair them with pupils who need more support.

- For pupils who need more time or support:
  - Pair them with pupils who are progressing faster, and have the latter support the former.
  - Set aside time to revise previously taught concepts while other pupils are working independently.
  - Organise extra lessons or private meetings to learn more about their progress and provide support.
  - Plan revision activities to be completed in the class or for homework.
  - Pay special attention to them in class, to observe their participation and engagement.

Lesson Title: Review of Number Bases and Indices	**Theme:** Review of SSS 1	
Lesson Number: M2-L001	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcomes By the end of the lesson, pupils will be able to: 1. Convert between number bases. 2. Apply the laws of indices to simplify expressions.	**Preparation** 1. Read note at the end of this lesson plan. 2. Write a few examples of number bases and indices on the board - 423_{three}, 423_3, $2a^2$	

Opening *(4 minutes)*

1. Tell pupils they will be revising two SSS 1 topics today: number bases and indices.
2. Ask pupils to explain what **number bases** are and allow them to discuss. (Example answer: They are numbers written with digits and a smaller number that gives the base such as 423_{three} or 423_3.)
3. Ask pupils to explain what **indices** are. (Example answer: They are expressions that have a base and a power.)

Teaching and Learning *(25 minutes)*

1. Explain basic facts about **number bases**:
   - Number bases are written with digits, and a smaller number in subscript that can be a digit or spelled out (Example: 423_{three} or 423_3, which is read as "423 base three").
   - Base ten has ten digits (0-9), base nine has nine digits (0-8), base eight has eight digits (0-7), and so forth.
   - When we see numbers without a base given, the number base is ten. Numbers are usually written in base ten.
2. Give a brief explanation of the process for each type of conversion (see the next three steps). Then, solve an example on the board and explain each step.
3. Explain how to **convert from another base to base 10:** Each digit of the number must be converted using powers of the base you are converting from.

   Convert 243_{five} to base ten:

$$\begin{aligned} 243_{five} &= (2\times 5^2)+(4\times 5^1)+(3\times 5^0) \\ &= (2\times 25)+(4\times 5)+(3\times 1) \\ &= 50+20+3 \\ &= 73_{ten} \\ &= 73 \end{aligned}$$

4. Explain how to **convert from base 10 to another base:** Successive division is used. The number in base 10 is divided repeatedly by the base number you are converting to. The answer is obtained by reading the remainders upwards.

Convert to 44ten base five:

$$\begin{array}{r|l} 5 & 44 \\ \hline & 8\ rem\ 4 \\ \hline & 1\ rem\ 3 \\ \hline & 0\ rem\ 1 \end{array}\uparrow$$

$$44_{ten} = 134_{five}$$

5. Explain how to **convert between two bases that are not 10:** First convert to base ten, then convert the result to the required base.

   Convert 236four to base six:

   Step 1. Change 236four to base ten:

$$\begin{aligned} 236_{four} &= (2 \times 4^2) + (3 \times 4^1) + (6 \times 4^0) \\ &= (2 \times 16) + (3 \times 4) + (6 \times 1) \\ &= 32 + 12 + 6 \\ &= 50_{ten} \\ &= 50 \end{aligned}$$

   Step 2. Change 50ten to base six:

$$\begin{array}{r|l} 6 & 50 \\ \hline & 8\ rem\ 2 \\ \hline & 1\ rem\ 2 \\ \hline & 0\ rem\ 1 \end{array}\uparrow$$

$$50_{ten} = 122_{six}$$

   Therefore, $236_{four} = 122_{six}$

6. Give an example problem, and ask pupils to solve it with seatmates: **Convert 307ten to base 6.**
7. Invite a volunteer to write and explain their solution on the board.

   Solution:

$$\begin{array}{r|l} 6 & 307 \\ \hline & 51\ rem\ 1 \\ \hline & 8\ rem\ 3 \\ \hline & 1\ rem\ 2 \\ \hline & 0\ rem\ 1 \end{array}\uparrow$$

$$307_{ten} = 1231_{six}$$

8. Explain basic facts about **indices:**

- Indices are numbers raised to a power. They have two numbers: A base and a power.

9. Write the laws of indices on the board and briefly explain each:
    1. $x^a \times x^b = x^{a+b}$
    2. $x^a \div x^b = x^{a-b}$
    3. $x^0 = 1, x \neq 0$
    4. $(x^a)^b = x^{ab}$
10. Simplify two expressions involving indices on the board:

    Example 1: Simplify $2a^2 \times 8a^3$

    Solution:

$$\begin{aligned} 2a^2 \times 8a^3 &= (2 \times 8)a^{2+3} && \text{Apply the first law of indices} \\ &= 16a^5 \end{aligned}$$

    Example 2: Simplify $\frac{(y^2)^3}{y^4}$

    Solution:

$$\begin{aligned} \frac{(y^2)^3}{y^4} &= \frac{y^{2\times3}}{y^4} && \text{Apply the fourth law of indices to the numerator} \\ &= \frac{y^6}{y^4} \\ &= y^{6-4} && \text{Apply the second law of indices} \\ &= y^2 \end{aligned}$$

11. Give an example problem, and ask pupils to solve it with seatmates: Simplify $b^2 \times 2b^3$
12. Invite a volunteer to write and explain their solution on the board.
    Solution:

$$\begin{aligned} b^2 \times 2b^3 &= (1 \times 2)b^{2+3} && \text{Apply the first law of indices} \\ &= 2b^5 \end{aligned}$$

Practice *(10 minutes)*

1. Write the following problems on the board. Ask pupils to solve them individually or with seatmates:
    1. Convert 441_{five} to base eight
    2. Simplify $(z^4)^3 \div z^7$
2. Invite 2 volunteers to write their solutions on the board. Other pupils should check their work.
    Solutions:

1. Convert 441_{five} to base eight
   Step 1. Convert 441_{five} to base ten:

$$\begin{aligned} 441_{five} &= (4\times5^2)+(4\times5^1)+(1\times5^0) \\ &= (4\times25)+(4\times5)+(1\times1) \\ &= 100+20+1 \\ &= 121_{ten} \\ &= 121 \end{aligned}$$

   Step 2. Convert 121 to base eight:

$$\begin{array}{r|l} 8 & 121 \\ \hline & 15\ rem\ 1 \\ \hline & 1\ rem\ 7 \\ \hline & 0\ rem\ 1 \end{array} \uparrow$$

$$121_{ten} = 171_{eight}$$

   Therefore, $441_{five} = 171_{eight}$

2. Simplify $(z^4)^3 \div z^7$

$$\begin{aligned} (z^4)^3 \div z^7 &= z^{4\times3} \div z^7 && \text{Apply the fourth law of indices} \\ &= z^{12} \div z^7 \\ &= z^{12-7} && \text{Apply the second law of indices} \\ &= z^5 \end{aligned}$$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM2-L001 in the Pupil Handbook.

[NOTE]

The first week of SSS 2 is spent revising selected topics from SSS 1. This is the first revision lesson. You may find additional explanations and examples of these topics in the SSS 1 Lesson Plans if needed.

<table>
<tr><td>Lesson Title: Review of Linear Equations</td><td colspan="2">Theme: Review of SSS 1</td></tr>
<tr><td>Lesson Number: M2-L002</td><td>Class: SSS 2</td><td>Time: 40 minutes</td></tr>
<tr><td>Learning Outcomes
By the end of the lesson, pupils will be able to:
1. Solve linear equations algebraically.
2. Graph linear functions.</td><td colspan="2">Preparation
None</td></tr>
</table>

Opening *(3 minutes)*

1. Tell pupils they will be revising linear equations today.
2. Ask pupils to explain what linear equations are in their own words. (Example answers: They are equations with 1 or 2 variables. They result in a line when graphed on the Cartesian plane.)

Teaching and Learning *(25 minutes)*

1. Write a linear equation on the board: $y = -2x + 1$
2. Remind pupils of some facts about linear equations:
   - x and y are variables
   - x is the independent variable, y is the dependent variable
   - -2 is the coefficient of x
   - 1 is a constant
   - Solutions to linear equations can be written as an ordered pair: (x, y)
3. Invite a volunteer to explain how to find a solution to the linear equation on the board. Allow time for discussion.
4. Explain: To find a solution to the equation, substitute any value of x and solve for y.
5. Substitute $x = 2$ into the equation on the board to find a solution:

$$\begin{aligned} y &= -2x + 1 \\ &= -2(2) + 1 \\ &= -4 + 1 \\ &= -3 \end{aligned}$$

6. Explain that the solution has two values, $x = 2$ and $y = -3$. This can be written as an ordered pair: $(2, -3)$.
7. Write the ordered pair on the board: $(2, -3)$.
8. Draw the table of values on the board, and explain that this is used to record multiple solutions before graphing them. Record the solution $(2, -3)$ in the table:

x	-2	-1	0	1	2
y					-3

9. Find another solution, and write it in the table:

$$
\begin{aligned}
y &= -2x + 1 \\
&= -2(-1) + 1 \\
&= 2 + 1 \\
&= 3
\end{aligned}
$$

x	-2	-1	0	1	2
y		3			-3

10. Ask pupils to work with seatmates to find the other y-values and complete the table. When they are finished, invite volunteers to write the solutions on the board:

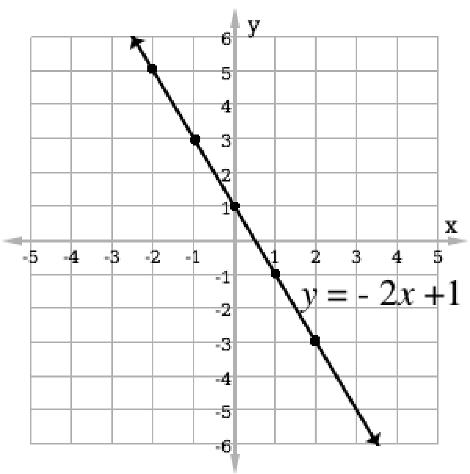
$$
\begin{aligned}
y &= -2(-2) + 1 \\
&= 4 + 1 \\
&= 5
\end{aligned}
$$

x	-2	-1	0	1	2
y	5	3	1	-1	-3

$$
\begin{aligned}
y &= -2(0) + 1 \\
&= 0 + 1 \\
&= 1
\end{aligned}
$$

$$
\begin{aligned}
y &= -2(1) + 1 \\
&= -2 + 1 \\
&= -1
\end{aligned}
$$

11. Draw a Cartesian plane on the board.
12. Explain that the table of values will be used to graph a line, and that each set of x- and y-values in the table represents a point on the graph.
13. Write each set of values on the board as an ordered pair to show this: $(-2, 5)$, $(-1, 3)$, $(0, 1)$, $(1, -1)$, $(2, -3)$.
14. Graph the first point $(-2, 5)$ in the table on the Cartesian plane.
15. Invite volunteers to come to the board and graph the other 4 points.
16. Draw a line through the 5 points, and label it with the equation: $y = -2x + 1$



17. Write another linear equation on the board: $6x - 2y = 4$

18. Explain that this equation will be easier to graph if we change the subject, and solve for y.
19. Ask them to solve for y with seatmates.
20. Invite a volunteer to write the solution on the board:

$$\begin{aligned} 6x - 2y &= 4 & \\ -2y &= 4 - 6x & \text{Transpose } 6x \\ \frac{-2y}{-2} &= \frac{4}{-2} - \frac{6x}{-2} & \text{Divide throughout by } -2 \\ y &= -2 + 3x & \end{aligned}$$

Note that this is the same as $y = 3x - 2$.

21. Graph the equation on the board as a class, following the same process of involving pupils:

$$\begin{aligned} y &= 3(-2) - 2 \\ &= -6 - 2 \\ &= -8 \end{aligned}$$

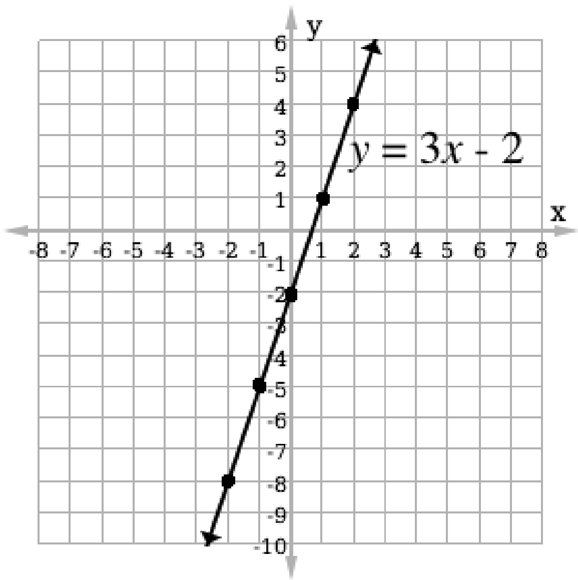
$$\begin{aligned} y &= 3(-1) - 2 \\ &= -3 - 2 \\ &= -5 \end{aligned}$$

$$\begin{aligned} y &= 3(0) - 2 \\ &= 0 - 2 \\ &= -2 \end{aligned}$$

$$\begin{aligned} y &= 3(1) - 2 \\ &= 3 - 2 \\ &= 1 \end{aligned}$$

$$\begin{aligned} y &= 3(2) - 2 \\ &= 6 - 2 \\ &= 4 \end{aligned}$$

x	-2	-1	0	1	2
y	-8	-5	-2	1	4



Practice *(10 minutes)*

1. Write the following problem on the board. Ask pupils to graph the equation individually or with seatmates:

   Using the given table of values, graph the linear equation $y + 2x = 3$.

x	-2	-1	0	1	2
y					

2. Ask pupils to compare answers with their neighbours and check their work. If time allows, invite volunteers to write the solution on the board.
   Solution:

   Solve the equation for y:

$$\begin{aligned} y + 2x &= 3 \\ y &= -2x + 3 \quad \text{Transpose } 2x \end{aligned}$$

   Find the solutions and fill the table of values before graphing:

$$\begin{aligned} y &= -2(-2) + 3 \\ &= 4 + 3 \\ &= 7 \end{aligned}$$

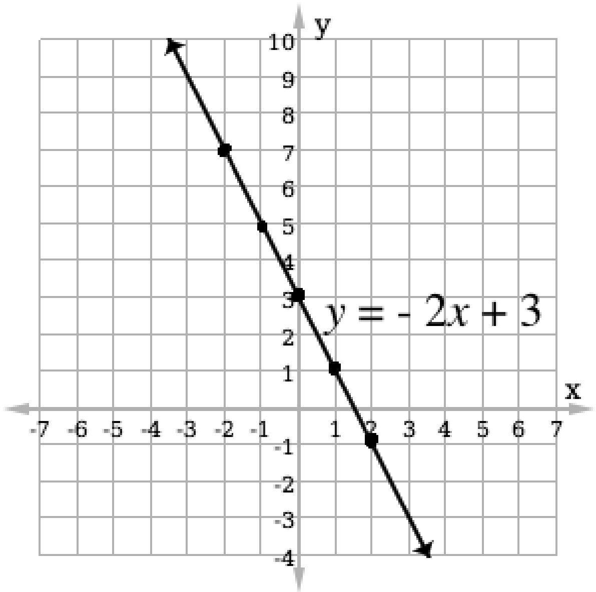
x	-2	-1	0	1	2
y	7	5	3	1	-1

$$\begin{aligned} y &= -2(-1) + 3 \\ &= 2 + 3 \\ &= 5 \end{aligned}$$

$$\begin{aligned} y &= -2(0) + 3 \\ &= 0 + 3 \\ &= 3 \end{aligned}$$

$$\begin{aligned} y &= -2(1) + 3 \\ &= -2 + 3 \\ &= 1 \end{aligned}$$

$$\begin{aligned} y &= -2(2) + 3 \\ &= -4 + 3 \\ &= -1 \end{aligned}$$



Closing *(2 minutes)*

1. Ask for volunteers to briefly describe the process for graphing a line. (Example answers: Solve for y, create a table of values, graph and connect points)
2. For homework, have pupils do the practice activity PHM2-L002 in the Pupil Handbook.

Lesson Title: Review of Quadratic Equations	**Unit:** Review of SSS 1	
Lesson Number: M2-L003	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcomes By the end of the lesson, pupils will be able to: 1. Solve quadratic equations algebraically. 2. Graph and interpret quadratic functions.	**Preparation** None	

Opening *(1 minute)*

1. Tell pupils that they will be revising quadratic equations today.
2. Ask pupils to explain what quadratic equations are in their own words and allow time for discussion. (Example answers: they are equations that contain the term x^2; when graphed they make parabolas; they can be solved by factorisation)

Teaching and Learning *(23 minutes)*

1. Write the following quadratic equation on the board: $y = x^2 + 4x + 3$
2. Remind pupils of some facts about quadratic functions and equations:
   - A "quadratic equation" is given in a way that allows you to solve for a variable and does not have a y-value. (Example: $x^2 + 4x + 3 = 0$)
   - A "quadratic function" has a y-value and can be graphed. (Example: $y = x^2 + 4x + 3$)
   - Quadratic equations and functions both contain the term x^2.
   - We can find the solutions to quadratic equations in several ways:
     - By graphing the related function
     - Using factorisation
     - Completing the square
     - Using the quadratic formula
   - The graph of a quadratic function is a parabola. The solutions (or roots) of a quadratic equation are the points where the parabola crosses the x-axis.
3. Tell pupils that we will first solve the quadratic equation on the board **graphically**. Draw the table of values shown below and a Cartesian plane on the board:

x	-4	-3	-2	-1	0
y					

4. Use the quadratic equation to solve for y and write the answer in the table of values.

   Solution:

$$\begin{aligned} y &= x^2 + 4x + 3 \\ &= (-4)^2 + 4(-4) + 3 \\ &= 16 - 16 + 3 \\ &= 3 \end{aligned}$$

x	-4	-3	-2	-1	0
y	3				

5. Ask pupils to work with seatmates to find the other y-values and complete the table. As they finish, invite volunteers to write the answers on the board.
Solutions:

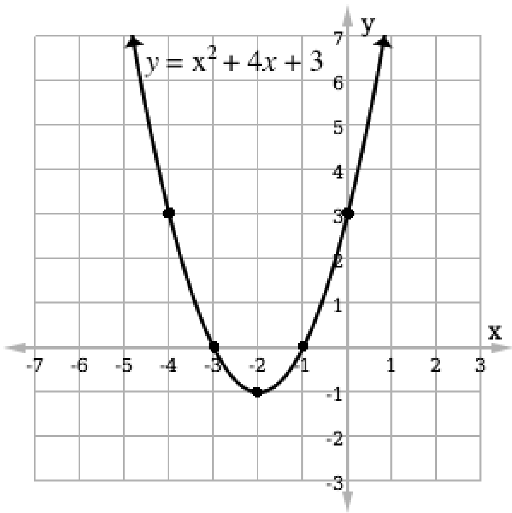
$$\begin{aligned} y &= (-3)^2 + 4(-3) + 3 \\ &= 9 - 12 + 3 \\ &= 0 \end{aligned}$$

x	-4	-3	-2	-1	0
y	3	0	-1	0	3

$$\begin{aligned} y &= (-2)^2 + 4(-2) + 3 \\ &= 4 - 8 + 3 \\ &= -1 \end{aligned}$$

$$\begin{aligned} y &= (0)^2 + 4(0) + 3 \\ &= 0 - 0 + 3 \\ &= 3 \end{aligned}$$

$$\begin{aligned} y &= (-1)^2 + 4(-1) + 3 \\ &= 1 - 4 + 3 \\ &= 0 \end{aligned}$$



6. Invite volunteers to come to the board and plot each of the 5 points in the Cartesian plane (Answer: See graph).
7. Connect the 5 points to make a smooth parabola. (Answer: See graph).
8. Ask pupils to identify the solutions to the quadratic equation. If needed, remind them that the x-intercepts give the solutions. (Answers: $x = -3$; $x = -1$)
9. Tell pupils that we will now solve the same problem by **factorisation**.
10. Review the process of factorisation:
    - Factorisation involves rewriting the quadratic equation in the form $x^2 + 4x + 3 = (x + a)(x + b)$, where a and b are integers.
    - a and b should sum to the coefficient of the second term of the equation, and multiply to get the third term. (In this case, $a + b = 4$ and $a \times b = 3$)
    - After finding a and b, set each binomial equal to 0 and solve to find the value of x. These values are the roots.
11. Solve the quadratic equation by factorisation, explaining each step:

$x^2 + 4x + 3 = (x + a)(x + b)$ Set up the equation

$a + b = 4$ $\quad$ $a \times b = 3$ Note that the values of a and b must be 3 and 1 to satisfy these equations

$x^2 + 4x + 3 = (x + 1)(x + 3)$ Substitute the values of a and b into the equation

$x + 1 = 0$ $\quad$ $x + 3 = 0$ $\quad$ Set each binomial equal to 0 and
$x = -1$ $\quad$ $x = -3$ $\quad$ solve for x. These are the roots.

12. Tell pupils that we will now solve the same problem using the **quadratic formula**.
13. Review the quadratic formula:
    - Write on the board: $ax^2 + bx + c = 0$
    - Write the quadratic formula on the board: $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$
    - Tell pupils that to find the roots of the equation, we can simply substitute the values of a, b, and c from a given equation into the formula and solve.
14. Solve the quadratic equation $x^2 + 4x + 3 = 0$ with the quadratic formula, explaining each step:

    Solution:

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \frac{-4 \pm \sqrt{4^2-4(1)(3)}}{2(1)}$ $\quad$ Substitute the values of a, b, and c

$= \frac{-4 \pm \sqrt{4}}{2}$ $\quad$ Simplify

$= \frac{-4 \pm 2}{2}$ $\quad$ Note: The $\pm$ symbol tells us that we can find 2 solutions: one by adding, and the other by subtracting

$= \frac{-4+2}{2}$ and $\frac{-4-2}{2}$

$x = -1$ and -3 $\quad$ Identify the 2 roots

5. Remind pupils that we have now found the same solutions using 3 methods: graphing, factorisation, and the quadratic formula.

Practice *(15 minutes)*

1. Write the problem below on the board.
2. Ask pupils to work individually or with seatmates.
    a. Using the given table of values, plot the quadratic function $y = x^2 - x - 2$.

x	-2	-1	0	1	2	3
y						

    b. Based on your graph, what are the roots of the equation $x^2 - x - 2 = 0$?
    c. Choose one algebraic method (factorisation or the quadratic formula) and use it to solve the quadratic equation.
3. Ask pupils to compare answers with seatmates and check their work. If time allows, invite volunteers to write the solutions on the board.

    Solutions:

    a. Table of values and graph:

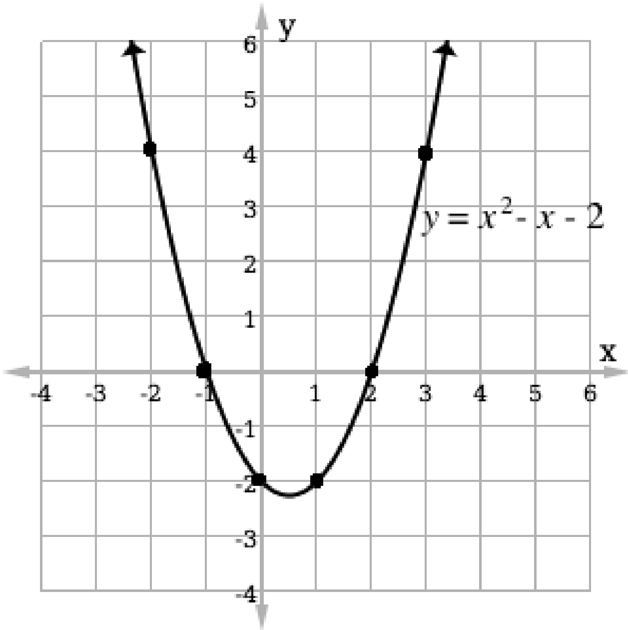
$y = (-2)^2 - (-2) - 2$

$= 4 + 2 - 2$

$= 4$

x	-2	-1	0	1	2	3
y	4	0	-2	-2	0	4

$$\begin{aligned} y &= (-1)^2 - (-1) - 2 \\ &= 1 + 1 - 2 \\ &= 0 \end{aligned}$$



$$\begin{aligned} y &= (0)^2 - (0) - 2 \\ &= 0 - 0 - 2 \\ &= -2 \end{aligned}$$

$$\begin{aligned} y &= (1)^2 - (1) - 2 \\ &= 1 - 1 - 2 \\ &= -2 \end{aligned}$$

$$\begin{aligned} y &= (2)^2 - (2) - 2 \\ &= 4 - 2 - 2 \\ &= 0 \end{aligned}$$

$$\begin{aligned} y &= (3)^2 - (3) - 2 \\ &= 9 - 3 - 2 \\ &= 4 \end{aligned}$$

b. The roots are $x = -1$ and $x = 2$ (the x-intercepts)

c. Pupils may solve by either factorisation or the quadratic formula:

Factorisation

$$\begin{aligned} x^2 - x - 2 &= (x + a)(x + b) \\ &= (x - 2)(x + 1) \end{aligned}$$

$x - 2 = 0$ $\quad$ $x + 1 = 0$

$x = 2$ $\quad$ $x = -1$

Quadratic Formula

$$\begin{aligned} x &= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-2)}}{2(1)} \\ &= \frac{1 \pm \sqrt{9}}{2} \\ &= \frac{1 \pm 3}{2} \\ &= \frac{1+3}{2} \text{ and } \frac{1-3}{2} \\ &= 2 \text{ and } -1 \end{aligned}$$

Closing *(1 minute)*

1. Revise the lesson by asking pupils to list the ways of solving a quadratic equation. (Example answers: graphically, factorisation, quadratic formula, completing the square (not covered in this lesson))
2. For homework, have pupils do the practice activity PHM2-L003 in the Pupil Handbook.

Lesson Title: Review of Angles and Triangles	**Theme:** Review of SSS 1	
Lesson Number: M2-L004	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcomes By the end of the lesson, pupils will be able to: 1. Identify types of angles and triangles. 2. Solve triangles by finding angle and side measures.	**Preparation** None	

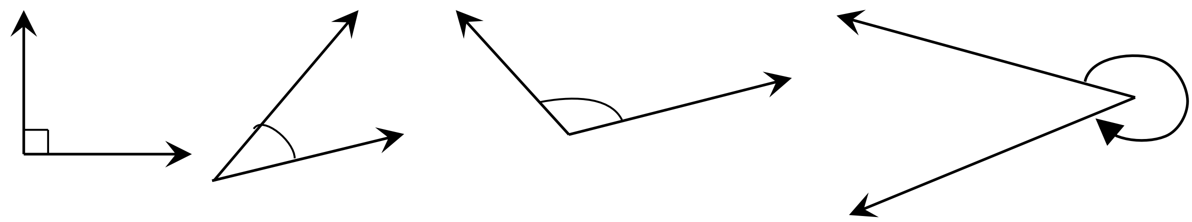
Opening *(3 minutes)*

1. Tell pupils that they will be revising angles and triangles today.
2. Ask pupils to call out the different types of triangles they know. (Example answers: right-angled triangle, scalene, isosceles)
3. Ask pupils to call out the different types of angles they know. (Example answers: right angle, obtuse angle, acute angle)

Teaching and Learning *(25 minutes)*

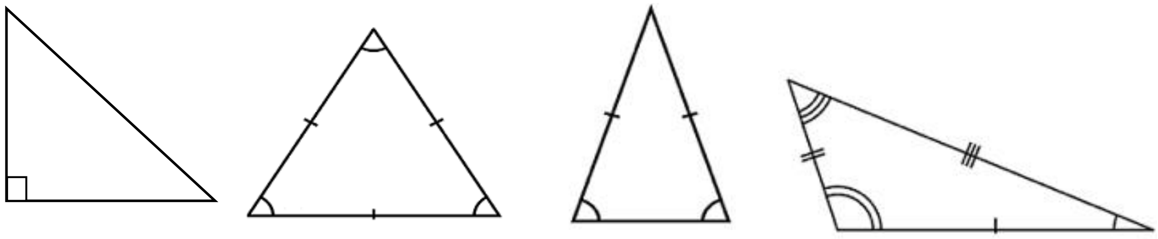
1. Invite 4 volunteers to come to the board and each draw one of the following **angles**: right, obtuse, acute, reflex. They may look at the examples in their pupil handbook.

   Examples:

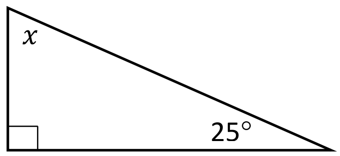


Right angle Acute angle Obtuse angle Reflex angle

2. Ask pupils to raise their hand to describe the characteristics of each type of angle. Make sure that they note the following:
   - Right angles are 90°
   - Acute angles are less than 90°
   - Obtuse angles are greater than 90°, but less than 180°
   - Reflex angles are greater than 180°, but less than 360°
3. Invite 4 volunteers to come to the board and each draw one of the following **triangles**: right, equilateral, isosceles, and scalene. They may look at the examples in their pupil handbook.

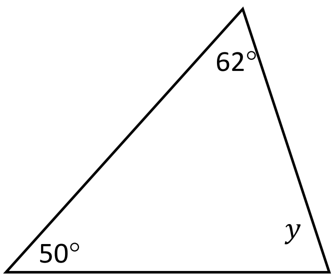


4. Ask for volunteers to give the characteristics of each type of triangle. Make sure that they note the following:
   - Right-angled triangles have one 90° (right) angle.
   - Equilateral triangles have all 3 sides of equal length. All 3 angles are equal, and are always equal to 60°.
   - Isosceles triangles have 2 sides of the same length, and 2 angles of the same measure. The equal angles are called base angles, and are opposite to the equal sides.
   - Scalene triangles have 3 different angles and 3 different sides.
5. Explain how to find missing angles in a triangle:
   - The angles of a triangle always add up to 180 degrees.
   - Subtract the known angles from 180 to find the measurement of a missing angle.
6. Draw the triangle at right on the board:

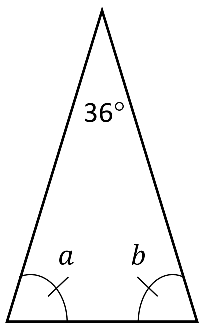


7. Have 1 pupil volunteer to explain how to find the missing angle, x. (Answer: Subtract the known angles, 90° and 25°, from 180°).
8. Write the solution on the board: $x = 180° - 90° - 25° = 65°$ ….
9. Draw the 2 examples shown below on the board. Ask pupils to work with seatmates to find the missing angles:

a.



b.



10. Invite 2 volunteers to solve for the missing angles on the board and explain their solutions.

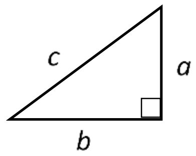
    Solutions:

    a. Subtract the two known angles from 180°: $y = 180° - 62° - 50° = 68°$

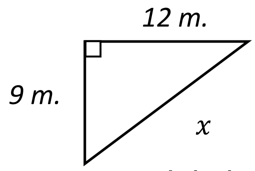
b. Subtract the known angle from 180: $180° - 36° = 144°$
We know that $a + b = 144°$, and that a and b are equal.
Divide $144°$ by 2 to find the measure of both a and b: $a = b = 144° \div 2 = 72°$

11. Remind pupils of Pythagoras' Theorem:
    - Pythagoras' Theorem is used to find unknown **sides** of **right-angled triangles.**
    - Pythagoras' Theorem is $a^2 + b^2 = c^2$, where c is the hypotenuse of the triangle, and a and b are the other sides.
    - Draw the following example on the board:

$a^2 + b^2 = c^2$



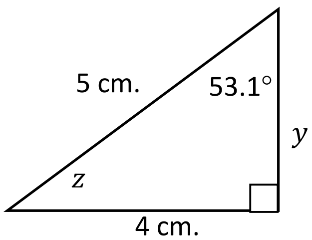
12. Draw the triangle shown below on the board:



13. Have 1 pupil volunteer to explain how to find the missing side, x. (Example answer: Substitute the values of the known sides, $a = 9$ m. and $b = 12$ m., into Pythagoras' Theorem. Solve for x.)
14. Write the solution on the board, explaining each step.
Solution:

c^2	=	$a^2 + b^2$	
x^2	=	$9^2 + 12^2$	Substitute the values
x^2	=	$81 + 144$	Simplify
x^2	=	225	
x	=	$\sqrt{225}$	Take the square root of both sides
c	=	15 m.	

15. Draw the triangle below on the board and write the following: Find the measure of the side y and angle z



16. Ask pupils to work with seatmates to find the missing angle and side. Walk around and check for understanding. Remind pupils to subtract known angle measures from 180° to find missing angle measures, and to use Pythagoras' Theorem to find side measures.
17. Invite volunteers to write and explain their solutions on the board.

Solutions:

Angle z: $z = 180° - 90° - 53.1° = 36.9°$

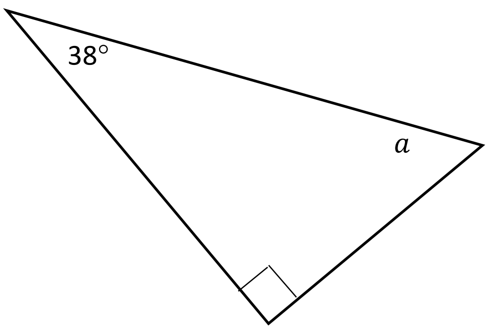
Side y:

5^2	=	$4^2 + y^2$	Substitute the values
25	=	$16 + y^2$	Simplify
$25 - 16$	=	y^2	Transpose 16
9		y^2	
$\sqrt{9}$	=	y	Take the square root of both sides
y	=	3 cm.	

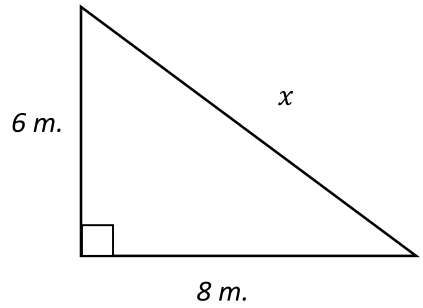
Practice *(10 minutes)*

1. Draw the triangles below on the board. Ask pupils to work individually or with seatmates to solve for a and x:

a.



b.



2. Invite volunteers to write the solutions on the board and explain their solutions.
Solutions:

a. $a = 180° - 90° - 38° = 52°$

b.

x^2	=	$6^2 + 8^2$
x^2	=	$36 + 64$
x^2	=	100
x	=	$\sqrt{100}$
x	=	10 m.

Closing *(2 minutes)*

1. Revise the lesson:
   - Ask pupils to describe how to find a missing angle. (Example answer: Subtract known angle measures from 180° to solve for missing angles.)
   - Ask pupils to describe how to find a missing side of a right-angled triangle. (Example answer: Substitute known values in Pythagoras' theorem and solve for missing sides.)
2. For homework, have pupils do the practice activity PHM2-L004 in the Pupil Handbook.

<table>
<tr><td>Lesson Title: Significant Figures</td><td colspan="2">Theme: Numbers and Numeration</td></tr>
<tr><td>Lesson Number: M2-L005</td><td>Class: SSS 2</td><td>Time: 40 minutes</td></tr>
<tr><td>Learning Outcome
By the end of the lesson, pupils will be able to round numbers to a given number of significant figures.</td><td colspan="2">Preparation
Write the problem in the Opening on the board.</td></tr>
</table>

Opening *(5 minutes)*

1. Write the following problem on the board to review the rounding of decimal numbers:

   Round 2.537 to: a. one decimal place; b. two decimal places

2. Ask pupils to work with seatmates to solve the problem.
3. Ask two volunteers to give and explain their answers.
   Answers and explanations:
   a. 2.5; round down because the number in the second decimal place is less than 5.
   b. 2.54; round up because the number in the third decimal place is greater than 5.
4. Tell pupils that the topic for the day is significant figures (s.f.). They will learn how to round to a given number of significant figures.
5. Explain that understanding significant figures helps us to give a precise and accurate number. This is useful for science subjects such as chemistry.

Teaching and Learning *(20 minutes)*

1. Explain the rules for identifying significant figures. You may write the example numbers on the board and explain, or ask pupils to look at the table in their pupil handbooks.
2. Mention that the number of significant figures is counted from left to right.

Rules	Examples	Number of s.f.
1. All non-zero digits are significant.	123	(3 s.f.)
2. Zeros between non-zero digits are significant	12.507 304	(5 s.f.) (3 s.f.)
3. Zeros to the left of the first non-zero digit are not significant.	1.02 0.12 0.012	(3 s.f.) (2 s.f.) (2 s.f.)
4. If a number ends in zeros to the right of the decimal point, those zeros are significant.	2.0 2.00	(2 s.f.) (3 s.f.)
5. Zeros at the end of a whole number are not significant, unless there is a decimal point.	4,300 4,300.0	(2 s.f.) (5 s.f.)

3. Write 3 numbers on the board: a. 0.00503; b. 4306; c.43,000
4. Underline the significant figures in each number, and use the rules to explain. (Answers: a. 0.00503; b. 4306 ; c. 43,000)
5. Write 2 more numbers on the board: a. 1,300; b. 43.006
6. Ask pupils to copy the numbers into their exercise books and underline the significant figures. They should use the rules detailed earlier in the lesson.
7. Invite 2 volunteers to write the answers on the board. (Answers: a.1,300; b. 43.006)
8. Explain how to round to a stated number of significant figures:
    a. Find the last significant figure you want to round to.
    b. Look at the next significant figure immediately to the right.
    c. If the next significant figure is less than 5, leave the last significant figure you want as it is. If the next significant figure is 5 or more, add 1 to the last significant figure you want.
9. Show examples of rounding significant figures on the board, and explain each:

    Example 1. Round 287,540 to: a. 4 s.f.; b. 3 s.f.; c. 2 s.f.

    Answers: a. 287,500; b. 288,000; c. 290,000

    Example 2. Round 0.0397 to 2 s.f.

    Answer: 0.040

10. Write the following 3 problems on the board, and ask pupils to solve with seatmates:
    a. Round 14,523 to 3 s.f.
    b. Correct 0.600701 to 5 s.f.
    c. Approximate 2,413.65 to 2 s.f.

11. Explain that “round”, “correct” and “approximate” have the same meaning in significant figure problems.
12. Walk around and check for understanding. Revise and explain as needed.
13. Ask volunteers to each give one of their answers and explain. (Answers: a. 14,500; b. 0.60070; c. 2,400)

Practice *(12 minutes)*

1. Write the following four problems on the board:
    a. Round 0.87751 to 3 s.f.
    b. Correct 439,800 to 1 s.f.
    c. Approximate 0.00034682 to 3 s.f.
    d. Round 0.0301 to 2 s.f.

2. Ask pupils to work independently to round the numbers.
3. Ask pupils to exchange books and check each other's answers.
4. Invite 4 volunteers to each call out one of the answers. (Answers: a. 0.878; b. 400,000; c. 0.000347; d. 0.030)

Closing *(3 minutes)*

1. Discuss as a class: When do you think it would be useful to know significant figures? What professionals do you think use significant figures in their work? (Example responses: when solving chemistry problems, engineers, scientists)
2. For homework, have pupils do the practice activity PHM2-L005 in the Pupil Handbook.

Lesson Title: Estimation	**Theme:** Numbers and Numeration	
Lesson Number: M2-L006	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to make a rough estimate of a calculation.	**Preparation** Write the problems in the Opening on the board.	

Opening *(3 minutes)*

1. Review rounding of significant figures. Write the following two problems on the board and ask pupils to find the answers:
    - Round 0.3041 to 2 s.f.
    - Round 29471 to 3 s.f.
2. Invite 2 volunteers to give their answers and explain. (Answers: a. 0.30; b. 29500)
3. Tell pupils that the topic for the day is estimation. Estimation skills are very important because they help us in everyday Maths.

Teaching and Learning *(20 minutes)*

1. Explain: When we estimate an amount, we find the approximate value or quantity. We find something **close** to the answer, but not the exact answer.
2. Give examples of times when estimation skills may be useful:
    - Estimating the cost of goods in the market.
    - Estimating how much fuel to buy for a trip.
    - Estimating how much your farm will produce.
3. Ask and allow pupils to discuss and share with the class: When do you use estimation skills? In what situations are estimation skills useful?
4. Write the following example problem on the board:
   Sia spends Le 29,500.00 per month on transportation. How much does she spend per year? Find a rough estimate.
5. Explain:
    - We could calculate the exact amount she spends per year by multiplying 29,500 by 12. We can perform this calculation within a few minutes.
    - We can save time by finding a rough estimate!
    - Rounding to 1 s.f., we find that Sia spends about 30,000 per month. We can multiply this by 12: $30{,}000 \times 12$.
    - We can solve this problem using mental Maths. We know that $12 \times 3 = 36$, so $30{,}000 \times 12 = 360{,}000$.
    - We have estimated that Sia spends roughly Le 360,000.00 per year on transportation.
    - This answer is an estimate, and it is correct to 2 significant figures.
6. Explain: We can often make a rough estimate of a calculation by rounding numbers first, then applying operations.

7. Write the problem show below on the board:

   Exact number of cars on the road on a specific day:

	Monday	Tuesday	Wednesday	Thursday	Friday
Cars	2,347	3,216	4,123	1,984	2,587
Estimate					

   - Round each number to 1 s.f., and fill the last row of the table.
   - Use your rounded values to estimate the total number of cars that drove on the road during the week.
8. Ask pupils to work with seatmates to solve the problem. Check for understanding, and provide explanations as needed.
9. Invite 5 volunteers to each give one of the estimates. Fill the table on the board with their answers.

   Answers:

	Monday	Tuesday	Wednesday	Thursday	Friday
Cars	2,347	3,216	4,123	1,984	2,587
Estimate	2,000	3,000	4,000	2,000	3,000

10. Invite a volunteer to give their answer to part (b) and explain. (Answer: by adding the estimated numbers, we get $2{,}000 + 3{,}000 + 4{,}000 + 2{,}000 + 3{,}000 = 14{,}000$).

Practice *(15 minutes)*

1. Write the two problems shown below on the board:
   a. Populations of the four Provinces of Sierra Leone (2015 census).

Province	Population	Estimate
Eastern Province	1,641,012	
Northern Province	2,502,805	
Southern Province	1,438,572	
Western Area Province	1,493,252	

   i. Round each number to 2 s.f., and fill the last column of the table.
   ii. Use your rounded values to estimate the total population of Sierra Leone in 2015.

   b. A newspaper headline reads, "Government Invests Le 21 Billion in Education". The author rounded the investment amount to 2 significant figures. Between what two amounts does this figure lie?
2. Ask pupils to work independently or with seatmates to find the answers.
3. Invite volunteers to come to the board to write the answers. Other pupils should check their work.

Answers:

a.

i.

Province	Population	Estimate
Eastern Province	1,641,012	1,600,000
Northern Province	2,502,805	2,500,000
Southern Province	1,438,572	1,400,000
Western Area Province	1,493,252	1,500,000

ii. Total population: $1{,}600{,}000 + 2{,}500{,}000 + 1{,}400{,}000 + 1{,}500{,}000 = 7{,}000{,}000$

Inform pupils that the actual population of Sierra Leone according to the 2015 census was 7,075,641. This is very close to their estimation!

b. The investment amount lies between Le 20.5 and Le 21.5 billion. Any number in that range would be rounded to 21 s.f.

Closing *(2 minutes)*

1. Discuss as a class: Now that you have practised your estimation skills, when do you think you will use these skills in the future?
2. For homework, have pupils do the practice activity PHM2-L006 in the Pupil Handbook.

Lesson Title: Percentage Error	**Unit:** Numbers and Numeration	
Lesson Number: M2-L007	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to calculate the percentage error when using rounded values.	**Preparation** Write the problem in the Opening on the board.	

Opening *(3 minutes)*

1. Review estimation. Write the following problem on the board:

   A group of pupils picked up trash in their community. They picked up 5,324 pieces one day and 2,148 pieces the next day. How many pieces of trash did they pick up in total? Estimate to 1 s.f.

2. Give pupils 1 minute to solve the problem. Remind them to round the numbers before applying operations.
3. Invite a volunteer to give the answer and explain. (Answer: $5000 + 2000 = 7000$ pieces of trash)
4. Tell pupils that the topic for the day is percentage error. Percentage error helps us to understand how accurate our estimations are.

Teaching and Learning *(25 minutes)*

1. Explain:
   - Estimated measurements are not exact, they are approximate. This means that we always have some error.
   - Percentage error tells us how close to the exact value our estimated value is.
   - A smaller percentage error means our estimate is **more** accurate, and a larger percentage error means our estimate is **less** accurate.
   - When using measurements, estimating with smaller units of measure gives us a smaller percentage error.
     - For example, estimating with centimetres is more accurate than estimating with metres. The percentage error will be smaller.
   - Percentage error is calculated from another amount called simply "error". We calculate error first, then percentage error.
2. Explain how to find error:
   - Error is found by first finding the range of numbers that a rounded quantity could fall between.
   - For example, we measure the width of a room and find that it is 2.5 metres to 2 s.f. The actual width could lie anywhere between 2.45 and 2.55. Any number in this range when rounded to 2 s.f. would give us 2.5.
   - The error of this measurement is the difference between the estimated amount and the minimum and maximum possible amounts.

- Write the following on the board and explain how to find the error in this case:

  Error: $2.55 - 2.5 = 0.05$ m. or $2.45 - 2.5 = -0.05$ m.

- Error is $+0.05$ m. or -0.05 m.

3. Explain how to find percentage error:
   - Percentage error is found by finding error as a percentage of the estimated measurement.
   - Write the formula shown below on the board:
     $$\text{Percentage error} = \frac{\text{error}}{\text{measurement}} \times 100\%$$
   - Remind pupils that in the previous example, the error is $\pm$0.05 m. and the measurement is 2.5 m.
   - Substitute these values into the formula and solve:

$$\begin{aligned} \text{Percentage error} &= \frac{\pm 0.05 \text{ m.}}{2.5 \text{ m.}} \times 100\% \\ &= \pm 2\% \end{aligned}$$

Write the example problem shown below on the board:

The length of a truck is measured as 5 metres to the nearest metre. What is the range of its actual length? Calculate the percentage error.

4. Ask pupils to work with seatmates to solve the first part of the problem, finding the range of the actual length of the truck.
5. Invite a volunteer to give their answer and explain. (Answer and explanation: The range of the actual length is 4.5 metres to 5.5 metres, because any value in that range would be rounded to 5 metres.)
6. Solve the second part of the problem on the board, calculating the percentage error. Involve pupils by asking them to give the error (± 0.5 m.) and the measurement (5 m.).
   Solution:

$$\begin{aligned} \text{Percentage error} &= \frac{\pm 0.5 \text{ m.}}{5 \text{ m.}} \times 100\% \\ &= \pm 10\% \end{aligned}$$

7. Write the problem shown below on the board:
   The length of a path is measured and given as 400 m. The path is measured accurately to the nearest 10 m. What is the percentage error?
8. Ask pupils to work with seatmates to find the error first. Remind them that the measurement is accurate to 10 m.
9. Invite a volunteer to give and explain their answer. (Answer and explanation: The range of the actual length is 395 metres to 405 metres, because any value in that

range would be rounded to 400 metres when rounded to the nearest 10 m. Thus, the error is $395-400=-5$ or $405-400=+5$. Therefore, the $\text{error}=\pm 5$.)

10. Ask pupils to work with seatmates to find the percentage error.
11. Invite a volunteer to write the solution and answer on the board. All other pupils should check their answers.

Solution:

$$\begin{aligned}\text{Percentage error} &= \frac{\pm 5\text{ m.}}{400\text{ m.}}\times 100\% \\ &= \pm 1.25\% \quad (\text{or} \quad \pm 1\tfrac{1}{4}\%)\end{aligned}$$

Practice *(10 minutes)*

1. Write the problem shown below on the board:
   The capacity of a bucket is 6.5 litres to 1 decimal place. What is the percentage error?
2. Ask pupils to work independently or with seatmates to find the answer.
3. Invite a volunteer to come to the board to write the solution and answer. Other pupils should check their work.

   Solution:

$$\begin{aligned}\text{error} &= 6.55\text{ l.}-6.5\text{ l.}=+0.05\text{ l.} \\ \text{or}\ \ \text{error} &= 6.45\text{ l.}-6.5\text{ l.}=-0.05\text{ l.}\end{aligned}$$

$$\begin{aligned}\text{Percentage error} &= \frac{\text{error}}{\text{measurement}}\times 100\% \\ &= \frac{\pm 0.05\text{ l.}}{6.5\text{ l.}}\times 100\% \\ &= \pm\frac{5}{6.5} \\ &= \pm 0.77\% \ \ (\text{or } \pm\tfrac{10}{13}\%)\end{aligned}$$

Closing *(2 minutes)*

1. Discuss as a class: What are some professions you know of that use measurements? Do you think it would be helpful for those professionals to understand percentage error? Why or why not?
2. For homework, have pupils do the practice activity PHM2-L007 in the Pupil Handbook.

Lesson Title: Degree of Accuracy	**Theme:** Numbers and Numeration	
Lesson Number: M2-L008	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to decide on the degree of accuracy that is appropriate for given data which may have been rounded.	**Preparation** Write the problem in the Opening on the board.	

Opening *(5 minutes)*

1. Review percentage error. Write the following problem on the board:

   A pencil is 5 cm. long. Issa measures it as 4.9 cm. What is the percentage error?

2. Give pupils 2-3 minutes to solve the problem.
3. Invite a volunteer to write and explain the solution on the board.
   Solution:

$$\text{error} = 4.9\text{ cm.} - 5\text{cm.} = -0.1\text{ cm.}$$

$$\begin{aligned}\text{Percentage error} &= \frac{\text{error}}{\text{measurement}} \times 100\% \\ &= \frac{-0.1\text{ cm.}}{5} \times 100\% \\ &= -2\%\end{aligned}$$

4. Tell pupils that the topic for the day is degree of accuracy. Understanding the degree of accuracy helps us to round our answers correctly.

Teaching and Learning *(20 minutes)*

1. Explain: "Degree of accuracy" refers to how many significant figures (s.f.) or decimal places a number has. Generally, using more digits makes a number more accurate.
2. Write the following on the board to explain degree of accuracy:
   Rounding 23547:

23550	23600	24000	20000
4 s.f.	3 s.f.	2 s.f.	1 s.f.

more accurate ⟶ less accurate

3. Explain:
   - The degree of accuracy that you should give in an answer depends on the degree of accuracy in the problem. The answer should not include more significant figures than the number of significant figures requested in the problem.

- It is not necessary to round to the given number of significant figures at each step or in the middle of a calculation. Numbers in a middle step of your calculation may have more significant figures than those in the problem and answer.

4. Write the following example problem on the board: Calculate the area of a circle with a 6.5 m radius
5. Ask pupils to raise their hand to give the formula for the area of a circle. Write it on the board: $A = \pi r^2$
6. Ask pupils to raise their hand to give the value of pi to two decimal places. (Answer: 3.14)
7. Solve the problem on the board.
   Solution:

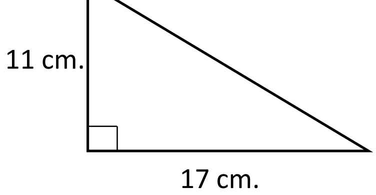
$$\begin{aligned} A &= \pi r^2 \\ &= (3.14)(6.5 \text{ m.})^2 \\ &= 38.465 \text{ m.}^2 \end{aligned}$$

8. Ask pupils to raise their hand to give the number of significant figures in this answer. (Answer: 5 s.f.)
9. Explain:
   - This answer should be rounded to 2 s.f. This is because the number in the problem, 3.5, has 2 s.f.
   - The number we used for pi (3.14) was a rounded number, and was not given in the problem. Rounding it to 3 s.f. gave us a more accurate result than if we had used only 2 s.f.
   - Remember that it’s okay to have more than 2 s.f. in the middle steps.
10. Write the answer on the board, rounded to 2 s.f.
    Solution:

$$\begin{aligned} A &= \pi r^2 \\ &= (3.14)(6.5 \text{ m.})^2 \\ &= 38.465 \text{ m.}^2 \\ &= 38 \text{ m.}^2 \text{ to 2 s.f.} \end{aligned}$$

11. Write another problem on the board:

    In the triangle below, the lengths of two sides are given to the nearest centimetre. Find the area of the triangle. Give your answer to 2 s.f.



12. Ask pupils to work with seatmates to solve the problem. If needed, remind them of the formula for area of triangle ($A = \frac{1}{2}bh$).

13. Invite a volunteer to solve the problem on the board:
    Solution:

$$\begin{aligned} A &= \tfrac{1}{2}(17\text{ cm.})(11\text{ cm.}) \\ &= \tfrac{1}{2}(187\text{ cm.}) \\ &= 93.5\text{ cm.} \\ &= 94\text{ cm. to 2 s.f.} \end{aligned}$$

Practice *(13 minutes)*

1. Write the following 2 problems on the board:
   a. Mr. Kamara wants to build a fence for his yard, which is a circle with a radius of 31 m. Find the circumference of his yard. (Take π to be 3.14, and use $C = 2\pi r$.)
   b. A ship travels 3000 km in 7 days. Estimate its speed in km/day.
2. Ask pupils to work independently or with seatmates to find the answers.
3. Invite volunteers to come to the board to write the solutions and answers. Other pupils should check their work.
   Solutions:

   a.

$$\begin{aligned} C &= 2\pi\text{r} \\ &= 2(3.14)(31\text{ m.}) \\ &= 194.68\text{ m.} \\ &= 190\text{ m. to 2 s.f.} \end{aligned}$$

   b.

$$\begin{aligned} \text{speed} &= \frac{3{,}000\text{ km.}}{7\text{ days}} \\ &= 428.57\text{ km./day} \\ &= 400\text{ km./day to 1 s.f.} \end{aligned}$$

Closing *(2 minutes)*

1. Discuss as a class: Why do you think it’s important to give answers to the correct degree of accuracy? Is this important in real life situations? If so, what situations?
2. For homework, have pupils do the practice activity PHM2-L008 in the Pupil Handbook.

Lesson Title: Simultaneous Linear Equations using Elimination	**Theme:** Algebraic Processes	
Lesson Number: M2-L009	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to solve simultaneous linear equations using elimination.	**Preparation** Write the problem in the Opening on the board.	

Opening *(3 minutes)*

1. Review solving linear equations. Write the following problem on the board: If $3x + 2y = 18$, find the value of y when $x = 2$.
2. Allow time for pupils to answer in their exercise books.
3. Invite a volunteer to come to the board to show the solution to the linear equation.
   Solution:

$$
\begin{aligned}
3x + 2y &= 18 \\
3(2) + 2y &= 18 && \text{Substitute } x = 2 \\
6 + 2y &= 18 \\
2y &= 18 - 6 && \text{Transpose} \\
2y &= 12 \\
\frac{2y}{2} &= \frac{12}{2} && \text{Divide both sides by 2} \\
y &= 6
\end{aligned}
$$

4. Explain to pupils that today's lesson is solving simultaneous linear equations using the method of elimination.

Teaching and Learning *(20 minutes)*

1. Write two linear equations on the board:

$$
\begin{aligned}
2x + 3y &= 10 \qquad (1) \\
2x + y &= 2 \qquad (2)
\end{aligned}
$$

2. Explain:
   - To solve for the unknown variable in one linear equation we use the given value as in our example on the board.
   - To solve for 2 linear equations with 2 unknown variables, x and y, we need to solve them together (simultaneously) so that they satisfy both equations.
   - We call that solving "simultaneous linear equations".
   - There are several methods: elimination, substitution and by graphing.
   - We will be using the method of elimination in this lesson.
3. Ask pupils to look for the variable with the same co-efficient in both equations.
4. Ask pupils to identify the variable. (Answer: x).
5. Ask pupils to give the value of the co-efficient. (Answer: 2)

6. Explain the solution step by step.
   Solution:

$2x + 3y$	$=$	10	(1)	
$-(2x + y$	$=$	$2)$	(2)	Subtract equation (2) from equation (1).
$0 + 2y$	$=$	8		Note: The negative sign will change all the signs across the bracket
$2y$	$=$	8		
$\frac{2y}{2}$	$=$	$\frac{8}{2}$		Divide both sides of the equation by the co-efficient of y (which is 2)
y	$=$	4		

7. Substitute the value of $y = 4$ in either equation 1 or 2 to find the value of x

$2x + 3y$	$=$	10	(1)	
$2x + 3(4)$	$=$	10		Substitute $y = 4$ in equation (1)
$2x + 12$	$=$	10		
$2x$	$=$	$10 - 12$		Transpose 12
$2x$	$=$	-2		
$\frac{2x}{2}$	$=$	$\frac{-2}{2}$		Divide throughout by 2
x	$=$	-1		

8. The solution is therefore $x = -1, y = 4$.
9. Write the solution on the board as an ordered pair: $(-1, 4)$
10. Ask pupils to check that the solution is correct by substituting the values for x and y in equation (2).
    Check:

$2x + y$	$=$	2	(2)	
$2(-1) + 4$	$=$	2		Substitute $x = -1, y = 4$ in equation (2)
$-2 + 4$	$=$	2		
2	$=$	2		
LHS	$=$	RHS		

11. Write on the board:

$2x + 5y$	$=$	2	(1)
$3x + 2y$	$=$	25	(2)

12. Ask the pupils what they notice about these two equations. (Answer: None of the variables have the same co-efficient).
13. Explain:
    - We eliminate the x-variable first and solve for the y-variable.
    - Make the coefficients of x the same by multiplying equation (1) throughout by the coefficient of x in equation (2), and multiplying equation (2) throughout by the coefficient of x in equation (1).

Solution:

$6x + 15y$	$=$	6	(3)	Multiply equation (1) × 3
$6x\ +\ 4y$	$=$	50	(4)	Multiply equation (2) × 2
$0 + 11y$	$=$	-44		Subtract equation (4) from equation (3).
$\frac{11y}{11}$	$=$	$\frac{-44}{11}$		Divide throughout by 11
y	$=$	-4		
$2x + 5y$	$=$	2	(1)	
$2x + 5(-4)$	$=$	2		Substitute $y = -4$ in equation (1)
$2x - 20$	$=$	2		
$2x$	$=$	$2 + 20$		Transpose -20
$2x$	$=$	22		
$\frac{2x}{2}$	$=$	$\frac{22}{2}$		Divide throughout by 5
x	$=$	11		

14. The solution is therefore $x = 11, y = -4$, or the ordered pair $(11, -4)$.
15. Ask pupils to work with seatmates to check the solution, $x = 11, y = -4$. They should substitute these values in equation (2).
16. Invite a volunteer to write their solution on the board.

    Solution:

$3x + 2y$	$=$	25	(2)	
$3(11) + 2(-4)$	$=$	25		Substitute $x = 11, y = -4$
$33 - 8$	$=$	25		
25	$=$	25		
LHS	$=$	RHS		

17. The solution is verified as $x = 11, y = -4$.
18. Explain:
    - It does not matter which variable we choose to eliminate first. This same procedure can be used to eliminate the y-variable first, then solve for the x-variable.
    - To make the coefficients of y the same, we multiply equation (1) throughout by the coefficient of y in equation (2), and multiply equation (2) throughout by the coefficient of y in equation (1).

Practice *(15 minutes)*

1. Ask pupils to solve the given pairs of equations using the method of elimination:

    a. $3x + 4y = -1$ b. $5x - 2y = 15$

$3x + 8y = 4$ $\qquad\qquad$ $3x + 5y = 9$

2. Ask pupils to exchange books and check each other's work.
3. Invite volunteers to come to the board to show their solutions.

   Solutions:

   a.
$$\begin{aligned}
3x + 4y &= -1 \\
3x + 8y &= 4 \\
\hline
0x - 4y &= -5 \\
-4y &= -5 \\
\frac{-4y}{-4} &= \frac{-5}{-4} \\
y &= \frac{5}{4}
\end{aligned}$$

$$\begin{aligned}
3x + 8y &= 4 \\
3x + 8(\tfrac{5}{4}) &= 4 \\
3x + 10 &= 4 \\
3x &= 4 - 10 \\
3x &= -6 \\
\frac{3x}{3} &= \frac{-6}{3} \\
x &= -2
\end{aligned}$$

$$x = -2, y = \frac{5}{4}$$

   b.
$$\begin{aligned}
5x - 2y &= 15 \\
3x + 5y &= 9
\end{aligned}$$

$$\begin{aligned}
15x - 6y &= 45 \\
15x + 25y &= 45 \\
\hline
0 - 31y &= 0 \\
y &= 0
\end{aligned}$$

$$\begin{aligned}
5x - 2y &= 15 \\
5x - 2(0) &= 15 \\
5x - 0 &= 15 \\
5x &= 15 \\
\frac{5x}{5} &= \frac{15}{5} \\
x &= 3
\end{aligned}$$

$$x = 3, y = 0$$

Closing *(2 minutes)*

1. Ask pupils to write in their exercise books the basic method to eliminate the x-variable from 2 linear equations. (Example answer: Make the coefficients of x the same by multiplying equation (1) throughout by the coefficient of x in equation (2), and multiplying equation (2) throughout by the coefficient of x in equation (1)).
2. Invite a volunteer to read out their answer. Correct any mistakes and confirm that pupils have written the method correctly in their exercise books.
3. For homework, have pupils do the practice activity PHM2-L009 in the Pupil Handbook.

Lesson Title: Simultaneous Linear Equations using Substitution	**Theme:** Algebraic Processes	
Lesson Number: M2-L010	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to solve simultaneous linear equations using substitution.	**Preparation** None	

Opening *(3 minutes)*

1. Review the previous lesson. Ask pupils to explain what simultaneous linear equations are. Allow them to discuss this with seatmates.
2. Remind pupils:
    - Linear equations have one or two variables and they don't have any powers on the variables.
    - "Simultaneous" means that we have more than one linear equation.
    - Generally, two simultaneous linear equations have two variables each.
3. Explain to pupils that today's lesson is solving simultaneous linear equations using the method of substitution.

Teaching and Learning *(20 minutes)*

1. Write the following two linear equations on the board:

$$\begin{aligned} a + 2b &= 13 \qquad (1) \\ 2a - 3b &= 5 \qquad (2) \end{aligned}$$

2. Explain:
    - To solve using the method of substitution, we must change the subject.
    - We should choose one of the given equations and make one of the variables the subject of the other one.
    - In this case, we can easily change the subject of equation (1) so that a is the subject. This is the easiest way, because a does not have a coefficient.
    - After changing the subject, we substitute the expression that is equal to a into the second linear equation.
3. Explain the solution step by step.
   Solution:

$a + 2b = 13$		(1)	
$a = 13 - 2b$			Change the subject of equation (1) by transposing $2b$
$2(13 - 2b) - 3b = 5$		(2)	Substitute equation (1) into equation (2)

$26 - 4b - 3b$	$=$	5	Simplify the left-hand side
$26 - 7b$	$=$	5	
$-7b$	$=$	$5 - 26$	Transpose 26
$-7b$	$=$	-21	
$\frac{-7b}{-7}$	$=$	$\frac{-21}{-7}$	Divide throughout by -7
b	$=$	3	

Substitute the value of $b = 3$ into the equation $a = 13 - 2b$ to find the value of a.

a	$=$	$13 - 2(3)$	Substitute $b = 3$ in the formula for a
a	$=$	$13 - 6$	
a	$=$	7	

4. The solution is therefore $a = 7, b = 3$.
5. Ask pupils to check that the solution is correct by substituting the values for a and b in equation (2).
   Check:

$2a - 3b$	$=$	5
$2(7) - 3(3)$	$=$	5
$14 - 9$	$=$	5
5	$=$	5
LHS	$=$	RHS

6. Write the following equations on the board:

$2x - y$	$=$	1	(1)
$x + 2y$	$=$	3	(2)

7. Invite a volunteer to explain what they would do as the first step. (Example answer: Change the subject of either equation so that one variable is in terms of the other.)
8. Ask pupils to work with seatmates to change the subject of equation (2) so that y is the subject of x.
9. Invite a volunteer to write their solution on the board.
   Solution:

$x + 2y$	$=$	3	(2)	
x	$=$	$3 - 2y$		Change the subject of equation (2)

10. Ask pupils to work with seatmates to substitute $x = 3 - 2y$ into equation (1) and solve for y.
11. Invite one group of seatmates to write their solution on the board.
    Solution:

$2(3 - 2y) - y$	$=$	1	(1)	Substitute equation (2) into equation (1)
$6 - 4y - y$	$=$	1		Simplify the left-hand side

$$\begin{aligned} 6-5y &= 1 \\ -5y &= 1-6 && \text{Transpose 6} \\ -5y &= -5 \\ \frac{-5y}{-5} &= \frac{-5}{-5} && \text{Divide throughout by } -5 \\ y &= 1 \end{aligned}$$

12. Invite a volunteer how to solve for x now that we know the value of y. (Answer: Substitute the value $y = 1$ into the formula for x, $x = 3 - 2y$.)
13. Ask pupils to work with seatmates to find the value of x.
14. Invite a volunteer to write their solution on the board.

    Solution:

$$\begin{aligned} x &= 3-2(1) && \text{Substitute } y=1 \text{ in the formula for } x \\ x &= 3-2 \\ x &= 1 \end{aligned}$$

Thus, the solution is $x = 1$, $y = 1$.

Practice *(15 minutes)*

1. Ask pupils to solve the given pairs of equations using the method of substitution:
   a. $x + 2y = 12$
      $3x - y = 1$
   b. $2a - b = 5$
      $3a + 2b = -24$
2. Ask pupils to exchange their books and check each other's work.
3. Invite volunteers to come to the board to show their solutions.

   Solutions:

a.

$$\begin{aligned} x+2y &= 12 && (1) \\ x &= 12-2y \\ \\ 3(12-2y)-y &= 1 && (2) \\ 36-6y-y &= 1 \\ 36-7y &= 1 \\ -7y &= 1-36 \\ -7y &= -35 \\ \frac{-7y}{-7} &= \frac{-35}{-7} \\ y &= 5 \end{aligned}$$

Substitute y into the formula for x:

$$\begin{aligned} x &= 12-2(5) \\ x &= 2 \end{aligned}$$

$$x = 2, y = 5$$

b.

$$\begin{aligned} 2a-b &= 5 && (1) \\ 2a-5 &= b \\ \\ 3a+2(2a-5) &= -24 && (2) \\ 3a+4a-10 &= -24 \\ 7a-10 &= -24 \\ 7a &= -14 \\ \frac{7a}{7} &= \frac{-14}{7} \\ a &= -2 \end{aligned}$$

Substitute a into the formula for b:

$$\begin{aligned} b &= 2(-2)-5 \\ b &= -4-5 \\ b &= -9 \end{aligned}$$

$$a = -2, b = -9$$

Closing *(2 minutes)*

1. Ask pupils to check the answer to problem 1. above ($x = 2, y = 5$). Remind them that they can check their answers at any time by substituting their solution into the equations in the question.
2. Invite a volunteer to write their work on the board:

$$\begin{aligned} x + 2y &= 12 \quad (1) \\ (2) + 2(5) &= 12 \\ 2 + 10 &= 12 \\ 12 &= 12 \\ \text{LHS} &= \text{RHS} \end{aligned} \qquad \text{or:} \qquad \begin{aligned} 3x - y &= 1 \quad (2) \\ 3(2) - 5 &= 1 \\ 6 - 5 &= 1 \\ 1 &= 1 \\ \text{LHS} &= \text{RHS} \end{aligned}$$

3. For homework, have pupils do the practice activity PHM2-L010 in the Pupil Handbook.

Lesson Title: Simultaneous Linear Equations using Graphical Methods – Part 1	**Theme:** Algebraic Processes	
Lesson Number: M2-L011	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to solve simultaneous linear equations using graphical methods.	**Preparation** None	

Opening *(4 minutes)*

1. Review the previous lesson. Ask pupils to explain the differences between solving simultaneous linear equations using elimination and using substitution.
2. Remind pupils:
    - To solve simultaneous equations by elimination, subtract one from the other. The coefficient on one of the variables should be the same in both equations, so they cancel out.
    - To solve simultaneous equations by substitution, solve one equation for a variable, and substitute the result into the other equation.
3. Explain to pupils that today's lesson is solving simultaneous linear equations by graphing the two lines.

Teaching and Learning *(25 minutes)*

1. Write the following two linear equations on the board:

$$-x + y = 1 \qquad (1)$$
$$2x + y = 4 \qquad (2)$$

2. Explain:
    - We can solve these simultaneous equations by graphing both lines. The solution of the simultaneous equations is the point where the lines intersect. At this point, the x-value and y-value satisfy both of the equations.
    - Remember to check your answer by substituting the values into the original equations.
    - When graphing, the answers may only be approximate values. The more accurately we draw our Cartesian planes, the more accurate the solutions will be. It is important to draw your plane precisely, with each point on the axes the same distance apart.
3. Draw a Cartesian plane on the board with axes up to –5 and 5. Ask pupils to do the same in their exercise books.
4. Write the two tables of values below on the board. Since the graphs are straight lines, it is only necessary to plot 3 points for each.

Equation (1)

x	0	1	2
y			

Equation (2)

x	0	1	2
y			

5. Fill the first table of values with solutions to equation (1), $-x + y = 1$. Remind pupils that it's easier to find the values of y after changing the subject.

$-x + y = 1$ (1)

$y = x + 1$ Change the subject

$y = (0) + 1$ Substitute each value of x and solve for y

$y = 1$

$y = (1) + 1$

$y = 2$

$y = (2) + 1$

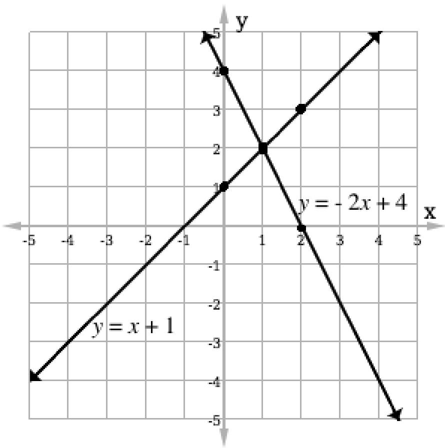
$y = 3$

Equation (1)			
x	0	1	2
y	1	2	3

6. Ask pupils to work with seatmates to fill the second table of values with solutions to equation (2), $2x + y = 4$.
   - Remind them to change the subject of the equation: $y = -2x + 4$
7. Invite volunteers to come to the board and fill the second table of values.

Equation (2)			
x	0	1	2
y	4	2	0

8. Invite volunteers to come to the board to plot each of the points in the tables on the Cartesian plane and draw the lines.



9. Ask pupils to identify the point at which the two lines intersect. (Answer: $(1, 2)$)
10. Explain that this is the solution to the simultaneous equations, and that we would get the same answer if we used either the elimination or substitution methods.
11. Write the equations below on the board:

$$\begin{aligned} 4x + y &= 3 && (1) \\ -x + y &= -2 && (2) \end{aligned}$$

12. Ask pupils to work with seatmates to fill the tables of values (below) and graph the two lines. They should then find the solution, or point of intersection.

Equation (1)

x	-1	0	1
y			

Equation (2)

x	-1	1	3
y			

13. Invite volunteers to fill the tables of values and draw the graph on the board.
Solutions:

Equation (1)

x	-1	0	1
y	7	3	-1

Equation (2)

x	-1	1	3
y	-3	-1	1

Change the subject of (1):

$$\begin{aligned} 4x + y &= 3 \\ y &= -4x + 3 \end{aligned}$$

Change the subject of (2):

$$\begin{aligned} -x + y &= -2 \\ y &= x - 2 \end{aligned}$$

Find y-values for (1):

$$\begin{aligned} y &= -4(-1) + 3 \\ y &= 7 \end{aligned}$$

$$\begin{aligned} y &= -4(0) + 3 \\ y &= 3 \end{aligned}$$

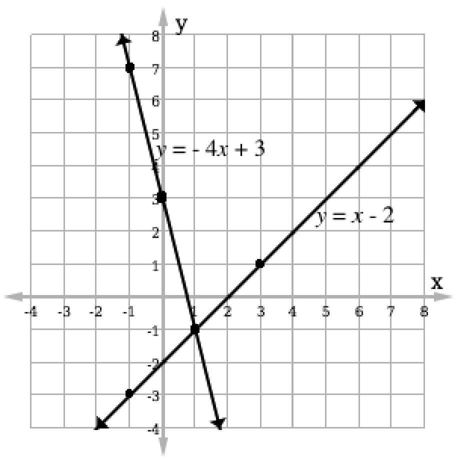
$$\begin{aligned} y &= -4(1) + 3 \\ y &= -1 \end{aligned}$$

Find y-values for (2):

$$\begin{aligned} y &= (-1) - 2 \\ y &= -3 \end{aligned}$$

$$\begin{aligned} y &= (1) - 2 \\ y &= -1 \end{aligned}$$

$$\begin{aligned} y &= (3) - 2 \\ y &= 1 \end{aligned}$$



14. Ask pupils to identify the solution. (Answer: $(1, -1)$)

15. Discuss: Did we all arrive at the same answer? If our answers are different, why might that be? (If pupils' answers are different, remind them that this can happen when graphs are not drawn accurately.)

Practice *(10 minutes)*

1. Ask pupils to solve the given pair of equations using the graphical method. Some of the points are already given in the tables of values. They should find the others and graph both equations.

$y = \frac{1}{2}x + 4$ (1)

$y = -x + 7$ (2)

Equation (1)

x	2	4	6
y	5		7

Equation (2)

x	2	4	6
y		3	

2. Invite volunteers to come to the board to fill the tables of values and graph the lines.

Solutions:

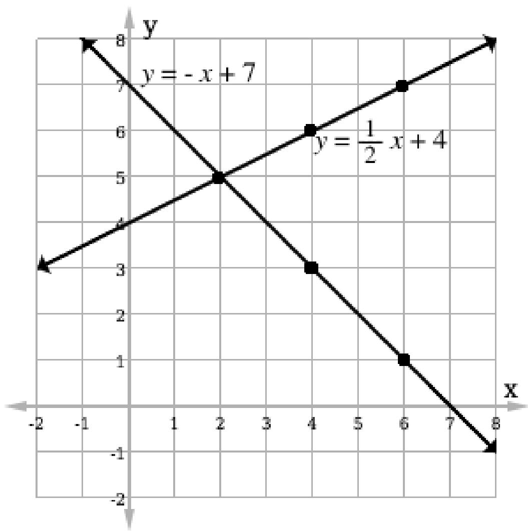
Equation (1)

x	2	4	6
y	5	**6**	7

Equation (2)

x	2	4	6
y	**5**	3	**1**

3. Ask pupils to identify the solution to the simultaneous equations (Answer: $(2, 5)$).



Closing *(1 minute)*

1. Discuss: Which method of solving simultaneous linear equations do you prefer: elimination, substitution, or graphing? Why?
2. For homework, have pupils do the practice activity PHM2-L011 in the Pupil Handbook.

Lesson Title: Simultaneous Linear Equations using Graphical Methods – Part 2	**Theme:** Algebraic Processes	
Lesson Number: M2-L012	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to solve simultaneous linear equations using graphical methods.	**Preparation** None	

Opening *(3 minutes)*

1. Review the previous the lesson. Ask pupils to explain how to solve simultaneous equations by graphing.
2. Remind pupils:
   - We can solve simultaneous equations by graphing both lines.
   - The solution of the simultaneous equations is the point where the lines intersect. It can be given as an ordered pair (x, y).
3. Explain to pupils that today's lesson is also solving simultaneous linear equations by graphing. We will practise more challenging problems today.

Teaching and Learning *(20 minutes)*

1. Explain:
   - In the previous lesson, we graphed lines with given tables of values. We then observed the intersection of the two lines.
   - Today you will be given simultaneous equations to solve. You will not be given the table of values.
2. Write the following problem on the board:

   Using graphical methods, find the solution of the simultaneous equations:

   $$-2x + y = 2 \qquad (1)$$
   $$-x + y = -1 \qquad (2)$$

3. Ask pupils to explain the steps in solving these simultaneous equations graphically.
   (Example answer:
   - Change the subject to make y the subject;
   - Draw a table of values and a Cartesian plane,
   - Find solutions and plot the points on the Cartesian plane;
   - Draw the lines and find the point of intersection.)
4. Draw a Cartesian plane on the board with axes up to –10 and 10. Ask pupils to do the same in their exercise books.

5. Draw two empty tables of values on the board. Remind pupils that they are not given a table of values in this problem. Tell them that they can choose any value of x, and find the corresponding y-value.
6. Ask pupils to suggest x-values to be used by the class (preferably small digits, between -3 and 3). Write these in the first rows of the tables. (See examples below)

Equation (1)

x	-2	0	2
y			

Equation (2)

x	-2	0	2
y			

7. Fill the first table of values with solutions to equation (1), $-2x + y = 2$. Remind pupils that it's easier to find the values of y if you change the subject of the equation.

$$-2x + y = 2 \quad (1)$$

$$y = 2x + 2$$ Change the subject

$$y = 2(-2) + 2$$
$$y = -2$$

Substitute each value of x and solve for y

$$y = 2(0) + 2$$
$$y = 2$$

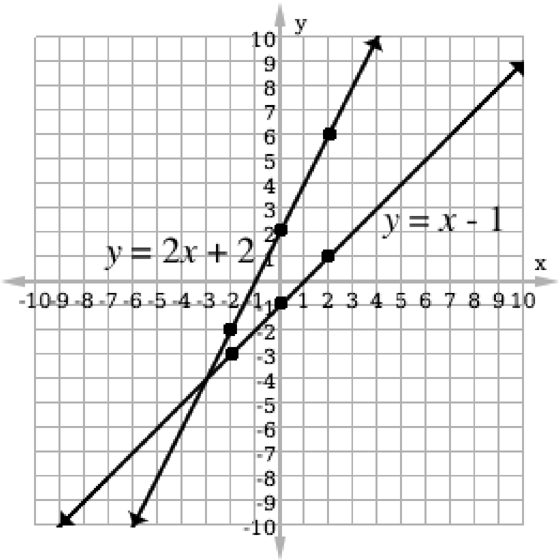
$$y = 2(2) + 2$$
$$y = 6$$

Equation (1)			
x	-2	0	2
y	-2	2	6

8. Ask pupils to work with seatmates to fill the second table of values with solutions to equation (2), $-x + y = -1$.
   - Remind them to change the subject of the equation: $y = x - 1$
9. Invite volunteers to come to the board and fill in the second table of values.

Equation (2)			
x	-2	0	2
y	-3	-1	1

10. Invite volunteers to come to the board to plot each of the points in the tables on the Cartesian plane and draw the lines.



11. Ask pupils to identify the point at which the two lines intersect. If the point of intersection is not within the range of points they plotted, they will need to extend their lines out using a straight edge. Show this on the board. (Answer: The point of intersection is at $(-3, -4)$)
12. Explain that this is the solution to the simultaneous equations, and that we would get the same answer if we used either the elimination or substitution methods.
13. Remind pupils that they can check their work by substituting the values of $x = -3$ and $y = -4$ into the original equations.
14. Ask pupils to work with seatmates to check the solution by substituting the values in both equation (1) and equation (2).
15. Invite 2 volunteers to come to the board to each show how their group checked the solution in one of the equations.
    Check:

$-2x + y$	$=$	2	(1)	$-x + y$	$=$	-1	(2)
$-2(-3) + (-4)$	$=$	2		$-(-3) + (-4)$	$=$	-1	
$6 - 4$	$=$	2		$3 - 4$	$=$	-1	
2	$=$	2		-1	$=$	-1	
LHS	$=$	RHS		LHS	$=$	RHS	

Practice *(15 minutes)*

1. Write the following problem on the board:

   On the same Cartesian plane, draw the graphs of: $y = 2x + 4$ and $y = 3x + 2$. From your graph, find the coordinates of the point of intersection of the two graphs.

2. Ask pupils to solve the problem with seatmates. Walk around to check for understanding. Make sure they are using appropriate x-values in their tables (values should not be too far from zero).
3. Invite volunteers to come to the board to fill the tables of values and graph the lines.
   Solution:
   Example tables of values (x-values chosen by pupils may differ):

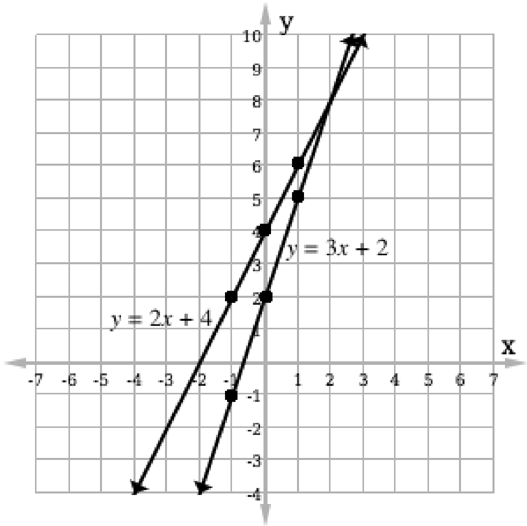
Equation (1)

x	-1	0	1
y	2	4	6

Equation (2)

x	-1	0	1
y	-1	2	5

Graph**:**



4. Ask pupils to identify the solution to the simultaneous equations. They may need to extend the straight lines using a straight edge to observe the solution (Answer: $(2, 8)$).

Closing *(2 minutes)*

1. Discuss: What are the challenges of solving simultaneous equations graphically? How can we overcome these challenges? (Example challenge: It is difficult to observe the precise point of intersection. Example solution: We can draw our Cartesian planes and plot points very precisely.)
2. For homework, have pupils do the practice activity PHM2-L012 in the Pupil Handbook.

Lesson Title: Words Problems on Simultaneous Linear Equations	**Theme:** Algebraic Processes	
Lesson Number: M2-L013	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to solve word problems leading to simultaneous linear equations.	**Preparation** None	

Opening *(5 minutes)*

1. Review solving simultaneous equations by giving pupils one problem to solve.
2. Write on the board: Solve using either the elimination method or the substitution method:

$$\begin{aligned} x+y &= 3 \\ x-y &= 1 \end{aligned}$$

3. Ask pupils to work with seatmates to solve the problem using **one** method. Walk around and check for understanding.
4. Invite two sets of volunteers to come to the board. One should write the solution using elimination, and the other should write the solution using substitution.
Solutions:

Elimination:

Subtract equations, solve for y:

$$\begin{aligned} x+y &= 3 && (1) \\ -(x-y &= 1) && (2) \\ \hline 0+2y &= 2 \\ 2y &= 2 \\ \frac{2y}{2} &= \frac{2}{2} \\ y &= 1 \end{aligned}$$

Substitute for y in equation 1:

$$\begin{aligned} x+(1) &= 3 && (1) \\ x &= 2 \end{aligned}$$

Solution: $(2,1)$

Substitution:

$$\begin{aligned} x+y &= 3 && (1) \\ x &= 3-y \end{aligned}$$

Substitute into equation 2:

$$\begin{aligned} (3-y)-y &= 1 && (2) \\ 3-2y &= 1 \\ -2y &= 1-3 \\ \frac{-2y}{-2} &= \frac{-2}{-2} \\ y &= 1 \end{aligned}$$

Substitute for y in equation 1:

$$\begin{aligned} x &= 3-(1) && (1) \\ x &= 2 \end{aligned}$$

5. Explain to pupils that today's lesson is solving simultaneous linear equations from word problems.

Teaching and Learning *(20 minutes)*

1. Explain:

- Algebra is used to solve word problems. We often need to identify the unknown variables in word problems, and use letters to represent them.
- In some cases, a word problem will give two equations with the same variables. These are simultaneous equations, and they can be solved using either elimination or substitution.

2. Write the following problem on the board: One pen and 2 exercise books cost Le 4,500.00. 2 pens and 3 exercise books cost Le 7,000.00. How much does each pen and exercise book cost?
3. Ask pupils to identify the unknown variables. (Answer: the cost of a pen, the cost of an exercise book)
4. Tell pupils that we will let the variable e represent the cost of an exercise book, and p represent the cost of a pen.
5. Write on the board: $p + 2e = 4{,}500$, where p is the cost of a pen, and e is the cost of an exercise book.
6. Explain: This equation tells us that the total cost of 1 pen and 2 exercise books is 4,500 Leones. We do not yet have enough information to solve this problem. We need another equation.
7. Ask pupils to look at the second sentence in the problem (2 pens and 3 exercise books cost Le 7,000.00). Ask pupils to work with seatmates to write an equation for this sentence.
8. Ask groups of seatmates to share and discuss their equations with the class.
9. Write on the board: $2p + 3e = 7{,}000$
10. Explain: This is the correct equation. It tells us that the cost of 2 pens and 3 exercise books is 7,000 Leones in total.
11. Ask pupils to work with seatmates to solve the system of equations. They may use either substitution or elimination.

$$\begin{aligned} p + 2e &= 4{,}500 \qquad (1) \\ 2p + 3e &= 7{,}000 \qquad (2) \end{aligned}$$

12. Ask one group of seatmates to write their solution on the board (either method).

Solutions:

Elimination:

$$\begin{aligned} 2(p + 2e &= 4{,}500) \qquad (1) \\ -(2p + 3e &= 7{,}000) \qquad (2) \\ \downarrow \qquad & \qquad \downarrow \\ 2p + 4e &= 9{,}000 \qquad (1)\text{x}2 \\ -(2p + 3e &= 7{,}000) \qquad (2) \\ 0 + e &= 2{,}000 \\ e &= 2{,}000 \\ p + 2(2{,}000) &= 4{,}500 \qquad (1) \\ p + 4{,}000 &= 4{,}500 \end{aligned}$$

Substitution:

$$\begin{aligned} p + 2e &= 4{,}500 \qquad (1) \\ p &= 4{,}500 - 2e \\ 2(4{,}500 - 2e) + 3e &= 7{,}000 \qquad (2) \\ 9{,}000 - 4e + 3e &= 7{,}000 \\ 9{,}000 - e &= 7{,}000 \\ 2{,}000 &= e \\ p &= 4{,}500 - 2(2{,}000) \qquad (1) \\ p &= 500 \end{aligned}$$

$$p = 500$$

Solution: $e = \text{Le } 2{,}000.00, \quad p = \text{Le } 500.00$

13. Write the following problem below on the board: The sum of the ages of Hawa and Fatu is 32 years. Hawa is 4 years older than Fatu. What are their ages?
14. Ask pupils to describe the unknown variables in this word problem. (Answers: the age of Hawa, the age of Fatu)
15. Write on the board: $h + f = 32$
16. Explain: This equation says that the sum of the ages of Hawa and Fatu is 32.
17. Ask pupils to look at the second sentence in the problem (Hawa is 4 years older than Fatu.). Ask them to work with seatmates to write an equation for this sentence.
18. Ask groups of seatmates to share and discuss their equations with the class.
19. Write on the board: $h = f + 4$
20. Explain: This is the correct equation. It tells us that Hawa's age is 4 years older than Fatu's age.
21. Remind pupils that this equation can be rewritten by transposing f (as $h - f = 4$).
22. Ask pupils to work with seatmates to solve the system of equations. They may use either substitution or elimination.

$$\begin{aligned} h + f &= 32 \qquad (1) \\ h - f &= 4 \qquad (2) \end{aligned}$$

23. Invite one group of seatmates to write their solution on the board.

Solutions:

Elimination:

$$\begin{aligned} h + f &= 32 \qquad (1) \\ -(h - f &= 4) \qquad (2) \\ \hline 0 + 2f &= 28 \\ 2f &= 28 \\ \frac{2f}{2} &= \frac{28}{2} \\ f &= 14 \\ \\ h + (14) &= 32 \qquad (1) \\ h &= 18 \end{aligned}$$

Substitution:

$$\begin{aligned} h + f &= 32 \qquad (1) \\ h &= 32 - f \\ \\ (32 - f) - f &= 4 \qquad (2) \\ 32 - 2f &= 4 \\ -2f &= 4 - 32 \\ \frac{-2f}{-2} &= \frac{-28}{-2} \\ f &= 14 \\ \\ h &= 32 - 14 \qquad (1) \\ h &= 18 \end{aligned}$$

Solution: $f = 14$ years old, $h = 18$ years old

Practice *(13 minutes)*

1. Write the two problems below on the board and ask pupils to solve them:
   a. The total age of 2 brothers is 112. One of the brothers is 14 years older than the other. What are their ages?
   b. The cost of a cup of sugar is x Leones, and the cost of a cup of flour is y Leones. If 1 cup of sugar and 4 cups of flour cost Le 7,000.00, and 3 cups of sugar and 2 cups of flour cost Le 6,000.00, find x and y.
2. Walk around to check for understanding. If pupils need support, work as a class to find the simultaneous equations and write them on the board:

1) $$\begin{aligned} x + y &= 112 \\ x - y &= 14 \end{aligned}$$

2) $$\begin{aligned} x + 4y &= 7{,}000 \\ 3x + 2y &= 6{,}000 \end{aligned}$$

3. Invite 2 volunteers to come to the board and each solve 1 problem.
   Solutions:
   Pupils do not need to solve both problems using 2 methods. One method is sufficient for each. Example solutions are below.

a. Using elimination:

$$\begin{aligned} x + y &= 112 && (1) \\ -(x - y &= 14) && (2) \\ \hline 0 + 2y &= 98 \\ 2y &= 98 \\ \frac{2y}{2} &= \frac{98}{2} \\ y &= 49 \\ \\ x + (49) &= 112 && (1) \\ x &= 63 \end{aligned}$$

Solution: Their ages are 49 and 63 years

b. Using substitution:

$$\begin{aligned} x + 4y &= 7{,}000 && (1) \\ x &= 7{,}000 - 4y \\ \\ 3(7{,}000 - 4y)) + 2y &= 6{,}000 && (2) \\ 21{,}000 - 12y + 2y &= 6{,}000 \\ 21{,}000 - 10y &= 6{,}000 \\ 21{,}000 - 6{,}000 &= 10y \\ 15{,}000 &= 10y \\ 1{,}500 &= y \\ \\ x &= 7{,}000 - 4y && (1) \\ &= 7{,}000 - 4(1{,}500) \\ &= 7{,}000 - 6{,}000 \\ x &= 1{,}000 \end{aligned}$$

Solution: $x = \text{Le } 1{,}000.00, y = \text{Le } 1{,}500.00$

Closing *(2 minutes)*

1. Discuss: When would solving simultaneous equations be useful in everyday life? What types of everyday problems could you solve? (Example answer: Simultaneous equations can help business owners solve problems related to the cost of goods.)
2. For homework, have pupils do the practice activity PHM2-L013 in the Pupil Handbook.

Lesson Title: Simultaneous Linear and Quadratic Equations using Substitution	**Theme:** Algebraic Processes	
Lesson Number: M2-L014	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to solve simultaneous linear and quadratic equations using substitution.	**Preparation** Write the equations in the Opening on the board.	

Opening *(3 minutes)*

1. Review solving simultaneous linear equations using substitution. Write the following set of simultaneous equations on the board:

$$\begin{aligned} -2x + y &= 3 \\ 3x + y &= -2 \end{aligned}$$

2. Discuss: Have pupils explain how to solve this set of equations using substitution. (Example answer: Make y the subject of equation (1) and substitute this into equation (2); solve for x and use this value of x to find the corresponding y-value)
    - For the sake of time, do not solve the problem.
3. Explain to pupils that today's lesson is solving simultaneous equations where one of them is a linear equation, and the other one is a quadratic equation.

Teaching and Learning *(20 minutes)*

1. Write the following equations on the board:

$$\begin{aligned} y &= x + 2 \qquad (1) \\ y &= x^2 \qquad (2) \end{aligned}$$

2. Ask pupils to identify the types of equations on the board. (Answer: (1) is linear; (2) is quadratic)
3. Explain:
    - These are simultaneous linear and quadratic equations. They can be solved using substitution or graphing.
    - Simultaneous linear and quadratic equations can have 0, 1 or 2 solutions. Most of the problems you will be asked to solve will have 2 solutions.
    - The solutions can be written as ordered pairs (x, y).
4. Explain how to **solve** simultaneous linear and quadratic equations:
    - Make y the subject of one equation, then substitute it into the other equation.
    - Simplify until it is in the standard quadratic equation form ($ax^2 + bx + c = 0$).
    - Solve the quadratic equation using any method. Today we will use the factorization method.
    - Each solution to the quadratic equation is an x-value. Substitute these into one of the original equations to find each corresponding y-value.

5. Solve the simultaneous equations on the board using substitution. Explain the solution step by step.
   Solution:

x^2	$=$	$x+2$	(1)	Substitute $y = x^2$ for y in equation (1)
$x^2 - x - 2$	$=$	0		Transpose x and 2
$(x-2)(x+1)$	$=$	0		Factorise the quadratic equation
$x-2=0$	or	$x+1=0$		Set each binomial equal to 0
$x=2$	or	$x=-1$		Transpose -2 and 1
y	$=$	$(2)+2$		Substitute $x=2$ into equation (2)
y	$=$	4		
y	$=$	$(-1)+2$		Substitute $x=-1$ into equation (2)
y	$=$	1		

6. Write another problem on the board:

$$y = x^2 - 5x + 7 \qquad (1)$$
$$y - 2x = 1 \qquad (2)$$

7. Ask pupils to describe how to solve this. Allow them to discuss and share.
8. Explain: Equation (1) is already written with y as the subject. We can substitute this into equation (2) and solve.
9. Solve on the board using substitution. Explain the solution step by step.
   Solution:

$(x^2-5x+7)-2x$	$=$	1	(2)	Substitute equation (1) into equation (2)
x^2-7x+7	$=$	1		Simplify
$x^2-7x+7-1$	$=$	0		Transpose 1
x^2-7x+6	$=$	0		
$(x-6)(x-1)$	$=$	0		Factorise the quadratic equation
$x-6=0$	or	$x-1=0$		Set each factor equal to 0
$x=6$	or	$x=1$		Transpose -6 and -1
$y-2(6)$	$=$	1		Substitute $x=6$ into equation (2)
$y-12$	$=$	1		
y	$=$	$1+12$		Transpose -12
y	$=$	13		
$y-2(1)$	$=$	1		Substitute $x=1$ into equation (2)
$y-2$	$=$	1		
y	$=$	$1+2$		Transpose -2
y	$=$	3		

Solutions: $(6,13)$ and $(1,3)$

Practice *(15 minutes)*

1. Write the following problem on the board and ask pupils to find the solutions:

$$\begin{aligned} y &= x^2 - 3 \qquad (1) \\ x &= y - 9 \qquad (2) \end{aligned}$$

2. Walk around to check for understanding. If pupils need support, remind them of the steps or allow them to work with seatmates.
3. Invite a volunteer to come to the board and write the solution. Other pupils should check their work.

   Solution:

x	$=$	$(x^2-3)-9$	(2)	Substitute equation (1) into equation (2)
x	$=$	x^2-12		Simplify
0	$=$	x^2-x-12		Transpose x
x^2-x-12	$=$	0		
$(x-4)(x+3)$	$=$	0		Factorise the quadratic equation
$x-4=0$	or	$x+3=0$		Set each factor equal to 0
$x=4$	or	$x=-3$		Transpose -4 and 3
y	$=$	4^2-3		Substitute $x=4$ into equation (1)
y	$=$	$16-3$		
y	$=$	13		
y	$=$	$(-3)^2-3$		Substitute $x=-3$ into equation (1)
y	$=$	$9-3$		
y	$=$	6		

   Solutions: $(4, 13)$ and $(-3, 6)$

Closing *(2 minutes)*

1. Discuss: What does it look like when we graph a quadratic equation on the coordinate plane? What does a linear equation look like? (Example answers: A quadratic equation is a parabola. A linear equation is a line.)
2. For homework, have pupils do the practice activity PHM2-L014 in the Pupil Handbook.

Lesson Title: Simultaneous Linear and Quadratic Equations using Graphical Methods - Part 1	**Theme:** Algebraic Processes	
Lesson Number: M2-L015	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to solve simultaneous linear and quadratic equations using graphical methods.	**Preparation** Write the equations in the Opening on the board.	

Opening *(2 minutes)*

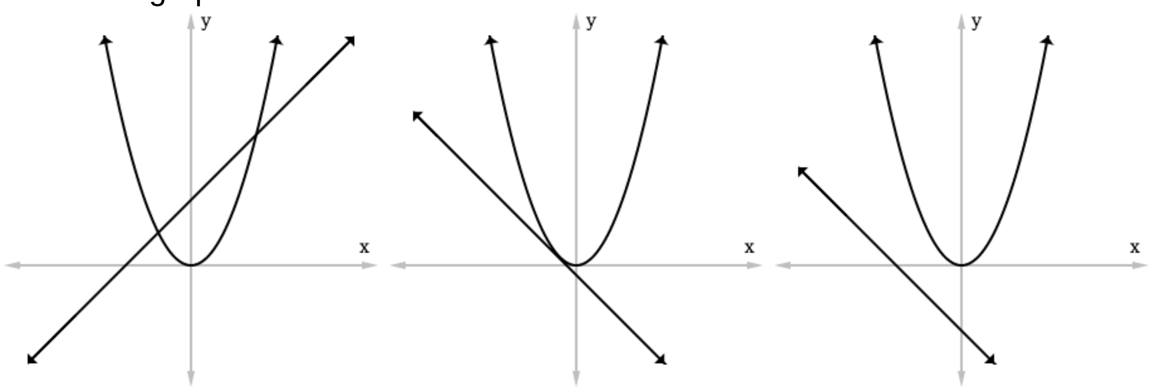
1. Write these simultaneous equations from the previous lesson on the board:

$$y = x + 2 \qquad (1)$$
$$y = x^2 \qquad (2)$$

2. Ask pupils to look at their notes from the previous lesson and give the solutions to these simultaneous equations. Write them on the board. (Answer: $(2, 4)$ and $(-1, 1)$)
3. Explain that in today's lesson we will solve systems of quadratic and linear equations by graphing. The answers that we find by graphing should be the same as the answers we find using substitution.

Teaching and Learning *(25 minutes)*

1. Explain:
   - To graph quadratic or linear equations, we need to find points on the curve and the line. We then plot the points and connect them.
   - The intersection points of the curve and line are the solutions to the simultaneous equations.
2. Sketch the graphs below on the board:



2 solutions **1 solution** **0 solutions**

3. Explain:

- When we graph a line and parabola, they have either 2 solutions, 1 solution, or no solutions.
- When they have 1 solution, the line touches the parabola at one point. This is also called a "tangent line".

4. Draw an empty Cartesian plane and empty tables of values on the board (see below for tables).
5. Solve the equations on the board by graphing. Explain each step.

Solution:

Fill the table of values for $y = x + 2$:

x	-2	-1	0	1	2
y	0	1	2	3	4

$$y = (-2) + 2 = 0$$

$$y = (-1) + 2 = 1$$

$$y = (0) + 2 = 2$$

$$y = (1) + 2 = 3$$

$$y = (2) + 2 = 4$$

Fill the table of values for $y = x^2$:

x	-2	-1	0	1	2
y	4	1	0	1	4

$$y = (-2)^2 = 4$$

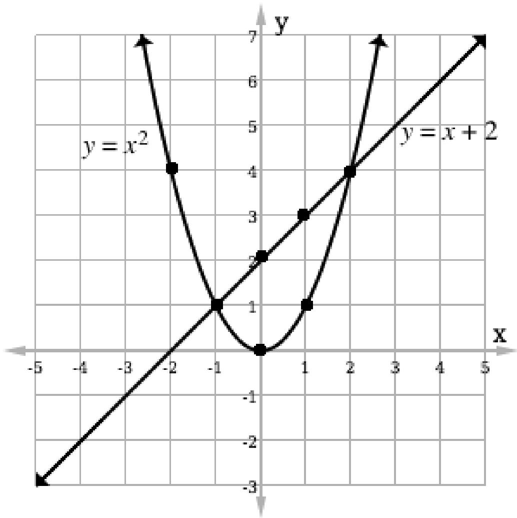
$$y = (-1)^2 = 1$$

$$y = (0)^2 = 0$$

$$y = (1)^2 = 1$$

$$y = (2)^2 = 4$$

Plot the points from both tables. Draw the line and curve:



Ask pupils to give the points of intersection. Write them on the board: $(-1, 1)$, $(2, 4)$

6. Remind pupils that this is the same solution that was reached using substitution.
7. Write another problem on the board:
   Solve the simultaneous equations by graphing. Use the given tables of values:

$$\begin{aligned} y &= -x^2 + 3 \qquad (1) \\ y &= 2x + 4 \qquad (2) \end{aligned}$$

$y = -x^2 + 3$

x	-2	-1	0	1	2
y					

$y = 2x + 4$

x	-2	-1	0	1	2
y					

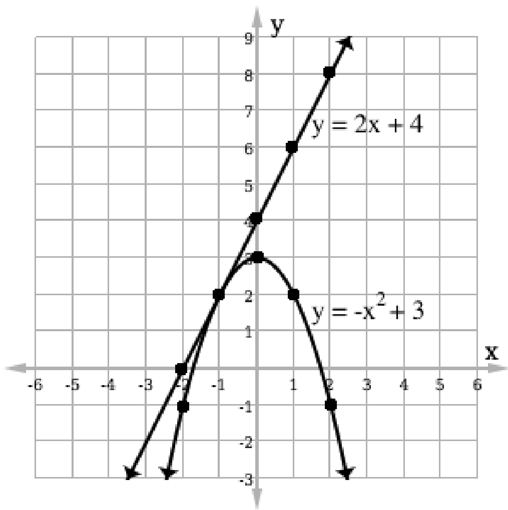
8. Ask pupils to work with seatmates to find the solution.
9. Invite a few volunteers to come to the board to write different parts of the solution. Ask different pupils to fill each table of values and graph the equations.
   Solution:

$y = -x^2 + 3$

x	-2	-1	0	1	2
y	-1	2	3	2	-1

$y = 2x + 4$

x	-2	-1	0	1	2
y	0	2	4	6	8



10. Invite a volunteer to give the point of intersection. Write it on the board: $(-1, 2)$
11. Explain that this is an example of simultaneous linear and quadratic equations with only 1 solution.

Practice *(10 minutes)*

1. Write the problem below on the board.
2. Ask pupils to solve the simultaneous equations by graphing. Use the given tables of values:

$$\begin{aligned} y &= x^2 + 1 \qquad (1) \\ y &= x - 1 \qquad (2) \end{aligned}$$

$y = x^2 + 1$

x	-2	-1	0	1	2
y					

$y = x - 1$

x	-2	-1	0	1	2
y					

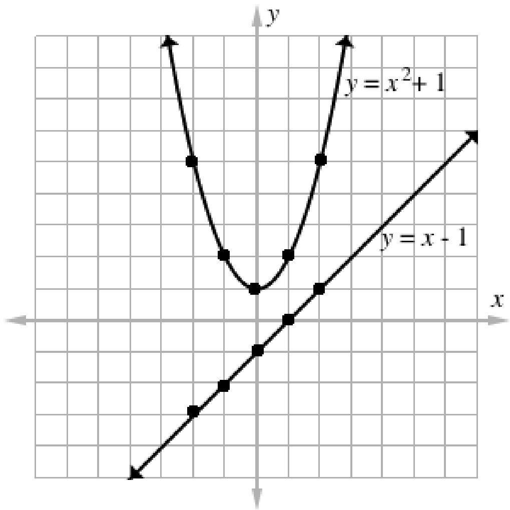
3. Walk around to check for understanding. If pupils need support, remind them of the steps or allow them to work with seatmates.
4. Invite a few volunteers to come to the board and write the solution. Other pupils should check their work.
   Solution:

$y = x^2 + 1$

x	-2	-1	0	1	2
y	5	2	1	2	5

$y = x - 1$

x	-2	-1	0	1	2
y	-3	-2	-1	0	1



No solution

5. Tell pupils that this is an example of a set of simultaneous equations with no solution. The graphs of the line and parabola do not intersect at any point.

Closing *(3 minutes)*

1. Review the lesson. Discuss: What does the graph of simultaneous linear and quadratic equations look like if there are 2 solutions? If there is 1 solution? If there are no solutions?
2. For homework, have pupils do the practice activity PHM2-L015 in the Pupil Handbook.

Lesson Title: Simultaneous Linear and Quadratic Equations using Graphical Methods - Part 2	**Theme:** Algebraic Processes
Lesson Number: M2-L016	**Class:** SSS 2 / **Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to solve simultaneous linear and quadratic equations using graphical methods.	**Preparation** None

Opening *(3 minutes)*

1. Review the previous lesson. Discuss:
    - What are the steps to solving simultaneous linear and quadratic equations by graphing? (Example answer: Fill in a table of values for each equation and plot the coordinates on the Cartesian plane. The points of intersection are the solutions.)
    - How many solutions can simultaneous linear and quadratic equations have? (Answer: 0, 1, or 2)
2. Explain that in today's lesson, we will continue solving systems of quadratic and linear equations by graphing.

Teaching and Learning *(15 minutes)*

1. Write the following problem on the board:

    Locate the points where $y = x^2 - 2x - 3$ intersects $y = x - 5$.

2. Explain:
    - In the previous lesson, you were given tables of values to help you graph the equations. Today you will make your own table of values.
    - If you plot the points, and the points of intersection are not clear, add more columns to the tables of values and plot more points.
3. Solve the problem on the board as a class. Ask for volunteers to describe each step. As they describe a step, perform the step on the board.

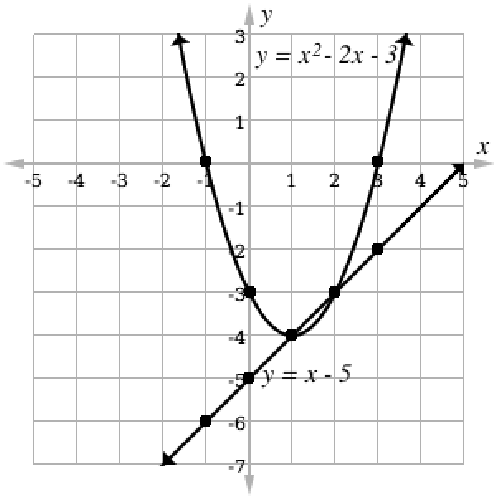
    Solution (tables of values and plotted points may be different):

$y = x^2 - 2x - 3$

x	-1	0	1	2	3
y	0	-3	-4	-3	0

$y = x - 5$

x	-1	0	1	2	3
y	-6	-5	-4	-3	-2



Solutions: $(1, -4)$ and $(2, -3)$

4. Explain: We can use substitution to check our answer.
5. Ask pupils to explain how to solve this problem using substitution. As they describe it, work it on the board.
   Solution:

$x - 5$	$=$	$x^2 - 2x - 3$	Substitute $y = x - 5$ in the quadratic equation
0	$=$	$x^2 - 3x + 2$	Transpose x and -2
$x^2 - 3x + 2$	$=$	0	
$(x - 2)(x - 1)$	$=$	0	Factorise the quadratic equation
$x - 2 = 0$	or	$x - 1 = 0$	Set each factor equal to 0
$x = 2$	or	$x = 1$	Transpose -2 and 1
y	$=$	$(2) - 5$	Substitute $x = 2$ into $y = x - 5$
y	$=$	-3	
y	$=$	$(1) - 5$	Substitute $x = 1$ into $y = x - 5$
y	$=$	-4	

Solutions: $(2, -3)$ and $(1, -4)$

Practice *(20 minutes)*

1. Write the two problems below on the board. Ask pupils to work independently to solve the problems. Allow them to discuss with seatmates if needed.
   Solve the simultaneous equations using graphing **and** substitution:

   a. $y = x^2 + 2x + 1$ (1)
   $y = 2x + 2$ (2)

   b. $y = x^2 - x - 2$ (1)
   $y = -2x + 2$ (2)

2. Walk around to check for understanding. If pupils need support, remind them of the steps or allow them to work with seatmates.
3. Invite a few volunteers to come to the board and write the solutions. Other pupils should check their work.

   Solutions:
   a. **Graphing:**

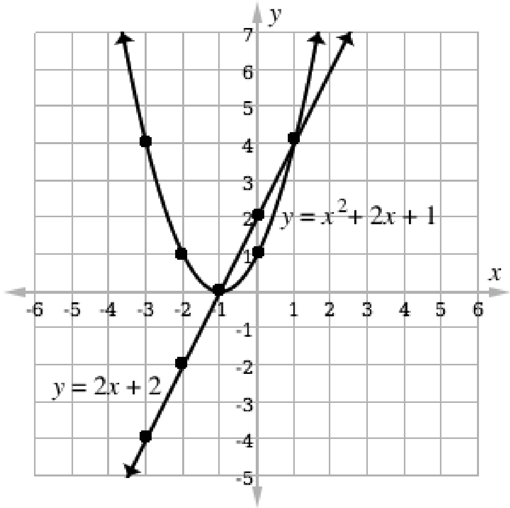
$y = x^2 + 2x + 1$

x	-3	-2	-1	0	1
y	4	1	0	1	4

$y = 2x + 2$

x	-3	-2	-1	0	1
y	-4	-2	0	2	4

Solutions: $(-1, 0)$ and $(1, 4)$



Substitution:

$2x + 2$	$=$	$x^2 + 2x + 1$	Substitute equation (2) into (1)
0	$=$	$x^2 - 1$	Transpose $2x$ and 2
$x^2 - 1$	$=$	0	
$(x + 1)(x - 1)$	$=$	0	Factorise the quadratic equation
$x + 1 = 0$	or	$x - 1 = 0$	Set each factor equal to 0
$x = -1$	or	$x = 1$	Transpose 1 and -1
y	$=$	$2(-1) + 2$	Substitute $x = -1$ into equation (2)
y	$=$	0	
y	$=$	$2(1) + 2$	Substitute $x = 1$ into equation (2)
y	$=$	4	

Solutions: $(-1, 0)$ and $(1, 4)$

b. **Graphing:**

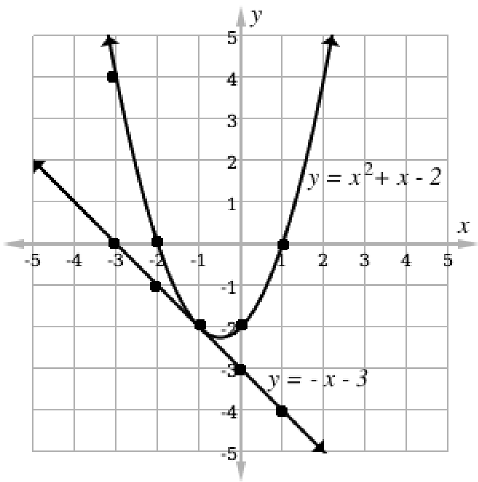
$y = x^2 + x - 2$

x	-3	-2	-1	0	1
y	4	0	-2	-2	0

$y = -x - 3$

x	-3	-2	-1	0	1
y	0	-1	-2	-3	-4

Solution: $(-1, -2)$



Substitution:

$-x-3$	$=$	x^2+x-2	Substitute equation (2) into (1)
0	$=$	x^2+2x+1	Transpose $-x$ and 3
x^2+2x+1	$=$	0	
$(x+1)(x+1)$	$=$	0	Factorise the quadratic equation
$(x+1)^2$	$=$	0	

$x+1=0$ — Set the factor equal to 0

$x=-1$ — Transpose 1

y	$=$	$-(-1)-3$	Substitute $x=-1$ into equation (2)
y	$=$	$1-3=-2$	

Solution: $(-1,-2)$

Closing *(2 minutes)*

1. Discuss: Which method of solving simultaneous linear and quadratic equations do you prefer, graphing or substitution? Why?
2. For homework, have pupils do the practice activity PHM2-L016 in the Pupil Handbook.

Lesson Title: Direct Variation	Theme: Algebraic Processes	
Lesson Number: M2-L017	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to solve numerical and word problems involving direct variation.	**Preparation** None	

Opening *(2 minutes)*

1. Give a simple example of direct variation from everyday life:
   - Tell pupils that the cost of one biscuit is Le 1,000.00
   - Ask pupils to give the cost of 2 biscuits. (Answer: Le 2,000.00)
   - Ask pupils to give the cost of 3 biscuits. (Answer: Le 3,000.00)
2. Tell pupils that with these examples, they have just solved problems related to direct variation, the topic for the day.

Teaching and Learning *(20 minutes)*

1. Explain:
   - The cost of biscuits **varies directly** with the number of biscuits.
   - This is the same as saying that the cost of biscuits **is proportional** *to* the number of biscuits.
   - As the number of biscuits increases, the cost also increases.
2. Write on the board: $C \propto B$
3. Explain:
   - C represents the cost of the biscuits, and B represents the number of biscuits.
   - The symbol $\propto$ represents variance or proportionality. It shows that the two quantities are directly proportional.
4. Write on the board: $C = kB$
5. Explain:
   - k is the constant of proportionality
   - In this case, the constant is the cost of one biscuit. Any time we add on the number of biscuits, the cost increases by Le 1,000.00.
   - In directly proportional relationships, one variable is a constant multiple of the other.
6. Write the following equation on the board with the constant: $C = 1{,}000B$
7. Substitute $B = 2$ (for 2 biscuits) on the board: $C = 1{,}000(2) = 2{,}000$
8. Explain:
   - With this equation we calculated that the cost of 2 biscuits is Le 2,000.00.

9. Ask pupils to use the equation to find the cost of 6 biscuits, and invite one volunteer to write the solution on the board. (Answer: $C = 1{,}000(6) = 6{,}000$ Leones)
10. Write the following problem on the board: If $c \propto n$ and $c = 5$ when $n = 20$, find the formula connecting c and n.
11. Solve on the board, explaining each step:

c	$\propto$	n	Identify the relationship between c and n
c	$=$	kn	
5	$=$	$k(20)$	Substitute known values for c and n
k	$=$	$\frac{5}{20}$	Solve for the constant, k
k	$=$	$\frac{1}{4}$	
c	$=$	$\frac{1}{4}n$	Write the formula

12. Write another problem on the board: If y varies directly as x and $y = 9$ when $x = 3$, find the value of y when $x = 4$.
13. Explain the process to find the solution:
    - Find the equation connecting the two variables.
    - Use the equation to find the missing value of y.
14. Solve on the board, explaining each step:

y	$\propto$	x	Identify the relationship between y and x
y	$=$	kx	
9	$=$	$k(3)$	Substitute known values for y and x
k	$=$	$\frac{9}{3}$	Solve for the constant, k
k	$=$	3	
y	$=$	$3x$	Write the formula
y	$=$	$3(4)$	Find y when $x = 4$
y	$=$	12	

Practice *(13 minutes)*

1. Write the following two problems on the board:
    a. If $M \propto L$ and $M = 6$ when $L = 2$, find:
        i. The equation connecting M and L.
        ii. The value of L when $M = 15$.
    b. A car travels 330 km in 5 hours at a uniform speed. In how many hours will it travel 4290 km? (Distance is directly proportional to time.)
2. Give pupils 10 minutes to solve the problems in their books. If needed, work as a class to set up the equation for question b. on the board before asking them to find the solution.

Closing *(5 minutes)*

1. Invite 3 volunteers to come to the board and simultaneously write their solutions to the practice exercises. Other pupils should check their work.
   Solutions:
   a. If $M \propto L$ and $M = 6$ when $L = 2$, find:
      i. The equation connecting M and L.

$$\begin{aligned} M &\propto L \\ M &= kL \\ 6 &= k(2) \\ k &= 3 \\ M &= 3L \end{aligned}$$

      ii. The value of L when $M = 15$.

$$\begin{aligned} 15 &= 3L \\ L &= \tfrac{15}{3} \\ L &= 5 \end{aligned}$$

   b. A car travels 330 km in 5 hours at a uniform speed. How many hours will it take to travel 4290 km?
      - If needed, remind pupils that because distance and time are directly proportional, we have the formula $d = kt$, where d is distance and t is time.

      Find the formula:

$$\begin{aligned} d &\propto t \\ d &= kt \\ 330 &= k(5) \\ k &= \tfrac{330}{5} = 66 \\ d &= 66t \end{aligned}$$

      Solve for t when $d = 4{,}290$ km:

$$\begin{aligned} 4{,}290 &= 66t \\ t &= \tfrac{4290}{66} \\ t &= 65 \end{aligned}$$

2. For homework, have pupils do the practice activity PHM2-L017 in the Pupil Handbook.

Lesson Title: Inverse Variation	**Theme:** Algebraic Processes	
Lesson Number: M2-L018	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to solve numerical and word problems involving inverse variation.	**Preparation** None	

Opening *(3 minutes)*

1. Review the previous lesson's topic, direct variation.
2. Ask pupils to explain 'direct variation' in their own words. Allow them to discuss this with seatmates. (Example answers: If two variables vary directly, one increases in proportion to an increase in the other. One variable is a constant multiple of the other.)
3. Write on the board: $A \propto B$
4. Ask pupils to describe what this says in words. (Example answers: A varies directly with B. A varies proportionally to B.)
5. Tell pupils that the topic for today is inverse variation.

Teaching and Learning *(25 minutes)*

1. Tell the following story: Foday was hired to brush a big yard. He estimates that it will take him 20 hours in total. However, he really wants to finish in one day. He realises that it's a lot of work for him to do alone.
2. Discuss:
    - If Foday invites one of his friends to help him, there will be 2 people. How long will it take them to brush the yard? (Answer: If his friend works as fast as Foday, it will take them 10 hours.)
    - If Foday invites 3 friends to help, there will be 4 people. How long will it take them to brush the yard? (Answer: If they work as fast as Foday, it will take them 5 hours.)
3. Explain:
    - The number of people brushing the yard **varies inversely** with the amount of time it takes to brush the yard.
    - As the number of people increases, the time it takes to finish the work decreases.
    - If two variables vary inversely, one variable decreases when the other variable increases. Alternately, one variable increases when the other variable decreases. The variables move in opposite directions.
4. Write on the board: $y \propto \frac{1}{x}$
5. Explain:
    - This means that y varies inversely with x, or y is inversely proportional to x.

- We use the same symbol (∝) to show variation.

6. Write on the board: $y = k\frac{1}{x}$ or $y = \frac{k}{x}$
7. Explain:
   - Inverse proportions are shown by this relationship. One value (y) is equal to another value ($\frac{1}{x}$) multiplied by the constant of proportionality (k).
8. Write the following problem on the board:
   If $y \propto \frac{1}{x}$, and $y = 4$ when $x = 2$, find:
   a. The formula that connects x and y.
   b. The value of y when $x = 16$.
9. Solve on the board, explaining each step:
   a.

y	$\propto$	$\frac{1}{x}$	Identify the relationship between y and x
y	$=$	$\frac{k}{x}$	
4	$=$	$\frac{k}{2}$	Substitute known values for y and x
k	$=$	4×2	Solve for the constant, k
k	$=$	8	
y	$=$	$\frac{8}{x}$	Write the formula

   b.

y	$=$	$\frac{8}{16}$	Substitute $x = 16$ into the formula
y	$=$	$\frac{1}{2}$	Simplify

10. Write another problem on the board: Juliet is traveling to Freetown. If she drives at the rate of 90 kph it will take her 2 hours. How long will it take her to reach Freetown if she drives at the rate of 60 kph?
11. Explain: Speed is inversely proportional to time. If Juliet drives faster, it will take less time to reach Freetown. If she drives slower, it will take more time.
12. Write the relationship between speed (s) and time (t) on the board:

$$s \propto \frac{1}{t}$$
$$s = \frac{k}{t}$$

13. Explain the steps to solve the problem:
    - Use the known values ($s = 90$ kph and $t = 2$ hr) to find the constant, k.
    - Then, use the equation to find the time when $s = 60$ kph.
14. Ask pupils to work with seatmates to solve the problem.
15. Invite a group of seatmates to volunteer to write the solution on the board.
    Solution:

90	$=$	$\frac{k}{2}$	Substitute known values for s and t
k	$=$	90×2	Solve for the constant, k

$$k = 180$$

$s = \frac{180}{t}$ Write the formula

$60 = \frac{180}{t}$ Substitute $s = 60$

$$t = \frac{180}{60}$$

$t =$ 3 hrs

16. Explain: If Juliet drives at 60 kph, it will take her 3 hours to reach Freetown. As her speed decreases, the time increases. Speed and time are inversely proportional.

Practice *(10 minutes)*

1. Write the following two problems on the board:
   a. A bag of rice can last 4 people for 30 days. How many days would the rice last if there were 6 people?
   b. 3 pupils can brush the schoolyard in 4 hours. If it needs to be done in 2 hours, how many pupils are needed in total?
2. Give pupils several minutes to solve the problems. If needed, work as a class to set up the equations for the questions before asking them to find the solutions.
3. Invite 2 volunteers to write the solutions on the board.
   Solutions:
   a. Let $p =$ people and $d =$ days.

$p \propto \frac{1}{d}$ Identify the relationship between p and d

$$p = \frac{k}{d}$$

$4 = \frac{k}{30}$ Substitute known values for p and d

$k = 4 \times 30$ Solve for the constant, k

$$k = 120$$

$p = \frac{120}{d}$ Write the formula

$6 = \frac{120}{d}$ Substitute $p = 6$

$$d = 120 \div 6$$

$d =$ 20 days

Answer: A bag of rice will last 6 people for 20 days.

b. Let $p =$ pupils and $t =$ time

$p \propto \frac{1}{t}$ Identify the relationship between p and t

$$p = \frac{k}{t}$$

$3 = \frac{k}{4}$ Substitute known values for p and t

k	$=$	3×4	Solve for the constant, k
k	$=$	12	
p	$=$	$\frac{12}{2}$	Write the formula
p	$=$	6 pupils	

Answer: 6 pupils are needed to brush the yard in 2 hours.

Closing *(2 minutes)*

1. Review direct and inverse variation. Discuss: What is the difference between direct and inverse variation? (Example answer: In direct variation, two values increase together or decrease together. In inverse variation, one value decreases as the other value increases.)
2. For homework, have pupils do the practice activity PHM2-L018 in the Pupil Handbook.

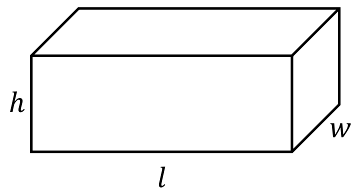
Lesson Title: Joint Variation	**Theme:** Algebraic Processes	
Lesson Number: M2-L019	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to solve numerical and word problems involving joint variation.	**Preparation** None	

Opening *(3 minutes)*

1. Review the previous two topics, direct and inverse variation.
2. Invite a volunteer to write the equation for direct variation on the board. (Example answer: $A = kB$)
3. Invite another volunteer to write the equation for inverse variation on the board. (Example answer: $A = k\frac{1}{B}$ or $A = \frac{k}{B}$)
4. Discuss: When you read a word problem, how can you tell if it deals with direct or inverse variation? (Example answer: If two quantities increase together, it is direct variation. If one quantity decreases as the other increases, it's inverse variation.)
5. Tell pupils that the topic for today is joint variation.

Teaching and Learning *(25 minutes)*

1. Draw the following box on the board. Label its length, width, and height.



2. Tell the following story: Fatu carries ground nuts to the market in a box and sells them. The box she is now using has a volume of 10 litres. She is getting more business everyday, and decides to get a bigger box.
3. Discuss:
   - If Fatu increases the width of her box, what will happen to the volume of the box? (Answer: It will also increase.)
   - If Fatu increases the length of her box, what will happen to the volume of the box? (Answer: It will also increase.)
4. Explain:
   - The volume of Fatu's box is directly proportional to both its length and width. As either increases, the volume of her box increases as well.
   - The volume of her box **varies jointly** with its width and length.
5. Write this relationship on the board: $V \propto lw$ and $V = klw$
6. Explain:

- Joint variation occurs when a variable varies directly or inversely with multiple variables.
- The problems you will work with today have 3 variables.

7. Write on the board and explain:
   - If x varies directly with both y and z, we have $x \propto yz$ or $x = kyz$.
   - If x varies directly with y and inversely with z, we have $x \propto \frac{y}{z}$ or $x = \frac{ky}{z}$.
8. Explain joint variation that involves inverse variation:
   - In some cases, a variable varies directly with another variable and varies inversely with another variable.
   - Write on the board: $s \propto \frac{d}{t}$.
   - For a moving object, its speed varies directly with distance and inversely with time.
9. Write the following problem on the board: z varies jointly with x and y. When $x = 3$, $y = 8$, and $z = 6$. Find z when $x = 6$ and $y = 4$.
10. Explain:
    - When the problem does not say whether it is a direct or inverse variation, assume it is direct.
11. Solve on the board, explaining each step:

z	$\propto$	xy	Identify the relationship between z, x and y
z	$=$	kxy	
6	$=$	$k(3)(8)$	Substitute known values for z, x and y
6	$=$	$24k$	Solve for the constant, k
k	$=$	$\frac{6}{24}$	
k	$=$	$\frac{1}{4}$	
z	$=$	$\frac{1}{4}xy$	Write the formula
z	$=$	$\frac{1}{4}(6)(4)$	Substitute $x = 6$ and $y = 4$ into the formula
z	$=$	$\frac{1}{4}(24)$	Simplify
z	$=$	6	

12. Write another problem on the board: x varies directly with y and inversely with z. If $x = 32$ when $y = 8$ and $z = 4$, find x when $y = 3$ and $z = 2$.
13. Ask pupils to give the relationship between x, y and z. Allow them to discuss with seatmates before inviting a volunteer to write the relationship on the board. (Answer: $x = \frac{ky}{z}$ or $x \propto \frac{y}{z}$)
14. Solve on the board, explaining each step:

x	$=$	$\frac{ky}{z}$	Identify the relationship between x, y and z
32	$=$	$\frac{k(8)}{(4)}$	Substitute known values for x, y and z

$32(4)$	$=$	$28k$	Solve for the constant, k
k	$=$	16	
x	$=$	$\frac{16y}{z}$	Write the formula
z	$=$	$\frac{16(3)}{(2)}$	Substitute $y = 3$ and $z = 2$ into the formula
z	$=$	24	Simplify

15. Write the following word problem on the board: Wind resistance varies jointly as the surface area and velocity of an object. A car with a surface area of 25 feet traveling at 20 kph experiences wind resistance of 250 newtons. If a truck has a surface area of 50 square feet, how fast must it travel in order to experience a wind resistance of 300 newtons?
16. Explain:
    - This question looks very difficult because it handles a topic that most people do not understand well: wind resistance.
    - However, it is not necessary to understand wind resistance or know its formula to solve this problem.
    - Identify the variables in the problem, and find their relationship. Write the equation first, then solve using the given values.
17. Write the variables on the board: R =wind resistance; S =surface area; V =velocity.
18. Ask pupils to give the relationship between R, S and V. Allow them to discuss with seatmates before inviting a volunteer to write the relationship on the board. (Answer: $R = kSV$ or $R \propto SV$)
19. Ask pupils to work with seatmates to find the value of constant k.
20. Invite a volunteer to write the solution on the board:

R	$=$	kSV	Identify the relationship between R, S and V
250	$=$	$k(25)(20)$	Substitute known values for R, S and V
250	$=$	$500k$	Solve for the constant, k
k	$=$	$\frac{1}{2}$	

21. Write the following equation on the board: $R = \frac{1}{2}SV$
22. Ask pupils to work with seatmates to find the value of V when $S = 50$ and $R = 300$ (solve the problem).
23. Invite a volunteer to write the solution on the board:

300	$=$	$\frac{1}{2}(50)V$	Substitute $S = 50$ and $R = 300$
$2(300)$	$=$	$50V$	Simplify
V	$=$	$\frac{600}{50}$	
V	$=$	12 kph	

Practice *(10 minutes)*

1. Write a problem on the board:
   c. The time t taken to withdraw money from a bank varies directly with the number of people P waiting in a queue, and inversely with the number bank employees E. In a bank with 3 employees, it took 30 minutes for 15 people to withdraw their money. How long would it take 40 people to withdraw money if there were 4 employees?
2. Give pupils several minutes to solve the problem. If needed, work as a class to set up the equation before asking them to find the solution.
3. Ask pupils to exchange books with seatmates and check their answers when they finish.
4. Invite a volunteer to write the solution on the board.
   Solution:

t	$=$	$\frac{kP}{E}$	Identify the relationship between t, P and E
30	$=$	$\frac{k(15)}{(3)}$	Substitute known values for t, P and E
$30(3)$	$=$	$15k$	Solve for the constant, k
k	$=$	$\frac{90}{15}$	
k	$=$	6	
t	$=$	$\frac{6P}{E}$	Write the formula
t	$=$	$\frac{6(40)}{(4)}$	Substitute $P = 40$ and $E = 4$ into the formula
t	$=$	60 minutes or 1 hour	Simplify

Closing *(2 minutes)*

1. Review joint variation. Discuss: What are the two types of joint variation? Explain in your own words. (Example answers: In one type, a variable varies directly with two other variables. In the other type, a variable varies directly with one variable and inversely with another variable.)
2. For homework, have pupils do the practice activity PHM2-L019 in the Pupil Handbook.

Lesson Title: Partial Variation	**Theme:** Algebraic Processes	
Lesson Number: M2-L020	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to solve numerical and word problems involving partial variation.	**Preparation** None	

Opening *(3 minutes)*

1. Review the previous topic, joint variation.
2. Discuss: Ask volunteers to explain joint variation in their own words. (Example answers: One quantity varies with two other quantities at the same time.)
3. Ask a volunteer to give the equation used when a variable varies **directly** with two other variables. (Answer: $x \propto yz$ or $x = kyz$)
4. Ask a volunteer to give the equation used when a variable varies **directly** with one variable, and **inversely** with another. (Answer: $x \propto \frac{y}{z}$ or $x = \frac{ky}{z}$)
5. Tell pupils that the topic for today is partial variation.

Teaching and Learning *(20 minutes)*

1. Tell the following story: Mohamed is a taxi driver. The amount that he charges to drive people in his taxi is partly fixed. He charges a base of Le 15,000.00 for each customer he drives. On top of that, he charges Le 1,000.00 for each kilometre he drives them.
2. Discuss:
   - If you call Mohamed to drive you 1 kilometre, how much will he charge you? (Answer: 16,000.00; his fixed charge of 15,000 plus 1,000 for the kilometre.)
   - If you call Mohamed to drive you 2 kilometres, how much will he charge you? (Answer: 17,000.00; his fixed charge of 15,000 plus 2,000 for the 2 kilometres.)
3. Write each answer on the board as an equation:

   1 kilometre: $15{,}000 + 1{,}000 = 16{,}000.00$

   2 kilometres: $15{,}000 + 2(1{,}000) = 17{,}000.00$
4. Explain:
   - The cost of riding in Mohamed's taxi is partly constant. The Le 15,000 is always constant in the equation.
   - The cost also partly varies based on the distance he drives. To find this part, we multiply the number of kilometres by his rate per kilometre.
5. Write on the board: $C = k_1 + kd$, where C is the cost, d is the distance driven, and k and k_1 are the constants.
6. Explain: This is the formula we use for partial variation. In all partial variation problems, there is a constant amount and an amount that varies.

7. Write the following problem on the board: Fatu hired Mohamed to take her to the market with her goods. His base rate is Le 15,000.00, and he charges an additional Le 1,000.00 per kilometre. If she traveled 7 kilometres, how much did she pay him?
8. Ask volunteers to assign numbers from the problem to the variables in the formula ($C = k_1 + kd$). Write these on the board. (Answers: $k_1 = 15{,}000$, $k = 1{,}000$, $d = 7$)
9. Solve the problem on the board, explaining each step.
Solution:

C	$=$	$k_1 + kd$	
	$=$	$15{,}000 + (1{,}000)(7)$	Substitute k_1, k, and d
	$=$	$15{,}000 + 7{,}000$	Simplify
	$=$	Le $22{,}000.00$	

10. Explain: The total amount that Fatu paid is Le 22,000.00.
11. Write another problem on the board: Mohamed drove one passenger 13 kilometres to reach the football stadium. How much did he charge?
12. Ask pupils to work with seatmates to solve the problem.
13. Invite a volunteer to write the solution on the board. All other pupils should check their work.
Solution:

C	$=$	$k_1 + kd$	
	$=$	$15{,}000 + (1{,}000)(13)$	Substitute k_1, k, and d
	$=$	$15{,}000 + 13{,}000$	Simplify
	$=$	Le $28{,}000.00$	

14. Explain: Mohamed charged Le 28,000.00 for 13 kilometres.
15. Write another problem on the board: x is partly constant and partly varies as y. When $y = 2$, $x = 20$, and when $y = 5$, $x = 35$.
    a. Find the relationship between x and y.
    b. Find x when $y = 4$.
16. Write the relationship on the board with variables: $x = k_1 + ky$
17. Explain:
    - We know some of the values of x and y. We need to find the values of the constants, k_1 and k, to write the relationship between x and y.
    - When you see this type of partial variation problem, you should recognise that you will be solving simultaneous equations.
18. Solve part a. on the board, explaining each step:
Step 1: Substitute the given values of x and y into $x = k_1 + ky$, which gives two equations:

x	$=$	$k_1 + ky$		
20	$=$	$k_1 + 2k$	(1)	Substitute $y = 2$ and $x = 20$
35	$=$	$k_1 + 5k$	(2)	Substitute $y = 5$ and $x = 35$

Step 2: Solve the simultaneous equations by subtracting (1) from (2):

$$\begin{array}{rcll} 35 & = & k_1 + 5k & (2) \\ -(20 & = & k_1 + 2k) & (1) \quad \text{Subtract each term} \\ \hline 15 & = & 3k & \end{array}$$

$$\begin{array}{rcll} \frac{15}{3} & = & \frac{3k}{3} & \text{Divide throughout by 3} \\ 5 & = & k & \end{array}$$

$$\begin{array}{rcll} 20 & = & k_1 + 2(5) \quad (1) & \text{Substitute } k = 5 \text{ into (1)} \\ 20 & = & k_1 + 10 & \text{Simplify} \\ 20 - 10 & = & k_1 & \text{Transpose 10} \\ 10 & = & k_1 & \end{array}$$

Step 3: Write the relationship: $x = 10 + 5y$

19. Solve part b. on the board, explaining each step:

$$\begin{array}{rcll} x & = & 10 + 5y & \text{Relationship} \\ x & = & 10 + 5(4) & \text{Substitute } y = 4 \\ x & = & 10 + 20 & \text{Simplify} \\ x & = & 30 & \end{array}$$

Practice *(15 minutes)*

1. Write the 2 problems below on the board:
   a. The cost of making a dress is partly constant and partly varies with time. The fabric has a constant cost of Le 25,000.00. The tailor charges Le 10,000.00 per hour of work.
      i. Find the relationship, using C for total cost and t for time.
      ii. If the dress takes 3 hours to make, what is the total cost?
   b. P is partly constant and partly varies as Q. When $Q = 4$, $P = 13$, and when $Q = 10$, $P = 25$.
      i. Find the relationship between P and Q.
      ii. Find P when $Q = 7$.
2. Give pupils several minutes to solve the problems. If needed, work as a class to set up the equations before asking them to find the solution.
3. Ask pupils to exchange books with their classmates and check their answers when they finish.
4. Invite volunteers to write the solutions on the board.

   Solutions:

   a.
   i. The relationship is $C = 25{,}000 + 10{,}000t$
   ii. Total cost is Le 55,000:

$$C = 25{,}000 + 10{,}000(3) \qquad \text{Substitute } t = 3$$

$$= \quad 25{,}000 + 30{,}000 \qquad \text{Simplify}$$
$$= \quad 55{,}000$$

b.

i. Find the relationship between P and Q:

Step 1: Substitute the given values of P and Q into $P = k_1 + kQ$, which gives two equations:

P	$=$	$k_1 + kQ$		
13	$=$	$k_1 + 4k$	(1)	Substitute $P = 13$ and $Q = 4$
25	$=$	$k_1 + 10k$	(2)	Substitute $P = 25$ and $Q = 10$

Step 2: Solve the simultaneous equations by subtracting (1) from (2):

25	$=$	$k_1 + 10k$	(2)	
$-(\ 13$	$=$	$k_1 + 4k)$	(1)	Subtract each term
12	$=$	$6k$		
$\frac{12}{6}$	$=$	$\frac{6k}{6}$		Divide throughout by 6
2	$=$	k		
13	$=$	$k_1 + 4(2)$	(1)	Substitute $k = 2$ into (1)
13	$=$	$k_1 + 8$		Simplify
$13 - 8$	$=$	k_1		Transpose 8
5	$=$	k_1		

Step 3: Write the relationship: $P = 5 + 2Q$

i. Find P when $Q = 7$.

P	$=$	$5 + 2Q$	Relationship
P	$=$	$5 + 2(7)$	Substitute $y = 4$
P	$=$	$5 + 14$	Simplify
P	$=$	19	

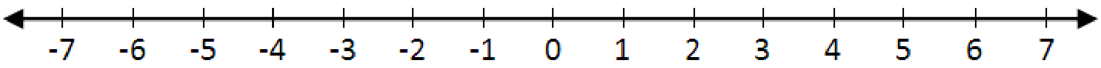
Closing *(2 minutes)*

1. Review partial variation. Discuss: What is partial variation? Explain in your own words. (Example answers: In partial variation, one quantity remains constant while another quantity varies.)
2. For homework, have pupils do the practice activity PHM2-L020 in the Pupil Handbook.

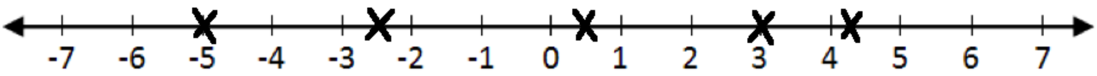
Lesson Title: Inequalities on a Number Line	**Theme:** Algebraic Processes	
Lesson Number: M2-L021	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to represent inequalities in one variable on a number line.	**Preparation** Draw three number lines for use in the Opening, Teaching and Learning, and Practice on the board.	

Opening *(3 minutes)*

1. Review the number line. Draw the following number line from -7 to 7 on the



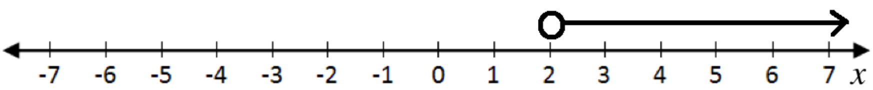
   board:
2. Give the numbers below, and invite volunteers to come to the board to mark them: $-5, 3, 0.5, -2.5, 4.2$
   Answers:



3. Ask comparison questions and discuss. For example: Which is greater, -5 or 4.2? (Answer: 4.2)
4. Remind pupils that numbers to the right on a number line are greater, and numbers to the left are less.
5. Tell pupils that today we will represent inequalities on a number line.

Teaching and Learning *(20 minutes)*

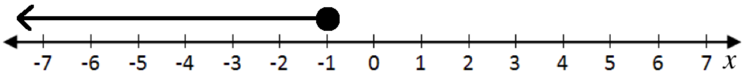
1. Write on the board: $x > 2$
2. Ask pupils to describe this expression with words. (Answer: "x is greater than 2")
3. Ask pupils to describe the meaning of this expression in their own words. (Example answer: x is an unknown variable, and it could be any number greater than 2.)
4. Explain: We can show this inequality on the number line.
5. Draw the diagram shown below on the board:



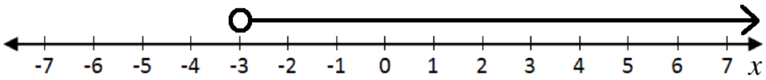
6. Explain:
   - This number line shows $x > 2$.
   - Use an open circle to show that 2 is not included. x cannot be equal to 2.
   - Draw an arrow pointing to the right to show that x could be any number to the right of 2. All numbers to the right of 2 are greater than 2.
   - Write "x" near the number line to show that this number line represents the possible values of x.

- The solution to an inequality can be called the "truth set".

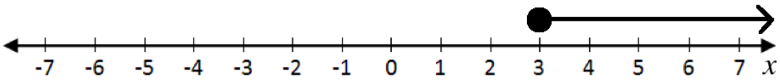
7. Write the following on the board: Show the truth set of $x \leq -1$ on a number line.
8. Draw the diagram shown below on the board:



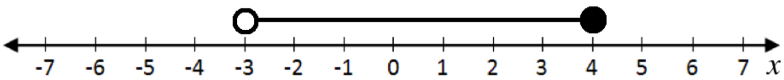
9. Explain:
    - The arrow is pointing to the left to show that x can be any value less than 1.
    - The circle is filled in because x can be equal to 1.
    - If the sign is "less than or equal to" or "greater than or equal to" the circle is filled in.
10. Draw the diagram shown below on the board:



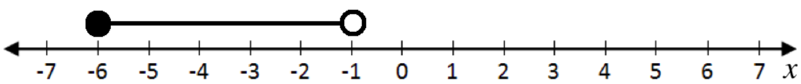
11. Ask pupils to work with seatmates to write the expression for this diagram. Invite a volunteer to write the answer on the board. (Answer: $x > -3$)
12. Write on the board: $x \geq 3$
13. Ask pupils to work with seatmates to draw the number line for this expression. Invite a volunteer to draw their answer on the board. (Answer: below)



14. Write on the board: $-3 < x \leq 4$
15. Ask pupils to explain in their own words the meaning of this expression. (Example answer: x can be any number greater than 3, but less than or equal to 4)
16. Draw the diagram below on the board:



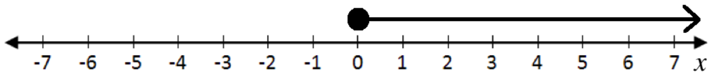
17. Write the following example on the board: $-6 \leq x < -1$
18. Ask pupils to work with seatmates to draw the number line for this expression. Invite a volunteer to draw their answer on the board. (Answer: below)



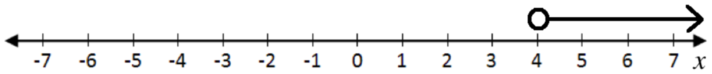
Practice *(15 minutes)*

1. Write the following problems on the board:
    a. Write the expression shown on each of the number lines:

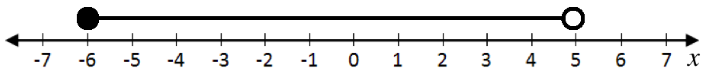
i.



ii.



iii.



b. Draw a number line showing the truth set of each expression:

i. $x > -3$

ii. $x \leq 1$

iii. $-2 < x \leq 2$

2. Ask pupils to solve the problems in their exercise books.
3. Ask pupils to exchange books with classmates and check their answers when they finish.
4. Invite 6 volunteers to each write a solution on the board.

Solutions:

a. Write the expression shown on each of the number lines:

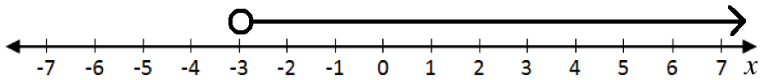
i. $x \geq 0$

ii. $x > 4$

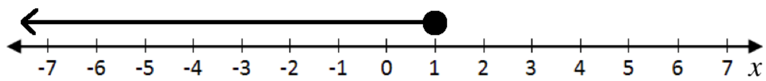
iii. $-6 \leq x < 5$

b. Draw a number line showing the truth set of each expression:

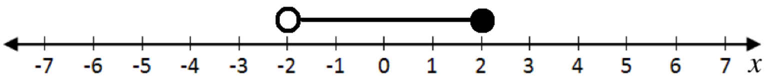
i.



ii.



iii.



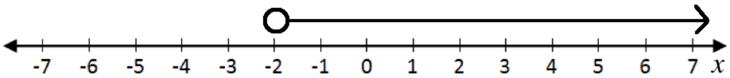
Closing *(2 minutes)*

1. Have a discussion to review this lesson. For example:
   a. How do we show "greater than or equal to" on a number line? (Answer: With a filled circle and an arrow to the right.)
   b. How do we show "less than" on a number line? (Answer: With an empty circle and an arrow to the left.)
2. For homework, have pupils do the practice activity PHM2-L021 in the Pupil Handbook.

Lesson Title:		
Lesson Title: Solutions of Inequalities	**Theme:** Algebraic Processes	
Lesson Number: M2-L022	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to solve inequalities on one variable.	**Preparation** Write the problem in the Opening on the board.	

Opening *(3 minutes)*

1. Review the previous lesson. Write the following problem on the board: Show the truth set of $x > -2$ on a number line.
2. Ask pupils to draw the answer in their exercise books.
3. Invite a volunteer to draw the answer on the board.
   Answer:



4. Tell pupils that today they will solve inequalities in one variable. They will be able to draw the solutions on a number line.

Teaching and Learning *(20 minutes)*

1. Write the following example on the board: $x + 3 > 5$
2. Ask pupils to describe the meaning of this expression in their own words. (Example answers: x is an unknown variable; when we add 3 to x the result will always be greater than 5)
3. Explain: To solve an inequality for a variable, you may add or subtract a number from both sides of the inequality. The inequality symbol does not change.
4. Solve the inequality on the board:

$$\begin{aligned} x + 3 &> 5 \\ x &> 5 - 3 \qquad \text{Transpose 3} \\ x &> 2 \end{aligned}$$

5. Write the following example on the board: $y - 12 \leq 3$
6. Ask pupils to work with seatmates to find the solution. Invite a volunteer to write the solution on the board.
   Solution:

$$\begin{aligned} y - 12 &\leq 3 \\ y &\leq 3 + 12 \qquad \text{Transpose } -12 \\ y &\leq 15 \end{aligned}$$

7. Write the following examples on the board: a) $3x > 12$ and b) $-3x > 12$
8. Ask for volunteers to describe the difference between these two expressions. (Answer: expression (a) has a positive 3 and expression (b) has a negative 3)

9. Explain:
    - To solve an inequality, you may multiply or divide both sides by a positive number. The sign of the inequality does not change.
    - You may also multiply or divide both sides by a **negative** number, but the direction of the inequality is **reversed**. Greater than becomes less than, and less than becomes greater than.
10. Write the solutions for the two inequalities on the board:

    Solution a.

$$\begin{aligned} 3x &> 12 \\ \frac{3x}{3} &> \frac{12}{3} && \text{Divide throughout by 3} \\ x &> 4 \end{aligned}$$

Solution b.

$$\begin{aligned} -3x &> 12 \\ \frac{-3x}{-3} &< \frac{12}{-3} && \text{Divide throughout by } -3 \text{ (reverse the direction of the inequality)} \\ x &< -4 \end{aligned}$$

11. Write the following example on the board: $-2x + 9 > 13$
12. Ask pupils to explain the steps needed to solve this. Ask for volunteers to share their ideas.
13. Explain:
    - Remember that when solving for a variable, we perform addition and subtraction first. Then we perform multiplication and division.
    - The first step is to subtract 9 from both sides. Then, we divide both sides by -2. Remember to change the direction of the inequality.
14. Ask pupils to solve the problem with seatmates. Invite a volunteer to write the solution on the board.

    Solution:

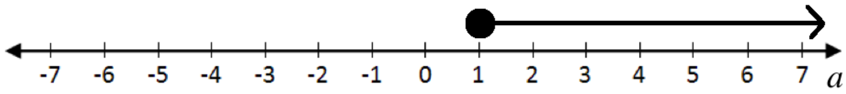
$$\begin{aligned} -2x + 9 &> 13 \\ -2x &> 13 - 9 && \text{Transpose 9} \\ -2x &> 4 \\ \frac{-2x}{-2} &< \frac{4}{-2} && \text{Divide throughout by } -2 \text{ (reverse the direction of the inequality)} \\ x &< -2 \end{aligned}$$

15. Write the following problem on the board: Solve $12a - 3 \geq 2a + 7$. Show the solution on a number line.
16. Explain: When you have a variable on both sides of the equation, solve in the same way that you would if there was an equals sign. $2a$ can be transposed or subtracted from both sides of the inequality.

17. Ask pupils to work in groups to find the solution. Invite one volunteer to write the solution on the board and another volunteer to draw the number line.

Solution:

$$
\begin{aligned}
12a - 3 &\geq 2a + 7 \\
12a - 2a - 3 &\geq 7 && \text{Transpose } 2a \\
10a - 3 &\geq 7 \\
10a &\geq 7 + 3 && \text{Transpose } -3 \\
10a &\geq 10 \\
\frac{10a}{10} &\geq \frac{10}{10} && \text{Divide throughout by } 10 \\
a &\geq 1
\end{aligned}
$$



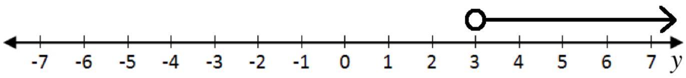
Practice *(15 minutes)*

1. Write the following problems on the board:
   Solve the following inequalities. Show each solution on a number line:
   a. $5y + 3 > 18$
   b. $-2x - 4 \leq 10$
   c. $3b - 4 \geq b + 6$
2. Ask pupils to solve the problems in their exercise books.
3. Ask pupils to exchange books with seatmates and check their answers when they finish.
4. Invite 3 volunteers to each write a solution on the board, and 3 volunteers to draw the number lines. They may write on the board simultaneously to save time.

Solutions:

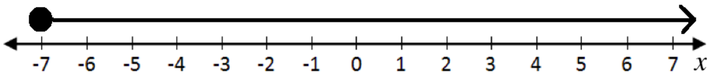
a.

$$
\begin{aligned}
5y + 3 &> 18 \\
5y &> 18 - 3 && \text{Transpose 3} \\
5y &> 15 \\
\frac{5y}{5} &> \frac{15}{5} && \text{Divide throughout by 5} \\
y &> 3
\end{aligned}
$$



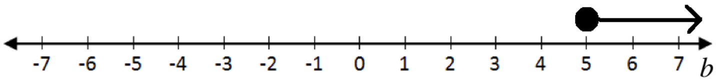
b.

$$\begin{array}{rcll} -2x-4 & \leq & 10 & \\ -2x & \leq & 10+4 & \text{Transpose - 4} \\ -2x & \leq & 14 & \\ \frac{-2x}{-2} & \geq & \frac{14}{-2} & \text{Divide throughout by } -2 \text{ (reverse the direction of the inequality)} \\ x & \geq & -7 & \end{array}$$



c.

$$\begin{array}{rcll} 3b-4 & \geq & b+6 & \\ 3b-b-4 & \geq & 6 & \text{Transpose } b \\ 2b-4 & \geq & 6 & \\ 2b & \geq & 6+4 & \text{Transpose } -4 \\ 2b & \geq & 10 & \\ \frac{2b}{2} & \geq & \frac{10}{2} & \text{Divide throughout by } 2 \\ b & \geq & 5 & \end{array}$$



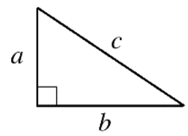
Closing *(2 minutes)*

1. Have a discussion to review this lesson. For example:
    a. What happens to the direction of the inequality if you divide throughout by a **positive** number? (Answer: Its direction does **not change**.)
    b. What happens to the direction of the inequality if you divide throughout by a **negative** number? (Answer: Its direction **reverses**.)
2. For homework, have pupils do the practice activity PHM2-L022 in the Pupil Handbook.

Lesson Title: Distance Formula	**Theme:** Algebraic Processes	
Lesson Number: M2-L023	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to apply the distance formula to find the distance from one point to another on a line.	**Preparation** Draw the right-angled triangle in the Opening, and a coordinate plane in Teaching and Learning on the board.	

Opening *(4 minutes)*

1. Review Pythagoras' Theorem. Draw the right-angled triangle shown below on the board:



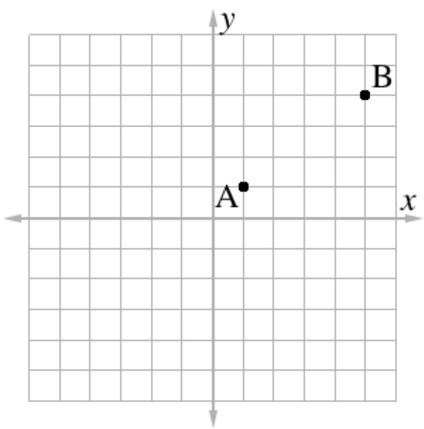
2. Ask pupils to write Pythagoras' Theorem in their exercise books.
3. Invite a volunteer to write the answer on the board. (Answer: $a^2 + b^2 = c^2$)
4. Write the following problem on the board: if $a = 6$ and $b = 8$, what is the length of c?
5. Ask pupils to find the solution in their exercise books.
6. Invite a volunteer to write the solution on the board:

$$\begin{aligned} (6)^2 + (8)^2 &= c^2 && \text{Substitute } a = 6 \text{ and } b = 8 \\ 36 + 64 &= c^2 && \text{Simplify} \\ 100 &= c^2 \\ \sqrt{100} &= \sqrt{c^2} && \text{Take the square root of both sides} \\ c &= 10 \end{aligned}$$

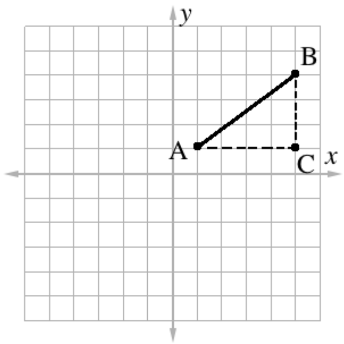
7. Tell pupils that today they will use the distance formula to find the distance between two points. They will see how this is related to Pythagoras' theorem.

Teaching and Learning *(25 minutes)*

1. Draw the coordinate plane on the board with points A $(1, 1)$ and B $(5, 4)$, as shown below:



2. Ask pupils to guess or estimate the distance between point A and point B. Allow pupils to discuss and write their guesses on the board. (For example, Guesses: 3, 4.5, 7)
3. Draw a line connecting A and B, and dotted lines to connect points A and B to point C $(5, 1)$. Label this as shown:



4. Explain:
   - I have drawn a right-angled triangle connecting points A and B.
   - The distance between A and B is the hypotenuse of my triangle. I can find the length of this hypotenuse using Pythagoras' Theorem.
   - To use Pythagoras' Theorem, I must first find the lengths of the other sides of the triangle, BC and AC.
5. Ask volunteers to give the lengths of the other 2 sides of the triangle. They can do this by looking at the points on the Cartesian plane and counting. (Answer: $|BC| = 3$ and $|AC| = 4$)
6. Explain: We can also calculate these distances using subtraction. As the triangle gets bigger, it will be easier to use subtraction than to count.
7. Write the following on the board:

   $|BC| = y_2 - y_1$
   $|AC| = x_2 - x_1$

   Where A is (x_1, y_1) and B is (x_2, y_2)

8. Explain: When calculating the distance between two points, subtract their y-values to find the length of one side of the triangle. Subtract their x-values to find the length of the other side.
9. Calculate the lengths of BC and AC on the board using the formulae:
   Since A is $(1, 1)$ and B is $(5, 4)$:

   $|BC| = 4 - 1 = 3$
   $|AC| = 5 - 1 = 4$

10. Explain: Now that we know the lengths of these two sides, we can use Pythagoras' Theorem to find the length of AB.
11. Solve on the board:

$\lvert BC\rvert^2 + \lvert AC\rvert^2$	$=$	$\lvert AB\rvert^2$	
$3^2 + 4^2$	$=$	$\lvert AB\rvert^2$	Substitute $\lvert BC\rvert = 3$ and $\lvert AC\rvert = 4$
$9 + 16$	$=$	$\lvert AB\rvert^2$	Simplify
25	$=$	$\lvert AB\rvert^2$	

$$\sqrt{25} = \sqrt{|AB|^2} \quad \text{Take the square root of both sides}$$
$$5 = |AB|$$

12. Tell pupils that the distance between A and B is 5.
13. Explain: We can write the distance formula and simply substitute the coordinates of any 2 points to find the distance between them.
14. Write the distance formula on the board:

$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ for any points A(x_1, y_1) and B(x_2, y_2)

15. Write the following problem on the board: Find the length of the line joining C $(0, -2)$ and D $(5, 10)$
16. Solve on the board, explaining each step to pupils:

$\lvert CD \rvert$	$=$	$\sqrt{(5-0)^2 + (10-(-2))^2}$	Substitute x- and y-values
	$=$	$\sqrt{5^2 + (10+2)^2}$	Simplify
	$=$	$\sqrt{5^2 + 12^2}$	
	$=$	$\sqrt{25 + 144}$	
	$=$	$\sqrt{169}$	
	$=$	13	

17. Write the following problem on the board: Find the distance d between (-1, 4) and (2, 6).
18. Ask pupils to work in groups to find the solution. If pupils need support, work as a class to substitute the values. Then, ask them to simplify.
19. Invite a volunteer to write the solution on the board.
Solution:

d	$=$	$\sqrt{(2-(-1))^2 + (6-4)^2}$	Substitute x- and y-values
	$=$	$\sqrt{(2+1)^2 + 2^2}$	Simplify
	$=$	$\sqrt{3^2 + 2^2}$	
	$=$	$\sqrt{9 + 4}$	
	$=$	$\sqrt{13}$	The answer cannot be simplified further

Practice *(10 minutes)*

1. Write the following 2 problems on the board:
   d. Calculate the distance between S (1, 2) and T (4, 6)
   e. Find the length of the line joining (-1, -4) and (-3, -2)
2. Ask pupils to solve the problems in their exercise books.

3. Invite 2 volunteers to each solve a problem on the board. Other pupils should check their work.

   Solutions:

   a.

$\|ST\|$	$=$	$\sqrt{(4-1)^2+(6-2)^2}$	Substitute x- and y-values
	$=$	$\sqrt{3^2+4^2}$	Simplify
	$=$	$\sqrt{9+16}$	
	$=$	$\sqrt{25}$	
	$=$	5	

   b.

d	$=$	$\sqrt{(-3-(-1))^2+(-2-(-4))^2}$	Substitute x- and y-values
	$=$	$\sqrt{(-3+1)^2+(-2+4)^2}$	Simplify
	$=$	$\sqrt{(-2)^2+2^2}$	
	$=$	$\sqrt{4+4}$	
	$=$	$\sqrt{8}$	
	$=$	$\sqrt{4\times 2}$	Review surds if needed
	$=$	$\sqrt{4}\times\sqrt{2}$	
	$=$	$2\sqrt{2}$	

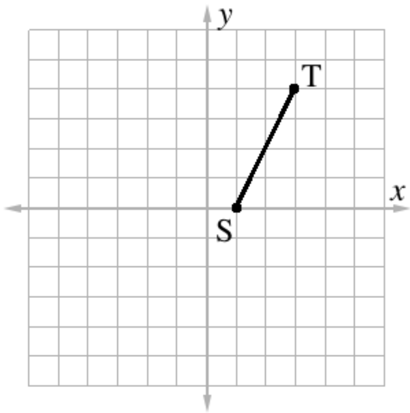
Closing *(1 minute)*

1. Have a discussion to review this lesson. Ask pupils to explain in their own words how Pythagoras' Theorem is related to the distance formula.
2. For homework, have pupils do the practice activity PHM2-L023 in the Pupil Handbook.

Lesson Title: Mid-point Formula	**Theme:** Algebraic Processes	
Lesson Number: M2-L024	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to apply the mid-point formula to find the mid-point of a line.	**Preparation** Draw the graph in the Opening on the board.	

Opening *(3 minutes)*

1. Draw the graph shown below on the board:



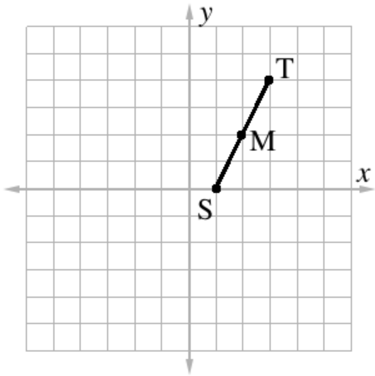
2. Explain: The topic of this lesson is finding the midpoint of a line using a formula. The midpoint is the point that is **exactly** midway, or in the middle, of two other points.
3. Ask pupils to guess the coordinates of the midpoint of line ST. Write their guesses on the board. (For example, Guesses: (1, 2), (2, 2))
4. Invite a volunteer to try drawing the midpoint M in exactly the middle of line ST.
5. Explain that although it is easy to find the midpoint of a short line by counting, we need to use a formula to find the midpoint of most lines.

Teaching and Learning *(20 minutes)*

1. Write the midpoint formula on the board: The midpoint of two points (x_1, y_1) and (x_2, y_2) is the point M found by the following formula:
$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$
2. Find the midpoint M of the line on the board (ST) using the formula.
   - Ask pupils to give the coordinates of S and T. Write the coordinates on the board. (Answer: S (1, 0), T (3, 4))
   - Solve for M, explaining each step:

$$\begin{aligned} M &= \left(\tfrac{1+3}{2}, \tfrac{0+4}{2}\right) && \text{Substitute } x\text{- and } y\text{-values} \\ &= \left(\tfrac{4}{2}, \tfrac{4}{2}\right) && \text{Simplify} \\ &= (2, 2) \end{aligned}$$

3. Explain that the midpoint of line ST is exactly $(2, 2)$. Make sure the point M is plotted at $(2, 2)$ on the board:



4. Write the following problem on the board: Find the midpoint M between $(-1, 4)$ and $(3, 6)$.
5. Solve the problem on the board, explaining each step:

M	$=$	$\left(\frac{-1+3}{2}, \frac{4+6}{2}\right)$	Substitute x- and y-values
	$=$	$\left(\frac{2}{2}, \frac{10}{2}\right)$	Simplify
	$=$	$(1, 5)$	

6. Write the following problem on the board: Find the midpoint of XY if the coordinates of X are $(-2, -3)$ and the coordinates of Y are $(4, 7)$.
7. Ask pupils to work with seatmates to find the solution.
8. Invite a volunteer to write the solution on the board.
   Solution:

M	$=$	$\left(\frac{-2+4}{2}, \frac{-3+7}{2}\right)$	Substitute x- and y-values
	$=$	$\left(\frac{2}{2}, \frac{4}{2}\right)$	Simplify
	$=$	$(1, 2)$	

9. Write the following problem on the board: Find the value of p such that $(-3, 6)$ is the midpoint between A$(-2, p)$ and B$(-4, 4)$.
10. Discuss: Ask pupils to share their ideas about how to solve this problem.
11. Write the solution on the board and explain each step:

M	$=$	$(-3, 6)$	← Use this fact
$(-3, 6)$	$=$	$\left(\frac{-2+(-4)}{2}, \frac{p+4}{2}\right)$	Apply the midpoint formula
$(-3, 6)$	$=$	$\left(\frac{-6}{2}, \frac{p+4}{2}\right)$	Simplify
$(-3, 6)$	$=$	$\left(-3, \frac{p+4}{2}\right)$	

Note that the x-coordinates already match. We know the y-coordinates are equal to each other too. Set them equal and solve for p:

6	$=$	$\frac{p+4}{2}$	
$2(6)$	$=$	$2\left(\frac{p+4}{2}\right)$	Multiply both sides by 2

$$
\begin{aligned}
12 &= p + 4 && \text{Transpose 4} \\
12 - 4 &= p \\
p &= 8
\end{aligned}
$$

Practice *(15 minutes)*

1. Write the following 3 problems on the board:
   a. Calculate the midpoint P of Q (1, 5) and R (7, 11)
   b. Find the midpoint M of (-8, -9) and (0, -15).
   c. Find the value of q such that $(1, 5)$ is the midpoint between A $(-2, 3)$ and B $(q, 7)$.
2. Ask pupils to solve the problems in their exercise books.
3. Invite 3 volunteers to each write a solution on the board. Other pupils should check their work.

   Solutions:

a.

$$
\begin{aligned}
P &= \left(\frac{1+7}{2}, \frac{5+11}{2}\right) && \text{Substitute x- and y-values} \\
&= \left(\frac{8}{2}, \frac{16}{2}\right) && \text{Simplify} \\
&= (4, 8)
\end{aligned}
$$

b.

$$
\begin{aligned}
P &= \left(\frac{-8+0}{2}, \frac{-9+(-15)}{2}\right) && \text{Substitute x- and y-values} \\
&= \left(\frac{-8}{2}, \frac{-24}{2}\right) && \text{Simplify} \\
&= (-4, -12)
\end{aligned}
$$

c.

$$
M = (1, 5) \qquad \leftarrow \text{Use this fact}
$$

$$
\begin{aligned}
(1, 5) &= \left(\frac{-2+q}{2}, \frac{3+7}{2}\right) && \text{Apply the midpoint formula} \\
(1, 5) &= \left(\frac{-2+q}{2}, \frac{10}{2}\right) && \text{Simplify} \\
(1, 5) &= \left(\frac{-2+q}{2}, 5\right)
\end{aligned}
$$

Set the x-coordinates equal and solve for q:

$$
\begin{aligned}
1 &= \frac{-2+q}{2} \\
2(1) &= 2\left(\frac{-2+q}{2}\right) && \text{Multiply both sides by 2} \\
2 &= -2 + q && \text{Transpose } -2 \\
2 + 2 &= q \\
q &= 4
\end{aligned}
$$

Closing *(2 minutes)*

1. Ask a volunteer to share the midpoint formula. Ask a second volunteer to share what the midpoint formula is used for.
2. For homework, have pupils do the practice activity PHM2-L024 in the Pupil Handbook.

Lesson Title: Gradient of a Line	**Theme:** Algebraic Processes	
Lesson Number: M2-L025	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcomes By the end of the lesson, pupils will be able to: 1. Find the gradient of a line using two points. 2. The formula $m = \frac{y_2 - y_1}{x_2 - x_1}$.	**Preparation** Write the problem in Opening, and draw a coordinate plane for Teaching and Learning and Practice on the board.	

Opening *(3 minutes)*

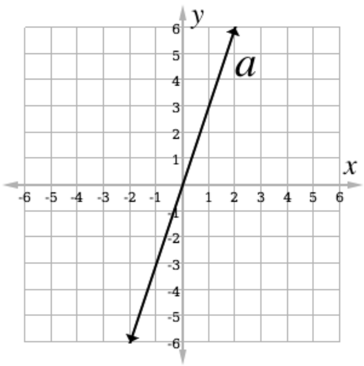
1. Review the previous lesson by writing the following problem on the board: Find the midpoint M of RS if R is (1, 5) and S is (3, 9).
2. Ask pupils to solve the problem independently.
3. Invite a volunteer to write the solution on the board. Other pupils should check their work.
   Solution:

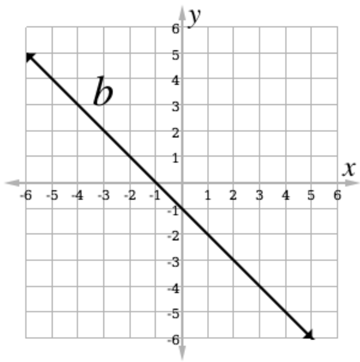
$$M = \left(\frac{1+3}{2}, \frac{5+9}{2}\right) \quad \text{Substitute } x\text{- and } y\text{-values}$$
$$= \left(\frac{4}{2}, \frac{14}{2}\right) \quad \text{Simplify}$$
$$= (2, 7)$$

4. Explain that the topic for today is also related to lines. Pupils will learn how to find the gradient.

Teaching and Learning *(25 minutes)*

1. Graph two lines (a and b) on the board, as shown:

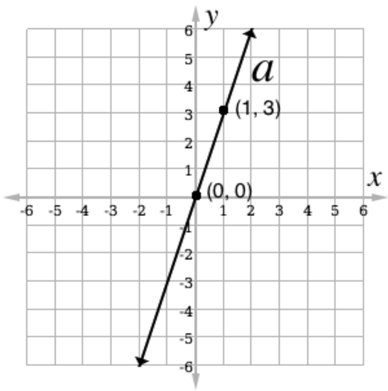




2. Explain:
   - Gradient is a number that tells us how steep a line is, and in which direction it increases.
   - Line a increases as it goes to the **right**, or the positive x-direction. Line a has a **positive** gradient.

- Line b increases as it goes to the **left**, or the negative x-direction. Line b has a **negative** gradient.

3. Label lines a and b on the board with their gradients (3 and -1, respectively).
4. Explain:
   - Line a is steep. It has a gradient of +3. Line b is not as steep as line a. It has a gradient of -1.
   - The greater the absolute value of the gradient, the steeper the line is.
5. Write on the board: $gradient = \frac{rise}{run} = \frac{change\ in\ y}{change\ in\ x}$
6. Explain:
   - We can calculate the gradient of any line using any 2 points on the line.
   - After choosing 2 points, find the change in y-values or how much the line "rises" up. Find the change in x-values or how much the line "runs" to the left or right.
   - To calculate the gradient, divide the change in y by the change in x. In other words, calculate the "rise over run".
7. Write the coordinates for two points on line a on the board: (0, 0) and (1, 3). Plot these points:



8. Ask pupils to observe how much the line "rises" (or the difference in y) between these 2 points. (Answer: 3)
9. Ask pupils to observe how much the line "runs" (or the difference in x) between these 2 points. (Answer: 1)
10. Write the following on the board: $gradient = \frac{rise}{run} = \frac{3}{1} = 3$
11. Explain:
    - In this case, it was easy to observe that the gradient is 3. In other cases, we must calculate the gradient with a formula.
    - The formula for the gradient is helpful because it can be used for any 2 points on a line, even if they are far apart.
12. Write the following on the board: Gradient m is given by the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ for any 2 points (x_1, y_1) and (x_2, y_2) on a line.
13. Calculate the gradient of line b using the formula. Write the coordinates on the board for two points on a line to use in the formula.

    $(x_1, y_1) = (0, -1)$

$(x_2, y_2) = (-2, 1)$

14. Substitute these coordinates into the formula and calculate the gradient:

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{1-(-1)}{-2-0} && \text{Substitute } x\text{- and } y\text{-values} \\ &= \frac{1+1}{-2} && \text{Simplify} \\ &= \frac{2}{-2} \\ &= -1 \end{aligned}$$

15. Write the following problem on the board: Calculate the gradient of the line passing through $(-3, -5)$ and $(5, 11)$.
16. Ask pupils to assign values to each of the x- and y-coordinates needed for the equation. (Answer: $(x_1, y_1) = (-3, -5)$ and $(x_2, y_2) = (5, 11)$)
17. Solve the problem on the board, explaining each step:

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{11-(-5)}{5-(-3)} && \text{Substitute } x\text{- and } y\text{-values} \\ &= \frac{11+5}{5+3} && \text{Simplify} \\ &= \frac{16}{8} \\ &= 2 \end{aligned}$$

18. Write the following problem on the board: Find the gradient of the line passing through $(4, 7)$ and $(5, 13)$.
19. Ask pupils to work with seatmates to find the solution.
20. Invite a volunteer to write the solution on the board.

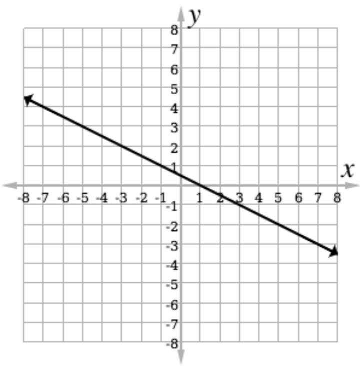
    Solution:

$$\begin{aligned} m &= \frac{13-7}{5-4} && \text{Substitute } x\text{- and } y\text{-values} \\ &= \frac{6}{1} && \text{Simplify} \\ &= 6 \end{aligned}$$

21. Discuss the meaning of the gradient:
    - In which direction does the line with gradient 6 increase? How do we know? (Answer: To the right, or the positive x-direction. We know this because the gradient is positive.)
    - If we calculated that the gradient of one line is 2 and the gradient of another line is 6, which line would be steeper? (Answer: The line with gradient 6. We know because it is the greater number.)
22. Tell learners that gradient is sometimes called "slope". These 2 words mean the same thing.

Practice *(10 minutes)*

1. Write the following 3 problems on the board:
   a. Find the gradient of the line →



   b. Find the gradient of the line passing through $(1,-2)$ and $(-2,4)$.
   c. Line AB has a gradient of 2, and line CD has a gradient of -4. Which line is steeper?
2. Ask pupils to solve the problems in their exercise books. Support them as needed. For example, for problem 1 you may identify points on the line as a class.
3. Invite 3 volunteers to each write a solution on the board. Other pupils should check their work.
   Solutions:

a.

$(x_1, y_1) = (-5, 3)$ Identify **any** two points on the line
$(x_2, y_2) = (1, 0)$

$$\begin{aligned} m &= \frac{0-3}{1-(-5)} && \text{Substitute } x\text{- and } y\text{-values} \\ &= \frac{-3}{1+5} && \text{Simplify} \\ &= \frac{-3}{6} \\ &= -\frac{1}{2} \end{aligned}$$

b.

$$\begin{aligned} m &= \frac{4-(-2)}{-2-1} && \text{Substitute } x\text{- and } y\text{-values} \\ &= \frac{4+2}{-3} && \text{Simplify} \\ &= -\frac{6}{3} \\ &= -2 \end{aligned}$$

c. Line CD is steeper, because $|CD| > |BC|$.

Closing *(2 minutes)*

1. Ask a volunteer to share the formula for gradient. Ask another volunteer to share how to determine using the gradient where one line is steeper than another line.
2. For homework, have pupils do the practice activity PHM2-L025 in the Pupil Handbook.

<table>
<tr><td>Lesson Title: Sketching Graphs of Straight Lines</td><td colspan="2">Theme: Algebraic Processes</td></tr>
<tr><td>Lesson Number: M2-L026</td><td>Class: SSS 2</td><td>Time: 40 minutes</td></tr>
<tr><td>Learning Outcome
By the end of the lesson, pupils will be able to sketch the graph of a straight line whose equation is $y = mx + c$ on the Cartesian plane, where m is the gradient of the line and c is the y-intercept.</td><td colspan="2">Preparation
Write the problem in Opening on the board, and draw a coordinate plane for Teaching and Learning and Practice on the board.</td></tr>
</table>

Opening *(3 minutes)*

1. Review the previous lesson. Write the following problem on the board: Find the gradient of the line passing through (-1, 4) and (3, 8).
2. Ask pupils to solve the problem independently.
3. Invite a volunteer to write the solution on the board. Other pupils should check their work.
   Solution:

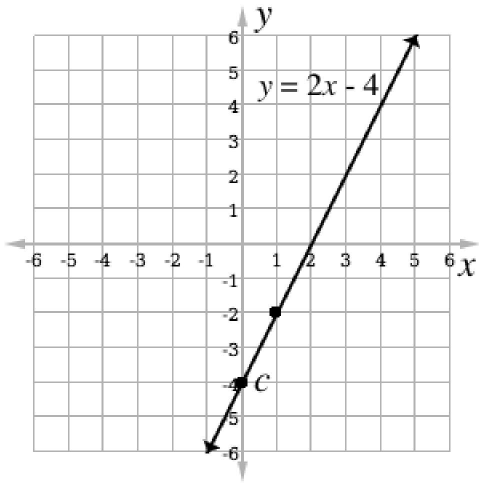
$$\begin{aligned} m &= \frac{y_2-y_1}{x_2-x_1} \\ &= \frac{8-4}{3-(-1)} && \text{Substitute } x\text{- and } y\text{-values} \\ &= \frac{4}{3+1} && \text{Simplify} \\ &= \frac{4}{4} \\ &= 1 \end{aligned}$$

4. Explain that the topic for today is also related to lines. Pupils will learn how to draw the graph of a straight line based on its equation.

Teaching and Learning *(25 minutes)*

1. Write the following equation on the board: $y = 2x - 4$
2. Discuss:
   - Ask pupils to tell what type of equation this is. (Answer: linear equation)
   - Ask pupils to describe the process for graphing a line. (Example answer: Find ordered pair solutions by substituting values for x and solving for y. Write the solutions in a table of values, and graph them.)
3. Explain:
   - We have graphed lines before by filling in a table of values with solutions. Today we will learn another way to graph lines.
   - The linear equation on the board is written in slope-intercept form.
4. Write the following on the board: Slope-intercept form: $y = mx + c$, where m is the gradient and c is the y-intercept of the line.
5. Explain: The y-intercept is the point where the line crosses the y-axis.

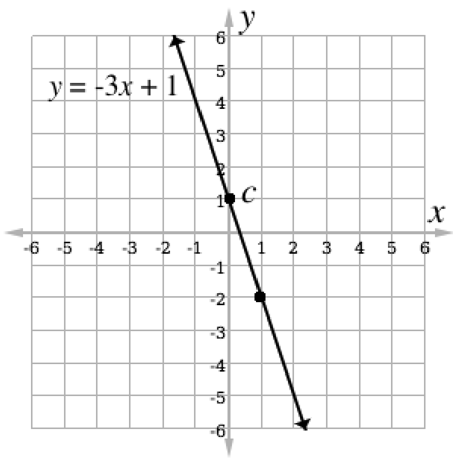
6. Ask pupils to look at the first equation on the board ($y = 2x - 4$) again, and find the gradient and y-intercept. (Answers: $m = 2$ and $c = -4$)
7. Graph the line on the board, explaining each step:
    - Identify and plot the y-intercept, $c = -4$.
    - Remind pupils that $m = \frac{rise}{run}$. In this case, $m = 2 = \frac{2}{1}$. From the y-intercept, count **up** (rise) 2 points in the y-direction. Count to the right (run) 1 point in the x-direction. Plot this point (1, -2) and draw a line that passes through this and the y-intercept.
    - Label the line $y = 2x - 4$.



8. Write the following problem on the board: Graph $y - 1 = -3x$
9. Discuss: Is this line in slope-intercept form? (Answer: No) How can we write it in slope-intercept form? (Answer: transpose -1)
10. Write the equation in slope-intercept form on the board:

$$y - 1 = -3x$$
$$y = -3x + 1 \qquad \text{Transpose -1}$$

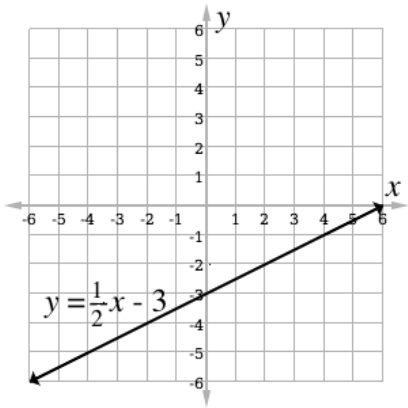
11. Graph the line, explaining each step to pupils:
    - Identify and plot the y-intercept, $c = 1$.
    - Using $m = -3 = \frac{-3}{1}$, find another point: Count **down** 3 places and to the right 1 place. Plot this point, (-2, 1).
    - Draw a line passing through the 2 plotted points.
    - Label the line $y = -3x + 1$.



12. Write the following problem on the board: Graph $y = \frac{1}{2}x - 3$
13. Ask pupils to work with seatmates to graph the line in their exercise books.
14. Invite a volunteer to draw the graph on the board and explain how they graphed it.

    Solution (and example explanation):
    - Identify and plot the y-intercept, $c = -3$.

- Using $m = \frac{1}{2}$, find another point: Count up 1 place and to the right 2 places. Plot this point, (2, -2).
- Draw a line passing through the 2 plotted points.
- Label the line $y = \frac{1}{2}x - 3$.

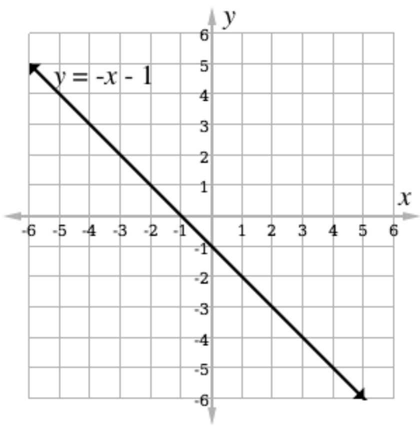


Practice *(10 minutes)*

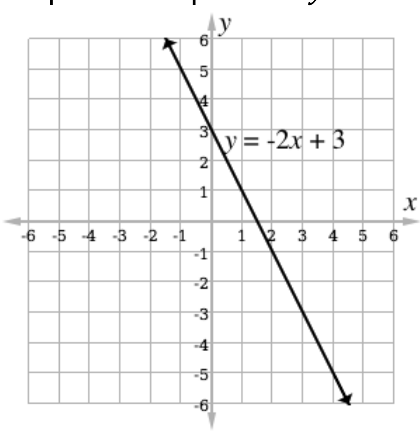
1. Write the following 2 problems on the board:
   d. Graph $y = -x - 1$
   e. Graph $y + 2x = 3$
2. Ask pupils to draw the graphs in their exercise books. Support them as needed. For example, for problem b) you may need to remind them to write the equation in slope-intercept form (by transposing $2x$).
3. Invite 2 volunteers to each graph a line on the board and explain the solutions. Other pupils should check their work.

   Solutions:

   a.



   b. Slope-intercept form: $y = -2x + 3$



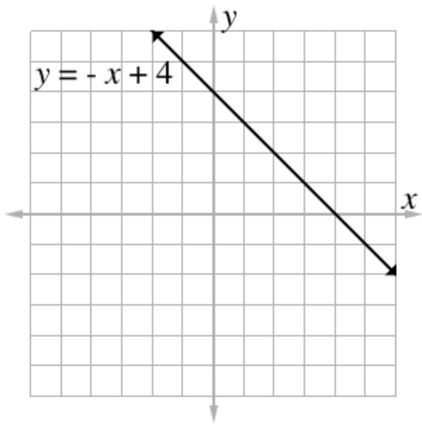
Closing *(2 minutes)*

1. Review the lesson. Discuss:
   a. What is the y-intercept of a line? How can you find the y-intercept in a linear equation? (Example answers: the point where the line crosses the y-axis; In slope-intercept form, y-intercept is the constant term c.)
   b. What is the gradient of a line? How can you find the gradient in a linear equation? (Example answers: It gives the direction in which the line increases and the steepness of the line; In slope-intercept form, gradient m is the coefficient of x.)
2. For homework, have pupils do the practice activity PHM2-L026 in the Pupil Handbook.

Lesson Title: Equation of a straight line	**Theme:** Algebraic Processes	
Lesson Number: M2-L027	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcomes By the end of the lesson, pupils will be able to: 1. Determine the equation of a straight line from the gradient and a given point. 2. Determine the equation of a straight line from two given points.	**Preparation** Draw an empty Cartesian plane on the board for the Opening problem.	

Opening *(5 minutes)*

1. Review the previous lesson. Write the following problem on the board: Graph the line $y = -x + 4$.
2. Ask pupils to work with seatmates to graph the line.
3. Invite a volunteer to draw the graph on the board. Other pupils should check their work.
   Solution:



4. Explain that the topic for today is also related to lines. Pupils will learn how to determine the equation of a straight line given the gradient and a point, or given any two points on the line.

Teaching and Learning *(23 minutes)*

1. Explain: There are different ways of writing linear equations. Pupils have been using slope-intercept form, $y = mx + c$. This lesson looks at another form.
2. Write the following on the board: $y - y_1 = m(x - x_1)$ where (x_1, y_1) is a specific point on the line. (x, y) is a general point on the line.
3. Explain: This is also an equation of a straight line. With this formula, we can use a given gradient and one point to find the linear equation.
4. Write the following problem on the board: Determine the equation of a straight line whose gradient is -3 and that passes through the point $(1, 4)$.
5. Assign a variable from the equation of a line to each number in the problem. Write them on the board: $m = -3$, $(x_1, y_1) = (1, 4)$

6. Solve the problem on the board, explaining each step:

$y-4$	$=$	$-3(x-1)$	Substitute values for m, x_1, and y_1
$y-4$	$=$	$-3x+3$	Simplify
y	$=$	$-3x+3+4$	Transpose -4
y	$=$	$-3x+7$	

7. Write the following problem on the board: Find the equation of a straight line with a gradient of 4, and which passes through $(-3, 2)$.
8. Ask pupils to work with seatmates to find the linear equation. Remind them to use the formula on the board, $y - y_1 = m(x - x_1)$.
9. Invite a volunteer to write the solution on the board.
Solution:

$y-2$	$=$	$4(x-(-3))$	Substitute values for m, x_1, and y_1
$y-2$	$=$	$4x+12$	Simplify
y	$=$	$4x+12+2$	Transpose -2
y	$=$	$4x+14$	

10. Explain: There is another way to write the equation of a line when given just two points on the line.
11. Ask pupils to give the formula for finding gradient, and write it on the board. (Answer: $m = \frac{y_2-y_1}{x_2-x_1}$)
12. Explain:
    - Given two points on a line, the first step is to find the gradient.
    - The second step is to substitute the gradient and either **one** of the points into the formula $y - y_1 = m(x - x_1)$.
13. Write the following problem on the board: Find the equation of the straight line passing through the points $(-1, -1)$ and $(3, 7)$.
14. Assign a variable from the equation of a line to each number in the problem. Write them on the board: $(x_1, y_1) = (-1, -1)$ and $(x_2, y_2) = (3, 7)$
15. Solve the problem on the board, explaining each step:

m	$=$	$\frac{7-(-1)}{3-(-1)}$	Substitute into the formula for m
	$=$	$\frac{7+1}{3+1}$	Simplify
	$=$	$\frac{8}{4}$	
m	$=$	2	
$y-y_1$	$=$	$m(x-x_1)$	Equation of a straight line
$y-7$	$=$	$2(x-3)$	Substitute $m=2$ and one point, $(3, 7)$
$y-7$	$=$	$2x-6$	
y	$=$	$2x-6+7$	Transpose -7
y	$=$	$2x+1$	Equation of the line

16. Write the following problem on the board: Find the equation of a straight line that passes through $(1, 4)$ and $(-1, -2)$.
17. Ask pupils to work with seatmates to find the linear equation. Remind them of the 2 steps: Find the gradient, then substitute the gradient and one point into the line formula.
18. Invite a volunteer to write the solution on the board.

Solution:

m	$=$	$\frac{-2-4}{-1-1}$	Substitute into the formula for m
	$=$	$\frac{-6}{-2}$	Simplify
m	$=$	3	
$y - y_1$	$=$	$m(x - x_1)$	Equation of a straight line
$y - 4$	$=$	$3(x - 1)$	Substitute $m = 3$ and one point, $(1, 4)$
$y - 4$	$=$	$3x - 3$	
y	$=$	$3x - 3 + 4$	Transpose -4
y	$=$	$3x + 1$	Equation of the line

Practice *(10 minutes)*

1. Write the following 2 problems on the board:
   a. Determine the equation of a straight line that passes through the point $(0, 5)$ and whose gradient is -1.
   b. Find the equation of the line passing through points $(-2, 3)$ and $(1, 12)$.
2. Ask pupils to solve the problems in their exercise books. Support them as needed.
3. Invite 2 volunteers to each write and explain their solution on the board. Other pupils should check their work.

Solutions:

a.

$m = -1$, $x_1 = 0$, $y_1 = 5$			Identify values for m, x_1, and y_1
$y - 5$	$=$	$-1(x - 0)$	Substitute
$y - 5$	$=$	$-x$	Simplify
y	$=$	$-x + 5$	Transpose -5

b.

$x_1 = -2$, $y_1 = 3$, $x_2 = 1$, and $y_2 = 12$			Identify values for x_1, y_1, x_2, and y_2
m	$=$	$\frac{12-3}{1-(-2)}$	Substitute into the formula for m
	$=$	$\frac{9}{3}$	Simplify
m	$=$	3	

$y - y_1$	$=$	$m(x - x_1)$	Equation of a straight line
$y - 3$	$=$	$3(x - (-2))$	Substitute $m = 3$ and one point, $(-2, 3)$
$y - 3$	$=$	$3x + 6$	
y	$=$	$3x + 6 + 3$	Transpose -3
y	$=$	$3x + 9$	Equation of the line

Closing *(2 minutes)*

1. Review the different ways to write linear equations. Ask volunteers to share three different ways, allowing them to look at their notes as needed.
2. Write the following equations on the board:
    a. $ax + by = c$ (simultaneous equation lessons)
    b. $y = mx + c$ (graphing lessons)
    c. $y - y_1 = m(x - x_1)$ (finding the equation of a line)
3. Explain: Any line can be written in any of these forms. These different forms are used for different types of problems.
4. For homework, have pupils do the practice activity PHM2-L027 in the Pupil Handbook.

Lesson Title: Practice with Straight Lines	**Theme:** Algebraic Processes	
Lesson Number: M2-L028	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to determine the equation of a straight line and graph it on the Cartesian plane.	**Preparation** Draw an empty Cartesian plane on the board for the Teaching and Learning problems.	

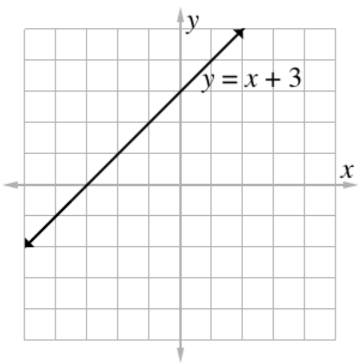
Opening *(4 minutes)*

1. Review the previous 3 lessons through discussion:
   - What form should a line be written in before graphing it? (Answer: slope-intercept form, $y = mx + c$)
   - What information do we need to find the equation of a line? (Answers: The gradient and a point on the line; any two points on the line.)
   - What is the formula for the gradient? (Answer: $m = \frac{y_2 - y_1}{x_2 - x_1}$)
   - What is the formula that we use to find the equation of a line given the gradient and a point? (Answer: $y - y_1 = m(x - x_1)$)
2. Explain that today pupils will combine and use all of this information. Pupils will find the equation of a line and graph it on the Cartesian plane.

Teaching and Learning *(15 minutes)*

1. Write the following 2 problems on the board:
   a. If the gradient of a line is 1 and $(-2, 1)$ is a point on the line, find its equation. Draw the line on the Cartesian plane.
   b. Find the equation of the line passing through points $(1, 0)$ and $(3, 6)$. Graph the line using its equation.
2. Ask pupils to explain how to solve the first problem. (Example answer: Substitute the gradient and coordinates into the formula $y - y_1 = m(x - x_1)$, then simplify. Graph the result on the Cartesian plane.)
3. Solve problem 1 on the board, and graph the line.
   Solution:

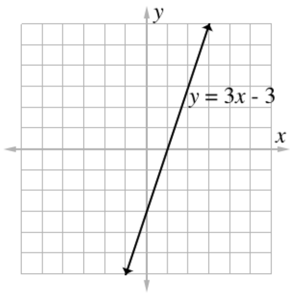
$$
\begin{aligned}
y - 1 &= 1(x - (-2)) && \text{Substitute values} \\
y - 1 &= x + 2 && \text{Simplify} \\
y &= x + 2 + 1 && \text{Transpose } -1 \\
y &= x + 3
\end{aligned}
$$



4. Ask pupils to explain how to solve the second problem. (Example answer: Substitute both points into the formula for the gradient; Use the gradient and one point to find the equation of the line; Graph the result on the Cartesian plane)
5. Ask pupils to work with seatmates to solve problem b. and graph the line.
6. Walk around and check for understanding. Support pupils as needed.
7. Invite 1 volunteer to write the solution on the board and 1 volunteer to draw the graph. All other pupils should check their work.

   Solution:

m	$=$	$\frac{6-0}{3-1}$	Substitute into the formula for m
	$=$	$\frac{6}{2}$	Simplify
m	$=$	3	
$y-y_1$	$=$	$m(x-x_1)$	Equation of a straight line
$y-0$	$=$	$3(x-1)$	Substitute $m=3$ and $(1,0)$
y	$=$	$3x-3$	



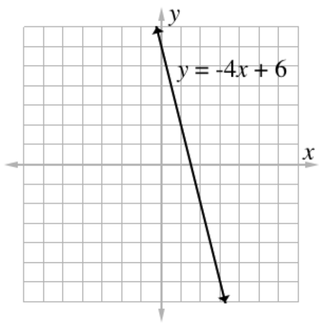
Practice *(20 minutes)*

1. Write the following 3 problems on the board:
   1) Determine the equation of a straight line that passes through the point $(1,2)$ and whose gradient is -4. Graph the line.
   2) Determine the equation of the line passing through points $(-4,0)$ and $(0,-8)$. Graph the line.
   3) Find the equation of the line passing through points $(-2,2)$ and $(2,4)$. Graph the line on the Cartesian plane.
2. Ask pupils to solve the problems in their exercise books. Support pupils as needed.
3. Invite 6 volunteers to come to the board (they may all come at the same time). Assign 3 volunteers to each write a solution, while the remaining 3 volunteers each draw a graph. Other pupils should check their work.

Solutions:

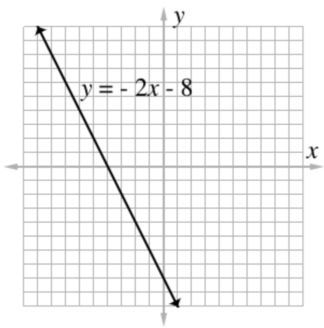
a.

$y-2$	$=$	$-4(x-1)$	Substitute values
$y-2$	$=$	$-4x+4$	Simplify
y	$=$	$-4x+4+2$	Transpose -2
y	$=$	$-4x+6$	



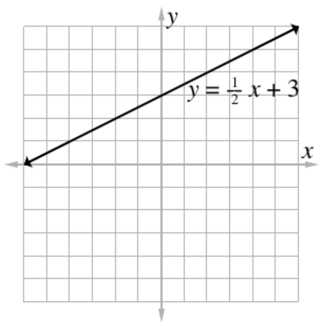
b.

m	$=$	$\frac{-8-0}{0-(-4)}$	Substitute into the formula for m
	$=$	$\frac{-8}{4}$	Simplify
m	$=$	-2	
$y-y_1$	$=$	$m(x-x_1)$	Equation of a straight line
$y-0$	$=$	$-2(x-(-4))$	Substitute $m=-2$ and $(-4,0)$
y	$=$	$-2(x+4)$	Simplify
y	$=$	$-2x-8$	



c.

m	$=$	$\frac{4-2}{2-(-2)}$	Substitute into the formula for m
	$=$	$\frac{2}{2+2}$	Simplify
m	$=$	$\frac{2}{4}=\frac{1}{2}$	
$y-y_1$	$=$	$m(x-x_1)$	Equation of a straight line
$y-2$	$=$	$\frac{1}{2}(x-(-2))$	Substitute $m=\frac{1}{2}$ and $(-2,2)$
$y-2$	$=$	$\frac{1}{2}(x+2)$	Simplify
$y-2$	$=$	$\frac{1}{2}x+1$	
y	$=$	$\frac{1}{2}x+1+2$	Transpose -2
y	$=$	$\frac{1}{2}x+3$	



Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM2-L028 in the Pupil Handbook.

<table>
<tr><td>Lesson Title: Gradient of a Curve – Part 1</td><td colspan="2">Theme: Algebraic Processes</td></tr>
<tr><td>Lesson Number: M2-L029</td><td>Class: SSS 2</td><td>Time: 40 minutes</td></tr>
<tr><td>Learning Outcomes
By the end of the lesson, pupils will be able to:
1. Draw the tangent to a curve at a given point.
2. Use the tangent to find an appropriate value for the gradient of a curve at a given point.</td><td colspan="2">Preparation
Bring anything straight and flat that can be used as a straight edge for drawing lines on the board. (such as a ruler or book)</td></tr>
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Opening *(3 minutes)*

1. Review quadratic functions briefly. Discuss:
    - What is the general form of a quadratic function? (Answer: $y = ax^2 + bx + c$)
    - What shape do we get when we graph a quadratic function? (Possible answers: a parabola; a curve)
    - How do we find the solutions (or roots) to a quadratic function by graphing? (Answer: Graph the function and find the x-intercepts.)
2. Explain that today's lesson is on finding the gradient of a curve.

Teaching and Learning *(25 minutes)*

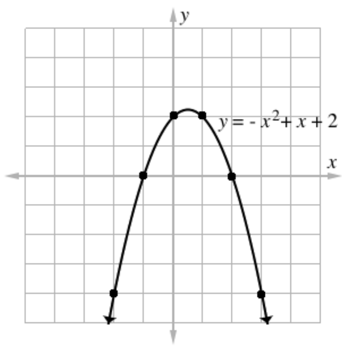
1. Review quadratic functions and parabolas by giving pupils one function to graph with seatmates. Write the problem and empty table of values below on the board:

   Graph $y = -x^2 + x + 2$ using the table of values:

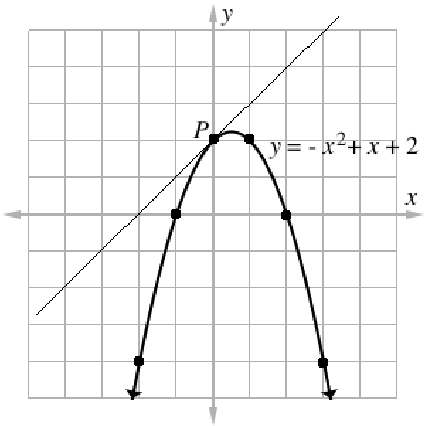
x	-2	-1	0	1	2	3
y						

2. Give pupils several minutes to fill the table and graph the parabola with seatmates.
3. Invite a volunteer to fill in the table, and another volunteer to graph the parabola on the board.
   Graph:

x	-2	-1	0	1	2	3
y	-4	0	2	2	0	-4



4. Explain:
    c. The gradient of a line stays the same along the entire line. The gradient of a curve changes from point to point.
    d. The gradient at any point on a curve is the same as the gradient of the tangent line at that exact point.
    e. A tangent line touches the curve at only one point.
    f. A tangent to a curve at point P can be drawn by placing a straight edge on the curve at P, then drawing a line. The “angles” between the curve and line should be nearly equal.
    g. Use anything straight for a straight edge, including paper or the side of an exercise book.
5. Label point $(0, 2)$ on the parabola as *P*. Draw a tangent to the parabola at this point, as shown:



6. Explain:
    - The parabola and tangent line must be drawn very accurately and clearly to find the gradient.
    - If pupils are drawing their own Cartesian plane, it is important to pay close attention when drawing the scale on the x- and y-axes (It is a common error to draw a large space between 0 and 1 or 0 and -1. The space between the origin and 1 or the origin and -1 should be the same size as every other space on the axes.)
    - The gradient found using a tangent is usually only an **approximate** gradient for the curve.
    - We use this tangent line to find the gradient of the curve at $(0, 2)$. We find any 2 points on the line and use them in the gradient formula.
7. Ask pupils to identify two points on the tangent line. (Example answers: $(-2, 0)$ and $(0, 2)$)
8. Assign variables to each of these from the gradient formula, and write them on the board. (Example: $(x_1, y_1) = (-2, 0)$ and $(x_2, y_2) = (0, 2)$)
9. Substitute these coordinates into the gradient formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{2-0}{0-(-2)}$$ Substitute x- and y-values.

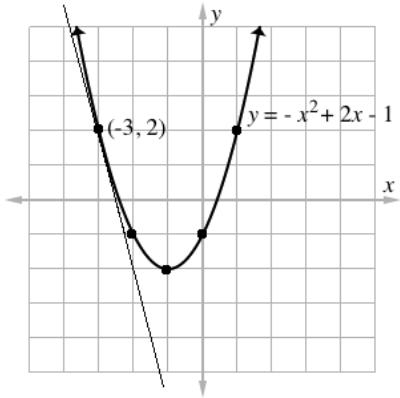
$$= \frac{2}{2} \quad \text{Simplify}$$
$$m = 1$$

10. Explain: By finding the gradient of the tangent line at point P we have also found the gradient of the curve $y = -x^2 + x + 2$ at this point.
11. Write another problem on the board: Graph $y = x^2 + 2x - 1$ using the table of values below. Find the gradient of the curve at $(-3, 2)$.

x	-3	-2	-1	0	1
y					

12. Ask pupils to work with seatmates to draw the parabola and tangent line at $(-3, 2)$.
13. Invite a volunteer to graph the parabola and tangent line on the board:

x	-3	-2	-1	0	1
y	2	-1	-2	-1	2



14. Discuss the tangent line with pupils. Check if this is the same as the tangent line they drew with seatmates. Clarify any misconceptions.
15. Ask volunteers to identify two points on the tangent line. (Example answers: $(x_1, y_1) = (-3, 2)$ and $(x_2, y_2) = (-2, -2)$)
16. Ask pupils to work with seatmates to find the gradient of the curve at $(-3, 2)$. Remind them to use two points on the tangent line.
17. Invite a volunteer to write the solution on the board.
Solution:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{-2-2}{-2-(-3)} \quad \text{Substitute } x\text{- and } y\text{-values.}$$
$$= \frac{-4}{-2+3} \quad \text{Simplify}$$
$$= \frac{-4}{1}$$
$$m = -4$$

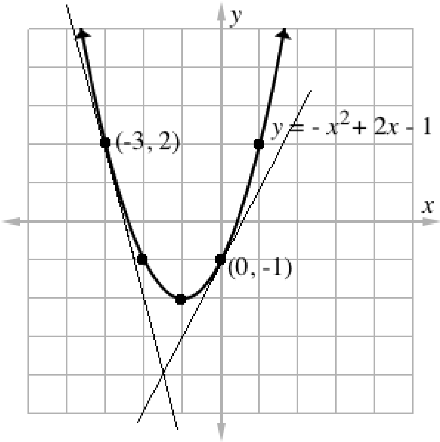
Practice *(10 minutes)*

1. Write the following problem on the board:

1) Draw the graph of $y = -x^2 + 2x - 1$ in your own exercise book. Find the gradient of the curve at $(0, -1)$ by drawing a tangent line.

2. Ask pupils to solve the problem in their exercise books. Support them as needed. Encourage them to take their time and draw the curve and tangent very precisely.
3. Invite a volunteer to come to the board and draw the tangent line on the parabola (which is already on the board). Invite another volunteer to come to the board and calculate the gradient of the curve at $(0, -1)$.

Solution:



Two points on the line are $(x_1, y_1) = (0, -1)$ and $(x_2, y_2) = (-1, -3)$. Note that pupils may choose different points.

$$
\begin{aligned}
m &= \frac{y_2 - y_1}{x_2 - x_1} \\
&= \frac{-3-(-1)}{-1-0} && \text{Substitute } x\text{- and } y\text{-values.} \\
&= \frac{-3+1}{-1} && \text{Simplify} \\
&= \frac{-2}{-1} \\
m &= 2
\end{aligned}
$$

Closing *(2 minutes)*

1. Discuss:
   a. We found the gradient of the curve $y = -x^2 + x + 2$ at two different points, $(-3, 2)$ and $(0, -1)$.
   b. What are the gradients at these 2 points? (Answers: -4 and 2)
   c. Why do you think the gradient of the curve is different at these two points?
   d. How many different gradients do you think this curve has? (Answer: It has infinitely many, because there are an infinite number of points on the curve and each has a different gradient.)
2. For homework, have pupils do the practice activity PHM2-L029 in the Pupil Handbook.

Lesson Title: Gradient of a Curve – Part 2	**Theme:** Algebraic Processes	
Lesson Number: M2-L030	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcomes By the end of the lesson, pupils will be able to: 1. Draw the tangent to a curve at a given point. 2. Use the tangent to find an appropriate value for the gradient of a curve at a given point.	**Preparation** Bring anything straight and flat that can be used as a straight edge for drawing lines on the board. (such as a ruler or book)	

Opening *(3 minutes)*

1. Review tangent lines. Discuss:
    - What is a tangent line? (Example answer: It is a line that only touches a curve at one point.)
    - What are the steps for drawing a tangent line to a quadratic function at point P? (Example answer: Use a straight edge to draw a line that only touches the curve at point P.)
    - How can we find the gradient of the curve at point P? (Example answer: Find the gradient of a tangent line that touches at point P, as the gradients for the curve and the tangent line are equal.)
2. Explain that today's lesson is also about finding the gradient of a curve.

Teaching and Learning *(15 minutes)*

1. Write the following problem and table of values on the board:

    a. Complete the table of values for the relation $y = x^2 - 2x - 3$.

x	-2	-1	0	1	2	3
y						

    b. Draw the graph of y for $-2 \leq x \leq 3$.
    c. From your graph:
        i. Find the roots of the equation $x^2 - 2x - 3 = 0$
        ii. Estimate the minimum value of y.
        iii. Calculate the gradient of the curve at the point $x = 2$.

2. Discuss:
    - This is what questions often look like on the WASSCE exam. There are multiple steps.
    - Do you know how to complete each step? What are the challenges?

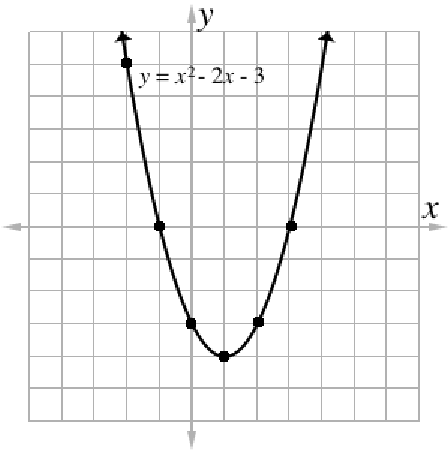
3. Ask pupils to work with seatmates to complete each step of the problem. If certain steps are challenging, invite volunteers to show them on the board while others are working.
4. Invite 5 volunteers to each write part of the solution on the board. Each pupil will need to wait until the pupil before them has finished their part.

Solutions:

a.

x	-2	-1	0	1	2	3
y	5	0	-3	-4	-3	0

b.

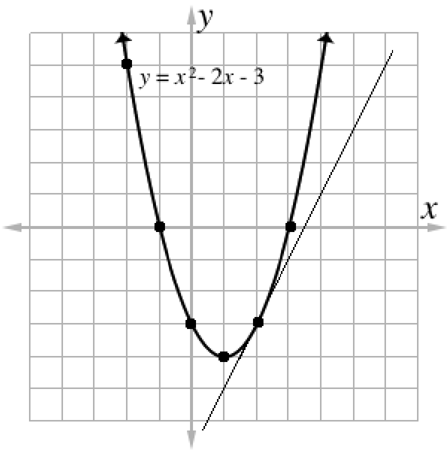


c.

i. The roots are the x-intercepts: $x = -1$ and $x = 3$

ii. The minimum value of y is $y = -4$

iii. Draw the tangent line at $x = 2$, and find its gradient:



Identify any two points on the tangent.

Example: $(x_1, y_1) = (2, -3)$ and $(x_2, y_2) = (4, 1)$

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{1-(-3)}{4-2} && \text{Substitute } x\text{- and } y\text{-values.} \\ &= \frac{1+3}{2} && \text{Simplify} \\ m &= 2 \end{aligned}$$

Practice *(20 minutes)*

1. Write the following 2 problems on the board:
   a. Find the gradient of $y = x^2$ at $x = -2$.
   b. Complete the following:
      i. Complete the table of values for the relation $y = x^2 - 5x + 4$.

x	0	1	2	3	4	5
y						

      ii. Draw the graph of y for $0 \leq x \leq 5$.

iii. From your graph:

1) Find the roots of the equation $x^2 - 5x + 4 = 0$
2) Estimate the minimum value of y.
3) Calculate the gradient of the curve at the point $x = 1$.

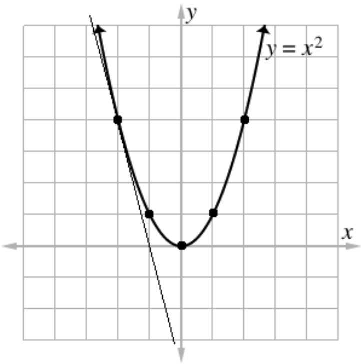
2. Ask pupils to solve the problems in their exercise books. Allow them to work with seatmates if needed.
3. Invite volunteers to come to the board and write the solutions. All other pupils should check their work.

Solutions:

a. Pupils need to create their own table of values using any points. For example:

x	-2	-1	0	1	2
y	4	1	0	1	4

Graph:



Pupils may use any 2 points on the tangent. For example: $(x_1, y_1) = (-2, 4)$ and $(x_2, y_2) = (-1, 0)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{0-4}{-1-(-2)}$$ Substitute x- and y-values.

$$= \frac{-4}{-1+2}$$ Simplify

$$= \frac{-4}{1}$$

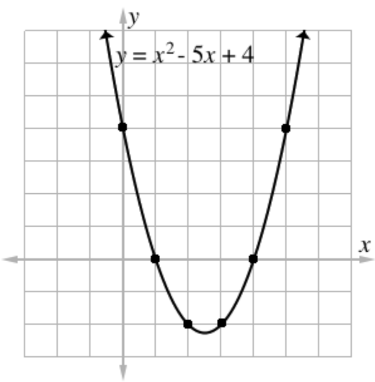
$$m = -4$$

b.

i. $y = x^2 - 5x + 4$

x	0	1	2	3	4	5
y	4	0	-2	-2	0	4

ii.

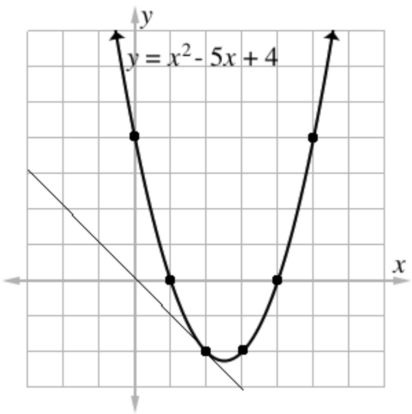


iii.

a) The roots are the x-intercepts: $x = 1$ and $x = 4$

b) The minimum value of y is slightly less than -2. Example estimate: $y = -2.2$

c) Draw the tangent line at $x = 2$, and find its gradient:



Identify any two points on the tangent.
Example: $(x_1, y_1) = (2, -2)$ and $(x_2, y_2) = (0, 0)$

$$
\begin{aligned}
m &= \frac{y_2 - y_1}{x_2 - x_1} \\
&= \frac{0-(-2)}{0-2} && \text{Substitute } x\text{- and } y\text{-values.} \\
&= \frac{2}{-2} && \text{Simplify} \\
m &= -1
\end{aligned}
$$

Closing *(2 minutes)*

1. Discuss: What are the challenges to solving problems with tangent lines? How can we overcome these challenges? (Example: It's challenging to draw the graph accurately enough to find the gradient. This can be overcome by taking time, practising, drawing an accurate scale on the axes, and using a straight edge.)
2. For homework, have pupils do the practice activity PHM2-L030 in the Pupil Handbook.

Lesson Title: Simplification of Algebraic Fractions – Part 1	**Theme:** Algebraic Processes	
Lesson Number: M2-L031	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to use factorisation to simplify algebraic fractions by reducing them to their lowest terms.	**Preparation** None	

Opening *(3 minutes)*

1. Write on the board: $\frac{wxy}{xyz}$
2. Discuss:
   - Ask pupils to describe the expression on the board. (Example answers: A fraction with variables; an algebraic expression; an algebraic fraction.)
   - Ask pupils to explain whether they can simplify the expression. (Example answer: Yes, variables can be canceled from the numerator and denominator.)
3. Explain that today's lesson is on algebraic fractions, like the one on the board. Pupils will learn how to simplify them.

Teaching and Learning *(20 minutes)*

1. Explain:
   - When simplifying algebraic fractions, first factor the numerator and denominator.
   - Find the highest common factor (HCF) of the numerator and denominator. Remember that the HCF is simply all of the common factors multiplied together.
   - Then, divide (cancel) the HCF from the numerator and denominator.
2. Discuss:
   - In the example on the board, are there any common factors in the numerator and denominator? If so, what are they? (Answer: Yes, x and y)
   - What is the HCF of the numerator and denominator? (Answer: xy)
3. Simplify the algebraic fraction on the board. Divide xy from both the numerator and denominator: $\frac{wxy \div xy}{xyz \div xy} = \frac{w}{z}$
4. Write on the board: Simplify $\frac{4x^2}{2x}$
5. Ask pupils to work with seatmates to identify the HCF of the numerator and denominator.
6. Invite a volunteer to give the HCF. (Answer: $2x$)
7. Divide both the numerator and denominator by the HCF: $\frac{4x^2}{2x} = \frac{2x}{1} = 2x$

8. Explain: Remember to apply the laws of indices correctly. When dividing indices, subtract the powers.
9. Write on the board: Simplify $\frac{3x^4y}{9x^3y^2}$
10. Ask pupils to work with seatmates to identify the HCF of the numerator and denominator.
11. Invite a volunteer to give the HCF. (Answer: $3x^3y$)
12. Divide both the numerator and denominator by the HCF: $\frac{3x^4y}{9x^3y^2} = \frac{x}{3y}$
13. Write on the board: Write the HCF for each, then simplify the expression:
    a. $\frac{16b^4}{12a^2b}$ b. $\frac{4xy^3}{6x^2y}$ c. $\frac{5a^2b^3c}{ab^3c^2}$
4. Ask pupils to work with seatmates to solve the 3 problems. Walk around and check for understanding.
5. Invite 3 volunteers to write their answers on the board and explain. All other pupils should check their work. (Answers: a. HCF: $4b$, simplified expression: $\frac{4b^3}{3a^2}$; b. HCF: $2xy$, simplified expression: $\frac{2y^2}{3x}$; c. HCF: ab^3c, simplified expression: $\frac{5a}{c}$)
6. Write on the board: $\frac{x+y}{x^2}$
7. Ask pupils if they can find the HCF of the numerator and denominator. Allow them to discuss.
8. Explain:
    - There are no common factors in the numerator and denominator. This fraction cannot be simplified.
    - When you see addition or subtraction connecting terms, that expression is considered one factor. In the numerator, $(x + y)$ is a factor. In the denominator, x and x are factors.
    - In the next lesson, we will handle algebraic fractions with addition and subtraction in the numerator and denominator.

Practice *(15 minutes)*

1. Write on the board:
    a. Simplify $\frac{3x^2y}{xy^2}$.
    b. Reduce $\frac{15m^2n^3}{5mn^2}$ to its lowest terms.
    c. Simplify $\frac{24c^5}{18a^2c^3}$.
    d. Reduce $\frac{100ab^2c}{50b^2c^2d}$ to its lowest terms.
    e. Simplify $\frac{u^9vw^3}{12u^2v^2w^2}$.
2. Ask pupils to solve the problems individually.
3. Invite volunteers to come to the board and write the answers. All other pupils should check their work. (Answers: a. $\frac{3x}{y}$; b. $3mn$; c. $\frac{4c^2}{3a^2}$; d. $\frac{2a}{cd}$; e. $\frac{u^7w}{12v}$)

Closing *(2 minutes)*

1. Ask pupils to revise the steps for simplifying fractions (Example answer: First factor the numerator and denominator. Then find the highest common factor (HCF). Next, cancel the HCF from the numerator and denominator).
2. For homework, have pupils do the practice activity PHM2-L031 in the Pupil Handbook.

Lesson Title: Simplification of Algebraic Fractions – Part 2	**Theme:** Algebraic Processes	
Lesson Number: M2-L032	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to use factorisation to simplify more complex algebraic fractions by reducing them to their lowest terms.	**Preparation** None	

Opening *(4 minutes)*

1. Revise the previous lesson. Write on the board: Simplify $\frac{2x^2z}{10xyz}$
2. Invite volunteers to give the common factors of the numerator and denominator. (Answer: 2, x, z)
3. Invite a volunteer to give the HCF. (Answer: $2xz$)
4. Ask pupils to solve the problem with seatmates.
5. Ask a volunteer to write the solution on the board. (Answer: $\frac{2x^2z}{10xyz} = \frac{x}{5y}$)
6. Explain that today's lesson is on simplifying more complex algebraic fractions.

Teaching and Learning *(20 minutes)*

1. Write on the board: Simplify $\frac{2x+xy}{2y+y^2}$
2. Explain: This is a more complicated fraction, but we follow the same process. Factor the numerator and denominator, and divide by the HCF.
3. Solve on the board, explaining each step:
   Step 1. Factor the numerator and denominator:
   $$\frac{2x+xy}{2y+y^2} = \frac{x(2+y)}{y(2+y)}$$
   Step 2. Identify that the HCF in the numerator and denominator is $(2 + y)$.
   Step 3. Divide (cancel) the numerator and denominator by the HCF:
   $$\frac{2x+xy}{2y+y^2} = \frac{x}{y}$$
4. Write another problem on the board: Reduce $\frac{x^2+4x+3}{x^2+3x}$ to its lowest terms.
5. Ask pupils to work with seatmates to factor both the numerator and denominator.
6. Invite volunteers to write the factorisation of each on the board. (Answers: Numerator: $x^2 + 4x + 3 = (x + 1)(x + 3)$; Denominator: $x^2 + 3x = x(x + 3)$)
7. Rewrite the algebraic fraction on the board: $\frac{x^2+4x+3}{x^2+3x} = \frac{(x+1)(x+3)}{x(x+3)}$
8. Ask volunteers to give the common factors. (Answer: $x + 3$).
9. Divide the numerator and denominator by $x + 3$: $\frac{x^2+4x+3}{x^2+3x} = \frac{(x+1)(x+3)}{x(x+3)} = \frac{(x+1)}{x}$
10. Write another problem on the board: Simplify $\frac{a^2-ab}{c^2-cb}$

11. Ask pupils to work with seatmates to factor both the numerator and denominator.
12. Invite volunteers to write the factorisation of each on the board. (Answers: Numerator: $a^2 - ab = a(a - b)$; Denominator: $c^2 - cb = c(c - b)$))
13. Rewrite the algebraic fraction on the board: $\frac{a^2-ab}{c^2-cb} = \frac{a(a-b)}{c(c-b)}$
14. Invite volunteers to give the common factors. (Answer: There are no common factors.)
15. This algebraic fraction cannot be simplified. It is already in its lowest terms.
16. Write on the board: Simplify each expression:
    a. $\frac{x^2-4x}{x^2-3x-4}$ b. $\frac{x^2-y^2}{(x-y)^2}$ c. $\frac{x^2-xy}{xz}$
17. Ask pupils to work with seatmates to solve the 3 problems. Walk around and check for understanding.
18. Invite 3 volunteers to write their answers on the board and explain. All other pupils should check their work. (Answers: a. $\frac{x^2-4x}{x^2-3x-4} = \frac{x(x-4)}{(x-4)(x+1)} = \frac{x}{x+1}$; b. $\frac{x^2-y^2}{(x-y)^2} = \frac{(x+y)(x-y)}{(x-y)(x-y)} = \frac{x+y}{x-y}$; c. $\frac{x^2-xy}{xz} = \frac{x(x-y)}{xz} = \frac{x-y}{z}$)

Practice *(15 minutes)*

1. Write on the board: Simplify the following fractions. If there is no simpler form, state “no simpler form”:
    a. $\frac{ab-b^2}{(a-b)^2}$
    b. $\frac{x+y}{x-y}$
    c. $\frac{5x^2}{5x-10xy}$
    d. $\frac{a^3b-a^2b^2}{a^3b+a^2b^2}$
    e. $\frac{x^2-2xy+y^2}{x^2-xy}$
2. Ask pupils to solve the problems individually.
3. Invite volunteers to come to the board and write the solutions. All other pupils should check their work.

    Solutions:

    a. $\frac{ab-b^2}{(a-b)^2} = \frac{b(a-b)}{(a-b)(a-b)} = \frac{b}{a-b}$
    b. $\frac{x+y}{x-y}$: no simpler form
    c. $\frac{5x^2}{5x-10xy} = \frac{5x^2}{5x(1-2y)} = \frac{x}{1-2y}$
    d. $\frac{a^3b-a^2b^2}{a^3b+a^2b^2} = \frac{a^2b(a-b)}{a^2b(a+b)} = \frac{a-b}{a+b}$
    e. $\frac{x^2-2xy+y^2}{x^2-xy} = \frac{(x-y)(x-y)}{x(x-y)} = \frac{x-y}{x}$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM2-L032 in the Pupil Handbook.

Lesson Title: Multiplication of Algebraic Fractions	**Theme:** Algebraic Processes	
Lesson Number: M2-L033	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to multiply algebraic fractions, reducing them to their lowest terms.	**Preparation** None	

Opening *(3 minutes)*

1. Revise the previous lesson. Write on the board: Simplify $\frac{x^2+x}{x^2+2x+1}$
2. Ask pupils to solve the problem with seatmates.
3. Invite a volunteer to write the solution on the board and explain. (Answer: $\frac{x^2+x}{x^2+2x+1} = \frac{x(x+1)}{(x+1)(x+1)} = \frac{x}{x+1}$)
4. Explain that today's lesson is on multiplying algebraic fractions.

Teaching and Learning *(20 minutes)*

1. Write on the board: Simplify $\frac{x^2+x}{x+4} \times \frac{xy+4y}{x+1}$.
2. Explain:
   - To multiply algebraic fractions, first factor the numerators and denominators.
   - Identify the common factors in the numerators and denominators.
   - Divide the numerators and denominators by the factors they have in common. In other words, cancel factors that are in both the numerator and denominator.
   - Leave the result as your answer. If there are brackets, you do not need to multiply them out.
3. Solve on the board, explaining each step:
   Step 1. Factor the numerators and denominators:
   $$\frac{x^2+x}{x+4} \times \frac{xy+4y}{x+1} = \frac{x(x+1)}{x+4} \times \frac{y(x+4)}{x+1}$$
   Step 2. Identify that common factors in the numerator and denominator are $(x+1)$ and $(x+4)$
   Step 3. Divide (cancel) the numerator and denominator by the factors:
   $$\frac{x\cancel{(x+1)}}{\cancel{x+4}} \times \frac{y\cancel{(x+4)}}{\cancel{x+1}}$$
   Step 4. Write the result: xy
4. Write another problem on the board: Simplify $\frac{x^2+4x+3}{x+1} \times \frac{x+4}{x^2+x-6}$.
5. Ask pupils to work with seatmates to factor both the numerator and denominator.

6. Invite a volunteer to write the factorisation on the board. (Answer: $\frac{(x+1)(x+3)}{x+1} \times \frac{x+4}{(x+3)(x-2)}$)
7. Ask volunteers to list factors that the numerator and denominator have in common. (Answers: $(x+1)$ and $(x+3)$)
8. Ask pupils to work with seatmates to cancel these factors and write the answer.
9. Invite a volunteer to write the solution on the board.
   Solution:
   $$\frac{\cancel{(x+1)}\cancel{(x+3)}}{\cancel{x+1}} \times \frac{x+4}{\cancel{(x+3)}(x-2)} = \frac{x+4}{x-2}$$
10. Write another problem on the board: Simplify $\frac{2b^3}{4b+8} \times \frac{3b^2-12}{6b^2}$
11. Ask pupils to work with seatmates to factor both the numerator and denominator.
12. Ask a volunteer to write the factorisation on the board. (Answer: $\frac{2b^3}{4b+8} \times \frac{3b^2-12}{6b^2} = \frac{2b^3}{4(b+2)} \times \frac{3(b+2)(b-2)}{6b^2}$)
13. Ask volunteers to list factors that the numerator and denominator have in common. (Answers: 2, 3, b^2, $(b+2)$)
14. Ask pupils to work with seatmates to cancel these factors and write the answer.
15. Invite a volunteer to write the solution on the board.
    Solution:
    $$\frac{\cancel{2}b^{\cancel{3}}}{4\cancel{(b+2)}} \times \frac{\cancel{3}\cancel{(b+2)}(b-2)}{\cancel{6b^2}} = \frac{b(b-2)}{4}$$
16. Write on the board: Multiply the fractions:
    a. $\frac{ab}{a-2b} \times \frac{2a-4b}{2a}$ b. $\frac{x^2-y^2}{xy} \times \frac{5y}{(x-y)^2}$ c. $\frac{m^2-mn}{mn} \times \frac{m+n}{m^2-n^2}$
17. Ask pupils to work with seatmates to solve the 3 problems. Walk around and check for understanding.
18. Invite 3 volunteers to write their solutions on the board and explain. All other pupils should check their work.
    Solutions:
    a. $\frac{ab}{a-2b} \times \frac{2a-4b}{2a} = \frac{\cancel{a}b}{\cancel{a-2b}} \times \frac{\cancel{2}\cancel{(a-2b)}}{\cancel{2a}} = b$
    b. $\frac{x^2-y^2}{xy} \times \frac{5y}{(x-y)^2} = \frac{(x+y)\cancel{(x-y)}}{x\cancel{y}} \times \frac{5\cancel{y}}{\cancel{(x-y)}(x-y)} = \frac{5(x+y)}{x(x-y)}$
    c. $\frac{m^2-mn}{mn} \times \frac{m+n}{m^2-n^2} = \frac{\cancel{m}\cancel{(m-n)}}{\cancel{m}n} \times \frac{\cancel{m+n}}{\cancel{(m+n)}\cancel{(m-n)}} = \frac{1}{n}$

Practice *(15 minutes)*

1. Write the following on the board:

   Multiply the following fractions:
   a. $\frac{6ab}{15cd} \times \frac{5bc}{d}$

b. $\frac{x^2+xy}{xy+y^2} \times \frac{2xy}{x}$

c. $\frac{cd}{5c-10d} \times \frac{c-2d}{cd^2}$

d. $\frac{x^2-4}{x^2-3x+2} \times \frac{5x}{x^2+2x}$

2. Ask pupils to solve the problems individually.
3. Invite volunteers to come to the board and write the solutions. All other pupils should check their work.

   Solutions:

   a. $\frac{6ab}{15cd} \times \frac{5bc}{d} = \frac{2ab^2}{d^2}$

   b. $\frac{x^2+xy}{xy+y^2} \times \frac{2xy}{x} = \frac{x(\cancel{x+y})}{\cancel{y}(\cancel{x+y})} \times \frac{2x\cancel{y}}{\cancel{x}} = \frac{2x}{1} = 2x$

   c. $\frac{cd}{5c-10d} \times \frac{c-2d}{cd^2} = \frac{\cancel{cd}}{5(\cancel{c-2d})} \times \frac{\cancel{c-2d}}{\cancel{c}d^2} = \frac{1}{5d}$

   d. $\frac{x^2-4}{x^2-3x+2} \times \frac{5x}{x^2+2x} = \frac{\cancel{(x+2)}\cancel{(x-2)}}{\cancel{(x-2)}(x-1)} \times \frac{5\cancel{x}}{\cancel{x}(\cancel{x+2})} = \frac{5}{x-1}$

Closing *(2 minutes)*

1. Ask the pupils to revise the steps for multiplying algebraic fractions. (Example answer: Factor the numerator and denominator. Identify the common factors in the numerator and denominator. Divide the numerator and the denominator by the common factors. The result is the answer.)
2. For homework, have pupils do the practice activity PHM2-L033 in the Pupil Handbook.

Lesson Title: Division of Algebraic Fractions	**Theme:** Algebraic Processes	
Lesson Number: M2-L034	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to divide algebraic fractions, reducing them to their lowest terms.	**Preparation** None	

Opening *(3 minutes)*

1. Revise division of numerical fractions. Write on the board: Divide $\frac{1}{3} \div \frac{1}{9}$
2. Ask pupils to solve the problem with seatmates.
3. Invite a volunteer to write the solution on the board and explain. (Answer: $\frac{1}{3} \div \frac{1}{9} = \frac{1}{3} \times \frac{9}{1} = \frac{9}{3} = 3$; Explanation: To divide, multiply the first fraction by the reciprocal of the second.)
4. Explain that today's lesson is on dividing algebraic fractions.

Teaching and Learning *(20 minutes)*

1. Write on the board: Simplify $\frac{x^2+2x+1}{x+4} \div \frac{x+1}{5x+15}$.
2. Explain:
   - To divide by an algebraic fraction, multiply by the reciprocal of the second fraction. This is the same step we take when dividing fractions with numbers.
   - Then, follow the steps for multiplication of algebraic fractions from the previous lesson.
3. Solve on the board, explaining each step:

   Step 1. Multiply by the reciprocal of the second fraction:

   $$\frac{x^2+2x+1}{x+4} \times \frac{5x+15}{x+1}$$

   Step 2. Factor the numerators and denominators:

   $$\frac{x^2+2x+1}{x+3} \times \frac{5x+15}{x+1} = \frac{(x+1)(x+1)}{x+3} \times \frac{5(x+3)}{x+1}$$

   Step 3. Identify that common factors in the numerator and denominator are $(x+1)$ and $(x+3)$

   Step 4. Divide (cancel) the numerator and denominator by the factors:

   $$\frac{(x+1)\cancel{(x+1)}}{\cancel{x+3}} \times \frac{5\cancel{(x+3)}}{\cancel{x+1}}$$

   Step 5. Write the result: $\frac{5(x+1)}{1} = 5(x+1)$
4. Write the following problem on the board: Simplify $\frac{x^2+4x-5}{x} \div \frac{x^2+2x-15}{x^2-3x}$.

5. Ask a volunteer to give the first step. Write the multiplication on the board. (Answer: $\frac{x^2+4x-5}{x} \times \frac{x^2-3x}{x^2+2x-15}$)
6. Ask pupils to work with seatmates to factor both the numerator and denominator.
7. Invite a volunteer to write the factorisation on the board. (Answer: $\frac{(x+5)(x-1)}{x} \times \frac{x(x-3)}{(x+5)(x-3)}$)
8. Ask volunteers to list factors that the numerator and denominator have in common. (Answers: x, $(x+5)$ and $(x-3)$)
9. Ask pupils to work with seatmates to cancel these factors and write the answer.
10. Invite a volunteer to write the solution on the board.
    Solution:
    $$\frac{\cancel{(x+5)}(x-1)}{\cancel{x}} \times \frac{\cancel{x(x-3)}}{\cancel{(x+5)}\cancel{(x-3)}}) = \frac{x-1}{1} = x-1$$
11. Write the following problem on the board: Simplify $\frac{x^2+2x+1}{3xy^2} \div \frac{xy+y}{6y^4}$
12. Ask pupils to work with seatmates to write the multiplication.
13. Ask a volunteer to write the multiplication on the board. (Answer: $\frac{x^2+2x+1}{3xy^2} \times \frac{6y^4}{xy+y}$)
14. Ask pupils to work with seatmates to factor both the numerator and denominator.
15. Invite a volunteer to write the factorisation on the board. (Answer: $\frac{(x+1)(x+1)}{3xy^2} \times \frac{6y^4}{y(x+1)}$)
16. Ask volunteers to list factors that the numerator and denominator have in common. (Answers: 3, y, $(x+1)$)
17. Ask pupils to work with seatmates to cancel these factors and write the answer.
18. Invite a volunteer to write the solution on the board.
    Solution:
    $$\frac{(x+1)\cancel{(x+1)}}{3x\cancel{y^2}} \times \frac{6\cancel{y^4}}{\cancel{y}\cancel{(x+1)}} = \frac{2y(x+1)}{x}$$
19. Write the following on the board:

    Divide the fractions:

    a. $\frac{2xy}{y-x} \div \frac{4x}{y^2-xy}$ b. $\frac{x^2-xy}{y^2-yz} \div \frac{y^2-xy}{xy-xz}$ c. $\frac{a^2+3a-10}{4a^2+16a} \div \frac{a^2-25}{a^2-a-20}$

20. Ask pupils to work with seatmates to solve the 3 problems. Walk around and check for understanding.
21. Invite 3 volunteers to write their solutions on the board and explain. All other pupils should check their work.

Solutions:

a. $\frac{2xy}{y-x} \div \frac{4x}{y^2-xy} = \frac{2xy}{y-x} \times \frac{y^2-xy}{4x} = \frac{\cancel{2x}y}{\cancel{y-x}} \times \frac{y\cancel{(y-x)}}{\cancel{4x}} = \frac{y^2}{2}$

b. $\frac{x^2-xy}{y^2-yz} \div \frac{y^2-xy}{xy-xz} = \frac{x^2-xy}{y^2-yz} \times \frac{xy-xz}{y^2-xy} = \frac{x(x-y)}{y\cancel{(y-z)}} \times \frac{x\cancel{(y-z)}}{y(y-x)} = \frac{x^2(x-y)}{y^2(y-x)}$

c. $\frac{a^2+3a-10}{4a^2+16a} \div \frac{a^2-25}{a^2-a-20} = \frac{a^2+3a-10}{4a^2+16a} \times \frac{a^2-a-20}{a^2-25} = \frac{\cancel{(a+5)}(a-2)}{4a\cancel{(a+4)}} \times \frac{\cancel{(a-5)}\cancel{(a+4)}}{\cancel{(a+5)}\cancel{(a-5)}} = \frac{a+2}{4a}$

Practice *(15 minutes)*

1. Write the following on the board:

   Divide the following fractions:

   e. $\frac{9x}{20yz} \div \frac{3xz}{5y}$

   f. $\frac{x^2+xy}{4x+4y} \div \frac{x}{8y^2}$

   g. $\frac{ab}{5a+5b} \div \frac{ab-b^2}{a^2-b^2}$

   h. $\frac{x^2+x-12}{x^3-2x^2-3x} \div \frac{x^2+9x+20}{x^2+5x}$

2. Ask pupils to solve the problems individually.
3. Invite volunteers to come to the board and write the solutions. All other pupils should check their work.

   Solutions:

   a. $\frac{9x}{20yz} \div \frac{3xz}{5y} = \frac{9\cancel{x}}{20\cancel{y}z} \times \frac{\cancel{5y}}{\cancel{3xz}} = \frac{3}{4z^2}$

   b. $\frac{x^2+xy}{4x+4y} \div \frac{x}{8y^2} = \frac{x^2+xy}{4x+4y} \times \frac{8y^2}{x} = \frac{\cancel{x(x+y)}}{\cancel{4(x+y)}} \times \frac{8y^2}{\cancel{x}} = 2y^2$

   c. $\frac{ab}{5a+5b} \div \frac{ab-b^2}{a^2-b^2} = \frac{ab}{5a+5b} \times \frac{a^2-b^2}{ab-b^2} = \frac{a\cancel{b}}{5\cancel{(a+b)}} \times \frac{\cancel{(a+b)}\cancel{(a-b)}}{\cancel{b}\cancel{(a-b)}} = \frac{a}{5}$

   d. $\frac{x^2+x-12}{x^3-2x^2-3x} \div \frac{x^2+9x+20}{x^2+5x} = \frac{x^2+x-12}{x^3-2x^2-3x} \times \frac{x^2+5x}{x^2+9x+20} = \frac{\cancel{(x+4)}\cancel{(x-3)}}{\cancel{x(x-3)}(x+1)} \times \frac{\cancel{x(x+5)}}{\cancel{(x+5)}\cancel{(x+4)}} = \frac{1}{x+1}$

Closing *(2 minutes)*

1. Ask pupils to revise the steps for dividing algebraic fractions. (Example answer: Multiple the first fraction by the reciprocal of the second fraction. Follow the steps for multiplying fractions. Simplify using factorisation.)
2. For homework, have pupils do the practice activity PHM2-L034 in the Pupil Handbook.

Lesson Title: Addition and Subtraction of Algebraic Fractions – Part 1	**Theme:** Algebraic Processes	
Lesson Number: M2-L035	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to add and subtract algebraic fractions to give a single algebraic fraction.	**Preparation** None	

Opening *(4 minutes)*

1. Revise addition of numerical fractions. Write on the board: Add $\frac{1}{2}+\frac{1}{3}$
2. Ask pupils to solve the problem with seatmates.
3. Invite a volunteer to write the solution on the board and explain. (Answer:$\frac{1}{2}+\frac{1}{3}=\frac{3}{6}+\frac{2}{6}=\frac{5}{6}$; Explanation: Change each fraction to have the LCM of 2 and 3 in the denominators, then add them.)
4. Remind pupils that fractions should have the same denominator before adding or subtracting them.
5. Explain that today's lesson is on adding and subtracting algebraic fractions. The following lesson will be on adding and subtracting more complex algebraic fractions.

Teaching and Learning *(25 minutes)*

1. Write on the board: Simplify $\frac{5}{x}+\frac{2}{3y}$.
2. Explain:
   - Factor the algebraic fractions if possible.
   - To add or subtract algebraic fractions, the denominators should be the same. Express each fraction with a denominator that is the LCM of the denominators in the problem.
   - Once the fractions have the same denominator, they are **like fractions** and can be added or subtracted.
   - Add the fractions, and combine like terms if possible.
   - If the answer can be simplified, simplify it. This requires factoring the answer.
3. Solve on the board, explaining each step:
   Step 1: Find the LCM of x and $3y$. x and $3y$ do not have common factors, so we find the LCM by multiplying them: $3xy$.
   Step 2: Change both denominators to the LCM:
   $$\frac{5}{x}+\frac{2}{3y}=\frac{5\times 3y}{x\times 3y}+\frac{2\times x}{3y\times x}=\frac{15y}{3xy}+\frac{2x}{3xy}$$
   Step 3: Add the fractions by adding the numerators:

$$\frac{15y}{3xy} + \frac{2x}{3xy} = \frac{15y+2x}{3xy}$$

4. Explain: The answer cannot be simplified. Remember to simplify your answer if possible.
5. Write the following problem on the board: Simplify $\frac{3x+2}{2} - \frac{x+6}{3}$.
6. Ask a volunteer to give the LCM of the denominators, 2 and 3. (Answer: 6)
7. Solve on the board, explaining each step:

$\frac{3x+2}{2} - \frac{x+6}{3}$	$= \frac{(3x+2)\times 3}{2\times 3} - \frac{(x+6)\times 2}{3\times 2}$	Change both denominators to the LCM
	$= \frac{9x+6}{6} - \frac{2x+12}{6}$	Simplify
	$= \frac{9x+6-(2x+12)}{6}$	Subtract the numerators
	$= \frac{9x+6-2x-12}{6}$	
	$= \frac{7x-6}{6}$	Combine like terms

8. Write the following problem on the board: $\frac{4x}{2} - \frac{3}{y}$:
9. Ask a volunteer to give the LCM of the denominators, 2 and y. (Answer: $2y$)
10. Ask pupils to work with seatmates to rewrite the fractions as like fractions.
11. Invite a volunteer to rewrite the fractions as like fractions on the board.

Answer:

$$\frac{4x}{2} - \frac{3}{y} = \frac{4x\times y}{2\times y} - \frac{3\times 2}{y\times 2}$$
$$= \frac{4xy}{2y} - \frac{6}{2y}$$

12. Ask volunteers to describe each step required to finish solving the problem. As they give the step, work it on the board.

Solution:

$\frac{4x}{2} - \frac{3}{y}$	$= \frac{4xy}{2y} - \frac{6}{2y}$	
	$= \frac{4xy-6}{2y}$	Subtract the numerators
	$= \frac{2(2xy-3)}{2y}$	Factor the numerator
	$= \frac{2xy-3}{y}$	Divide by 2 (cancel 2) in the numerator and denominator

13. Write the following problem on the board: $\frac{x}{4} - \frac{x+2}{x}$
14. Ask pupils to solve the problem with seatmates.
15. Invite a volunteer to write the solution on the board.

Solution:

$\frac{x}{4} - \frac{x+2}{x}$	$= \frac{x\times x}{4\times x} - \frac{(x+2)\times 4}{x\times 4}$	Change both denominators to the LCM
	$= \frac{x^2}{4x} - \frac{4x+8}{4x}$	Simplify

$$= \frac{x^2-(4x+8)}{4x}$$ Subtract the numerators

$$= \frac{x^2-4x-8}{4x}$$

Practice *(10 minutes)*

1. Write on the board: Simplify the following:
   i. $\frac{2}{3z}+\frac{1}{4y}$
   j. $\frac{x+3}{4}-\frac{x-2}{6}$
2. Ask pupils to solve the problems individually.
3. Invite volunteers to come to the board and write the solutions. All other pupils should check their work.

   Solutions:

   a.

   $$\frac{2}{3z}+\frac{1}{4y} = \frac{2\times 4y}{3z\times 4y}+\frac{1\times 3z}{4y\times 3z}$$ Change both denominators to the LCM

   $$= \frac{8y}{12yz}-\frac{3z}{12yz}$$ Simplify

   $$= \frac{8y-3z}{12yz}$$ Add the numerators

   b.

   $$\frac{x+3}{4}-\frac{x-2}{6} = \frac{(x+3)\times 3}{4\times 3}-\frac{(x-2)\times 2}{6\times 2}$$ Change both denominators to the LCM

   $$= \frac{3x+9}{12}-\frac{2x-4}{12}$$ Simplify

   $$= \frac{3x+9-(2x-4)}{12}$$ Subtract the numerators

   $$= \frac{3x+9-2x+4}{12}$$ Simplify

   $$= \frac{x+13}{12}$$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM2-L035 in the Pupil Handbook.

Lesson Title: Addition and Subtraction of Algebraic Fractions – Part 2	**Theme:** Algebraic Processes	
Lesson Number: M2-L036	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to add and subtract algebraic fractions to give a single algebraic fraction.	**Preparation** None	

Opening *(4 minutes)*

1. Revise addition of simple algebraic fractions. Write the following problem on the board: Add $\frac{2}{x} + \frac{6}{y}$
2. Ask pupils to solve the problem with seatmates.
3. Invite a volunteer to write the solution on the board and explain.
   Solution:

$\frac{2}{x} + \frac{6}{y}$	$=$	$\frac{2\times y}{x\times y} + \frac{6\times x}{y\times x}$	Change both denominators to the LCM
	$=$	$\frac{2y}{xy} + \frac{6x}{xy}$	Simplify
	$=$	$\frac{2y+6x}{xy}$	Add the numerators
	$=$	$\frac{2(y+3x)}{xy}$	Factor

4. Explain that today's lesson is also on adding and subtracting algebraic fractions. The problems that pupils will solve today are more complex.

Teaching and Learning *(25 minutes)*

1. Ask volunteers to explain in their own words the process for adding or subtracting algebraic fractions. Allow them to discuss. (Example answer: Rewrite the problem with like fractions using the LCM. Add the like fractions, then factor and simplify the answer.)
2. Explain: We will follow the same process today. The problems in today's lesson will require more factoring.
3. Write the following problem on the board: Simplify $\frac{x}{x+1} + \frac{x^2-4}{3x+3}$.
4. Solve on the board, explaining each step:
   Step 1. Factor the numerators and denominators:
   $$\frac{x}{x+1} + \frac{x^2-4}{3x+3} = \frac{3}{x+1} + \frac{(x+2)(x-2)}{3(x+1)}$$
   Step 2. Find the LCM of $x + 1$ and $3(x + 1)$ by multiplying their common factors, 3 and $x + 1$. The LCM is $3(x + 1)$.
   Step 3. Change the denominators to the LCM:
   $$\frac{x}{x+1} + \frac{(x+2)(x-2)}{3(x+1)} = \frac{x\times 3}{(x+1)\times 3} + \frac{(x+2)(x-2)}{3(x+1)} = \frac{3x}{3(x+1)} + \frac{(x+2)(x-2)}{3(x+1)}$$
   Step 4. Add the fractions by adding the numerators:

$\frac{3x+(x+2)(x-2)}{3(x+1)}$

Step 5. Multiply out the numerator so like terms can be combined:

$\frac{3x+(x+2)(x-2)}{3(x+1)} = \frac{3x+x^2+2x-2x-4}{3(x+1)}$

Step 6. Combine like terms in the numerator:

$\frac{x^2+3x-4}{3(x+1)}$

Step 7. Factor:

$\frac{(x+4)(x-1)}{3(x+1)}$

5. Explain: This answer cannot be simplified. Remember to simplify your answer if possible.
6. Write the following problem on the board: Simplify $\frac{x+2}{x^2-2x} - \frac{x+1}{x^2-x-2}$.
7. Ask pupils to work with seatmates to factor the 2 denominators: $x^2 - 2x$ and $x^2 - x - 2$.
8. Invite volunteers to write the factorisation of each on the board: (Answers: $x^2 - 2x = x(x-2)$ and $x^2 - x - 2 = (x-2)(x+1)$)
9. Ask volunteers to list the factors in the denominators. (Answer: x, $x-2$ and $x+1$)
10. Multiply the factors together to give the LCM: $x(x-2)(x+1)$
11. Solve on the board, explaining each step:

$\frac{x+2}{x^2-2x} - \frac{x+1}{x^2-x-2}$	$= \frac{x+2}{x(x-2)} - \frac{x+1}{(x-2)(x+1)}$	Factor the denominators
	$= \frac{(x+2)(x+1)}{x(x-2)(x+1)} - \frac{x(x+1)}{x(x-2)(x+1)}$	Change denominators to the LCM
	$= \frac{(x+2)(x+1)-x(x+1)}{x(x-2)(x+1)}$	Subtract the numerators
	$= \frac{x^2+3x+2-x^2-x}{x(x-2)(x+1)}$	Multiply out the brackets
	$= \frac{2x+2}{x(x-2)(x+1)}$	Combine like terms
	$= \frac{2(x+1)}{x(x-2)(x+1)}$	Cancel $(x+1)$
	$= \frac{2}{x(x-2)}$	

12. Write the following problem on the board: $\frac{a-2}{a^2-4a} - \frac{a-3}{a^2-2a-8}$
13. Invite volunteers to write the factorisation of each on the board: (Answers: $a^2 - 4a = a(a-4)$ and $a^2 - 2a - 8 = (a-4)(a+2)$)
14. Ask volunteers to list the factors in the denominators. (Answer: a, $a-4$ and $a+2$)
15. Multiply the factors together to give the LCM: $a(a-4)(a+2)$
16. Ask volunteers to describe each step required to solve the problem. As they give the step, work it on the board.

Solution:

$$\frac{a-2}{a^2-4a} - \frac{a-3}{a^2-2a-8} = \frac{a-2}{a(a-4)} - \frac{a-3}{(a-4)(a+2)}$$ Factor the denominators

$$= \frac{(a-2)(a+2)}{a(a-4)(a+2)} - \frac{a(a-3)}{a(a-4)(a+2)}$$ Change denominators to the LCM

$$= \frac{(a-2)(a+2)-a(a-3)}{a(a-4)(a+2)}$$ Subtract the numerators

$$= \frac{a^2-2a+2a-4-a^2+3a}{a(a-4)(a+2)}$$ Multiply out the brackets

$$= \frac{3a-4}{a(a-4)(a+2)}$$ Combine like terms

17. Write the following problem on the board: $\frac{x}{2x-4} + \frac{4}{x^2-4}$
18. Ask pupils to solve the problem with seatmates.
19. Invite a volunteer to write the solution on the board.

Solution:

$$\frac{x}{2x-4} + \frac{4}{x^2-4} = \frac{x}{2(x-2)} + \frac{4}{(x-2)(x+2)}$$ Factor the denominators

$$= \frac{x(x+2)}{2(x-2)(x+2)} + \frac{2(4)}{2(x-2)(x+2)}$$ Change denominators to the LCM

$$= \frac{x(x+2)+8}{2(x-2)(x+2)}$$ Add the numerators

$$= \frac{x^2+2x+8}{2(x-2)(x+2)}$$ Multiply out the brackets

Practice *(10 minutes)*

1. Write the following on the board:

   Simplify:

   a. $\frac{a}{2a+4b} + \frac{b}{a+2b}$
   b. $\frac{x}{x^2+x} - \frac{1}{x+3}$
2. Ask pupils to solve the problems individually.
3. Invite volunteers to come to the board and write the solutions. All other pupils should check their work.

Solutions:

a.

$$\frac{a}{2a+4b} + \frac{b}{a+2b} = \frac{a}{2(a+2b)} + \frac{b}{a+2b}$$ Factor the denominators

$$= \frac{a}{2(a+2b)} + \frac{2b}{2(a+2b)}$$ Change denominators to the LCM

$$= \frac{a+2b}{2(a+2b)}$$ Add the numerators

$$= \frac{1}{2}$$ Cancel $a + 2b$

b.

$\frac{x}{x^2+x} - \frac{1}{x+3}$	$= \frac{x}{x(x+1)} - \frac{1}{x+3}$	Factor the denominators
	$= \frac{x(x+3)}{x(x+1)(x+3)} - \frac{x(x+1)}{x(x+1)(x+3)}$	Change denominators to the LCM
	$= \frac{x(x+3)-x(x+1)}{x(x+1)(x+3)}$	Subtract the numerators
	$= \frac{x^2+3x-x^2-x}{x(x+1)(x+3)}$	Multiply out the brackets
	$= \frac{2x}{x(x+1)(x+3)}$	Combine like terms
	$= \frac{2}{(x+1)(x+3)}$	Cancel x

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM2-L036 in the Pupil Handbook.

Lesson Title: Substitution in Algebraic Fractions	**Theme:** Algebraic Processes	
Lesson Number: M2-L037	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to use the substitution of numerical values or algebraic terms to simplify given algebraic fractions.	**Preparation** None	

Opening *(4 minutes)*

1. Revise basic substitution. Write the following problem on the board: If $x = -3$, evaluate $y = x^2 - x$
2. Ask pupils to solve the problem with seatmates.
3. Invite a volunteer to write the solution on the board and explain.
   Solution:

$$\begin{aligned} y &= x^2 - x \\ &= (-3)^2 - (-3) \qquad \text{Substitute } x = -3 \\ &= 9 + 3 \\ &= 12 \end{aligned}$$

4. Explain that today's lesson is on substitution in algebraic fractions. There are 2 different types of problems pupils will learn to solve.

Teaching and Learning *(25 minutes)*

1. Write the following problem on the board: If $\frac{x}{y} = \frac{1}{2}$, evaluate $\frac{4x-y}{2x+3y}$.
2. Explain:
   - We know the value of $\frac{x}{y}$. However, there is no $\frac{x}{y}$ in the given formula.
   - Divide the numerator and denominator of the formula by y. That is, divide each term in the numerator and denominator by y. Each x term will become a fraction with $\frac{x}{y}$, and each y will be eliminated.
3. Solve on the board, explaining each step:
   Step 1: Divide the numerator and denominator of $\frac{4x-y}{2x+3y}$ by y:

$$\frac{4x-y}{2x+3y} = \frac{4\left(\frac{x}{y}\right)-1}{2\left(\frac{x}{y}\right)+3}$$

   Step 2: Substitute $\frac{x}{y} = \frac{1}{2}$ into the expression:

$$= \frac{4\left(\frac{1}{2}\right)-1}{2\left(\frac{1}{2}\right)+3}$$

   Step 3: Simplify:

$$= \frac{\frac{4}{2}-1}{\frac{2}{2}+3}$$

$$= \frac{2-1}{1+3} = \frac{1}{4}$$

4. Write the following problem on the board: Given that $x\!:\!y = 3\!:\!2$, evaluate $\frac{8x+2y}{x-2y}$.
5. Then write on the board: If $x\!:\!y = 3\!:\!2$ then $\frac{x}{y} = \frac{3}{2}$
6. Explain that although the question used a ratio instead of a fraction, it will be solved in the same way.
7. Ask volunteers to describe the first step. (Example answer: Divide the numerator and denominator by y to get the fraction $\frac{x}{y}$ in the formula.)
8. Solve on the board, explaining each step:

$\frac{8x+2y}{x-2y}$	$=$	$\frac{8\left(\frac{x}{y}\right)+2}{\left(\frac{x}{y}\right)-2}$	Divide throughout by y
	$=$	$\frac{8\left(\frac{3}{2}\right)+2}{\left(\frac{3}{2}\right)-2}$	Substitute $\frac{x}{y} = \frac{3}{2}$
	$=$	$\frac{12+2}{\left(\frac{3}{2}\right)-2}$	Simplify
	$=$	$\frac{14}{\frac{3}{2}-\frac{2}{1}} = \frac{14}{\frac{3}{2}-\frac{4}{2}} = \frac{14}{-\frac{1}{2}}$	Subtract the denominator
	$=$	$14 \times \left(-\frac{2}{1}\right)$	Divide
	$=$	-28	

9. Write another problem on the board: $a\!:\!b = 4\!:\!3$, evaluate $\frac{3a-2b}{6a+b}$
10. Ask pupils to solve the problem with seatmates.
11. Invite a volunteer to write the solution on the board.

Solution:

$\frac{3a-2b}{6a+b}$	$=$	$\frac{3\left(\frac{a}{b}\right)-2}{6\left(\frac{a}{b}\right)+1}$	Divide throughout by b
	$=$	$\frac{3\left(\frac{4}{3}\right)-2}{6\left(\frac{4}{3}\right)+1}$	Substitute $\frac{a}{b} = \frac{4}{3}$
	$=$	$\frac{4-2}{8+1}$	Simplify
	$=$	$\frac{2}{9}$	

12. Write another problem on the board: If $x = \frac{z+1}{z-1}$, express $\frac{x+1}{x-1}$ in terms of z.
13. Explain:
    - When you see a problem with "in terms of", you want to find the result with only the given variable (in this case z).
    - Substitute $x = \frac{z+1}{z-1}$ into the formula $\frac{x+1}{x-1}$ for each x, and simplify the result.
14. Solve on the board, explaining each step:

Step 1: Substitute $x = \frac{z+1}{z-1}$ for each x in the given expression.

$$\frac{x+1}{x-1} = \frac{\left(\frac{z+1}{z-1}\right)+1}{\left(\frac{z+1}{z-1}\right)-1}$$

Step 2: Multiply the numerator and denominator by $z - 1$ (this will eliminate the fractions):

$$= \frac{(z+1)+(z-1)}{(z+1)-(z-1)}$$

Step 3: Combine like terms and simplify:

$= \frac{z+1+z-1}{z+1-z+1}$
$= \frac{2z}{2}$
$= z$

15. Write another problem on the board: If $x = \frac{2a+3}{2a-3}$, express $\frac{x-2}{x+2}$ in terms of a.
16. Ask pupils to work with seatmates to do the first step, substitute $x = \frac{2a+3}{2a-3}$ into the given formula.
17. Invite a volunteer to write the answer on the board. (Answer: $\frac{\left(\frac{2a+3}{2a-3}\right)-2}{\left(\frac{2a+3}{2a-3}\right)+2}$)
18. Ask volunteers to describe each step. Work the problem on the board as they explain:

$\frac{x-2}{x+2}$	$=$	$\frac{\left(\frac{2a+3}{2a-3}\right)-2}{\left(\frac{2a+3}{2a-3}\right)+2}$	Substitute $x = \frac{2a+3}{2a-3}$
	$=$	$\frac{(2a+3)-2(2a-3)}{(2a+3)+2(2a-3)}$	Multiply throughout by $2a-3$
	$=$	$\frac{2a+3-4a+6}{2a+3+4a-6}$	Remove brackets
	$=$	$\frac{-2a+9}{6a-3}$	Combine like terms
	$=$	$\frac{-2a+9}{3(2a-1)}$	Factor. Answer cannot be simplified.

19. Write another problem on the board: If $b = \frac{m+1}{m-1}$, express $\frac{2b-1}{3b+2}$ in terms of m.
20. Ask pupils to solve the problem with seatmates.
21. Invite a volunteer to write the solution on the board.

Solution:

$\frac{2b-1}{3b+2}$	$=$	$\frac{2\left(\frac{m+1}{m-1}\right)-1}{3\left(\frac{m+1}{m-1}\right)+2}$	Substitute $b = \frac{m+1}{m-1}$
	$=$	$\frac{2(m+1)-(m-1)}{3(m+1)+2(m-1)}$	Multiply throughout by $m-1$
	$=$	$\frac{2m+2-m+1}{3m+3+2m-2}$	Remove brackets
	$=$	$\frac{m+3}{5m+1}$	Combine like terms

Practice *(10 minutes)*

1. Write on the board: Solve the following:
   a. If $x{:}y = 3{:}5$, evaluate $\frac{x-2y}{10x+y}$
   b. If $a = \frac{2x+1}{x-1}$, express $\frac{2a+1}{3a-1}$ in terms of x.
2. Ask pupils to solve the problems individually.
3. Invite volunteers to come to the board and write the solutions. All other pupils should check their work.

Solutions:

a.

$\frac{x-2y}{10x+y}$	$=$	$\frac{\left(\frac{x}{y}\right)-2}{10\left(\frac{x}{y}\right)+1}$	Divide throughout by y

$$= \frac{\left(\frac{3}{5}\right)-2}{10\left(\frac{3}{5}\right)+1}$$ Substitute $\frac{x}{y} = \frac{3}{5}$

$$= \frac{\left(\frac{3}{5}\right)-2}{6+1}$$ Simplify

$$= \frac{\frac{3}{5}-\frac{2}{1}}{7} = \frac{\frac{3}{5}-\frac{10}{5}}{7} = \frac{\frac{-7}{5}}{7}$$ Subtract the denominator

$$= \frac{-7}{5\times7}$$ Simplify

$$= -\frac{1}{5}$$

b.

$$\frac{2a+1}{3a-1} = \frac{2\left(\frac{2x+1}{x-1}\right)+1}{3\left(\frac{2x+1}{x-1}\right)-1}$$ Substitute $a = \frac{2x+1}{x-1}$

$$= \frac{2(2x+1)+(x-1)}{3(2x+1)-(x-1)}$$ Multiply throughout by $x-1$

$$= \frac{4x+2+x-1}{6x+3-x+1}$$ Remove brackets

$$= \frac{5x+1}{5x+4}$$ Combine like terms

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM2-L037 in the Pupil Handbook.

Lesson Title: Equations with Algebraic Fractions	**Theme:** Algebraic Processes	
Lesson Number: M2-L038	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to solve equations that contain algebraic fractions.	**Preparation** None	

Opening *(4 minutes)*

1. Revise LCM. Write two algebraic fractions on the board: $\frac{2}{x+1}$ and $\frac{y^2}{x-4}$
2. Ask pupils to write the LCM of the denominators in their exercise books.
3. Ask a volunteer to give the answer. (Answer: LCM= $(x+1)(x-4)$)
4. Follow the same process with another example: $\frac{y+1}{x^2+x}$ and $\frac{y-1}{x}$. (Answer: LCM= $x(x+1)$)
5. Explain that today's lesson is on solving equations with algebraic fractions. This involves identifying the LCM of the denominators.

Teaching and Learning *(20 minutes)*

1. Write the following problem on the board: Solve: $\frac{2}{x} = x - 1$
2. Explain:
   - This is an equation that involves an algebraic fraction.
   - To solve equations with algebraic fractions, multiply throughout by the LCM of the denominators of any fractions in the equation. This eliminates the fractions.
   - Then, balance the equation and solve for the variable.
3. Solve on the board, explaining each step:
   Step 1. Multiply throughout by the LCM, x:
   $$x\left(\frac{2}{x}\right) = x(x-1)$$
   $$2 = x^2 - x$$
   Step 2. Balance the equation:
   $$x^2 - x - 2 = 0$$
   Step 3. Solve for x:
   $$(x-2)(x+1) = 0$$
   $x - 2 = 0$ or $x + 1 = 0$
   Answer: $x = 2, -1$
4. Write the following problem on the board: Solve $\frac{4}{x+3} - \frac{3}{x+2} = 0$
5. Remind pupils of the first step: multiply throughout by the LCM of the denominators. Ask a volunteer to give the LCM. (Answer: $(x+3)(x+2)$)
6. Solve on the board, explaining each step:
   $$\frac{4}{x+3} - \frac{3}{x+2} = 0$$

$$\frac{4(x+3)(x+2)}{x+3} - \frac{3(x+3)(x+2)}{x+2} = 0 \quad \text{Multiply throughout by the LCM}$$
$$4(x+2) - 3(x+3) = 0 \quad \text{Cancel } (x+3) \text{ and } (x+2)$$
$$4x + 8 - 3x - 9 = 0 \quad \text{Remove brackets}$$
$$x - 1 = 0 \quad \text{Combine like terms}$$
$$x = 1 \quad \text{Solve for } x$$

7. Write the following problem on the board: $\frac{2}{2b+1} = \frac{6}{b-2}$
8. Ask volunteers to give the LCM of the denominators. (Answer: $(2b+1)(b-2)$)
9. Ask volunteers to describe each step. Work the problem on the board as they explain:

$$\frac{2}{2b+1} = \frac{6}{b-2}$$
$$\frac{2(2b+1)(b-2)}{2b+1} = \frac{6(2b+1)(b-2)}{b-2} \quad \text{Multiply throughout by the LCM}$$
$$2(b-2) = 6(2b+1) \quad \text{Cancel } (2b+1) \text{ and } (b-2)$$
$$2b - 4 = 12b + 6 \quad \text{Remove brackets}$$
$$2b - 12b = 6 + 4 \quad \text{Balance the equation}$$
$$-10b = 10$$
$$b = \frac{10}{-10} = -1$$

10. Write the following problem on the board: $\frac{y-3}{y+5} = y$
11. Ask pupils to solve the problem with seatmates.
12. Invite a volunteer to write the solution on the board.

Solution:

$$\frac{y-3}{y+5} = y$$
$$\frac{(y-3)(y+5)}{y+5} = y(y+5) \quad \text{Multiply throughout by the LCM}$$
$$y - 3 = y(y+5) \quad \text{Cancel } (y+5)$$
$$y - 3 = y^2 + 5y \quad \text{Remove brackets}$$
$$0 = y^2 + 5y - y + 3 \quad \text{Balance the equation}$$
$$0 = y^2 + 4y + 3$$
$$0 = (y+3)(y+1)$$

Answer: $y = -3, -1$

Practice *(15 minutes)*

1. Write the following on the board:
   Solve the following:
   c. $3x = \frac{6}{x+1}$
   d. $\frac{12}{a-3} + \frac{6}{a+2} = 0$
2. Ask pupils to solve the problems individually.
3. Invite volunteers to come to the board and write the solutions. All other pupils should check their work.

   Solutions:

   a.

$$3x = \frac{6}{x+1}$$

$$3x(x+1) = \frac{6(x+1)}{x+1}$$ Multiply throughout by the LCM

$$3x(x+1) = 6$$ Cancel $(x+1)$

$$3x^2 + 3x = 6$$ Remove brackets

$$3x^2 + 3x - 6 = 0$$ Balance the equation

$$3(x^2 + x - 2) = 0$$ Solve for x

$$3(x+2)(x-1) = 0$$

Answer: $x = -2, 1$

b.

$$\frac{12}{a-3} + \frac{6}{a+2} = 0$$

$$\frac{12(a-3)(a+2)}{a-3} + \frac{6(a-3)(a+2)}{a+2} = 0$$ Multiply throughout by the LCM

$$12(a+2) + 6(a-3) = 0$$ Cancel $(a-3)$ and $(a+2)$

$$12a + 24 + 6a - 18 = 0$$ Remove brackets

$$18a + 6 = 0$$ Combine like terms

$$18a = -6$$ Balance the equation

$$a = \frac{-6}{18} = -\frac{1}{3}$$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM2-L038 in the Pupil Handbook.

Lesson Title: Undefined algebraic fractions	Theme: Algebraic Processes	
Lesson Number: M2-L039	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to determine the values that make an algebraic fraction undefined.	**Preparation** None	

Opening *(4 minutes)*

1. Revise the previous lesson. Write on the board: Solve $\frac{4}{x+3} = \frac{3}{x}$
2. Ask a volunteer to give the LCM of the denominators. (Answer: LCM= $x(x+3)$)
3. Ask pupils to solve the problem in their exercise books.
4. Invite a volunteer to write the solution on the board. Other pupils should check their work.

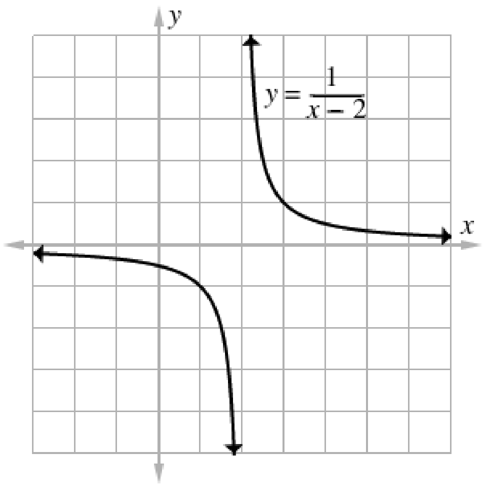
   Solution:

$$\frac{4}{x+3} = \frac{3}{x}$$
$$\frac{4x(x+3)}{x+3} = \frac{3x(x+3)}{x} \quad \text{Multiply throughout by the LCM}$$
$$4x = 3(x+3) \quad \text{Cancel } (x+3) \text{ and } x$$
$$4x = 3x + 9 \quad \text{Remove brackets}$$
$$x = 9 \quad \text{Balance the equation}$$

5. Explain that today's lesson is on undefined algebraic fractions.

Teaching and Learning *(25 minutes)*

1. Write the following function on the board and copy its graph from below: $y = \frac{1}{x-2}$
   - While graphing, note that the curve approaches $x = 2$ but never touches it.



2. Explain:

- This function approaches $x = 2$ from the left and from the right. It never touches $x = 2$.
- The function y is said to be **undefined** at $x = 2$ because it has no value there.

3. Substitute $x = 2$ into y on the board, and evaluate: $y = \frac{1}{2-2} = \frac{1}{0}$
4. Explain:
   - At $x = 2$, we have $y = \frac{1}{0}$. It is **impossible** to divide by zero.
   - The fraction $\frac{1}{x-2}$ is undefined when $x = 2$.
   - Any fraction is undefined when the denominator equals zero.
5. Write the following problem on the board: Find the value of x for which $\frac{x-3}{x+4}$ is not defined.
6. Ask pupils to explain in their own words how to find where $\frac{x-3}{x+4}$ is undefined. (Answer: Set the denominator equal to zero and solve for x.)
7. Solve on the board, explaining each step:
   Step 1. Set the denominator equal to zero:
   $x + 4 = 0$
   Step 2. Solve for x by transposing 4:
   $x = -4$
8. Write the following problem on the board: Find the values of x for which $\frac{x^2+2x+1}{x^2-4x-5}$ is undefined.
9. Explain: In this case, the denominator must be factored before finding the values at which the fraction is undefined.
10. Solve on the board, explaining each step:
    Step 1. Set the denominator equal to zero:
    $x^2 - 4x - 5 = 0$
    Step 2. Factor the denominator:
    $(x - 5)(x + 1) = 0$
    Step 3. Solve for x in each factor:
    $x - 5 = 0 \rightarrow x = 5$
    $x + 1 = 0 \rightarrow x = -1$
11. Write the following problem on the board: Find the values of x for which $\frac{1}{x+3} + \frac{1}{x}$ is undefined.
12. Explain:
    - If an expression contains an undefined fraction, the whole expression is undefined.
    - The expression is undefined where any of the fractions has a denominator of zero.
13. Solve the problem on the board:
    Step 1. Set **each** denominator equal to zero:
    $x = 0$ and $x + 3 = 0$

Step 2. Solve for x in the second expression by transposing 3:

$x = -3$

14. Explain: The two values of x for which $\frac{1}{x+3} + \frac{1}{x}$ is undefined are 0 and -3.
15. Write the following on the board: Find the values of x for which the following fractions are undefined:
    a. $\frac{9}{x-12}$
    b. $\frac{2x+3}{4x-2}$
    c. $\frac{5x+1}{x^2+4x+4} + \frac{1}{x-1}$
16. Ask pupils to work with their seatmates to solve the 3 problems. Walk around and check for understanding.
17. Invite 3 volunteers to write their solutions on the board and explain. All other pupils should check their work.

    Solutions:

    a. $x - 12 = 0$ → undefined at $x = 12$
    b. $4x - 2 = 0$ → $4x = 2$ → $x = \frac{2}{4}$ → undefined at $x = \frac{1}{2}$
    c. First fraction: $x^2 + 4x + 4 = 0$ →$(x + 2)(x + 2) = 0$→ $x = -2$
       Second fraction: $x - 1$→ $x = 1$
       The expression is undefined at two points, $x = -2$ and $x = 1$.

Practice *(10 minutes)*

1. Write the following on the board:
   Find the values of x for which the following fractions are not defined:
   a. $\frac{x+6}{x^2-6x}$
   b. $\frac{x}{x^2-4} + \frac{x+1}{x-3}$
2. Ask pupils to solve the problems individually.
3. Invite volunteers to come to the board and write the solutions. All other pupils should check their work.

   Solutions:

   a. $x^2 - 6x = 0$ → $x(x - 6) = 0$ → undefined at $x = 0, 6$
   b. First fraction: $x^2 - 4 = 0$ → $(x + 2)(x - 2) = 0$ → $x = 2, -2$
      Second fraction: $x - 3 = 0$ → $x = 3$
      The expression is undefined at three points, $x = 2, -2, 3$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM2-L039 in the Pupil Handbook

Lesson Title: Algebraic fraction problem solving	**Theme:** Algebraic Processes	
Lesson Number: M2-L040	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to solve various problems that contain algebraic fractions.	**Preparation** Write the problems at the start of the **Teaching and Learning** section on the board.	

Opening *(2 minutes)*

1. Revise algebraic fraction topics briefly. Discuss:
   - What topics have we covered in algebraic fractions? (Answers: Simplifying algebraic fractions; operations on algebraic fractions; substitution; solving equations; undefined algebraic fractions)
   - Which topics were most challenging? Which topics did you enjoy most? Why?
2. Explain that the WASSCE exam usually has at least one question on algebraic fractions. In today's lesson, pupils will practise solving various WASSCE-style questions on algebraic fractions.

Teaching and Learning *(20 minutes)*

1. Write the following problems on the board:
   a. Express $5-\left(\frac{a-b}{b}\right)$ as a single fraction.
   b. Simplify: $\frac{32a^2-8}{2a-1}$.
   c. If $\frac{3}{x-4}-\frac{4}{x-3}$ is equal to $\frac{S}{(x-4)(x-3)}$, find S.
      A. $-x-7$ B. $x-7$ C. $-x-25$ D. $7-x$
   d. Simplify $\frac{a}{b}+\frac{(a-2)}{3b}-\frac{(a-4)}{6b}$.
   e. Given that $y=1-\frac{x}{x-5}$, find the value of x for which y is undefined.
      A. $x=1$ B. $x=-1$ C. $x=5$ D. $x=-5$
2. Explain: Some of the problems are multiple choice. Solve the problem and give your answer as one of the letter choices.
3. Ask pupils to work with seatmates to solve the problems on the board. Encourage them to use their Pupil Guidebooks and notes from previous lessons to solve the problems.
4. Walk around to check for understanding and clear misconceptions. Provide guidance if pupils struggle with a particular problem or topic.
5. Invite 5 volunteers to each solve one of the problems on the board. They should explain the solution. Allow other pupils to ask questions.
   Solutions:
   a. Express $5-\left(\frac{a-b}{b}\right)$ as a single fraction.
      Solution:

Note that $5 = \frac{5}{1}$, and the LCM of the denominators is b.

$5 - \left(\frac{a-b}{b}\right) = \frac{5 \times b}{b} - \left(\frac{a-b}{b}\right)$ Change both denominators to the LCM

$= \frac{5b-(a-b)}{b}$ Subtract the numerators

$= \frac{5b-a+b}{b}$

$= \frac{6b-a}{b}$ Combine like terms

b. Simplify: $\frac{32a^2-8}{2a-1}$.

Solution:

$\frac{32a^2-8}{2a-1} = \frac{8(4a^2-1)}{2a-1}$ Factor the numerator and denominator

$= \frac{8(2a+1)(2a-1)}{2a-1}$

$= 8(2a+1)$ Divide by (cancel) $2a-1$

c. If $\frac{3}{x-4} - \frac{4}{x-3}$ is equal to $\frac{S}{(x-4)(x-3)}$, find S.

A. $-x-7$ B. $x-7$ C. $-x-25$ D. $7-x$

Solution:

Note that the LCM of the denominators is $(x-4)(x-3)$.

$\frac{3}{x-4} - \frac{4}{x-3} = \frac{3(x-3)}{(x-4)(x-3)} - \frac{4(x-4)}{(x-4)(x-3)}$ Change denominators to the LCM

$= \frac{3(x-3)-4(x-4)}{(x-4)(x-3)}$ Subtract the numerators

$= \frac{3x-9-4x+16}{(x-4)(x-3)}$ Multiply out the brackets

$= \frac{-x+7}{(x-4)(x-3)}$ Combine like terms

The numerator is $S = -x + 7$, which is the same as D. $7 - x$.

d. Simplify $\frac{a}{b} + \frac{(a-2)}{3b} - \frac{(a-4)}{6b}$.

Solution:

Note that the LCM of the denominators is $6b$.

$\frac{a}{b} + \frac{(a-2)}{3b} - \frac{(a-4)}{6b} = \frac{6a}{6b} + \frac{2(a-2)}{6b} - \frac{(a-4)}{6b}$ Change denominators to the LCM

$= \frac{6a+2(a-2)-(a-4)}{6b}$ Add and subtract the numerators

$= \frac{6a+2a-4-a+4}{6b}$ Multiply out the brackets

$= \frac{7a}{6b}$ Combine like terms

e. Given that $y = 1 - \frac{x}{x-5}$, find the value of x for which y is undefined.

Solution:

Note that a function is undefined when any part is undefined. Thus, y is undefined when $x - 5 = 0$. Transposing -5 gives answer C. $x = 5$.

Practice *(17 minutes)*

1. Write the following problems on the board:
   a. Simplify: $\frac{2x^2-5x-3}{4x^2-1}$
   b. For which value of x is the function $y = \frac{3x-1}{x+7}$ undefined?
      (a) $\frac{1}{3}$ B. -7 C. 7 D. $\frac{1}{7}$
   c. Simplify: $\frac{4}{3xy} - \frac{3}{2yz}$
2. Ask pupils to solve the problems individually. Allow them to use notes and discuss with seatmates as needed.
3. Invite volunteers to come to the board and write the solutions. All other pupils should check their work.

   Solutions: Type equation here.

   a. Simplify: $\frac{2x^2-5x-3}{4x^2-1}$

   Solution:

$\frac{2x^2-5x-3}{4x^2-1}$	$=$	$\frac{(2x+1)(x-3)}{(2x+1)(2x-1)}$	Factor the numerator and denominator
	$=$	$\frac{x-3}{2x-1}$	Divide by (cancel) $2x+1$

   b. For which value of x is the function $y = \frac{3x-1}{x+7}$ undefined?
      A. $\frac{1}{3}$ B. -7 C. 7 D. $\frac{1}{7}$

   Solution:

   $y = \frac{3x-1}{x+7}$ is undefined when $x + 7 = 0$. Transposing 7 gives answer B. $x = -7$.

   c. Simplify: $\frac{4}{3xy} - \frac{3}{2yz}$

   Solution:

   Note that the LCM of the denominators is $6xyz$.

$\frac{4}{3xy} - \frac{3}{2yz}$	$=$	$\frac{4\times 2z}{3xy\times 2z} - \frac{3\times 3x}{2yz\times 3x}$	Change both denominators to the LCM
	$=$	$\frac{8z}{6xyz} - \frac{9x}{6xyz}$	
	$=$	$\frac{8z-9x}{6xyz}$	Subtract the numerators

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM2-L040 in the Pupil Handbook.

Lesson Title: Simple Statements	**Theme:** Logical Reasoning	
Lesson Number: M2-L041	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcomes By the end of the lesson, pupils will be able to: 1. Identify and form open and closed simple statements. 2. Deduce the truth or otherwise of simple statements.	**Preparation** None	

Opening *(4 minutes)*

1. Write the following statement on the board: a: Banana is the best fruit.
2. Ask pupils to raise their hands if statement a is **true**. Then, ask them to raise their hands if sentence a is **false**.
3. Discuss: Did everyone raise their hand? How many times did you raise your hand? (Example answers: Everyone should have raised their hand once, because the statement is either true or false. No pupils should have raised their hand twice.)
4. Explain that this sentence can be true or false. It depends on each person's taste. It cannot be both true **and** false – a banana is either the best, or it's not!
5. Explain that today's lesson is on simple statements. The next 8 lessons will focus on different types of statements and logic.

Teaching and Learning *(20 minutes)*

1. Explain:
    - The statement on the board is an example of an open simple statement.
    - An open simple statement can be either true or false. It cannot be true and false at the same time.
    - We can assign letters to represent statements.
2. Write another example on the board: S: y is a negative number.
3. Discuss: Is statement S a simple statement? Why or why not? (Example answer: Yes, because it can be true or false, but it cannot be true and false. We don't know the value of y, but it is either negative or it is not.)
4. Write another statement on the board: E: 2 is an even number.
5. Discuss: Is this an open simple statement? Why or why not? (Example answer: It is not an open simple statement, because it is always true. It cannot be false.)
6. Explain: A simple statement that is either always true or always false is a **closed** simple statement.
7. Write the following on the board:

Open	It is not known if the statement is true or false. It can be either true or false, but cannot be both true and false.
Closed	The statement is always true or always false.

8. Write more example statements on the board:
   q: Freetown is the capital city of Sierra Leone.
   r: Bingo is a bad dog.
   s: z is a positive number.
   t: Quadrilaterals have 5 sides.
9. Discuss each statement as a class:
   - Ask pupils to identify whether each statement is open or closed, and give the reason.
   - Clear any misconceptions. For example, statement t is false, but it is still a closed statement.
10. Write "open" or "closed" next to each statement:
   q: Freetown is the capital city of Sierra Leone. – **Closed**
   r: Bingo is a bad dog. – **Open**
   s: z is a positive number. – **Open**
   t: Quadrilaterals have 5 sides. – **Closed**
11. Ask pupils to work with seatmates to write 2 **open** simple statements and 2 **closed** simple statements. Give them several minutes to work.
12. Ask each group of seatmates to exchange their paper with the row in front or behind them. Allow groups of seatmates to check each other's work.
13. Invite a few volunteers to stand and give an example statement from their group.
14. Make sure pupils understand the difference between open and closed simple statements.

Practice *(15 minutes)*

1. Write the following on the board:
   a. Determine whether each statement is open or closed. Mark the correct answer:

	Statement	Open	Closed
a:	5 is greater than -6.		
b:	x is an even number.		
c:	Right angles measure 90 degrees.		
d:	Ama is an excellent Maths teacher.		
e:	Bo is the capital of Sierra Leone.		

   b. Write 2 open simple statements and 2 closed simple statements.
2. Ask pupils to work independently.
3. Ask pupils to exchange exercise books with seatmates and check each other's work.
4. Invite volunteers to come to the board and select the answers to question a.

Answers:

	Statement	Open	Closed
a:	5 is greater than -6.		✓
b:	x is an even number.	✓	
c:	Right angles measure 90 degrees.		✓
d:	Ama is an excellent Maths teacher.	✓	
e:	Bo is the capital of Sierra Leone.		✓

5. Ask some volunteers to read out some statements they wrote for question b. Allow the class to discuss each statement and decide whether it is open or closed.

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM2-L041 in the Pupil Handbook.

Lesson Title: Negation	**Theme:** Logical Reasoning	
Lesson Number: M2-L042	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to form the negation of a simple statement.	**Preparation** None	

Opening *(4 minutes)*

1. Write the following statement on the board: a: 3 is an even number.
2. Ask pupils to identify the type of statement. (Answer: closed simple statement)
3. Discuss: Is this statement true? Why or why not? (Example answer: It is false, because 3 is not an even number.)
4. Remind pupils that statements can be closed simple statements even if they are false.
5. Ask pupils to rewrite the statement so that it is true. After a minute, ask volunteers to share their answers with the class. (Example answers: 3 is not an even number. 3 is an odd number.)
6. Explain that today's lesson is on negation of simple statements.

Teaching and Learning *(20 minutes)*

1. Explain:
   - "Negation" is related to the word "negative". It means to change the meaning of a statement to make it the opposite.
   - We can write the negation of a sentence by inserting the word "not".
2. Write on the board: $\sim a$: 3 is not an even number.
3. Explain:
   - We use this symbol (~) to show that a statement is the negation. This is called the negation of a.
   - The negation of a statement has the opposite meaning of the original statement.
   - We write the negation of a by inserting the word "not" into the statement.
   - We can write the negation of closed and open statements.
4. Write the following open statement on the board: S: x is an even number.
5. Ask pupils to work with seatmates to write the negation of S.
6. Invite a volunteer to write the negation of S on the board. (Answer: $\sim S$: x is not an even number.)
7. Write another statement on the board: F: He went to Freetown.
8. Ask pupils to work with seatmates to write the negation of this statement.
9. Invite a volunteer to write the negation of F on the board. (Answer: $\sim F$: He did not go to Freetown.)
10. Explain:

- Sometimes it is necessary to add or change words when adding "not" to a sentence.
- It is important to write grammatically correct sentences. (Adding "not" gives us the statement "He not went to Freetown," which is not grammatically correct.)

11. Write the following two statements on the board:
    m: Bingo is a good dog.
    n: Bingo is not a good dog.
12. Explain:
    - Although different letters are used for the two statements, one is still the negation of the other.
    - We can say that "n is the negation of m".
    - If n is true, then m must be false. If n is false, then m must be true.
13. Write on the board: $n = \sim m$
14. Explain: This is how we would write "n is the negation of m" using Maths terms.
15. Ask pupils to work with seatmates to write 2 statements: 1 open, and 1 closed. They should not write any negation for now.
16. Ask pupils to exchange papers with the row of seatmates in front or behind them. Explain that each group of seatmates should write the negation of each of the 2 sentences written by another group.

    Example:
    The first group of seatmates writes 2 statements:

a: Angle B is an obtuse angle. b: Pentagons have 7 sides.

    The second group of seatmates writes the negation of the statements:

$\sim a$: Angle B is **not** an obtuse angle. $\sim b$: Pentagons **do not** have 7 sides.

17. Invite a few volunteers to stand and give an example of a statement and its negation from their group.

Practice *(15 minutes)*

1. Write the following on the board:
   a. Write the negation of each statement:

      q: Sierra Leone is a large country.

      r: There are forests in Sierra Leone.

      s: x is a number between -5 and 7.

t: Fatu has a large house.

u: He passed all of his classes.

v: Angle z is a reflex angle.

b. Write your own simple statement and its negation.

2. Ask pupils to work independently.
3. Ask pupils to exchange exercise books with seatmates and check each other's work.
4. Invite volunteers to come to the board and write the answers to question 1.
   Answers:

   $\sim q$: Sierra Leone is **not** a large country.

   $\sim r$: There are **not** forests in Sierra Leone.

   $\sim s$: x is **not** a number between -5 and 7.

   $\sim t$: Fatu does **not** have a large house.

   $\sim u$: He did **not** pass all of his classes.

   $\sim v$: Angle z is **not** a reflex angle.

5. Ask volunteers to each read out the simple statement and the negation that they wrote for question 2. Allow the class to discuss each statement.

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM2-L042 in the Pupil Handbook.

Lesson Title: Compound statements	**Theme:** Logical Reasoning	
Lesson Number: M2-L043	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to distinguish between simple and compound statements.	**Preparation** Write the problem in the Opening on the board.	

Opening *(4 minutes)*

1. Review simple statements. Write the following on the board:

   Identify whether each statement is open or closed. Write the negation of each.

   a: 5 is greater than 12.

   b: 3 is less than x.

2. Ask pupils to work with seatmates to complete the task.
3. Ask volunteers to give their answers and explain their reasons.
   Answers:
   a is a closed statement. Its negation is $\sim a$: 5 is **not** greater than 12.
   b is an open statement. Its negation is $\sim b$: 3 is **not** less than x.
4. Explain that today's lesson is on compound statements.

Teaching and Learning *(20 minutes)*

1. Write the following on the board: Sami is a cat and he likes to eat fish.
2. Discuss: How is this statement different than the other statements we have seen? Do you think it's a simple statement? Why or why not?
3. Explain:
   - This sentence is a compound statement. It has two parts: "Sami is a cat" and "he likes to eat fish".
   - The two parts of the sentence are connected by the word "and".
   - The two parts are both simple statements on their own. When connected, they become a compound statement.
4. Write another statement on the board: Kofi loves bananas but he hates pineapples.
5. Ask pupils to identify the two simple statements in this compound statement (Answer: "Kofi loves bananas" and "he hates pineapples")
6. Ask pupils to identify the word that connects these two statements. (Answer: "but")
7. Explain:
   - There are several words that connect simple statements to make them into compound statements.
   - Examples of these words are "and", "but", "or", "if", "if and only if".

8. Write another statement on the board: He likes rice if and only if it is served with cassava leaf.
9. Discuss the meaning of the statement: How does he like to eat rice? Are there any other ways he likes to eat rice? (Example answer: He only likes rice if it is with cassava leaf. He doesn't like to eat rice any other way.)
10. Ask pupils to identify the two simple statements in this compound statement (Answer: "He likes rice" and "it is served with cassava leaf")
11. Ask pupils to identify the words that connect these two statements. (Answer: "if and only if")
12. Write another statement on the board: x is a number greater than -3 but less than 4.
13. Discuss: Is this a simple or compound statement? If it is a compound statement, what are the simple statements it is made from?
14. Explain:
    - This is a compound statement.
    - The two simple statements are: "x is a number greater than -3" and "x is a number less than 4".
    - The connecting word is "but".
    - When we connect two simple statements, we can sometimes leave words out of the second statement. In this case, "x is a number" is left out of the second statement because it repeats the first statement.
15. Write the following on the board:
    Copy each compound statement. Circle the connecting word or words in each one. Write two simple statements from each one:
    a: The rectangle has width 4 cm and length 12 cm.
    b: If there are two linear equations then you can solve using substitution.
    c: A triangle is equilateral if and only if all 3 sides are equal.
16. Ask pupils to work with seatmates to complete the task.
    Answers:
    a: The rectangle has width 4 cm and length 12 cm.
    Simple statements:
    The rectangle has width 4 cm.
    The rectangle has length 12 cm.
    b: If there are two linear equations then you can solve using substitution.
    Simple statements:
    There are two linear equations.
    You can solve using substitution.
    c: A triangle is equilateral if and only if all 3 sides are equal.
    Simple statements:
    A triangle is equilateral.
    All 3 sides are equal.

Practice *(15 minutes)*

1. Write the following on the board:

   Make compound statements by connecting the simple statements:

1.	You can be a doctor.	+	You go to medical school.
2.	You can solve for y.	+	You substitute the value of x.
3.	He likes fruit.	+	He does not like vegetables.
4.	She is wearing green.	+	She is wearing blue.
5.	A triangle is scalene.	+	It has 3 unequal sides.
6.	z is greater than 3.	+	z is less than 25.

2. Ask pupils to work independently.
3. Ask pupils to exchange exercise books with seatmates and check each other's work.
4. Invite volunteers to come to the board and write the compound statements. Note that some may have more than one answer (For example, problem 1 could use "if" or "if and only if".)

   Answers:

   1. You can be a doctor if you go to medical school.
   2. You can solve for y if you substitute the value of x.
   3. He likes fruit but he does not like vegetables.
   4. She is wearing green and blue.
   5. A triangle is scalene if and only if it has 3 unequal sides.
   6. z is greater than 3 but less than 25.

5. Allow pupils to discuss each statement. If they wrote a different compound statement for an answer, allow them to read it out.

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM2-L043 in the Pupil Handbook.

Lesson Title: Implication	Theme: Logical Reasoning	
Lesson Number: M2-L044	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to draw conclusions from a given implication.	**Preparation** Write the compound statement in the Opening on the board.	

Opening *(3 minutes)*

1. Review compound statements. Write the following on the board: Foday likes to wear hats and glasses.
2. Ask pupils to give the connecting word. (Answer: "and")
3. Ask pupils to write 2 simple statements from the compound statement in their exercise books.
4. Ask a volunteer to give their answers and explain. (Answers: "Foday likes to wear hats" and "Foday likes to wear glasses")
5. Explain that today's lesson is on implication.

Teaching and Learning *(20 minutes)*

1. Write on the board: If Hawa lives in Freetown, then she lives in Sierra Leone.
2. Discuss: What type of statement is this? How do you know? (Example answers: compound statement; it is connected by "if…then")
3. Explain:
   - This is a compound statement. It has two parts: "Hawa lives in Freetown" and "she lives in Sierra Leone".
   - The two parts of the sentence are connected by the words "if" and "then"
   - This is an example of an implication. Implications can be written as "if-then" statements.
4. Write each simple statement on the board and label it:
   A: Hawa lives in Freetown.
   B: She lives in Sierra Leone.
5. Write the implication on the board using symbols: $A \Rightarrow B$
6. Explain:
   - This statement is read "A implies B".
   - This means that "Hawa lives in Freetown" implies that "she lives in Sierra Leone". If she lives in Freetown, then we know that she must live in Sierra Leone because that's where the city is located.
7. Write additional examples of implications on the board:
   a. If Sam passes the exam, then he will graduate.
   b. If $4x - 2 = 6$, then $x = 2$.
   c. My head will hurt if I hit it on a wall.
8. Ask pupils to identify the 2 simple statements in implication a. (Answers: "Sam passes the exam" and "he will graduate")

9. Label these on the board:
    - E: Sam passes the exam.
    - G: He will graduate.
10. Ask pupils to work with seatmates to write an implication for this statement using letters and symbols.
11. Invite one volunteer to write the answer on the board. (Answer: $E \Rightarrow G$)
12. Ask pupils to identify the 2 statements in implication b. (Answer: They are mathematical statements, $4x - 2 = 6$ and $x = 2$.)
13. Label these on the board:
    - S: $4x - 2 = 6$
    - T: $x = 2$
14. Ask pupils to work with seatmates to write an implication for this statement using letters and symbols.
15. Invite one volunteer to write the answer on the board. (Answer: $S \Rightarrow T$)
16. Explain: The first equation implies the second equation. If we solve the first equation, we will find that the second equation is definitely true.
17. Ask pupils to identify the 2 statements in implication c. (Answer: "My head will hurt" and "I hit it on a wall")
18. Label these on the board:
    - X: My head will hurt.
    - Y: I hit it on a wall.
19. Ask pupils to work with seatmates to write an implication for this statement using letters and symbols.
20. Invite one volunteer to write the answer on the board. (Answer: $Y \Rightarrow X$)
21. Explain:
    - This statement is written in a different way than the others. Instead of "if…then", it uses only "then".
    - In this statement Y implies X.
    - In other words, "I hit it on a wall" implies "My head will hurt".
    - We can say this another way: "If I hit it on a wall, then my head will hurt." This implication has the same meaning as the one in the problem.
22. Write on the board: Is it true to say $x^2 = 9 \Rightarrow x = 3$?
23. Allow pupils to discuss the question with seatmates for a moment.
24. Discuss:
    - Is this implication a true statement? Why or why not?
    - Does the first statement ($x^2 = 9$) always make the second statement ($x = 3$) true? (Answer: No, $x = -3$ could also result in $x^2 = 9$)
25. Explain:
    - $x^2 = 9$ does not imply that x is 3. It could be either 3 or -3.
    - We can rewrite this statement in a way that makes it true.
26. Write on the board: True implication: $x^2 = 9 \Rightarrow x = \pm 3$

Practice *(14 minutes)*

1. Write the following on the board:

   Match each statement in the left column with a statement in the right column. Write each as an implication using words and symbols.

A:	Fatu lives in Bo.	U	She eats fruit.
B:	$3x = 12$	V	It is day time.
C:	Shape S is a square.	W	She lives in Sierra Leone.
D:	Hawa eats bananas.	X	$x = 4$
E:	$2x + 3 = 15$	Y	$x = 6$
F:	The sun is shining.	Z	It has 4 sides.

2. Write an example on the board so pupils understand: If Fatu lives in Bo, then she lives in Sierra Leone. $A \Rightarrow W$
3. Ask pupils to work independently.
4. Ask pupils to exchange exercise books with seatmates and check each other's work.
5. Invite volunteers to come to the board and write each implication.
   Answers:

   If $3x = 12$, then $x = 4$. $B \Rightarrow X$

   If shape S is a square, then it has 4 sides. $C \Rightarrow Z$

   If Hawa eats bananas, then she eats fruit. $D \Rightarrow U$

   If $2x + 3 = 15$, then $x = 6$. $E \Rightarrow Y$

   The sun is shining, then it is day time. $F \Rightarrow V$

6. Allow pupils to discuss each statement. If they wrote a different implication for an answer, allow them to read it out. Discuss whether each one is correct.

Closing *(3 minutes)*

1. Ask pupils to write an implication of their own in their exercise book. Challenge them to make it interesting or funny!
2. Ask volunteers to read their implications aloud to the class.
3. For homework, have pupils do the practice activity PHM2-L044 in the Pupil Handbook.

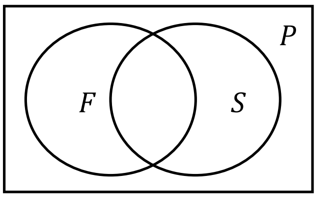
Lesson Title: Conjunction and Disjunction	**Theme:** Logical Reasoning	
Lesson Number: M2-L045	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to distinguish between conjunction and disjunction, representing them on truth tables.	**Preparation** Write the problem in the Opening on the board.	

Opening *(3 minutes)*

1. Review implication. Write the following problem on the board: Connect the statements in an implication. Write the implication using words and symbols.
   A: I work hard.
   B: I earn money.
2. Ask pupils to write the answer in their exercise books.
3. Ask a volunteer to give their answer and explain. (Answer**:** If I work hard, then I earn money. $A \Rightarrow B$)
4. Explain that today's lesson is on conjunction and disjunction.

Teaching and Learning *(22 minutes)*

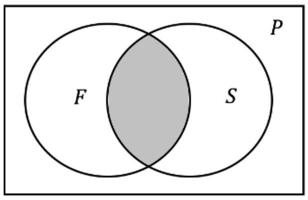
1. Write the following on the board:
   P: People who live in Freetown
   F: Females
   S: Students
2. Explain: There are more than 1 million people living in Freetown. Among them are females, and also among them are students.
3. Draw the Venn diagram on the board:



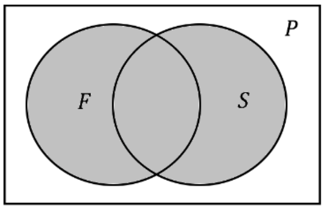
4. Explain the diagram:
   - The rectangle is the population of Freetown.
   - The left circle is the female population. The right circle is the population who are pupils.
   - Where the circles intersect, we have the population that are both female and are pupils.
5. Write the compound statements on the board:
   A_1: people who are female and are pupils
   A_2: people who are either female, or are pupils, or both
6. Explain A_1:
   - A_1 is a conjunction.

- The two statements “people who are female” and “people who are pupils” are linked by the word **“and”**.
- In the diagram, this is where the two circles intersect. This is the population of Freetown who fit into both of the categories.

7. Shade the centre section of the Venn diagram to show A_1:



8. Write the following on the board: $A_1 = F \cap S$ or $A_1 = F \wedge S$
9. Explain: These are the two ways to write the conjunction A_1.
10. Explain A_2:
    - A_2 is a disjunction.
    - The two statements “people who are female” and “people who are pupils” are linked by the words **“either – or – or both”**.
    - In the diagram, this is all of the space inside of the two circles, including where they intersect.
11. Shade both circles of the Venn diagrams to show A_2:



12. Write the following on the board: $A_2 = F \cup S$ or $A_2 = F \vee S$
13. Explain: These are the two ways to write the conjunction A_1.
14. Draw the truth table for A_1 on the board:

F	S	$F \wedge S$
T	T	T
T	F	F
F	T	F
F	F	F

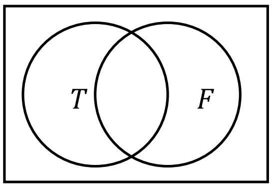
15. Explain:
    - This truth table says that statement A_1 is only true if both of the sub-statements are true (“people who are female” and “people who are students”).
    - If either of the sub-statements is false, then statement A_1 is also false.
16. Draw the truth table for A_2 on the board:

F	S	$F \vee S$
T	T	T
T	F	T
F	T	T
F	F	F

17. Explain:
    - This truth table says that A_2 is true if either or both of the sub-statements are true.
    - A_2 is false only if both sub-statements are false.
18. Write the following problem on the board:
    Consider the following statements about some people living in a village:
    T: Some people are teachers.
    F: Some people have farms.

    Based on this information, prepare truth tables that describe:

    a. A person in the village who is either a teacher, or has a farm, or both.
    b. A person in the village who is both a teacher and has a farm.
19. Discuss: Do you think many people in the village have farms? Do you think teachers can also have farms?
20. Draw the Venn diagram on the board to help pupils visualise the problem:



21. Ask volunteers to explain whether a. is a conjunction or disjunction. (Answer: It is a disjunction, because it uses the word "either".)
22. Draw a truth table on the board with the first 2 columns filled:

T	F	$T \vee F$
T	T	
T	F	
F	T	
F	F	

23. Remind pupils: $T \vee F$ means that the person is either a teacher or has a farm. Only one column in the table must be true.
24. Discuss each row as a class. Ask volunteers to say whether $T \vee F$ is true or false, and write the correct answer in the row.
    Answers:

T	F	$T \vee F$
T	T	T
T	F	T
F	T	T
F	F	F

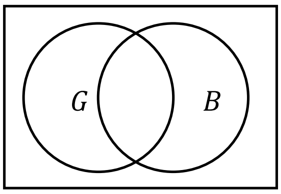
25. Follow the same process to fill the truth table for $T \wedge F$. Fill each row as a class.
Answers:

T	F	$T \wedge F.$
T	T	T
T	F	F
F	T	F
F	F	F

Practice *(14 minutes)*

1. Write the following problem on the board: Bentu has a farm with many types of animals in many different colours. Consider the following statements:
   G: Some animals are goats.
   B: Some animals are brown.

   Based on this information, prepare truth tables that describe
      a. An animal on the farm that is a brown goat.
      b. An animal on the farm that is either brown, or a goat, or both.
2. Ask pupils to work independently. Support them as needed. For example, if they have difficulty imagining the problem, draw the Venn diagram on the board:



3. Ask pupils to exchange exercise books with seatmates and check each other's work.
4. Invite volunteers to come to the board and draw each truth table.
Answers:

B	G	$B \wedge G$
T	T	T
T	F	F
F	T	F
F	F	F

B	G	$B \vee G$
T	T	T
T	F	T
F	T	T
F	F	F

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM2-L045 in the Pupil Handbook.

Lesson Title: Equivalence and Chain rule	**Theme:** Logical Reasoning	
Lesson Number: M2-L046	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcomes By the end of the lesson, pupils will be able to: 1. Recognise equivalent statements and apply them to arguments. 2. Recognise the chain rule and apply it to arguments.	**Preparation** Write the problem in the Opening on the board.	

Opening *(5 minutes)*

1. Review conjunction and disjunction. Write the following problem on the board:
   P: The man went to the market and sold 5 pineapples.
   Use appropriate symbols and truth tables to describe the conditions for P to be true.
2. Ask a volunteer to give the two sub-statements. Write them on the board. (Answers: M: The man went to the market. S: The man sold 5 pineapples.)
3. Ask a volunteer to tell whether P is a conjunction or disjunction. (Answer: Conjunction; both M and S must be true for P to be true.)
4. Invite a volunteer to write the conjunction on the board using symbols. (Answer: $P = M \wedge S$)
5. Ask pupils to draw the truth table for P in their exercise books.
6. Invite a volunteer to draw the truth table on the board.

   Answer:

M	S	$M \wedge S$
T	T	T
T	F	F
F	T	F
F	F	F

7. Explain that today's lesson is on implication and equivalence. Pupils will be applying equivalence and the chain rule to arguments.

Teaching and Learning *(24 minutes)*

1. Write the following ion the board:
   A: $x^2 = 16$
   B: $x = \pm 4$
2. Ask pupils to write an implication in their exercise books based on these statements.
3. Invite volunteers to come to the board and write their implications. (Answers: $A \Rightarrow B$ or $x^2 = 16 \Rightarrow x = \pm 4$; $B \Rightarrow A$ or $x = \pm 4 \Rightarrow x^2 = 16$)

4. Check that both implications ($A \Rightarrow B$ and $B \Rightarrow A$) are written on the board. If one is not written on the board, write it.
5. Revise implication by asking pupils to describe each implication as an "if-then" sentence. (Answers: $A \Rightarrow B$: If $x^2 = 16$ then $x = \pm 4$; $B \Rightarrow A$: If $x = \pm 4$ then $x^2 = 16$)
6. Explain:
   - We can form 2 different implications. Statement A implies B, and statement B implies A. Both of these implications are true.
   - A and B are called **equivalent** statements.
   - If the reverse of an implication is also true, then the reverse is called the **converse**. In the case on the board, $B \Rightarrow A$ is the converse of $A \Rightarrow B$.
   - Note that statements A and B are equivalent **if and only if** both $A \Rightarrow B$ and its converse $B \Rightarrow A$ are true.
7. Write on the board: If $X \Rightarrow Y$ and $Y \Rightarrow X$, then X and Y are equivalent and we write $X \Leftrightarrow Y$. The implication $Y \Rightarrow X$ is the converse of $X \Rightarrow Y$.
8. Write on the board: If $X \Rightarrow Y$, then $\sim Y \Rightarrow \sim X$ is an equivalent statement.
9. Explain:
   - If X implies Y, then the negation of Y implies the negation of X.
   - Give an example:
     - Consider the implication: "If Fatu lives in Freetown, then she lives in Sierra Leone".
     - This is an equivalent statement: "If Fatu does not live in Sierra Leone, then she does not live in Freetown".
10. Write the following on the board:
    Consider the statements below. If $X \Rightarrow Y$, write 2 valid statements:
    X: Hawa studies hard.
    Y: Hawa passes exams.
11. Ask pupils to brainstorm with seatmates and write any possible valid statements from this implication in their exercise books.
12. Invite volunteers to write their statements on the board. Allow all other pupils to check if the statements are valid and correct them.
    Answers: There are only 2 valid statements.
    a. If Hawa studies hard, then she passes exams. ($X \Rightarrow Y$)
    b. If Hawa does not pass exams, then she does not study hard. ($\sim Y \Rightarrow \sim X$)
13. Explain: Remember that these 2 are equivalent statements.
14. Write statement Z on the board, under X and Y:
    X: Hawa studies hard.
    Y: Hawa passes exams.
    Z: Hawa graduates from secondary school.
15. Write on the board: $X \Rightarrow Y$ and $Y \Rightarrow Z$
16. Ask volunteers to give each implication in words. (Answers: $X \Rightarrow Y$: "If Hawa studies hard then she passes exams"; $Y \Rightarrow Z$: "If Hawa passes exams then she graduates from secondary school.")

17. Discuss: If X implies Y and Y implies Z, what can we say about X and Z? Is there any relationship between X and Z?
18. Allow pupils to make their observations, then explain:
    - If X, Y and Z are 3 statements such that $X \Rightarrow Y$ and $Y \Rightarrow Z$, then $X \Rightarrow Z$.
    - This is called the **chain rule.**
    - In this problem, it gives us the statement "If Hawa studies hard, then she graduates secondary school".
19. Write the following on the board:

    A: The school team practises football.

    B: The team wins the match.

    C: The team qualifies for the championship.

    $A \Rightarrow B$ and $B \Rightarrow C$. Write as many statements as possible with this information.
20. Ask pupils to work with seatmates to write statements in symbols and words. Encourage them to use equivalence and the chain rule.
21. Invite volunteers to write their statements on the board. Allow all other pupils to check if the statements are valid and correct them.

    Possible answers:

$A \Rightarrow B$	If the school team practises football, then the team wins the match.
$B \Rightarrow C$	If the team wins the match, then the team qualifies for the championship.
$A \Rightarrow C$	If the school team practises football, then the team qualifies for the championship.
$\sim B \Rightarrow \sim A$	If the team does not win a match, then the team does not practise football.
$\sim C \Rightarrow \sim B$	If the team does not qualify for the championship, then the team does not win the match.
$\sim C \Rightarrow \sim A$	If the team does not qualify for the championship, then the team does not practise football.

Practice *(10 minutes)*

1. Write a problem on the board: If $R \Rightarrow S$ and $S \Rightarrow T$, write 6 true statements.

    R: Michael lives in a big city.

    S: Michael travels far to work.

    T: Michael's transport is expensive.
2. Ask pupils to work independently. Support them as needed.
3. Ask pupils to exchange exercise books with seatmates and check each other's work.
4. Invite volunteers to come to the board and write their statements.

Answers:

$R \Rightarrow S$	If Michael lives in a big city, then he travels far to work.
$S \Rightarrow T$	If Michael travels far to work, then his transport is expensive.
$R \Rightarrow T$	If Michael lives in a big city, then his transport is expensive.
$\sim S \Rightarrow \sim R$	If Michael does not travel far for work, then he does not live in a big city.
$\sim T \Rightarrow \sim S$	If Michael's transport is not expensive, then he does not travel far to work.
$\sim T \Rightarrow \sim R$	If Michael's transport is not expensive, then he does not live in a big city.

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM2-L046 in the Pupil Handbook.

Lesson Title: Venn diagrams	Theme: Logical Reasoning	
Lesson Number: M2-L047	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to use Venn diagrams to demonstrate connections between statements	**Preparation** Write the problem in the Opening on the board.	

Opening *(4 minutes)*

1. Review equivalence. Write the following on the board:

   S: Francis is often absent from school.
   T: Francis' marks are low.
   $S \Rightarrow T$ is valid.

2. Discuss: Read each statement below aloud. Allow pupils to discuss whether the statement is valid and give their reasons.
   a. If Francis is often absent from school, then his marks are low. (Answer: valid; $S \Rightarrow T$)
   b. If Francis is not often absent from school, then his marks are not low. (Answer: not valid; $\sim S \Rightarrow \sim T$ is not an equivalent statement to $S \Rightarrow T$)
   c. If Francis' marks are not low, then Francis is not often absent from school. (Answer: valid; $\sim T \Rightarrow \sim S$ is an equivalent statement to $S \Rightarrow T$)
3. Explain that today's lesson is on Venn diagrams. Pupils will draw Venn diagrams to show connections between statements.

Teaching and Learning *(25 minutes)*

1. Revise the different types of Venn diagrams (from SSS 1 Term 2 lessons on sets). Draw the following on the board:

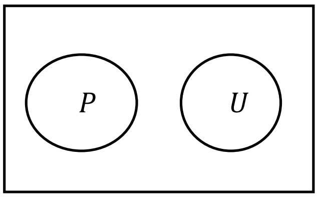


Figure A

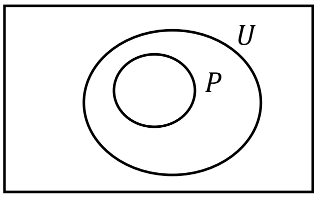


Figure B

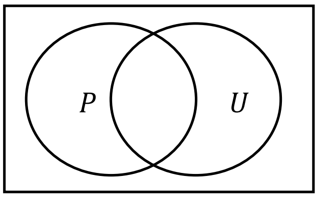
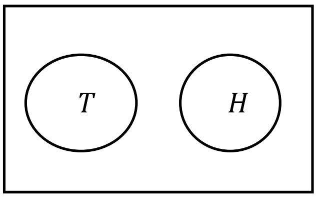


Figure C

2. Ask volunteers to answer the following questions:
   - Which diagram shows a subset? (Answer: Figure B)
   - Which diagram shows disjoint sets? (Answer: Figure A)
   - Which diagram shows intersecting sets? (Answer: Figure C)
3. Label each figure with its set notation. Read each aloud and explain as needed:

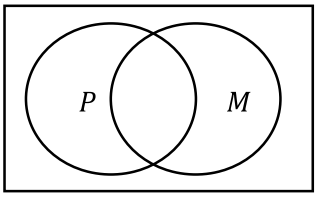
- **Figure A.** $P \cap U = \emptyset$ ("P and U are disjoint sets. There is no intersection between P and U.")
- **Figure B.** $P \subset U$ ("P is a subset of U. Every element in P is also in U.")
- **Figure C.** $P \cap U$ ("P intersects with U. Some elements of P are also in U.")

4. Write the following 3 statements on the board:
   X: All police officers wear uniforms.
   Y: Some police officers wear uniforms.
   Z: No police officers wear uniforms.
5. Explain: There are 2 sets of people in these diagrams. People who are police officers are shown with P. People who wear uniforms are shown with U.
6. Ask volunteers to answer the following questions and give their reason. Allow discussion:
   - Which Venn diagram shows statement X? (Answer: Figure B shows X, because P is a subset of U. All of the police are in the set of people who wear uniforms.)
   - Which Venn diagram shows statement Y? (Answer: Figure C shows Y. The sets intersect, and there are some police who don't wear uniforms.)
   - Which Venn diagram shows statement Z? (Answer: Figure A shows Z. The sets are separate, because none of the police officers wear uniforms and no one wearing a uniform is a police officer.)
7. Write the following 3 statements on the board:
   A: No teachers work at the hospital.
   B: Some pupils sell goods in the market.
   C: All pupils have exercise books.
8. Ask volunteers to give 2 sets related to statement A. Write them on the board. (Answers: $T =$ {teachers} and $H =${people who work at the hospital})
9. Ask volunteers to describe what the Venn diagram will look like. Draw it on the board:

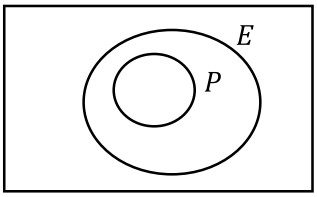


10. Explain:
    - "Teachers" and "people who work at the hospital" are separate sets.
    - When you read a statement, "no" and "never" are key words that indicate a separate set.
11. Ask volunteers to give 2 sets related to statement B. Write them on the board. (Answers: P={pupils} and M={people who sell in the market})

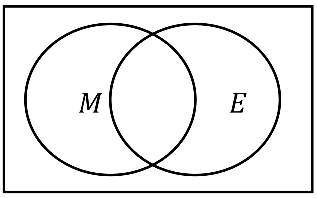
12. Ask volunteers to describe what the Venn diagram will look like. Draw it on the board:



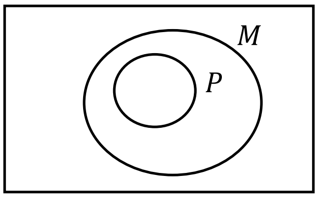
13. Explain:
    - "Pupils" and "people who sell in the market" are intersecting sets.
    - When you read a statement, "some", "most" or "not all" are key words that indicate an intersection.
14. Ask volunteers to give 2 sets related to statement C. Write them on the board. (Answers: $P=$\{pupils\} and $E=$\{people who have exercise books\})
15. Ask volunteers to describe what the Venn diagram will look like. Draw it on the board:



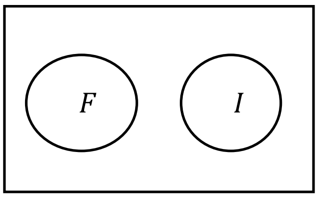
16. Explain:
    - "Pupils" is a subset of the set "people who have exercise books".
    - When you read a statement, "all, "no…not" or "if…then" are key words that indicate a subset.
17. Write 3 more statements on the board:
    R: Some Maths textbooks are expensive.
    S: All of the pupils have mobile phones.
    T: None of the football team members are injured.
18. Ask pupils to work with seatmates to draw a Venn diagram for each statement.
19. Invite volunteers to come to the board and draw the Venn diagrams and give an explanation. All other pupils should check their work.
    Answers (Example explanations are below):



R: Intersection of the sets $M=$\{Maths textbooks\} and $E=$ \{expensive books\}; $M \cap E$



S: $P=$ \{pupils with mobile phones\} is a subset of $M=$ \{people with mobile phones\}; $P \subset M$

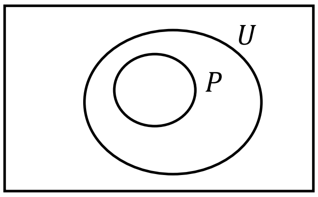


T: $F=$ \{football team members\} and $I=$\{injured people\} are disjoint sets; $F \cap I = \emptyset$

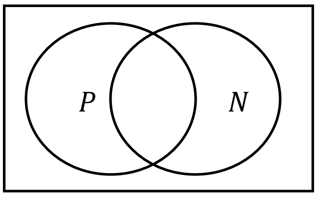
Practice *(10 minutes)*

1. Write the following on the board:
   Draw a Venn diagram for each of the statements:
    a. X: All senior secondary pupils wear a uniform.
    b. Y: Some senior secondary pupils live near the school.
    c. Z: No senior secondary pupils are rich.
2. Ask pupils to work independently. Support pupils as needed.
3. Ask pupils to exchange exercise books with seatmates and check each other's work.
4. Invite volunteers to come to the board and draw their diagrams. Note that pupils could have selected different letters to represent the sets.
   Answers:

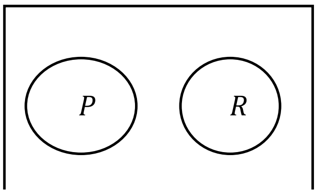
a.



b.



c.



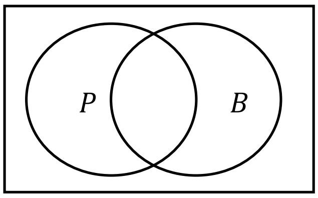
Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM2-L047 in the Pupil Handbook.

Lesson Title: Validity	**Theme:** Logical Reasoning	
Lesson Number: M2-L048	**Class:** SSS 2	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to determine the validity of an argument.	**Preparation** Write the problem in the Opening on the board.	

Opening *(4 minutes)*

1. Revise Venn diagrams. Write the following on the board:
   Draw a Venn diagram for the statement: X: Some pupils ride bicycles. Let P ={pupils} and B ={people who ride bicycles}.
2. Allow pupils to work with seatmates to draw the diagram.
3. Invite a volunteer to draw the diagram on the board:
   Answer:

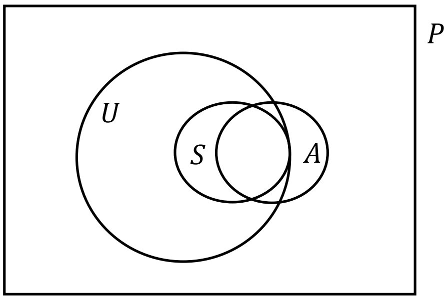


4. Explain that today's lesson is on validity. Pupils are familiar with the term "validity" from previous lessons. Today they will practise determining the validity of arguments using Venn diagrams.

Teaching and Learning *(25 minutes)*

1. Explain:
   - An argument is valid if and only if the conclusion can be drawn from other statements.
   - We have practised drawing conclusions from statements during the lesson on equivalence and the chain rule.
   - Note that the actual **truth does not matter** when determining whether a statement is valid.
2. Write the following on the board:
   A: Monrovia is in Sierra Leone.
   B: Sierra Leone is in Asia.
3. Discuss:
   - Are these statements true? Why or why not? (Answer: They are not true. Monrovia is in Liberia, and Sierra Leone is in Africa.)
   - Can you draw any conclusions from these statements? Allow pupils to share their ideas.
4. Explain:
   - Although these statements are false, a valid conclusion can be drawn.
   - Based on A and B, it is valid to say "Monrovia is in Asia".
5. Write on the board: C: Monrovia is in Asia.

6. Explain:
    - This conclusion is reached using the chain rule.
    - The validity of an argument can be determined using information you already know. This includes:
        - A Venn diagram.
        - The fact that if $p \Rightarrow q$, then $\sim q \Rightarrow \sim p$ is an equivalent statement.
        - The chain rule.
7. Write the following problem on the board:
   Consider the following statements and answer the questions:
   X: All senior secondary pupils wear uniforms.
   Y: Most senior secondary pupils have high attendance.
    a. Draw one Venn diagram to illustrate both statements.
    b. Determine which of the following implications are valid based on X and Y:
        i. Fatu wears a uniform ⇒ Fatu is a senior secondary pupil.
        ii. Michael is a senior secondary pupil ⇒ He has high attendance.
        iii. Hawa does not wear a uniform ⇒ She is not a senior secondary pupil.
8. Draw the Venn diagram for part a. on the board:
   Answer: a.



   Where P ={all pupils}, U ={pupils who were uniforms}, S ={senior secondary pupils}, A ={pupils with high attendance}
9. Explain the Venn diagram:
    - From statement X, we know that all senior secondary pupils wear uniforms. S is a subset of U, so the entire circle is inside U.
    - From statement Y, we know that most senior secondary pupils have high attendance. S and A intersect.
    - We do not have information about the other pupils, such as junior secondary and those who don't wear uniforms. This is the best we can draw the Venn diagram with the information given.
10. Explain part b: We can determine which statements are valid by looking at the Venn diagram, or by considering equivalence or the chain rule.
11. Ask pupils to read statement i. with seatmates, and discuss whether it is valid. Allow volunteers to share their ideas with the class.
12. Explain: **Statement i. is not valid**. All senior secondary pupils wear uniforms. However, all pupils who wear uniforms are not necessarily senior secondary pupils. Fatu may be in U, but outside of S.
13. Follow the same process with statement ii. and statement iii. Explain:

- **Statement ii. is not valid.** Although most senior secondary pupils have high attendance, not all of them do. Michael might be in set S, but outside of set A.
- **Statement iii. is valid.** If Hawa does not wear a uniform, she is outside of set U. She must also be outside of set S, which is a subset of U.

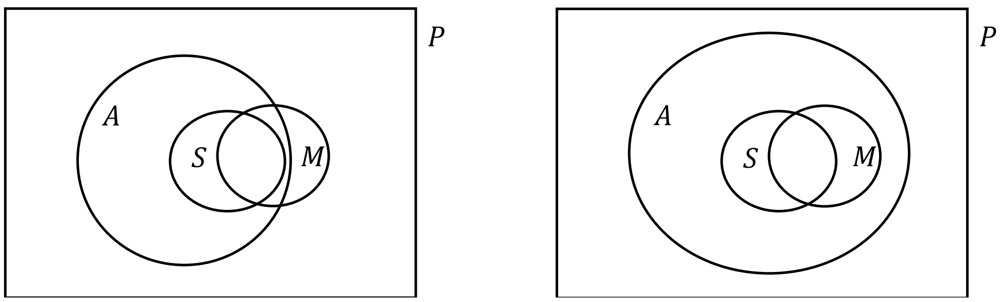
14. Write another problem on the board:
    Consider the following statements:

    S: All serious pupils have high attendance.

    T: Most pupils with high marks are serious.

    a. Illustrate the information above in a Venn diagram.
    b. Using the Venn diagram, or otherwise, determine whether or not each of the following statements is valid:
        i. Martin has high attendance ⇒ Martin is serious.
        ii. Bentu is serious ⇒ Bentu has high attendance.
        iii. Issa has high marks ⇒ Issa is serious.
        iv. Aminata is serious ⇒ Aminata has high marks.
        v. Ama has high marks and is serious ⇒ Ama has high attendance.
        vi. Mohammed has high attendance and high marks ⇒ Mohamed is serious.
15. Ask pupils to work with seatmates to draw the diagram (in part a.), and answer i. and ii. of part b.
16. Invite a volunteer to draw their group's Venn diagram on the board. Allow pupils to discuss. If anyone drew it differently, allow them to also draw theirs on the board.

    Possible answers: a. Either answer is correct:



Where P ={all pupils}, S ={pupils who are serious}, A ={ pupils with high attendance}, and M ={pupils with high marks}

17. Ask volunteers to explain whether each statement (i. and ii.) is valid.

    Answers:
    i. Statement i. is **not valid**. Some pupils with high attendance are not serious. Martin might not be serious.
    ii. Statement ii. **valid**. All serious pupils have high attendance. If Bentu is serious, she must have high attendance.

Practice *(10 minutes)*

1. Ask pupils to work independently to finish the problem on the board. They should determine whether statements iii. – vi. are valid. Support pupils as needed.

2. Allow pupils to discuss their answers with seatmates when they finish.
3. Ask volunteers to give their answers and explain to the class. All other pupils should check their work.

   Answers (and example explanations):

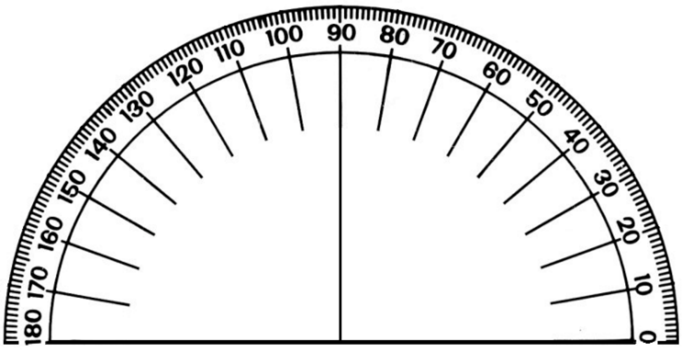
   iii. **Statement iii. is not valid.** Most pupils with high marks are serious, but not all of them. Issa could be a pupil with high marks who is not serious.
   iv. **Statement iv. is not valid.** Pupils who are serious and those with high marks intersect, but not all pupils who are serious have high marks.
   v. **Statement v. is valid.** All pupils who are serious have high attendance. Therefore, Aminata's marks are not important to validity. She must have high attendance if she is serious.
   vi. **Statement vi. is not valid.** There are some pupils who have high attendance and high marks but are not serious.

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM2-L048 in the Pupil Handbook.

Appendix I: Protractor

You can use a protractor to measure angles. If you do not have a protractor, you can make one with paper. Trace this protractor with a pen onto another piece of paper. Then, cut out the semi-circle using scissors.



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